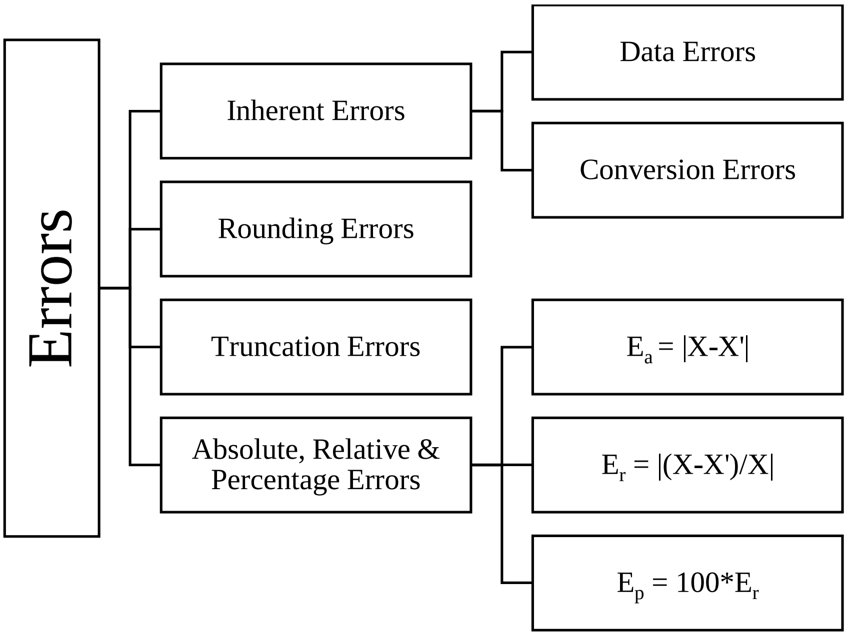
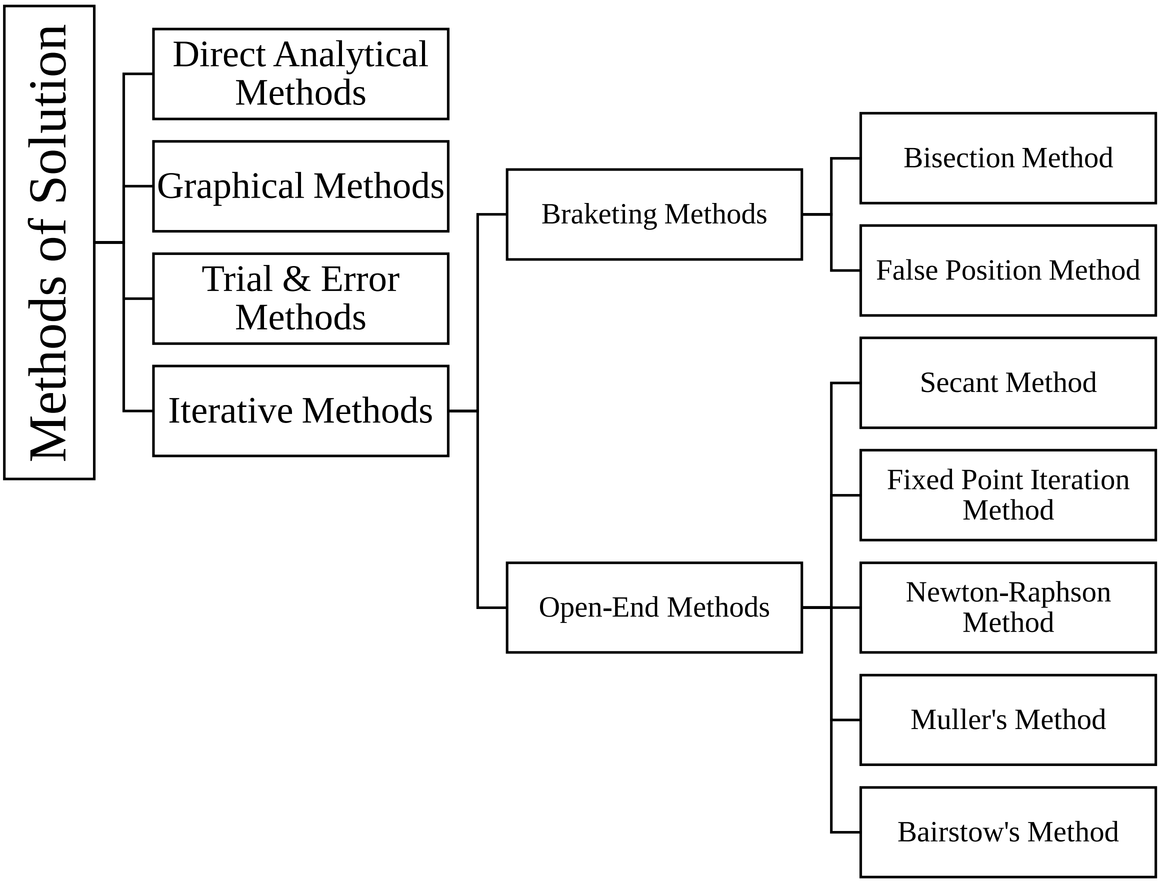
Chapter 1 & 2

Types of Error



Roots of Non-linear Equations



Stopping Criterion

Based on behavior of the function, we can terminate iteration. Various tests are:

1. **Test Absolute Error in x**

   $|x_{i+1} - x_i| \leq E_a$

2. **Test Relative Error in x**

   $\frac{|x_{i+1} - x_i|}{x_{i+1}} \leq E_r, x \neq 0$

3. **Test Value of Function at Root**

   $|f_{i+1}| \leq E$

4. **Test Difference in Function Value**

   $|f_{i+1} - f_i| \leq E$

5. **Test Large Function Value**

   $|f(x)| \leq F_{max}$

6. **Test Large Value of x**

   $|x_i| \leq XL$

7. **Limit on Iterations**

   $Iterations \leq N$

Bisection Method

1. Find a real root of the following equations by Bisection Method correct to 4-decimal places:
   a. $x^2 - 4x - 10 = 0$ b. $\sin(x) = 1/x$

$x^2 - 4x - 10 = 0$

Let f(x) = $\boldsymbol{x^2 - 4x - 10 = 0}$

Let two initial guesses are **a = -2** and **b = -1**

Since, f(a) = f(-2) = $(-2)^2$ - 4*(-2) – 10 = 2 i.e. +ve

f(b) = f(-1) = $(-1)^2$ - 4*(-1) – 10 = -5 i.e. -ve

Optional

Therefore a root lies between -2 and -1.

Let Error Tolerance, Et = 0.0001

$$No.of\ iterations, n = \frac{\log(b-a) - \log(Et)}{log2} = 13.28 \cong 14$$

Therefore, first approximation to the root is

$$x = \frac{a+b}{2} = \frac{-2-1}{2} = -1.5$$

Thus, f(x) = f(-1.5) = $(-1.5)^2$ - 4*(-1.5) – 10 = -1.75 i.e. -ve

Therefore, the root lies between x and -2.

Likewise, calculating other roots by tabulation method:

Iteration	a	b	f(a)	f(b)	x=(a+b)/2	f(x)	Status
1	-2	-1	2.0000	-5.0000	-1.5000	-1.7500	-ve
2	-2.0000	-1.5000	2.0000	-1.7500	-1.7500	0.0625	+ve
3	-1.7500	-1.5000	0.0625	-1.7500	-1.6250	-0.8594	-ve
4	-1.7500	-1.6250	0.0625	-0.8594	-1.6875	-0.4023	-ve
5	-1.7500	-1.6875	0.0625	-0.4023	-1.7188	-0.1709	-ve
6	-1.7500	-1.7188	0.0625	-0.1709	-1.7344	-0.0544	-ve
7	-1.7500	-1.7344	0.0625	-0.0544	-1.7422	0.0040	+ve
8	-1.7422	-1.7344	0.0040	-0.0544	-1.7383	-0.0253	-ve
9	-1.7422	-1.7383	0.0040	-0.0253	-1.7402	-0.0106	-ve
10	-1.7422	-1.7402	0.0040	-0.0106	-1.7412	-0.0033	-ve
11	-1.7422	-1.7412	0.0040	-0.0033	-1.7417	0.0003	+ve
12	-1.7417	-1.7412	0.0003	-0.0033	-1.7415	-0.0015	-ve
13	-1.7417	-1.7415	0.0003	-0.0015	**-1.7416**	-0.0006	-ve
14	-1.7417	-1.7416	0.0003	-0.0006	**-1.7416**	-0.0001	-ve

Here in 13th & 14th Iterations, value up to four decimal place is same. So, we stopped the iteration.

*Therefore, **root = -1.7416***

$\boldsymbol{sin(x) = 1/x}$

Let f(x) = $\boldsymbol{x sin(x) - 1 = 0}$

Let two initial guesses are **a = 1** and **b = 1.5**

Since, f(a) = f(1) = 1*sin(x) - 1 = -0.1584 i.e. -ve

f(a) = f(1.5) = 1.5*sin(1.5) - 1 = 0.49625 i.e. +ve

Therefore, a root lies between 1 and 1.5.

First approximation to the root is

$$x = \frac{a+b}{2} = \frac{1+1.5}{2} = 1.25$$

Likewise, calculating other roots by tabulation method:

Iteration	a	b	f(a)	f(b)	x=(a+b)/2	f(x)	Status
1	1	2	-0.1585	0.4962	1.2500	0.1862	+ve
2	1.0000	1.2500	-0.1585	0.1862	1.1250	0.0151	+ve
3	1.0000	1.1250	-0.1585	0.0151	1.0625	-0.0718	-ve
4	1.0625	1.1250	-0.0718	0.0151	1.0938	-0.0284	-ve
5	1.0938	1.1250	-0.0284	0.0151	1.1094	-0.0066	-ve
6	1.1094	1.1250	-0.0066	0.0151	1.1172	0.0042	+ve
7	1.1094	1.1172	-0.0066	0.0042	1.1133	-0.0012	-ve
8	1.1133	1.1172	-0.0012	0.0042	1.1152	0.0015	+ve
9	1.1133	1.1152	-0.0012	0.0015	1.1143	0.0001	+ve
10	1.1133	1.1143	-0.0012	0.0001	1.1138	-0.0005	-ve
11	1.1138	1.1143	-0.0005	0.0001	1.1140	-0.0002	-ve
12	1.1140	1.1143	-0.0002	0.0001	1.1141	0.0000	-ve
13	1.1141	1.1143	0.0000	0.0001	**1.1142**	0.0001	+ve
14	1.1141	1.1142	0.0000	0.0001	**1.1142**	0.0000	+ve

Here in 13th & 14th Iterations, value up to four decimal place is same. So, we stopped the iteration.

Therefore, ***root = 1.1142***

False Position Method

(**Regula-falsi/Interpolation Method**)

1. Using Regula-falsi method, compute the real root of the following equations correct to 3-decimal places:
    a. $cos(x) = xe^x$

Let f(x) = $cos(x) - xe^x = 0$

Let two initial guesses are **x1 = 0** and **x2 = 1**

Since, f(x1) = f(0) = $cos(0) - 0e^0$ = 1 i.e. +ve

f(x2) = f(1) = $cos(1) - 1e^1$ = -2.178 i.e. -ve

Therefore, a root lies between 0 and 1.

First approximation to the root is

$$x0 = x1 - \frac{f(x1)(x2 - x1)}{f(x2) - f(x1)} = 0 - \frac{1 * (1 - 0)}{-2.178 - 1} = 0.3147$$

Likewise, calculating other roots by tabulation method:

Iteration	x1	x2	f(x1)	f(x2)	x0	f(x0)
1	0	1	1.0000	-2.1780	0.3147	0.5199
2	0.3147	1.0000	0.5199	-2.1780	0.4467	0.2035
3	0.4467	1.0000	0.2035	-2.1780	0.4940	0.0708
4	0.4940	1.0000	0.0708	-2.1780	0.5099	0.0236
5	0.5099	1.0000	0.0236	-2.1780	0.5152	0.0078
6	0.5152	1.0000	0.0078	-2.1780	0.5169	0.0025
7	0.5169	1.0000	0.0025	-2.1780	0.5175	0.0008
8	0.5175	1.0000	0.0008	-2.1780	**0.5177**	0.0003
9	0.5177	1.0000	0.0003	-2.1780	**0.5177**	0.0001

Here in 8th & 9th Iterations, value up to four decimal place is same.
So, we stopped the iteration.

Therefore, ***root = 0.5177***

Secant Method

1. Find a root of the following equations correct up to 3 decimal places by the Secant Method:
   a. $xlogx = 1.9$

Let f(x) = $xlogx - 1.9 = 0$

Let two initial guesses are **x1 = 3** and **x2 = 4**

Since, f(x1) = f(3) = $3 * \log(3) - 1.9$ = -0.4686<0

f(x2) = f(4) = $4 * \log(4) - 1.9$ = 0.5082>0

Therefore, a root lies between 3 and 4.

First approximation to the root is

$$x0 = \boldsymbol{x2} - \frac{\boldsymbol{f(x2)}(x2 - x1)}{f(x2) - f(x1)} = 4 - \frac{0.5082 * (4 - 3)}{0.5082 - (-0.4686)} = 3.4797$$

Likewise, calculating other roots by tabulation method:

Iteration	x1	x2	f(x1)	f(x2)	x0	f(x0)
1	3	4	-0.4686	0.5082	3.4797	-0.0156
2	3.47973	4.00000	-0.01557	0.50824	3.49519	-0.00046
3	3.49519	4.00000	-0.00046	0.50824	**3.49565**	-0.00001
4	3.49565	4.00000	-0.00001	0.50824	**3.49567**	0.00000

Here in 3rd & 4th Iterations, value up to four decimal place is same. So, we stopped the iteration.

Therefore, ***root = 3.49567***

Fixed Point Iteration Method

(**Iteration/Successive Approximation/Direct Substitution/Fixed Point Method**)

1. Use the iteration method to find a root of the following equations to four decimal places:
    a. $x^2 - 2x - 3 = 0$

Given: f(x) = x^2 – 2x – 3 = 0 – Eq. I

For fixed point iteration method, arranging Eq. I in terms of g(x) = x^2 – 2x – 3.

Or, x^2 – 2x – 3=0

Or, x^2 – 2x = 3

Or, x(x-2) = 3

Or, x = 3/(x-2)

By iteration formula, $x_{n+1} = g(x_n)$

Let x0 = 0 be the initial approximation to the root. Then,

$$x1 = \frac{3}{x0 - 2} = \frac{3}{0 - 2} = -1.5$$

Likewise, calculating other roots by tabulation method:

Iteration	**Xn**	**X(n+1)**	**Error**
1	0.000000	-1.500000	1.000000
2	-1.500000	-0.857143	0.750000
3	-0.857143	-1.050000	0.183673
4	-1.050000	-0.983607	0.067500
5	-0.983607	-1.005495	0.021768
6	-1.005495	-0.998172	0.007336
7	-0.998172	-1.000610	0.002436
8	-1.000610	-0.999797	0.000813
9	-0.999797	-1.000068	0.000271
10	-1.000068	-0.999977	0.000090
11	-0.999977	-1.000008	0.000030
12	-1.000008	-0.999997	0.000010
13	-0.999997	-1.000001	0.000003
14	-1.000001	-1.000000	0.000001
15	-1.000000	-1.000000	0.000000
			END

Since, error is less than 0.000001, so we stopped iteration.

Therefore, ***root = -1.000000***