

1. **(Alternative Problem for Example 1 in Week 9 Notes)**

TXU stock is has the following probability distribution:

Probability	Return
0.25	8%
0.55	10%
0.20	12%

What are its expected return and standard deviation?

Solution)

- Expected Return

$$\begin{aligned} E[R] &= (0.25)(0.08) + (0.55)(0.10) + (0.20)(0.12) \\ &= 0.020 + 0.055 + 0.024 = 0.099 = 9.9\% \end{aligned}$$

- Standard Deviation

$$\begin{aligned} SD(R) &= \sqrt{(0.25)(0.08 - 0.099)^2 + (0.55)((0.10 - 0.099)^2 + (0.20)((0.12 - 0.099)^2} \\ &= \sqrt{0.00009025 + 0.00000055 + 0.0000882} \\ &= \sqrt{0.000179} = 1.338\%. \end{aligned}$$

2. **(Alternative Problem for Example 2 in Week 9 Notes)**

What were the realized annual returns for NRG stock in 2012 and in 2018?

Date	Price (\$)	Dividend (\$)	Return	Date	Price(\$)	Dividend(\$)	Return
12/31/2011	58.69			12/31/2017	6.73	0	
1/31/2012	61.44	0.26		3/31/2018	5.72	0	
4/30/2012	63.94	0.26		6/30/2018	4.81	0	
7/31/2012	48.50	0.26		9/30/2018	5.20	0	
10/31/2012	54.88	0.29		12/31/2018	2.29	0	
12/31/2012	53.31						

Solution)

We compute each period's return using Eq. (4). For example, the return from December 31, 2011, to January 31, 2012, is

$$\frac{61.44 + 0.26}{58.69}$$

We then determine annual returns using Eq. (5):

$$R_{2012} = (1.0513)(1.0449)(0.7626)(1.1375)(0.9714) - 1 = -7.43\%$$

$$R_{2018} = (0.8499)(0.8409)(1.0811)(0.440) - 1 = -66.0\%$$

Note that, since NRG did not pay dividends during 2018, the return can also be computed as follows:

$$\frac{2.29}{6.73} - 1 = -66.0\%$$

Date	Price (\$)	Dividend (\$)	Return	Date	Price(\$)	Dividend(\$)	Return
12/31/2011	58.69			12/31/2017	6.73	0	
1/31/2012	61.44	0.26	5.13%	3/31/2018	5.72	0	-15.01%
4/30/2012	63.94	0.26	4.49%	6/30/2018	4.81	0	-15.91%
7/31/2012	48.50	0.26	-23.74%	9/30/2018	5.20	0	8.11%
10/31/2012	54.88	0.29	13.75%	12/31/2018	2.29	0	-55.96%
12/31/2012	53.31		-2.86%				

3. (Alternative Problem for Example 3 in Week 9 Notes)

Using the data from Table 2, what are the variance and standard deviation of Microsoft's returns from 2008 to 2017?

Year End	S&P 500 Index	Dividends Paid*	S&P 500 Realized Return	Microsoft Realized Return	1-Month T-Bill Return
2004	1211.92				
2005	1248.29	23.15	4.9%	−0.9%	3.0%
2006	1418.30	27.16	15.8%	15.8%	4.8%
2007	1468.36	27.86	5.5%	20.8%	4.7%
2008	903.25	21.85	−37.0%	−44.4%	1.5%
2009	1115.10	27.19	26.5%	60.5%	0.1%
2010	1257.64	25.44	15.1%	−6.5%	0.1%
2011	1257.61	26.59	2.1%	−4.5%	0.0%
2012	1426.19	32.67	16.0%	5.8%	0.1%
2013	1848.36	39.75	32.4%	44.3%	0.0%
2014	2058.90	42.47	13.7%	27.6%	0.0%
2015	2043.94	43.45	1.4%	22.7%	0.0%
2016	2238.83	49.56	12.0%	15.1%	0.2%
2017	2673.61	53.99	21.8%	40.7%	0.8%

* Total dividends paid by the 500 stocks in the portfolio, based on the number of shares of each stock in the index, adjusted until the end of the year, assuming they were reinvested when paid.

Source: Standard & Poor's, Microsoft and U.S. Treasury Data

Solution)

First, we need to calculate the average return for Microsoft's over that time period, using Eq. (6):

$$\bar{R} = \frac{1}{10}(-44.4\% + 60.5\% - 6.5\% - 4.5\% + 5.8\% + 44.3\% + 27.6\% + 22.7\% + 15.1\% + 40.7\%) = 16.1\%$$

Next, we calculate the variance using Eq. (7):

$$\begin{aligned} Var(R) &= \frac{1}{T-1} \sum_t (R_t - \bar{R})^2 \\ &= \frac{1}{10-1} [(-44.4\% - 16.1\%)^2 + (60.5\% - 16.1\%)^2 + \dots + (27.5\% - 16.1\%)^2] \\ &= 9.17\% \end{aligned}$$

The standard deviation is therefore

$$SD(R) = \sqrt{Var(R)} = \sqrt{9.17\%} = 30.28\%$$

4. **(Alternative Problem for Example 4 in Week 9 Notes)**

Using the data from Problem 3, what is the 95% confidence interval you would estimate for Microsoft's expected return?

Solution)

The 95% confidence interval for Microsoft's expected return is calculated as follows:

$$16.1\% \pm 2 \left(\frac{30.28\%}{\sqrt{10}} \right) = 16.1\% \pm 19.1\%$$

Or a range from -3.0% to 35.3% .

5. **(Alternative Problem for Example 8 in Week 9 Notes)**

Suppose the market portfolio tends to increase by 52% when the economy is strong and decline by 21% when the economy is weak. What is the beta of a type S firm whose return is 55% on average when the economy is strong and -24% when the economy is weak? What is the beta of a type I firm that bears only idiosyncratic, firm-specific risk?

Solution)

- The systematic risk of the strength of the economy produces a $52\% - (-21\%) = 73\%$ change in the return of the market portfolio.
- The type S firm's return changes by $55\% - (-24\%) = 79\%$ on average.
- Thus the firm's beta is $\beta_s = \frac{79\%}{73\%} = 1.082$.
- That is, each 1% change in the return of the market portfolio leads to a 1.082% change in the type S firm's return on average.
- The return of a type I firm has only firm-specific risk, however, and so is not affected by the strength of the economy. Its return is affected only by factors specific to the firm.
- Because it will have the same expected return, whether the economy is strong or weak,
$$\beta_1 = \frac{0\%}{73\%} = 0.$$

6. (Alternative Problem for Example 9 in Week 9 Notes)

Assume the economy has a 60% chance of the market return will be 15% next year and a 40% chance the market return will be 5% next year. Assume the risk-free rate is 6%. If Microsoft's beta is 1.18, what is its expected return next year?

Solution)

$$E[R_{Mkt}] = (60\% \times 15\%) + (40\% \times 5\%) = 11\%$$

$$E[R] = r_f + \beta \times (E[R_{Mkt}] - r_f)$$

$$E[R] = 6\% + 1.18(11\% - 6\%)$$

$$E[R] = 6\% + 5.9\% = 11.9\%$$

7. You bought a stock one year ago for \$50 per share and sold it today for \$55 per share. It paid a \$1 per share dividend today.
- a. What was your realized return?
 - b. How much of the return came from dividend yield and how much came from capital gain?

Solution)

Compute the realized return and dividend yield on this equity investment.

a.

$$R = \frac{1 + (55 - 50)}{50} = 0.12 = 12\%$$

b.

$$R_{div} = \frac{1}{50} = 2\%$$

$$R_{capital\ gain} = \frac{55 - 50}{50} = 10\%$$

The realized return on the equity investment is 12%. The dividend yield is 10%.

8. Consider an economy with two types of firms, S and I. S firms all move together. I firms move independently. For both types of firms, there is a 60% probability that the firms will have a 15% return and a 40% probability that the firms will have a -10% return. What is the volatility (standard deviation) of a portfolio that consists of an equal investment in 20 firms of (a) type S, and (b) type I?

Solution)

a. $E[R] = 0.15(0.6) - 0.1(0.4) = 0.05$

$$Standard\ Deviation = \sqrt{(0.15 - 0.05)^2 0.6 + (-0.10 - 0.05)^2 0.4} = 0.12247$$

Because all S firms in the portfolio move together there is no diversification benefit. So the standard deviation of the portfolio is the same as the standard deviation of the stocks—12.25%.

b. $E[R] = 0.15(0.6) - 0.1(0.4) = 0.05$

$$Standard\ Deviation = \sqrt{(0.15 - 0.05)^2 0.6 + (-0.10 - 0.05)^2 0.4} = 0.12247$$

Type I stocks move independently. Hence the standard deviation of the portfolio is

$$SD(\text{Portfolio of 20 Type I stocks}) = \frac{0.12247}{\sqrt{20}} = 2.74\%$$