Foundation Algebra for Physical Sciences & Engineering CELEN036

Practice Problems SET-3 Sample Solution

Type 1: Remainder and Factor theorems

1. Two cubic polynomials are defined by $f(x) = x^3 + (a-3)x + 2b$, $g(x) = 3x^3 + x^2 + 5ax + 4b$, where a and b are constants. Given that $f(x)$ and $g(x)$ have a common factor of $(x-2)$. Find the value of a and b.

Solution:

As $(x-2)$ is a factor for both $f(x)$ and $g(x)$,

$f(2) = 0$ and $g(2) = 0$.

$\therefore \quad 2^3 + 2 \times (a-3) + 2b = 0 \implies a + b = -1$

$3 \times 2^3 + 2^2 + 5 \times a \times 2 + 4b = 0 \implies 5a + 2b = -14 \therefore \quad a = -4, \quad b = 3$

Type 2: Method of long and synthetic division

6. Perform the following divisions:

(i) $(2x^3 + 2x - 1) \div (x - 1)$

Solution:

$$
\begin{array}{r|rrrr}
 & 2x^2 & +2x & +4 & \\
\cline{2-5}
x-1 & 2x^3 & +0x^2 & +2x & -1 \\
- & & & & \\
 & 2x^3 & -2x^2 & & \\
\hline
 & & 2x^2 & +2x & -1 \\
 & & - & & \\
 & & 2x^2 & -2x & \\
\hline
 & & & 4x & -1 \\
 & & & - & \\
 & & & 4x & -4 \\
\hline
 & & & & 3
\end{array}
$$

7. Use the method of synthetic division to find the quotient $q(x)$ and the remainder $r(x)$ that result when $p(x)$ is divided by $s(x)$.

   (i) $p(x) = 2x^4 + 3x^3 - 17x^2 - 27x - 9\ ;\ s(x) = x + 4$

   Solution:

$$\begin{array}{r|rrrrr} -4.0 & 2 & 3 & -17 & -27 & -9 \\ & & -8 & 20 & -12 & 156 \\ \hline & 2 & -5 & 3 & -39 & 147 \end{array}$$

$$\frac{2x^4 + 3x^3 - 17x^2 - 27x - 9}{x + 4} = 2x^3 - 5x^2 + 3x - 39 + \frac{147}{x + 4}$$

Type 3: Polynomial factorisation and solving

15. Let $\cos\theta = x$ to find the all the solution for θ of the equation $4\cos^3\theta - 7\cos\theta - 3 = 0$ for $0 \leq \theta \leq 2\pi$. Solution:

Let $\cos\theta = x$, therefore $4\cos^3\theta - 7\cos\theta - 3 = 0 \implies 4x^3 - 7x - 3 = 0$ for $-1 \leq x \leq 1$

Let $p(x) = 4x^3 - 7x - 3$

As the constant term of $p(x)$ is -3, have divisors of $\pm 1, \pm 3$

$p(1) = 4(1)^3 - 7(1) - 3 = -1 \neq 0$ then try the next divisor,

$p(-1) = 4(-1)^3 - 7(-1) - 3 = -0$ therefore $(x - (-1)) = (x + 1)$ is a factor of $p(x)$

Use synthetic division to find $\dfrac{p(x)}{(x+1)}$:

$$\begin{array}{r|rrrr} -1.0 & 4 & 0 & -7 & -3 \\ & & -4 & 4 & 3 \\ \hline & 4 & -4 & -3 & 0 \end{array}$$

$\therefore p(x) = (x + 1)(4x^2 - 4x - 3) = (x + 1)(2x + 1)(2x - 3) = 0$

$\implies x = -1, x = -\dfrac{1}{2}, x = \dfrac{3}{2},$ as $-1 \leq x \leq 1$

$\therefore \cos\theta = x = -1$ or $-\dfrac{1}{2}$

$\therefore \cos\theta = \pi, \dfrac{2\pi}{3}, \dfrac{4\pi}{3}$

Type 4: Partial fraction

17. Express the following as the sum of partial fractions: $(i)\ \dfrac{13}{(x^2+1)(2x+3)}$

Solution:

Let $\dfrac{13}{(x^2+1)(2x+3)} = \dfrac{Ax+B}{(x^2+1)} + \dfrac{C}{(2x+3)}$

Multiply both sides by $(x^2+1)(2x+3)$

$\therefore 13 = (Ax+B)(2x+3) + C(x^2+1)$

Re-arrange the equation $\implies 13 = 2Ax^2 + 2Bx + 3Ax + 3B + Cx^2 + C$

$\therefore (2A+C)x^2 + (2B+3A)x + (3B+C) = 13$

As LHS is always equals to RHS

$\therefore 2A+C=0,\ 2B+3A=0,\ 3B+C=13$

$\therefore A=-2,\ B=3,\ C=4$

Therefore $\dfrac{13}{(x^2+1)(2x+3)} = \dfrac{-2x+3}{(x^2+1)} + \dfrac{4}{(2x+3)}$