Derivatives

共享题干题

【题干】Sonal Johnson is a risk manager for a bank. She manages the bank's risks using a combination of swaps and forward rate agreements (FRAs).Johnson prices a three-year Libor-based interest rate swap with annual resets using the present value factors presented in Exhibit 1.

Exhibit 1. Present Value Factors

Maturity (years)	Present Value Factors
1	0.990099
2	0.977876
3	0.965136

Johnson also uses the present value factors in Exhibit 1 to value an interest rate swap that the bank entered into one year ago as the pay-fixed (receive-floating) party. Selected data for the swap are presented in Exhibit 2. Johnson notes that the current equilibrium two-year fixed swap rate is 1.12%.

Exhibit 2. Selected Data on Fixed for Floating Interest Rate Swap

Swap notional amount	$50,000,000
Original swap term	Three years, with annual resets
Fixed swap rate (since initiation)	3.00%

One of the bank's investments is exposed to movements in the Japanese yen, and Johnson desires to hedge the currency exposure. She prices a one-year fixed-for-fixed currency swap involving yen and US dollars, with a quarterly reset. Johnson uses the interest rate data presented in Exhibit 3 to price the currency swap.

Exhibit 3. Selected Japanese and US Interest Rate Data

Days to Maturity	Yen Spot Interest Rates	US Dollar Spot Interest Rate
90	0.05%	0.20%
180	0.10%	0.40%
270	0.15%	0.55%
360	0.25%	0.70%

Johnson next reviews an equity swap with an annual reset that the bank entered into six months ago as the receive-fixed, pay-equity party. Selected data regarding the equity swap, which is linked to an equity index, are presented in Exhibit 4. At the time of initiation, the underlying equity index was trading at 100.00.

The equity index is currently trading at 103.00, and relevant US spot rates, along with their associated present value factors, are presented in Exhibit 5.

Exhibit 4. Selected Data on Equity Swap	
Swap notional amount	$20,000,000
Original swap term	Five years, with annual resets
Fixed swap rate	2.00%

Exhibit 5. Selected US Spot Rates and Present Value Factors

Maturity (years)	Spot Rate	Present Value Factors
0.5	0.40%	0.998004
1.5	1.00%	0.985222
2.5	1.20%	0.970874
3.5	2.00%	0.934579
4.5	2.60%	0.895255

Johnson reviews a 6×9 FRA that the bank entered into 90 days ago as the pay-fixed/receive-floating party. Selected data for the FRA are presented in Exhibit 6, and current Libor data are presented in Exhibit 7. Based on her interest rate forecast, Johnson also considers whether the bank should enter into new positions in 1 × 4 and 2 × 5 FRAs.

Exhibit 6. 6 × 9 FRA Data	
FRA term	6 × 9
FRA rate	0.70%
FRA notional amount	US$20,000,000
FRA settlement terms	Advanced set, advanced settle

Three months later, the 6 × 9 FRA in Exhibit 6 reaches expiration, at which time the three-month US dollar Libor is 1.10% and the six-month US dollar Libor is 1.20%. Johnson determines that the appropriate discount rate for the FRA settlement cash flows is 1.10%.

1.【单项选择题】Based on Exhibit 1, Johnson should price the three-year Libor-based interest rate swap at a fixed rate closest to:

A. 0.34%.

B. 1.16%.

C. 1.19%.

参考答案：C

【莽学解析】The swap pricing equation is

Exhibit 7. Current Libor	
30-day Libor	0.75%
60-day Libor	0.82%
90-day Libor	0.90%
120-day Libor	0.92%
150-day Libor	0.94%
180-day Libor	0.95%
210-day Libor	0.97%
270-day Libor	1.00%

$$r_{FIX}=\frac{1-PV_{0,t_n}(1)}{\sum_{i=1}^{n}PV_{0,t_i}(1)}$$

That is, the fixed swap rate is equal to 1 minus the final present value factor (in this case, Year 3) divided by the sum of the present values (in this case, the sum of Years 1, 2, and 3). The sum of present values for Years 1, 2, and 3 is calculated as

$$\sum_{i=1}^{n}PV_{0,t_i}(1)=0.990099+0.977876+0.965136=2.933111$$

Thus, the fixed-swap rate is calculated as

$$r_{FIX}=\frac{1-0.965136}{2.933111}=0.01189\ or\ 1.19\%$$

2.【单项选择题】From the bank's perspective, using data from Exhibit 1, the current value of the swap described in Exhibit 2 is closest to:

A. –$2,951,963.

B. –$1,849,897.

C. –$1,943,000.

参考答案：B

【莽学解析】The value of a swap from the perspective of the receive-fixed party is calculated as

$$V=NA(FS_0-FS_t)\sum_{i=1}^{n'}PV_{t,t_i}$$

The swap has two years remaining until expiration. The sum of the present val?ues for Years 1 and 2 is

$$\sum_{i=1}^{n'} PV_{t:t_i} = 0.990099 + 0.977876 = 1.967975$$

Given the current equilibrium two-year swap rate of 1.00% and the fixed swap rate at initiation of 3.00%, the swap value per dollar notional is calculated asV = (0.03 - 0.0112)×1.967975 = 0.0336998The current value of the swap, from the perspective of the receive-fixed party, is \$50,000,000 × 0.035998 = \$1,849,897.From the perspective of the bank, as the receive-floating party, the value of the swap is －\$1,849,897.

3.【单项选择题】Based on Exhibit 3, Johnson should determine that the annualized equilibrium fixed swap rate for Japanese yen is closest to:

A.0.0624%.

B.0.1375%.

C.0.2496%.

参考答案：C

【莽学解析】The equilibrium swap fixed rate for yen is calculated as

$$r_{FIX,JPY} = \frac{1 - PV_{0:t_4},JPY(1)}{\sum_{i=1}^{4} PV_{0:t_4},JPY(1)}$$

The yen present value factors are calculated as

$$PV_{0:t_i}(1) = \frac{1}{1 + r_{Spot_i}\left(\frac{NAD_i}{NTD}\right)}$$

90-day PV factor = 1/ [1 0.0005(90/360)] = 0.999875.180-day PV factor = 1/ [1 0.0010(180/360)] = 0.999500.270-day PV factor = 1/ [1 0.0015(270/360)] = 0.998876.360-day PV factor = 1/ [1 0.0025(360/360)] = 0.997506.Sum of present value factors = 3.995757.Therefore, the yen periodic rate is calculated as

$$r_{FIX,JPY} = \frac{1 - 0.997506}{3.995757} = 0.000624 \text{ or } 0.0624\%$$

The annualized rate is (360/90) times the periodic rate of 0.0624%, or 0.2496%.

4.【单项选择题】From the bank's perspective, using data from Exhibits 4 and 5, the fair value of the equity swap is closest to:

A. －\$1,139,425.

B. －\$781,323.

C. －\$181,323.

参考答案：B
【莽学解析】The value of an equity swap is calculated as

$$V_{EQ,t} = V_{FIX}(C_0) - (S_t/S_{t-1})NA_E - PV(Par - NA_E).$$

The swap was initiated six months ago, so the first reset has not yet passed; thus, there are five remaining cash flows for this equity swap. The fair value of the swap is determined by comparing the present value of the implied fixed-rate bond with the return on the equity index. The fixed swap rate of 2.00%, the swap notional amount of $20,000,000, and the present value factors in Exhibit 5 result in a present value of the implied fixed-rate bond’s cash flows of $19,818,677:

Date (in years)	PV Factors	Fixed Cash Flow	PV (fixed cash flow
0.5	0.998004 or 1/[1 + 0.0040(180/360)]	$400,000	$399,202
1.5	0.985222 or1 /[1 + 0.0100(540/360)]	$400,000	$394,089
2.5	0.970874 or 1/[1 + 0.0120(900/360)]	$400,000	$388,350
3.5	0.934579 or 1/[1 + 0.0200(1,260/360)]	$400,000	$373,832
4.5	0.895255 or 1/[1 + 0.0260(1,620/360)]	$20,400,000	$18,263,205
Total		$19,818,677	

The value of the equity leg of the swap is calculated as (103/100) ($20,000,000) = $20,600,000. Note the swap’s notional amount and the implied fixed-rate bond’s par value are both $20,000,000; therefore, the term – PV(Par – NA_E) reduces to zero.
The swap was designed to profit if rates fell or equities declined. Neither happened, so the swap value will be negative for the bank. The fair value of the equity swap, from the perspective of the bank (receive-fixed, pay-equity party) is calculated asV_{EQ} = $19,818,677 – $20,600,000 = －781,323

5.【单项选择题】Based on Exhibit 5, the current value of the equity swap described in Exhibit 4 would be zero if the equity index was currently trading the closest to:
A. 97.30.
B. 99.09.
C. 100.00.
参考答案：B
【莽学解析】The equity index level at which the swap’s fair value would be zero can be

calculated by setting the swap valuation formula equal to zero and solving for St:

$$V_{EQ,t} = V_{FIX}(C_0) - (S_t/S_{t-1})NA_E = 0.$$

The value of the fixed leg of the swap has a present value of $19,818,677, or 99.0934% of par value:

Date (years)	PV Factors	Fixed Cash Flow	PV (fixed cash flow)
0.5	0.998004	$400,000	$399,202
1.5	0.985222	$400,000	$394,089
2.5	0.970874	$400,000	$388,350
3.5	0.934579	$400,000	$373,832
4.5	0.895255	$20,400,000	$18,263,205
Total		$19,818,677	

Treating the swap notional value as par value and substituting the present value of the fixed leg and S0 into the equation yields

$$0 = 99.0934 - \left(\frac{St}{100}\right)100$$

Solving for *St* yields

$$St = 99.0934$$

6.【单项选择题】From the bank's perspective, based on Exhibits 6 and 7, the value of the 6 × 9 FRA 90 days after inception is closest to:

A. $14,817.

B. $19,647.

C. $29,635.

参考答案：A

【莽学解析】The current value of the 6 × 9 FRA is calculated as

$$V_g(0,h,m) = \{[FRA(g,h-g,m) - FRA(0,h,m)]t_m\}$$
$$/[1 + D_g(h+m-g)t_{h+m-g}]$$

The 6 × 9 FRA expires six months after initiation. The bank entered into the FRA 90 days ago; thus, the FRA will expire in 90 days. To value the FRA, the first step is to compute the new FRA rate, which is the rate on Day 90 of an FRA that expires in 90 days in which the underlying is the 90-day Libor, or FRA (90,90,90):

$$FRA(g,h-g,m)$$
$$=\left\{\left[1+Lg(h-g+m)t_{h-g+m}\right]/\left[1+L_0(h-g)t_{h-g}\right]-1\right\}/t_m$$
$$FRA(90,90,90)$$
$$=\left\{\begin{matrix}\left[1+L_{90}(180-90+90)(180/360)\right]\div\\ \left[1+L_{90}(180-90)(90/360)\right]-1\end{matrix}\right\}/(90/360)$$
$$FRA(90,90,90)$$
$$=\left\{\left[1+L_{90}(180)(180/360)\right]/\left[1+L_{90}(90)(90/360)\right]-1\right\}$$
$$/(90/360)$$

Exhibit 7 indicates that L90 (180) = 0.95% and L90 (90) = 0.90%, soFRA (90, 90,90) = {[1 0.0095(180/360)]/[1 0.0090(90/360)] - 1}/(90/360)FRA (90, 90,90) = [(1.00475/1.00225) - 1](4) = 0.009978, or 0.9978%Therefore, given the FRA rate at initiation of 0.70% and notional principal of $20 million from Exhibit 1, the current value of the forward contract is calculated asVg (0,h,m) = V90(0,180,90)V90 (0,180,90) = $20,000,000[(0.009978 - 0.0070)(90/360)]/[1 0.0095(180/360)].V90 (0,180,90) = $14,887.75/1.00475 = $14,817.37.

7.【单项选择题】Based on Exhibit 7, the no-arbitrage fixed rate on a new 1 × 4 FRA is closest to:

A.0.65%.

B.0.73%.

C.0.98%.

参考答案：C

【莽学解析】The no-arbitrage fixed rate on the 1 × 4 FRA is calculated as

$$FRA(0,h,m)=\left\{\left[1+L_0(h+m)t_{h+m}\right]/\left[1+L_0(h)t_h\right]-1\right\}/t_m$$

For a 1 × 4 FRA, the two rates needed to compute the no-arbitrage FRA fixed rate are L(30) = 0.75% and L(120) = 0.92%. Therefore, the no-arbitrage fixed rate on the 1 × 4 FRA rate is calculated asFRA (0, 30, 90) = {[1 0.0092(120/360)]/[1 0.0075(30/360)] - 1}/(90/360).FRA (0,30,90) = [(1.003066/1.000625) - 1]4 = 0.009761, or 0.98% rounded.

8.【单项选择题】Based on Exhibit 7, the fixed rate on a new 2 × 5 FRA is closest to:

A.0.61%.

B.1.02%.

C.1.71%.

参考答案：B

【莽学解析】The fixed rate on the 2 × 5 FRA is calculated as

For a 2 × 5 FRA, the two rates needed to compute the no-arbitrage FRA fixed rate are L(60) =

$$FRA(0,h,m)=\{[1+L_0(h+m)t_{h+m}]/[1+L_0(h)t_h]-1\}/t_m$$

0.82% and L(150) = 0.94%. Therefore, the no-arbitrage fixed rate on the 2 × 5 FRA rate is calculated asFRA (0,60,90) = {[1 0.0094(150/360)]/[1 0.0082(60/360)] - 1}/(90/360)FRA (0,60,90) = [(1.003917/1.001367) - 1]4 = 0.010186, or 1.02% rounded

9.【单项选择题】Based on Exhibit 6 and the three-month US dollar Libor at expiration, the payment amount that the bank will receive to settle the 6 × 9 FRA is closest to:

A.$19,945.

B.$24,925.

C.$39,781.

参考答案：A

【莽学解析】Given a three-month US dollar Libor of 1.10% at expiration, the settlement amount for the bank as the receive-floating party is calculated as

$$Settlement\ amount\ (receive\ floating)$$

$$=NA\{[L_h(m)-FRA(0,h,m)]t_m\}/[1+D_h(m)t_m]$$

$$Settlement\ amount\ (receive\ floating)$$

$$=\$20,000,000[(0.011-0.0070)(90/360)]/[1+0.011(90/360)]$$

$$Settlement\ amount\ (receive\ floating)$$

$$=\$20,000/1.00275=\$19,945.15$$

Therefore, the bank will receive $19,945 (rounded) as the receive-floating party.

【题干】Trident Advisory Group manages assets for high-net-worth individuals and family trusts.Alice Lee, chief investment officer, is meeting with a client, Noah Solomon, to discuss risk management strategies for his portfolio. Solomon is concerned about recent volatility and has asked Lee to explain options valuation and the use of options in risk management.Options on StockLee begins: “We use the Black-Scholes-Merton (BSM) model for option valuation. To fully understand the BSM model valuation, one needs to understand the assumptions of the model. These assumptions include normally distributed stock returns, constant volatility of return on the underlying, constant interest rates, and continuous prices” Lee uses the BSM model to price TCB, which is one of Solomon's holdings. Exhibit 1 provides the current stock price (S), exercise price (X), risk-free interest rate (r), volatility (σ), and time to expiration (T) in years as well as selected outputs from the BSM model. TCB does not pay a dividend.
Options on FuturesThe Black model valuation and selected outputs for options on another of Solomon's holdings, the GPX 500 Index (GPX), are shown in Exhibit 2. The spot index level for the GPX is 187.95, and the index is assumed to pay a continuous dividend at a rate of 2.2% (5) over the life of the options being valued, which expire in 0.36 years. A futures contract on the GPX also expiring in 0.36 years is currently priced at 186.73.

Exhibit 1 BSM Model for European Options on TCB

BSM Inputs

S	X	r	Σ	T
$57.03	55	0.22%	32%	0.25

BSM Outputs

d_1	$N(d_1)$	d_2	$N(d_2)$	BSM Call Price	BSM Put Price
0.3100	0.6217	0.1500	0.5596	$4.695	$2.634

Exhibit 2. Black Model for European Options on the GPX Index

Black Model Inputs

GPX Index	X	r	σ	T	δ Yield
187.95	180	0.39%	24%	0.36	2.2%

Black Model Call Value	Black Model Put Value	Market Call Price	Market Put Price
$14.2089	$7.4890	$14.26	$7.20

Option Greeks

Delta (call)	Delta (put)	Gamma (call or put)	Theta (call) daily	Rho (call) per %	Vega per % (call or put)
0.6232	-0.3689	0.0139	-0.0327	0.3705	0.4231

After reviewing Exhibit 2, Solomon asks Lee which option Greek letter best describes the changes in an option's value as time to expiration declines. Solomon observes that the market price of the put option in Exhibit 2 is $7.20. Lee responds that she used the historical volatility of the GPX of 24% as an input to the BSM model, and she explains the implications for the implied volatility for the GPX.Options on Interest Rates

Solomon forecasts the three-month Libor will exceed 0.85% in six months and is considering using options to reduce the risk of rising rates. He asks Lee to value an interest rate call with a strike price of 0.85%. The current three-month Libor is 0.60%, and an FRA for a three-month Libor loan beginning in six months is currently 0.75%.

Hedging Strategy for the Equity Index

Solomon's portfolio currently holds 10,000 shares of an exchange-traded fund (ETF) that tracks the GPX. He is worried the index will decline. He remarks to Lee, "You have told me how the BSM model can provide useful information for reducing the risk of my GPX position" Lee suggests a delta hedge as a strategy to protect against small moves in the GPX Index.

Lee also indicates that a long position in puts could be used to hedge larger moves in the GPX. She notes that although hedging with either puts or calls can result in a delta-neutral position, they would need to consider the resulting gamma.

10.【单项选择题】Based on Exhibit 1 and the BSM valuation approach, the initial portfolio required to replicate the long call option payoff is:

A.Long 0.3100 shares of TCB stock and short 0.5596 shares of a zero-coupon bond.

B.Long 0.6217 shares of TCB stock and short 0.1500 shares of a zero-coupon bond.

C.Long 0.6217 shares of TCB stock and short 0.5596 shares of a zero-coupon bond.

参考答案：C

【莽学解析】The no-arbitrage approach to creating a call option involves buying Delta = N (d_1) = 0.6217 shares of the underlying stock and financing with -N (d_2) =-0.5596 shares of a risk-free bond priced at exp (-rt)(X) = exp(-0.0022 x 0.25) (55) = \$54.97 per bond. Note that the value of this replicating portfolio is $n_SS + n_BB$ = 0.6217(57.03) - 0.5596(54.97) = \$4.6943 (the value of the call option with slight rounding error).

11.【单项选择题】To determine the long put option value on TCB stock in Exhibit 1, the correct BSM valuation approach is to compute:

A.0.4404 times the present value of the exercise price minus 0.6217 times the price of TCB stock.

B.0.4404 times the present value of the exercise price minus 0.3783 times the price of TCB stock.

C.0.5596 times the present value of the exercise price minus 0.6217 times the price of TCB stock.

参考答案：B

【莽学解析】The formula for the BSM price of a put option isp = $e^{-rt}XN(-d_2)-SN(-d_1)$. N $(-d_1)$ = 1 - N(d_1) = 1 - 0.6217 = 0.3783, and N$(-d_2)$ = 1 - N(d_2) = 1 - 0.5596 = 0.4404.Note that the BSM model can be represented as a portfolio of the stock (n_SS) and zero-coupon bonds (N_BB). For a put, the number of shares is n_S = -N $(-d_1)$ < 0 and the number of bonds is n_B=-N (d_2) > 0. The value of the replicating portfolio is $n_SS + n_BB$ = -0.3783(57.03) + 0.4404(54.97) = \$2.6343 (the value of the put option with slight rounding error). B is a risk-free bond priced at exp (-rt)(X) = exp(-0.0022 x 0.25)(55) = \$54.97.

12.【单项选择题】What are the correct spot value (S) and the risk-free rate (r) that Lee should use as inputs for the Black model?

A. 186.73 and 0.39%, respectively
B. 186.73 and 2.20%, respectively
C. 187.95 and 2.20%, respectively
参考答案：A
【莽学解析】Black's model to value a call option on a futures contract is $c = e^{-rT}[F_0(T)N(d_1) - XN(d_2)]$. The underlying F_0 is the futures price (186.73). The correct discount rate is the risk-free rate, r = 0.39%.

13.【单项选择题】Which of the following is the correct answer to Solomon's question regarding the option Greek letter?
A. Vega
B. Theta
C. Gamma
参考答案：B
【莽学解析】Lee is pointing out the option price's sensitivity to small changes in time. In the BSM approach, option price sensitivity to changes in time is given by the option Greek theta.

14.【单项选择题】Based on Solomon's observation about the model price and market price for the put option in Exhibit 2, the implied volatility for the GPX is most likely:
A. Less than the historical volatility.
B. Equal to the historical volatility.
C. Greater than the historical volatility.
参考答案：A
【莽学解析】The put is priced at $7.4890 by the BSM model when using the historical volatility input of 24%. The market price is $7.20. The BSM model overpricing suggests the implied volatility of the put must be lower than 24%.

15.【单项选择题】The valuation inputs used by Lee to price a call reflecting Solomon's interest rate views should include an underlying FRA rate of:
A. 0.60% with six months to expiration.
B. 0.75% with nine months to expiration.
C. 0.75% with six months to expiration.
参考答案：C
【莽学解析】Solomon's forecast is for the three-month Libor to exceed 0.85% in six months. The correct option valuation inputs use the six-month FRA rate as the underlying, which currently has a rate of 0.75%.

16.【单项选择题】The strategy suggested by Lee for hedging small moves in Solomon's ETF position would most likely involve:
A. Selling put options.
B. Selling call options.
C. Buying call options.
参考答案：B
【莽学解析】Because selling call options creates a short position in the ETF that would hedge his current long position in the ETF. Exhibit 2 could also be used to answer the question. Solomon owns 10,000 shares of the GPX, each with a delta of +1; by definition, his portfolio

delta is +10,000. A delta hedge could be implemented by selling enough calls to make the portfolio delta neutral:

$$N_H = \frac{\text{Portfolio delta}}{\text{Delta}_H} = \frac{+10,000}{+0.6232} = -16,046\ calls$$

17.【单项选择题】Lee's put-based hedge strategy for Solomon's ETF position would most likely result in a portfolio gamma that is:

A. Negative.

B. Neutral.

C. Positive.

参考答案：C

【莽学解析】Because the gamma of the stock position is 0 and the put gamma is always non-negative, adding a long position in put options would most likely result in a positive portfolio gamma. Gamma is the change in delta from a small change in the stock's value. A stock position always has a delta of +1. Because the delta does not change, gamma equals 0. The gamma of a call equals the gamma of a similar put, which can be proven using put－call parity.

【题干】Bruno Sousa has been hired recently to work with senior analyst Camila Rocha. Rocha gives him three option valuation tasks. Alpha CompanySousa's first task is to illustrate how to value a call option on Alpha Company with a one-period binomial option pricing model. It is a non-dividend-paying stock, and the inputs are as follows. ● The current stock price is 50, and the call option exercise price is 50. ● In one period, the stock price will either rise to 56 or decline to 46. ● The risk-free rate of return is 5% per period. Based on the model, Rocha asks Sousa to estimate the hedge ratio, the risk-neutral probability of an up move, and the price of the call option. In the illustration, Sousa is also asked to describe related arbitrage positions to use if the call option is overpriced relative to the model. Beta CompanyNext, Sousa uses the two-period binomial model to estimate the value of a European-style call option on Beta Company's common shares. The inputs are as follows.

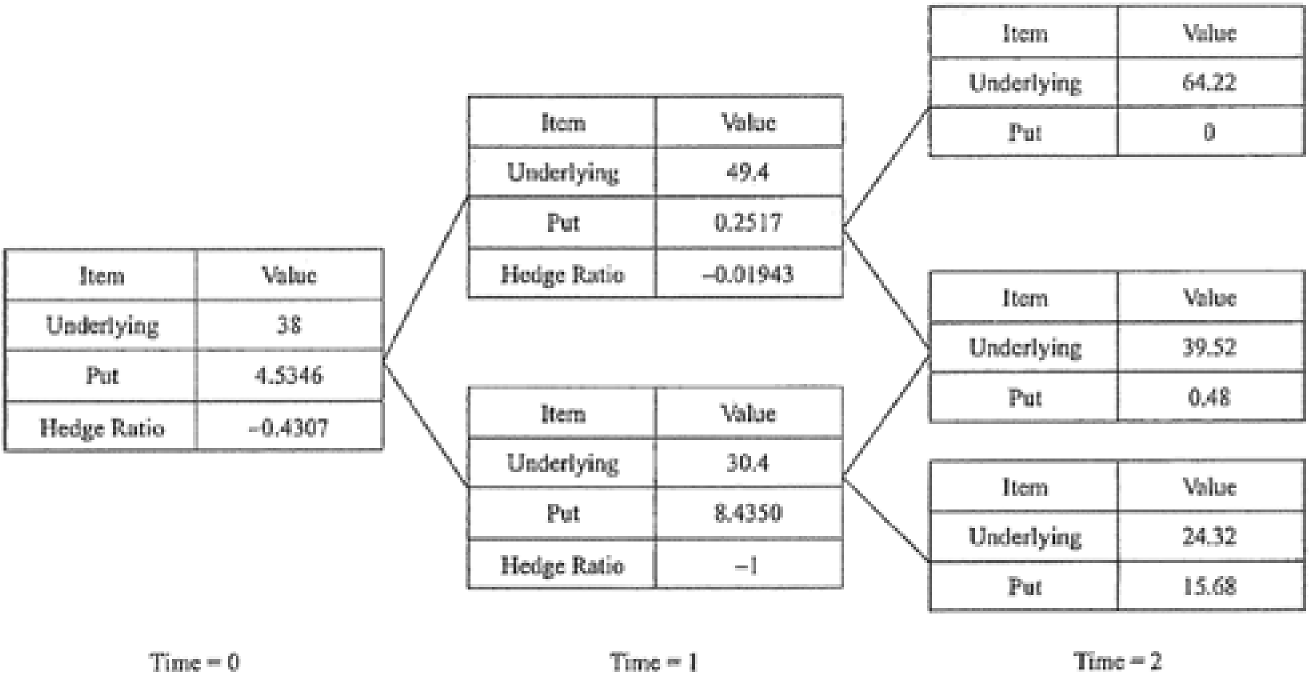
● The current stock price is 38, and the call option exercise price is 40.

● The up factor (u) is 1.300, and the down factor (d) is 0.800.

● The risk-free rate of return is 3% per period.

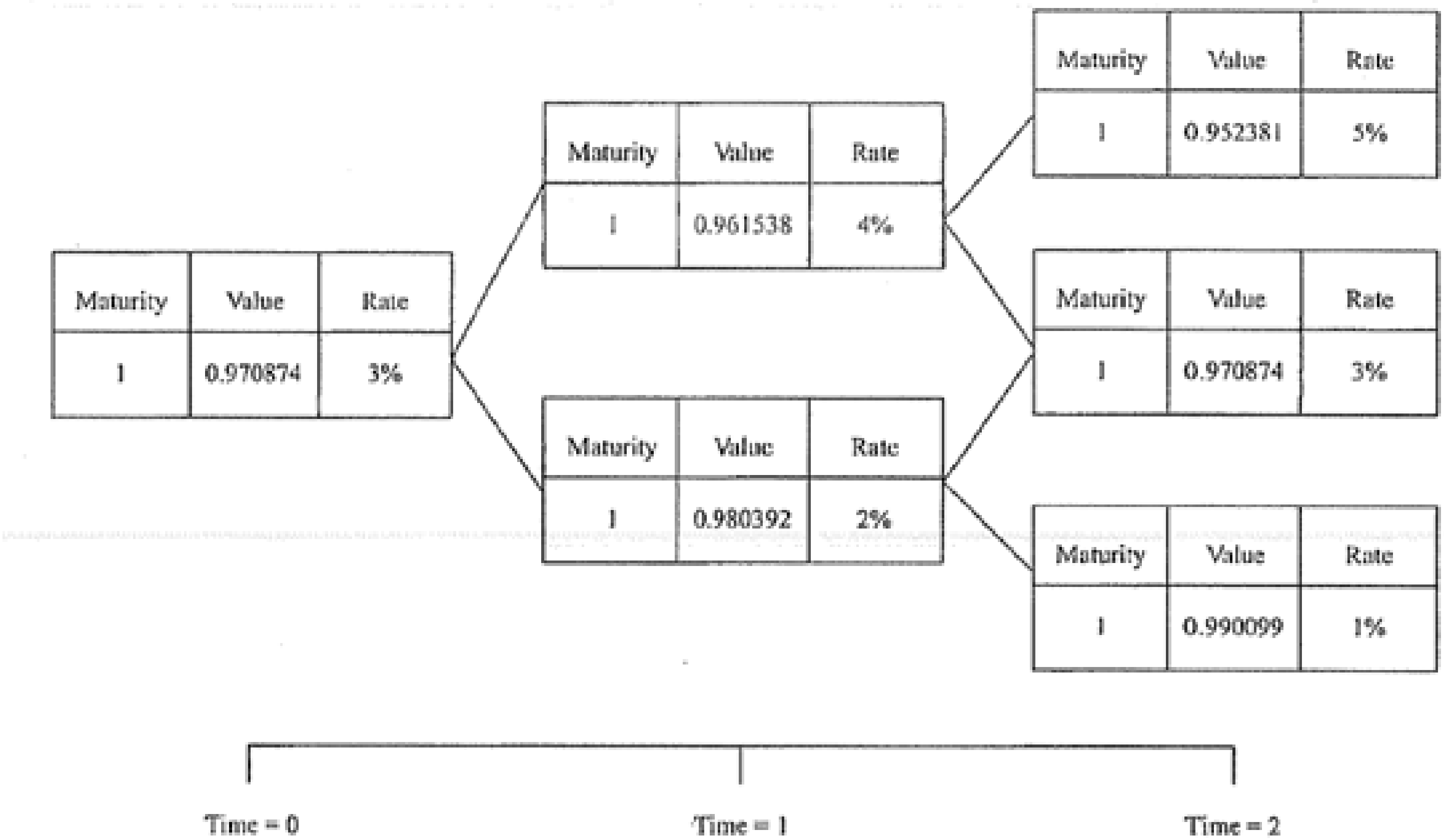
Sousa then analyzes a put option on the same stock. All of the inputs, including the exercise price, are the same as for the call option. He estimates that the value of a European-style put option is 4.53. Exhibit 1 summarizes his analysis. Sousa next must determine whether an American-style put option would have the same value. Exhibit 1. Two-period Binomial European-Style Put Option on Beta Company

Sousa makes two statements with regard to the valuation of a European-style option under the expectations approach. Statement 1: The calculation involves discounting at the risk-free rate. Statement 2: The calculation uses risk-neutral probabilities instead of true probabilities. Rocha asks Sousa whether it is ever profitable to exercise American options prior to maturity. Sousa answers, "I can think of two possible cases. The first case is the early exercise of an American call option on a dividend-paying stock. The second case is the early



exercise of an American put option.”

Interest Rate OptionThe final option valuation task involves an interest rate option. Sousa must value a two-year, European-style call option on a one-year spot rate. The notional value of the option is 1 million, and the exercise rate is 2.75%. The risk-neutral probability of an up move is 0.50. The current and expected one-year interest rates are shown in Exhibit 2, along with the values of a one-year zero-coupon bond of 1 notional value for each interest rate.Exhibit 2. Two-Year Interest Rate Lattice for an Interest Rate Option



Rocha asks Sousa why the value of a similar in-the-money interest rate call option decreases if the exercise price is higher. Sousa provides two reasons.Reason 1: The exercise value of the call option is lower. Reason 2: The risk-neutral probabilities are changed.

18.【单项选择题】The optimal hedge ratio for the Alpha Company call option using the one-period binomial model is closest to:

A. 0.60.

B. 0.67.

C. 1.67.

参考答案：A

【莽学解析】The hedge ratio requires the underlying stock and call option values for the up move and down move. S⁺ = 56, and S⁻ = 46. c⁺ = Max(0,S⁺ - X) =Max(0,56 - 50) = 6, and c⁻ = Max(0,S⁻ - X) = Max(0,46 - 50) = 0. The hedge ratio is

$$h=\frac{c^{+}-c^{-}}{s^{+}-s^{-}}=\frac{6-0}{56-46}=\frac{6}{10}=0.60$$

19.【单项选择题】The risk-neuvtral probability of the up move for the Alpha Company stock is closest to:

A. 0.06.

B. 0.40.

C. 0.65.

参考答案：C

【莽学解析】For this approach, the risk-free rate is r = 0.05, the up factor is u = S⁺/S = 56/50 = 1.12, and the down factor is d = S⁻/S = 46/50 = 0.92. The risk- neutral probability of an up move is

$$\pi=[FV(1)-d]/(u-d)=(1+r-d]/(u-d)$$

$$\pi=(1+0.05-0.92)/(1.12-0.92)=0.13/0.20=0.65$$

20.【单项选择题】The value of the Alpha Company call option is closest to:

A. 3.71.

B. 5.71.

C. 6.19.

参考答案：A

【莽学解析】The call option can be estimated using the no-arbitrage approach or the expectations approach. With the no-arbitrage approach, the value of the call option is c = hS + PV (-hS⁻ + c⁻). h = (c⁺ - c⁻) / (S⁺ - S⁻) = (6 - 0)/ (56 - 46) = 0.60. c = (0.60 x 50) + (1/1.05) x [(-0.60 x 46) + 0]. c = 30 - [(1/1.05) x 27.6] = 30 - 26.286 = 3.714.

Using the expectations approach, the risk-free rate is r = 0.05, the up factor is u

=S⁺/S = 56/50 = 1.12, and the down factor is d = S⁻/S = 46/50 = 0.92. The value of the call option is

$$c = PV \times \left[nc^{+} + (1-n)c^{-} \right].$$

$$n = [FV(1) - d] / (u - d) = (1.05 - 0.92) / (1.12 - 0.92) = 0.65.$$

$$c = (1/1.05)\ x\ \left[0.65(6) + (1-0.65)(0)\right] = (1/1.05)(3.9) = 3.714.$$

Both approaches are logically consistent and yield identical values.

21.【单项选择题】For the Alpha Company option, the positions to take advantage of the arbitrage opportunity are to write the call and:

A. Short shares of Alpha stock and lend.

B. Buy shares of Alpha stock and borrow.

C. Short shares of Alpha stock and borrow.

参考答案：B

【莽学解析】You should sell (write) the overpriced call option and then go long (buy) the replicating portfolio for a call option. The replicating portfolio for a call option is to buy h shares of the stock and borrow the present value of (hS⁻ - c⁻). c = hS + PV (-hS⁻ + c⁻). h = (c⁺ - c⁻)/(S⁺ - S⁻) = (6 - 0)/(56 - 46) = 0.60. For the example in this case, the value of the call option is 3.714. If the option is overpriced at, say, 4.50, you short the option and have a cash flow at Time 0 of +4.50. You buy the replicating portfolio of 0.60 shares at 50 per share (giving you a cash flow of -30) and borrow (1/1.05) x [(0.60 x 46) - 0] = (1/1.05) x 27.6 = 26.287. Your cash flow for buying the replicating portfolio is -30 + 26.287 = -3.713. Your net cash flow at Time 0 is + 4.50 - 3.713 = 0.787. Your net cash flow at Time 1 for either the up move or down move is zero. You have made an arbitrage profit of 0.787. In tabular form, the cash flows are as follows:

Transaction	Time Step 0	Time Step 1 Down Occurs	Time Step 1 Up Occurs
Sell the call option	4.50	0	-6.00
Buy *h* shares	-0.6 x 50 = -30	0.6 x 46 = 27.6	0.6 x 56 = 33.6
Borrow -PV(-hS⁻ + c⁻)	-(1/1.05) x [(-0.6 x 46) + 0] = 26.287	-0.6 x 46 = -27.6	-0.6 x 46 = -27.6
Net cash flow	0.787	0	0

22.【单项选择题】The value of the European-style call option on Beta Company shares is closest to:

A. 4. 83.
B. 5. 12.
C. 7. 61.
参考答案：A
【莽学解析】

A is correct. Using the expectations approach, the risk-neutral probabilit
up move is

$$\pi = [FV(1) - d]/(u - d) = (1.03 - 0.800)/(1.300 - 0.800) = 0.46.$$

The terminal value calculations for the exercise values at Time Step 2 are

$$c^{++} = \text{Max}(0, u^2S - X) = \text{Max}[0, 1.30^2(38) - 40] = \text{Max}(0, 24.22) = 24.2$$
$$c^{-+} = \text{Max}(0, udS - X) = \text{Max}[0, 1.30(0.80)(38) - 40] = \text{Max}(0, -0.48) =$$
$$c^{--} = \text{Max}(0, d^2S - X) = \text{Max}[0, 0.80^2(38) - 40] = \text{Max}(0, -15.68) = 0.$$

Discounting back for two years, the value of the call option at Time Step

$$c = PV[\pi^2 c^{++} + 2\pi(1 - \pi)c^{-+} + (1 - \pi)^2 c^{--}].$$
$$c = [1/(1.03)]^2[0.46^2(24.22) + 2(0.46)(0.54)(0) + 0.54^2(0)].$$
$$c = [1/(1.03)]^2[5.1250] = 4.8308.$$

23.【单项选择题】The value of the American-style put option on Beta Company shares is closest to:
A. 4. 53.
B. 5. 15.
C. 9. 32.
参考答案：B
【莽学解析】Using the expectations approach, the risk-neutral probability of an up move is

$$\pi = [FV(1) - d]/(u - d) = (1.03 - 0.800)/(1.300 - 0.800) = 0.46.$$

An American-style put can be exercised early. At Time Step 1, for the up move, p⁺ is 0.2517 and the put is out of the money and should not be exercised early (X < S, 40 < 49.4). However, at Time Step 1, p⁻ is 8.4350 and the put is in the money by 9.60 (X - S = 40 - 30.40). So, the put is exercised early, and the value of early exercise (9.60) replaces the value of not exercising early (8.4350) in the binomial tree. The value of the put at Time Step 0 is now
Following is a supplementary note regarding Exhibit 1. The values in Exhibit 1 are calculated as follows.

$$p = PV\left[\pi p^{+} + (1-\pi)p^{-}\right] = \left[1/(1.03)\right]\times$$
$$\left[0.46(0.2517) + 0.54(9.60)\right] = 5.1454.$$

At Time Step 2:

$$p^{++} = \text{Max}(0,X - u^2S) = \text{Max}[0,40 - 1.300^2(38)] = \text{Max}(0,40 - 64.2$$
$$p^{-+} = \text{Max}(0,X - udS) = \text{Max}[0,40 - 1.300(0.800)(38)] = \text{Max}(0,40$$
$$39.52) = 0.48.$$
$$p^{--} = \text{Max}(0,X - d^2S) = \text{Max}[0,40 - 0.800^2(38)] = \text{Max}(0,40 - 24.3$$
$$15.68.$$

At Time Step 1:

$$p^{+} = \text{PV}[\pi p^{++} + (1-\pi)p^{-+}] = [1/(1.03)][0.46(0) + 0.54(0.48)] = 0.25$$
$$p^{-} = \text{PV}[\pi p^{-+} + (1-\pi)p^{--}] = [1/(1.03)][0.46(0.48) + 0.54(15.68)] =$$
$$8.4350.$$

At Time Step 0:

$$p = \text{PV}[\pi p^{+} + (1-\pi)p^{-}] = [1/(1.03)][0.46(0.2517) + 0.54(8.4350)] =$$

24.【单项选择题】Which of Sousa’s statements about binomial models is correct?

A.Statement 1 only

B.Statement 2 only

C.Both Statement 1 and Statement 2

参考答案：C

【莽学解析】Both statements are correct. The expected future payoff is calculated using risk-neutral probabilities, and the expected payoff is discounted at the risk-free rate.

25.【单项选择题】Based on Exhibit 2 and the parameters used by Sousa, the value of the interest rate option is closest to:

A.5,251.

B.6,236.

C.6,429.

参考答案：C

【莽学解析】

Using the expectations approach, per 1 of notional value, the values of the call option at Tim

Step 2 are

$c^{++} = \text{Max}(0, S^{++} - X) = \text{Max}(0, 0.050 - 0.0275) = 0.0225.$

$c^{+-} = \text{Max}(0, S^{+-} - X) = \text{Max}(0, 0.030 - 0.0275) = 0.0025.$

$c^{--} = \text{Max}(0, S^{--} - X) = \text{Max}(0, 0.010 - 0.0275) = 0.$

At Time Step 1, the call values are

$c^{+} = PV[\pi c^{++} + (1 - \pi) c^{+-}] = 0.961538[0.50(0.0225) + (1 - 0.50)(0.0025)] = 0.012019$

$c^{-} = PV[\pi c^{+-} + (1 - \pi) c^{--}]$

$c^{-} = 0.980392[0.50(0.0025) + (1 - 0.50)(0)] = 0.001225.$

At Time Step 0, the call option value is $c = PV[\pi c^{+} + (1 - n) c^{-}]$.

$c = 0.970874[0.50(0.012019) + (1 - 0.50)(0.001225)] = 0.006429.$

The value of the call option is this amount multiplied by the notional value, or 0.006429

1,000,000 = 6,429.

26.【单项选择题】Which of Sousa's reasons for the decrease in the value of the interest rate option is correct?

A. Reason 1 only

B. Reason 2 only

C. Both Reason 1 and Reason 2

参考答案：A

【莽学解析】Reason 1 is correct: A higher exercise price does lower the exercise value (payoff) at Time 2. Reason 2 is not correct because the risk-neutral probabilities are based on the paths that interest rates take, which are determined by the market and not the details of a particular option contract.

【题干】Tim Doyle is a portfolio manager at BestFutures Group, a hedge fund that frequently enters into derivative contracts either to hedge the risk of investments it holds or to speculate outside of those investments. Doyle works alongside Diane Kemper, a junior analyst at the hedge fund. They meet to evaluate new investment ideas and to review several of the firm's existing investments.Carry Arbitrage ModelDoyle and Kemper discuss the carry arbitrage model and how they can take advantage of mispricing in bond markets. Specifically, they would like to execute an arbitrage transaction on a Eurodollar futures contract in which the underlying Eurodollar bond is expected to make an interest payment in two months. Doyle makes the following statements:Statement 1: If the Eurodollar futures price is less than the price suggested by the carry arbitrage model, the futures contract should be purchased.Statement 2: Based on the cost of carry model, the futures price would be higher if the underlying Eurodollar bond's upcoming interest payment was expected in five months instead of two. Three-Year Treasury Note Futures ContractKemper then presents two investment ideas to Doyle. Kemper's first investment idea is to purchase a three-year Treasury note futures contract. The underlying 1.5%, semi-annual three-year Treasury note is quoted at a clean price of 101. It has

been 60 days since the three-year Treasury note' s last coupon payment, and the next coupon payment is payable in 120 days. Doyle asks Kemper to calculate the full spot price of the underlying three-year Treasury note.10-Year Treasury Note Futures ContractKemper' s second investment idea is to purchase a 10-year Treasury note futures contract. The underlying 2%, semi-annual 10-year Treasury note has a dirty price of 104.17. It has been 30 days since the 10-year Treasury note' s last coupon payment. The futures contract expires in 90 days. The quoted futures contract price is 129. The current annualized three-month risk-free rate is 1.65%. The conversion factor is 0.7025. Doyle asks Kemper to calculate the equilibrium quoted futures contract price based on the carry arbitrage model.
Japanese Government BondsAfter discussing Kemper' s new investment ideas, Doyle and Kemper evaluate one of their existing forward contract positions. Three months ago, BestFutures took a long position in eight 10-year Japanese government bond (JGB) forward contracts, with each contract having a contract notional value of 100 million yen. The contracts had a price of JPY153 (quoted as a percentage of par) when the contracts were purchased.Now, the contracts have six months left to expiration and have a price of JPY155. The annualized six-month interest rate is 0.12%. Doyle asks Kemper to value the JGB forward position.
Interest Rate SwapsAdditionally, Doyle asks Kemper to price a one-year plain vanilla swap. The spot rates and days to maturity at each payment date are presented in Exhibit 1.

Exhibit 1 Selected US Spot Rate Data

Days to Maturity	Spot Interest Rates (%)
90	1.90
180	2.00
270	2.10
360	2.20

Finally, Doyle and Kemper review one of BestFutures' s pay-fixed interest rate swap positions. Two years ago, the firm entered into a JPY5 billion five-year interest rate swap, paying the fixed rate. The fixed rate when BestFutures entered into the swap two years ago was 0.10%. The current term structure of interest rates for JPY cash flows, which are relevant to the interest rate swap position, is presented in Exhibit 2.
Doyle asks Kemper to calculate the value of the pay-fixed interest rate swap.

27.【单项选择题】Which of Doyle' s statements regarding the Eurodollar futures contract price is correct?
A.Only Statement 1.
B.Only Statement 2
C.Both Statement 1 and Statement 2.
参考答案：C
【莽学解析】Doyle' s first statement is correct. Unless the Eurodollar futures contract' s quoted price is equal to the no-arbitrage futures price, there is an arbitrage opportunity. Moreover, if the quoted futures price is less than the no-arbitrage futures price, then to take

Exhibit 2 Selected Japanese Interest Rate Data

Maturity (Years)	Yen Spot Interest Rates (%)	Present Value Factors
1	0.03	0.9997
2	0.06	0.9988
3	0.08	0.9976
Sum		2.9961

advantage of the arbitrage opportunity, the Eurodollar futures contract should be purchased and the underlying Eurodollar bond should be sold short. Doyle would then lend the short sale proceeds at the risk-free rate. The strategy that comprises those transactions is known as reverse carry arbitrage.

Doyle’s second statement is also correct. Based on the cost of carry model, the futures price is calculated as the future value of the sum of the underlying plus the underlying carry costs minus the future value of any ownership benefits. If the Eurodollar bond’s interest payment was expected in five months instead of two, the benefit of the cash flow would occur three months later, so the future value of the benefits term would be slightly lower. Therefore, the Eurodollar futures contract price would be slightly higher if the Eurodollar bond’s interest payment was expected in five months instead of two months.

A is incorrect because Doyle’s Statement 2 is correct (not incorrect). Based on the cost of carry model, the futures price would be higher if the underlying Eurodollar bond’s interest payment took place in five months instead of two months.

B is incorrect because Doyle’s Statement 1 is correct (not incorrect). If the Eurodollar’s futures contract price is less than the price suggested by the carry arbitrage model, the futures contract should be purchased.

28.【单项选择题】The full spot price of the three-year Treasury note is:

A. 101.00.

B. 101.25.

C. 101.50.

参考答案：B

【莽学解析】The full spot price of the three-year Treasury note is calculated as:

S_0 = Quoted bond price + Accrued interest = B_0 + AI_0.

Accrued interest (AI) = Accrual period × Periodic coupon amount = (NAD / NTD) × (C / n)

AI = (60/180) × (0.015/2) = 0.25.

S_0 = 101 + 0.25 = 101.25.

A is incorrect because 101 is the quoted clean (not the full spot) price of the three-year Treasury note. The clean price excludes accrued interest; the full price, also referred to as the dirty price, includes accrued interest.

C is incorrect because the number of days until the next coupon payment (instead of the accrual period) is incorrectly used to compute accrued interest:

AI = (120/180) × (0.015/2) = 0.50.

S_0 = 101 + 0.50 = 101.50.

29.【单项选择题】The equilibrium 10-year Treasury note quoted futures contract price is closest to:

A.147.94.

B.148.89.

C.149.78.

参考答案：A

【莽学解析】The equilibrium 10-year quoted futures contract price based on the carry arbitrage model is calculated as:

Q₀ = (1/CF) × [FV(B₀ + AI₀)—FVCI].

CF = 0.7025.

B₀ = 104.00.

AI₀ = 0.17.

AI_T = (120/180 × 0.02/2) = 0.67.

FVCI = 0.

Q₀ = (1/0.7025) × [(1+0.0165)$^{3/12}$ (104.17)—0]=147.94

B is incorrect because accrued interest at expiration is not subtracted in the equilibrium quoted futures contract price formula:

Q₀ = (1/0.7025) × [(1+0.0165)$^{3/12}$(104.17)—0.67—0] =149.78.

30.【单项选择题】The value of the JGB long forward position is closest to:

A.JPY15,980,823.

B.JPY15,990,409.

C.JPY16,000,000.

参考答案：B

【莽学解析】The value of the JGB forward position is calculated as:

V_t = PV[F_t— F_0] = (155-153) / (1+0.0012) = 1.9980.

0.019980 × (JPY100,000,000) × 8 = JPY15,980,823.

C is incorrect because the absolute difference (not the present value of the difference) between the price when the contracts were purchased and the current price of the contracts was computed:

V_t = [F_t—F_0] = (155-153) = 2.

0.02 (JPY100,000,000) × 8 = JPY16,000,000.

31.【单项选择题】Based on Exhibit 1, the fixed rate of the one-year plain vanilla swap is closest

A.0.12%.

B.0.55%.

C.0.72%.

参考答案：B

【莽学解析】

32.【单项选择题】Based on Exhibit 2, the value of the pay-fixed interest rate swap is closest to:

The swap's fixed rate is calculated as:

$$r_{FIX} = \frac{1-PV_n(1)}{\sum_{i=1}^{n} PV_i(1)}$$

$$PV_i(1) = \frac{1}{1+r_{Spot_i}\left(\frac{NAD_i}{NTD}\right)}$$

90 − day PV factor = 1/[1 + 0.019 × (90/360)] = 0.9953.

180 − day PV factor = 1/[1 + 0.020 × (180/360)] = 0.9901.

270 − day PV factor = 1/[1 + 0.021 × (270/360)] = 0.9845.

360 − day PV factor = 1/[1 + 0.022 × (360/360)] = 0.9785..

$$\sum_{i=1}^{4} PV_i(1) = 0.9953 + 0.9901 + 0.9845 + 0.9785 = 3.9483.$$

r_{FIX} = (1 − 0.9785)/3.9483 = 0.0055 = 0.55%.

A. －JPY6,491,550.
B. －JPY2,980,500.
C. －JPY994,793.
参考答案：B
【莽学解析】

The value of the pay-fixed interest rate swap is calculated as

$-Vswap_t = NA \times (FS_t - FS_0) \times \sum_{i=1}^{n} PVi$

$FS_t = r_{FIX} = [1\text{-}PV_n(1)] / \sum_{i=1}^{3} PVi(1) = (1\text{-}0.9976)/2.9961 = 0.000801 = 0.08\%$

$-Vswap_t = NA \times (FS_t - FS_0) \times \sum_{i=1}^{3} PVi$

=JPY 5billion × (0.000801-0.001) ×2.9961

=-JPY2,980,500

Given that rates have declined since the inception of the swap, the value of the pay-fixed, rece

floating position is currently a loss of JPY2,980,500.