Lab 7: Estimating Causal Effects via Instrumental Variables

Welcome to the seventh DS102 lab!

The goals of this lab is to implement and get better understanding of instrumental variables discussed in Discussion 06 (https://piazza.com/class/k5ofad3nps24c1?cid=199). We highly recommend that you watch the video for Discussion 06 (https://piazza.com/class/k5ofad3nps24c1?cid=199) before attempting this lab.

The code you need to write is commented out with a message "TODO: fill in".

Course Policies

Collaboration Policy

Data science is a collaborative activity. While you may talk with others about the labs, we ask that you **write your solutions individually.** If you do discuss the assignments with others please **include their names** by adding a cell below.

Submission: to submit this assignment, rerun the notebook from scratch (by selecting Kernel > Restart & Run all), and then print as a pdf (File > download as > pdf) and submit it to Gradescope.

This assignment should be completed and submitted before Thursday March 19, 2020 at 11:59 PM.

Write collaborator names here.

In [56]:
```
import matplotlib.pyplot as plt
import numpy as np
import statsmodels.api as sm
%matplotlib inline
```

Instrumental Variables Background

Suppose that we measure X_1, the number of books a student read in the last year, and we are intrested in determing how X_1 affects an observed target outcome Y, the student's SAT score. The effect we are interested in is **causal** because we want to know how Y changes if all randomness other than X_1 remains fixed, and only X_1 changes. We will refer to X_1 as the "treatment". In general, X_1 might be multi-dimensional, however for the purpose of this exercise we take $X_1 \in \mathbb{R}$.

Suppose there's also a confounder X_2, which is the income of the student's family. We don't observe X_2, but it affects both the number of books the student reads (wealthier families may have more access to books) and the student's SAT score (wealthier students may have more access to SAT tutoring).

We assume that the outcome is generated as a linear function of the confounder X_2 and treatment X_1, with additive noise ϵ:

$$Y = \beta_1 X_1 + \beta_2 X_2 + \epsilon.$$

The goal is to estimate β_1, the true causal effect of the number of books a student reads on their SAT score.

Danger of bias

As we saw in Discussion 06, if the confounder X_2 is highly correlated with X_1, performing ordinary least squares (OLS) on the observed data X_1, Y can lead to very biased results.

Instrumental variables (IVs) and two-stage least squares (2SLS)

One way to get around this issue is by using **instrumental variables (IVs)**. A valid instrument Z is a variable which is independent of the confounder X_2, and affects Y only through X_1. For example, as discussed in Discussion 06, we can create such an instrument Z by employing *encouragement design*, where we randomly select students and encourage them to read by organizing a “readathon” at their school. Let Z be the number of days that the organized readathon lasts for the given student.

Using the instrumental variable Z, we can estimate β_1 by first "guessing" X_1 from Z using ordinary least squares (OLS) (denoted $\hat{X}_1$), and then regressing Y onto $\hat{X}_1$ (instead of X_1) using OLS as well. This procedure is known as **two-stage least squares (2SLS)**.

In this lab, we will observe the bias that can occur when naively performing OLS on the observed data X_1, Y, and also how employing 2SLS can achieve a better estimate of β_1.

1. Model setup

Suppose that we have historical data from $n = 10,000$ different students. Suppose we observe the following variables:

$X_1^{(i)}$ = number of books the student read in the last year,

$Z^{(i)}$ = whether or not there was a "readathon" at the student's school,

$Y^{(i)}$ = the student's SAT score.

Suppose that the student's family income $X_2^{(i)}$ affects both $X_1^{(i)}$ and $Y^{(i)}$, but is not observed.

In [57]:
```
n = 10000 # sample size
```

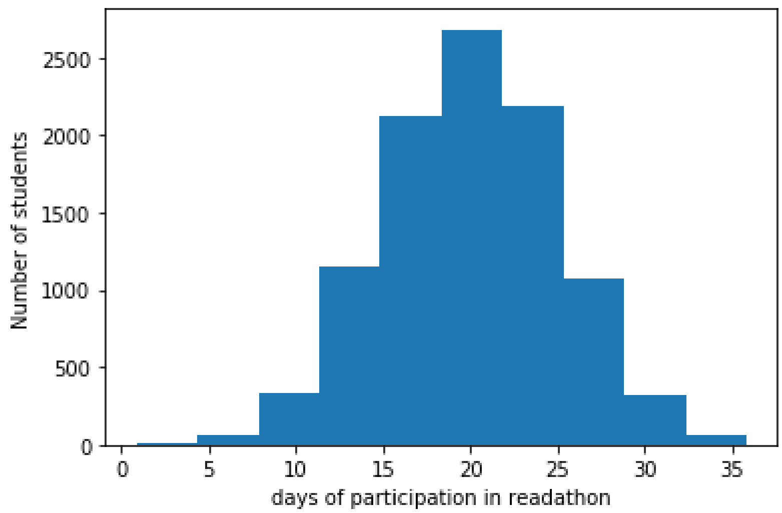
Generate the data

Each student participates in a readathon that lasts for 20 days on average, with a standard deviation of 5 days:

$$Z^{(i)} \sim \text{Normal}(20, 5)$$

In [58]:
```
# numpy array of length n, where each entry is Z^{(i)}.
Z = np.random.normal(20,5,n)

plt.xlabel("days of participation in readathon")
plt.ylabel("Number of students")
plt.hist(Z)
plt.show()
```

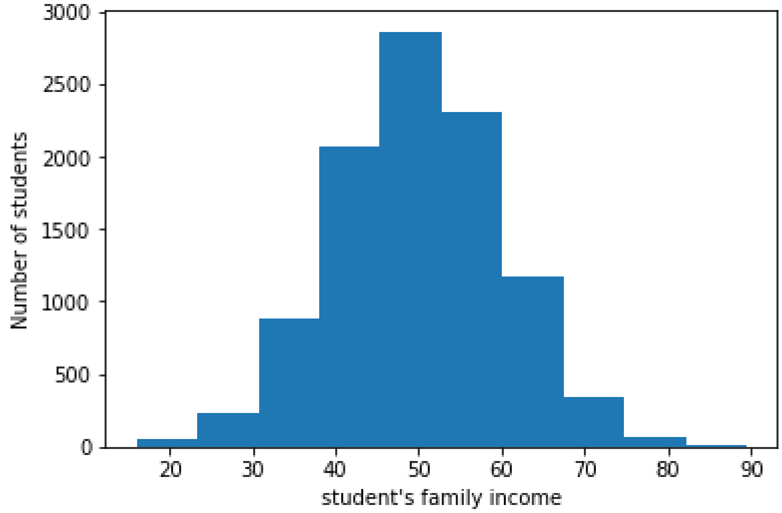


Student i's family income is normally distributed, where $X_2^{(i)}$ is the family's annual income (in thousands):

$$X_2^{(i)} \sim \text{Normal}(50, 10)$$

In [59]:
```
# numpy array of length n, where the ith entry is student i's annual inc
ome
X_2 = np.random.normal(50,10,n)

plt.xlabel("student's family income")
plt.ylabel("Number of students")
plt.hist(X_2)
plt.show()
```

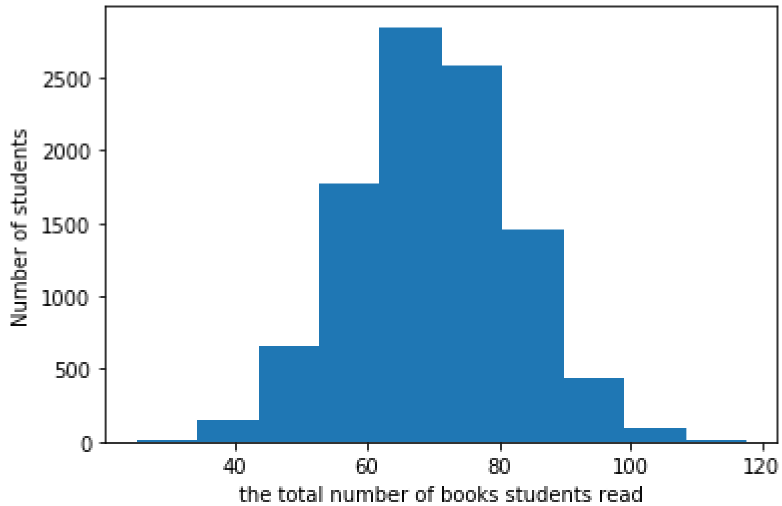


The number of books a student reads is linear in whether or not there was a readathon and the student's family income:

$$X_1 = \gamma_1 Z + \gamma_2 X_2 + \epsilon',$$

In [60]:
```
gamma_1 = 1 # number of additional books read per day when there's a rea
dathon
gamma_2 = 1 # number of books read vs. family income
eps_prime = np.random.normal(0,5,n) # noise for number of books read
X_1 = gamma_1 * Z + gamma_2 * X_2 + eps_prime

plt.xlabel("the total number of books students read")
plt.ylabel("Number of students")
plt.hist(X_1)
plt.show()
```

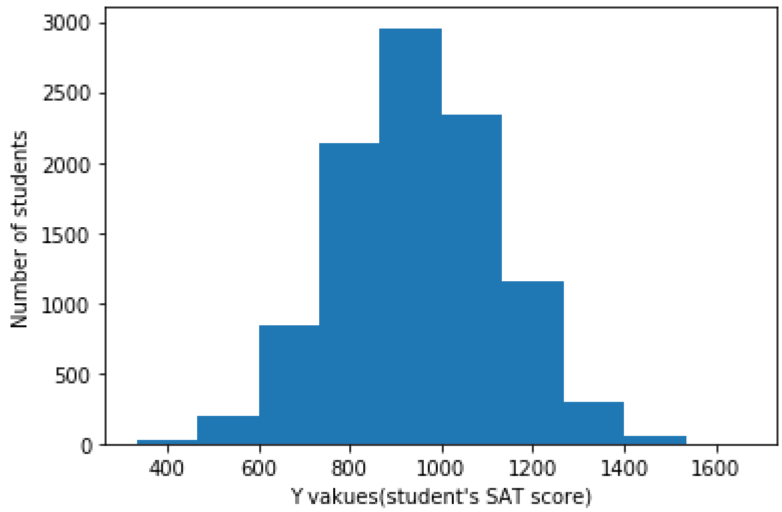


The student's SAT score is linear in the number of books the student read and the student's family income:

$$Y = \beta_1 X_1 + \beta_2 X_2 + \epsilon.$$

In [61]:
```
beta_1 = 5 # true relationship between books read and SAT score
beta_2 = 12 # relationship between SAT score and family income
eps = np.random.normal(0,10,n) # noise for SAT score
Y = beta_1 * X_1 + beta_2 * X_2 + eps

plt.xlabel("Y vakues(student's SAT score)")
plt.ylabel("Number of students")
plt.hist(Y)
plt.show()
```



Goal: estimate β_1, the true causal effect of the number of books a student reads on their SAT score.

2. Naive OLS: OLS on the observed variables X_1, Y.

The confounding variable X_2 (family income) is unfortunately unobserved. We will start by somewhat "naively" attempting to estimate the causal effect β_1 by using plain linear regression (OLS) on the observed variables X_1 and Y.

In [62]:

```
# No TODOs here, just run this cell and understand what this function is doing.
def beta_hat_with_intercept(input_array, target_array):
    """Calculates the OLS estimator parameters. The returned parameters include an intercept term.

    Args:
      input_array: numpy array with n entries, where each entry corresponds with a feature value for a given student.
      target_array: numpy array with n entries, where each entry corresponds with a label value for a given student.

    Returns:
      numpy array with 2 entries, where the entries are [intercept, beta_hat]. The intercept is a constant term,
      so the final OLS predictions should be predictions = intercept + beta_hat * input_array.
    """
    ols_features_w_const = sm.add_constant(input_array) # prepend a constant feature for intercept term
    ols_model = sm.OLS(target_array, ols_features_w_const)
    ols_results = (ols_model.fit()).params # predicted coefficients
    return ols_results
```

In [63]:

```
# TODO: Fit OLS parameters to predict Y from X_1.
beta_hat_naive_with_intercept = beta_hat_with_intercept(input_array=X_1, target_array=Y)

beta_hat_naive = beta_hat_naive_with_intercept[1]
print("Naive OLS estimate of beta_1:", beta_hat_naive)
print("True beta_1:", beta_1)
print("Absolute error:", np.abs(beta_hat_naive - beta_1))
```

```
Naive OLS estimate of beta_1: 13.08255810705653
True beta_1: 5
Absolute error: 8.08255810705653
```

3. Instrumental variables and 2SLS

To eliminate the bias, we turn to instrumental variables. In the first stage, we "predict" the number of books a student read X_1 from whether or not they had a readathon, Z, producing an estimate $\hat{X}_1$. Then, in the second stage, we regress the SAT score Y onto the predicted number of books read $\hat{X}_1$.

In [64]:
```
# No TODOs here, just run this call and understand what this function is doing.
def compute_OLS_predictions(input_array, input_params):
    """Calculates OLS predictions from fitted OLS parameters, input_params.

    Args:
      input_array: numpy array with n entries, where each entry corresponds with a feature value for a given student.
      input_params: numpy array with 2 entries, where the entries are [intercept, beta_hat].
        The intercept is a constant term, so the final OLS predictions should be
        predictions = intercept + beta_hat*input_array.

    Returns:
      numpy array with n entries containing predictions from input_array.
    """
    predictions = input_params[0] + input_params[1] * input_array
    return predictions
```

3a) Stage 1: Predict treatment variable $\hat{X}_1$ from instrumental variable Z

In [65]:
```
# Stage 1 of 2SLS.
# TODO: Fit OLS parameters to predict X_1 from Z
X_1_hat_params = beta_hat_with_intercept(input_array=Z, target_array=X_1)
X_1_hat_params
```

Out[65]:
```
array([49.27852086,  1.03396629])
```

In [66]:
```
# TODO: Compute predictions using the fitted parameters above.
X_1_hat = compute_OLS_predictions(input_params=X_1_hat_params, input_array=Z)
X_1_hat
```

Out[66]:
```
array([71.62849789, 61.27638515, 73.14730963, ..., 62.30211587,
       64.79467698, 74.13722379])
```

3b) Stage 2: Estimate target Y from predicted treatment variable $\hat{X}_1$

In [67]:
```
# Stage 2 of 2SLS.
# TODO: Fit OLS parameters to predict Y from the predicted X_1_hat.
beta_hat_2SLS_with_itercept = beta_hat_with_intercept(input_array=X_1_ha
t, target_array=Y)

beta_hat_2SLS = beta_hat_2SLS_with_itercept[1]
print("2SLS estimate of beta_1:", beta_hat_2SLS)
print("True beta_1:", beta_1)
print("Absolute error:", np.abs(beta_hat_2SLS - beta_1))
```

```
2SLS estimate of beta_1: 5.440906393026446
True beta_1: 5
Absolute error: 0.4409063930264461
```

3c) Question: Which technique produced a better estimate of β_1, naive OLS (part 2) or 2SLS (part 3)?

I think 2SLS produced a better estimate of β_1. We can see that the absolute error is only 0.046 for 2SLS but meanwhile the absolute error is 8.005695428687707 for naive OLS. This is because the 2SLS takes part in considering the relationship and dependence in X_1 and X_2 as input vlaues but the naive OLS fail to do so(it simply assume independency). That's why the absolute error is much higher is naive OLS and the 2SLS is a better technique.

In [ ]:
```

```