University of Nottingham
UK | CHINA | MALAYSIA

Foundation Algebra for Physical Sciences & Engineering CELEN036

Practice Problems SET-2 **Topic:** Quadratics, exponentials and logarithms

Type 1: Quadratic equations

1. Solve the following quadratic equations.

(i)	$2x^2 - 6x + 4 = 0$	(ii)	$x^2 + 4x - 8 = 0$	(iii)	$2x^2 + 7x + 3 = 0$
(iv)	$x^2 - 2ax + b = 0$	(v)	$6x^2 + 11x - 35 = 0$	(vi)	$x^2 + x - 4 = 0$
(vii)	$2x^2 - 5x + 1 = 0$	$(viii)$	$x^2 + x - 6 = 0$	(ix)	$4x^2 - 10x - 7 = 0$

2. Determine the nature of the roots of the following quadratic equations (without finding the roots).

(i)	$4x^2 - 7x + 3 = 0$	(ii)	$x^2 + ax + a^2 = 0$	(iii)	$x^2 - px - q^2 = 0$
(iv)	$x^2 - 6x + 9 = 0$	(v)	$x^2 - 6x + 10 = 0$	(vi)	$2x^2 - 5x + 3 = 0$
(vii)	$3x^2 + 4x + 2 = 0$	$(viii)$	$4x^2 - 12x + 9 = 0$	(ix)	$9x^2 + 24x + 16 = 0$

3. Find the value(s) of k if the roots of the equation $3x^2 + kx + 12 = 0$ are equal (ie. repeated roots).

4. Find the relation between p and q if the roots of $px^2 + qx + 1 = 0$ are equal.

5. For what values of m, will the polynomial $p(x) = m^2x^2 + 2(m+1)x + 4$ have exactly one zero?

6. By completing the square, find the range of the following functions for $x \in \mathbb{R}$:
   (i) $f(x) = x^2 - 2x - 8$ (ii) $g(x) = 6 - 3x - x^2$ (iii) $h(x) = 2x^2 - 6x + 1$

7. Completing the square for $f(x) = x^2 + x - 2$ and hence sketch the graph of $f(x)$.

Type 2: Quadratic inequalities

8. Determine the values of x for which the following quadratic inequalities hold.

(i)	$x^2 > 7x - 12$	(ii)	$x^2 + 5 < 2x$	(iii)	$b^2 + a^2x^2 > 2abx;\ a \neq 0$
(iv)	$6x^2 - 5x - 4 < 0$	(v)	$3x^2 + 5x < 2$		

Type 3: Exponential functions

9. Simplify $(1 + x^{a-b})^{-1} + (1 + x^{b-a})^{-1}$.

10. Prove that $\dfrac{1}{1 + x^{a-b} + x^{a-c}} + \dfrac{1}{1 + x^{b-c} + x^{b-a}} + \dfrac{1}{1 + x^{c-a} + x^{c-b}} = 1.$

11. A function f is defined by $f(x) = \dfrac{1}{2}(10^x + 10^{-x});\ x \in \mathbb{R}$. Show that:
    (i) $2[f(x)]^2 = f(2x) + 1$ (ii) $2f(x)f(y) = f(x+y) + f(x-y)$

12. Solve the following equations:

(i)	$e^{2x+3} = 500$	(ii)	$2e^x - 1 = 83$	(iii)	$e^{5-x} = 10$
(iv)	$4e^{3x+2} = 78$	(v)	$e^{4x+1} = e^{1-x}$	(v)	$10^x = 59$

13. By substituting $e^x = t$, solve the following equations:

    (i) $e^{2x} - 5e^x + 6 = 0$ $\quad(ii)$ $e^x + e^{-x} = 2$

    (iii) $12e^{2x} + 6 = 17e^x$ $\quad(iv)$ $3e^{2x} - 5e^x = 2$

14. Newton's Law of Cooling states that the temperature T of a cooling object is modeled by: $T = A + (I - A)e^{-kt}$. Where A is the ambient temperature in Celsius, I is the initial object's temperature, $k = 0.018$ is the heat transfer coefficient and t is the time in minutes. How long will it take for boiling water ($100°$C) to cool down to body temperature ($38°$C) at a room temperature of $20°$C?

Type 4: Logarithmic functions

15. Use rules of the logarithm to simplify:

    (i) $\ln 3x^2 + \ln 2x - \ln 6x^3$ $\quad(ii)$ $\log 5x^2 - \log 10x^2 + \log 4x$

    (iii) $3\log x - \log x^2$ $\quad(iv)$ $\log x - 3\log 2x + 2\log 4x$

    (v) $2\ln 3x - \frac{1}{2}\ln 16x^2$ $\quad(vi)$ $\frac{1}{3}\left(\log 9x + \log 3x^2\right)$

16. Using the change of base rule and the power rule, simplify: $\dfrac{\log_2 128 \cdot \log_9 243}{\log_{125} 625}$.

17. Prove that $\log\left(\frac{a^2}{bc}\right) + \log\left(\frac{b^2}{ac}\right) + \log\left(\frac{c^2}{ab}\right) = 0$.

18. Prove that $\log\left(\sqrt{x^2+1} + x\right) + \log\left(\sqrt{x^2+1} - x\right) = 0$.

19. Prove that $\log_a x + \log_{a^2} x^2 + \log_{a^3} x^3 + \cdots + \log_{a^n} x^n = \log_a x^n$.

20. Solve for $x \in \mathbb{R}^+$:

    (i) $\log(x-1) + \log(x+1) = 2$ $\quad(ii)$ $\ln(2x+5) = \ln(14-x)$

    (iii) $\dfrac{\log 2x}{\log x} = 2$ $\quad(iv)$ $\ln\left(\dfrac{x^2}{2}\right) - \ln x = 0.7$

Answers

1 (i) $1, 2$ $\quad(ii)$ $-2 \pm 2\sqrt{3}$ $\quad(iii)$ $-\frac{1}{2}, -3$ $\quad(iv)$ $a \pm \sqrt{a^2 - b}$ $\quad(v)$ $\frac{5}{3}, -\frac{7}{2}$

(vi) $\dfrac{-1 \pm \sqrt{17}}{2}$ $\quad(vii)$ $\dfrac{5 \pm \sqrt{17}}{4}$ $\quad(viii)$ $2, -3$ $\quad(ix)$ $\dfrac{5 \pm \sqrt{53}}{4}$

2 Roots are real and distinct: (i), (iii), (vi); Roots are real and equal: (iv), $(viii)$, (ix); No real roots: (ii), (v), (vii).

3 ± 12

4 $q^2 = 4p$

5 $m = 1$ or $-\frac{1}{3}$

6 (i) $R_f = [-9, +\infty)$ $\quad(ii)$ $R_f = (-\infty, \frac{33}{4}]$ $\quad(iii)$ $R_f = [-\frac{7}{2}, +\infty)$

7 $\left(x + \frac{1}{2}\right)^2 - \frac{9}{4}$

8 (i) $x < 3$ or $x > 4$ $\quad(ii)$ No solution $\quad(iii)$ $x \in \mathbb{R}, x \neq \frac{b}{a}$ $\quad(iv)$ $-\frac{1}{2} < x < \frac{4}{3}$ $\quad(v)$ $-2 < x < \frac{1}{3}$

9 1

12 (i) 1.61 $\quad(ii)$ 3.74 $\quad(iii)$ 2.70 $\quad(iv)$ 0.32 $\quad(v)$ 0 $\quad(vi)$ 1.77

13 (i) $\ln 2$ or $\ln 3$ $\quad(ii)$ 0 $\quad(iii)$ $\ln\left(\frac{2}{3}\right)$ or $\ln\left(\frac{3}{4}\right)$ $\quad(iv)$ $\ln 2$

14 82.87

15 (i) 0 $\quad(ii)$ $\log 2x$ $\quad(iii)$ $\log x$ $\quad(iv)$ $\log 2$ $\quad(v)$ $\log\left(\frac{9x}{4}\right)$ $\quad(vi)$ $\log 3x$

16 $\dfrac{105}{8}$

20 (i) 10.05 $\quad(ii)$ 3 $\quad(iii)$ 2 $\quad(iv)$ 4.03