Foundation Algebra for Physical Sciences & Engineering CELEN036

Practice Problems SET-5 Sample Solution

Type 1: Addition and Factor Formulae

1. Prove the following trigonometric identities: $(i)\ \cos(270^\circ - \theta) = -\sin\theta$

Solution:

$$\begin{aligned}
LHS &= \cos(270^\circ - \theta) \\
&= \cos 270^\circ \cos\theta + \sin 270^\circ \sin\theta \\
&= 0 \cdot \cos\theta - 1 \cdot \sin\theta \\
&= -\sin\theta \\
&= RHS
\end{aligned}$$

Type 2: Multi-angle Formulae

5. Prove the following results: $(i)\ \dfrac{1 - \cos 2\theta + \sin 2\theta}{1 + \cos 2\theta + \sin 2\theta} = \tan\theta$

Solution:

$$\begin{aligned}
LHS &= \frac{1 - \cos 2\theta + \sin 2\theta}{1 + \cos 2\theta + \sin 2\theta} \\
&= \frac{1 - (1 - 2\sin^2\theta) + 2\sin\theta\cos\theta}{1 + (2\cos^2\theta - 1) + 2\sin\theta\cos\theta} \\
&= \frac{2\sin^2\theta + 2\sin\theta\cos\theta}{2\cos^2\theta + 2\sin\theta\cos\theta} \\
&= \frac{2\sin\theta(\sin\theta + \cos\theta)}{2\cos\theta(\sin\theta + \cos\theta)} \\
&= \frac{\sin\theta}{\cos\theta} \\
&= \tan\theta \\
&= RHS
\end{aligned}$$

Type 3: Inverse Trigonometric Functions

12. Without using a calculator, find the values of: $(i)\ \cos\left[\sin^{-1}\left(-\frac{1}{2}\right)\right]$

Solution:

Let $\alpha = \sin^{-1}\left(-\frac{1}{2}\right)$

$\therefore \sin\alpha = -\frac{1}{2}$

And $\alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\implies \cos\alpha > 0$

$\therefore \cos\alpha = \sqrt{1-\sin^2\alpha} = \sqrt{1-(-\frac{1}{2})^2} = \frac{\sqrt{3}}{2}$

Type 4: Expressing $a\cos x + b\sin x$ in the form $r\cos(x\pm\theta)$ or $r\sin(x\pm\theta)$

20. Express $5\sin x + 12\cos x$ in the form $R\sin(x+\theta)$, where $R > 0$ and $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ are to be determined. Solution:

As $R\sin(x+\theta) = R\sin x\cos\theta + R\cos x\sin\theta$

$\therefore 5\sin x + 12\cos x \equiv R\sin(x+\theta) = R\sin x\cos\theta + R\cos x\sin\theta$

$\therefore 5 = R\cos\theta \cdots\cdots (1)$

$12 = R\sin\theta \cdots\cdots (2)$

$(1)^2 + (2)^2 \implies R^2\sin^2\theta + R^2\cos^2\theta = R^2(\sin^2\theta + \cos^2\theta) = R^2 = 5^2 + 12^2 = 169\ \therefore\ R = 13;$

$(2) \div (1) \implies \frac{R\sin\theta}{R\cos\theta} = \tan\theta = \frac{12}{5}$

$\therefore \theta = \tan^{-1}\left(\frac{12}{5}\right) \approx 1.18$

$\therefore 5\sin x + 12\cos x \equiv 13\sin(x+1.18)$