1. (a) take $\psi = e^{\gamma}\phi_0$

$$E\psi = H\psi = (P_1\sigma^1 + P_2\sigma^2 + m(x)\sigma^3)\psi$$
$$= (-i\sigma^1(\partial_1\gamma) - i\sigma^2(\partial_2\gamma) + m(x)\sigma^3)\psi$$
$$= e^{\gamma}(i\sigma^1 m(x) + \sigma^2 P_2 + m(x)\sigma^3)\phi_0$$

$\therefore$ we must have: $(\sigma_3 + i\sigma_1)\phi_0 = 0$

$[\sigma_2, \sigma_3 + i\sigma_1] = 2(\sigma_3 + i\sigma_1)$

$\therefore (\sigma_2 - 1)\phi_0 = 0$

(b): $\psi = (\theta(-x_1)e^{iP_{1-}x_1} + \theta(x_1)e^{iP_{1+}x_1})e^{iP_2x_2}\phi$.

$$\therefore H\psi = \sigma_1(P_{1-}\theta(-x_1)e^{iP_{1-}x_1} + P_{1+}\theta(x_1)e^{iP_{1+}x_1})e^{iP_2x_2}\phi + \sigma_2P_2\psi + (\theta(x_1)e^{iP_{1+}x_1} - \theta(-x_1)e^{iP_{1-}x_1})e^{iP_2x_2})\sigma_3\phi$$

$\therefore \sigma_1P_{1+} + \sigma_3,\ \sigma_1P_{1-} - \sigma_3$ and σ_2 are the generates.

$\therefore P_{1-} = -i$, and $P_{1+} = i$

2. (a): $L_3 = -i\partial_\theta = -i(\partial_\theta z)\partial_z - i(\partial_\theta \bar{z})\partial_{\bar{z}}$
$$= z\partial_z - \bar{z}\partial_{\bar{z}}$$

$$\therefore L_3\Psi_m = (z\partial_z - \bar{z}\partial_{\bar{z}})(z^m\exp(-\frac{\bar{z}z}{4l^2}))$$
$$= zmz^{m-1}\exp(-\frac{\bar{z}z}{4l^2}) - (\frac{\bar{z}z}{4l^2})z^m\exp(-\frac{\bar{z}z}{4l^2}) + (\frac{\bar{z}z}{4l^2})z^m\exp(-\frac{\bar{z}z}{4l^2})$$
$$= m\Psi_m$$

(b): $L_z \Psi_m = \sum_{i=1}^{N} (z_i \partial_{z_i} - \bar{z}_i \partial_{\bar{z}_i}) \prod (z_j - z_k)^m \exp(-\sum_{j=1}^{N} \frac{\bar{z}_j z_j}{4l^2})$

$= \cdot \quad - \quad - \quad - \cdot$

3、time-reversal: $k \to -k$, Γ^x

$(i\sigma^y)(\sigma^z)(-i\sigma^y) = -\sigma^z$

$\therefore T\Gamma^x T^{-1} = (i\sigma^y)\Gamma^x(-i\sigma^y) = -\Gamma^x$

Γ^y is imaginary and commutes with σ^y.

$\therefore T\Gamma^y T^{-1} = -(i\sigma^y)\Gamma^y(-i\sigma^y) = -\Gamma^y$. Γ^0 is real.

$\therefore T\Gamma^0 T^{-1} = (i\sigma^y)\Gamma^0(-i\sigma^y) = \Gamma^0$

$\therefore TH(k)T^{-1} = T\Gamma^x T^{-1}\sin(-k_x) + T\Gamma^y T^{-1}\sin(-k_y) + T\Gamma^0 T^{-1}(2-M-\cos(-k_x)-\cos(-k_y))$

$= \Gamma^x \sin k_x + (-1)^2 \Gamma^y \sin k_y + \Gamma^0(2-M-\cos k_x - \cos k_y)$.

$= H(k)$.

4、$H\psi = E\psi$, then $(THT^{-1})T\psi = ET\psi$.

$a\psi + b\psi' = 0$

$\therefore 0 = T(a\psi + b\psi') = a^*\psi' - b^*\psi$.

$\therefore \begin{pmatrix} a & b \\ b^* & a^* \end{pmatrix} \begin{pmatrix} \psi \\ \psi' \end{pmatrix} = 0$

$\det = |a|^2 + |b|^2 \Rightarrow \psi, \psi'$ is independt