1. the generalisation:

$$Ec_m = \varepsilon_0 c_n + V\left(\sum_l (D_l + D_l^{-1}) c_n\right) + W\delta_{mn} c_m$$

$$Ec_k = \varepsilon_0 c_k + 2V\left(\sum_l \cos k_l\right) c_k + W e^{ik\cdot m} c_m$$

$$c_k = \sum_n e^{ik\cdot n} c_n$$

$$\left(E - \varepsilon_0 - 2V \sum_l \cos k_l\right) c_k = W e^{ik\cdot m} c_m$$

$\therefore$

$$c_m = \frac{1}{(2\pi)^d} \int \frac{d^d k\, W}{E - \varepsilon_0 - 2V \sum_l \cos k_l} c_m$$

and $W\langle G(E)\rangle = 1 \Rightarrow \langle G(E)\rangle = \int \frac{d^d k}{(2\pi)^d} \frac{1}{E - \varepsilon(k)}$

$\varepsilon(k) = \varepsilon_0 + 2V \sum_l \cos k_l$

$\therefore$ we have:

$$\langle G(E)\rangle = \int d\varepsilon \int \frac{d^d k}{(2\pi)^d} \frac{1}{E - \varepsilon} \delta(\varepsilon - \varepsilon(k))$$

$$= \int \frac{d\varepsilon}{E - \varepsilon} \int \frac{d^d k}{(2\pi)^d} \delta(\varepsilon - \varepsilon(k))$$

$$= \int \frac{N(\varepsilon) d\varepsilon}{E - \varepsilon}$$

$\therefore$ if $|E - \varepsilon_0| < 2Vd$, the integral diverges

$E > \varepsilon_0 + 2Vd$ there are no poles in the integral

$E < \varepsilon_0 - 2Vd$, no poles.

If $W > 0$, $\langle G(E)\rangle > 0$, vice versa.

2. for a single defect:

$$\omega C_b = \varepsilon_b C_b + V_b (C_{b+1} + C_{b-1})$$

$$C_{b+1} = \frac{V_b}{V_a} e^{ik} C_b, \quad C_{b-1} = \frac{V_b}{V_a} e^{-ik} C_b$$

$$\therefore \omega = \varepsilon_b + \frac{V_b^2}{V_a} (e^{ik} + e^{-ik})$$

and $\omega C_a = \varepsilon_a C_a + V_a (e^{ik} + e^{-ik}) C_a$

$$\therefore \cos k = \frac{V_a(\varepsilon_a - \varepsilon_b)}{2(V_b^2 - V_a^2)}$$

$\therefore$ the energy is: $\omega = \dfrac{V_b^2 \varepsilon_a - V_a^2 \varepsilon_b}{V_b^2 - V_a^2}$

3. $$P = e^{-\int_T^{T\phi} \frac{\lambda_F^{d-1} V_F dt}{(D_0 t)^{\frac{d}{2}}}}$$

$$= \exp\left(-\frac{2\lambda_F V_F}{D_0} \int_l^{L\phi} \frac{\lambda_F^{d-2} dl}{l^{d-1}}\right)$$

4. $$\because \frac{d \log g}{d \log L} = d - 2 - \frac{A_d}{g}$$

$$\frac{g\, d(\log g)}{g(d-2) - A_d} = d \log L$$

$$\frac{dg}{g(d-2) - A_d} = \frac{dL}{L}$$

$$(d-2)g - A_d = ((d-2)g_0 - A_d)\left(\frac{L}{L_0}\right)^{d-2}$$

$$g_c = \frac{Ad}{d-2}$$

$$(g - g_c) = (g_0 - g_c)\left(\frac{L}{L_0}\right)^{d-2}$$

$$\left(\frac{g - g_c}{g_0 - g_c}\right)^{\frac{1}{d-2}} = \frac{L}{L_0}$$

$$\xi = L_0\left(\frac{g_0 - g_c}{g_c}\right)^{\frac{1}{d-2}}$$

$$\left(\frac{g - g_c}{g_c}\right)^{\frac{1}{d-2}} = \frac{1}{\xi}$$

$$g = g_c\left(1 + \left(\frac{2}{\xi}\right)^{d-2}\right)$$

$\therefore$ for $d=1$, $g \to \infty$

$d=3$, $g_c \sim g$, $L \to \infty$

for $r = \frac{1}{d-2}$, $s=1$, $\therefore$ valid expansion about $d=2$.