

<자료구조 기말 강의자료 요약>

CH5-1. TREES

* 기본적인 tree terms *

- height : 세대
= depth = number of levels
- node degree : 자녀 수
= # of children
- tree degree : 부모 노드가 가진 자녀의 수 중 최댓값
= max node degree

<conditions>

- tree : a finite set of one or more nodes (empty X)
- leaf (terminal) : node degree = 0 (child X)

<Binary Tree>

- def) a finite set of nodes that is either empty or consists of a root and two disjoint binary trees (left / right subtree)
- empty 가능 (nonempty의 경우, root 존재)
- full binary tree의 k번째 깊이의 node의 개수 $\Rightarrow 2^k - 1$ nodes
 \Rightarrow array representations
- complete binary tree with n nodes $\Rightarrow \text{depth}(\text{height}) \leq \log_2 n + 1$
- parent의 index = i 일 경우, leftChild는 2i번째, rightChild는 2i+1번째에 존재
(단, $2i < n$, $2i+1 < n$ 일 때)
- node structure (linked list ver.)

```
typedef struct node *tree_pointer;
```

```
typedef struct node {  
  
    int data; tree_pointer left_child, right_child;  
  
}
```

<Binary Tree Traversal>

1) preorder => V L R

```
void preorder (treePointer ptr){
```

```
    if (ptr){
```

```
        visit(ptr);
```

=> V

```
        preorder(ptr->leftchild);
```

=> L

```
        preorder (ptr->rightchild);
```

=> R

```
    }
```

```
} **** visit = print ****
```

2) inorder => L V R

3) postorder = > L R V

4) level order => FIFO queue

```
void level_order(treePointer ptr){
```

```
    int front=rear=0;
```

```
    treePointer queue[MAX_SIZE];
```

```
    if(!ptr) return;
```

```
    addq(front,&rear, ptr);
```

```
    for (;;) {
```

```
        ptr = deleteq(&front,rear);
```

```
        if(ptr) {
```

```
            printf("%d",ptr->data);
```

```
            if(ptr->leftChild) addq(front,&rear,ptr->leftchild);
```

```
            if(ptr->rightChild) addq(front,&rear,ptr->rightchild);
```

```
        }
```

```
        else break;
```

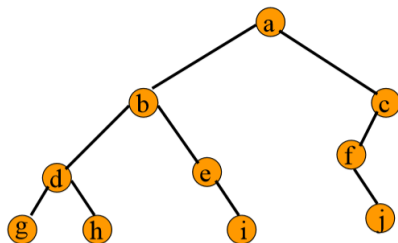
```
    } // for문 fin. }
```

5) Iterative Inorder => LIFO stack

* left node가 null을 만날때까지 stack에 push -> stack pop -> right node가 push

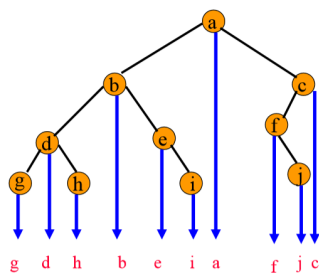
```
void iter_inorder (treePointer node){  
    treePointer stack[MAX_SIZE];  
  
    for (;;) {  
        for (; node; node = node->leftChild) push(&top, node);  
  
        node = pop(&top);  
  
        if (!node) break;  
  
        printf("%d", node->data);  
  
        node = node->rightChild; } // outer for문 fin.  
}
```

** 출력 순서



1) preorder : a b d g h e i c f j

2) inorder : g d h b e i a f j c => projection (squishing)



3) postorder : g h d i e b j f c a

4) level order : a b c d e f g h i j

<Binary Search Tree>

- (key, value)로 이루어짐
- x를 가진 노드의 left subtree => x보다 작은 값들
- x를 가진 노드의 right subtree => x보다 큰 값들

<Time complexity of BST>

1) ascending operation => inorder traversal : $O(n)$

2) searching operation : $O(\text{height})$

```
① treePointer search (treePointer root, int key){  
    if (!root) return NULL;  
    if (key == root->data) return root;  
    if (key < root->data) return search (root->leftChild, key);  
    return search (root->rightChild, key);
```

```
} // recursive
```

```
② treePointer search2 (treePointer tree, int key) {  
    while (tree) {  
        if (key == tree->data) return tree;  
        if (key < tree->data) tree = tree->leftChild;  
        else tree = tree->rightChild;  
        return NULL;  
    }
```

```
} // iterative
```

3) Inserting operation : $O(\text{height})$

```
void insert_node (treePointer *node, int num){  
    treePointer ptr, temp = modified_search(*node, num);  
    // modified_search : empty/해당 숫자가 존재=> NULL  
    나머지 경우 => tree의 last node
```

```

if (temp || !(*node)) {

    ptr = (treePointer)malloc(sizeof(node));

    ptr->data = num;

    ptr->leftChild = ptr->rightChild = NULL;

    if (*node) {

        if (num < temp->data) temp->leftChild = ptr;

        else temp->rightChild = ptr; }

    else *node = ptr;

}
}

```

4) Deleting operation

4-1) element가 없을 때

4-2) 자식이 없는 노드 (leaf)

4-3) 1개의 자식이 있는 노드 (degree 1)

=> 그냥 지워버리고 남은 parent와 child를 연결시켜준다.

★4-4) 2개의 자식이 있는 노드 (degree 2)

=> 해당 노드를 left subtree의 largest key나, right subtree의 smallest key로 대체

(위의 노드들은 항상 degree 0 이거나 1)

대체하고자 하는 대상의 원래 노드를 삭제

Time Complexity : $O(\text{height})$

- Height of a binary search tree => $\log_2 n$

<Winner Tree>

1) Min winner Tree => smaller element wins

height = $\log_2 n$

$O(1)$: playing match at each match node ($n-1$ 개의 match nodes)

$O(n)$: initialize n player winner tree

- sorting할 때의 min winner tree 사용하기 -> sorted array 이용

1) initialize n player winner tree : $O(n)$

2) remove winner & replay : $O(\log n)$

3) repeat 2번 for n times : $O(n \log n)$

<Loser Tree>

- left child winner를 저장한다 => $O(n)$

- replay matches => $O(\log n)$

<Forest>

- find(i) : i를 원소로 가지는 tree의 root에 있는 element를 리턴

-> table[i]에서 시작해서 root node에 있는 element를 리턴

- simple union => `parent[i] = j; // make one tree a subtree of the other`

- simple find => `while (parent[i] >= 0) i = parent[i]; // move up the tree`

* union

-> height rule : height가 더 작은 tree가 subtree가 된다.

-> weight rule : node의 개수가 더 작은 tree가 subtree가 된다.

- root노드의 parent는 -1이다.

* void weightedUnion (int i, int j){

 int temp = parent[i] + parent[j];

 if (parent[i] > parent[j]) {

 parent[i] = j; parent[j] = temp;

 } // j가 새로운 root

 else {

 parent[j] = i; parent[i] = temp;

 } // i가 새로운 root

} // 위로 갈수록 값이 작아지는 tree

$O(n + m \log_2 n) \rightarrow n-1$ union , m find operations

<CH 5.4-5.6>

- Binaty Tree의 equality 확인하기

1) !first && !second

2) first && second && first->data == second->data && equal (first->leftChild,second->leftChild)
&& equal (first->rightChild && second->rightChild)

<Threaded Binary Trees>

Thread => left : Inorder Predecessor / right : Inorder Succesor

thread 0 == Child 존재 / thread 1 == Ancestor

* Inorder Succesor

```
threaded_pointer insucc(threaded_pointer tree){  
    threaded_pointer temp;  
    temp = tree->right_child;  
    if(! tree->rightthread) while(!temp->left_thread) temp= temp->left_child;  
    return temp;  
}
```

* InsertRight in Threaded Binary Tree

```
child->rightChild = parent->rightChild;  
child->rightThread = parent->rightThread;  
child->leftChild = parent;  
child->leftThread = 1;  
parent->rightChild = child;  
parent->rightThread = 0;  
if (!child->rightThread){  
    temp = insucc(child);  
    temp->leftChild = child;  
}
```

<Insertion into a max heap> => $O(\log_2 n)$

```
void insert_max_heap(element item, int *n) {  
    /* insert item into a max heap of current size *n */  
    int i = ++(*n);  
    while ((i != 1) && (item.key > heap[i/2].key)) { heap[i] = heap[i/2]; i /= 2; }  
    heap[i] = item; }
```

<Deletion in a max heap> => $O(\log_2 n)$

```
element delete_max_heap(int *n){  
    /* save value of the element with the largest key */  
    item = heap[1];  
    /* use last element in heap to adjust heap */  
    temp = heap[(*n)--]; parent = 1; child = 2;  
    while (child <= *n) { /* find the larger child of the current parent */  
        if ((child < *n) && (heap[child].key < heap[child+1].key)) child++;  
        if (temp.key >= heap[child].key) break;  
        /* move to the next lower level */  
        heap[parent] = heap[child]; parent = child; child *= 2; }  
    heap[parent] = temp; return item;  
}
```