University of Nottingham
UK | CHINA | MALAYSIA

Foundation Algebra for Physical Sciences & Engineering CELEN036

Practice Problems SET-8 **Topic:** Binomial theorem

Type 1: Expansion using the Binomial theorem with $\begin{pmatrix} n \\ r \end{pmatrix} = \dfrac{n!}{r!(n-r)!}$

1. Evaluate and simplify:

   $(i)\quad \begin{pmatrix} n \\ n-1 \end{pmatrix}$ $\qquad (ii)\quad \begin{pmatrix} n+1 \\ n-1 \end{pmatrix}$

   $(iii)\quad \dfrac{\begin{pmatrix} n \\ k+1 \end{pmatrix}}{\begin{pmatrix} n \\ k \end{pmatrix}}$ $\qquad (iv)\quad \dfrac{\begin{pmatrix} n+1 \\ r \end{pmatrix}}{\begin{pmatrix} n \\ r-1 \end{pmatrix}}$

2. Prove that $\begin{pmatrix} n \\ r \end{pmatrix} = \begin{pmatrix} n-1 \\ r-1 \end{pmatrix} + \begin{pmatrix} n-1 \\ r \end{pmatrix}$.

3. Expand the following expressions using the Binomial theorem:

   $(i)\quad (1+x)^7 \qquad (ii)\quad \left(1+\dfrac{x}{4}\right)^5 \qquad (iii)\quad \left(1-\dfrac{x}{2}\right)^6$

   $(iv)\quad (2x+5)^4 \qquad (v)\quad (4-3x)^4 \qquad (vi)\quad (2x+y)^4$

Type 2: To find the coefficient of certain term in the expansion

4. Find the coefficient of x^4 in the expansion of $(2x+3)^9$.

5. Find the coefficient of x^2 in the expansion of $(7-3x)^7$.

6. Find the coefficient of x^3 in the expansion of $\left(1-\dfrac{x}{5}\right)^{15}$.

7. Find the coefficient of x^3 in the expansion of $\left(2x-\dfrac{1}{3x}\right)^9$.

8. Find the coefficient of x^4y^4 in the expansion of $\left(2x-y^2\right)^6$.

9. Expand the expression $(\sqrt{2}+1)^5+(\sqrt{2}-1)^5$ using Binomial expansion.

10. In the Binomial expansion of $(x+ky)^8$, the coefficient of (x^5y^3) is -1512. Find the value of $k \in \mathbb{R}$.

11. Find the value of x if the 5th term in $\left(\frac{1}{2\sqrt{x}} - \frac{1}{2}\right)^{10}$ is 105.

12. Consider the expansion of $(m+8)^n$, where m is a positive constant, n is a positive integer. Determin the value of m and n, given that the ratio between the coefficients of 12th term and 14th term is equal to $\dfrac{637}{4640}$ and that the ratio between the coefficients of 7th term and 9th term is $\dfrac{49}{1360}$.

Type 3:Application of the generalized Binomial theorems

13. Assuming that $|x| < 1$, obtain the Binomial expansions of the following expressions, up to the term with x^3 :

    (i) $(1+x)^{-6}$ (ii) $(1-x)^{-3}$

14. Expand $\dfrac{1}{\sqrt{1-x^2}}$ using generalized Binomial theorem up to the term of x^6.

15. Find the coefficient of x^2 in $\dfrac{1+x}{(1-2x)^5}$. $\left(\text{Hint:} \dfrac{1+x}{(1-2x)^5} = (1-2x)^{-5} + x(1-2x)^{-5}\right)$

16. Use generalized Binomial theorem to show that $\sqrt{\dfrac{1+4x}{1-x}} \approx 1 + \dfrac{5}{2}x - \dfrac{5}{8}x^2$, as $|x| < \dfrac{1}{4}$.

17. Find the expansion of $\dfrac{\sqrt{1-\frac{1}{2}x}}{(1+x)^2}$ in ascending powers of x, up to and including the term in x^2.

18. Apply the Binomial theorem to approximate the following values. Correct to 4 decimal places using the first **four** terms in the expansion.

    (i) $(1.01)^{-3}$ (ii) $(1.03)^{-2}$ (iii) $\sqrt{1.03}$ (iv) $\dfrac{1}{\sqrt[3]{1.02}}$

19. Use the first four terms of the Binomial expansion of $\left(1-\dfrac{1}{50}\right)^{1/2}$ to derive an approximation of $\sqrt{2}$ to 5 decimal places.

20. The radius of a sphere is measured as r, with an error of $\delta r = 1.2\%$ of r. The volume of the sphere $V = \dfrac{4}{3}\pi r^3$ is then calculated using the measured r. Use the approximation $(1+x)^n \approx 1 + nx + \frac{n(n-1)}{2} \cdot x^2$ to find the resulting error δV in the calculated volume.

Answers

1 (i) n (ii) $\dfrac{(n+1)\,n}{2}$ (iii) $\dfrac{n-k}{k+1}$ (iv) $\dfrac{n+1}{r}$

3 (i) $1+7x+21x^2+35x^3+35x^4+21x^5+7x^6+x^7$ (ii) $1+\dfrac{5}{4}x+\dfrac{5}{8}x^2+\dfrac{5}{32}x^3+\dfrac{5}{256}x^4+\dfrac{1}{1024}x^5$

(iii) $1-3x+\dfrac{15}{4}x^2-\dfrac{5}{2}x^3+\dfrac{15}{16}x^4-\dfrac{3}{16}x^5+\dfrac{1}{64}x^6$ (iv) $16x^4+160x^3+600x^2+1000x+625$

(v) $256-768x+864x^2-432x^3+81x^4$ (vi) $x^{10}-5x^8y+10x^6y^2-10x^4y^3+5x^2y^4-y^5$

4 489888

5 3176523

6 $-\dfrac{91}{25}$

7 $-\dfrac{1792}{9}$

8 240

9 $58\sqrt{2}$

10 $k=-3$

11 $\dfrac{1}{8}$

12 $m=7,\ n=41$

13 (i) $1-6x+21x^2-56x^3+\cdots$ (ii) $1+3x+6x^2+10x^3+\cdots$

14 $1+\dfrac{1}{2}x^2+\dfrac{3}{8}x^4+\dfrac{5}{16}x^6+\cdots$

15 70

17 $1-\dfrac{9}{4}x+\dfrac{111}{32}x^2+\cdots$

18 (i) 0.9706 (ii) 0.9426 (iii) 1.0149 (iv) 0.9934

19 1.414214

20 $\delta V \approx 3.6432\% V$