DATA 605 HW 8

11 on page 303.

A company buys 100 lightbulbs, each of which has an exponential lifetime of 1000 hours. What is the expected time for the first of these bulbs to burn out?

Solution: If $X_i \sim Exp(\lambda_i)$,

Then

$$X_i \sim Exp(\sum \lambda_i)$$

According to the problem,

$$\lambda_i = \frac{1}{1000}$$

$$\sum \lambda_i = \frac{1}{10}$$

Thus, the expected lifetime of the first of these bulbs to burn out is 10 hours.

14 on page 303

14. Assume that X_1 and X_2 are independent random variables, each having an exponential density with parameter λ. Show that $Z = X_1 - X_2$ has density

$$f_Z(z) = (1/2)\lambda e^{-\lambda|z|}$$

$$f_Z(z) = \int_{-\infty}^{\infty} f_{X_1}(x_1) f_{-X_2}(z - x_1)\, dx_1$$

$$= \int_{-\infty}^{\infty} f_{X_1}(x_1) f_{X_2}(x_1 - z)\, dx_1$$

$z<0$, $f_Z(z) = \int_0^{\infty} \lambda e^{-\lambda x_1} \lambda e^{-\lambda(x_1 - z)} dx_1$

$$= \lambda e^{\lambda z} \int_0^{\infty} \lambda e^{-2\lambda x_1} dx_1$$

$$= \lambda e^{\lambda z} \left(\left(-\frac{1}{2}\right) e^{-2\lambda x_1} \Big|_0^{\infty} \right)$$

$$= \frac{\lambda}{2} e^{\lambda z}, \quad z<0.$$

$z \ge 0$ $\quad f_Z(z) = f_Z(-z).$

$$= \frac{\lambda}{2} e^{-\lambda z}, \quad z \ge 0$$

so $f_Z(z) = \begin{cases} \frac{\lambda}{2} e^{\lambda z}, & z \le 0 \\ \frac{\lambda}{2} e^{-\lambda z}, & z \ge 0 \end{cases}$

$$f_Z(z) = \frac{1}{2} \lambda e^{-\lambda|z|}.$$

#1 on page 320 & 321

1. Let X be a continous random variable with mean $\mu = 10$ and variance $\sigma^2 = 100/3$. Using Chebyshev's Inequality, find an upper bound for the following probabilities.

$$P(|X - \mu| \geq \epsilon) \leq \frac{V(X)}{\epsilon^2}$$

(a) $P(|X-10| \geq 2) \leq \frac{100/3}{2^2} = 8.33\dot{3}$

So the upper bound for $P(|X-10| \geq 2)$ is 1

(b) $P(|X-10| \geq 5) \leq \frac{100/3}{25} = 1.3\dot{3}$

So the upper bound for $P(|X-10| \geq 5)$ is 1.

(c) $P(|X-10| \geq 9) \leq \frac{100/3}{81} = \frac{100}{243}$

So the upper bound for c is $\frac{100}{243}$.

(d) $P(|X-10| \geq 20) \leq \frac{100/3}{400} = \frac{1}{12}$

So the upper bound for d is $1/12$.