1. In the lattice, the Hamiltonian:

$$H_{tb} = -t \sum_{\langle i,j \rangle \sigma} c^{\dagger}_{iA\sigma} c_{jB\sigma} + h.c.$$

So that we have:

$$c_{iA\sigma} = \frac{1}{\sqrt{N}} \sum_{k} e^{ik\cdot R_i} \psi_{kA\sigma}, \quad c_{jB\sigma} = \frac{1}{\sqrt{N}} \sum_{k} e^{ik\cdot (R_i+\delta_i)} \psi_{kB\sigma}$$

take $\delta_l = r_i - r_j$

$$\therefore H_{tb} = \frac{1}{N} \sum_{\cdots} e^{i(k-k')R_i} \left(-t \sum_{l} e^{ik\cdot\delta_l}\right) \psi^{\dagger}_{kA\sigma} \psi_{kB\sigma} + h.c.$$

take $\Delta_k = -t \sum e^{ik\delta_l} = -t e^{-ik_x a}\left(1 + 2e^{i\frac{3k_x a}{2}} \cos \frac{\sqrt{3}k_y a}{2}\right)$

$$\mathcal{H}_{tb} = \sum_{k} \Delta_k \psi^{\dagger}_{kA\sigma} \psi_{kB\sigma} + h.c. = \begin{pmatrix} 0 & \Delta_k \\ \Delta_k^* & 0 \end{pmatrix}$$

$\therefore$ the dispersion relation:

$$\varepsilon_k = \pm|\Delta_k| = \pm t\left(1 + 4\cos\frac{3k_x a}{2}\cos\frac{\sqrt{3}k_y a}{2} + 4\cos^2\frac{\sqrt{3}k_y a}{2}\right)$$

for $\varepsilon_k = 0$, $\Delta_k = 0$, $e^{i\frac{3k_x a}{2}} = 1$, $\cos\frac{\sqrt{3}k_y a}{2} = -\frac{1}{2}$

or $e^{i\frac{k_x a}{2}} = -1$, $\cos\frac{\sqrt{3}k_y a}{2} = \frac{1}{2}$

$$\therefore k = \frac{2\pi}{3a}\left(2n, \pm\frac{2}{\sqrt{3}}\right), \frac{2\pi}{3a}\left((2n+1), \pm\frac{1}{\sqrt{3}}\right)$$

$$\Delta_{k+q} = q\cdot\nabla_k \Delta_k + O(\,) = \frac{3t}{2a} e^{\frac{3\pi i}{0}} (q_x + iq_y) + O(\,)$$

$\therefore$ the effective Hamiltonian near zero is

$$H = \hbar v_F \begin{pmatrix} 0 & q_x + iq_y \\ q_x - iq_y & 0 \end{pmatrix} = \hbar v_F \sigma\cdot q.$$

$$v_F = \frac{3t}{2\hbar a}$$