
 Marwadi University Marwadi Chandarana Group 	Marwadi University Faculty of Engineering & Technology Department of Information and Communication Technology	
Subject: Programming With Python (01CT1309)	Aim: Analysis of LTI System Responses to Standard Inputs Using Python	
Experiment No: 19	Date:	Enrollment No: 92510133025

Aim: Analysis of LTI System Responses to Standard Inputs Using Python

IDE:

Analyzing Discrete-Time Systems Using Z-Transform

The Z-transform is used for analyzing discrete-time signals and systems. The Z-transform of a discrete-time signal $x[n]$ is given by:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

where z is a complex variable, $X(z)$ represents the Z-transform of the signal.

Z-Transform Function

For an LTI system, the Z-transform function $H(z)$ is defined as:

$$H(z) = \frac{N(z)}{D(z)} = \frac{N_0 + N_1 z^{-1} + \dots + N_M z^{-M}}{D_0 + D_1 z^{-1} + \dots + D_N z^{-N}}$$

where $N(z)$ is the numerator polynomial, $D(z)$ is the denominator polynomial.

Stability

A discrete-time system is stable if all poles of its Z-transfer function lie inside the unit circle in the Z-plane. To check stability:

Calculate the poles of $H(z)$



Check if the magnitude of each pole is less than 1.

Causality

A system is causal if its impulse response $h[n]$ is zero $n < 0$. This generally means that the numerator polynomial should not have terms that depend on future values.

Time Invariance

A system is time-invariant if a time shift in the input results in an equivalent time shift in the output. For LTI systems, if the system is defined properly, it is generally assumed to be time-invariant.

 Marwadi University Marwadi Chandarana Group 	Marwadi University Faculty of Engineering & Technology Department of Information and Communication Technology	
Subject: Programming With Python (01CT1309)	Aim: Analysis of LTI System Responses to Standard Inputs Using Python	
Experiment No: 19	Date:	Enrollment No: 92510133025

Example

$$H(z) = \frac{(z^2 + 0.5)}{(z^2 - 1.5z + 0.5)}$$

Bode Plot Analysis

Stability:

- Check the gain and phase margins.
- Ensure that both margins are positive for stability.

Causality:

- Examine the magnitude and phase at low frequencies.
- Confirm that the system behaves as a causal system (magnitude starts lower, phase starts near 0 and decreases).

Time Invariance:

- If the system is LTI, it is inherently time-invariant.
- Analyse the impulse response (if available) to verify consistent responses to delayed inputs.

Python Implementation

```
import numpy as np
```



```
import matplotlib.pyplot as plt
```

```
from scipy.signal import TransferFunction, lti
```

```
def analyze_z_transfer_function(num, den):
```

```
    # Create a Transfer Function object
```

```
    system = TransferFunction(num, den)
```

 Marwadi University Marwadi Chandarana Group 	Marwadi University Faculty of Engineering & Technology Department of Information and Communication Technology	
Subject: Programming With Python (01CT1309)	Aim: Analysis of LTI System Responses to Standard Inputs Using Python	
Experiment No: 19	Date:	Enrollment No: 92510133025

```
# Get the poles and zeros

zeros = system.zeros

poles = system.poles

print("Zeros:", zeros)

print("Poles:", poles)

# Stability Analysis

stable = all(np.abs(pole) < 1 for pole in poles)

print("Stability:", "Stable" if stable else "Unstable")

# Causality Analysis

causal = all(num[i] == 0 for i in range(len(num) - 1) if num[i + 1] == 0)

print("Causality:", "Causal" if causal else "Non-Causal")

# Time Invariance Analysis

time_invariant = True # For Z-transforms, generally time-invariant if system defined properly

print("Time Invariance:", "Time Invariant" if time_invariant else "Time Variant")

# Bode plot (magnitude and phase)



w, mag, phase = bode(system)

# Plot Bode plot

plt.figure(figsize=(12, 8))

plt.subplot(2, 1, 1)

plt.semilogx(w, mag) # Bode magnitude plot
```

 Marwadi University Marwadi Chandarana Group 	Marwadi University Faculty of Engineering & Technology Department of Information and Communication Technology	
Subject: Programming With Python (01CT1309)	Aim: Analysis of LTI System Responses to Standard Inputs Using Python	
Experiment No: 19	Date:	Enrollment No: 92510133025

```
plt.title('Bode Magnitude Plot')
```

```
plt.xlabel('Frequency [rad/s]')
```

```
plt.ylabel('Magnitude [dB]')
```

```
plt.grid()
```

```
plt.subplot(2, 1, 2)
```

```
plt.semilogx(w, phase) # Bode phase plot
```

```
plt.title('Bode Phase Plot')
```

```
plt.xlabel('Frequency [rad/s]')
```

```
plt.ylabel('Phase [degrees]')
```

```
plt.grid()
```

```
plt.tight_layout()
```

```
plt.show()
```



```
# Example: Analyzing a specific system  $H(z) = (z^2 + 0.5)/(z^2 - 1.5z + 0.5)$ 
```

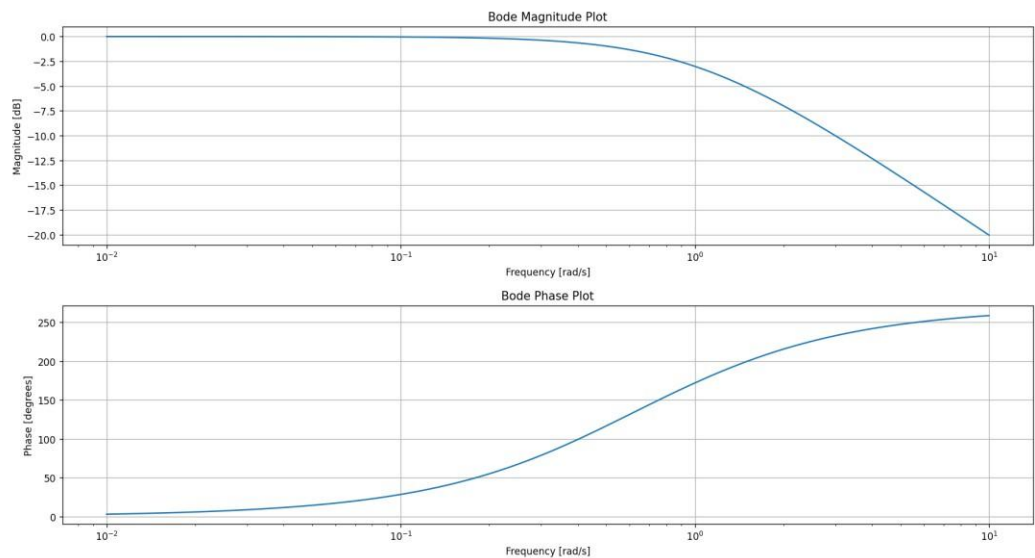
```
num = [1, 0.5] # Numerator coefficients
```

```
den = [1, -1.5, 0.5] # Denominator coefficients
```

```
analyze_z_transfer_function(num, den)
```

```
Zeros: [-0.5]
Poles: [1.  0.5]
Stability: Unstable
Casuality: Casual
Time invariance: Time Invariant
PS E:\python codes> □
```

 Marwadi University Marwadi Chandarana Group 	Marwadi University Faculty of Engineering & Technology Department of Information and Communication Technology	
Subject: Programming With Python (01CT1309)	Aim: Analysis of LTI System Responses to Standard Inputs Using Python	
Experiment No: 19	Date:	Enrollment No: 92510133025



Transfer Function:

$$H(z) = \frac{0.5}{1 - 0.8z^{-1}}$$

Causality: This system is causal because the denominator has a non-negative exponent (i.e., all powers of z^{-1} are non-negative).

Stability: The system is stable if the poles (the roots of the denominator) lie inside the unit circle. Here, the pole is $z = 0.8$, which is inside the unit circle, so the system is stable.



Time Invariance: The system is time-invariant because the coefficients do not depend on time.
Transfer function:

$$H(z) = \frac{1 - 0.5z^{-1}}{1 - 0.5z^{-1}}$$

Causality: This system is causal.

Stability: The pole at $z = 0.5$ is inside the unit circle, making the system stable.

Time Invariance: The system is time-invariant

 Marwadi University Marwadi Chandarana Group 	Marwadi University Faculty of Engineering & Technology Department of Information and Communication Technology	
Subject: Programming With Python (01CT1309)	Aim: Analysis of LTI System Responses to Standard Inputs Using Python	
Experiment No: 19	Date:	Enrollment No: 92510133025

Post Lab Exercise:

- Write a Python function to compute the Z-transform of an unit step function. verify whether the system is stable or unstable.

Code:

```
import sympy as sp

import numpy as np

import matplotlib.pyplot as plt

n = 10

z_transform = [1] * n

poles = np.roots(z_transform)

is_stable = True

for pole in poles:

    if abs(pole) >= 1:

        is_stable = False

        break

print("Is the system stable?", is_stable)



plt.figure(figsize=(6, 6))

plt.plot(np.real(poles), np.imag(poles), 'x')

plt.xlabel('Real axis')

plt.ylabel('Imaginary axis')

plt.title('Poles of the Z-transform')
```

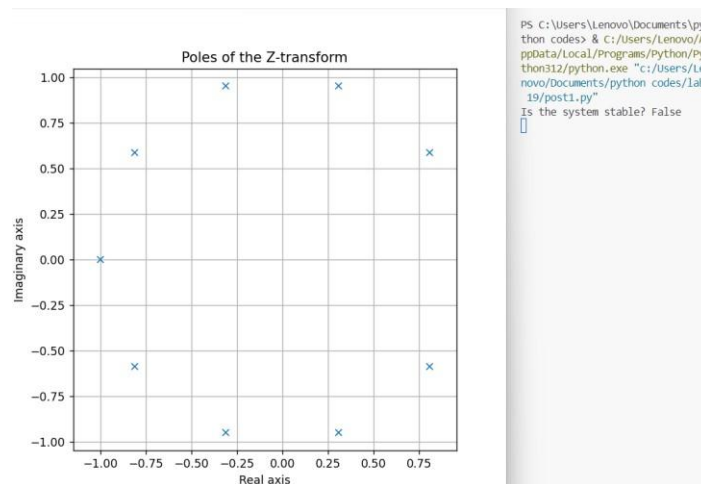
 Marwadi University Marwadi Chandarana Group 	Marwadi University Faculty of Engineering & Technology Department of Information and Communication Technology	
Subject: Programming With Python (01CT1309)	Aim: Analysis of LTI System Responses to Standard Inputs Using Python	
Experiment No: 19	Date:	Enrollment No: 92510133025

```
plt.grid(True)
```

```
plt.axis('equal')
```

```
plt.show()
```

Output:



- Implement this for the system $H(z) = \frac{0.5(z - 0.7)(z - 0.9)}{(z - 0.6)(z - 0.4)}$ and verify whether the system is stable or unstable.

Code:

```
import numpy as np
```



```
import matplotlib.pyplot as plt
```

```
num_coeffs = [0.5, -0.65, 0.315]
```

```
den_coeffs = [1, -1, 0.24]
```

```
def H_z(z):
```

```
    num = np.polyval(num_coeffs[::-1], z)
```

 Marwadi University Marwadi Chandarana Group 	Marwadi University Faculty of Engineering & Technology Department of Information and Communication Technology	
Subject: Programming With Python (01CT1309)	Aim: Analysis of LTI System Responses to Standard Inputs Using Python	
Experiment No: 19	Date:	Enrollment No: 92510133025

```

den = np.polyval(den_coeffs[::-1], z)

return num / den

def find_roots(coeffs):

    return np.roots(coeffs)

poles = find_roots(den_coeffs)

print("Poles:", poles)

stable = all(np.abs(pole) < 1 for pole in poles)

print("System is", "stable" if stable else "unstable")

w = np.linspace(0, np.pi, 1000)

z = np.exp(1j * w) H =

H_z(z)

plt.figure(figsize=(12, 6))

plt.subplot(121)

plt.plot(w, np.abs(H))

plt.xlabel('Frequency (rad/sample)')

plt.ylabel('Magnitude')

plt.title('Magnitude Response')



plt.grid()

plt.subplot(122) plt.plot(w,

np.angle(H)) plt.xlabel('Frequency

(rad/sample)')

```


 Marwadi University Marwadi Chandarana Group 	Marwadi University Faculty of Engineering & Technology Department of Information and Communication Technology	
Subject: Programming With Python (01CT1309)	Aim: Analysis of LTI System Responses to Standard Inputs Using Python	
Experiment No: 19	Date:	Enrollment No: 92510133025

```
plt.ylabel('Phase (rad)')

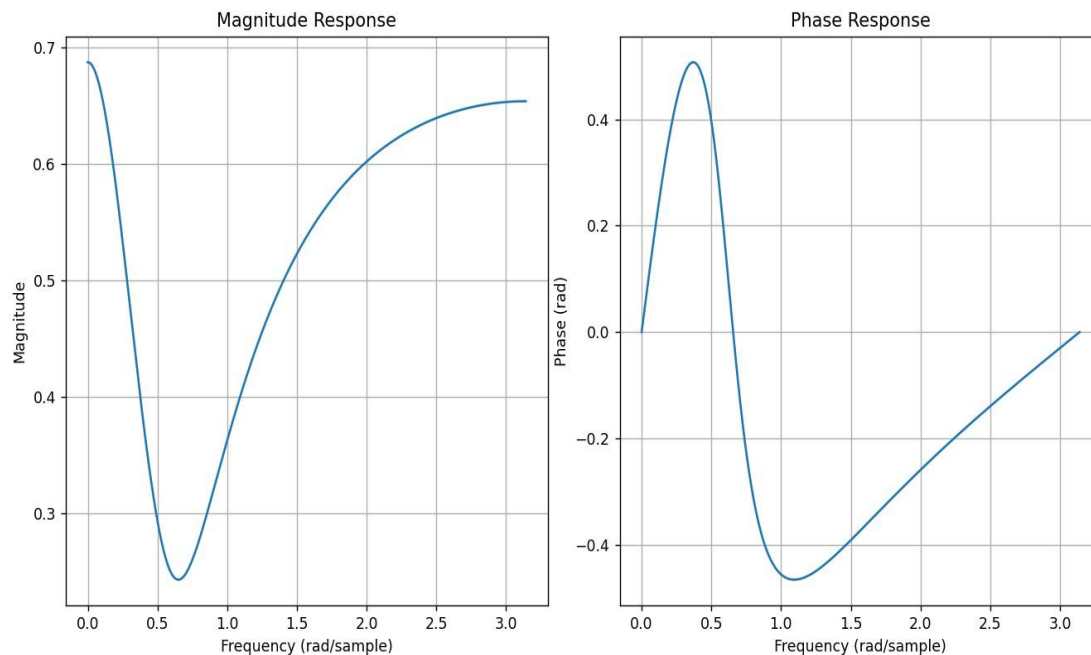
plt.title('Phase Response')

plt.grid()

plt.tight_layout()

plt.show()
```

Output:



```
PS C:\Users\Lenovo\Document
:/Users/Lenovo/AppData/Local
python312/python.exe "c:/Use
/python codes/lab 19/post2.
Poles: [0.6 0.4]
System is stable
```

Github link: https://github.com/JenishDesai5115/PWP_postlabs