11 | Map Projection

to simplify computation, we use sphere instead of ellipsoid

Coordinate operations

map projection

- coordinate conversion = Same CRS, make operations, but don't change parameters of the ellipsoid.
- coordinate transformation: target and source are different CRS ⇒ different datum and different ellipsoid.

(spherical/Ellipsoid → plane)

Projection = translating the 3D real surface of the Earth to a 2D picture (map)

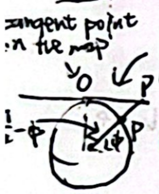
point will not distort

Shape, area, Angles, distance, direction ⇒ at least one aspect of the real world distort

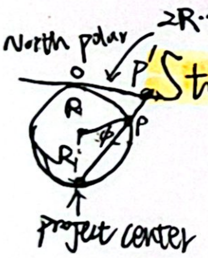
☆ 大类 小类

[Planar or Azimuthal Projection] (directions from a central point are preserved) (⇒great circles through the central point are represented by straight line on the map)

Orthographic: It is constructed from a point of perspective at an infinite distance from the tangent point



Centrographic (Gnomonic): It is constructed by using a point of perspective at the center of the Earth



Stereographic: It is constructed by using the tangent point's antipode as the point of perspective.

They are true (真实的) perspective projection, which means that they can be constructed projecting the surface of the Earth by extending lines from a point of perspective onto the plane

Azimuthal equidistant project (等距方位投影) = distance from the tangent point on map is proportional to surface distance on the earth

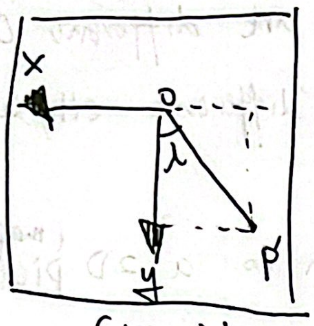
$$X = R\left(\frac{\pi}{2} - \phi\right)\sin\lambda$$

$$y = -R\left(\frac{\pi}{2} - \phi\right)\cos\lambda$$

ϕ: latitude parallel 纬线

λ: longitude meridian 经线

From the top: convention



Greenwich

已经算出 $OP' = 2R\tan\left(\frac{\pi}{4} - \frac{\phi}{2}\right)$

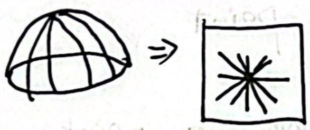
$$\Rightarrow \begin{cases} x = -2R\tan\left(\frac{\pi}{4} - \frac{\phi}{2}\right)\cdot\sin\lambda \\ y = 2R\tan\left(\frac{\pi}{4} - \frac{\phi}{2}\right)\cdot\cos\lambda \end{cases}$$

$$\Rightarrow y = -\frac{x}{\sin\lambda}\cdot\cos\lambda = -\cot\lambda\cdot x$$

For parallel: $x^2+y^2 = 4R^2\tan^2\left(\frac{\pi}{4} - \frac{\phi}{2}\right)$

$\forall\lambda$, if $\phi = \bar{\phi} \leftarrow$ constant, represent circle on the map.

For ~~west~~ meridians: $\frac{y}{x} = -\cot\lambda \Rightarrow \underline{y = -\cot\lambda\cdot x}$ line equation.

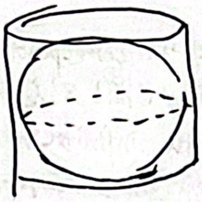


$\forall\phi$, if $\lambda = \bar{\lambda}$, represent a straight line passing the origiean.

[Cylindrical or Conical Projection] Cylinder 圆柱 Cone 圆锥
投影到 Cylinder or Cone Surface

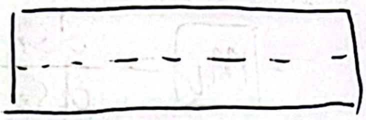
①

envolop the earth/ Sphere/ellipsoid with Cylinder/Cone.

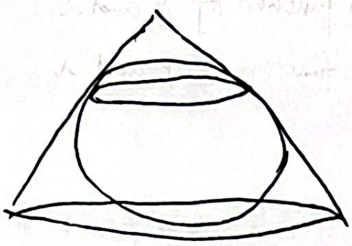


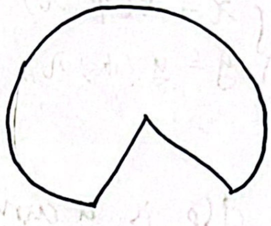
the first step project on the surface with distortion

②



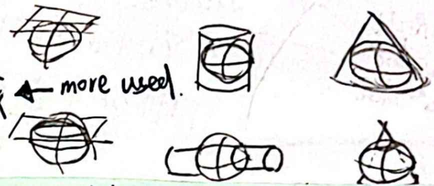
the second step, in cutting and opening, there is no more distortion





Planar, cylinder, cone are three main surface of projection to envelop the earth.

Intersection with the reference surface] ① tangent 切线
② secant 割线 ← more used.



Orientation of the reference surface] ① normal
② transverse
③ oblique /ə,bli:k/ 倾斜

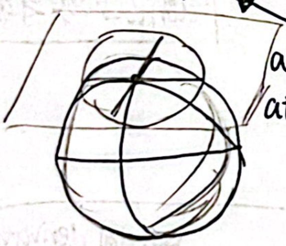
• normal (inline with the earth's axis)

• transverse (at right angles to the earth's axis)

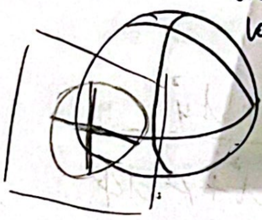
• oblique (any angle in between)

normal

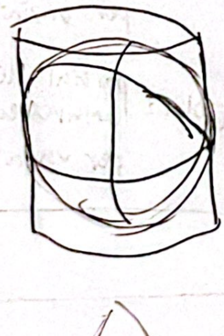
compute or map a region which is at a certain point

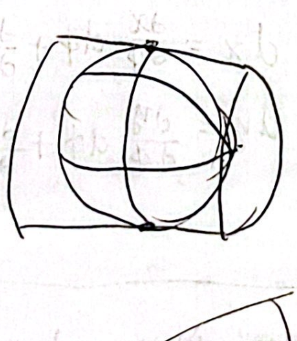


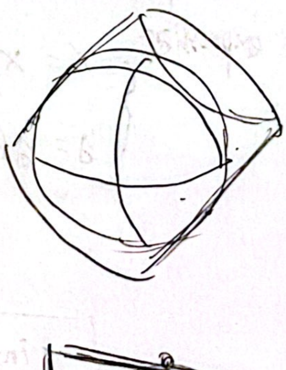
transverse 适合 defin (regi long

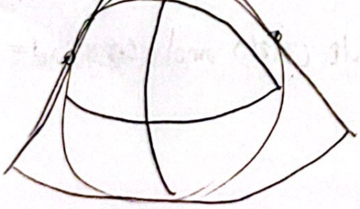


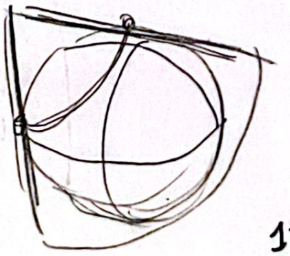
compare with tangent, secant is more used. because the tangency features (points, lines) are exactly where we have distortion.
where the projection surface is intersecting (secant), the model of the earth is no distortion. the surface coincide and therefore we have no distortion.
And the farther we go with respect to this intersection, the greater is the distortion.











以上的n种选择 was simply pure geometric projection (以前的研究工作).

⇒ 转变为后来的 $\boxed{\text{Analytical Projection}}$ 有3种 distortion

① linear distortion
② Angular distortion
③ surface distortion.

define an index △ linear distortion !!

describing the distortion with respect to curve. → $\boxed{m_l} = \dfrac{dS_r}{dS_e}$

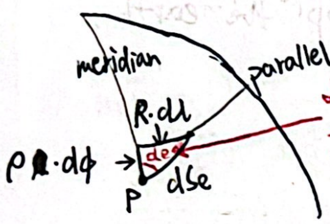
- m_l: Linear
- dS_r ← representation ← length of the corresponding infinitesimal element (arc) on the map. 地图上对应无穷小元素长度.
- dS_e ← earth. ← length of the infinitesimal arc on the reference surface 椭圆体上无穷小元素长度.

this is what I want to derive. ⇒ $\begin{cases} x = x(\phi, \lambda) \\ y = y(\phi, \lambda) \end{cases}$

we want x is a function of ϕ and λ

we want y is a function of ϕ and λ.

[Surface of earth]. dS_e is a curve.



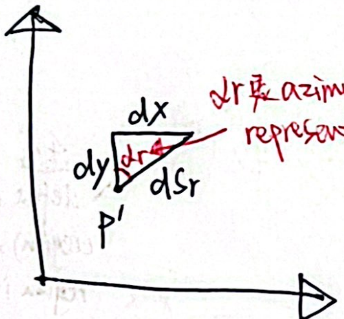
α_e是 azimuth on the earth 方位角(从正北方向顺时针到目标方向之间的角度)

r: radius of the parallel 纬线半径.

ρ: radius of the meridian 子午线半径.

~~$dS_e = R^2 d\phi^2$~~ $dS_e^2 = r^2 d\lambda^2 + \rho^2 d\phi^2$.

[projection] map dS_r is segment.



α_r是 azimuth under representation.

$dS_r^2 = dx^2 + dy^2$

differential 微分
derivatives 导数

$$\Rightarrow m_l^2 = \frac{dS_r^2}{dS_e^2} = \frac{dx^2 + dy^2}{r^2 d\lambda^2 + \rho^2 d\phi^2}$$

derive a general expression $\begin{cases} x = x(\phi, \lambda) \\ y = y(\phi, \lambda) \end{cases} \Rightarrow \begin{aligned} dx &= \frac{\partial x}{\partial \phi} d\phi + \frac{\partial x}{\partial \lambda} d\lambda \\ dy &= \frac{\partial y}{\partial \phi} d\phi + \frac{\partial y}{\partial \lambda} d\lambda \end{aligned}$

$\frac{\partial x}{\partial \phi} d\phi$: partial derivative of x with respect to ϕ, per variation in ϕ

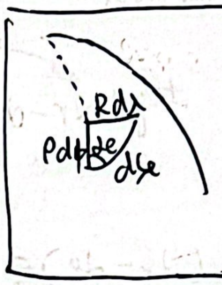
$\frac{\partial x}{\partial \lambda} d\lambda$: partial derivative of x with respect to λ, per variation in λ

Infinitesimal changes in x and y can be expressed using total differentials.
Change in x depends on the change in latitude ($d\phi$) and longitude ($d\lambda$), weighted by the partial derivatives of x

$$\begin{cases} dx^2 = \left(\frac{\partial x}{\partial \phi} d\phi + \frac{\partial x}{\partial \lambda} d\lambda\right)^2 \\ \quad = \left(\frac{\partial x}{\partial \phi}\right)^2 d\phi^2 + \left(\frac{\partial x}{\partial \lambda}\right)^2 d\lambda^2 + 2\frac{\partial x^2}{\partial \phi \partial \lambda} d\phi d\lambda \\ dy^2 = \left(\frac{\partial y}{\partial \phi} d\phi + \frac{\partial y}{\partial \lambda} d\lambda\right)^2 \\ \quad = \left(\frac{\partial y}{\partial \phi}\right)^2 d\phi^2 + \left(\frac{\partial y}{\partial \lambda}\right)^2 d\lambda^2 + 2\frac{\partial y^2}{\partial \phi \partial \lambda} \cancel{d\lambda} d\phi d\lambda . \end{cases}$$

$$\Rightarrow ds_r^2 = dx^2 + dy^2 = \underbrace{\left[\left(\frac{\partial x}{\partial \phi}\right)^2 + \left(\frac{\partial y}{\partial \phi}\right)^2\right]}_{e} d\phi^2 + \underbrace{\left[\left(\frac{\partial x}{\partial \lambda}\right)^2 + \left(\frac{\partial y}{\partial \lambda}\right)^2\right]}_{g} d\lambda^2 + 2\underbrace{\left[\frac{\partial x^2 + \partial y^2}{\partial \phi \partial \lambda}\right]}_{f} d\phi d\lambda$$

$$= e\, d\phi^2 + 2f\, d\lambda d\phi + g\, d\lambda^2$$



$ds_e \cdot \sin\alpha_e = R\, d\lambda$

$ds_e \cdot \cos\alpha_e = \rho\, d\phi$

$\Rightarrow tg\,\alpha_e = \frac{R\, d\lambda}{\rho\, d\phi}$

$$\Rightarrow m_l^2 = \frac{ds_r^2}{ds_e^2} = \frac{e\, d\phi^2 + 2f\, d\lambda d\phi + g\, d\lambda^2}{R^2 d\lambda^2 + \rho^2 d\phi^2}$$

$$= \frac{\cancel{\rho^2 d\phi^2}\left[\frac{e}{\rho^2} + \frac{2f R d\lambda}{\rho^2 d\phi R} + \frac{g R^2 d\lambda^2}{\rho^2 d\phi^2 R^2}\right]}{\cancel{\rho^2 d\phi^2}\left[1 + \frac{R^2 d\lambda^2}{\rho^2 d\phi^2}\right]} = \frac{\frac{e}{\rho^2} + \frac{2f}{\rho R}\, tg\alpha_e + \frac{g}{R^2}\, tg^2\alpha_e}{1 + tg^2\alpha_e}$$

$$\Rightarrow m_l = \sqrt{\frac{\frac{e}{\rho^2} + \frac{2f}{\rho R}\, tg\alpha_e + \frac{g}{R^2}\, tg^2\alpha_e}{1 + tg^2\alpha_e}}$$

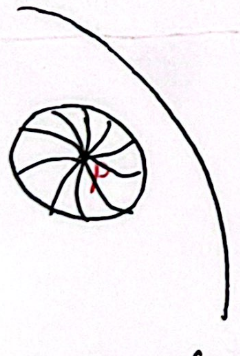
$m_l(\alpha_e = 0) = \sqrt{\frac{e}{\rho^2}} = \frac{\sqrt{e}}{\rho} \rightarrow dy$

$\tan\alpha_e = 0$

$m_l(\alpha_e \rightarrow \frac{\pi}{2}) \rightarrow \frac{\sqrt{g}}{\sqrt{R^2}} = \frac{\sqrt{g}}{R} \rightarrow dx$

$\tan\alpha_e \rightarrow$ infinite

☆ the linear distortion depends on
① the point of t
② ellipsoid.
③ azimuth.



if we are on ellipsoid and consider po
circle
⇒ the direction we have on the earth ar
map (representation), because the linear distortion depends on azimuth.

⇒ the infinitesimal circle on the ellipsoid doesn't correspond to an infinitesimal circle on the map.

⇒ the equivalent of a small circle on the earth becomes an ellipsoid on the map.

△ Areal distortion = Surface distortion.

$$m_A = \frac{dA_r}{dA_e}$$

dA_r ← infinitesimal surface (area) on the map.

dA_e ← infinitesimal surface (area) on the earth.

不用推导

$\Rightarrow m_A = \frac{1}{\rho r}\sqrt{eg-f^2}$ $\Rightarrow$ depend on ① location ② ellipsoid.

★ <u>independ on</u> azimuth.

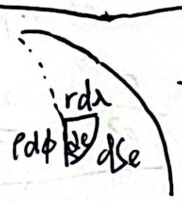
△ Angular Distortion

$$m_\alpha = tg(\alpha_r - \alpha_e)$$

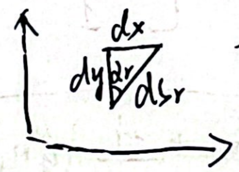
α_r ← angle between two infinitesimal segment on the map.

α_e ← angle between two infinitesimal arc on the earth.

$$\Rightarrow \underline{\underline{m_\alpha}} = tg(\alpha_r - \alpha_e) = \frac{tg\alpha_r - tg\alpha_e}{1 + tg\alpha_r \cdot tg\alpha_e}$$



$tg\alpha_e = \frac{r d\lambda}{\rho d\phi}$



$tg\alpha_r = \frac{dx}{dy}$

$$= \frac{\frac{dx}{dy} - \frac{r d\lambda}{\rho d\phi}}{1 + \frac{dx}{dy} \cdot \frac{r d\lambda}{\rho d\phi}}$$

$$m_l = \frac{dS_r}{dS_e} = \frac{dx}{r d\lambda} = \frac{dy}{\rho d\phi}$$

$\Rightarrow m_l(\alpha = \frac{\pi}{2}) = \frac{dx}{r d\lambda} \Rightarrow dx = m_l(\alpha = \frac{\pi}{2}) \cdot r d\lambda = \frac{\sqrt{e}}{r} \cdot r d\lambda = \underline{\sqrt{e}\, d\lambda}$

$m_l(\alpha = 0) = \frac{dy}{\rho d\phi} \Rightarrow dy = m_l(\alpha=0) \cdot \rho d\phi = \frac{\sqrt{g}}{\rho} \cdot \rho d\phi = \underline{\sqrt{g}\, d\phi}$

$$= \frac{\frac{\rho\sqrt{e}\, r d\lambda}{r\sqrt{g}\, \rho d\phi} - tg\alpha_e}{1 + \frac{\rho\sqrt{e}\, r d\lambda}{r\sqrt{g}\, \rho d\phi} \cdot tg\alpha_e}$$

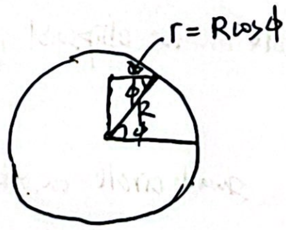
$$= \underline{\underline{\frac{\left(\frac{\rho}{r}\sqrt{\frac{e}{g}} - 1\right) tg\alpha_e}{\frac{\rho}{r}\sqrt{\frac{e}{g}}\, tg^2\alpha_e + 1}}}$$

$\Rightarrow$ depend on ① location ② ellipsoid ③ azimuth.

If we compute sphere surface instead of ellipsoid surface:

$\rho = R$

$r = R\cos\phi$



△ Conformal Map 保角投影 : maintain the angles and shape.

① maintain the Angles

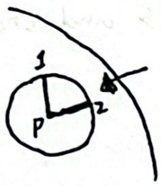
we derive in page 5.

Angular distortion $m_\alpha = tg(\alpha_r - \alpha_e) = 0$ And $m_\alpha = \dfrac{(\frac{\rho}{r}\sqrt{\frac{g}{e}} - 1)\, tg\alpha_e}{1 + \frac{\rho}{r}\sqrt{\frac{g}{e}}\, tg^2\alpha_e}$

$\Rightarrow \frac{\rho}{r}\sqrt{\frac{g}{e}} - 1 = 0 \Rightarrow \boxed{g = \frac{r^2 e}{\rho^2}}$

② maintain the shape.

if it is conformal, it is not equivalent



∵ ① location ✓
② ellipsoid ✓ ⇒ P1和P2投影后长度不等 ⇒ 不是保角投影
③ azimuth ✗ 圆变成椭圆

保角投影必须 maintain the shape ⇒ the only possibility is the ~~the~~ linear distortion doesn't dependent on azimuth !!!

⇒ constant with respect to azimuth.

$$\begin{cases} g = \dfrac{r^2 e}{\rho^2} \\ m_l^2 = \dfrac{\frac{e}{\rho^2} + \frac{2f}{\rho \cdot r} tg\alpha_e + \frac{g}{r^2} tg^2\alpha_e}{1 + tg^2\alpha_e} \end{cases} \Rightarrow m_l^2 = \frac{\frac{e}{\rho^2} + \frac{2f}{\rho \cdot r} tg\alpha_e + \frac{r^2 e}{\rho^2 r^2} tg^2\alpha_e}{1 + tg^2\alpha_e}$$

$$= \frac{\frac{e}{\rho^2}(1 + tg^2\alpha_e) + \frac{2f}{\rho \cdot r} tg\alpha_e}{1 + tg^2\alpha_e}$$

constant

$$= \boxed{\frac{e}{\rho^2}} + \boxed{\frac{\frac{2f}{\rho \cdot r} tg\alpha_e}{1 + tg^2\alpha_e}} = 0 \Rightarrow f = 0$$

we want this part dispear

$\Rightarrow \boxed{f = 0}$

①+② ⇒ we have two condition $\begin{cases} g = \frac{r^2 e}{\rho^2} & \langle 1 \rangle \\ f = 0 & \langle 2 \rangle \end{cases}$

we use isometric latitude 等量代换

$du = \frac{\rho}{r} d\phi \Rightarrow \frac{du}{d\phi} = \frac{\rho}{r}$

so $\frac{dx}{d\phi} = \frac{dx}{du} \cdot \frac{du}{d\phi} = \frac{dx}{du} \cdot \frac{\rho}{r}$

$\Rightarrow (\frac{dx}{d\phi})^2 = (\frac{dx}{du})^2 \cdot \frac{\rho^2}{r^2}$

⟨1⟩ $g = \frac{r^2 e}{\rho^2} \Rightarrow (\frac{\partial x}{\partial \lambda})^2 + (\frac{\partial y}{\partial \lambda})^2 = \frac{r^2}{\rho^2}[(\frac{dx}{d\phi})^2 + (\frac{dy}{d\phi})^2]$

$= \cancel{\frac{r^2}{\rho^2}}[(\frac{dx}{du})^2 \cancel{\frac{\rho^2}{r^2}} + (\frac{dy}{du})^2 \cancel{\frac{\rho^2}{r^2}}] = (\frac{dx}{du})^2 + (\frac{dy}{du})^2$

⟨2⟩ $f = 0 \Rightarrow$ 同乘 $\frac{r}{\rho}$ $\frac{r}{\rho}\frac{dx}{d\phi} \cdot \frac{\partial x}{\partial \lambda} + \frac{r}{\rho}\frac{dy}{d\phi} \cdot \frac{dy}{d\lambda} = 0 = \frac{dx}{du}\frac{dx}{d\lambda} + \frac{dy}{du}\frac{dy}{d\lambda}$

$du = \frac{\rho}{r}d\phi$

令 $\begin{cases} \frac{dx}{d\lambda} = a & \frac{dy}{d\lambda} = b \\ \frac{dx}{du} = c & \frac{dy}{du} = d \end{cases} \Rightarrow$ ⟨1⟩: $a^2 + b^2 = c^2 + d^2$

⟨2⟩: $ac + bd = 0 \Rightarrow a = -\frac{bd}{c}$

⟨2⟩代入⟨1⟩: $\frac{b^2 d^2}{c^2} + b^2 = c^2 + d^2$

$\Rightarrow b^2 d^2 + b^2 c^2 = c^4 + c^2 d^2$

$\Rightarrow (b^2 - c^2) d^2 + c^2(b^2 - c^2) = 0$

$\Rightarrow (b^2 - c^2)(c^2 + d^2) = 0$

$\Rightarrow b^2 - c^2 = 0 \Rightarrow (b + c)(b - c) = 0$

① $b = -c$ $a = d$ ✓

② $b = c$ $a = -d$ ✗

then we have $\boxed{\text{differential equation of conformal Map}}$

$$\begin{cases} \dfrac{dx}{du} = -\dfrac{dy}{d\lambda} \\ \dfrac{dx}{d\lambda} = \dfrac{dy}{du} \end{cases}$$

differential equation of Conformal Map 等角投影的微分方程. if a map is conformal

trick for memory

① related to parallel

② related to latitude

$$\frac{\partial x}{\partial u} = -\frac{\partial y}{\partial \lambda}$$

just remember which is minus. it cannot be equivalent.

③ latitude is connected to the meridian

$$\frac{\partial x}{\partial \lambda} = \frac{\partial y}{\partial u}$$

parallel, meridian, longitude, latitude

⇒ they are not coherent each other

⇒ put the sign minus on the opposite.

if we have $f(x_0+\varepsilon)$ (finite, infinitesimal)

using Taylor Series to express it (approximate it):

$f(x_0+\varepsilon) \cong$ ← approximate

$f(x_0) + \frac{df}{dx}\big|_{x_0}\cdot\varepsilon + \frac{d^2f}{dx^2}\big|_{x_0}\cdot\frac{\varepsilon^2}{2!} + \dots$

For checking if a map is conformal or not, try to apply the equation of the map.

→ satisfy these two differential equation ⇒ conformal Map and maintain angles and shape.

Cauchy-Riemman Equation (which admit the complex solution)

虚数单位 imaginary unit

Small value of λ given in radiant (express λ as angle between 0 and 360)

$$y + ix = f(u + i\lambda)$$

λ is longitude, it means that my projection is valid only for a certain longitude.

real variable

Approximate with Taylor Series:

incremental square, $i^2=-1$, $i^3=-i$, λ power three, $i^4=1$

$$y + ix = f(u) + i\lambda\cdot f'(u) - \frac{\lambda^2}{2!}f''(u) - i\frac{\lambda^3}{3!}f'''(u) + \frac{\lambda^4}{4!}f''''(u) + \dots$$

the factorial 2阶乘

four derivative of u

On the two sides of the equation, we have some terms representing the real coefficient of the complex number and some parts representing the imaginary part of the complex number

⇒ So we split in the 2 equation and also remove the i due to the imaginary part.

represent what happened along the parallel: odd derivative.

$$x = \lambda\cdot f'(u) - \frac{\lambda^3}{3!}f'''(u) + \frac{\lambda^5}{5!}f'''''(u) + \dots$$

$$y = f(u) - \frac{\lambda^2}{2!}f''(u) + \frac{\lambda^4}{4!}f''''(u) + \dots$$

represent what happened along the meridian

⇒ what we have to decide is how to define the $f(u)$

how we want to characterize our map with respect to latitude

⇒ some times we have x and y and need to derive ϕ and λ (x, y are coordinates on the map; ϕ, λ are location on the earth)

inverse solution $\quad u + i\lambda = g(y + ix)$

$$u + i\lambda = g(y) + ix\cdot g'(y) - \frac{x^2}{2}\cdot g''(y) - i\frac{x^3}{3!}\cdot g'''(y) + \dots$$

△ For conformal maps, the distortions are given by : 已知 conformal map 计算 m_l, m_α, m_A

two equation with respect to ϕ and λ

If prof ask what is the Angular distortion of conformal map, don't do computation, it's always $m_\alpha = 0$

$$\begin{cases} m_l = \sqrt{\frac{e}{\rho^2}} = \sqrt{\frac{g}{r^2}} \\ m_\alpha = 0 \\ m_A = m_l^2 \end{cases}$$

$e = \frac{\rho^2}{r^2} \cdot g$

during the exam, prof will write the equation of map and ask to check if the map is conformal or not / ask what is the linear distortion of this map
=> we need to depend on the map and decide if doing the derivative w.r.t ϕ or λ.
If I think about stereographic map in term q is $\arctan(\frac{\pi}{4} - \frac{\phi}{2})$ and λ is only cos and
=> so probably it's easier to do the derivat w.r.t ~~to~~ cos and sin => we use $m_l = \sqrt{\frac{g}{r^2}}$

In conformal map, we don't really ~~do~~ to compute the area distortion.

△ In case of ~~Con~~ Conformal Maps on the Sphere $\begin{cases} \rho = R \\ r = R\cos\phi \end{cases}$

$$\begin{cases} \frac{\partial x}{\partial \lambda} = \cos\phi \frac{\partial y}{\partial \phi} \\ \cos\phi \frac{\partial x}{\partial \phi} = -\frac{\partial y}{\partial \lambda} \end{cases}$$

$$du = \frac{\rho}{r} d\phi = \frac{R}{R\cos\phi} d\phi = \frac{1}{\cos\phi} d\phi$$

$$m_l = \sqrt{\frac{e}{R^2}} = \sqrt{\frac{(\partial x/\partial\phi)^2 + (\partial y/\partial\phi)^2}{R^2}}$$

R = radius of reference sphere.

△ Equal Area Maps.

$\sqrt{eg - f^2}$

$$m_A = \frac{dA_r}{dA_e}$$

Ellipsoid: $\frac{\partial y}{\partial \phi}\frac{\partial x}{\partial \lambda} - \frac{\partial x}{\partial \phi}\frac{\partial y}{\partial \lambda} = \rho \cdot r$ $\Rightarrow m_A = \frac{1}{\rho \cdot r}\sqrt{eg - f^2}$

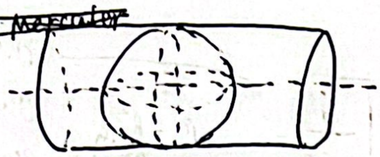
Sphere: $\frac{\partial y}{\partial \phi}\frac{\partial x}{\partial \lambda} - \frac{\partial x}{\partial \phi}\frac{\partial y}{\partial \lambda} = R^2\cos\phi$

The surface distortion is everywhere: $m_A = 1$.

ΔGauss Projection : Cylindrical transverse projection. 圆柱横向投影.

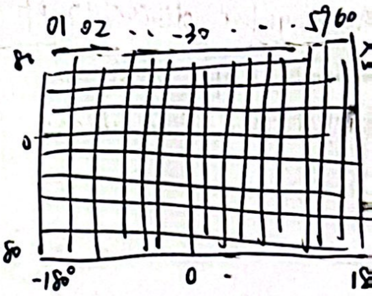
~~UTM map : universal Transverse Mercator~~

using Gauss Projection



It was designed as a conformal map

$$\begin{cases} m_l \\ m_\alpha = 0 \\ m_A = m_l^2 \end{cases}$$

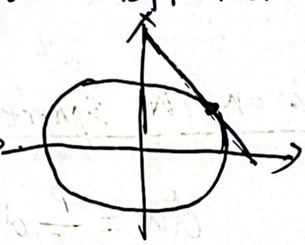


应用 UTM map : Universal transverse Mercator

divide longitude ~~into~~ from 01 to 60 of amplitude 6°

divide latitude into 20 bands of amplitude 8° from c to x (no I and O)

Lambert map : Conical Projection 圆锥投影



It ~~was~~ is conformal map

应用 Mercator map : a case of Lambert Conic Conformal Map Projection.

15 GIS advanced functionalities: exploring data / field analysis.

Vector Data: represented by coordinate. using point, line, surface as primitives. 基元.

Raster Data: represented the world by regular grid of small units, called pixel.

we distinguish two possible views of phenomena. 现象区分为两种不同的表现方式.

Object: characterized by distinct entities having exact boundary, we can model the world by objects consisting of geometric primitives, as points, lines, surfaces, using vector model.
清晰的边界与实体.

☆ Field: phenomena are widespread and continuously distributed.
One possibility is to use a raster model (grid with a regularly shaped pixels)
Another possibility is to use TIN

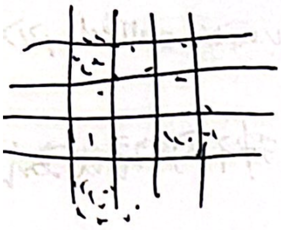
analysing the world from the field view: it means that the world consists of properties which
- vary continuously
- can be measured everywhere in space.

The steps for ~~##~~ field description are:

- sampling the real surface
- exploring data
- interpolating the observations to give a continuous surface representation

Sampling the real surface.

△ Random Sampling: selecting measurement points uniformly (均匀) and independently over the reference area.

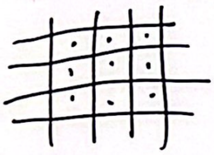


drawbacks ① points must be located and selected independently from each other and that could be more time consuming than a ~~grid~~ regular grid sampling.

② It can lead to a non uniform distribution unless we sample a great number of points and in that case will increase the cost.

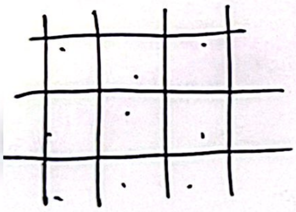
△ Regular Grid / ~~Random Stratified~~ sampling.

Distribution on a regular grid can affect of distortion (bias)



in case the sampling grid coincides in frequency with a phenomena regularly distributed in space.

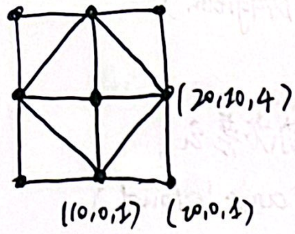
△ Random Stratified (分层) Sampling.



Compromise between the Random Sampling and Regular Grid.

we have locations randomly selected within blocks regularly covering the area of interest

TIN



for the lowest right triangle the facet is given by:
分面.

$$z = ax + by + c$$

$$\begin{cases} 1 = 10a + 0b + c \\ 1 = 20a + 0b + c \\ 4 = 20a + 10b + c \end{cases} \Rightarrow \begin{cases} a = 0 \\ b = \frac{3}{10} \\ c = 1 \end{cases} \Rightarrow z = \frac{3}{10}y + 1.$$

$$\text{Slope} = \arctan\sqrt{f_x^2 + f_y^2} \qquad \text{aspect} = \arctan\left(\frac{f_y}{f_x}\right)$$

$$f_x = \frac{\partial f}{\partial x} \qquad f_y = \frac{\partial f}{\partial y}$$

where the function is $z = f(x,y) = ax + by + c$

$f_x = a \quad f_y = b$

$\Rightarrow$ slope $= \arctan\sqrt{a^2 + b^2}$ aspect $= \arctan\left(\frac{b}{a}\right)$

~~aspect = arctan $\sqrt{\frac{b}{a}}$~~

Contour: the output is given by polyline that connect points of equal value.
输出由连接相等值的点的折线给出.

Slope: the value of the cell in the output slope raster gives the slope value, ~~obtain~~ obtained by calculating the maximum rate of charge between each cell and its neighbours.

The lower the slope value $\Rightarrow$ the flatter is the terrain.

the higher the slope value $\Rightarrow$ the steeper is the terrain

Aspect: the value of each cell in the output aspect raster indicates the direction the cell faces

Flat slope have no direction and given a value of -1.

Aspect is measured clockwise (顺时针) in degrees from 0 (north) to 360

Exploring Data.

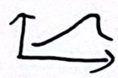
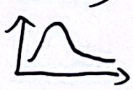
- Data Distribution (summary statistics, Histogram, Diagram)
- Global trends
- Spatial autocorrelation (semivariance / covariance cloud)
  半变异函数 / 协方差云.
- clue for outlier presence
  异常值

Summary statistics

位置特征. Data set Location
- minimum, maximum
- mean (average)
- median

离散性. Data set variability
- Standard ~~deviation~~ deviation 标准差: $\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(z_i-\bar{z})^2}$
- Range 极差: max − min
- Average absolute deviation $AAD = \sum_{i=1}^{N}\frac{|z_i-\bar{z}|}{N}$
- Median Absolute deviation $MAD = med|z_i - z_{med}|$
- Interquantile Range 四分位距. 75%分位数 − 25%分位数

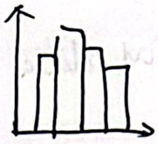
对称性. Data set symmetry. skewness 偏度 $= \dfrac{\frac{1}{N}\sum_{i=1}^{N}(z_i-\bar{z})^3}{(N-1)\sigma^3}$

- 0 → 完全对称 (正态)
- <0 左偏 (长尾在左) 
- >0 右偏 (长尾在右) 

平坦度/尖锐度

Data Set flatness or peakness Kurtosis 峰度 $= \dfrac{\frac{1}{N}\sum_{i=1}^{N}(z_i-\bar{z})^4}{(N-1)\sigma^4}$

Histogram 直方图



to visualize data distribution

the main problem is to determine the optimal bin size or the number if bin.

① $n_b = \lceil \frac{max - min}{3.49\sigma} \sqrt[3]{N} \rceil$

② $n_b = \cancel{log} \lceil \log_2 N + 1 \rceil$

Diagram Probability plot, normal and general Quantile Quantile Plot QQ Plot

L能QQ图.

Probability Plot assess whether a data set follows a given distribution such as normal.

If the data set follow the chosen distribution, points will form approximately straight line. ~~Depat~~ Departures from the straight line indicate departures from the chosen distribution.

Normal Probability Plot: follow normal Distribution.

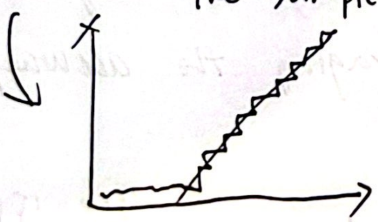
Normal QQ Plot is formed by plotting the quantile values for the selected attributes (horizontal axis) versus the quantile values for a standard normal distribution (vertical axis).

Gener QQ Plot: to determine if two data sets with a common distribution.

It is formed by plotting the quantile values for the 1st selected attribute (horizontal axis) versus the quantile values for the 2nd selected attribute (vertical axis). Both axis are in units of their respective data sets.

The sample sizes are not required to be equal

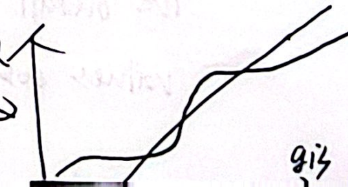
Normal QQ plot



服从正态分布

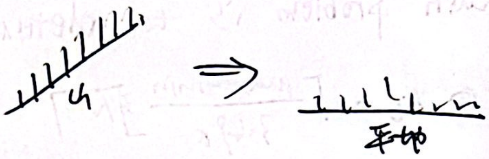
follow normal Distribution

不服从正态



Global trend : Global variation in spatial data caused by environmental factor.

Why remove the global trend.



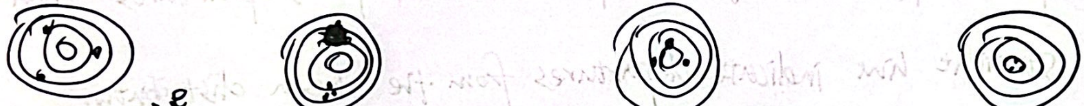
The mean changes with location and violates kriging stationary assumption

Method: { classification
polynomial interpolation

Precision = the level of refinement in the performance of a method used to obtain a result.

Accuracy : the degree of conformity (一致性) with the "true"

~~Acept~~



Neither precise nor accurate | Precise but not accurate | Accurate but not precise | Precise and accurate

Accuracy assessment : cross-validation.

the dataset is partitioned in two ~~dataset~~ subset (ramdom sampling)

first one is training set to train the model

Second one is test set to test the quality of model.

Leave-one-out cross-validation.

partition the original dataset in k subsets of equal size.

The model is trained k times, using each subsets in turn as the test set, with the remaining subsets being the training sets.

The overall accuracy is obtained by averaging the accuracy values computed on each subset.

~~Sat Spatial autocorrection~~

Spatial autocorrelation.

~~Is~~ measures the similarity of spatially near objects.

A tool to analyze the spatial autocorrelation is the covariance/semivariogram cloud.

semivariogram is to verify if two points close to each other take on close values and if, on the contrary, points at long distance show greater ~~variance~~ variation.

The variability with distance can be measured with the variogram cloud which is the plot of all sampled points pairs giving as x and y axis

$$X\text{-axis} = d_{ij} = |x_i - x_j|$$

$$Y\text{-axis}: V_{ij} = \frac{1}{2}(z_i - z_j)^2.$$

box plot consists of 25%, 50%, 75%, maximum ~~A~~ values.

histogram 和 box plot ~~可以~~ are clue of outlier.

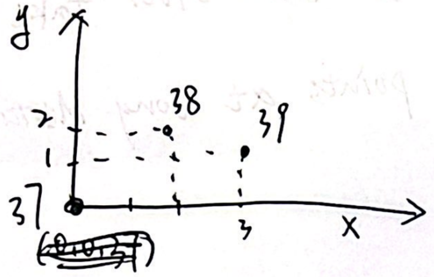
Serivariogram (derive it from observation)

① compute the distance ~~bt~~ btw points $\sqrt{x_0^2+y_0^2}$

② compute corresponding ~~half~~ halved squared difference in Z value

③ assign each pair to corresponding range.

④ calculate the mean from the N_h pairs falling in each range. plot that value at the midpoint distance of each range.



⇒ A (0,0,37)
B (2,2,38)
C (3,1,39)

A与B ① distance : $\sqrt{2^2+2^2}=\sqrt{8}$ $h=\sqrt{8}$

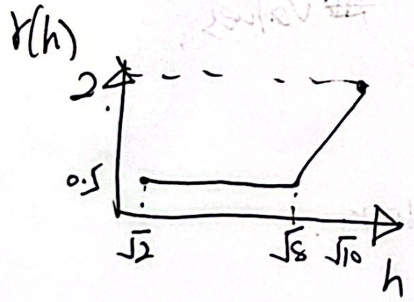
② difference : $\frac{1}{2}\times(37-38)^2=0.5$ $\gamma(h)=0.5$

A与C ① distance : $\sqrt{3^2+1^2}=\sqrt{10}$ $h=\sqrt{10}$

② difference : $\frac{1}{2}\times(37-39)^2=2$ $\gamma(h)=2$

B与C ① distance $=\sqrt{1^2+1^2}=\sqrt{2}$ $h=\sqrt{2}$

② difference $\frac{1}{2}\times(38-39)^2=0.5$ $\gamma(h)=0.5$



~~$\hat{\gamma}(h) = \frac{}{2N_h}$~~

$$\hat{\gamma}(h) = \underset{\|x_i-x_j\|\approx h}{\text{Mean}}\left\{\frac{1}{2}[Z(x_i)-Z(x_j)]^2\right\}$$

if 两点离得近，值差不多 ⇒ 空间相关性强

if 两点离得远，值差很多 ⇒ 空间相关性弱.

Kriging 是一种利用空间相关性预测的插值方法.

↑入权重的计算依赖于半变异函数

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17 Spatial Interpolation

classification of interpolation methods.

Deterministic Method : Do not use Probability theory, the result is determined. and always the same.

Stochastic Method: Based on the concept of randomness, the interpolated surface is one of the many possible that could have produced from the observed data set. Stochastic methods include different kinds of kriging.

/stəʊˈkæstɪk/
随机的

Global Method: A single function interpolated for the whole region, the value of a single point affects the entire map.

Local Method: An algorithm applies to a subset of the total points. it affects only the portion of the map closer to it.

Exact Method: ~~Interpl~~ ~~Inte~~ Interpolate surface passes through all points whose values are known.

Approximate Method: Based on least squares approach, which interpolate the surface to reduce the effects of error.

Inverse Distance Weighting (IDW) 反距离权重.

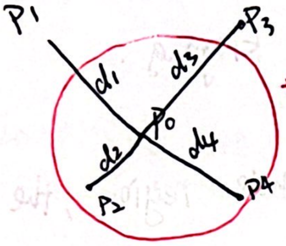
对称点会有误差

is a deterministic exact interpolator.

↑ the same input always produces the same result (no randomness)

↑ the interpolated surface passes through all known data points.

IDW estimates the value at an unknown point P_0 by averaging the values of nearby known points $P_1, P_2 \ldots P_n$, giving more weight to closer points.
The closer a point is to P_0, the greater its influence on the estimated value.



moving window: only points within a certain radius from P_0 are used

the known value at point P_i

$$\hat{Z}(P_0) = \frac{\sum_{i=1}^{n} \frac{Z(X_i)}{d_{i0}^{r}}}{\boxed{\sum_{i=1}^{n} \frac{1}{d_{i0}^{r}}}}$$

interpolated ~~point~~ value at the unknown point P_0

to normalize the weights
⇒ make the sum of the weights = 1

power parameter

$$d_{i0}^{r} = |P_i - P_0|^{r}$$

distance btw P_i and P_0

To optimize the power value (r), the root mean square prediction error (RMSPE) $= \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\hat{y}_i - y_i)^2}$ has to be minimized.

The RMSPE is calculated from cross-validation
交叉验证

Cross-Validation is used to evalute how well a model can predict unknown dat

one of cross-validation method is leave-one-out 留一法.

↑ removes one point at a time, predicts its value using all the other points, and compute the error btw estimated and true values, repeat this for all points.

Adv: Only make decision on power parameter.

No assumption required of the data.

Disad: No assessment of prediction error.

It can produce "bulls eyes" patterns of concentric contour around the data points.

9/3

Kriging 克里金插值法.

It provides linear regression estimate, which is unbiased and has minimum error variance.

It assume data from a stationary (平稳的) stochastic process.

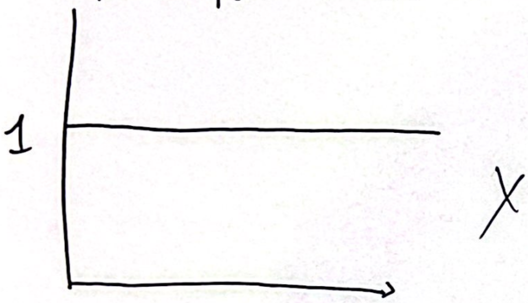
{ constant mean.
  variance only depends on distance btw pairs, independent of positi

$$Z(x_0) = \sum_{i=1}^{N} \hat{\lambda}_i \cdot Z(x_i)$$

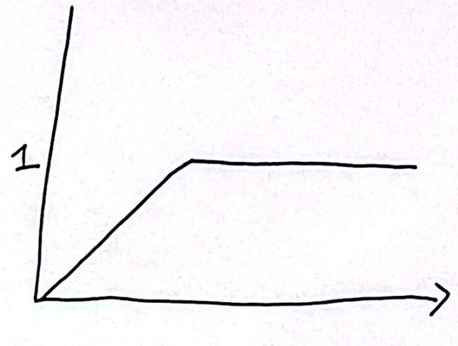
unknown weight for the measured value at the i-th location.
got from Kriging.

In Kriging the weights are based on the spatial autocorrelation (空间自相关). : Semivariogram (to build the function about λ) modeling is a key step.

~~no~~ nugget model

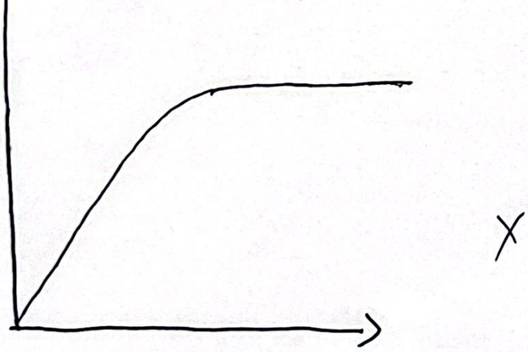


linear with threshold ~~threshold~~.



describing the spatial autocorrelatio about data,
flat: no dependency

Spherical model



exponential model.