

Homework 14 (30/09/24)

$$\text{Info}(D) = I(2,3) = -\frac{2}{5} \log_2 \left(\frac{2}{5} \right) - \frac{3}{5} \log_2 \left(\frac{3}{5} \right) = 0.5288 + 0.4922 = 0.971$$

$$\begin{aligned} \text{Info}_{\text{income}}(D) &= \frac{2}{5} I(0,2) + \frac{1}{5} I(1,1) + \frac{1}{5} I(1,0) \\ &= \frac{2}{5} \left[-\frac{0}{2} \log_2 \left(\frac{0}{2} \right) - \frac{2}{2} \log_2 \left(\frac{2}{2} \right) \right] + \frac{1}{5} \left[-\frac{1}{1} \log_2 \left(\frac{1}{1} \right) - \frac{1}{1} \log_2 \left(\frac{1}{1} \right) \right] \\ &\quad + \frac{1}{5} \left[-\frac{1}{1} \log_2 \left(\frac{1}{1} \right) - \frac{0}{1} \log_2 \left(\frac{0}{1} \right) \right] \\ &= 0 + \frac{2}{5} (0.5 + 0.5) + 0 = 0.40 \end{aligned}$$

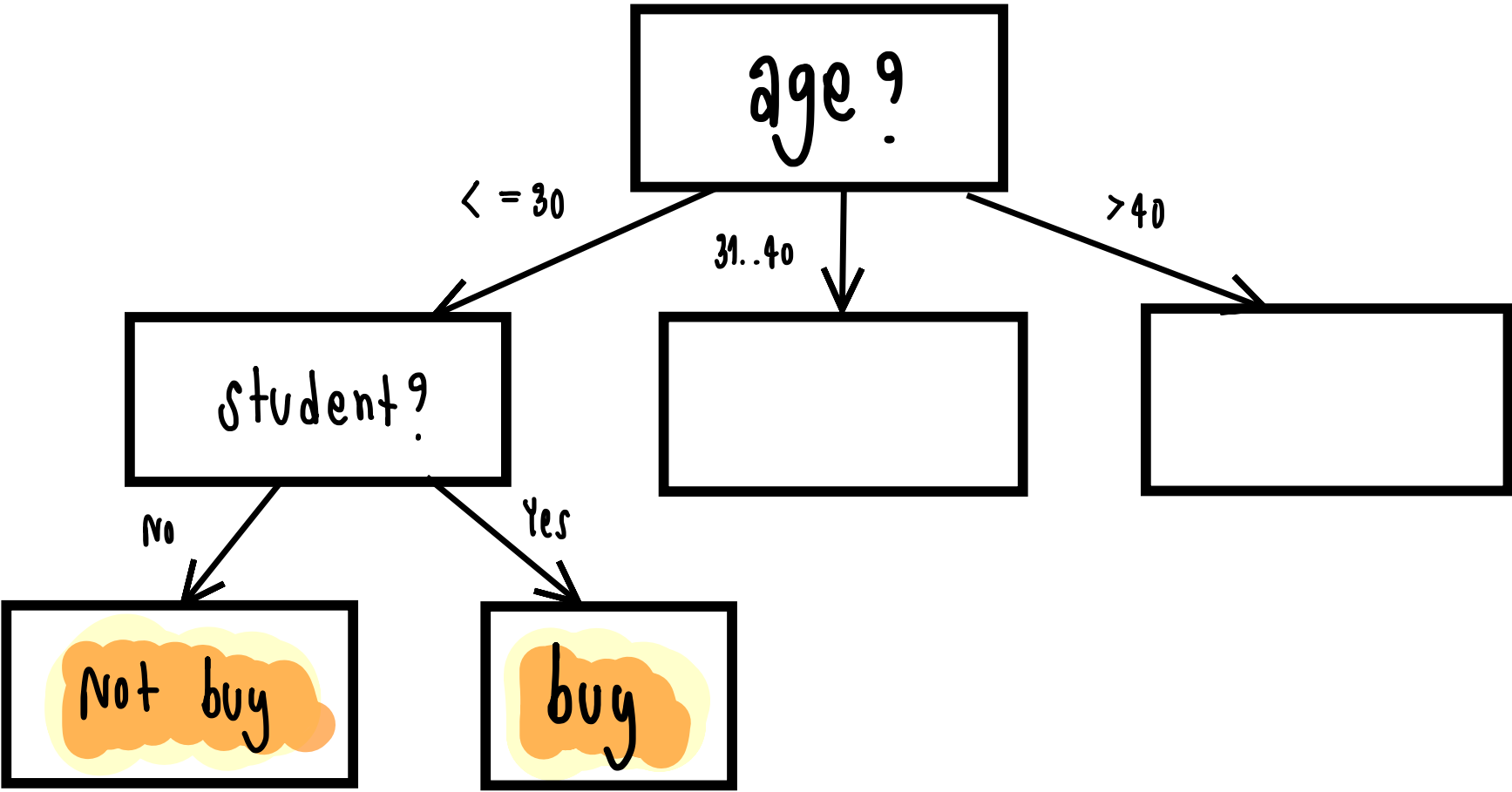
$$\begin{aligned} \text{Info}_{\text{student}}(D) &= \frac{2}{5} I(2,0) + \frac{3}{5} I(0,3) \\ &= \frac{2}{5} \left[-\frac{2}{2} \log_2 \left(\frac{2}{2} \right) - \frac{0}{2} \log_2 \left(\frac{0}{2} \right) \right] + \frac{3}{5} \left[-\frac{0}{3} \log_2 \left(\frac{0}{3} \right) - \frac{3}{3} \log_2 \left(\frac{3}{3} \right) \right] \end{aligned}$$

$$\begin{aligned} \text{Info}_{\text{credit}}(D) &= \frac{3}{5} I(1,2) + \frac{2}{5} I(1,1) \\ &= \frac{3}{5} \left[-\frac{1}{3} \log_2 \left(\frac{1}{3} \right) - \frac{2}{3} \log_2 \left(\frac{2}{3} \right) \right] + \frac{2}{5} \left[-\frac{1}{2} \log_2 \left(\frac{1}{2} \right) - \frac{1}{2} \log_2 \left(\frac{1}{2} \right) \right] \\ &= \frac{3}{5} \left[(-0.3333)(-1.5851) + (0.6667)(0.5849) \right] + \frac{2}{5} \left[(-0.5)(0.1) + (-0.5)(-1) \right] \\ &= 0.5510 + 0.4 = 0.9510 \end{aligned}$$

$$\text{Gain}(\text{income}) = \text{Info}(D) - \text{Info}_{\text{income}}(D) = 0.971 - 0.40 = 0.571$$

$$\text{Gain}(\text{student}) = \text{Info}(D) - \text{Info}_{\text{student}}(D) = 0.971 - 0 = 0.971$$

$$\text{Gain}(\text{credit}) = \text{Info}(D) - \text{Info}_{\text{credit}}(D) = 0.971 - 0.9510 = 0.02$$



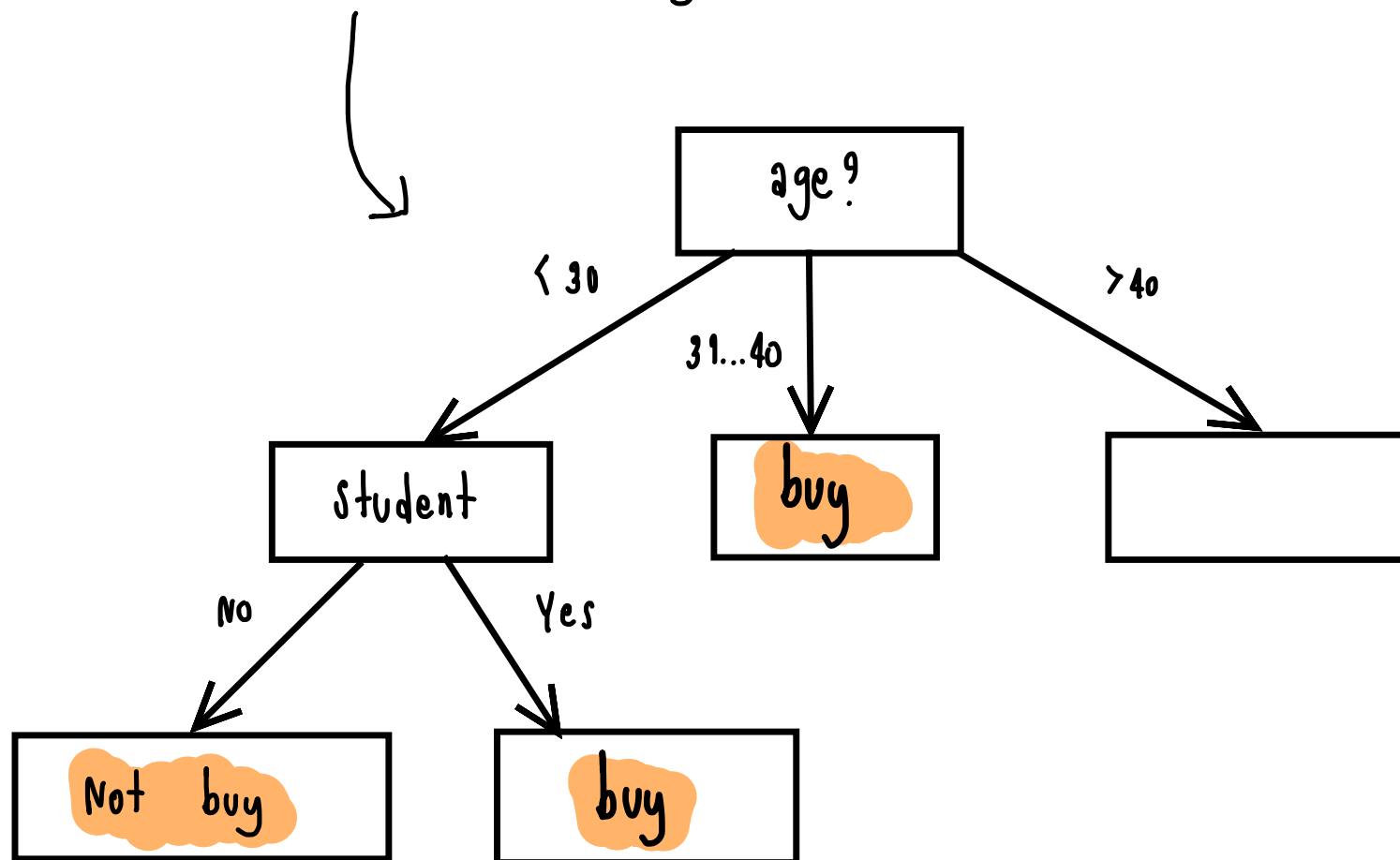
$$\text{Info}(D) = I(4,0) = -\frac{4}{4} \log_2\left(\frac{4}{4}\right) - \frac{0}{4} \log_2\left(\frac{0}{4}\right) = 0$$

$$\begin{aligned} \text{Info}_{\text{income}}(D) &= \frac{2}{4} I(2,0) + \frac{1}{4} I(1,0) + \frac{1}{4} I(1,0) \\ &= \frac{2}{4} \left[-\frac{2}{0} \log_2\left(\frac{2}{0}\right) - \frac{0}{2} \log_2\left(\frac{0}{2}\right) \right] + \frac{1}{4} \left[-\frac{1}{0} \log_2\left(\frac{1}{0}\right) - \frac{0}{1} \log_2\left(\frac{0}{1}\right) \right] \\ &\quad + \frac{1}{4} \left[-\frac{1}{0} \log_2\left(\frac{1}{0}\right) - \frac{0}{1} \log_2\left(\frac{0}{1}\right) \right] = 0 \end{aligned}$$

$$\begin{aligned} \text{Info}_{\text{credit}}(D) &= \frac{2}{4} I(2,0) + \frac{2}{4} I(2,0) \\ &= \frac{2}{4} \left[-\frac{2}{0} \log_2\left(\frac{2}{0}\right) - \frac{0}{2} \log_2\left(\frac{0}{2}\right) \right] + \frac{2}{4} \left[-\frac{2}{0} \log_2\left(\frac{2}{0}\right) - \frac{0}{2} \log_2\left(\frac{0}{2}\right) \right] \end{aligned}$$

$$\text{Gain}(\text{income}) = \text{Info}(D) - \text{Info}_{\text{income}}(D) = 0 - 0 = 0$$

$$\text{Gain}(\text{credit}) = \text{Info}(D) - \text{Info}_{\text{credit}}(D) = 0 - 0 = 0$$



$$\text{Info}(D) = I(3,2) = -\frac{3}{5} \log_2 \left(\frac{3}{5} \right) - \frac{2}{5} \log_2 \left(\frac{2}{5} \right) = 0.4422 + 0.5288 = 0.9710$$

$$\begin{aligned} \text{Info}_{\text{income}}(D) &= \frac{0}{5} I(0,0) + \frac{3}{5} I(2,1) + \frac{2}{5} I(1,1) \\ &= \frac{3}{5} \left[-\frac{2}{5} \log_2 \left(\frac{2}{5} \right) - \frac{1}{3} \log_2 \left(\frac{1}{3} \right) \right] + \frac{2}{5} \left[-\frac{1}{2} \log_2 \left(\frac{1}{2} \right) - \frac{1}{2} \log_2 \left(\frac{1}{2} \right) \right] \\ &= 0.5510 + 0.4 = 0.9510 \end{aligned}$$

$$\begin{aligned} \text{Info}_{\text{credit}}(D) &= \frac{3}{5} I(3,0) + \frac{2}{5} I(0,2) \\ &= \frac{3}{5} \left[-\frac{3}{5} \log_2 \left(\frac{3}{5} \right) - \frac{0}{3} \log_2 \left(\frac{0}{3} \right) \right] + \frac{2}{5} \left[-\frac{0}{2} \log_2 \left(\frac{0}{2} \right) - \frac{2}{0} \log_2 \left(\frac{2}{0} \right) \right] \\ &= 0 \end{aligned}$$

$$\text{Gain}(\text{income}) = \text{Info}(D) - \text{Info}_{\text{income}}(D) = 0.9710 - 0.9510$$

$$\text{Gain}(\text{credit}) = \text{Info}(D) - \text{Info}_{\text{credit}}(D) = 0.9710 - 0 = 0.9710$$

