

## 0.1 Metabolic and coinfection models

### 0.1.1 Full coinfection model

$$\begin{aligned}
\frac{dB}{dt} &= \underbrace{r\left(1 - \frac{N}{K}\right)B}_{\text{growth}} - \underbrace{dBP}_{\text{infection}} + \underbrace{sL}_{\text{cure}} \\
\frac{dP}{dt} &= \underbrace{c\mu_p(B, L)I_p}_{\text{burst function}} - \underbrace{dBP}_{\text{Infection}} - \underbrace{dI_nP}_{\text{coinfection}} - \underbrace{mP}_{\text{viral decay}} - \underbrace{\chi dPL}_{\text{lys. immunity}} \\
\frac{dI_n}{dt} &= \underbrace{dBP}_{\text{infection}} - \underbrace{dI_nP}_{\text{coinfection}} - \underbrace{\mu_{ld}(B, L)I_n}_{\text{Infected to } \Phi \text{ prod.}} \\
\frac{dI_p}{dt} &= \underbrace{\mu_{ld}(B, L)I_n}_{\text{Infected to } \Phi \text{ prod.}} - \underbrace{\mu_p(B, L)I_p}_{\text{lysed cells}} + \underbrace{\mu_i L}_{\text{induction}} \\
\frac{dL}{dt} &= \underbrace{r\left(1 - \frac{N}{K}\right)L}_{\text{growth}} + \underbrace{dI_nP}_{\text{coinfection}} - \underbrace{\mu_i L}_{\text{induction}} - \underbrace{sL}_{\text{cure}}
\end{aligned}$$

### 0.1.2 Basic coinfection model

$$\begin{aligned}
\frac{dB}{dt} &= \underbrace{r\left(1 - \frac{N}{K}\right)B}_{\text{growth}} - \underbrace{dBP}_{\text{infection}} \\
\frac{dP}{dt} &= \underbrace{c\mu_p(B, L)I_p}_{\text{burst function}} - \underbrace{dBP}_{\text{Infection}} - \underbrace{dI_nP}_{\text{coinfection}} - \underbrace{mP}_{\text{viral decay}} \\
\frac{dI_n}{dt} &= \underbrace{dBP}_{\text{infection}} - \underbrace{dI_nP}_{\text{coinfection}} - \underbrace{\mu_{ld}(B, L)I_n}_{\text{Infected to } \Phi \text{ prod.}} \\
\frac{dI_p}{dt} &= \underbrace{\mu_{ld}(B, L)I_n}_{\text{Infected to } \Phi \text{ prod.}} - \underbrace{\mu_p(B, L)I_p}_{\text{lysed cells}} + \underbrace{\mu_i L}_{\text{induction}} \\
\frac{dL}{dt} &= \underbrace{r\left(1 - \frac{N}{K}\right)L}_{\text{growth}} + \underbrace{dI_nP}_{\text{coinfection}} - \underbrace{\mu_i L}_{\text{induction}}
\end{aligned}$$

### 0.1.3 Metabolic model

$$\begin{aligned}
\frac{dB}{dt} &= \underbrace{rH_{O_2}H_{DOC}H_{eDAR}B}_{\text{growth}} - \underbrace{dBP}_{\text{Infection}} \\
\frac{dP}{dt} &= \underbrace{c\mu_p[1 - \mathcal{P}(L)]I_n}_{\text{burst}} - \underbrace{dBP}_{\text{infection}} - \underbrace{mP}_{\text{viral decay}} + \underbrace{c\mu_i L}_{\text{burst induction}} \\
\frac{dI_n}{dt} &= \underbrace{dBP}_{\text{Infection}} - \underbrace{[1 - \mathcal{P}(L)]I_n}_{\text{lysis}} - \underbrace{\mathcal{P}(L)I_n}_{\text{lysogeny}} \\
\frac{dL}{dt} &= \underbrace{rH_{O_2}H_{DOC}H_{eDAR}L}_{\text{growth}} + \underbrace{\mathcal{P}(L)I_n}_{\text{lysogeny}} - \underbrace{\mu_i L}_{\text{induction}}
\end{aligned}$$