Problem 1d: Solution to Kinematics Problem

The solution provided in the original sample solutions is only approximate. You can still use the graph to solve this exercise but here I provide a solution based on the equations of constant acceleration motion.

Given:

- Initial position $x_0 = -10$ m
- Initial velocity $v_0 = 8$ m/s
- Final velocity at $t = 8$ s is $v = -6$ m/s
- Motion has constant acceleration

Step 1: Find acceleration

Using the velocity equation for constant acceleration:

$$\begin{aligned}
v &= v_0 + at \\
-6 &= 8 + a(8) \\
-14 &= 8a \\
a &= -1.75 \text{ m/s}^2
\end{aligned}$$

Step 2: Find time when velocity is zero

This will help us calculate total distance:

$$\begin{aligned}
0 &= v_0 + at \\
0 &= 8 + (-1.75)t \\
t &= 4.57 \text{ s}
\end{aligned}$$

Step 3: Find position at t = 4.57 s

Using the position equation:

$$\begin{aligned}
x &= x_0 + v_0 t + \frac{1}{2}at^2 \\
x &= -10 + 8(4.57) + \frac{1}{2}(-1.75)(4.57)^2 \\
x &= -10 + 36.56 - 18.27 \\
x &= 8.29 \text{ m}
\end{aligned}$$

Step 4: Find position at t = 8 s

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$
$$x = -10 + 8(8) + \frac{1}{2}(-1.75)(8)^2$$
$$x = -10 + 64 - 56$$
$$x = -2 \text{ m}$$

Step 5: Calculate displacement (s)

$$s = x_{\text{final}} - x_{\text{initial}}$$
$$s = -2 - (-10)$$
$$s = 8 \text{ m}$$ to the right

Step 6: Calculate total distance (d)

The particle moves:

- From $x = -10$ m to $x = 8.29$ m: $|18.29|$ meters
- From $x = 8.29$ m to $x = -2$ m: $|10.29|$ meters

Total distance:

$$d = |18.29| + |10.29|$$
$$d = 28.58 \text{ m}$$

Final Values at t = 8 s

- Total distance (d) = 28.58 m
- Displacement (s) = 8 m to the right
- Position (x) = -2 m
- Velocity (v) = -6 m/s
- Acceleration (a) = -1.75 m/s^2

Question 2(a): Physics Energy Conservation Problem

The problem here was at the last part of the question (iv) which did not consider the friction.

Given Information:

- Mass $m = 10$ kg
- Initial height $h = 2.0$ m
- Incline angle $\theta = 30$
- Spring constant $k = 500$ N/m
- Maximum spring compression $x = 0.75$ m
- Horizontal surface is frictionless

(i) Speed at bottom of incline (V_1)

From spring maximum compression:

$$\begin{aligned}\text{Spring potential energy} &= \frac{1}{2}kx^2 \\ &= \frac{1}{2} \times 500 \times (0.75)^2 \\ &= 140.63 \text{ J}\end{aligned}$$

This equals kinetic energy at bottom:

$$\begin{aligned}\frac{1}{2}mV_1^2 &= 140.63 \\ V_1 &= \sqrt{\frac{2 \times 140.63}{10}} \\ &= 5.30 \text{ m/s}\end{aligned}$$

(ii) Work of friction

Initial gravitational potential energy:

$$\begin{aligned}E_h &= mgh \\ &= 10 \times 9.8 \times 2 \\ &= 196 \text{ J}\end{aligned}$$

Work of friction:

$$\begin{aligned} E_f &= E_h - E_k \\ &= 196 - 140.63 \\ &= 55.37 \text{ J} \end{aligned}$$

(iii) Speed when returning to base

Since horizontal surface is frictionless, spring energy is conserved:

$$V_2 = V_1 = 5.30 \text{ m/s}$$

(iv) Distance up incline

Calculate frictional force f:

$$\begin{aligned} f &= \frac{E_f}{h/\sin(30)} \\ &= \frac{55.37}{2/\sin(30)} \\ &= \frac{55.37}{4} \\ &= 13.84 \text{ N} \end{aligned}$$

Using energy conservation for upward motion:

$$\begin{aligned} mgd\sin(30) + fd &= 140.63 \\ 10 \times 9.8 \times d \times 0.5 + 13.84d &= 140.63 \\ 49d + 13.84d &= 140.63 \\ 62.84d &= 140.63 \\ d &= 2.24 \text{ m} \end{aligned}$$

Question3: RC Circuit Analysis

The problem here was at (iii) and (iv) please check.

Problem

A $500\,\Omega$ resistor, an uncharged $1.50\,\mu\text{F}$ capacitor, and a $6.16\,\text{V}$ emf are connected in series.

Solutions

(i) Initial current [2 marks]

$$\begin{aligned} I &= \frac{V}{R} \\ &= \frac{6.16\,\text{V}}{500\,\Omega} \\ &= 0.012\,32\,\text{A} \\ &= 12.32\,\text{mA} \end{aligned}$$

(ii) RC time constant [2 marks]

$$\begin{aligned} \tau &= RC \\ &= 500\,\Omega \times 1.50\,\mu\text{F} \\ &= 500 \times (1.50 \times 10^{-6}) \\ &= 750\,\mu\text{s} \end{aligned}$$

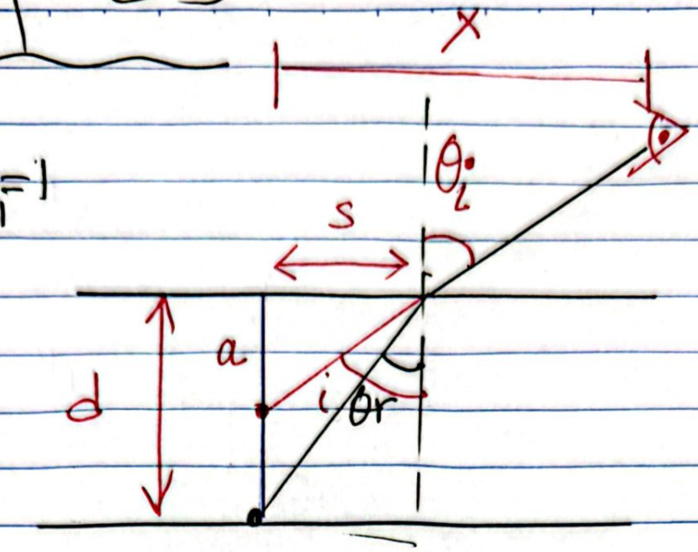
(iii) Current after one time constant [4 marks]

$$\begin{aligned} I(\tau) &= I_0 e^{-1} \\ &= 0.012\,32\,\text{A} \times e^{-1} \\ &= 0.012\,32\,\text{A} \times 0.368 \\ &= 0.004\,54\,\text{A} \\ &= 4.54\,\text{mA} \end{aligned}$$

(iv) Voltage on capacitor after one time constant [3 marks]

$$\begin{aligned} V &= V_0(1 - e^{-t/\tau}) \\ &= 6.16\,\text{V}(1 - e^{-1}) \\ &= 6.16\,\text{V}(1 - 0.368) \\ &= 6.16\,\text{V} \times 0.632 \\ &= 3.89\,\text{V} \end{aligned}$$

Question 5b



$$n_1 \sin\theta_i = n_2 \sin\theta_r$$

When viewing from above $x \to 0$

θ_i, θ_r = small

$$\sin\theta \simeq \tan\theta \qquad (\cos\theta \simeq 1)$$

$$\sin\theta_i \simeq \tan\theta_i = \frac{s}{a}$$

$$\sin\theta_r \simeq \tan\theta_r = \frac{s}{d}$$

Hence $1 \cdot \frac{\cancel{s}}{a} = \frac{4}{3} \frac{\cancel{s}}{d}$

$$\Rightarrow 3d = 4a$$

$$d = \frac{4a}{3} = \frac{4 \cdot 4}{3} = \underline{5.33\,m}$$

Problem 6(a)

The problem here was that there was no reason to add the radius to the distance - given that the distance is from the centre of the sphere.

A hollow metal sphere with a diameter of 10 cm has a net charge of 4 µC distributed uniformly across its surface. What is the magnitude of the field a distance 2.0 m from the center of the sphere? [4 marks]

For a point charge Q, electric field $E = k\dfrac{Q}{r^2}$

where $k = 9 \times 10^9\ \mathrm{N \cdot m^2/C^2}$

Given:

$Q = 4\ \mu\mathrm{C} = 4 \times 10^{-6}\ \mathrm{C}$

$r = 2.0\ \mathrm{m}$

$$E = \frac{(9 \times 10^9)(4 \times 10^{-6})}{(2.0)^2}$$

$$= \frac{36 \times 10^3}{4}$$

$$= 9{,}000\ \mathrm{N/C}$$