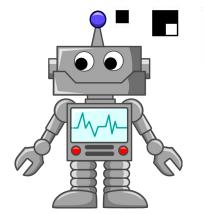
## Structure is all you need learning compressed RL policies via gradient sensing

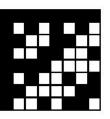
UNIVERSITY OF CAMBRIDGE

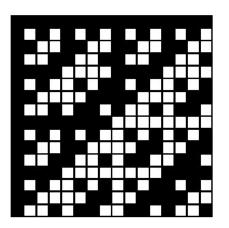
Google Brain Robotics: Krzysztof Choromanski, Vikas Sindhwani

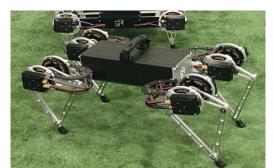
University of Cambridge and Alan Turing Institute: Mark Rowland, Richard E. Turner, Adrian Weller



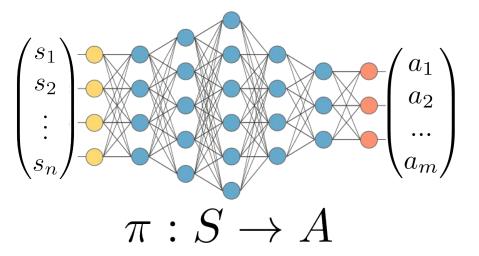


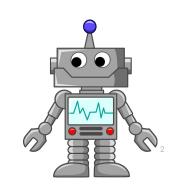


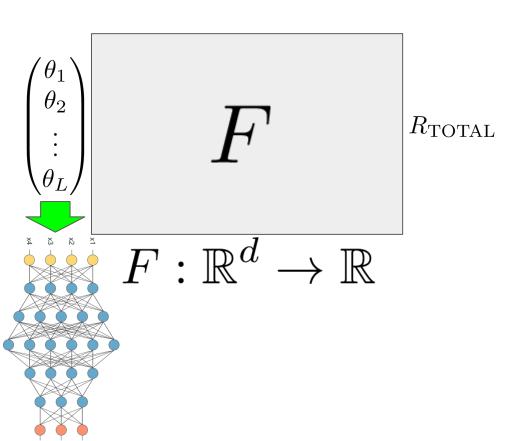


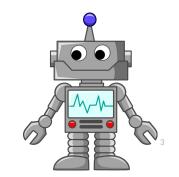


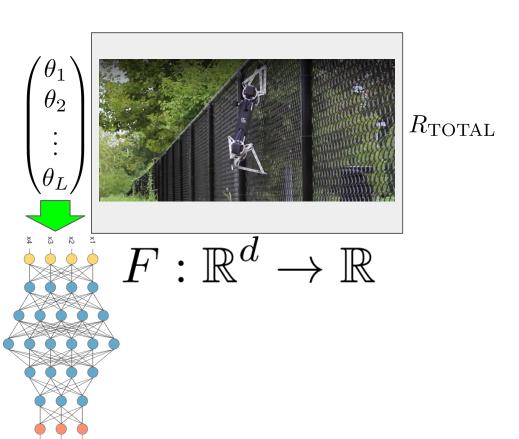
INNOVATION + ASSI

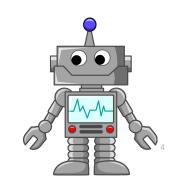


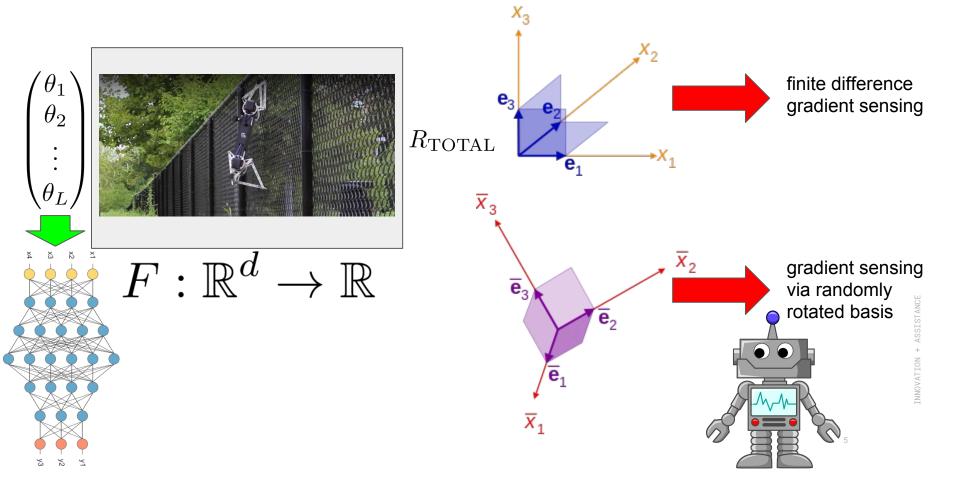


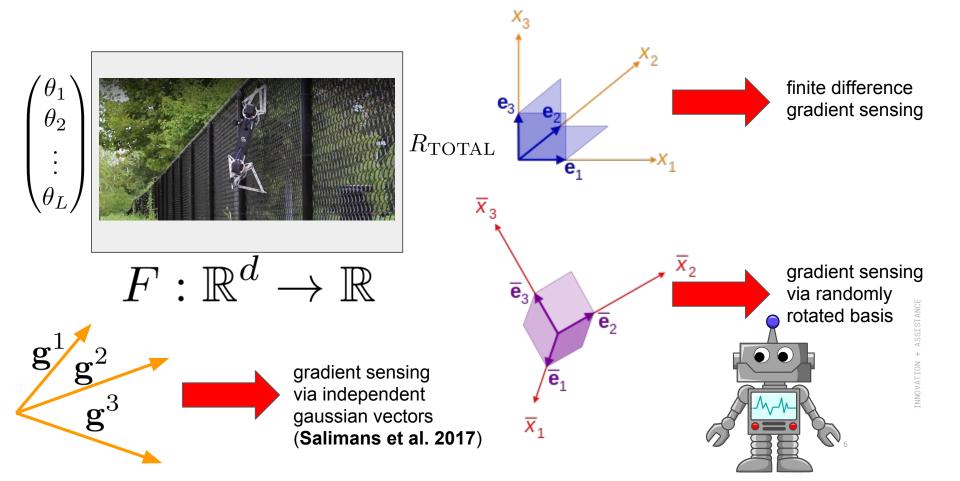


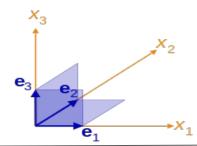




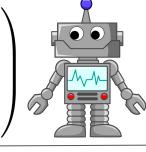


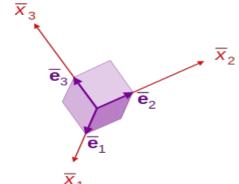




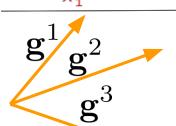


$$\mathbf{P} = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 \end{pmatrix}$$





$$\mathbf{G}_{\text{ort}} = \begin{pmatrix} g_{1,1}^{\text{ort}} & g_{1,2}^{\text{ort}} & \dots & g_{1,n}^{\text{ort}} \\ g_{2,1}^{\text{ort}} & g_{2,2}^{\text{ort}} & \dots & g_{2,n}^{\text{ort}} \\ \dots & \dots & \dots & \dots \\ g_{m,1}^{\text{ort}} & g_{m,n}^{\text{ort}} & \dots & g_{m,n}^{\text{ort}} \end{pmatrix}$$



$$\mathbf{G} = \begin{pmatrix} g_{1,1} & g_{1,2} & \dots & g_{1,n} \\ g_{2,1} & g_{2,2} & \dots & g_{2,n} \\ \dots & \dots & \dots & \dots \\ g_{m,1} & g_{m,n} & \dots & g_{m,n} \end{pmatrix}$$

## $F: \mathbb{R}^d \to \mathbb{R}$

Towards smooth relaxations

$$\max_{\mu \in \mathcal{P}(\mathbb{R}^d)} \mathbb{E}_{\theta \sim \mu}[F(\theta)]$$

family of probabilistic distributions on  $\mathbb{R}^d$ 

### Gaussian smoothings

 $\max_{\theta \in \mathbb{R}^d} J(\theta) = \mathbb{E}_{\phi \sim N(\theta, \sigma^2 I)} \left[ F(\phi) \right]$ 

- infinitely differentiable objective (Nesterov & Spokoiny, 2017)
- optimal value lower-bounds
   That of the original problem

#### ES-style gradient estimator (Salimans et al. 2017):

$$\widehat{\nabla}_N J(\theta) = \frac{1}{N\sigma} \sum_{i=1}^N F(\theta + \sigma \epsilon_i) \epsilon_i \qquad \text{used in many evolutionary strategies papers, no control variate}$$

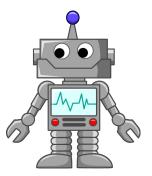
#### Gradient estimator with antithetic pairs (Salimans et al. 2017):

$$\widehat{\nabla}_N J(\theta) = \frac{1}{2N\sigma} \sum_{i=1}^N (F(\theta + \sigma \epsilon_i) \epsilon_i - F(\theta - \sigma \epsilon_i) \epsilon_i) \qquad \text{used for variance reduction}$$

#### **FD-style gradient estimator:**

samples

$$\widehat{\nabla}_{\overline{N}}J(\theta) = \frac{1}{N\sigma}\sum_{i=1}^{N}(F(\theta+\sigma\epsilon_i)\epsilon_i - F(\theta)\epsilon_i)$$
 randomized version of a standard finite difference method



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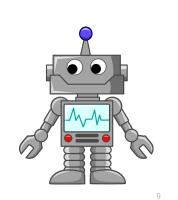
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 randomized version of a standard finite difference method

randomized version

reduction

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used for variance

$$egin{aligned} 
abla_N^{ ext{ort}} J( heta) \ \{\epsilon_1^{ ext{ort}},...,\epsilon_N^{ ext{ort}}\} \end{aligned}$$

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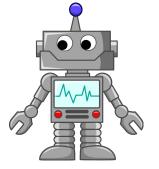
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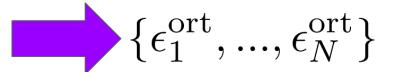
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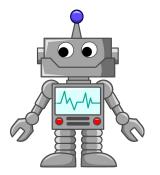
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 number of

- all three estimators are unbiased
- empirically tested that none of them dominates the others
- how to learn optimal control variates?





INNOVATION + ASSISTANC

#### ES-style gradient estimator (Salimans et al. 2017):

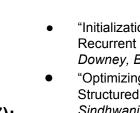
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$$\widehat{\nabla}_N J(\theta) = \frac{1}{N\sigma} \sum_{i=1}^N (F(\theta + \sigma \epsilon_i) \epsilon_i - F(\theta) \epsilon_i)$$



Some recent success stories of structured random estimators (very biased selection!):

- "Initialization matters: Orthogonal Predictive State Recurrent Neural Networks", Choromanski, Downey, Boots (to appear at ICLR'18)
- "Optimizing Simulations with Noise-Tolerant Structured Exploration", Choromanski, Iscen, Sindhwani, Tan, Coumans (to appear at ICRA'18)
- "The Geometry of Random Features", Choromanski, Rowland, Sarlos, Sindhwani, *Turner, Weller* (to appear at AISTATS'18)
- "The Unreasonable Effectiveness of Structured Random Orthogonal Embeddings", Choromanski, Rowland, Weller (NIPS'17)
- "Structured adaptive and random spinners for fast machine learning computations", Bojarski, Choromanska, Choromanski, Fagan, Gouy-Pailler, Morvan, Sakr, Sarlos, Atif (AISTATS'17)
- "Orthogonal Random Features", Yu, Suresh, Choromanski, Holtmann-Rice, Kumar (NIPS'16) "Recycling Randomness with Structure for
- Sindhwani (ICML'16) "Binary embeddings with structured hashed projections", Choromanska, Choromanski, Bojarski, Jebara, Kumar, LeCun (ICML'16)

Sublinear time Kernel Expansion", *Choromanski*,

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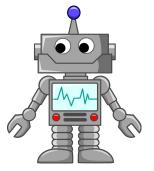
$$\widehat{\nabla}_N J(\theta) = \frac{1}{N\sigma} \sum_{i=1}^N (F(\theta + \sigma \epsilon_i) \epsilon_i - F(\theta) \epsilon_i)$$

Theorem (Choromanski, Rowland, Sindhwani, Turner, Weller'18)

The orthogonal gradient estimator  $\widehat{\nabla}_N^{\mathrm{ort}} J(\theta)$  is unbiased and yields lower MSE than the unstructured estimator  $\widehat{\nabla}_N J(\theta)$ , namely:

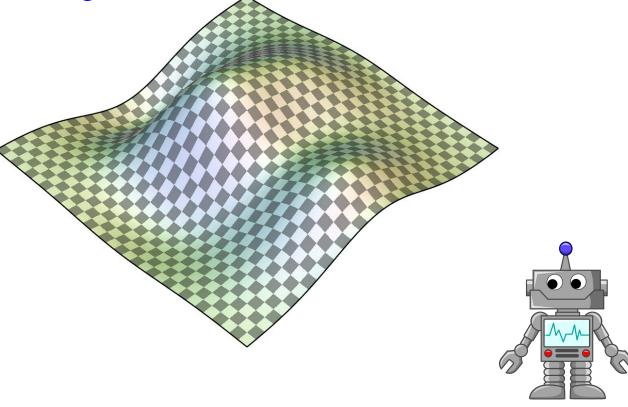
$$MSE(\widehat{\nabla}_N^{\mathrm{ort}}J(\theta)) =$$

$$MSE(\widehat{\nabla}_N J(\theta)) - \frac{N-1}{N} \|\nabla J(\theta)\|_2^2.$$

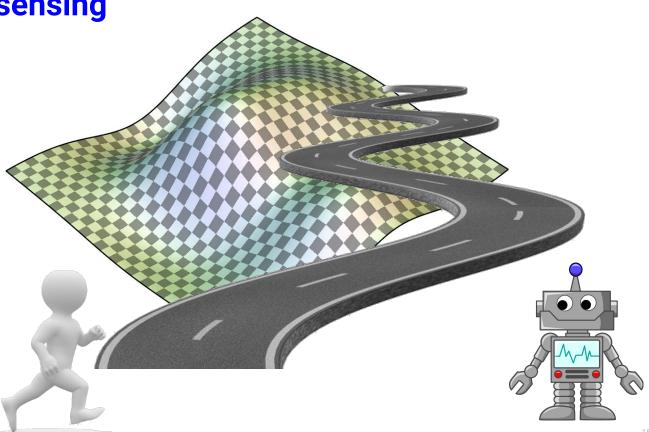


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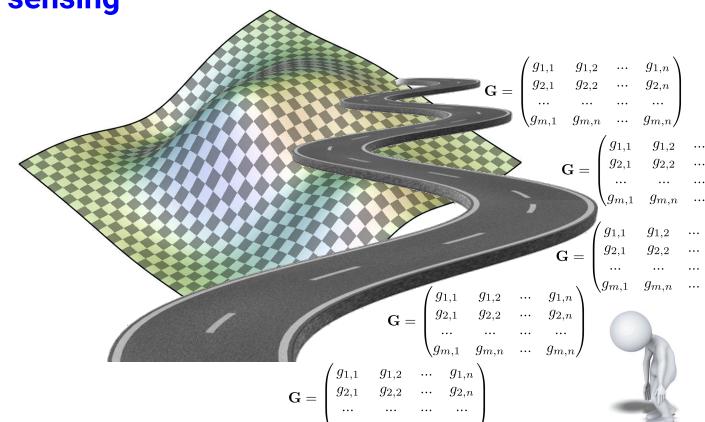
for gradient sensing

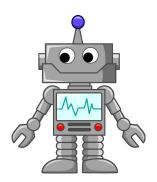


for gradient sensing



for gradient sensing

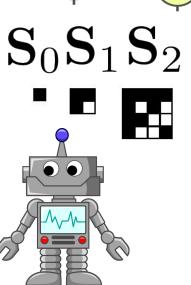


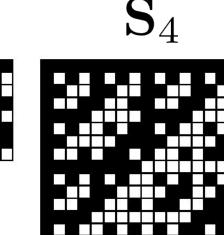


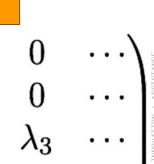
for gradient sensing - discrete space sensing



$$\mathbf{M}_{\mathbf{S}\mathcal{R}}^{(k)} = \prod_{i=1}^{n} \mathbf{S} \mathbf{D}_{i}^{(\mathcal{R})} o |\lambda_{i}| = 0$$

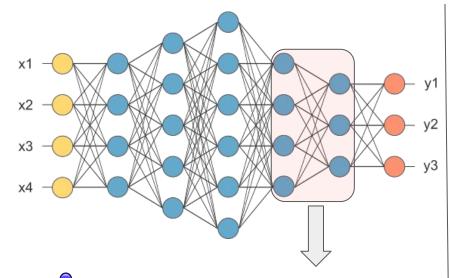


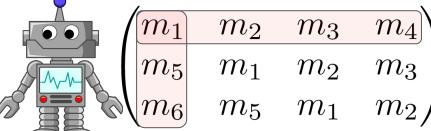




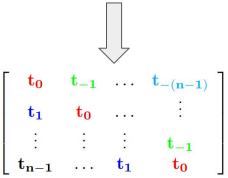
 $\lambda_i \sim Unif\{-1,+1\}$ 

# Efficiency of the orthogonal space exploration for gradient sensing - structured neural networks



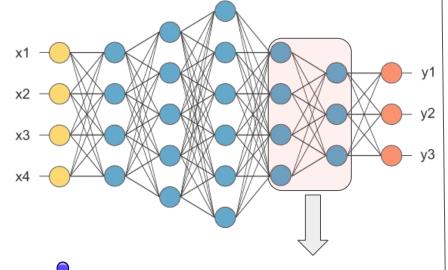


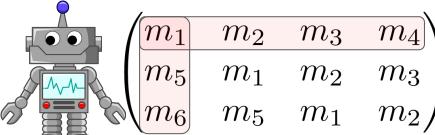
#### Structured Matrices: Toeplitz



- $\blacktriangleright n \times n$  matrix parameterized by 2n-1 numbers: constant diagonal values
- $ightharpoonup O(n \log n)$  matrix-vector products, Linear Systems
- Applications
  - Implements One-dimensional Linear Convolutions
  - Arises naturally in time series analysis and dynamical systems.
  - Related matrix: Hankel antidiagonals are constant

gradient sensing - structured neural networks





### **Sylvester Displacement and Unit-Circulants**

► The Sylvester displacement operator is defined by,

$$\nabla_{\mathbf{A},\mathbf{B}}[\mathbf{M}] = \mathbf{A}\mathbf{M} - \mathbf{M}\mathbf{B}$$

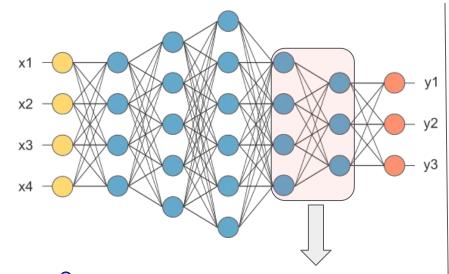
where  $\mathbf{A} \in \mathbb{R}^{m \times m}$ ,  $\mathbf{B} \in \mathbb{R}^{n \times n}$  are fixed operator matrices.

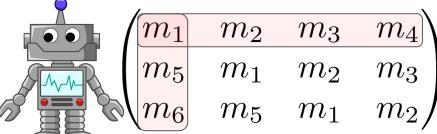
- Design displacement operators by carefully choosing A, B.
- ► Shift-and-Scale matrices are called *f*-unit Circulant matrices:

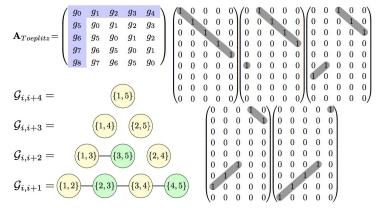
$$\mathbf{Z}_f = \left[egin{array}{cccc} 0 & 0 & \dots & f \ 1 & 0 & \dots & 0 \ dots & dots & dots & dots \ 0 & \dots & 1 & 0 \end{array}
ight], \quad \mathbf{Z}_f \left(egin{array}{c} v_1 \ v_2 \ v_3 \ dots \ v_n \end{array}
ight) = \left(egin{array}{c} f v_n \ v_1 \ v_2 \ dots \ v_{n-1} \end{array}
ight)$$

- $egin{aligned} &- \mathbf{Z}_f^n = f \mathbf{I} \ &- ext{ Upshifts with } \mathbf{Z}_f^T \ &- \mathbf{Z}_f^{-1} = \mathbf{Z}_{f-1}^T \end{aligned}$

## Efficiency of the orthogonal space exploration for gradient sensing - structured neural networks







#### Very Low-displacement Rank Property



Structured Matrix ${f M}$	A	В	$rank( abla_{\mathbf{A},\mathbf{B}}[\mathbf{M}])$
Toeplitz and its inverse	$\mathbf{Z}_1$	$\mathbf{Z}_{-1}$	$\leq 2$
Hankel and its inverse	$\mathbf{Z}_1$	$\mathbf{Z}_0^T$	$\leq 2$
${\sf Toeplitz} + {\sf Hankel}$	$\mathbf{Z}_0 + \mathbf{Z}_0^T$	$\mathbf{Z}_0 + \mathbf{Z}_0^T$	$\leq 4$
Vandermonde $V(\mathbf{v})$	$diag(\mathbf{v})$	$\mathbf{Z}_0$	$\leq 1$
Inverse of Vandermonde i.e. $V(\mathbf{v})^{-1}$	$\mathbf{Z}_0$	$diag(\mathbf{v})$	$\leq 1$
Transpose of Vandermonde i.e. $V(\mathbf{v})^T$	$\mathbf{Z}_0^T$	$diag(\mathbf{v})$	$\leq 1$
Cauchy $\mathbf{C}(\mathbf{s},\mathbf{t})$	$diag(\mathbf{s})$	$diag(\mathbf{t})$	$\leq 1$
Inverse of Cauchy i.e. $\mathbf{C}(\mathbf{s},\mathbf{t})^{-1}$	$diag(\mathbf{t})$	$diag(\mathbf{s})$	$\leq 1$

## **Experimental results: orthogonal blackbox**

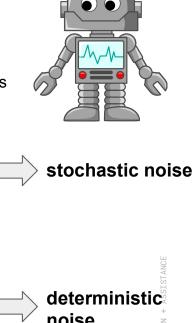
DFO benchmarking suite, More & Wild (2009)

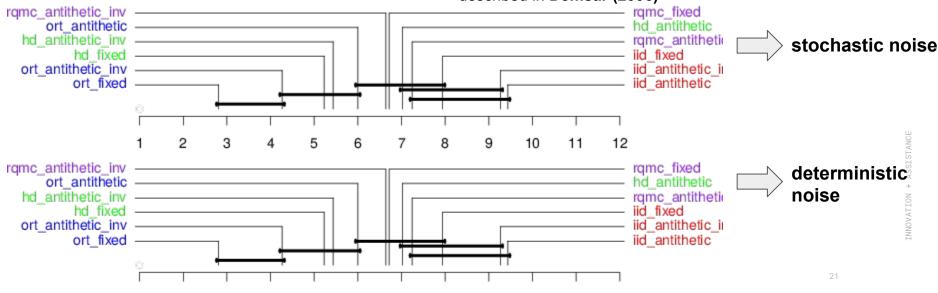
## gradient sensing for low-dimensional problems

53 different optimization problems, four settings:

- stochastic
- deterministic noise
- smooth problems
- non-differentiable problems

- we compute average ranking of the methods against each other in terms of quality of final objective value in each optimisation task
- we then compare these average rankings using multiple hypothesis testing as described in **Demsar (2006)**





## **Experimental results: orthogonal blackbox**

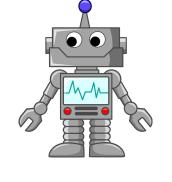
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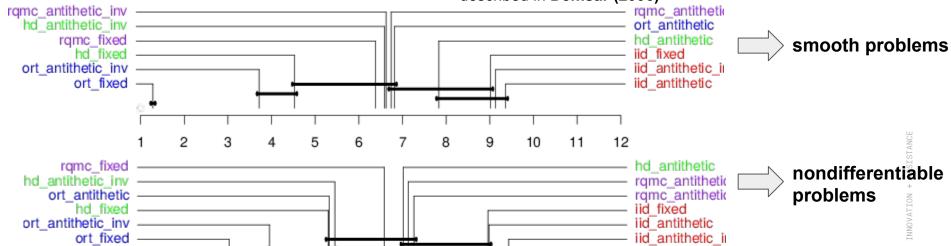
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## **Experimental results: learning compressed**

## policies for RL tasks

#### **Optimization setting:**

- AdamOptimizer with  $\alpha=0.01$  and  $\sigma=0.02$
- no heuristics used (e.g no fitness shaping)
- Base compressed setting: two hidden layers of size h=41

#### **Tested variants:**

- structured architectures + structured exploration (STST)
- structured architectures + unstructured exploration (STUN)
- unstructured architectures + unstructured exploration (UNUN)

#### Structured space exploration strategies:

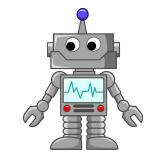
- Gaussian orthogonal matrices
- matrices HD (k=1); 256- or
   512-dimensional parameter vectors



#### **Tested environments:**

- Swimmer
- Ant
- HalfCheetah
- Hopper
- Humanoid
- Walker2d
- Pusher
- Reacher
- Striker
- Thrower
- ContMountainCar
- Pendulum
- Minitaur walking





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Learns reward

1842

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- Minitaur walking

Distributed TF training on at most 400 machines

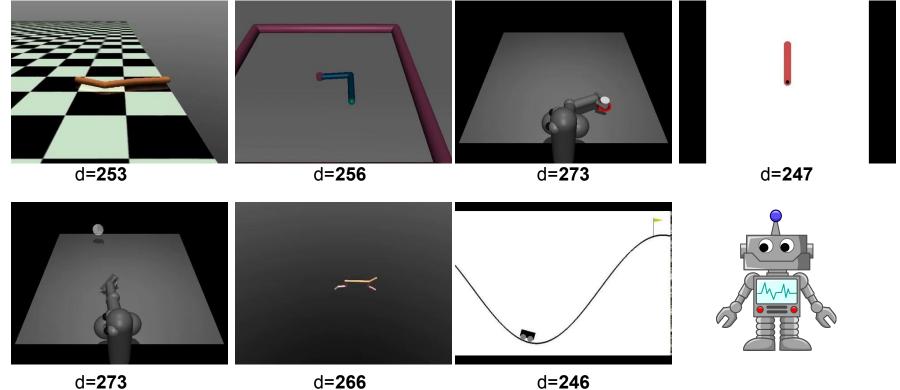
	GaussOrt	Hadamard	baseline			
Swimmer	253	253	1408			
Ant	362	254	4896			
HalfCheetah	266	254	2174			
Hopper	257	254	1536			
Humanoid	636	510	13664			
Walker2d	266	254	1824			
Pusher	273	255	2048			
Reacher	256	256	1189			
Striker	273	255	2048			
Thrower	273	255	2048			
ContMountCar	246	246	1184			
Pendulum	247	247	1216			
Minitaur	279	256	2240			

279-dimensional neural network learns reward **4.83** for rollouts of length

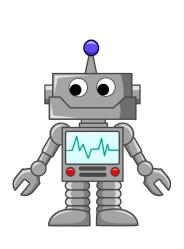
**500** 

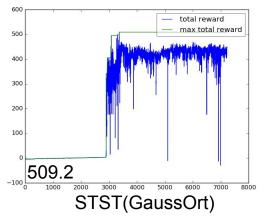


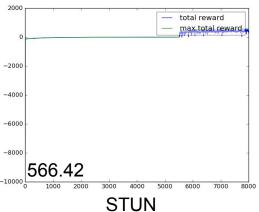
# **Experimental results: learning compressed policies for RL tasks**

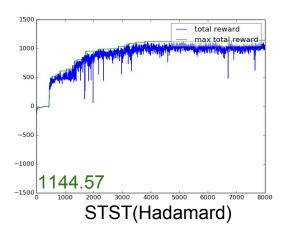


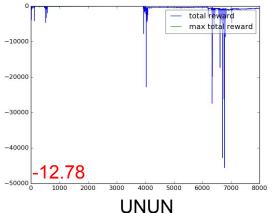
## **Experimental results: learning curves - Ant**



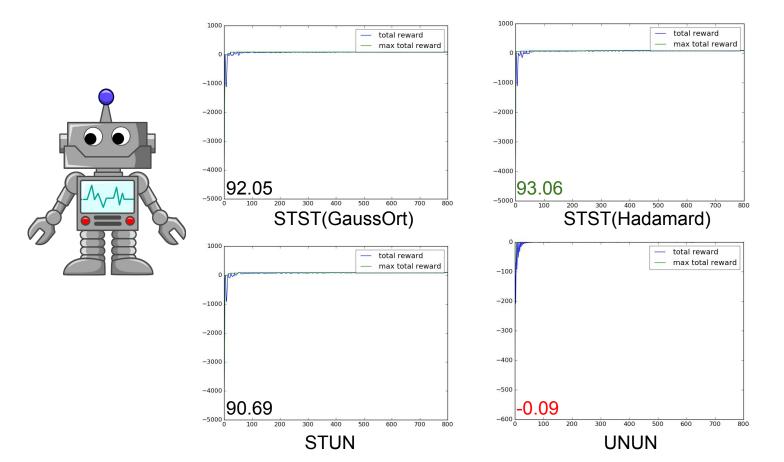




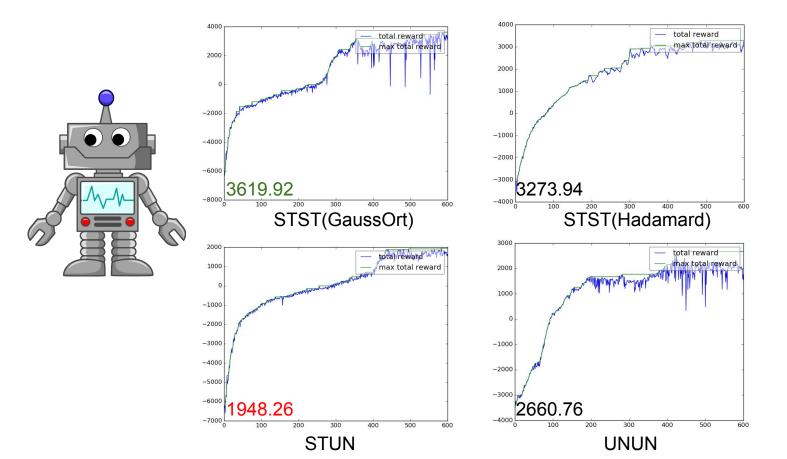




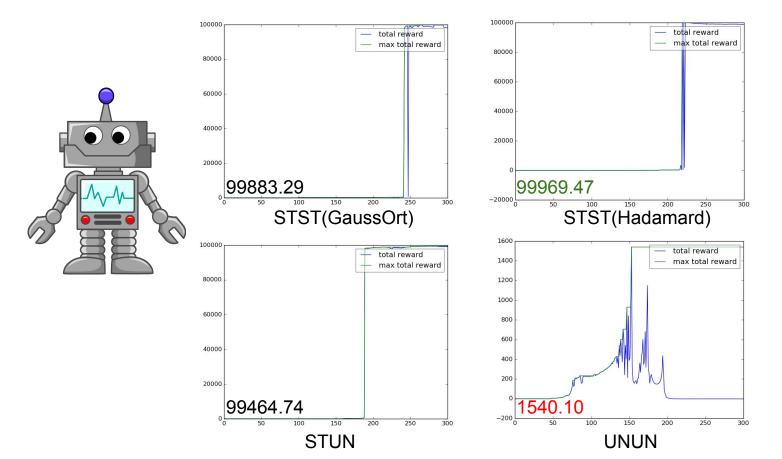
## **Experimental results: learning curves - Cont. Mountain Car**



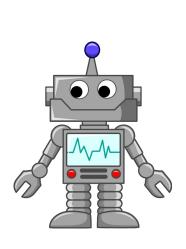
## **Experimental results: learning curves - HalfCheetah**

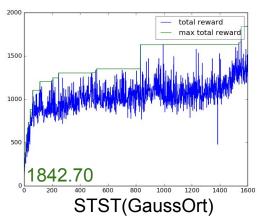


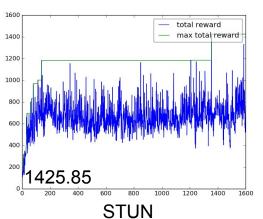
## **Experimental results: learning curves - Hopper**

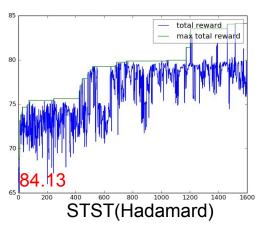


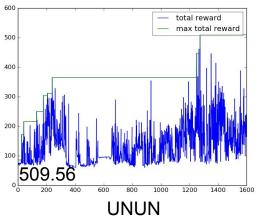
## **Experimental results: learning curves - Humanoid**



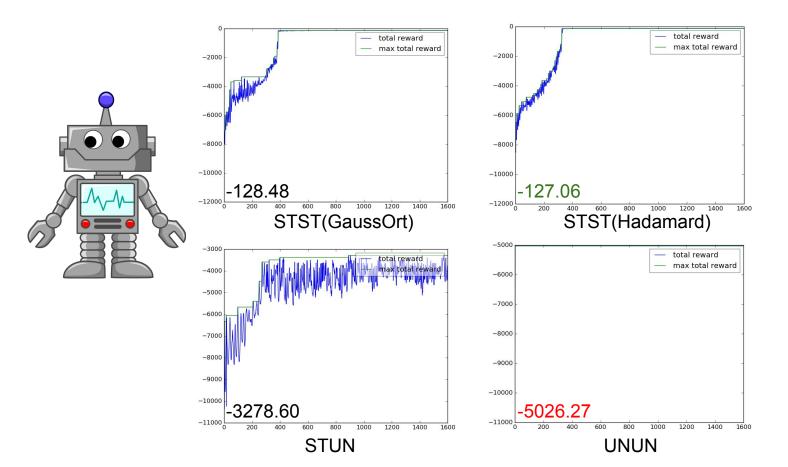




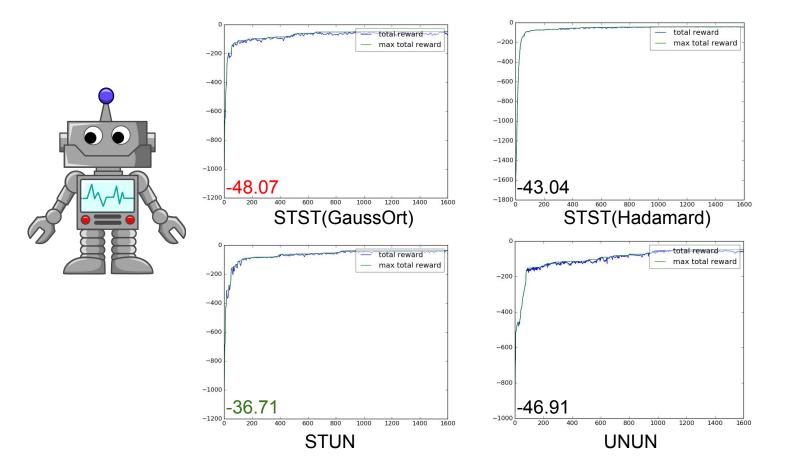




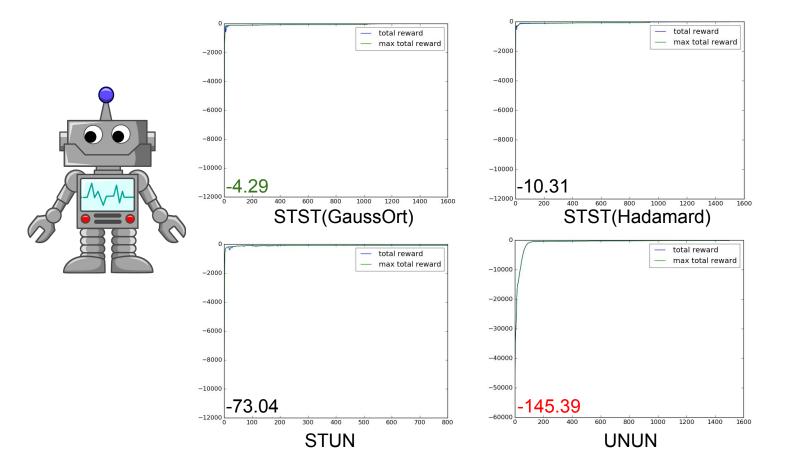
## **Experimental results: learning curves - Pendulum**



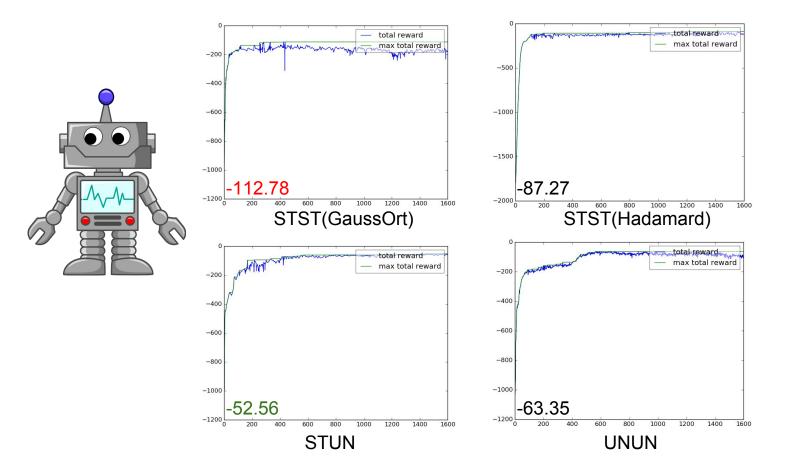
## **Experimental results: learning curves - Pusher**



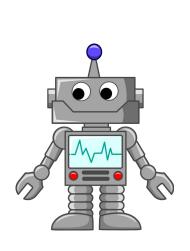
## **Experimental results: learning curves - Reacher**

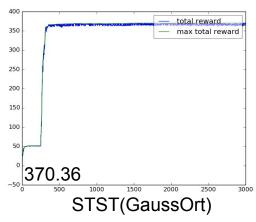


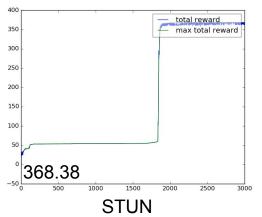
## **Experimental results: learning curves - Striker**

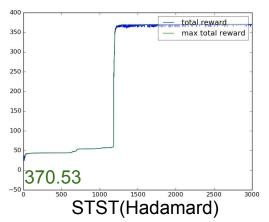


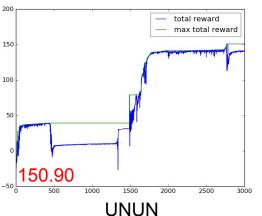
## **Experimental results: learning curves - Swimmer**



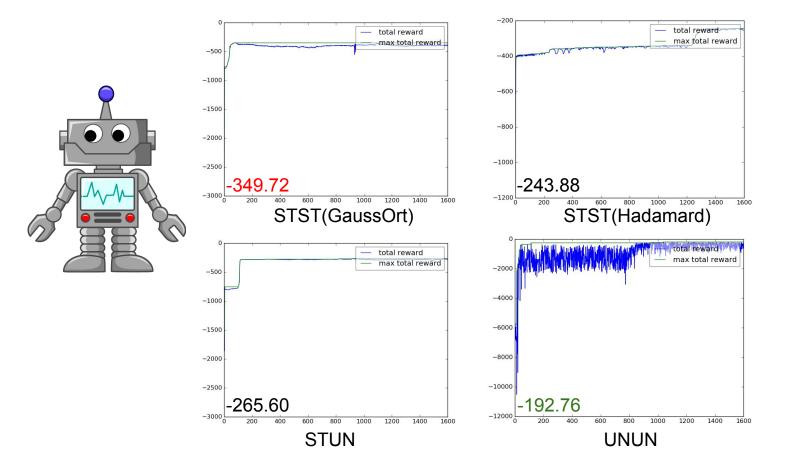




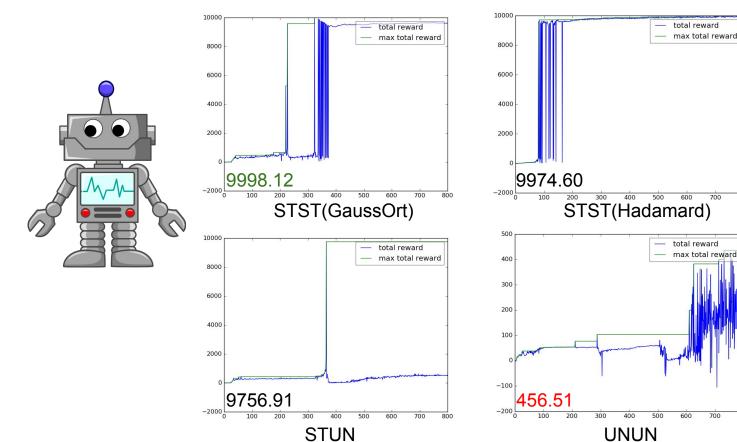




## **Experimental results: learning curves - Thrower**



## **Experimental results: learning curves - Walker2d**



## Success stories - teaching "Smoky" to walk (ICRA'18)

joint work with: Atil Iscen, Vikas Sindhwani, Jie Tan and Erwin Coumans

