Assignment#6

Numerical Integration

1. Given that:

x	4.0	4.2	4.4	4.6	4.8	5.0	5.2
log x	1.3963	1.4351	1.4816	1.5261	1.5686	1.6094	1.6487

   Evaluate $\int_4^{5.2} \log x \, dx$ by

       a. **Trapezoidal** rule   b. **Simpson's 1/3ʳᵈ** rule
       c. **Simpson's 3/8ᵗʰ** rule & also find the error in each case

2. Write an **algorithm** to calculate the definite integral $\int_a^b f(x)dx$ using **composite Simpson's 1/3ʳᵈ rule**.

3. Evaluate $\int_0^2 f(x)dx$ for the function f(x) = eˣ + sin(2x) using **Simpson's 3/8ᵗʰ** rule taking h = **0.4**.

4. Evaluate $\int_0^3 (\sin(x) + \cos(x) + 12)dx$ using **Simpson's 3/8ᵗʰ** rule taking h = **0.5**, Determine the **percent error** by comparing the result with the result with **exact solution**.

5. Evaluate $\int_0^1 \frac{1}{1+x^2} dx$ using
       a. Trapezoidal rule taking h = ¼ = 0.25
       b. Simpson's 1/3ʳᵈ rule taking n=4, h = ¼ = 0.25
       c. Simpson's 3/8ᵗʰ rule taking n=6, h = 1/6

   Hence compute an approximate value of **π** in each case. [Hint: generate two tables]

6. The velocity v of a particle at a distance s from a point on its path is given in the table below:

s (ft)	4.0	0	10	20	30	40	50	60
v (ft/sec)	1.3963	47	58	64	65	61	52	38

   Estimate the time taken to travel a distance of 60ft by using Simpson's 1/3ʳᵈ rule. Compare the result with Simpson's 3/8ᵗʰ rule. [**Ans:** $I_{1/3}$ = 1.06352, $I_{3/8}$ = 1.06445]

7. Write a pseudo-code to integrate a given function within given limits using Simpson's 3/8ᵗʰ rule.

8. Use **Romberg's Integration** method to compute $\int_0^{0.5} \frac{x}{\sin x} dx$, correct to 3 decimal places.

9. Evaluate following using **Gaussian Quadrature 2-point & 3-point** formula:
       a. $\int_0^1 \frac{\tan^{-1} x}{x} dx$
       b. $\int_0^{\pi/2} e^{\sin x} dx$
       c. $\int_{0.2}^{1.5} e^{-x^2} dx$
       d. $\int_2^3 \frac{\cos 2x}{1+\sin x} dx$
       e. $\int_{0.2}^{1.2} (\log(x + 1) + \sin 2x) \, dx$

10. Use **Gauss-Legendre** four-point formula to evaluate $\int_2^4 (x^4 + 1)dx$