

### Problem 9.3

#### Exercise 1

Let  $X$  be a continuous random variable with mean  $\mu(x)$  and variance  $\sigma^2(x)$  and let  $x^* = (x - \mu)/\sigma$  be its standardized version. Verify directly that  $\mu(x^*) = 0$  and  $\sigma^2(x^*) = 1$ .

$$\begin{aligned}\text{Solution: } E(x^*) &= \frac{1}{\sigma} (E(x) - \mu) \\ &= \frac{1}{\sigma} (\mu - \mu) = 0\end{aligned}$$

$$\begin{aligned}\sigma^2(x^*) &= E\left(\frac{x - \mu}{\sigma}\right)^2 \\ &= \frac{1}{\sigma^2} \sigma^2 = 1\end{aligned}$$