2 Basic Definition and Law of Electromagnetic Radiation

Source of electromagnetic energy: the Sun.

$\begin{cases} \text{atmosphere interaction.} \\ \text{surface interaction} \\ \text{sensor interaction.} \end{cases}$

Electric Magnetic Radiation (EMR) travell in 真空 vacuum at the speed of light c 光速.

电场 磁场 辐射

$\Rightarrow \lambda v = c$ ← the speed of light

λ ← wavelength, v ← frequency

The frequency is inversely proportional to the wavelength.

All object (above absolute zero temperature) emit EMR at different wavelength.

no reflection so Black Body is black.

Black Body: ideal physical object absorbs all incident radiation and emitted all of them.

Lambertian Surface: idealized surface that reflects light equally in all direction.

Spectrum: the set of wavelengths.

bands = Intervals of wavelengths.

the visible Spectrum = our eyes are sensible to the energy within 0.4–0.7 um, blue, green, red are bands of this portion of the spectrum.

(Q) Energy captured by sensor depends on $\begin{cases} \text{the time required to sense it} \\ \text{the total area seen by the sensor.} \\ \text{the sensor aperture} \Rightarrow \text{孔径影响立体角 solid angle} \end{cases}$

$q_i = h v_i$, h is Planck constant

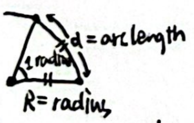
$v_i = \frac{c}{\lambda_i} = \frac{\text{光速}}{\text{波长}} = \text{频率}$ $\Rightarrow q_i = h\frac{c}{\lambda}$

$Q = \sum q_i$

什么是 ~~stereo radian~~.

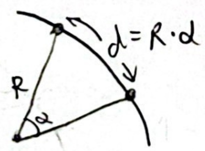
$\boxed{\text{2 dimension}}$ radian

单位弧度: 圆弧长度等于半径时的圆心角.



$d = R \Rightarrow \alpha = \frac{d}{R} = 1$ radian

半圆 $= \frac{\pi R}{R} = \pi$ radian



the unit of angle in 2D is Radian

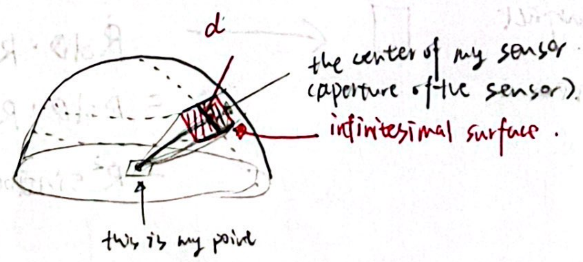
Radian $\alpha = \frac{d}{R}$.

3D完整球面面积为$4\pi R^2$

立体角 $w = \frac{A_1}{R^2} = \frac{4\pi R^2}{4\pi R^2}$

$= 4\pi$

$\boxed{\text{3 dimension}}$ stereo radian



the unit of angle is stereo radian

stereo radian $w = \frac{A_1}{R^2} = \frac{d^2}{R^2}$

Radiance : the electromagnetic energy per unit time, per unit area and per unit solid angle.

radiance $L = \frac{d^3Q}{dA\,dw\,dt}$.

Q: sensor collect electromagnetic energy.
A: emitted from a surface area.
t: during a time interval
W: arriving at the sensor operature with a solid angle.

radiant flux $\phi = \frac{dQ}{dt}$

发射 radiant exitance $M = \frac{d\phi}{dA}$

接收到. irradiance $E = \frac{d\phi}{dA}$

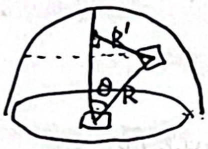
radiance $L = \frac{dE}{dw} = \frac{dE}{\pi}$

suppose the satellite is flying ascending along the sphere.

θ : angle between vertical to the surface

$d\theta$: ~~无穷~~ infinitesimal incremental in theta

angle.



$R' = R\sin\theta$

$d\lambda$: infinitesimal incremental in λ

angle

arc length I ⟵ $Rd\theta$ small variation in ascending direction along the sphere.

⟼ ⟵ $R'd\lambda$
$= R\sin\theta \cdot d\lambda$

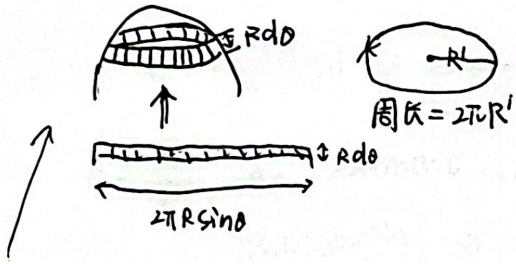
Surface Area. □ ⟵ $Rd\theta \cdot R'd\lambda$
$= Rd\theta \cdot R\sin\theta d\lambda$
$= R^2 \sin\theta\, d\theta\, d\lambda$

$$L = \frac{dE}{d\Omega \cos\theta} = \frac{dE}{d\omega} = \frac{dE}{\pi}$$

capital omega is the aperture of the sensor, which is infinitesimal and we approximate it with square. $\square$

omega : stereo angle.

If I consider an aperture along ~~spherical~~ spheric ring :



⇒ infinitesimal spheric ring (area):

$$d\Omega = 2\pi R \sin\theta \cdot R\, d\theta = 2\pi R^2 \sin\theta\, d\theta$$

In this case : the quantities exactly the same, because it depends only on θ, the λ is vary btw 0 and 2π.

⇒ ① spheric ~~Ring~~ ring w.r.t λ (area).

$$\int_0^{2\pi} R\sin\theta\, d\lambda \cdot R\, d\theta$$

$$= R^2 \sin\theta\, d\theta \int_0^{2\pi} d\lambda = 2\pi R^2 \sin\theta\, d\theta.$$

If we want to compute the complete effect, we need to make the ~~integer~~ integral also with respect to θ.

⇒ ② Energy ~~w.r.t~~ w.r.t θ (area).

θ vary from 0 to $\frac{\pi}{2}$.

$$\int_0^{\frac{\pi}{2}} d\Omega \cos\theta\, d\theta$$

$$= \int_0^{\frac{\pi}{2}} 2\pi R^2 \sin\theta \cos\theta\, d\theta = -2\pi R^2 \int_0^{\frac{\pi}{2}} \cos\theta\, d\cos\theta = -2\pi R^2 \cdot \frac{1}{2} \cos^2\theta \Big|_0^{\frac{\pi}{2}} = \pi R^2$$

Surface area.

then divide by R^2 for computing the stereo radian.

$$d\omega = \frac{\text{area}}{R^2} = \frac{\pi R^2}{R^2} = \pi.$$

⇒ Radiance $L = \frac{dE}{d\omega} = \frac{dE}{\pi}$.

One typical way of modeling our signal is <u>Fourier Transform</u> (sin + cos).
(Fourier integral decomposition).

The <u>Fourier Transform</u> of a signal $f(t)$ in terms of the <u>frequency</u> ν is given by.

$$\hat{f}(\nu) = \int_{-\infty}^{\infty} f(t)\, e^{-i2\pi\nu t}\, dt . \qquad \underset{(\text{Euler})}{e^{i\omega t}} = \cos(\omega t) + i\sin(\omega t) .$$

given in the frequency domain — given in the time domain

Inverse Fourier Transform

$$f(t) = \int_{-\infty}^{\infty} \hat{f}(\nu)\, e^{i2\pi\nu t}\, d\nu$$

the integral is taken over all frequencies, reconstructing the signal $f(t)$ in the time domain from its frequency components.

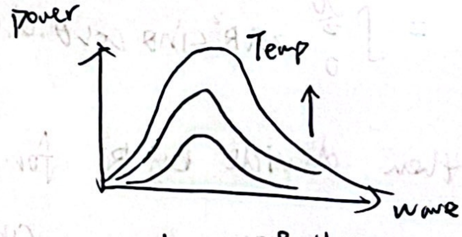
<u>Electromagnetic Signals</u> represented by Fourier Integral

$$\Phi = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \Phi_\lambda \, d\lambda$$

$$E = \frac{1}{2\pi} \int_{-\infty}^{+\infty} E_\lambda \, d\lambda$$

$$L = \frac{1}{2\pi} \int_{-\infty}^{+\infty} L_\lambda \, d\lambda$$

The law of electromagnetic radiation



Temperature

radiant exitance $M_{\lambda,B}(\lambda, T) = \dfrac{2\pi h c^2}{\lambda^5 \left(e^{\frac{hc}{\lambda k T}} - 1\right)}$

B: black body, absorbs all incident radiation and emitted all of them.
This law only valid for Black Body

h, k, c : constant , λ, T unknown.

$$L = \frac{M}{\pi}$$

Passing from exitance to radiance $\Rightarrow L_{\lambda,B}(\lambda, T) = \dfrac{2hc^2}{\lambda^5 \left(e^{\frac{hc}{\lambda k T}} - 1\right)}$

(1) given in terms of the wavelength.

$$M_{\lambda,B}(\lambda,T) = \frac{2\pi h c^2}{\lambda^5 (e^{\frac{hc}{\lambda kT}} - 1)}$$

(2) given in terms of the frequency.

$$M_{\nu,B}(\nu,T) = \frac{2\pi h \nu^3}{c^2 (e^{\frac{h\nu}{kT}} - 1)}$$

} Planck's law.

$(M_\lambda = M_\nu)$.

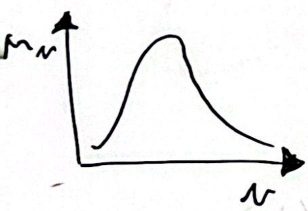
derive: M expressed in terms of wavelength = M expressed in terms of frequency

$c = \frac{\lambda}{\nu}$, an increment in the frequency corresponds to a decrement in the wavelength $\Rightarrow d\lambda = -d\nu$ 此升彼降.

$$M_{\lambda,B}(\lambda,T)\, d\lambda = -M_{\nu,B}(\nu,T)\, d\nu.$$

$$\frac{d\lambda}{d\nu} = \frac{d}{d\nu}\left(\frac{c}{\nu}\right) = \cancel{\frac{c}{\nu}} - \frac{c}{\nu^2}$$

$$\Rightarrow M_{\nu,B}(\nu,T) = -M_{\lambda,B}(\lambda,T)\boxed{\frac{d\lambda}{d\nu}}$$



$$= -M_{\lambda,B}(\lambda,T)\cdot\left(-\frac{c}{\nu^2}\right)$$

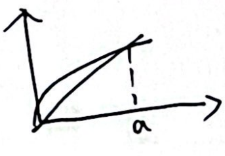
$$= \frac{2\pi h c^3 \cdot c^2}{\lambda^5 (e^{\frac{hc}{\lambda kT}} - 1)\nu^2 \cdot c^2} = \frac{2\pi h \nu^5}{(e^{\frac{h\nu}{kT}} - 1)\nu^2 c^2} = \frac{2\pi h \nu^3}{(e^{\frac{h\nu}{kT}} - 1)c^2}$$

compute the derivative w.r.t ν to find the maximum.

$$\frac{dM_\nu}{d\nu} = 0 \quad \Rightarrow \quad \frac{d}{d\nu}\left(\frac{\nu^3}{e^{\frac{h\nu}{kT}} - 1}\right) = 0$$

$$\Rightarrow 3\nu^2 (e^{\frac{h\nu}{kT}} - 1) - e^{\frac{h\nu}{kT}} \cdot \frac{h}{kT} \cdot \nu^3 = 0$$

$$\Rightarrow 3(e^x - 1) - e^x \cdot x = 0$$



$x = 0$

$x = a$.

we can also derivative w.r.t λ.

$$\Rightarrow \nu_{max} = \cdots \cdot T$$

$$\lambda_{max} = \cdots \cdot \frac{1}{T}$$

Stefan-Bolzmann law

exitance is directly proportional to power four of the Black Body temperature.

(total energy radiated per unit surface area, per unit time).

$$M_B(T) = \sigma T^4. \qquad \text{(In Black Body case)}.$$

$$M(T) = \varepsilon \sigma T^4 \leftarrow \text{In general case, not black body}$$

ε is emissivity 发射率 $0<\varepsilon<1$

Wien's law

$$\lambda_{max} = \frac{C_3}{T}$$ ← constant. (第5页 Planck law 求极值已证).

Energy conservation principle 能量守恒定律.

absorbed reflected transmitted.

$$\Phi_i(\lambda) = \Phi_a(\lambda) + \Phi_r(\lambda) + \Phi_t(\lambda)$$

$$\Rightarrow 1 = \frac{\Phi_a(\lambda)}{\Phi_i(\lambda)} + \frac{\Phi_r(\lambda)}{\Phi_i(\lambda)} + \frac{\Phi_t(\lambda)}{\Phi_i(\lambda)} = \underbrace{\alpha(\lambda)}_{\text{absorptivity}} + \overbrace{\rho(\lambda)}^{\text{reflectivity}} + \underbrace{\tau(\lambda)}_{\text{transmissivity}}.$$

3 satellite orbit | derive coordinate (ϕ, θ)

Circular orbit.

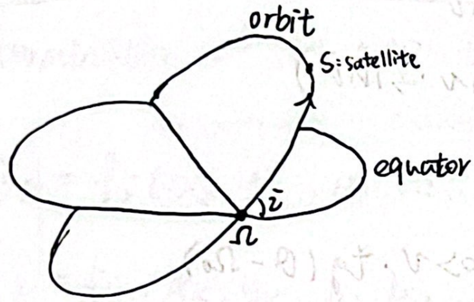
- constant angular velocity 角速度.
- revolution period depending on the orbital radius.
  公转周期.

centrifugal force $\Rightarrow$ $\boxed{\omega^2 r = \frac{GM}{r^2}}$ $\Leftarrow$ Gravity force.

ω = angular velocity.
r = distance btw satellite and the center of the earth.
M = mass of the earth.

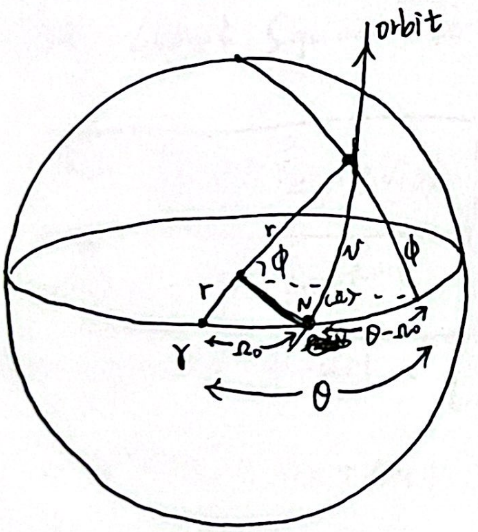
where $\omega = \frac{2\pi}{T}$

$$\Rightarrow \frac{4\pi^2}{T^2} r = \frac{GM}{r^2} \Rightarrow T = \sqrt{\frac{4\pi^2 r^3}{GM}}$$



i = Inclination: the angle btw orbital plane and equator

Ω: ascending node: intersection node btw the orbit and equator. In this node the satellite is going towards ascending direction.



~~that~~ true anomaly $\nu = w(t - T_0)$ angular velocity of the satellite.

$\Omega = N$ = ascending node

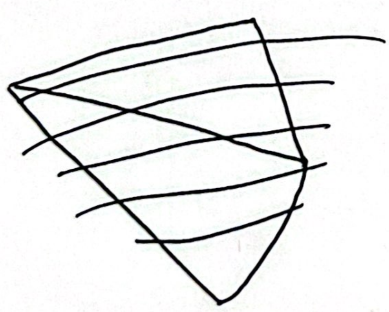
γ: intersection btw the ecliptic and celestial equator

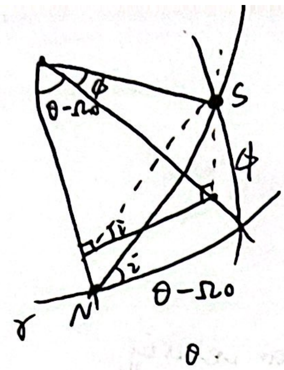
θ: the longitude of the satellite w.r.t the γ point.

ϕ = the latitude of the satellite.

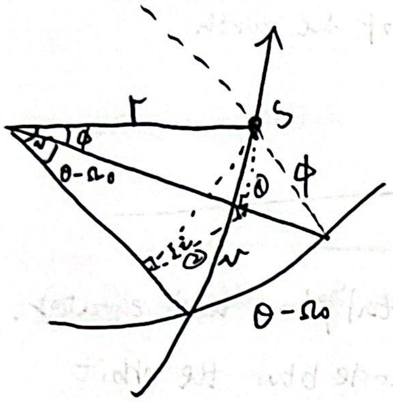
Ω_0: longitude of the ascending node.

r: ~~orbit~~ radius.





satellite coordinate (θ, ϕ)



作 ~~sate~~ satellite 垂直线于 equator

作垂直点的垂直线于 ν ascending node 所在线.

For latitude ϕ:

① $r\sin\phi = r\sin\nu \cdot \sin i$

$\Rightarrow \phi = \arcsin(\sin\nu \cdot \sin i)$

For longitude θ:

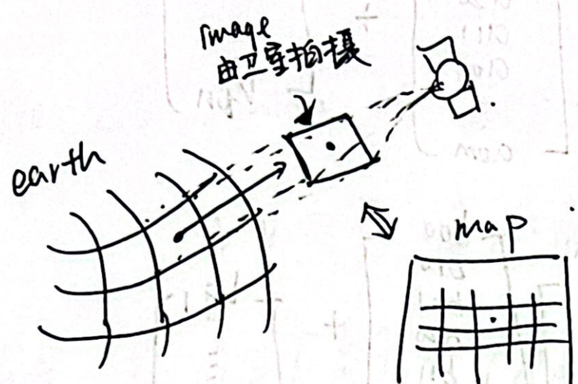
② $r\sin\nu \cdot \cos i = r\cos\nu \cdot tg(\theta - \Omega_0)$

$\Rightarrow \theta = \Omega_0 + arctg(tg\nu \cdot \cos i)$

Satellite coordinate (θ, ϕ)

5 Image Registration | 图像匹配

Georeferencing : from image to map.



Coordinates on ← 经纬度.

- earth (λ, ϕ)
- ☆ image (i, j) ← i and j should be integers 整数 → because they are rows and columns of the pixel in the image.
- ☆ map (x, y) ← x and y are real numbers 实数 ⇒ because they are coordinate of points on the map.

(image ⇄ map).

image registration : create relationship btw points of map and corresponding points of images. 将 image 中的像素位置转换到 map 中的位置.

Determination of the map to image transformation.

〈1〉 the corresponding coordinate on the map.

$$i_k = f_i(x_k, y_k, a_1, a_2, \dots, a_m) + \boxed{v_i}$$
$$j_k = f_j(x_k, y_k, b_1, b_2, \dots, b_m) + \boxed{v_j}$$

row and column in the image

← noise/error = stochastic variables.

k range from $0 \to n$

⇒ the number of the points n must be larger than the number of parameter

⇒ $n > m$.

〈2〉 Use Least Square to determine the transformation parameters $\hat{a} \dots \hat{b}$:

$$\boxed{\min \sum_k (v_i)_k^2 + (v_j)_k^2} \Rightarrow \hat{a}_1, \hat{b}_1, \hat{a}_2 \dots \hat{b}_m$$

↑ minimizing this function

we obtain the estimate of these parameter.

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transformation Model : $\boxed{\text{Polynomial Model}}$

$$\begin{cases} i = i(x,y) = a_{00} + a_{10}x + a_{01}y + a_{20}x^2 + a_{11}xy + a_{02}y^2 + \dots a_{m0}x^m + \dots a_{0m}y^m \\ j = j(x,y) = b_{00} + b_{10}x + b_{01}y + b_{20}x^2 + b_{11}xy + b_{02}y^2 + \dots b_{m0}x^m + \dots b_{0m}y^m \end{cases}$$

~~⇒~~ (i, j) and (x, y) are known. a and b are unknown ⇒ need to be estimated.

Another possible model = rototranslation with scale factor

$$\begin{bmatrix} i \\ j \end{bmatrix} = s \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_i \\ t_j \end{bmatrix}$$

It is a sub case of polynomial model when $m = 1$.

In general format.

$$i = \begin{bmatrix} i_1 \\ \vdots \\ i_n \end{bmatrix} = \begin{bmatrix} 1 & x_1 & y_1 & x_1^2 & x_1y_1 & y_1^2 & \cdots & y_1^m \\ \vdots \\ 1 & x_n & y_n & x_n^2 & x_ny_n & y_n^2 & \cdots & y_n^m \end{bmatrix} \cdot \overbrace{\begin{bmatrix} a_{00} \\ a_{10} \\ a_{01} \\ a_{20} \\ a_{11} \\ a_{02} \\ \vdots \\ a_{0m} \end{bmatrix}}^{a} + \overbrace{\begin{bmatrix} V_{i1} \\ \vdots \\ V_{in} \end{bmatrix}}^{V_i}$$

$$j = \begin{bmatrix} j_1 \\ \vdots \\ j_n \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & x_1 & y_1 & x_1^2 & x_1y_1 & y_1^2 & \cdots & y_1^m \\ \vdots \\ 1 & x_n & y_n & x_n^2 & x_ny_n & y_n^2 & \cdots & y_n^m \end{bmatrix}}_{W} \underbrace{\begin{bmatrix} b_{00} \\ b_{10} \\ b_{01} \\ b_{20} \\ b_{11} \\ b_{02} \\ \vdots \\ b_{0m} \end{bmatrix}}_{b} + \underbrace{\begin{bmatrix} V_{j1} \\ \vdots \\ V_{jn} \end{bmatrix}}_{V_j}$$

more compact:

$$\Rightarrow \begin{cases} \underline{i} = W\underline{a} + V_i \\ \underline{j} = W\underline{b} + V_j \end{cases} \quad \Rightarrow \begin{bmatrix} i \\ j \end{bmatrix} = \underbrace{\begin{bmatrix} W & 0 \\ 0 & W \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} a \\ b \end{bmatrix}}_{Z} + \underbrace{\begin{bmatrix} V_i \\ V_j \end{bmatrix}}_{V} = A \cdot Z + V$$

$[i]_{n\times1}$ $\quad [V_i]_{n\times1}$ $\quad [W]_{n\times ?}$

$[j]_{n\times1}$ $\quad [V_j]_{n\times1}$ $\quad [a],[b]_{?\times1}$

?: the number of coefficients of polynomial for the order m.

How to derive ? the number of coefficient?

A polynomial of order m with 2 variables include all possible monomials where the sum of the exponents of x and y is $\leq m$. $\boxed{a_{ij}x^{ij}}$ $\quad i,j \geq 0$, $i+j \leq m$

How many pairs (i,j) satisfies the condition $i+j \leq m$?

let us consider $l = i+j$, $l = 0 \dots m$

$\forall l$, the number of pairs is $\boxed{l+1}$

① $l=0 \Rightarrow i=0 \; j=0 \Rightarrow 1$ pair

② $l=1 \Rightarrow \begin{cases} i=1, j=0 \\ i=0, j=1 \end{cases} \Rightarrow 2$ pairs

③ $l=2 \Rightarrow \begin{cases} i=2 \; j=0 \\ i=1 \; j=1 \\ i=0 \; j=2 \end{cases} \Rightarrow 3$ pairs

$\Rightarrow$ therefore the total number of pair is

$$\sum_{l=0}^{m}(l+1) = \sum_{l=0}^{m} l + \sum_{l=0}^{m} 1 = \frac{m(m+1)}{2} + (m+1) = \frac{(m+1)(m+2)}{2}$$

$$? = \frac{(m+1)(m+2)}{2}$$

$\Rightarrow [W]_{n\times\frac{(m+1)(m+2)}{2}}$

$[a],[b]_{\frac{(m+1)(m+2)}{2}\times1}$

$\Rightarrow$ 对于 i和j合起来的整个方程 $\begin{bmatrix} i \\ j \end{bmatrix} = \cdots$ 来说.

the total ~~to~~number of equator (known) $= n+n = 2n$.

the total number of estimation (unknown) $= \frac{(m+1)(m+2)}{2} + \frac{(m+1)(m+2)}{2}$

$= (m+1)(m+2)$ $\quad$ Eq. (+6)

$\begin{bmatrix} i \\ j \end{bmatrix} = Az + v$, the total number of equator : $2n$
the total number of estimation : $(m+1)(m+2)$.

The L.S solution is obtained minimizing :

$$v^T v = \sum_k v_{ik}^2 + v_{jk}^2 = min .$$

参数估计 $\Rightarrow \hat{z} = \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = (A^T A)^{-1} A^T \begin{bmatrix} i \\ j \end{bmatrix} = \begin{bmatrix} (w^T w)^{-1} w^T i \\ (w^T w)^{-1} w^T j \end{bmatrix}$

残差估计 $\Rightarrow \hat{v} = \begin{bmatrix} i \\ j \end{bmatrix} - \begin{bmatrix} w & 0 \\ 0 & w \end{bmatrix} \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix}$

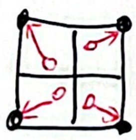
$$\Rightarrow \hat{\sigma}_0^2 = \frac{\hat{v}^T \hat{v}}{2n-(m+1)(m+2)} .$$

∵ image: (i,j) is integer , map (x,y) is real 但 registration后 map → image 得到的 (i,j)是 real.

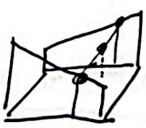
⇒ 所以需要将 real 变成 integer . making integer i and j (Resample)
real → integer .

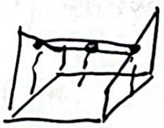
Three Approaches :

① Nearest Neighbour Resampling (map pixel takes values of closest image pixel)



② Bilinear Resampling





③ Bicubic Resampling .

6 Atmospheric Corrections

Correction for the error ① due to the atmospheric interaction with solar radiation
② due to the Earth Surface corrugation 起皱
③ measurement errors of each sensor.

Atmospheric interaction { ~~abso~~ absorption 吸收
~~dis~~ diffusion 散射 . { Rayleigh scattering $d \ll \lambda$ 蓝天
Mie Scattering $d \gg \lambda$ 雾霾天 .

散射  向各个方向进行散射 .

Rayleigh Scattering: applies when the particles causing the scattering are much smaller than the wavelength of the incoming light. ($d \ll \lambda$, d is the particle diameter)

From classical electromagnetism, the power radiated ~~by~~ in free space follows:

$$P \propto \frac{1}{\lambda^4}$$

=> This means that shorter wavelength scatter much more strongly than longer wavelengths.

→ blue 最小 蓝光散射更厉害 !

<effect>

① blue has shorter wavelength => blue ~~is~~ is scattered much more strongly
=> blue sky 蓝天

② when sunset, the sun is closer to the ~~ho~~ horizon
=> the atmosphere is thicker, lots of shorter wavelengths are scattered
=> remaining longer wavelength can passing through the atmosphere
=> sunset, the sky becomes red 夕阳 .

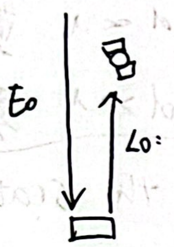
Mie Scattering ① The water droplets that make up clouds are of a comparable size to the wavelength in visible light ② all wavelengths of visible light are scattered approximately identically 云中水滴与可见光波长相当, 所有可见光都 Mie 散射 .
=> clouds appear to be white or gray .

Atmospheric correction: to derive the imaged Earth surface reflectivity $\rho_{surface}(\lambda)$ from the radiance $L_{sensor}(\lambda)$ registered by the sensor. $\frac{\text{reflected}}{\text{incident}}$

从传感器接收到的 radiance 推导出地球表面的真实反射率.

Atmospheric influence

<1> Ideal situation ①太阳,传感器在正上方 ② flat terrain ③ no atmosphere.



$$E_r = \rho E_0$$

$$L_0 = \frac{E_r}{\pi} = \frac{\rho E_0}{\pi}$$

E_0: incident irradiance 地表接收到

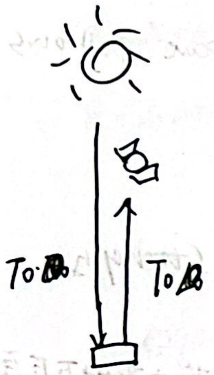
E_r: reflected irradiance 地表发射.

ρ: reflectivity 发射率

L_0 = radiance (recorded at sensor)

 π = solid angle of the upper half space where E_r is diffused (Lambertinian surface).

<2> with atmosphere



$E_0 \to T_0 E_0$

$L_0 \to T_0 L_0$

$$E = \rho T_0 E_0$$

$$L_s = T_0 \cdot \frac{E}{\pi} = T_0 \cdot \frac{\rho T_0 E_0}{\pi}$$

incident irradiance E_0 reduced by a factor T_0

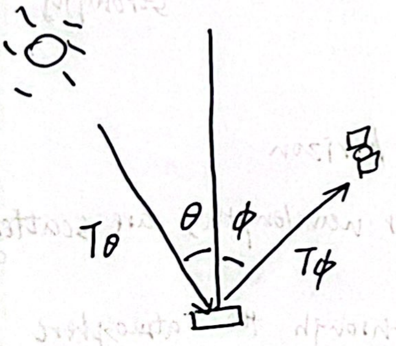
reflected irradiance L_0 reduced by a factor T_0

T = absorbance.

L_s: radiance (recorded at sensor).

<3> Sun at Zenith angle θ

Sensor at Zenith angle ϕ



$E_0 \to E = E_0 \cos\theta \to T_\theta E$

~~$\to E = T_\theta E_0 \cos\theta$~~

~~$E = T_\theta E_0 \cos\theta$~~

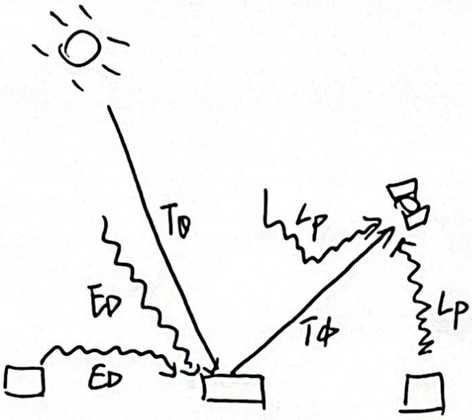
$$E = T_\theta E_0 \cos\theta$$

$$L_T = T_\phi \frac{\rho E}{\pi} = T_\phi \frac{\rho T_\theta E_0 \cos\theta}{\pi}$$

E = incident irradiance reduced by a factor $T_\theta > T_0$ (passing thicker layer) and by a factor $\cos\theta$ (spread over a larger area)

L_T = radiance (recorded at sensor) reduced by a factor $T_\phi > T_0$

<4> Additional incident irradiance E_D diffused from atmosphere ①
Additional radiance L_p diffused from atmosphere. ②



<3>

① $E_G = E + E_D$
$= T_\theta E_0 \cos\theta + E_D$

irradiance diffused from atmosphere.

② $L_s = T_\phi \frac{\rho E_G}{\pi} + L_p$
$= T_\phi \frac{\rho(T_\theta E_0 \cos\theta + E_D)}{\pi} + L_p$

radiance diffused from atmosphere

整理一下 : final $\boxed{L_s = T_\phi \frac{\rho}{\pi}(T_\theta E_0 \cos\theta + E_D) + L_p}$.

Radiometric Correction

变形.

✓ Radiance arriving at sensor:

$$L_s = T_\phi \cos\theta \left[T_\theta + \frac{E_D}{E_0 \cos\theta} \right] \frac{E_0}{\pi} \rho + L_p$$

express the true radiance (L_s) as a function of the ideal radiance (L_0) with two coefficients α and β.

ideal $L_0 = \frac{\rho E_0}{\pi} \cos\theta$

true $L_s = \alpha L_0 + \beta$

atmospheric condition parameters

$\alpha = \alpha(\theta, \phi, a) = T_\phi [T_\theta + \frac{E_D}{E_0 \cos\theta}]$
$\beta = \beta(a) = L_p$

✓ recorded at sensor: $x = k L_s + c$, k and c are parameters of the sensor

✓ ~~ideal~~ instead of ideal: $x_0 = k_0 L_0 + C_0$

Radiometric correction: determining x_0 from x.

for atmospheric influence $L_0 = \frac{L_s - \beta}{\alpha}$ $x_0 = k L_0 + C_0$

7 Histogram manipulation 直方图调整

n is the number of bits used ~~memories~~ memorize the radiance of each pixel.

Digital Number (DN) 数字编号 = values btw 0 and 2^n-1

Image Histogram: 用于表示图像中每个像素值DN出现的频率 (灰度值).

x-axis: 像素值 (DN: $0-2^n-1$)   relative frequency.

y-axis: 该~~像~~像素值在整幅图像中出现的相对频率 f_i

Density function

$$f_i = \frac{f_{ai}}{N}$$

the number of DN ~~tot~~ corresponding to i
像素值为i的像素个数.

the total number of pixels 总像素数.
行数×列数

$(\sum f_i = 1)$

Cumulative function

cumulative frequency → $F_i = \sum_{k=0}^{i} f_k$

$F_0 = f_0$

$F_1 = f_0 + f_1$

$F_2 = f_0 + f_1 + f_2$

⋮

$f_i = F_i'$

Example

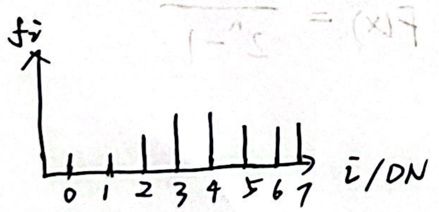
1	2	3	3
4	4	5	0
6	3	7	4
2	5	6	7

$n=3 \Rightarrow L = 2^n - 1 = 7 \Rightarrow$ DN 从 $0 \to 7$

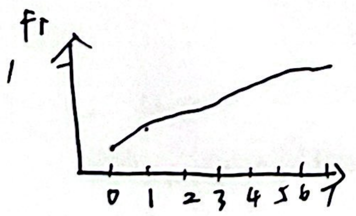
$N = 4\times4 = 16$ (rows × columns)

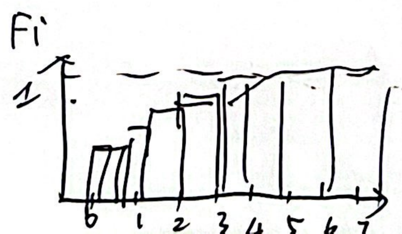
$i=$	0	1	2	3	4	5	6	7
f_{ai}	1	1	2	3	3	2	2	2
f_i	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{2}{16}$	$\frac{2}{16}$
F_i	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{4}{16}$	$\frac{7}{16}$	$\frac{10}{16}$	$\frac{12}{16}$	$\frac{14}{16}$	$\frac{16}{16}$

Histogram
(Density function)



cumulative frequency

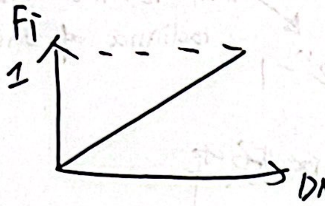




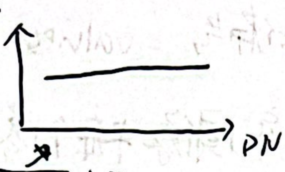
the best cumulative image

⇒ all different gray level weigh exactly ~~itis~~ in the same way.

Cumulative function



⇒ Density function



Contrast = difference btw maximum and minimum gray level of image.

⇒ the lower the constrast, the less readable is the image.

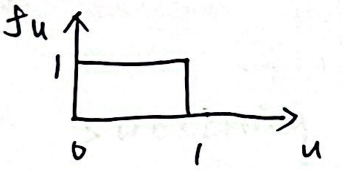
△ Contrast enhancement can be obtained by manipulating the image histogram.

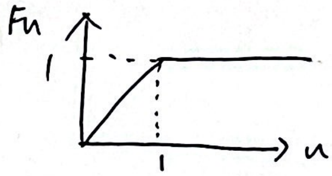
① Histogram equalization

(transformation of the values of pixels, making image histogram uniform 均匀)

Random Variables $U \sim Unif\,[0,1]$

uniform distribution in $(0,1)$: pdf and cdf.





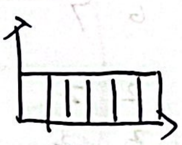
$f(u) = F'(u)$

pdf: probability density function

cdf: cumulative density function

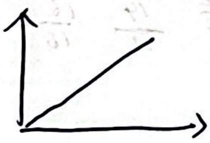
Image with optimal contrast = all values of gray equally present.

$f(x)$

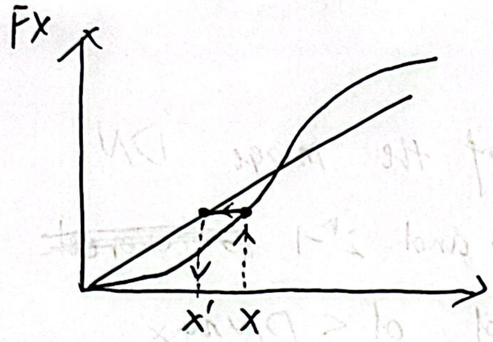


~~f(x)~~ $f(x) = \text{constant} = \frac{1}{2^n - 1}$

$F(x)$



$F(x) = \frac{x}{2^n - 1}$



Each pixel value x is replaced with a new value x' such that :

$$x' = L \cdot F_x(x) \quad (L = 2^n-1)$$

$$\underline{F_x(x')} = \frac{x'}{2^n-1} = \frac{L \cdot F_x(x)}{2^n-1} = \underline{F_x(x)}$$

Example

1	2	3	3
4	4	5	0
6	3	7	4
2	5	6	7

$n=3 \quad L = 2^n-1 = 7$

$N = 4 \times 4 = 16$

$i' = [L \cdot F]$

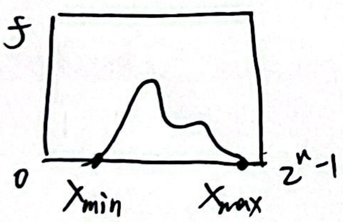
i	fai	f_i	F_i	i'
0	1	1/16	1/16	1/16×7
1	1	1/16	2/16	2/16×7
2	2	2/16	4/16	4/16×7
3	3	3/16	7/16	7/16×7
4	3	3/16	10/16	10/16×7
5	2	2/16	12/16	12/16×7
6	2	2/16	14/16	14/16×7
7	2	2/16	16/16	16/16×7=7

⇒ After histogram equalization, the new cumulative function we get may ~~will~~ not be uniform, but very closer to uniform !!

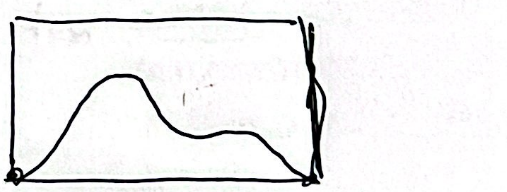
② Linear stretching

we look for a linear transformation of image DN ~~such~~ such that the whole interval btw 0 and 2^n-1

before f



Linear stretching ⟹ **after**



$$\boxed{DN' = a\,DN + b}$$

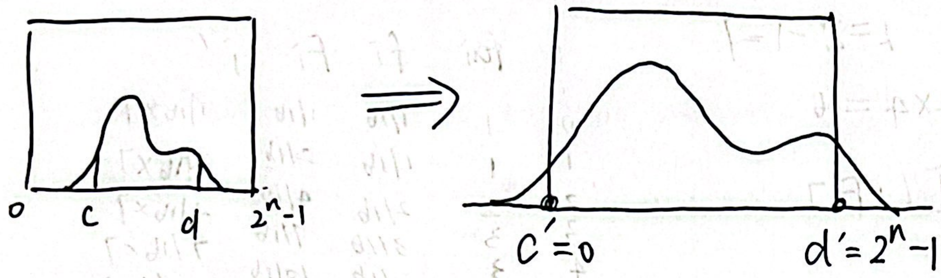
$$\begin{cases} a \cdot DN_{min} + b = 0 \\ a \cdot DN_{max} + b = 2^n-1 \end{cases} \Rightarrow \begin{cases} a = \dfrac{2^n-1}{DN_{max}-DN_{min}} \\ b = \dfrac{2^n-1}{DN_{max}-DN_{min}} \cdot DN_{min} \end{cases}$$

$$\Rightarrow DN' = \frac{2^n-1}{DN_{max}-DN_{min}} \cdot (DN - DN_{min})$$

③ ~~SE~~ Saturated Linear Stretching

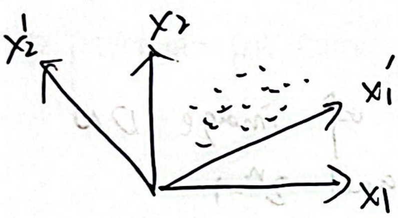
we look for linear transformation of the image DN such that the whole interval btw 0 and 2^n-1 ~~is covered~~ with the DN btw $c > DN_{min}$ and $d < DN_{max}$.

(不从最小值和最大值拉伸)



$$\Rightarrow DN' = \frac{2^n-1}{d-c}(DN-c) \qquad c<DN<d$$

10 principal Components.



two dimension:

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x^1 \\ x^2 \end{bmatrix}$$

$$\boxed{x' = R(\theta)\, x}$$

three Dimension:

$$x' = R(\theta_1, \theta_2, \theta_3)\, x$$

Finding the best possible rotation.

8 Template Filter

We take a window, play it pixel by pixel, and ~~recursing~~ recursively apply it to all the pixels of the image.

- remove undesired effects (noise, isolated pixels)
- emphasize interest features (roads, boundaries).

radiometric resolution

image of ~~dimes~~ dimension $I \times J = N$, ranging btw 1 and $2^{p}-1$

i : row index , f_{ij} : the original digital number in position (i, j)

j = column index , g_{ij} : transformed values.

Template filters are localized, position-invariant linear transformations of an image.

$\Rightarrow$ linear :
$$g_{ij} = \sum_{k}\sum_{m} h_{ij;km} \cdot f_{km}$$

$\Rightarrow$ position-invariant :
$$h_{ij;km} = h_{k-i,m-j}$$

the effect of moving window doesn't depend on the specific point where I'm moving. It depends only on the distance of the value of window

$$g_{ij} = \sum_{k}\sum_{m} h_{k-i,m-j} \cdot f_{km}$$

$\Rightarrow$ localized :
$$g_{ij} = \sum_{k=i-p}^{i+p}\sum_{m=j-p}^{j+p} h_{k-i,m-j} \cdot f_{km}$$

以ij为中心，在输入图像中取出(2p+1)(2p+1)的小区域，对每个像素乘上模板核，再求和，得到输出

using a (2p+1)(2p+1) template ij

Combination of all properties.

$$\cancel{g_{ij} = \sum_{k'=-p}^{p}\sum_{m'=-p}^{p} h_{k'm'} f_{i+k',j+m'}} \qquad g_{ij} = \sum_{k=-p}^{p}\sum_{m=-p}^{p} h_{km} \cdot f_{i+k,j+m}$$

center of the template (计算结果存在模板中心覆盖的位置上).

$$\boxed{g_{00} = \sum_{k=-p}^{p}\sum_{m=-p}^{p} h_{km} f_{km}}$$

$g_{ij} = f_{i-1,j-1} h_{-1,-1} + f_{i-1,j} h_{-1,0} + f_{i-1,j+1} h_{-1,1}$

filter

P8 ⑰

low-pass filter : ($\sum_{k=-p}^{p} \sum_{m=-p}^{p} h_{km} = 1$) Smoth our image

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

high-pass filter ($\sum_{k=-p}^{p} \sum_{m=-p}^{p} h_{km} = 0$) emphasize the difference of the image

$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix} \quad \begin{bmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{bmatrix}$$

9 band Algebra

Single band transformation

- Image Registration.
- Radiometric corrections
- Histogram manipulation
- Template Filters

~~multiple~~ Multispectral transformation.

- Band algebra
- Vegetation indexes
- Principal Componants.
- Classification.

Vegetation indexes $X_i = \dfrac{X_i^k - X_i^m}{X_i^k + X_i^m}$ k,m: band

near-infra-red wavelength. visible waveleng

Normalised Difference Vegetation Index $NDVI = \dfrac{NIR - V}{NIR + V}$