B^+-Tree Index Files

B^+-tree indices are an alternative to indexed-sequential files.

- Disadvantage of indexed-sequential files: performance degrades as file grows, since many overflow blocks get created. Periodic reorganization of entire file is required.
- Advantage of B^+-tree index files: automatically reorganizes itself with small, local, changes, in the face of insertions and deletions. Reorganization of entire file is not required to maintain performance.
- Disadvantage of B^+-trees: extra insertion and deletion overhead, space overhead.
- Advantages of B^+-trees outweigh disadvantages, and they are used extensively.

B^+-Tree Index Files (Cont.)

A B^+-tree is a rooted tree satisfying the following properties:

- All paths from root to leaf are of the same length
- Each node that is not a root or a leaf has between $\lceil n/2 \rceil$ and n children.
- A leaf node has between $\lceil (n-1)/2 \rceil$ and $n-1$ values
- Special cases: if the root is not a leaf, it has at least 2 children. If the root is a leaf (that is, there are no other nodes in the tree), it can have between 0 and $(n-1)$ values.

B$^+$-Tree Node Structure

- Typical node

P_1	K_1	P_2	$\ldots$	P_{n-1}	K_{n-1}	P_n

  - K_i are the search-key values
  - P_i are pointers to children (for non-leaf nodes) or pointers to records or buckets of records (for leaf nodes).

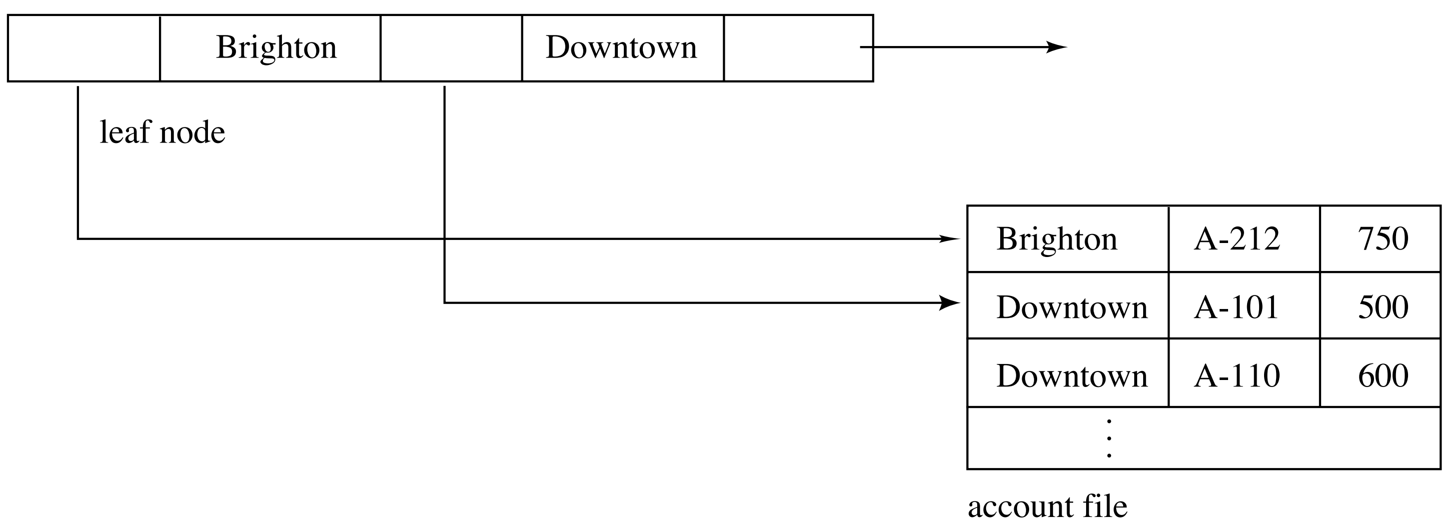
- The search-keys in a node are ordered

$$K_1 < K_2 < K_3 < \ldots < K_{n-1}$$

Leaf Nodes in B^{+}-Trees

Properties of a leaf node:

- For $i = 1, 2, \ldots, n-1$, pointer P_i either points to a file record with search-key value K_i, or to a bucket of pointers to file records, each record having search-key value K_i. Only need bucket structure if search-key does not form a primary key.
- If L_i, L_j are leaf nodes and $i < j$, L_i's search-key values are less than L_j's search-key values
- P_n points to next leaf node in search-key order

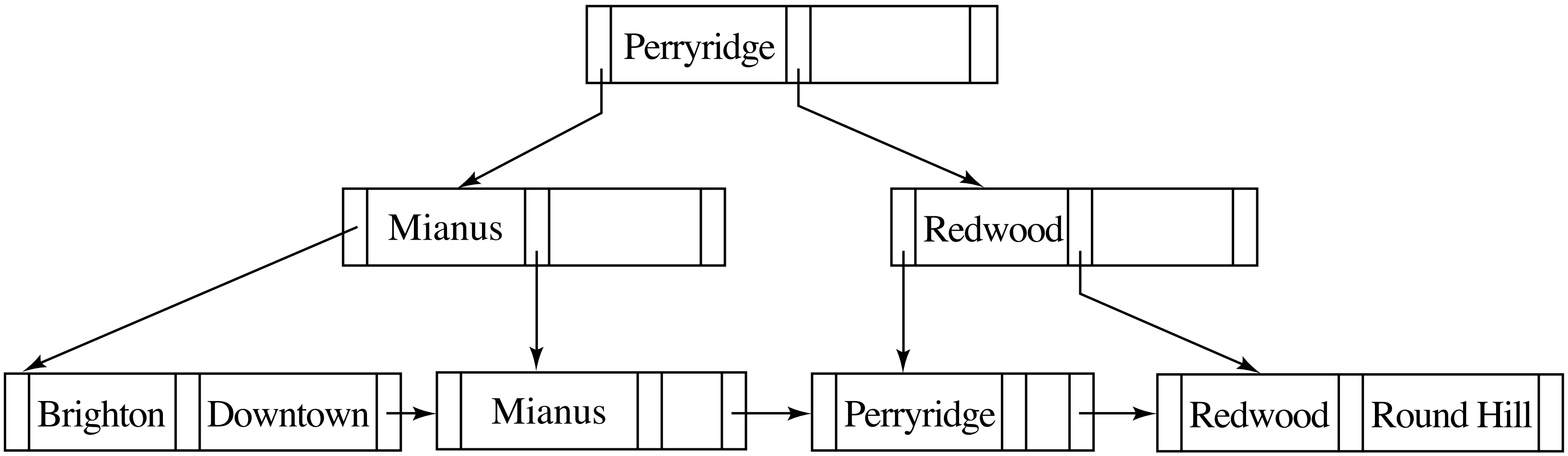


Non-Leaf Nodes in B^+-Trees

- Non leaf nodes form a multi-level sparse index on the leaf nodes. For a non-leaf node with m pointers:
  - All the search-keys in the subtree to which P_1 points are less than K_1
  - For $2 \leq i \leq n - 1$, all the search-keys in the subtree to which P_i points have values greater than or equal to K_{i-1} and less than K_i
  - All the search-keys in the subtree to which P_m points are greater than or equal to K_{m-1}

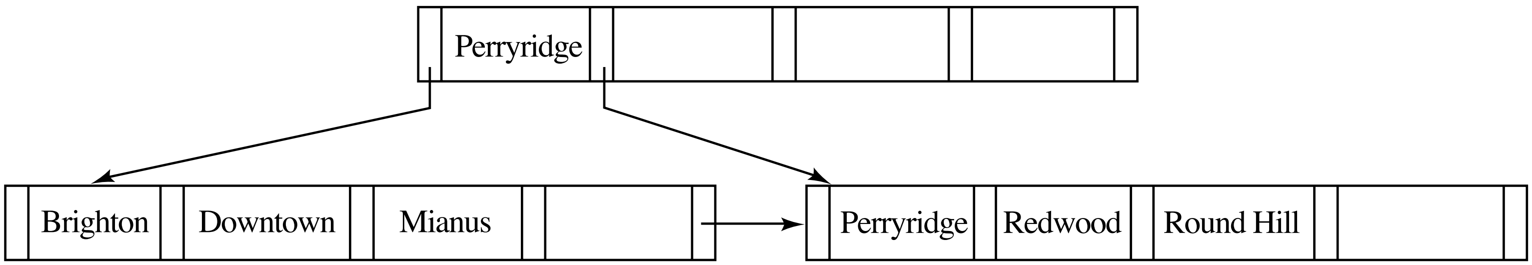
P_1	K_1	P_2	...	P_{n-1}	K_{n-1}	P_n

Example of a B+-tree



B+-tree for *account* file ($n = 3$)

Example of a B^+-tree



B^+-tree for *account* file ($n = 5$)

- Leaf nodes must have between 2 and 4 values ($\lceil (n-1)/2 \rceil$ and $n - 1$, with $n = 5$).
- Non-leaf nodes other than root must have between 3 and 5 children ($\lceil n/2 \rceil$ and n with $n = 5$).
- Root must have at least 2 children

Observations about B$^+$-trees

- Since the inter-node connections are done by pointers, there is no assumption that in the B$^+$-tree, the “logically” close blocks are “physically” close.
- The non-leaf levels of the B$^+$-tree form a hierarchy of sparse indices.
- The B$^+$-tree contains a relatively small number of levels (logarithmic in the size of the main file), thus searches can be conducted efficiently.
- Insertions and deletions to the main file can be handled efficiently, as the index can be restructured in logarithmic time (as we shall see).

Queries on B$^+$-Trees

- Find all records with a search-key value of k.
  - Start with the root node
    - Examine the node for the smallest search-key value $> k$.
    - If such a value exists, assume it is K_i. Then follow P_i to the child node
    - Otherwise $k \geq K_{m-1}$, where there are m pointers in the node. Then follow P_m to the child node.
  - If the node reached by following the pointer above is not a leaf node, repeat the above procedure on the node, and follow the corresponding pointer.
  - Eventually reach a leaf node. If key $K_i = k$, follow pointer P_i to the desired record or bucket. Else no record with search-key value k exists.

Queries on B$^+$-Trees (Cont.)

- In processing a query, a path is traversed in the tree from the root to some leaf node.
- If there are K search-key values in the file, the path is no longer than $\lceil \log_{\lceil n/2 \rceil}(K) \rceil$.
- A node is generally the same size as a disk block, typically 4 kilobytes, and n is typically around 100 (40 bytes per index entry).
- With 1 million search key values and $n = 100$, at most $log_{50}(1,000,000) = 4$ nodes are accessed in a lookup.
- Contrast this with a balanced binary tree with 1 million search key values — around 20 nodes are accessed in a lookup
  - above difference is significant since every node access may need a disk I/O, costing around 30 millisecond!

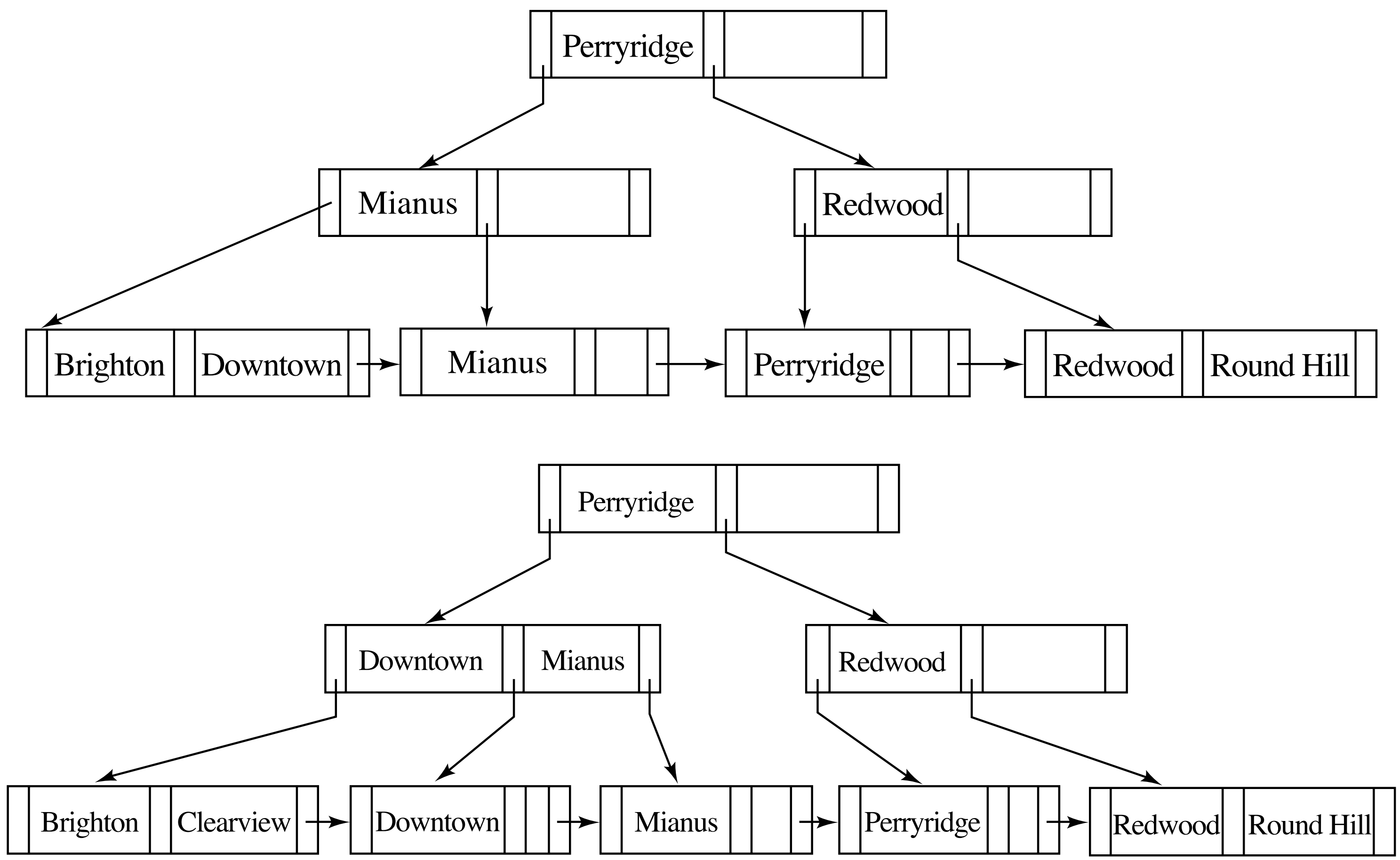
Updates on B^+-Trees: Insertion

- Find the leaf node in which the search-key value would appear
- If the search-key value is already there in the leaf node, record is added to file and if necessary pointer is inserted into bucket.
- If the search-key value is not there, then add the record to the main file and create bucket if necessary. Then:
  - if there is room in the leaf node, insert (search-key value, record/bucket pointer) pair into leaf node at appropriate position.
  - if there is no room in the leaf node, split it and insert (search-key value, record/bucket pointer) pair as discussed in the next slide.

Updates on B$^+$-Trees: Insertion (Cont.)

- Splitting a node:
  - take the n (search-key value, pointer) pairs (including the one being inserted) in sorted order. Place the first $\lceil n/2 \rceil$ in the original node, and the rest in a new node.
  - let the new node be p, and let k be the least key value in p. Insert (k, p) in the parent of the node being split. If the parent is full, split it and propagate the split further up.
- The splitting of nodes proceeds upwards till a node that is not full is found. In the worst case the root node may be split increasing the height of the tree by 1.

Updates on B$^+$-Trees: Insertion (Cont.)



B$^+$-Tree before and after insertion of “Clearview”

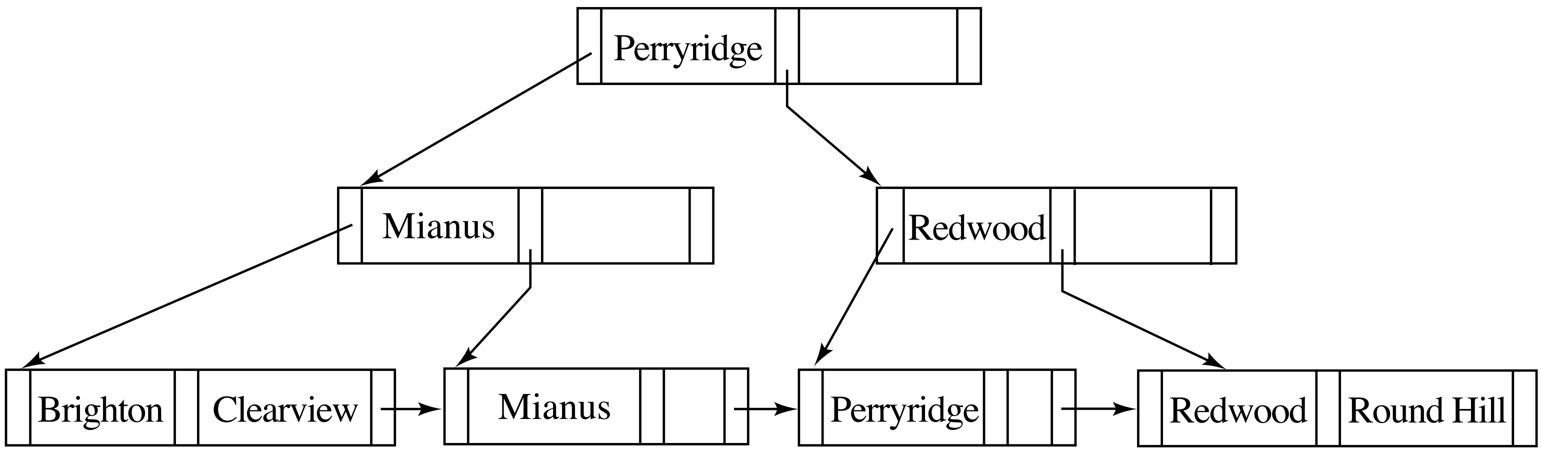
Updates on B$^+$-Trees: Deletion

- Find the record to be deleted, and remove it from the main file and from the bucket (if present)
- Remove (search-key value, pointer) from the leaf node if there is no bucket or if the bucket has become empty
- If the node has too few entries due to the removal, and the entries in the node and a sibling fit into a single node, then
  - Insert all the search-key values in the two nodes into a single node (the one on the left), and delete the other node.
  - Delete the pair (K_{i-1}, P_i), where P_i is the pointer to the deleted node, from its parent, recursively using the above procedure.

Updates on B$^+$-Trees: Deletion

- Otherwise, if the node has too few entries due to the removal, and the entries in the node and a sibling fit into a single node, then
  - Redistribute the pointers between the node and a sibling such that both have more than the minimum number of entries.
  - Update the corresponding search-key value in the parent of the node.
- The node deletions may cascade upwards till a node which has $\lceil n/2 \rceil$ or more pointers is found. If the root node has only one pointer after deletion, it is deleted and the sole child becomes the root.

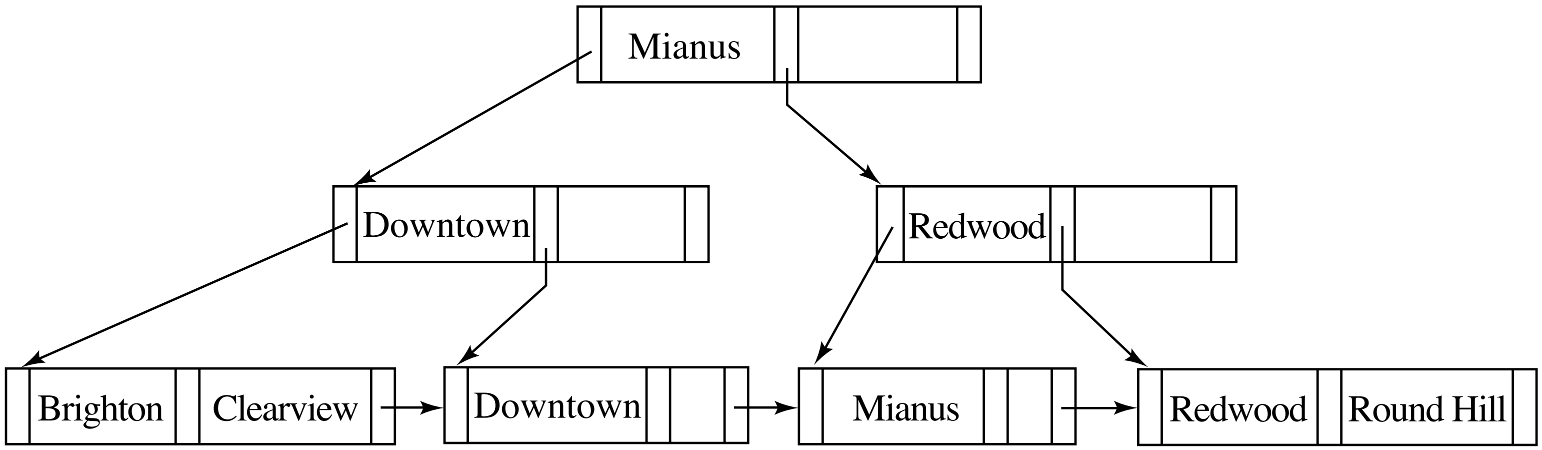
Examples of B$^+$-Tree Deletion



Result after deleting "Downtown" from *account*

- The removal of the leaf node containing "Downtown" did not result in its parent having too little pointers. So the cascaded deletions stopped with the deleted leaf node's parent.

Examples of B$^+$-Tree Deletion (Cont.)



Deletion of “Perryridge” instead of “Downtown”

- The deleted “Perryridge” node’s parent became too small, but its sibling did not have space to accept one more pointer. So redistribution is performed. Observe that the root node’s search-key value changes as a result.

B^+-Tree File Organization

- Index file degradation problem is solved by using B^+-Tree Indices. Data file degradation problem is solved by using B^+-Tree File Organization.
- The leaf nodes in a B^+-tree file organization store records, instead of pointers.
- Since records are large than pointers, the maximum number of records that can be stored in a leaf node is less than the number of pointers in a nonleaf node.
- Leaf nodes are still required to be half full.
- Insertion and deletion are handled in the same way as insertion and deletion of entries in a B^+-tree index.
- Good space utilization is important since records use more space than pointers. To improve space utilization, involve more sibling nodes in redistribution during splits and merges.

B-Tree Index Files

- Similar to B^+-tree, but B-tree allows search-key values to appear only once; eliminates redundant storage of search keys.
- Search keys in nonleaf nodes appear nowhere else in the B-tree; an additional pointer field for each search key in a nonleaf node must be included.
- Generalized B-tree leaf node

P_1	K_1	P_2	...	P_{n-1}	K_{n-1}	P_n

- Nonleaf node – pointers B_i are the bucket or file record pointers.

P_1	B_1	K_1	P_2	B_2	K_2	...	P_{m-1}	B_{m-1}	K_{m-1}	P_m

B-Tree Index Files (Cont.)

- Advantages of B-Tree indices:
  - May use less tree nodes than a corresponding B^+-Tree.
  - Sometimes possible to find search-key value before reaching leaf node.
- Disadvantages of B-Tree indices:
  - Only small fraction of all search-key values are found early
  - Non-leaf nodes are larger, so fan-out is reduced. Thus B-Trees typically have greater depth than corresponding B^+-Tree
  - Insertion and deletion more complicated than in B^+-Trees
  - Implementation is harder than B^+-Trees.
- Typically, advantages of B-Trees do not outweigh disadvantages.