

Foundation Physics

Lecture 6: Circular Motion and Simple Harmonic Motion

Aims of today's lecture

1. Circular Motion
2. Centripetal Acceleration and Force
3. Periodic/Oscillatory Motion
4. Simple Harmonic Motion
5. The Simple Pendulum

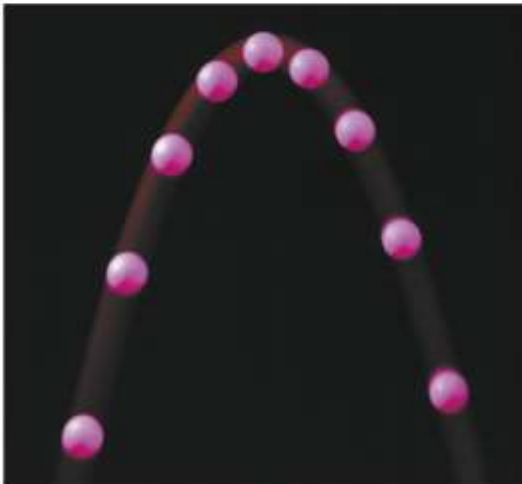
Types of Motion



Linear motion



Circular motion



Projectile motion



Rotational motion

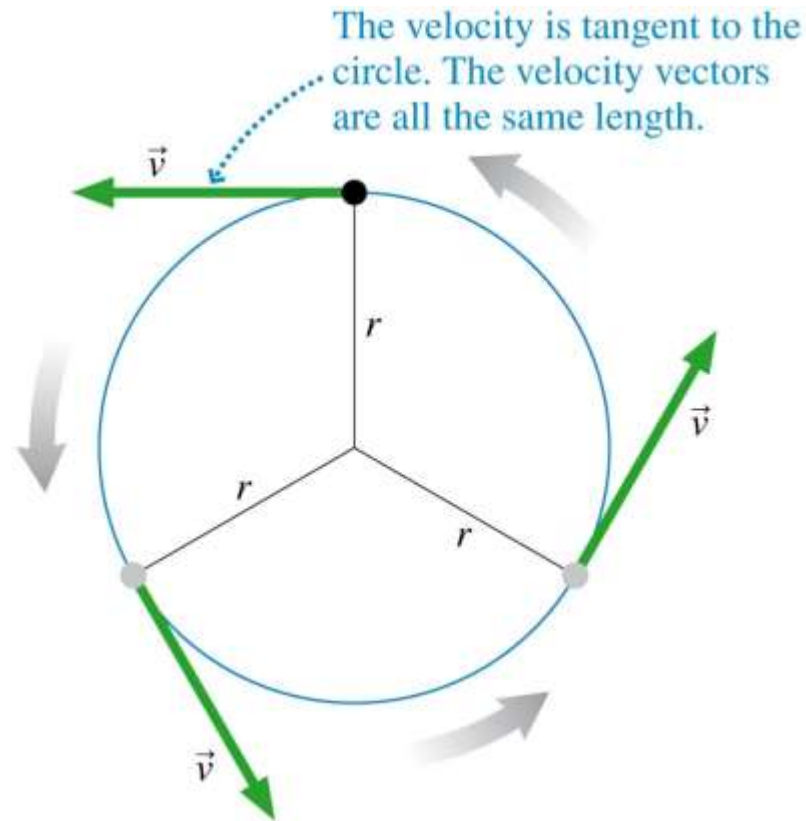


**Periodic
(or oscillatory motion)**

- Our focus for this course.

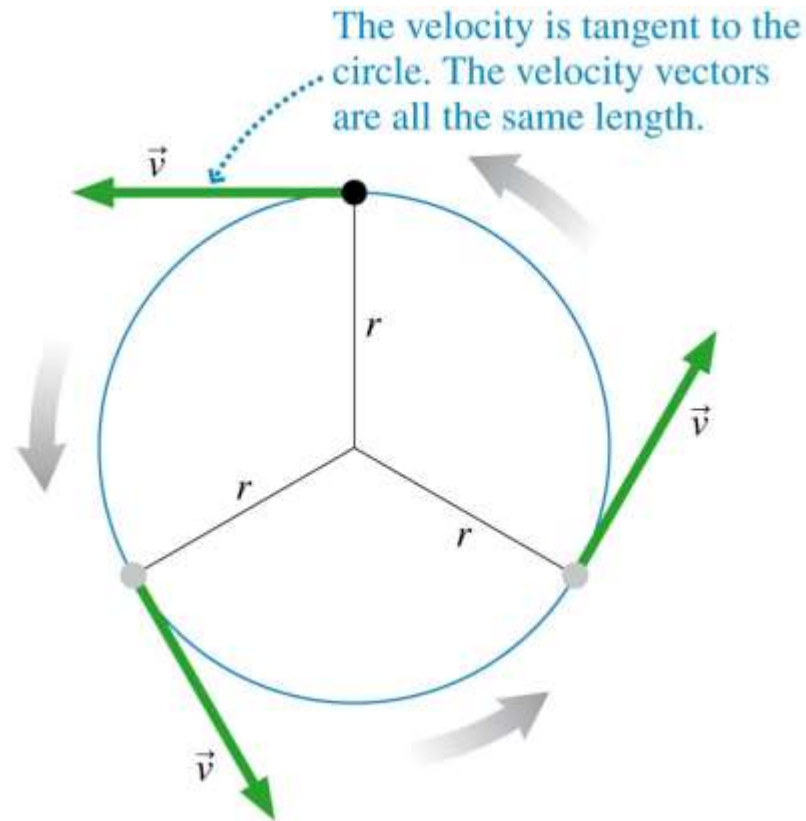
1. Circular Motion

Uniform Circular Motion



- If the object has a **constant tangential velocity**, we can say that circular motion is **uniform**.
- We can also say that the time interval to complete one revolution, called the period, **T** , is fixed.

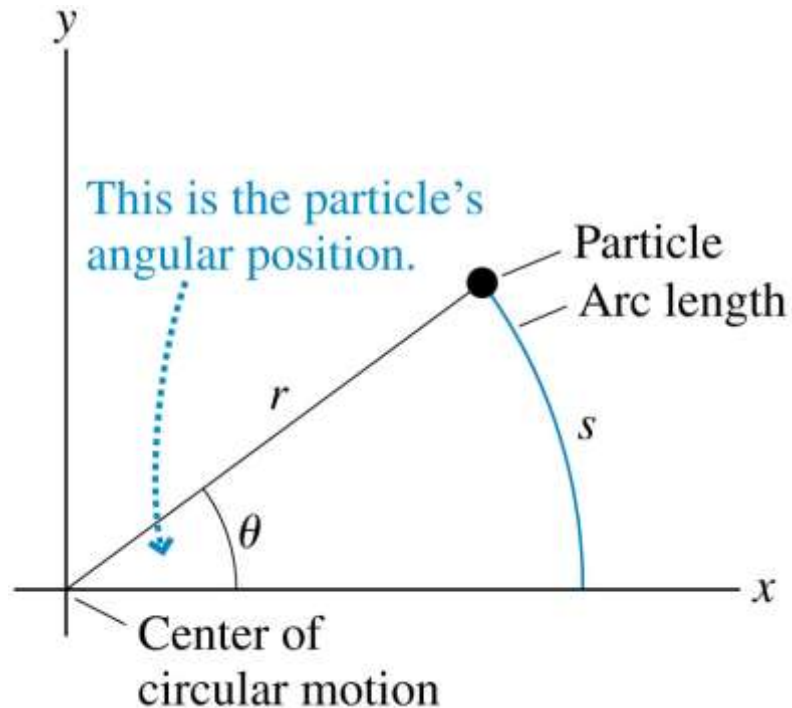
Uniform Circular Motion



- The relationship between the period, T , and the tangential speed, v , is:

$$v = \frac{1 \text{ circumference}}{1 \text{ period}} = \frac{2\pi r}{T}$$

Angular Position

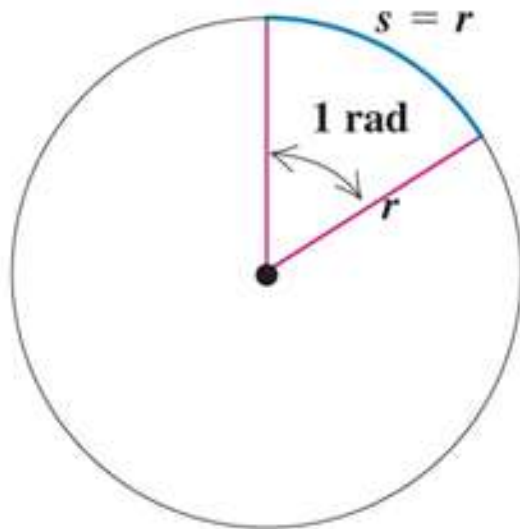


- Consider an object/particle at a distance r from the origin, at an angle θ from the positive x -axis.
- The angle may be measured in degrees, revolutions (rev) or **radians** (rad), that are related by:

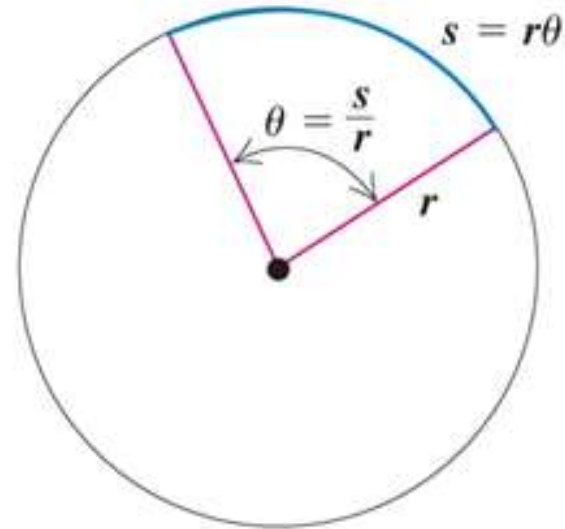
$$1 \text{ rev} = 360^\circ = 2\pi \text{ rad}$$

Radians

One radian is the angle at which the arc s has the same length as the radius r .



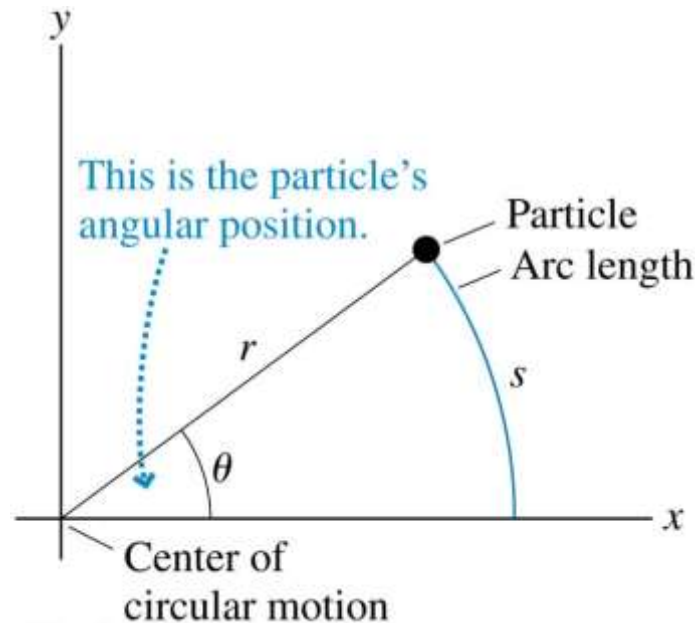
An angle θ in radians is the ratio of the arc length s to the radius r .



N.B. Because the length of the circumference of a circle is $2\pi r$, dividing this length by r , gives us 2π , which means that there are 2π radians in a circle.

$$\frac{2\pi r}{r} = 2\pi \text{ rad}$$

Angular Position

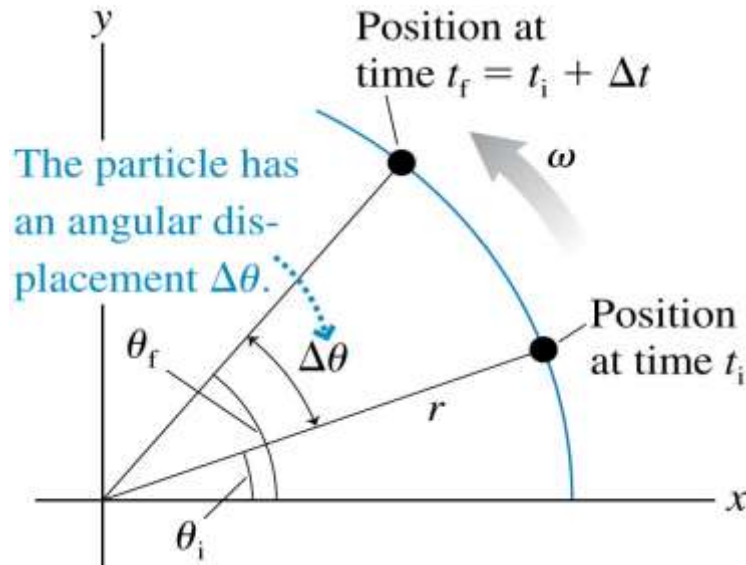


- Thus, if the angle is measured in radians, we can describe the **angular position** of the object, using the relationship below.

$$s = r\theta \quad (\text{with } \theta \text{ in rad})$$

- We can then go one step further, and describe the velocity of the object, referred to as **angular velocity**, because the object is moving in a circle; let's now consider this idea in more detail.

Angular Velocity

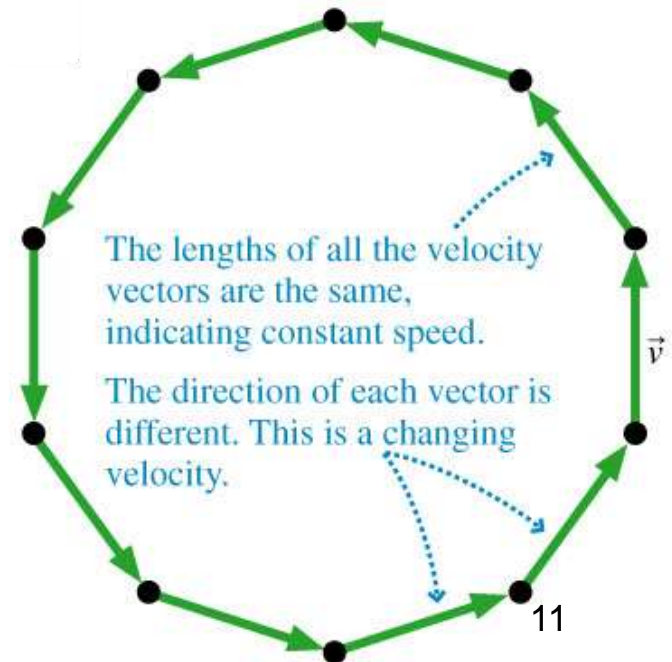


- A particle on a circular path moves through an **angular displacement** $\Delta\theta = \theta_f - \theta_i$ in a time interval $\Delta t = t_f - t_i$.
- Similar to linear motion, we can define average angular velocity, ω :
- The definition of **instantaneous angular velocity** is

$$\omega = \lim_{\Delta t \rightarrow 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} \quad (\text{angular velocity})$$

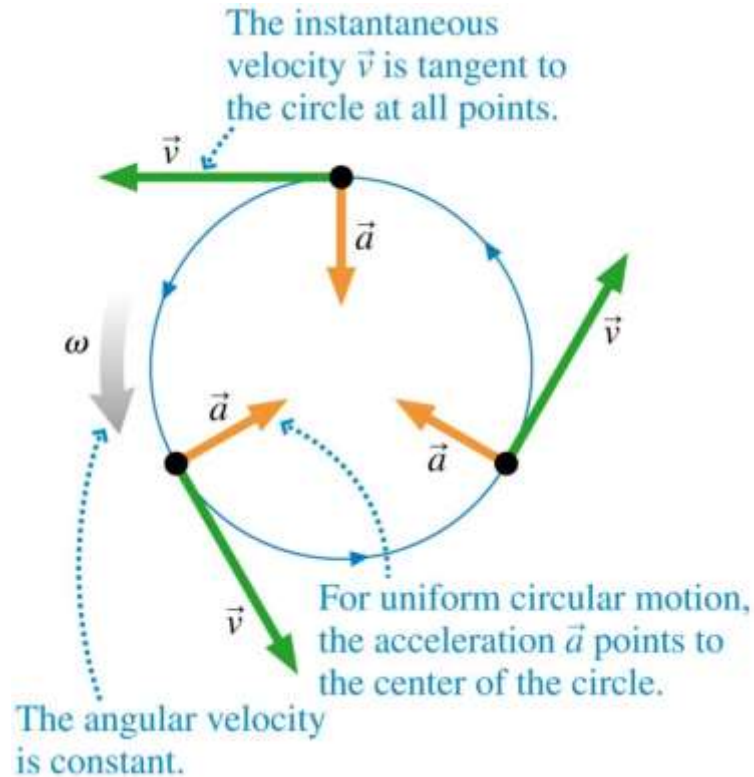
Uniform Circular Motion

- The figure to the right shows a motion diagram for a Ferris wheel.
- It has **constant speed**, but not **constant velocity** (because its direction is changing), so it is accelerating.
- We can choose a point on the wheel to track.
- For every pair of adjacent velocity vectors, we can subtract them to find the average acceleration near that point; we call this acceleration **centripetal acceleration**.

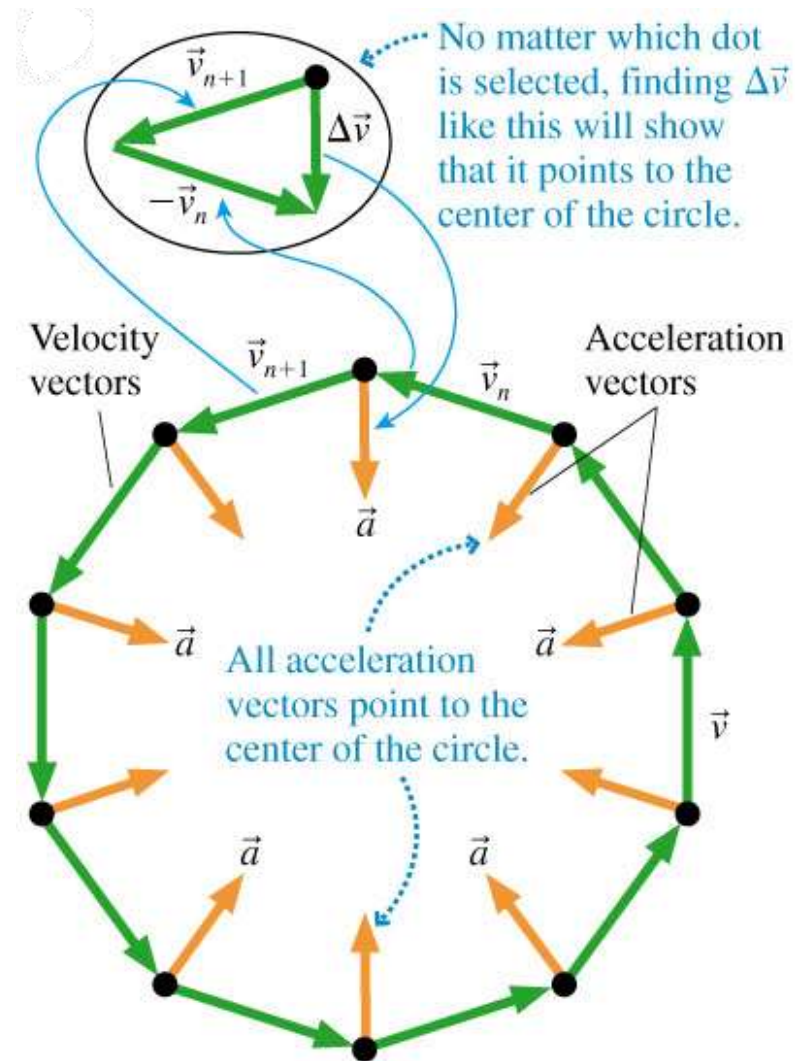
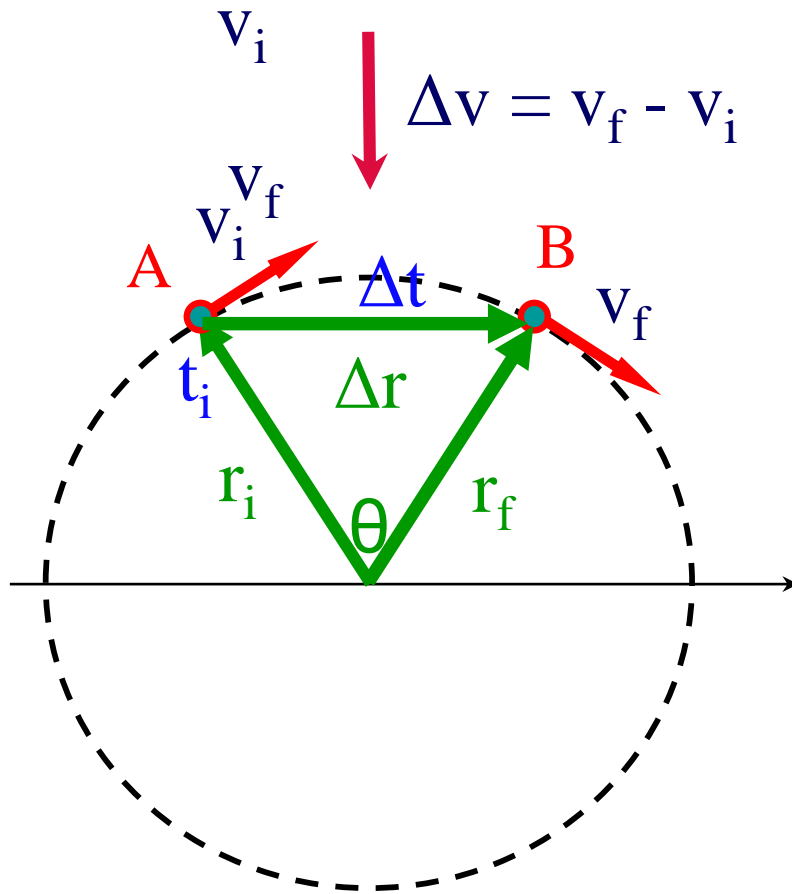


2. Centripetal Acceleration and Force

Centripetal Acceleration

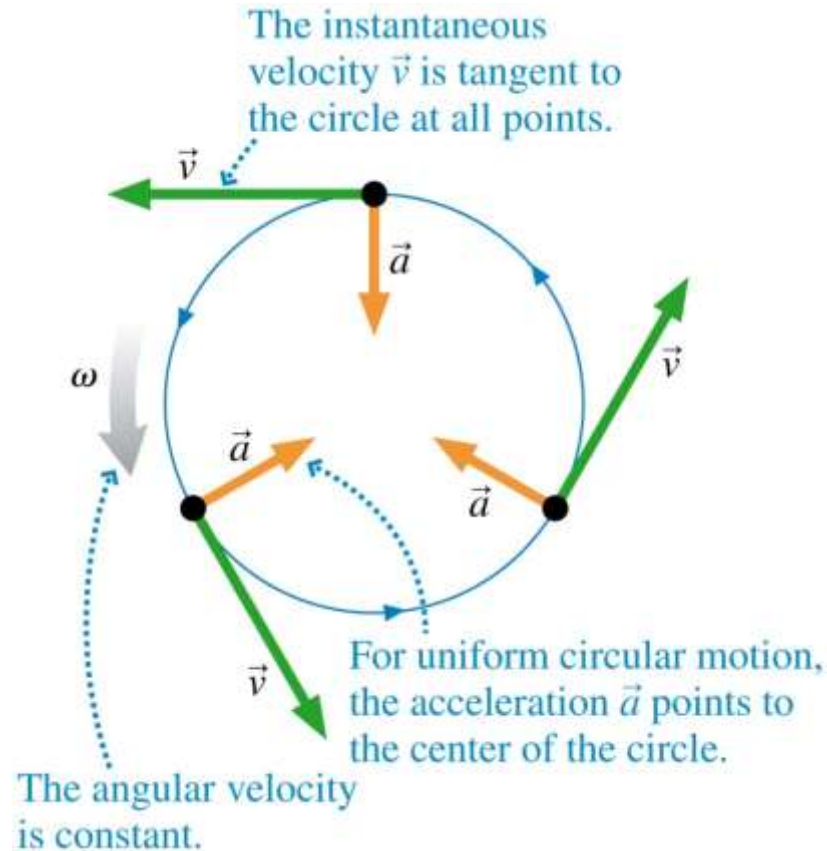


- In uniform circular motion, although the speed is constant, there is an **acceleration** because the **direction** of the velocity vector is always changing.
- The acceleration of uniform circular motion is called **centripetal acceleration**.



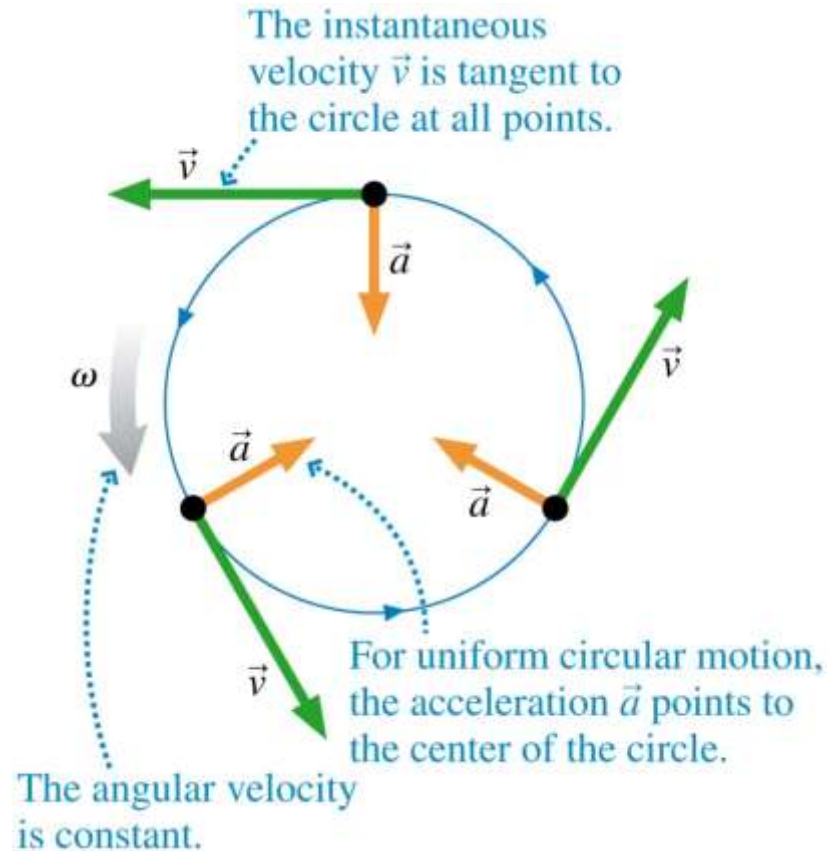
- At every point, the acceleration points toward the centre of the circle.
- This is an acceleration due to changing direction, not to changing speed.

Centripetal Acceleration



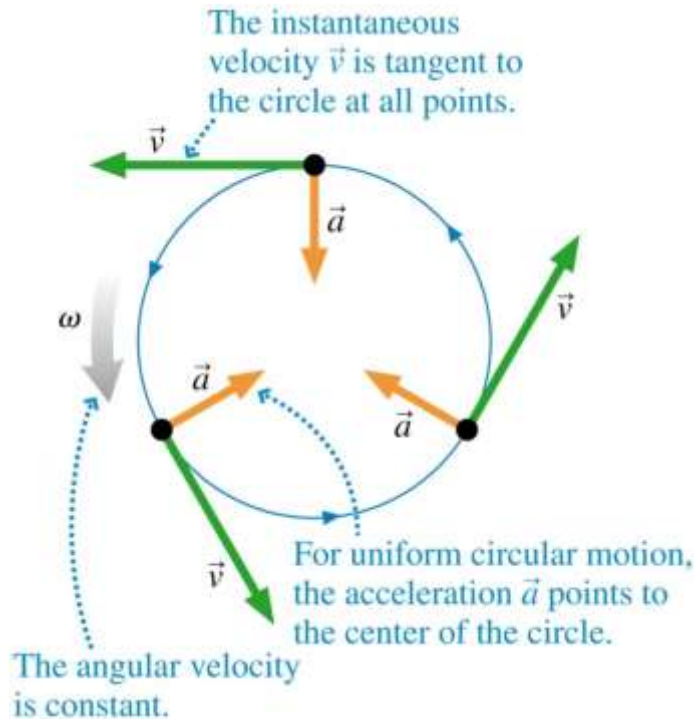
- The velocity vector \vec{v} for uniform circular motion is always tangent to the circle.
- In other words, the velocity vector has only a tangential component, which we can call v_t .

Centripetal Acceleration



- The tangential velocity component v_t is the rate at which the particle moves around the circle, where s is the arc length measured from the positive x -axis.
- The arc length is $s = r\theta$

Centripetal Acceleration



$$s = r\theta \quad \omega = \frac{\theta}{t}$$

$$\frac{s}{r} = \theta \quad \omega = \frac{s}{r} \cdot \frac{1}{t}$$

$$\omega = \frac{1}{r} \cdot \frac{s}{t} \quad \omega = \frac{1}{r} \cdot v_t$$

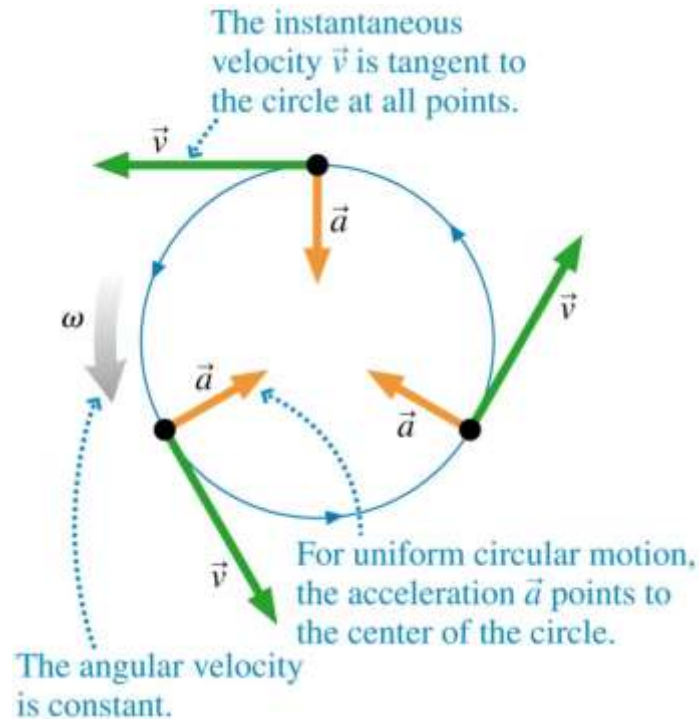
$$\omega r = v_t$$

- The angular velocity is given by ω . Thus, the tangential velocity and the angular velocity are related by

$$v_t = \omega r \text{ (with } \omega \text{ in } \frac{\text{rad}}{\text{s}} \text{)}$$

- The tangential velocity v_t is positive for counterclockwise (ccw) motion, while it is negative for clockwise motion (cw).

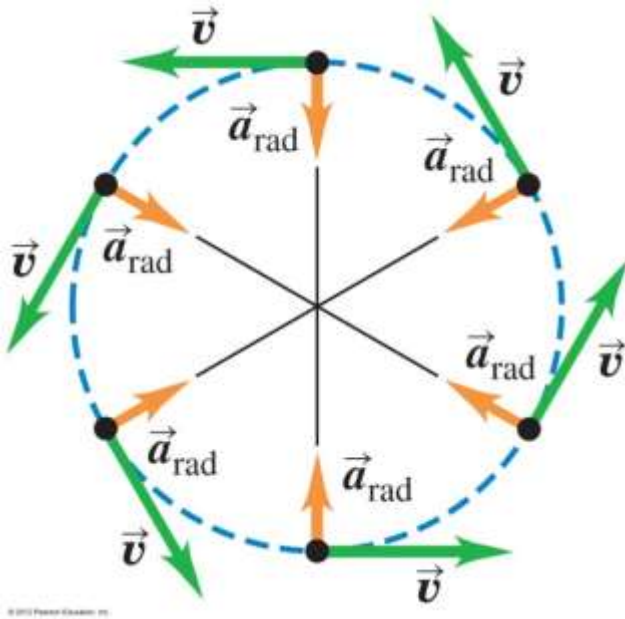
Centripetal Acceleration



$$\vec{a} = \left(\frac{v^2}{r}, \text{toward center of circle} \right) \quad (\text{centripetal acceleration})$$

Centripetal acceleration can be written in terms of angular velocity as: $a = \omega^2 r$.

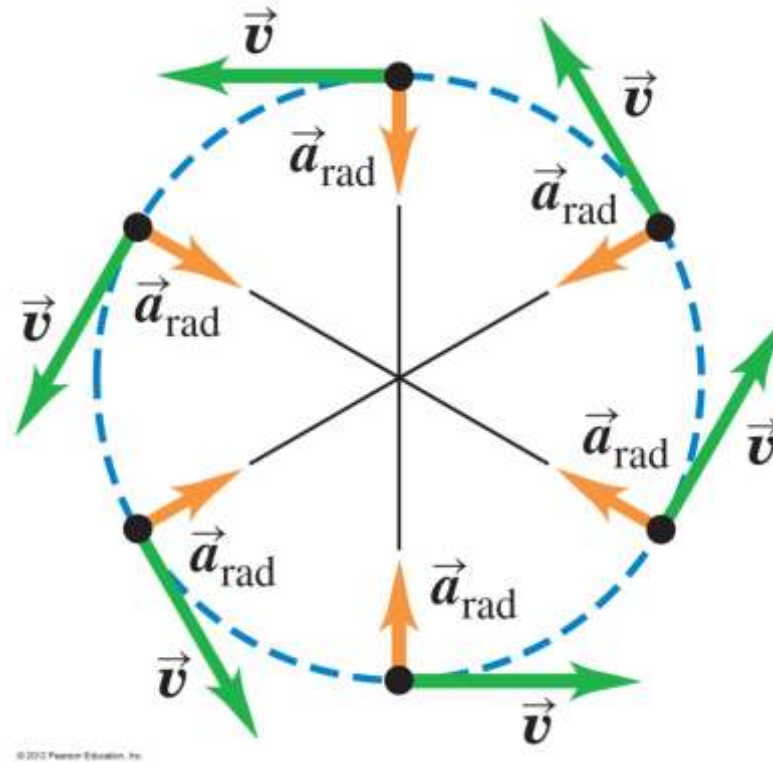
Centripetal Force



$$F_{\text{net}} = m \frac{v^2}{R}$$

- Building on the idea of centripetal acceleration
- we can see that the object's acceleration toward the centre of the circle must be caused by a force, or **several forces, such that their vector sum $\sum \vec{F}$ is a vector** that is always directed toward the centre, with constant magnitude.

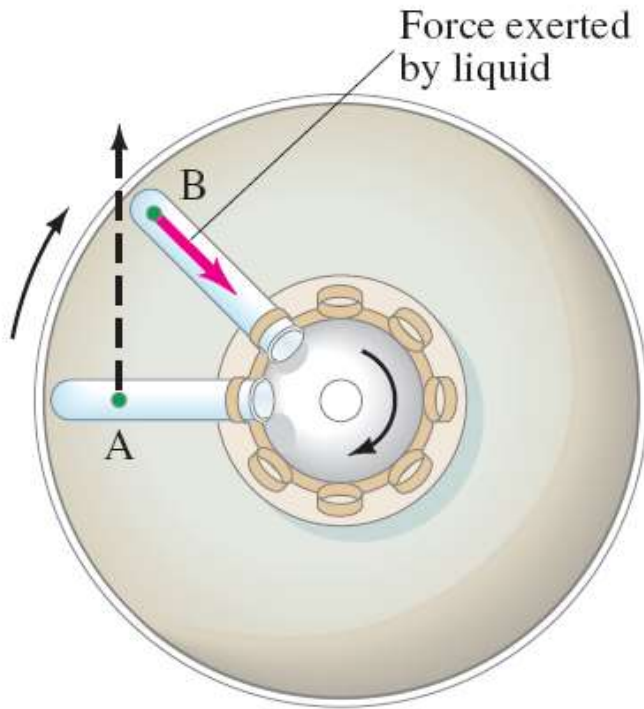
Centripetal Force



N.B.

When a particle moves in a circular path with constant speed (or uniform circular motion), there is no component of acceleration parallel to its instantaneous velocity; otherwise the particle's speed would change. Let's now look at some **applications of circular motion**.

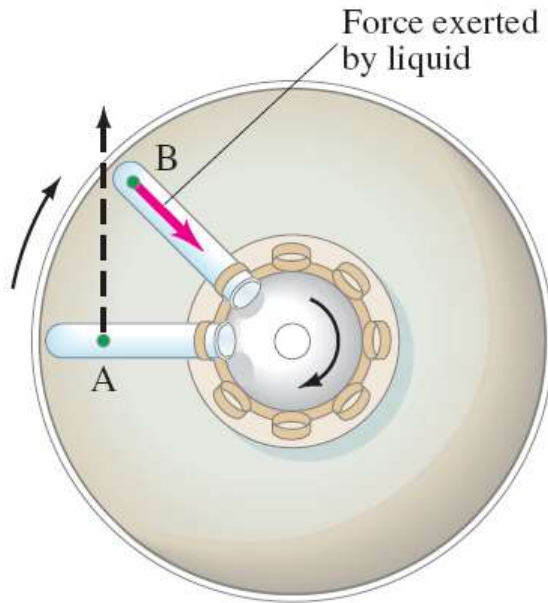
A Centrifuge



Centrifugation is a technique that can be used to separate Insoluble particles in solution based on density, size, viscosity, or shape.

- Centrifuges are used to separate materials.
- Test tubes held in the centrifuge rotor are accelerated to very high rotational speeds.
- The small green dot represents a small particle, in a fluid-filled test tube.

A Centrifuge



In centrifugal as we rotate the substance experience centrifugal force away from the center of rotation which forces the materials against the walls of the tube.

$$F_f = -m \frac{v^2}{R} \quad F_p = m \frac{v^2}{R}$$

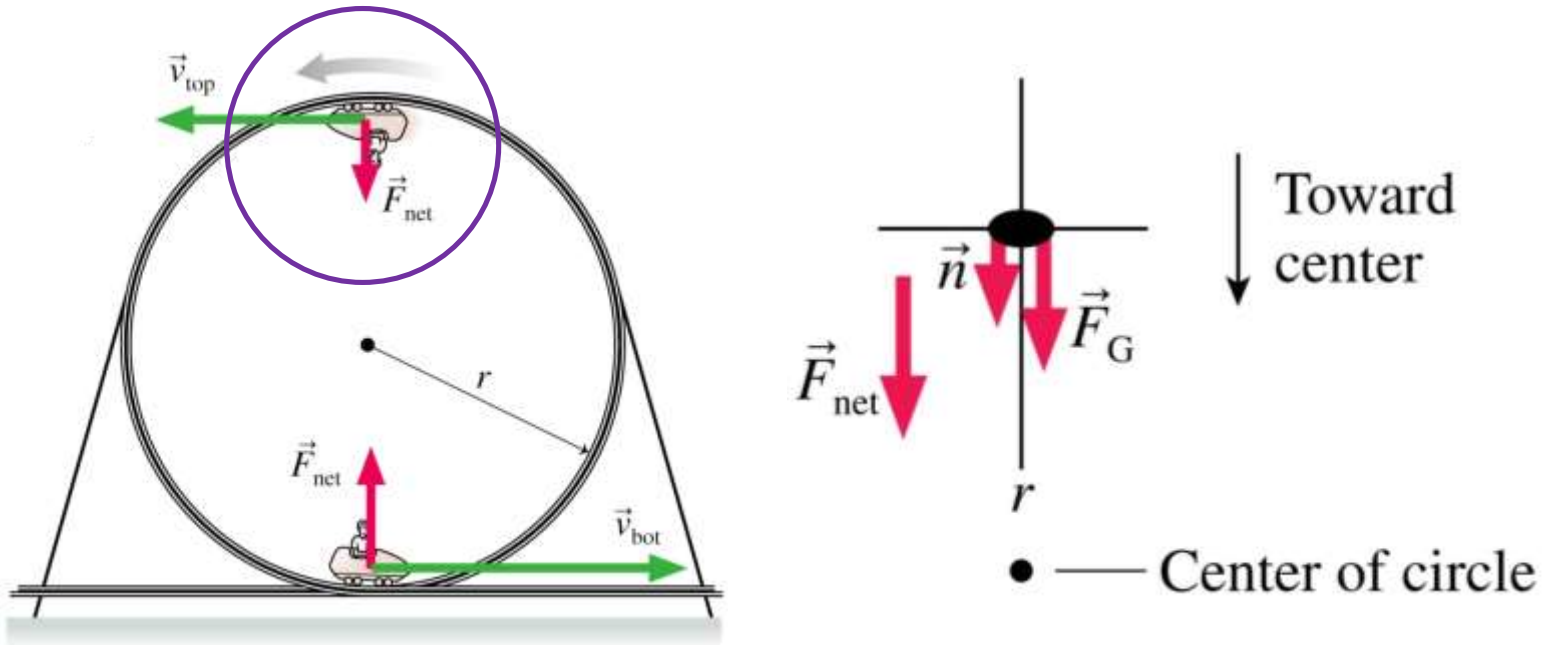
- At position A, the particle has a tendency to move in a straight line, but the fluid resists the motion of the particles, exerting a centripetal force that keeps the particles moving nearly in a circle.
- The resistive force exerted by the fluid usually does not quite equal mv^2/r , and the particle moves slowly toward the bottom of the tube.
- Let's look at another interesting application, that of the **rollercoaster**.

A Rollercoaster



Q. Why doesn't the rollercoaster fall off the tracks when doing a loop-the-loop?

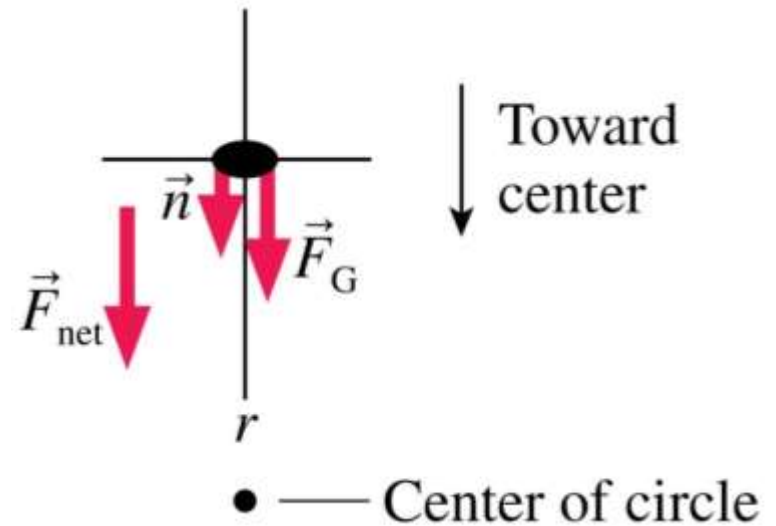
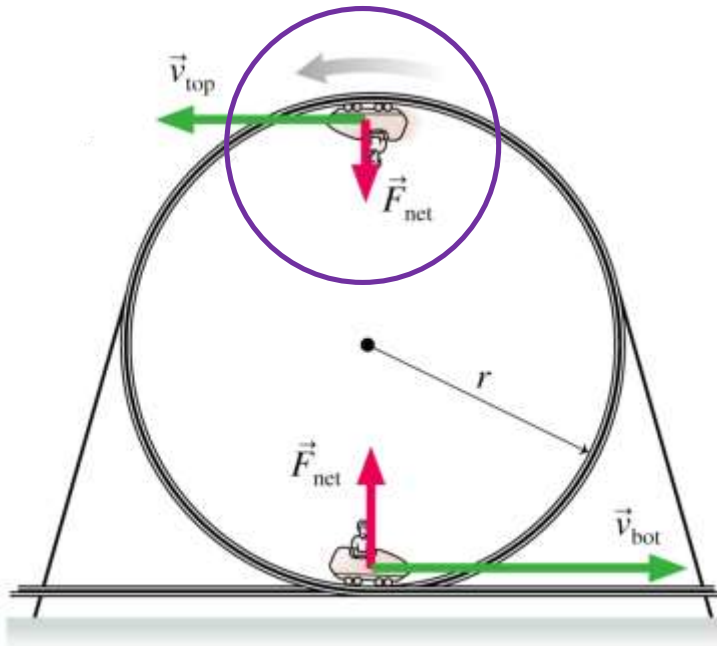
A Loop-the-Loop



- The figure shows the roller-coaster free-body diagram at the top of the loop.
- Now, the normal force acts downward.
- The car is still moving in a circle, so the net force is also downward:

$$\sum F_r = n_r + (F_G)_r = n + mg = \frac{m(v_{\text{top}})^2}{r}$$

A Loop-the-Loop



$$\Rightarrow n = \frac{m(v_{top})^2}{r} - mg$$

- As v_{top} decreases, there comes a point when n reaches zero.
- The speed at which $n = 0$ is the minimum speed you must travel at to avoid falling off:

$$v_{minimum} = \sqrt{\frac{rmg}{m}} = \sqrt{rg}$$

A Loop-the-Loop

- Obviously, when designing your rollercoaster, you should plan for it to travel a bit faster than this minimum speed.
- Health and safety is important 😊.

3. Periodic/Oscillatory Motion

Periodic/Oscillatory Motion



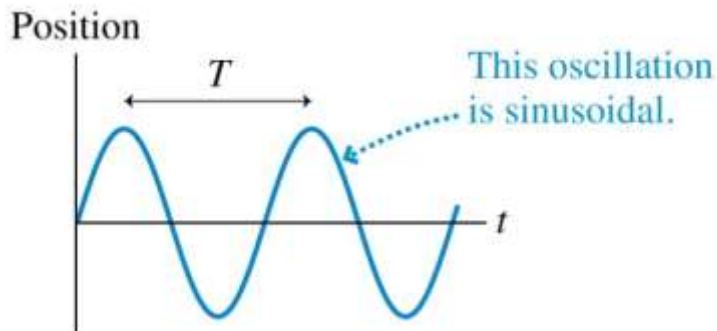
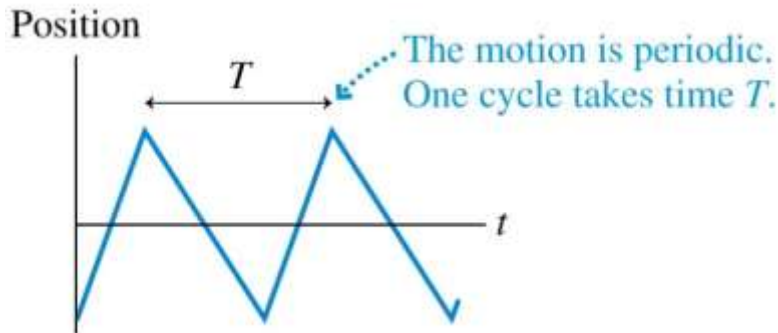
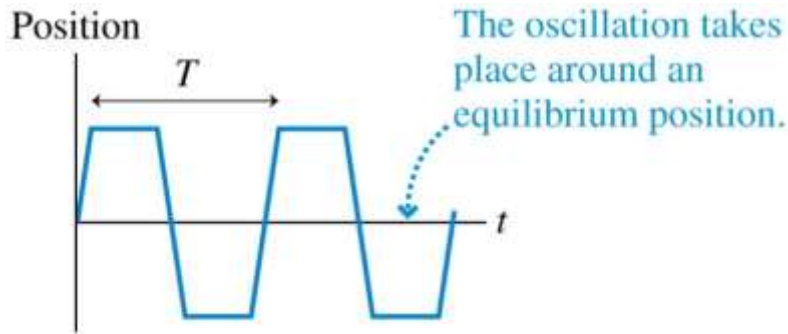
- Many everyday objects, such as springs and grandfather clocks, appear to move in a repeating pattern.
- We refer to such repeating movement as **periodic motion**.
- A mechanical system that undergoes periodic motion always has a stable **equilibrium position**.

Periodic/Oscillatory Motion



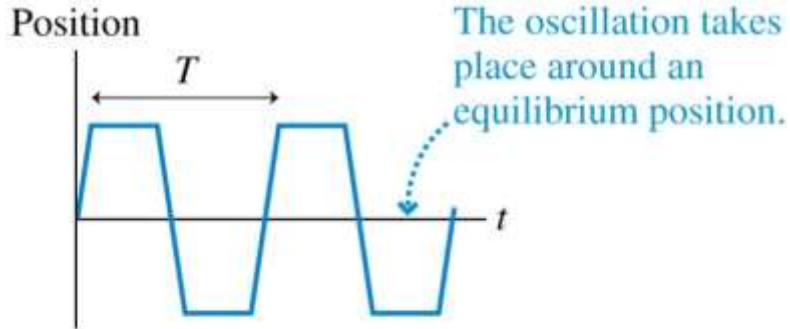
- When the object is moved away from its equilibrium/'average' position, a **force** comes into play that attempts to **return the object** to this position.
- However, by the time it reaches its equilibrium position, it has picked up **kinetic energy**.
- The object will slow down, and eventually stop. It will then start to **accelerate** back to its equilibrium position again.

Periodic/Oscillatory Motion



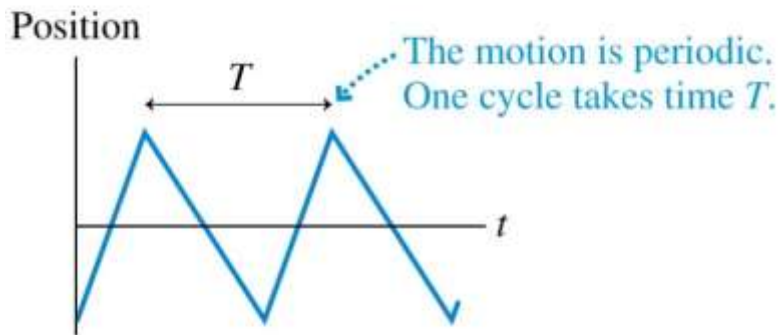
- In general, objects that undergo any type of repetitive motion back-and-forth around an equilibrium position are called **oscillators**.
- The time to complete one full cycle, or one oscillation, is called the **period T** .

Periodic/Oscillatory Motion

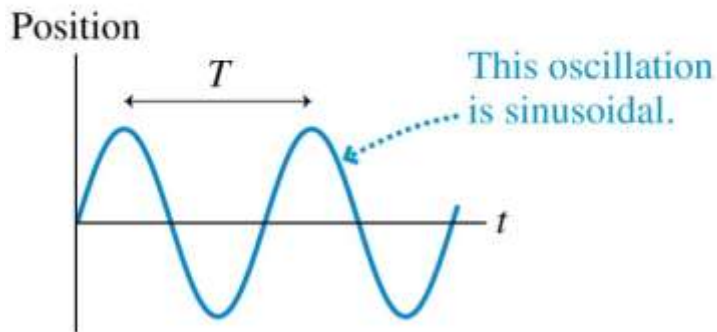


- The number of cycles per second is called the **frequency f** , measured in Hz:

$$f = \frac{1}{T} \quad \text{or} \quad T = \frac{1}{f}$$



- 1 Hz = 1 cycle per second = 1 s⁻¹

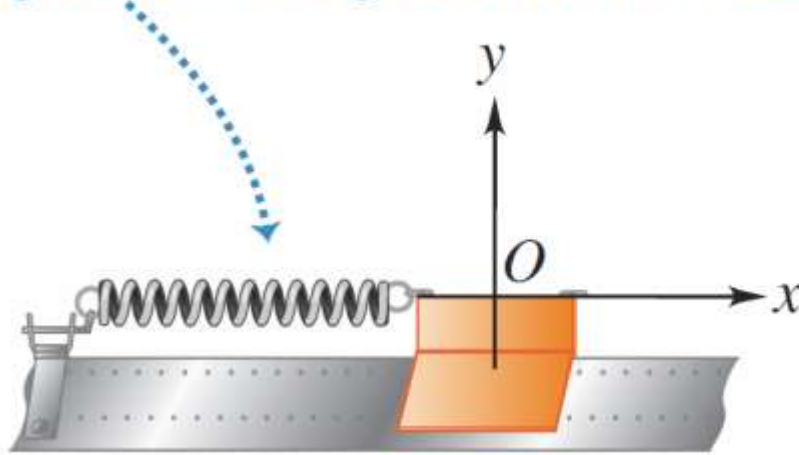


- The simplest type of periodic motion is called **simple harmonic motion**, so let's have a look at it in more detail.

4. Simple Harmonic Motion

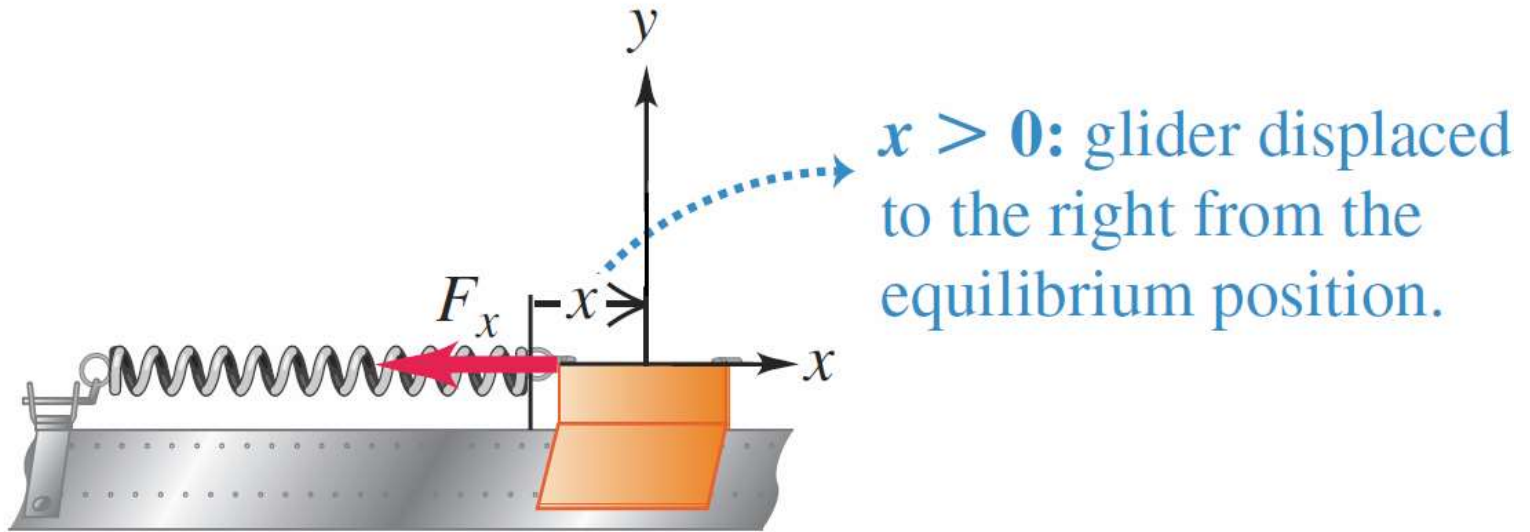
Simple Harmonic Motion

$x = 0$: The relaxed spring exerts no force on the glider, so the glider has zero acceleration.



- Consider the example of an air track glider attached to a spring.
- **We define the origin, O , as the equilibrium position of the system.**
- We ignore friction, assuming it to be negligible.

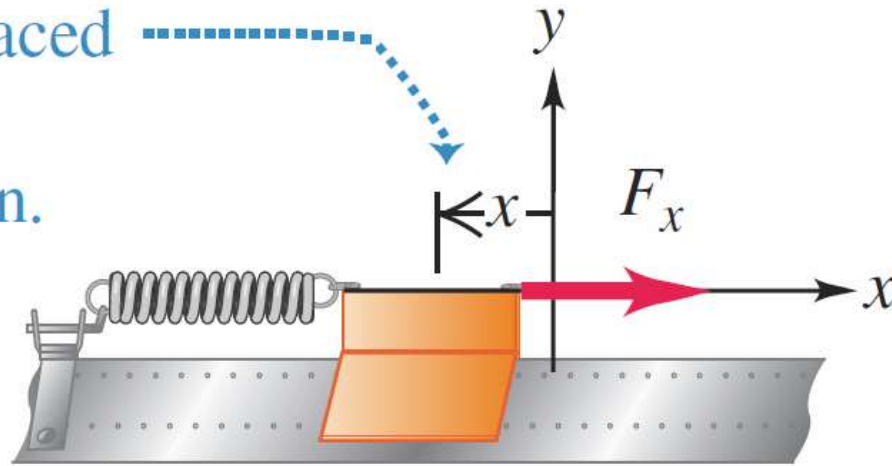
Simple Harmonic Motion



- When the glider is displaced to the right, a force will act to the left as the spring is stretched (**remember Hooke's law**).
- We call this force the '**restoring force**' because the force wants to restore the object/glider to its original/equilibrium position.
- Since our coordinate system defines $x > 0$ then $F_x < 0$.

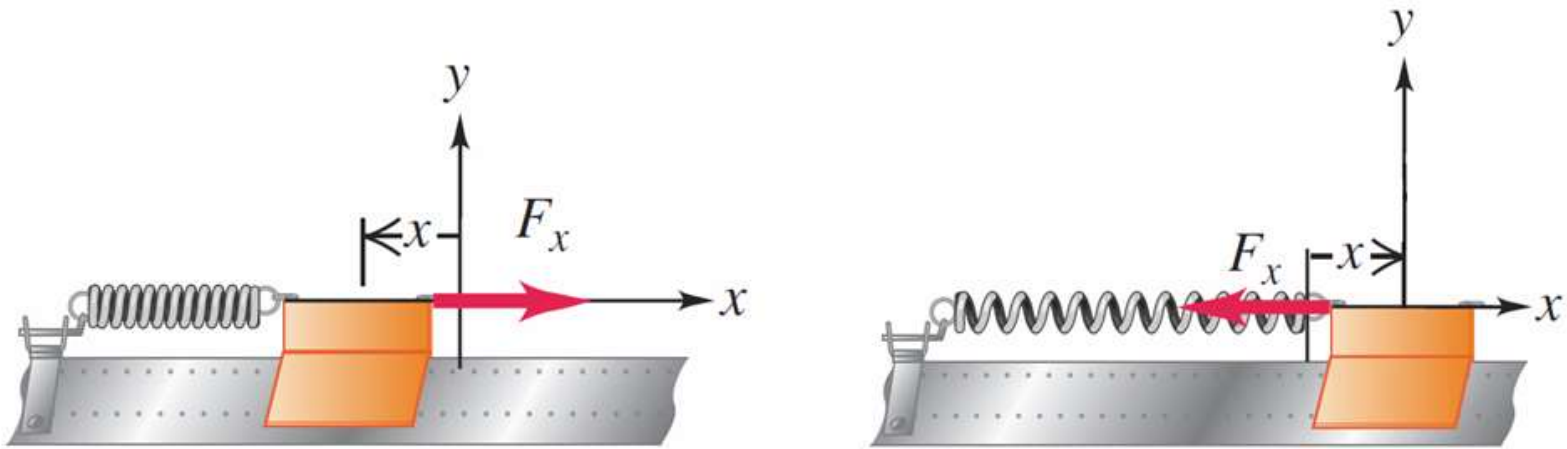
Simple Harmonic Motion

$x < 0$: glider displaced to the left from the equilibrium position.



- And conversely, when the glider is displaced to the left, a restoring force will act to the right as the spring is compressed.
- Since our coordinate system defines $x < 0$ then $F_x > 0$.

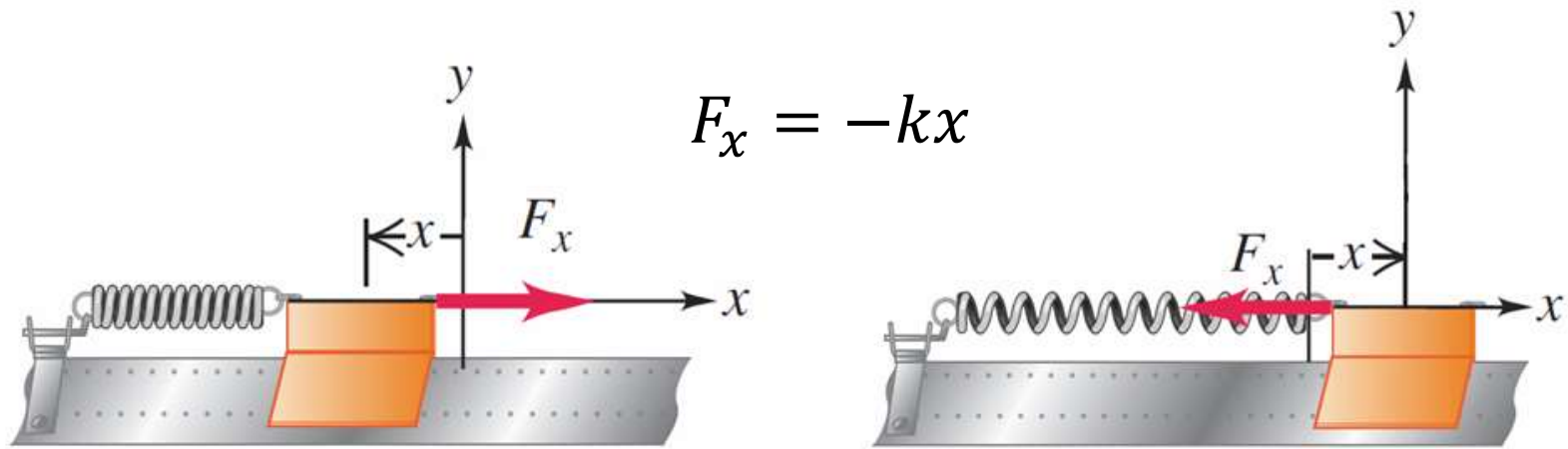
Simple Harmonic Motion



- When the system is displaced from equilibrium, the restoring force F_x will **always act in the opposite direction to the displacement** x .
- The equation for the restoring force (**Hooke's law**) is therefore

$$F_x = -kx$$

Simple Harmonic Motion



- We can use our understanding of Newton's 2nd law to obtain an equation for the acceleration of the glider/object:

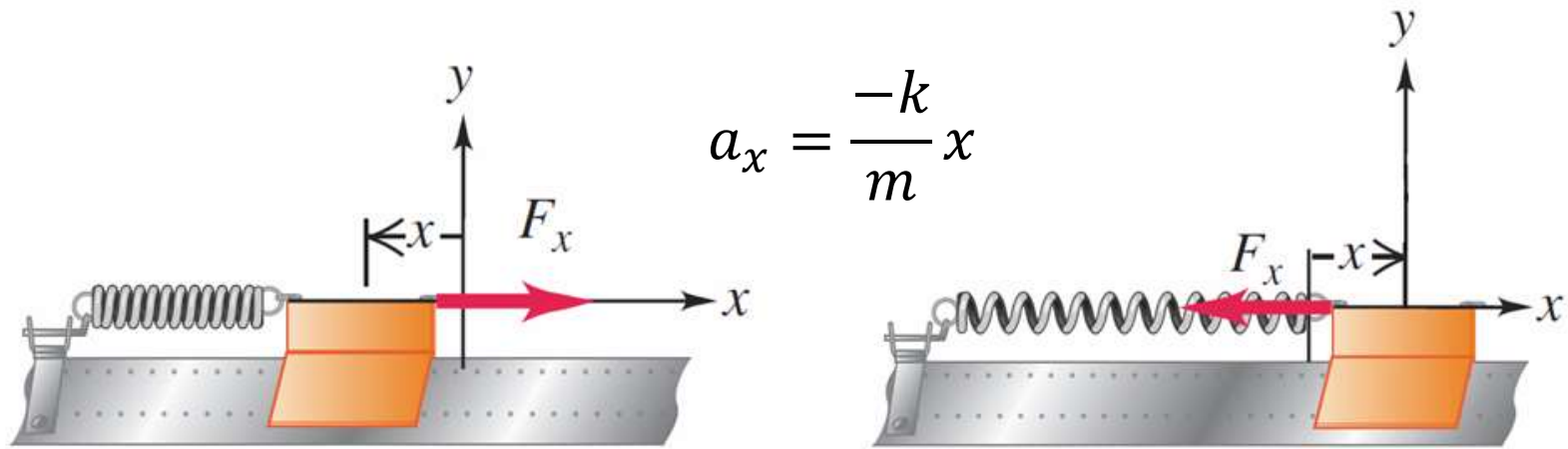
$$F_x = ma_x$$

$$ma_x = -kx$$

$$a_x = \frac{-k}{m}x$$

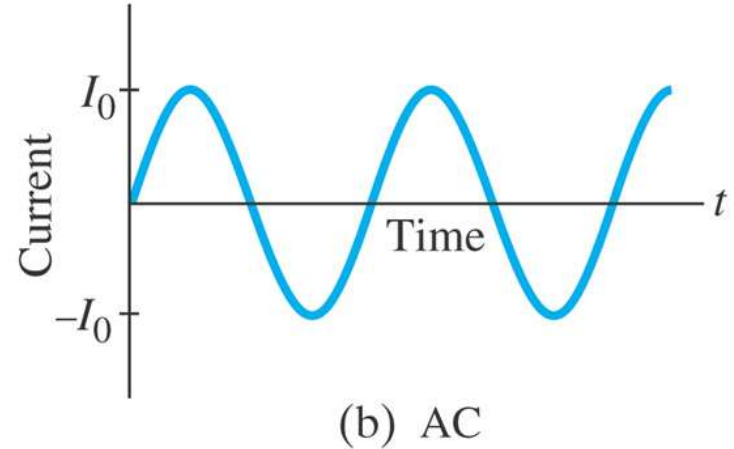
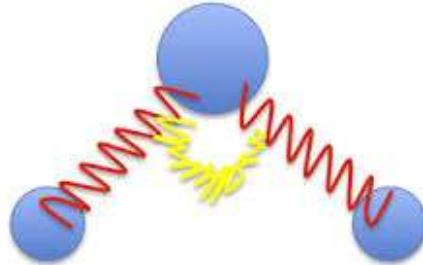
where $\frac{-k}{m}$ is a constant.

Simple Harmonic Motion



- Thus, the defining feature of an object which undergoes simple harmonic motion is that its **acceleration is directly proportional to its displacement from its equilibrium position**.
- Consequently, the restoring force acting on the object is directly proportional to the object's **displacement from its equilibrium position**.
- In more complex types of periodic motion, the restoring force may not be directly proportional to the displacement.

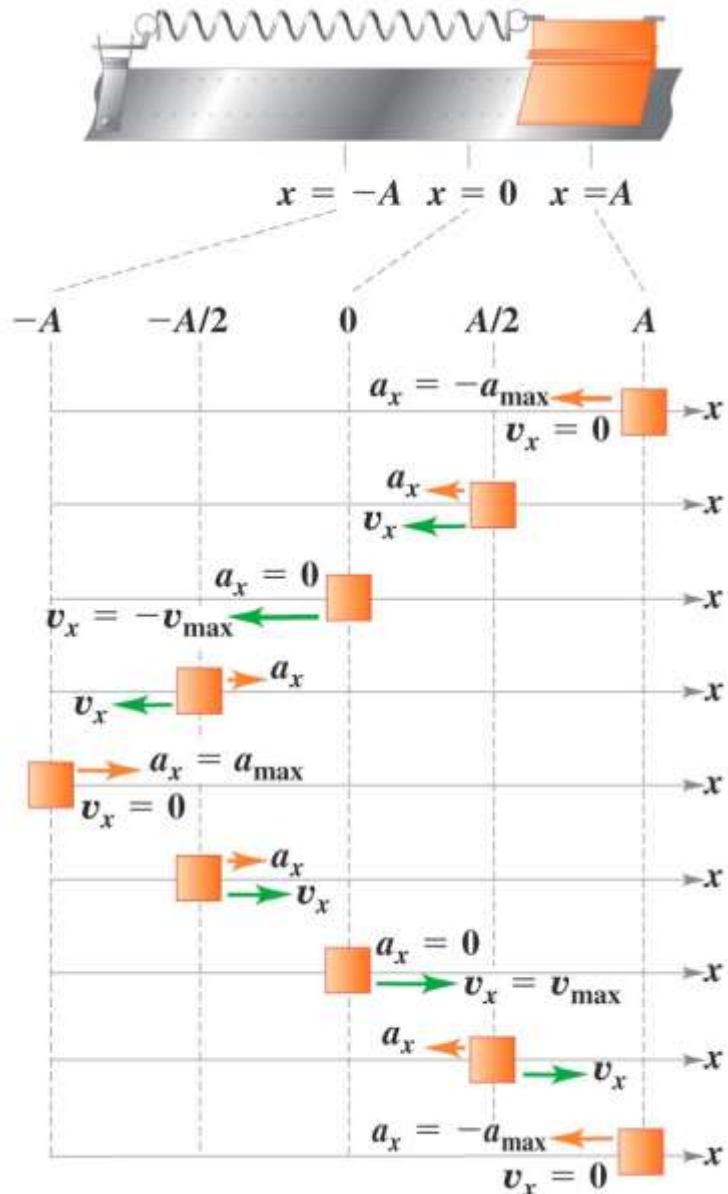
Simple Harmonic Motion



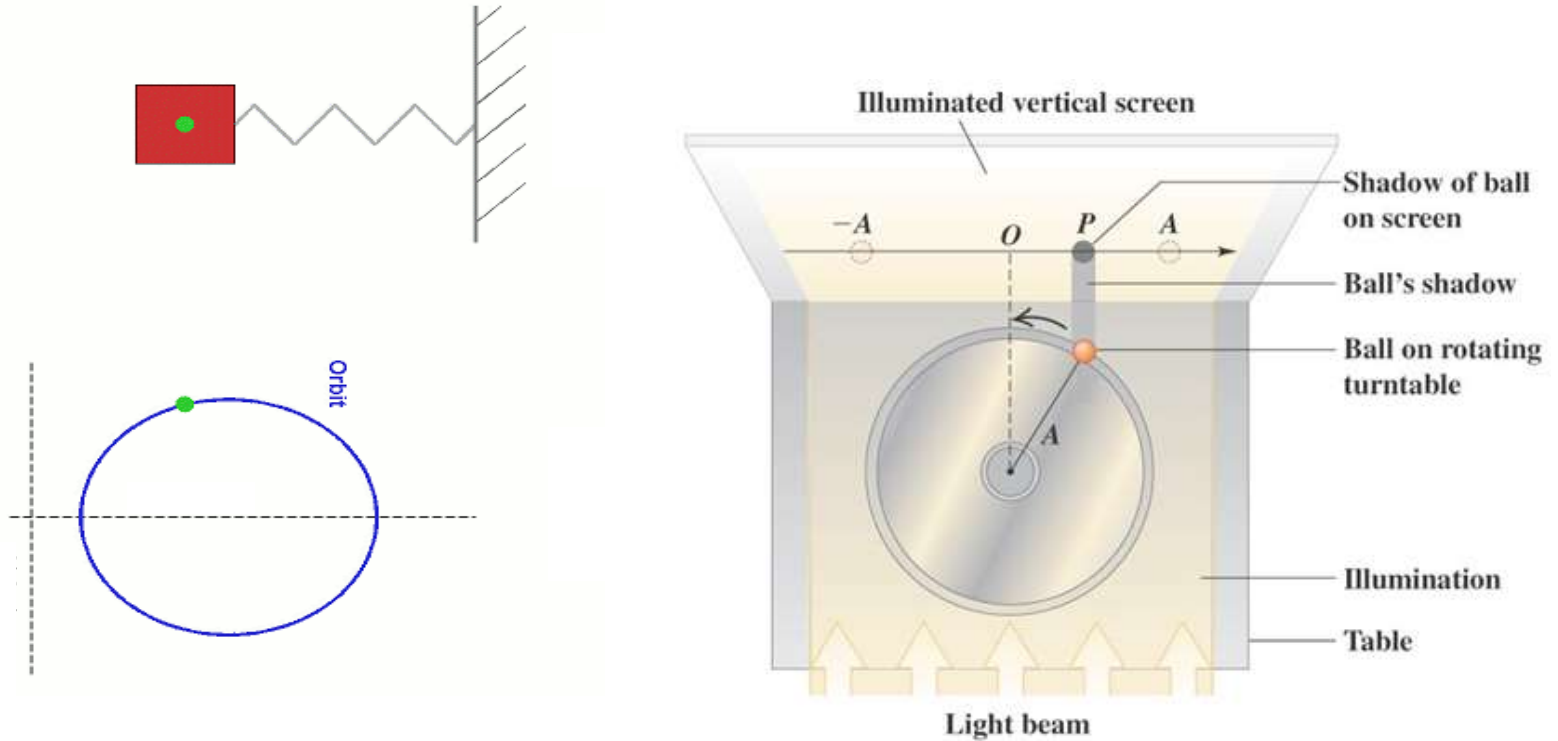
- However, many periodic motions are approximately simple harmonic if the **maximum displacement from the equilibrium position is small**.
- Examples of objects undergoing periodic motion that can be considered simple harmonic motion include the **simple pendulum**, **molecular bonds**, and, as we will see later, alternating **electric current**.
- Let's now look at some more quantities and equations that can be used to represent certain features of **simple harmonic motion**.

Features of Simple Harmonic Motion

- The amplitude (A) is the **maximum magnitude** of the displacement.
- Since simple harmonic motion is **symmetrical**, this means the overall motion is over a distance of $2A$.
- A **cycle (also known as an oscillation)** is one complete round trip. For example, from A to $-A$ and then back to A .

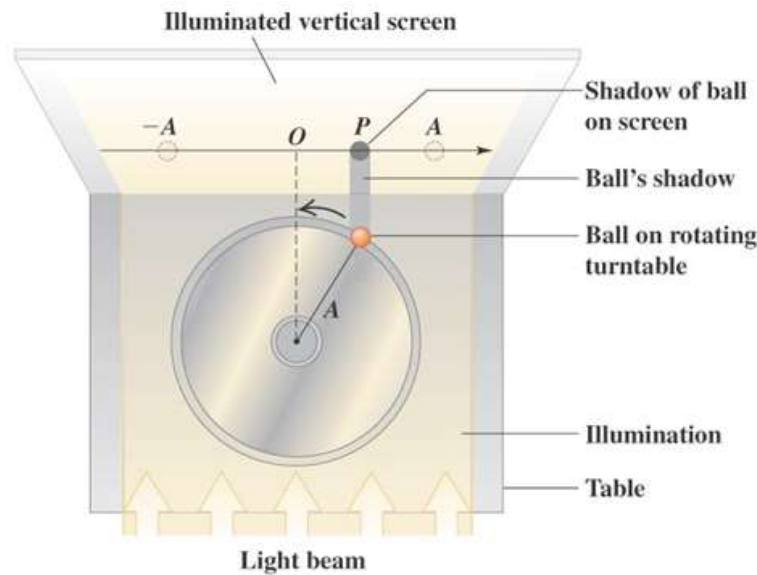


Features of Simple Harmonic Motion



- For a body undergoing simple harmonic motion, T is the **time/period** for one complete oscillation.
- This oscillation, when referenced to the uniform circular motion of a circle, (as in the **orange ball** on the rotating turntable) corresponds to one complete revolution of the circle.

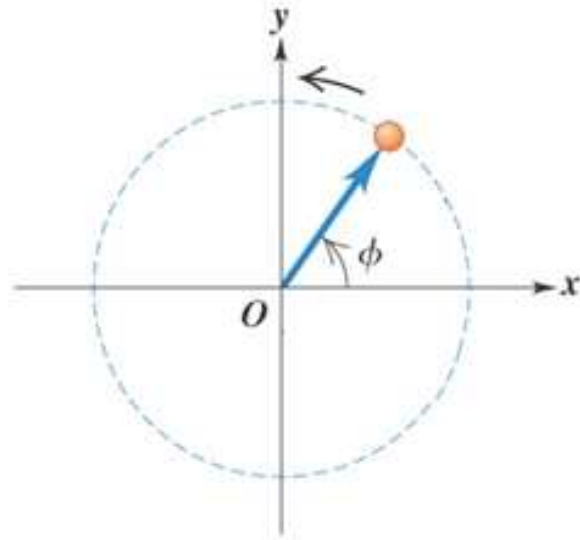
Features of Simple Harmonic Motion



$$\begin{aligned} & \frac{2\pi \text{ radians}}{T} \\ &= 2\pi \text{ radians} \times f \\ &= 2\pi f = \omega \end{aligned}$$

- If we divide the number of radians in a circle (2π) by the time it takes for one complete revolution of the circle (T), we can construct a useful quantity called the **angular frequency (ω)**.
- It represents the rate of change of the angle, φ , with time, and is always measured in radians per second (rads/s).

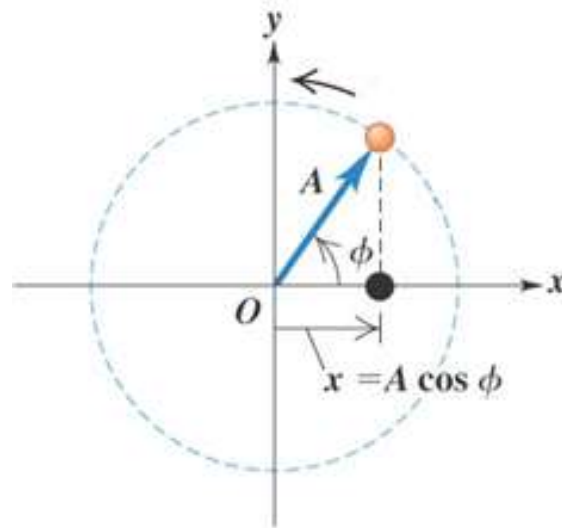
Features of Simple Harmonic Motion



- The **orange ball** moves at a constant speed, tracing an angle, ϕ , with ϕ measured in radians.
- ω is the rate of change of the angle, ϕ , with time:

$$\omega = \frac{\Delta\phi}{\Delta t} = 2\pi f$$

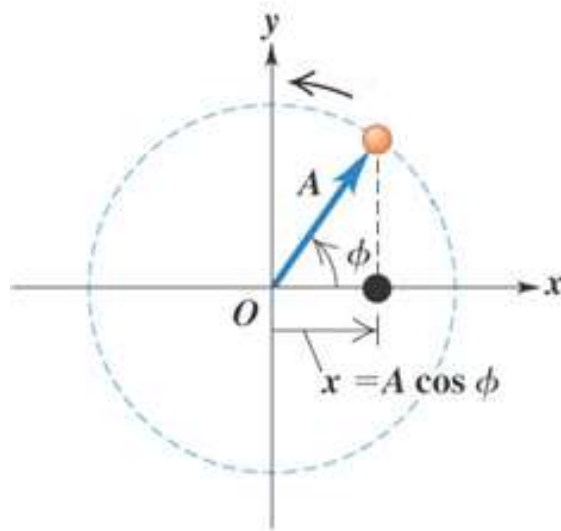
Features of Simple Harmonic Motion



- In the above picture, the **black ball** (and the function used to describe the position of this **black ball** on the x-axis) represents the position of the **orange ball** relative to the x-axis.
- x is the position of the **black ball** relative to the origin; A is the maximum displacement (or amplitude) of the ball relative to the origin; and ϕ is the angle that the **ball** makes relative to the origin.

Features of Simple Harmonic Motion

$$\omega = \lim_{\Delta t \rightarrow 0} \frac{\Delta \theta}{\Delta t} = \frac{d\theta}{dt}$$



- Because ω is constant, ϕ increases uniformly with time; that is:

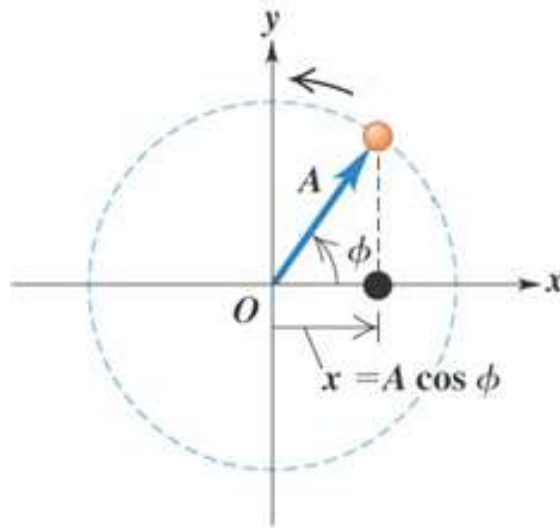
$$\phi = \omega t$$

- Since $x = A \cos \phi$, then $x = A \cos \omega t$
- We have defined $\omega = 2\pi f$, so we can also rewrite x as;

$$x = A \cos 2\pi f t$$

- This is the position of the black ball at time t .

Features of Simple Harmonic Motion



- From the position function for the ball, we can derive the equations for both its velocity and acceleration, using calculus:

$$x = A \cos \omega t$$

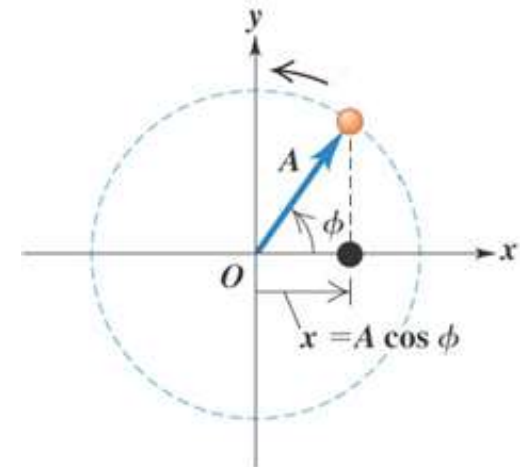
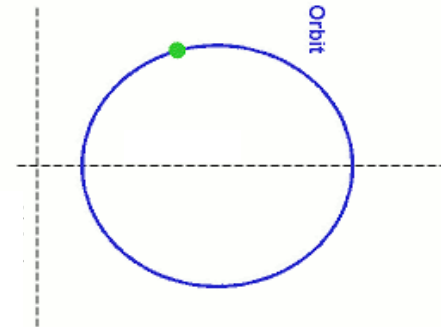
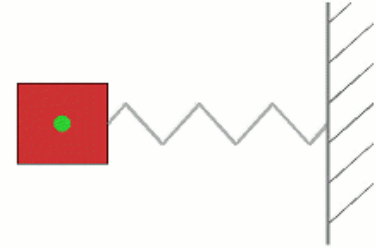
$$v = -\omega A \sin \omega t$$

$$a = -(\omega)^2 A \cos \omega t$$

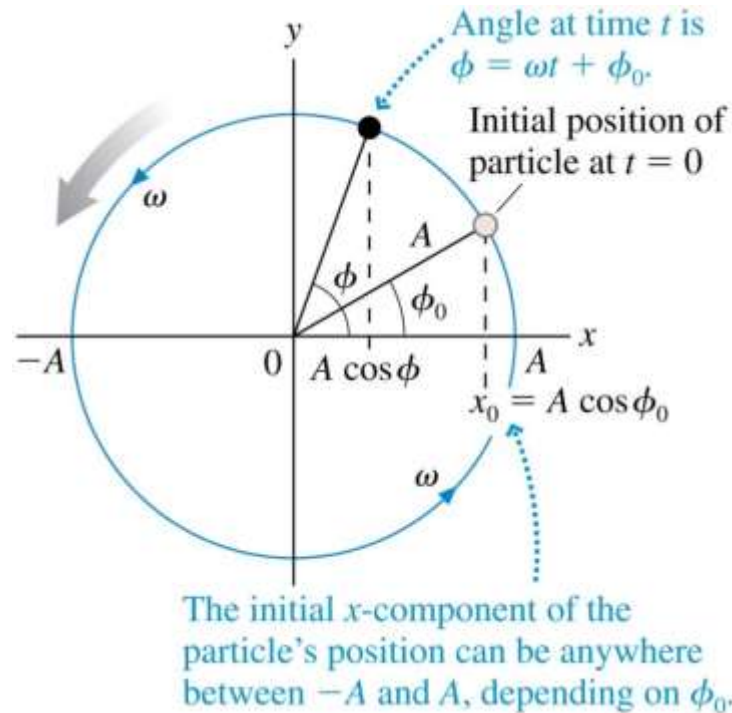
- By substituting in for x , we can also write acceleration as

$$a = -(\omega)^2 x$$

Features of Simple Harmonic Motion



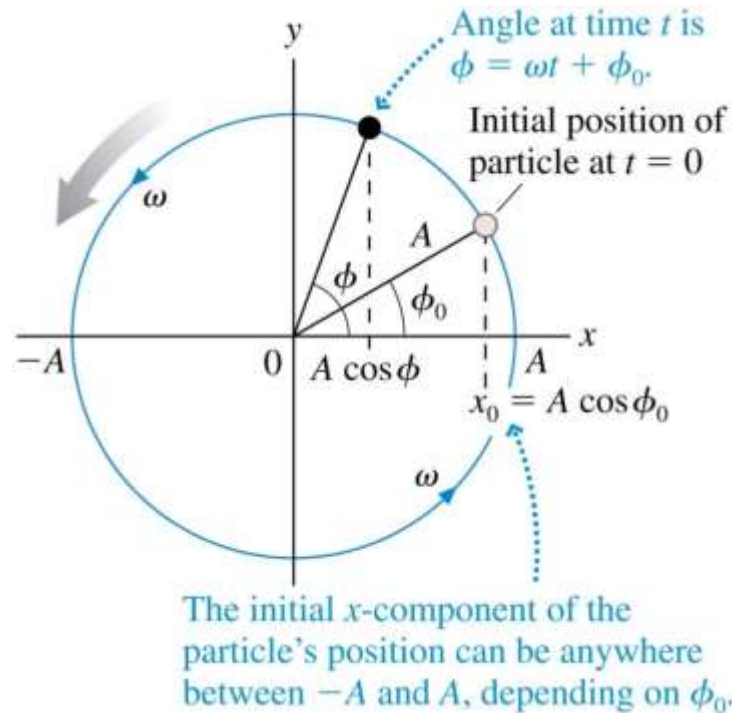
The Phase Constant



Q. What if an object in SHM is not initially at rest (equilibrium) at $x = A$ when $t = 0$?

A. Well, we can still use the cosine function, but we must add an extra term to it, called a **phase constant (ϕ_0)**, measured in radians.

The Phase Constant

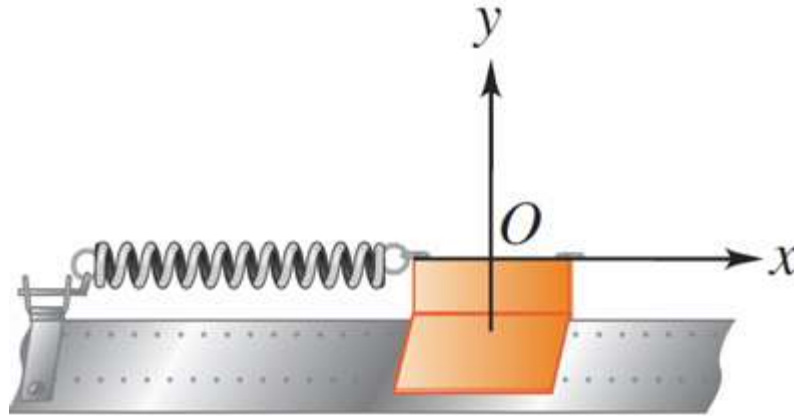


- In this case, the position function and velocity function are as follows:

$$x(t) = A \cos(\omega t + \phi_0)$$

$$v_x(t) = -\omega A \sin(\omega t + \phi_0) = -v_{\max} \sin(\omega t + \phi_0)$$

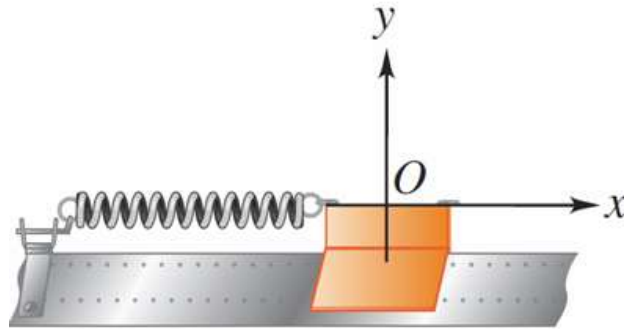
Energy in Simple Harmonic Motion



- Since we are ignoring friction, the **mechanical energy of the system is conserved**. Hence

$$E = \frac{1}{2}mv^2 + \frac{kx^2}{2} = \text{constant}$$

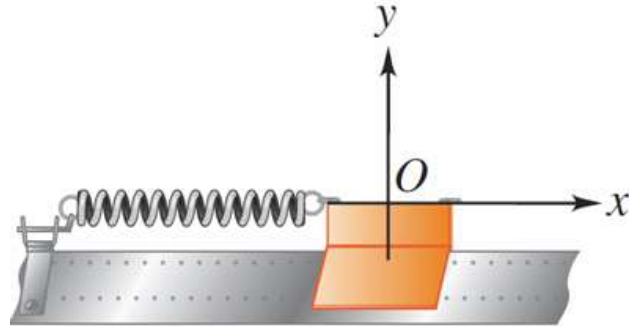
Energy in Simple Harmonic Motion



- When $x = A$ or $x = -A$

$$E = \frac{kA^2}{2} = \text{constant}$$

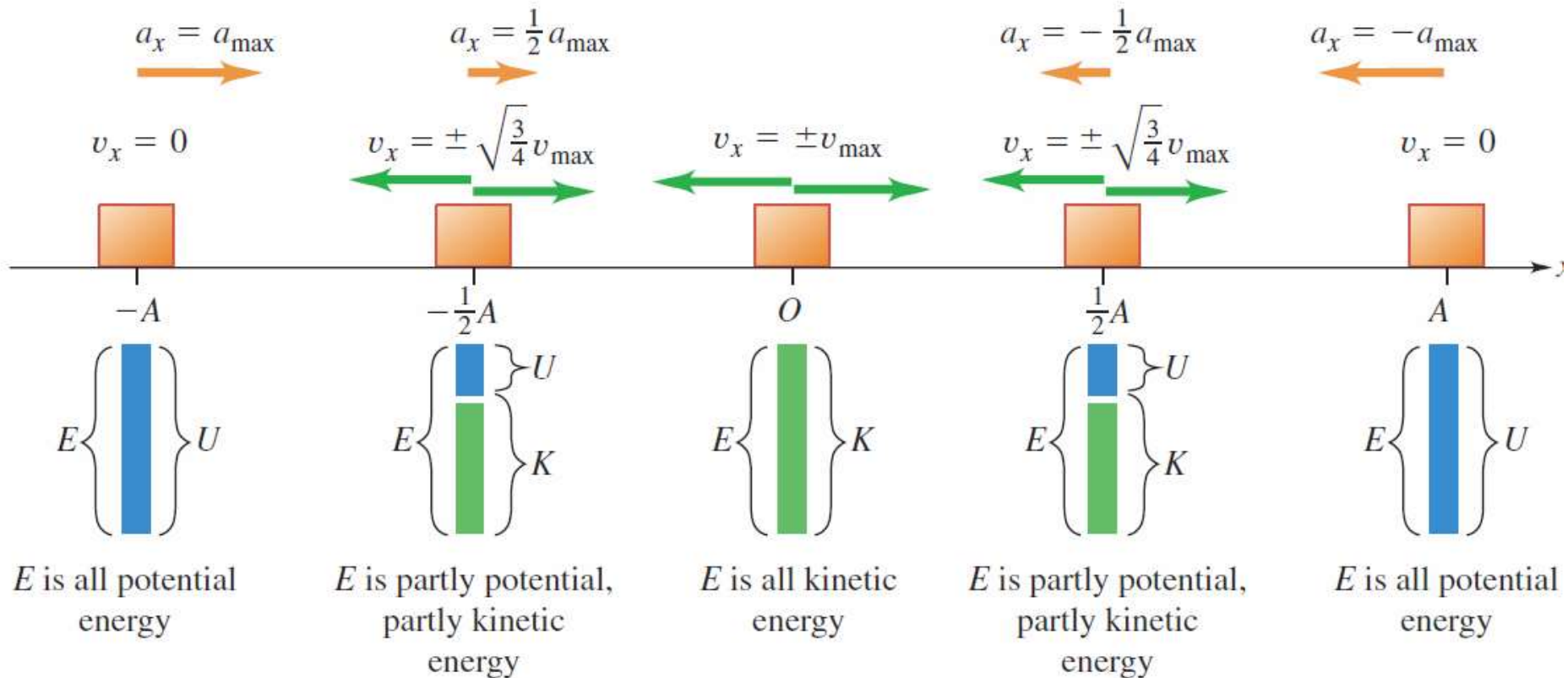
Energy in Simple Harmonic Motion



$$v_x = \pm \sqrt{\frac{k}{m}} A$$

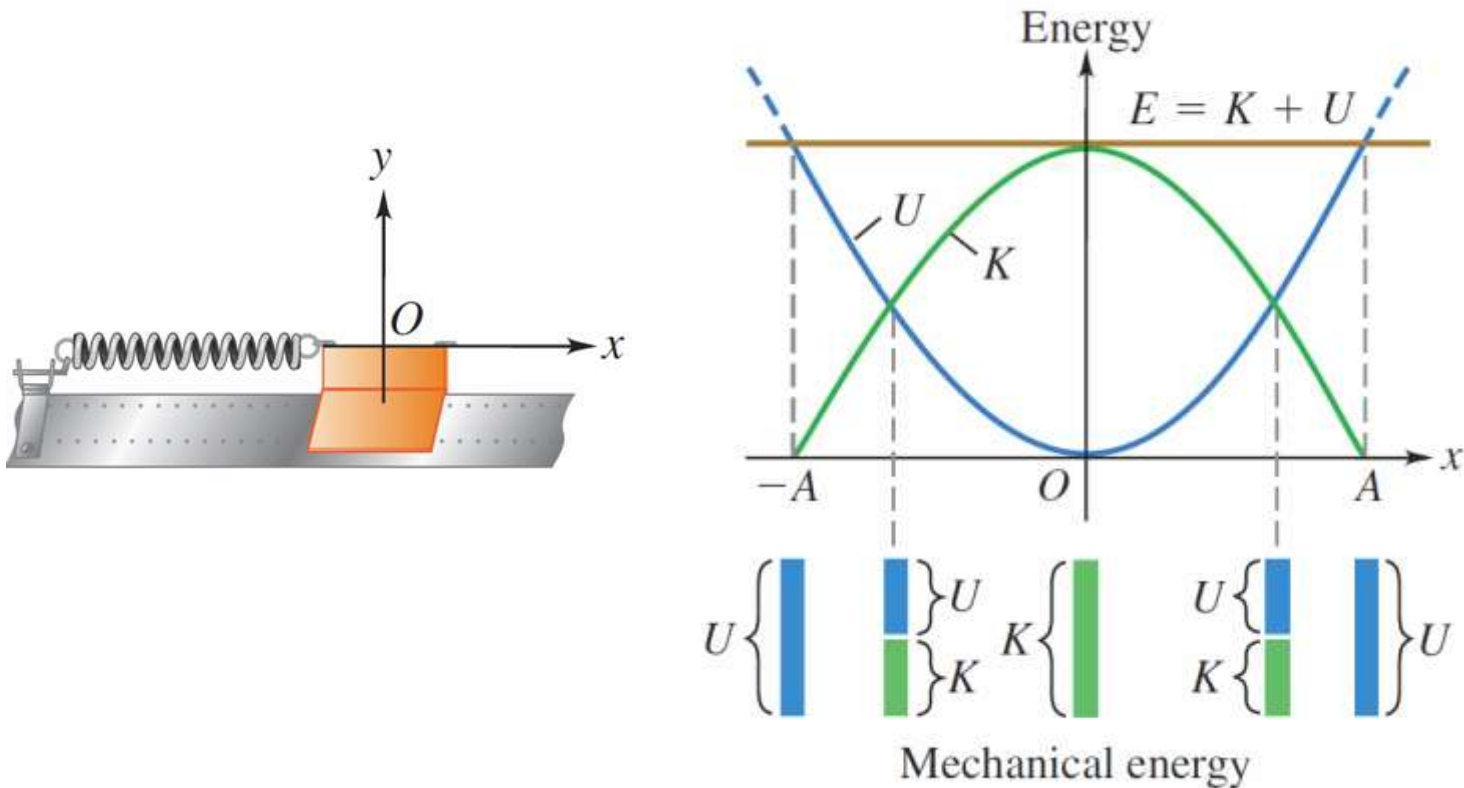
The **maximum speed** of the object occurs at the **equilibrium position**.

Energy in Simple Harmonic Motion



- As the object travels from A to $-A$, the energy is constantly changing from kinetic energy to elastic potential energy, and *vice versa*.

Energy in Simple Harmonic Motion



- The total energy, is always constant.
- This is true for all forms of SHM (where friction is ignored).

5. The Simple Pendulum

The Simple Pendulum

- The simple pendulum is a **mathematical model** based on a few simplifications/assumptions.



- The pendulum is a **point mass**.
 - The pendulum is suspended from a **weightless string**.
 - The string is **inextensible**.
 - The pendulum is in a **uniform gravitational field**.
- With these in place, we can study the simple pendulum, and determine if its motion is simple harmonic.

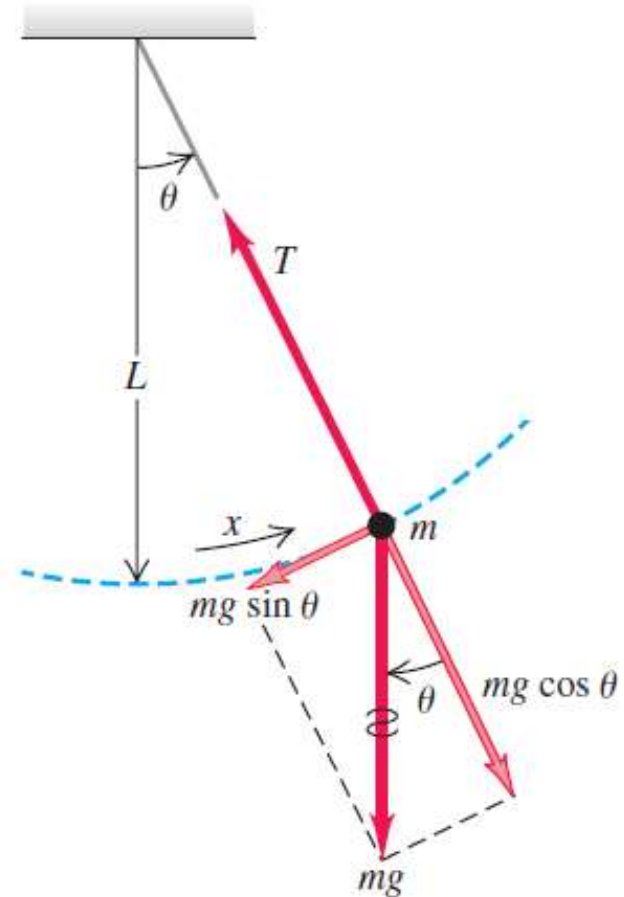
The Simple Pendulum

- The path of the simple pendulum is an arc of a circle.
- So the displacement x measured along the arc is given by

$$x = L\theta$$

- For small angles of θ , the acceleration, a is proportional to θ , and hence x .
- If a pendulum performs SHM, then

$$a \propto -\theta$$



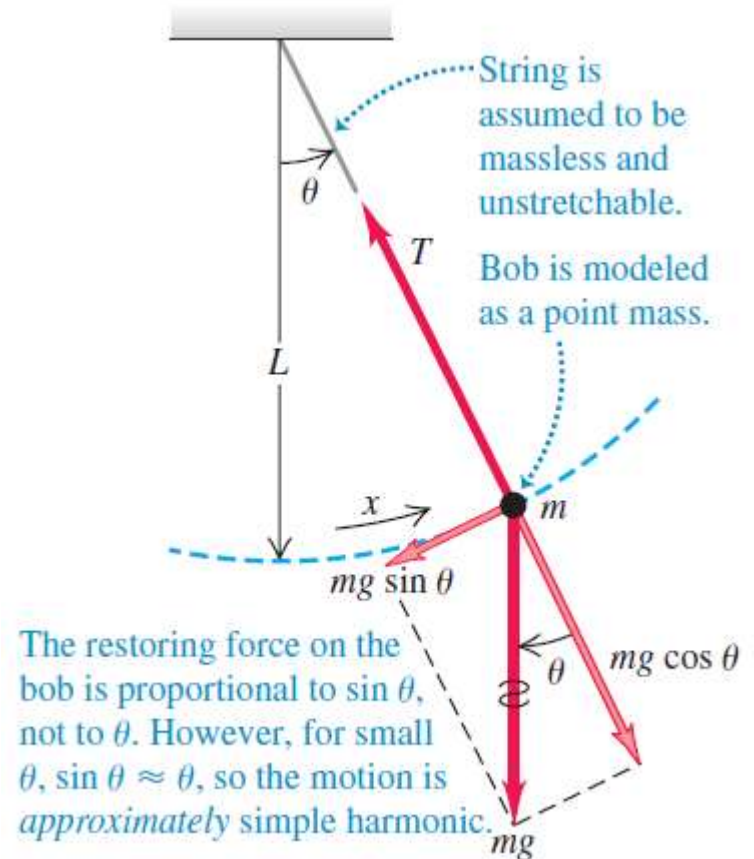
The Simple Pendulum

- Since $\sin\theta \sim \theta$ for small values

$$F = -mg\sin\theta$$

$$ma = -mg\theta$$

$$a = -g \frac{x}{L}$$



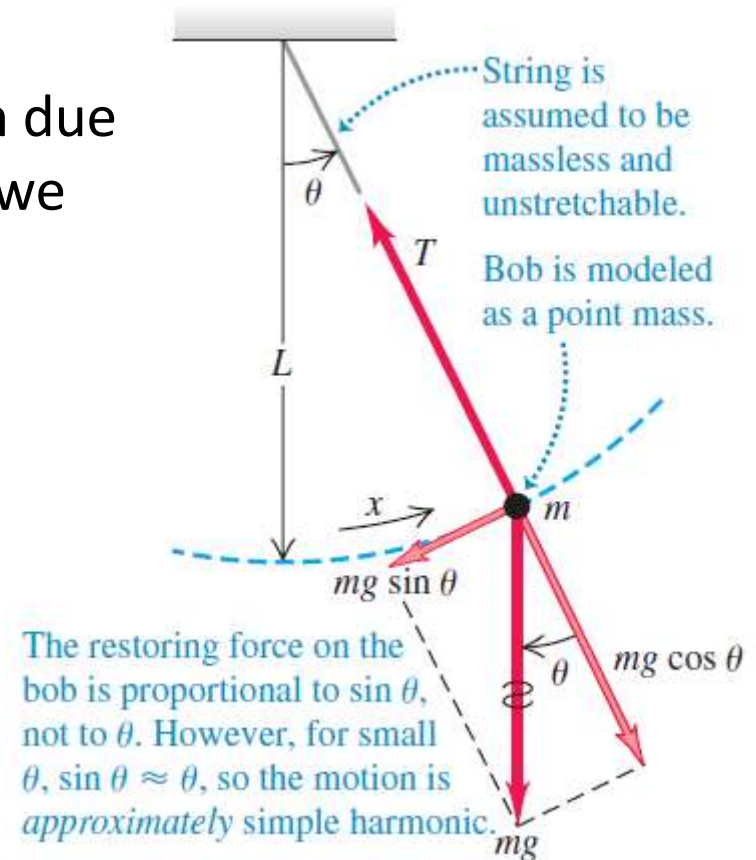
The Simple Pendulum

- We can then calculate the period (T) by comparing the equation for acceleration due to simple harmonic motion to the form we derived on the other slide:

$$a = -\frac{g}{L}x$$
$$a = -(\omega)^2x$$

- Therefore $\omega = \sqrt{\frac{g}{L}}$

$$\frac{2\pi}{T} = \sqrt{\frac{g}{L}} \Rightarrow T = 2\pi \sqrt{\frac{L}{g}}$$



Simple Harmonic Motion thus Far

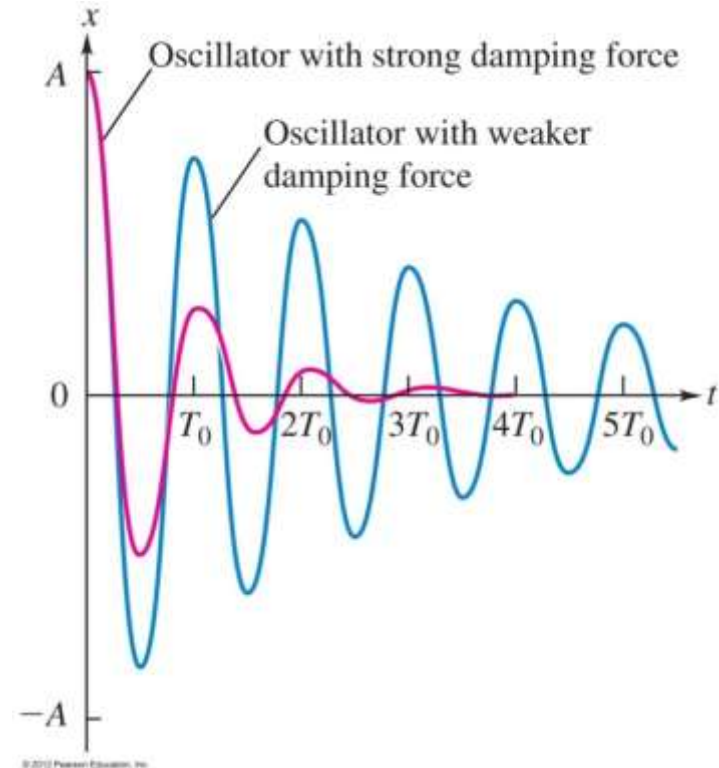


- The idealised oscillating systems (simple harmonic motion systems) that we've discussed so far are assumed to be frictionless, with no **non-conservative** forces.
- In other words, we've assumed that the total mechanical energy is constant, and such a system set into motion continues oscillating forever with no decrease in amplitude.
- The real world is different, however. Most oscillating systems undergo what we call **damping**.

Damping



(a) A real pendulum



- Real-world systems always have some friction (a non-conservative, or dissipative force, in other words), and oscillations do die-out with time unless we provide some means for replacing the mechanical energy lost to friction.
- In a situation such as this, we call the oscillations **damped oscillations**.

Announcement

Seminar 3

1. Students should solve all the seminar **3** questions in a **brand-new BOOK** and bring the solutions to the seminar class.
2. Students should come to the seminar class with their calculators, pens, pencils, and notebooks.
3. Phones, laptops, and tablets will not be used in the seminar classrooms.

Summary of today's Lecture

1. Circular Motion
2. Centripetal Force
3. Periodic/Oscillatory Motion.
4. Simple Harmonic Motion.
5. Representing Features of Simple Harmonic Motion
6. Energy in Simple Harmonic Motion.
7. The Simple Pendulum.

Lecture 8: Recommended Readings

- **Ch. 5.2**, Uniform Circular Motion; p.141-144
- **Ch. 5.3**, Dynamics of Uniform Circular Motion; p.144-148
- **Ch. 14.1**, Oscillations of a Spring; p.428-429
- **Ch. 14.2**, Simple Harmonic Motion (SHM); p.430-435
- **Ch. 14.3**, Energy in the Simple Harmonic Oscillator; p.435-436
- **Ch. 14.4**, SHM Related to Uniform Circular Motion; p.437

Online weekly Quizzes

Do not forget to complete Quiz 1

**The deadline for this assessment is Friday,
18th October 2024, 3pm.**