Assignment#6

Numerical Integration

1. Given that:

x	4.0	4.2	4.4	4.6	4.8	5.0	5.2
log x	1.3963	1.4351	1.4816	1.5261	1.5686	1.6094	1.6487

Evaluate $\int_4^{5.2} \log x\, dx$ by

   a. **Trapezoidal** rule b. **Simpson's 1/3rd** rule
   c. **Simpson's 3/8th** rule & also find the error in each case

2. Write an **algorithm** to calculate the definite integral $\int_a^b f(x)dx$ using **composite Simpson's 1/3rd rule**.
3. Evaluate $\int_0^2 f(x)dx$ for the function f(x) = $e^x + \sin(2x)$ using **Simpson's 3/8th** rule taking h = **0.4**.
4. Evaluate $\int_0^3 (\sin(x) + \cos(x) + 12)dx$ using **Simpson's 3/8th** rule taking h = **0.5**, Determine the **percent error** by comparing the result with the result with **exact solution**.
5. Evaluate $\int_0^1 \frac{1}{1+x^2} dx$ using
   a. Trapezoidal rule taking h = ¼ = 0.25
   b. Simpson's 1/3rd rule taking n=4, h = ¼ = 0.25
   c. Simpson's 3/8th rule taking n=6, h = 1/6

   Hence compute an approximate value of π in each case. [Hint: generate two tables]

6. The velocity v of a particle at a distance s from a point on its path is given in the table below:

s (ft)	0	10	20	30	40	50	60
v (ft/sec)	47	58	64	65	61	52	38

Estimate the time taken to travel a distance of 60ft by using Simpson's 1/3rd rule. Compare the result with Simpson's 3/8th rule. [**Ans:** $I_{1/3}$ = 1.06352, $I_{3/8}$ = 1.06445]

7. Write a pseudo-code to integrate a given function within given limits using Simpson's 3/8th rule.
8. Use **Romberg's Integration** method to compute $\int_0^{0.5} \frac{x}{sinx} dx$, correct to 3 decimal places.
9. Evaluate following using **Gaussian Quadrature 2-point & 3-point** formula:
   a. $\int_0^1 \frac{\tan^{-1} x}{x} dx$
   b. $\int_0^{\pi/2} e^{sinx}\, dx$
   c. $\int_{0.2}^{1.5} e^{-x^2}\, dx$
   d. $\int_2^3 \frac{cos2x}{1+sinx} dx$
   e. $\int_{0.2}^{1.2} (\log(x+1) + sin2x)\, dx$
10. Use **Gauss-Legendre** four-point formula to evaluate $\int_2^4 (x^4 + 1)dx$