Introduction to Mathematical Software and Programming

Session 9

(w/c 21 April 2025)

GeoGebra

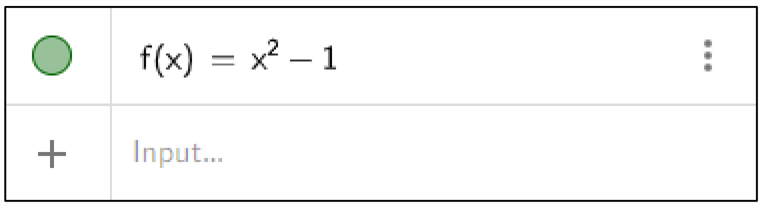
- Free software that is available on multiple platforms
- Has applications in different subjects of mathematics: algebra, geometry, calculus, statistics…
- We will mainly learn how to use GeoGebra for visualizing graphs, and solving math questions

Function

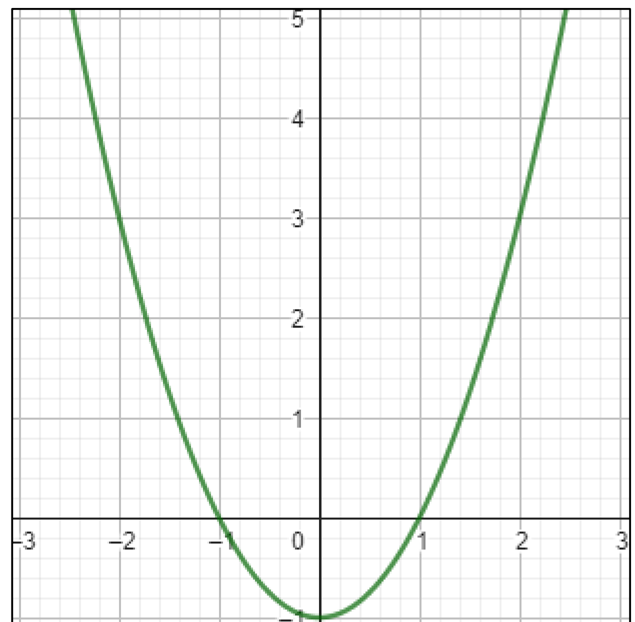
Function can be defined in the algebra window, by inputting its expression.

Graph of function will be shown on the graphics window.

algebra window



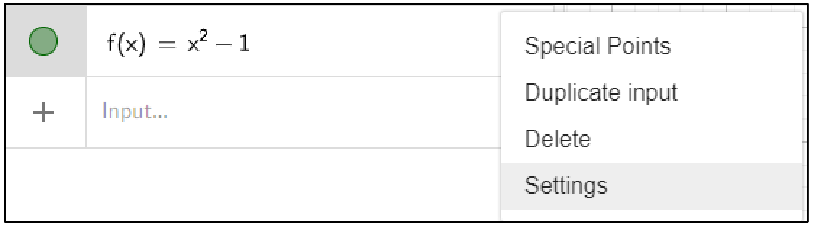
graphics window



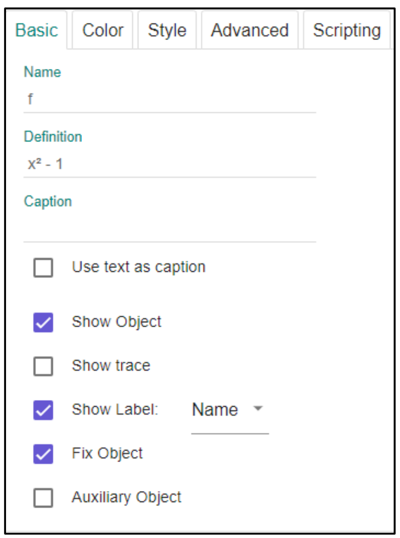
Type x^2-1 and press Enter.

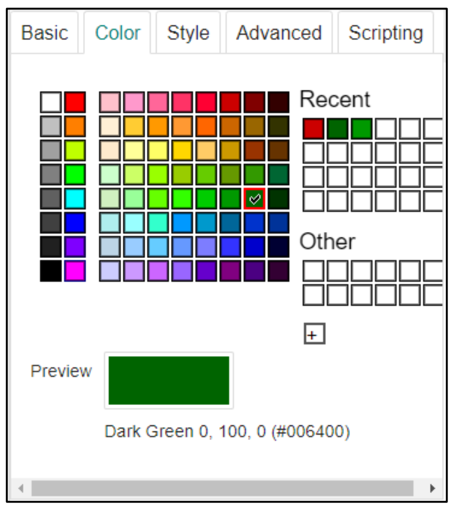
By default, GeoGebra will name functions using conventional names $f(x), g(x), h(x)$ ...

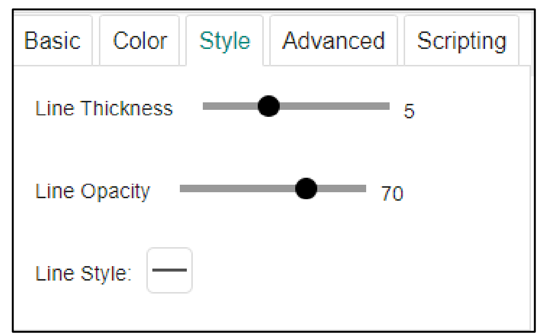
Formatting object



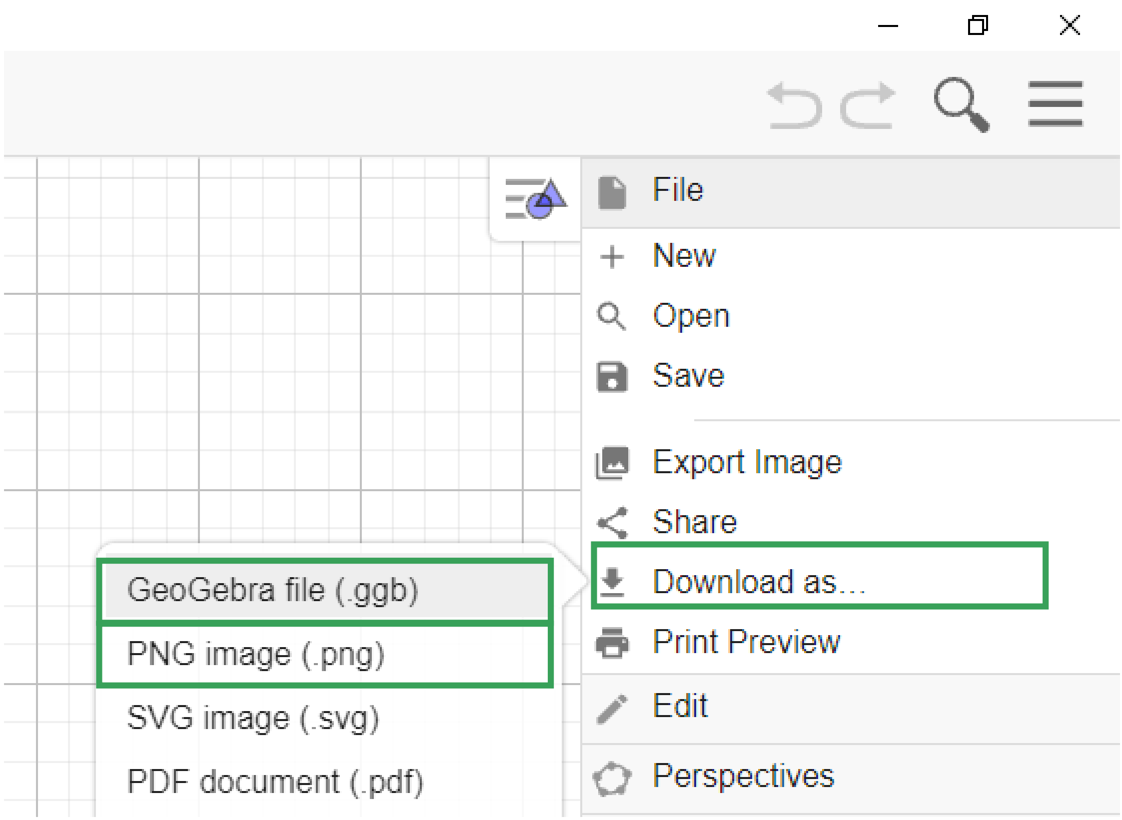
We can edit the format of created objects (e.g., functions) in the settings.







Saving file

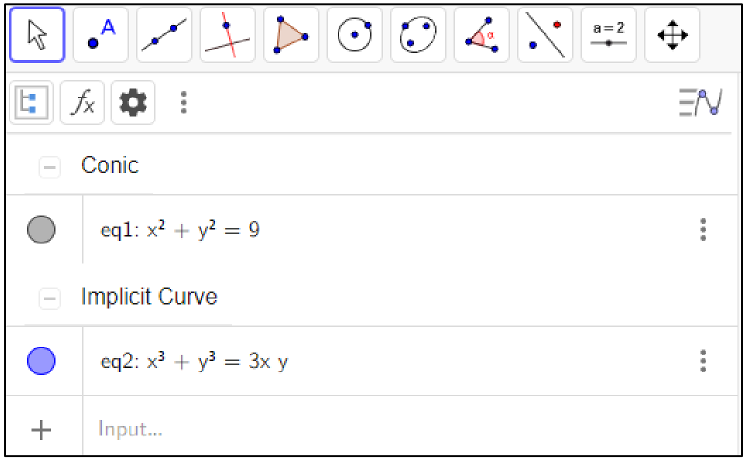


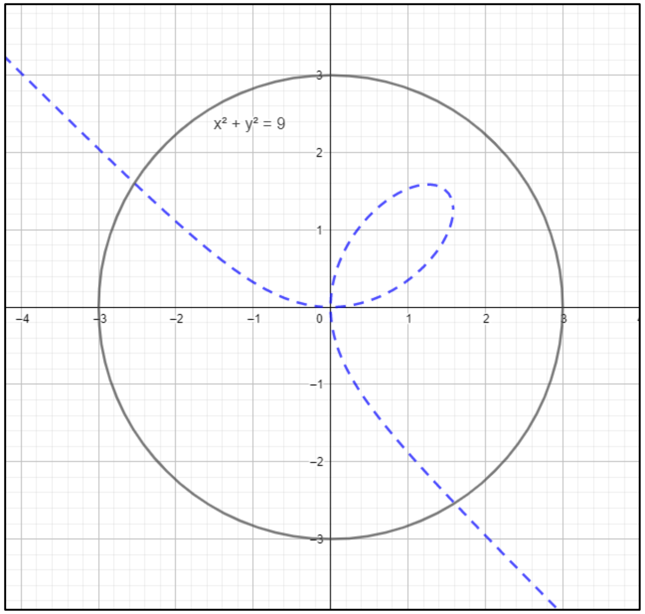
GeoGebra work can be saved with a file extension .ggb.

We can also download the graphics window as an PNG image and insert the figure into LaTeX file.

Equation

If we enter an equation with variables x and y, its graph (i.e., a collection of all points (x, y) satisfying this equation) will also be shown in the graphics window.





Parametric equations

Parametric equations can be visualized using the Curve command.

Curve

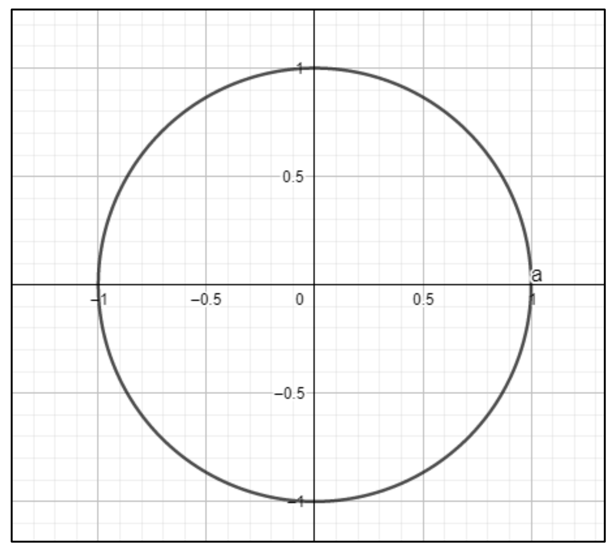
Curve(Expression, Expression, Parameter Variable, Start Value, End Value)

Parametric Curve

$a = \text{Curve}(\cos(t), \sin(t), t, 0, 2\pi)$

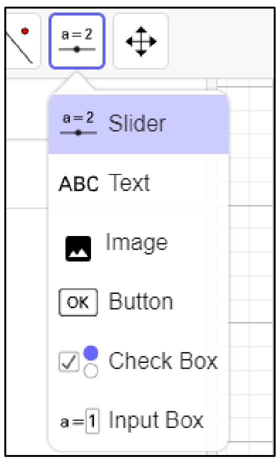
$$= \left.\begin{array}{l} x = \cos(t) \\ y = \sin(t) \end{array}\right\} 0 \le t \le 6.2832$$

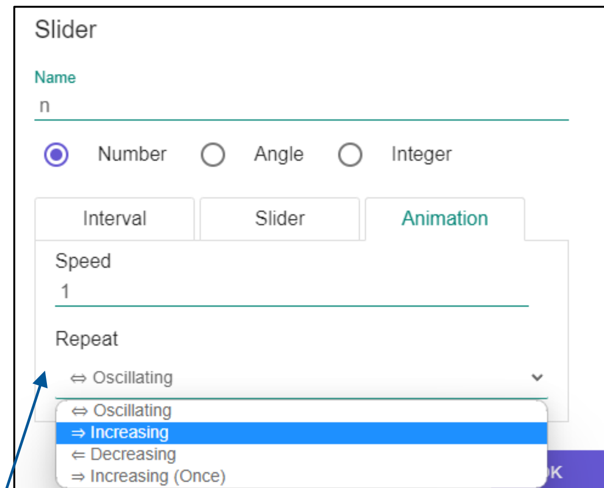
Input...



Slider

One good feature of GeoGebra is the Slider, which can define a changing variable and create animation.

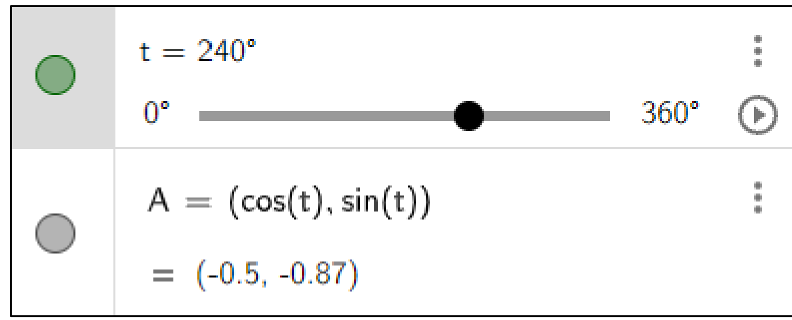




Specifications on the slider’s range and animation behavior.

Example

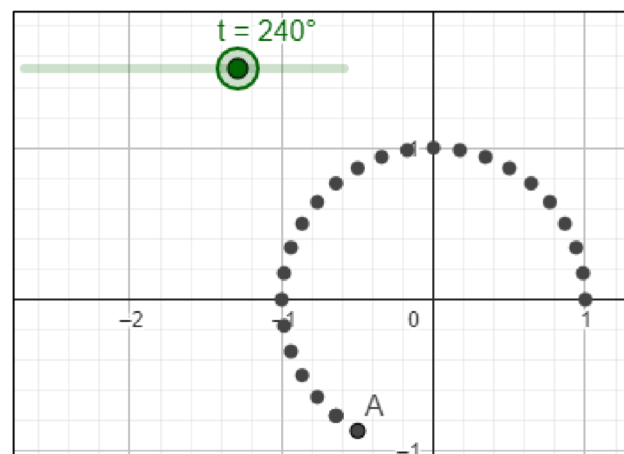
Create a slider t with minimum value 0° and maximum value 360°, with increment 10°.



Create a point $A = (\cos(t), \sin(t))$

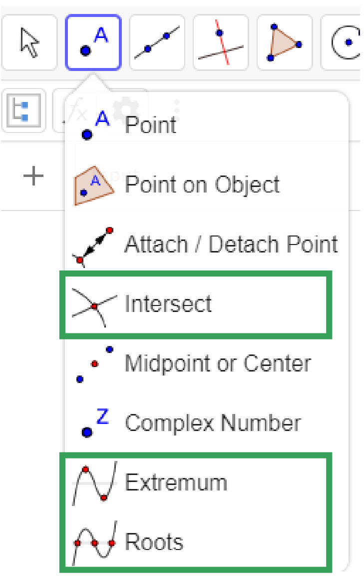
Right click on the point object, tick “Show trace” in the setting.

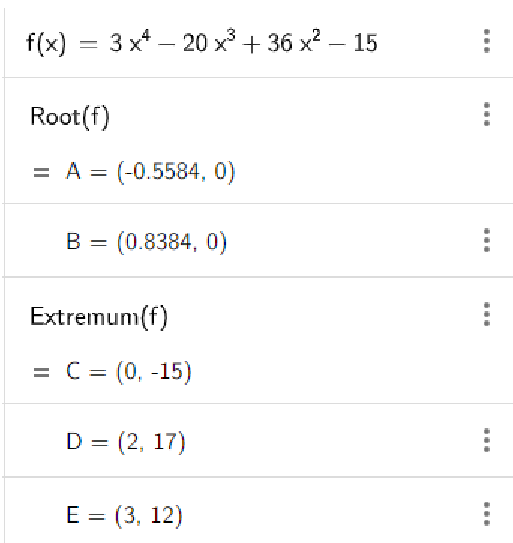
Play the slider and you can see the trajectory of point A as the parameter t value varies.

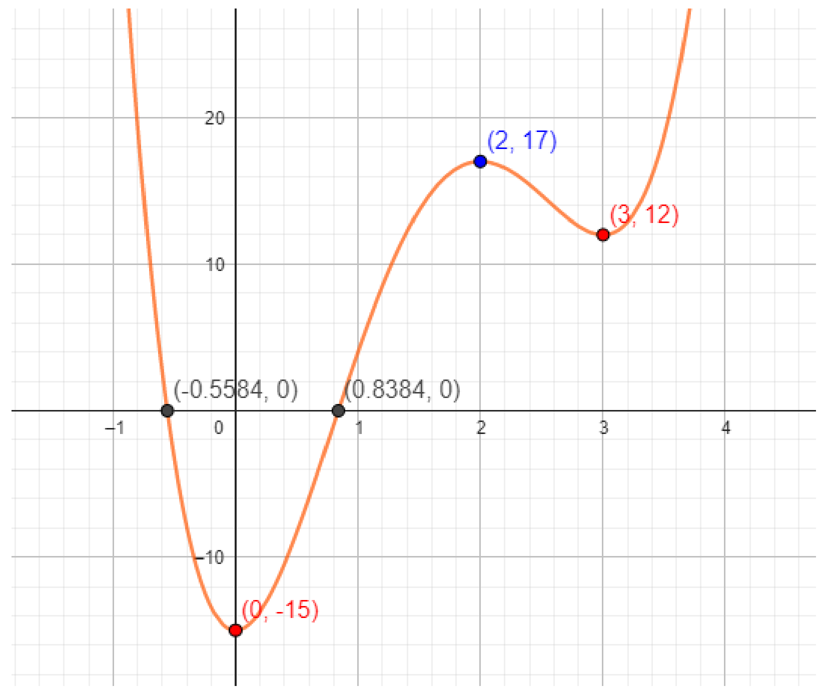


Roots and extreme values

For an explicit function $y = f(x)$, the following information can be obtained from function’s graph or using Roots and Extremum from GeoGebra toolbox.



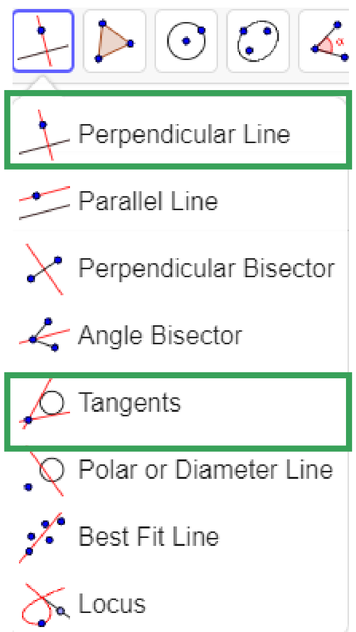




Note:
In Menu-Settings, we can change the global rounding for displaying given decimal places (e.g., 4.d.p.)

Tangent and normal lines

Tangent line and normal line (and their equations) at a given point on the curve can be obtained directly using Tangents and Perpendicular Line from GeoGebra toolbox.



$a : x^3 + y^3 = 3\,x\,y$

$= x^3 - 3x\,y + y^3 = 0$

$A = \text{Point}(a)$

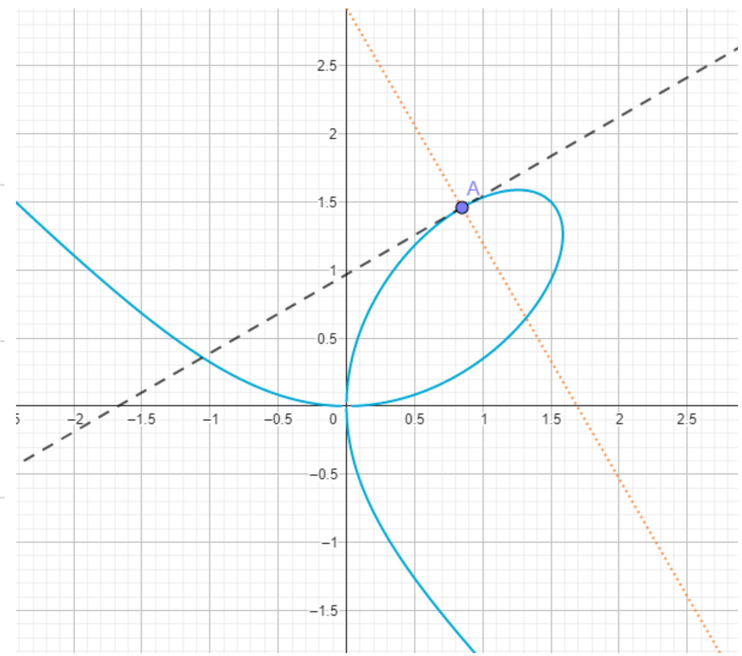
$= (0.8479,\ 1.4579)$

$f : \text{Tangent}(A, a)$

$= y = 0.5784x + 0.9675$

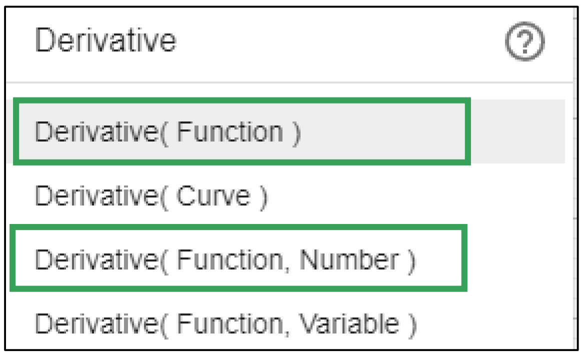
$g : \text{PerpendicularLine}(A, f)$

$= -3.833x - 2.2172y + 6.4822 = 0$



Derivative

For an explicit function $y = f(x)$, the derivative and higher order derivative can be computed using the Derivative command.



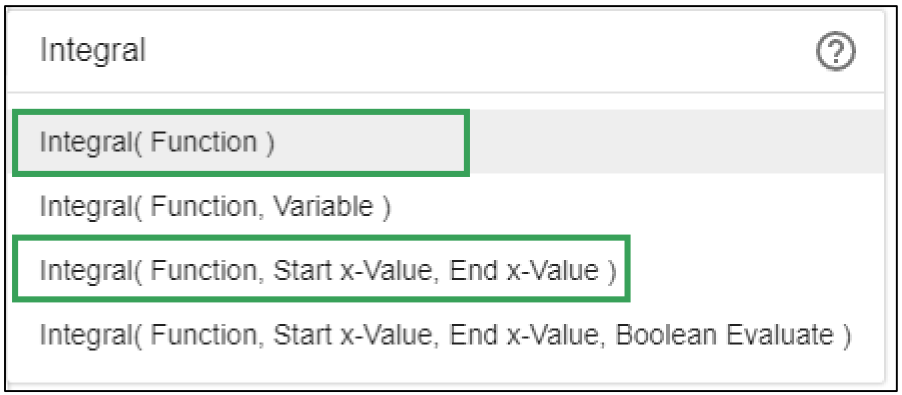
Note:
If f(x) is defined and has all higher order derivatives. We can evaluate those derivative functions at given values directly by calling
f'(2), f'''(1), f''''(0).

Derivative(f(x)) Compute the first order derivative $\frac{d}{dx}f(x)$

Derivative(f(x),n) Compute the n-th order derivative $\frac{d^n}{dx^n}f(x)$

Integral

For an explicit function $y = f(x)$, the indefinite integral (anti-derivative) and definite integral can be computed using the Integral command.



Note:
Definite integral represents signed area bounded by curves.

Integral(f(x)) Compute anti-derivative $\int f(x)\,dx$. The result is a function.

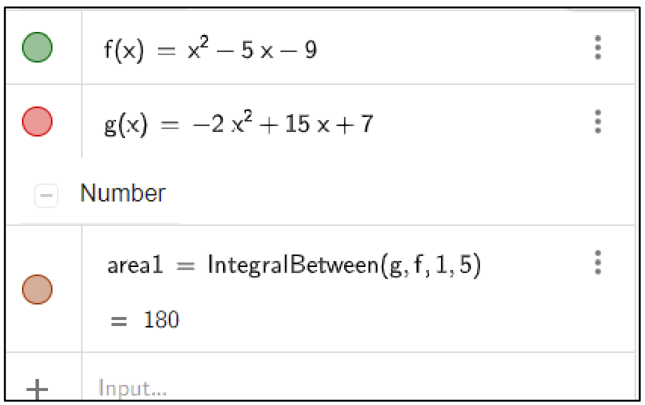
Integral(f(x), a, b) Compute definite integral $\int_a^b f(x)dx$. The result is a value.

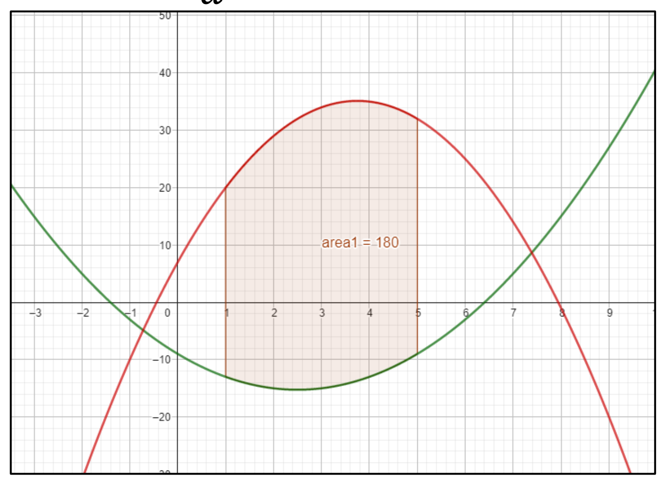
Area bounded by curves

For two functions $y = f(x)$ and $y = g(x)$, the area of region bounded by these two functions on the interval [a, b] can be computed using IntegralBetween command:

IntegralBetween(f(x), g(x), a, b)

$$\int_a^b [f(x) - g(x)]\ dx$$



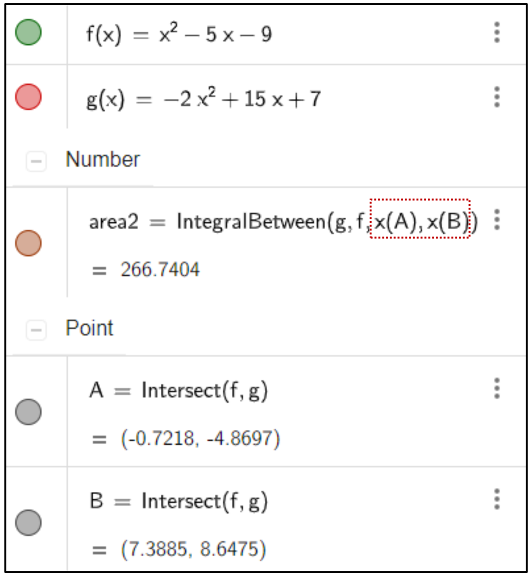


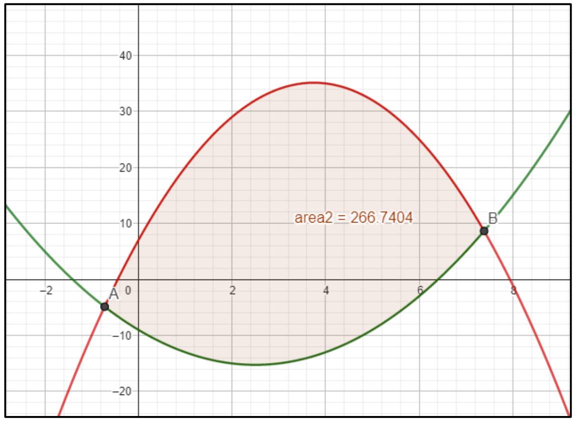
To get a positive value for the area, set the top function as the first input argument, and bottom function as second input argument.

Note:
IntegralBetween(f(x), g(x), a, b) has the same computing value as using Integral (f(x)-g(x), a, b)

Area bounded by curves

If $f(x)$ and $g(x)$ has two intersections points A and B, we can find the intersection points using Intersect from GeoGebra toolbox.





Then the region enclosed by $y = f(x)$ and $y = g(x)$ can be computed using

IntegralBetween(f(x), g(x), x(A), x(B))

Note:
Assume $f(x)$ is on top of $g(x)$, A is to the left of B. Otherwise, adjust the order of input arguments correspondingly.

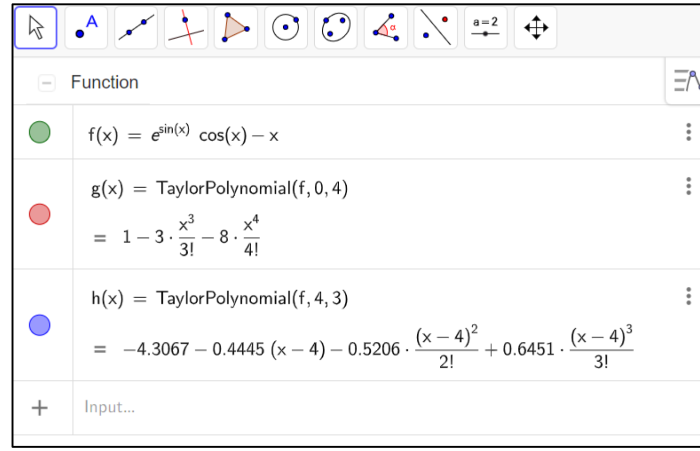
Polynomial approximation

Maclaurin's expansion (up to x^n term) to $f(x)$ around $x = 0$:

TaylorPolynomial(f, 0, n)

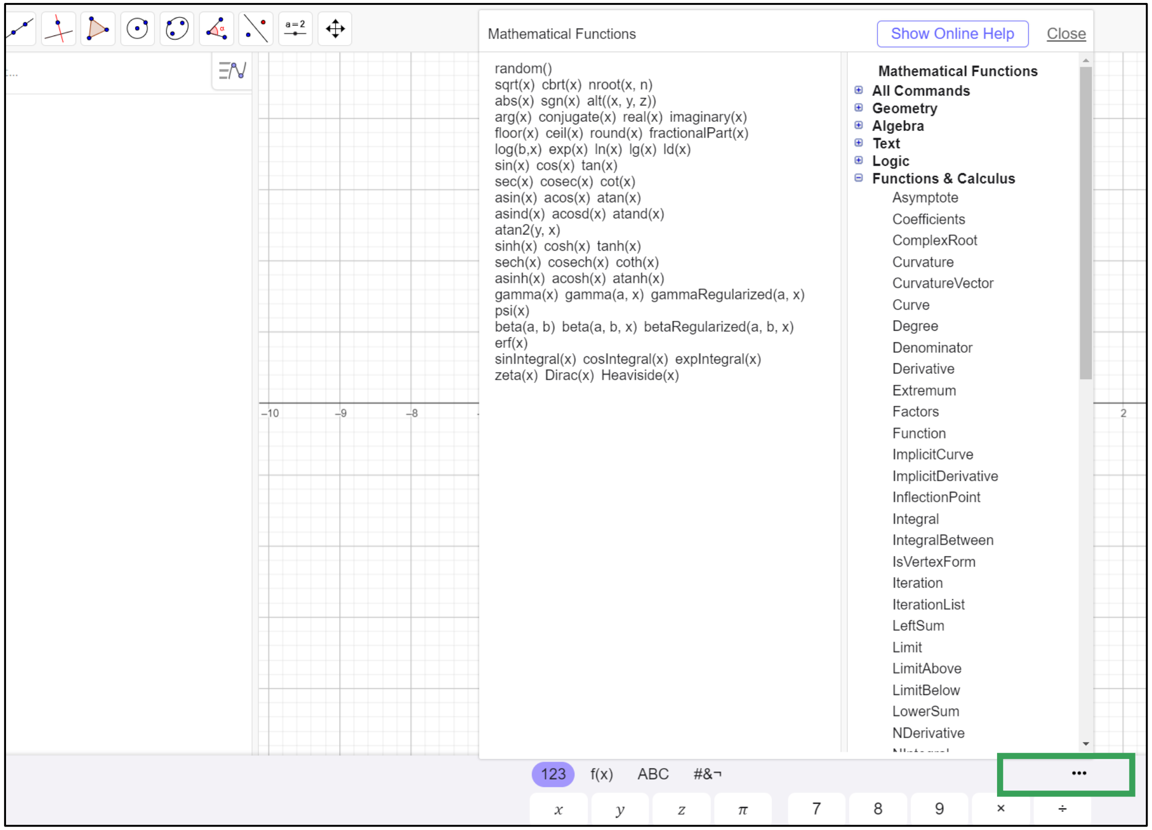
Taylor's expansion (up to x^n term) to $f(x)$ around $x = a$:

TaylorPolynomial(f, a, n)



Command lists

A comprehensive command lists can be found by clicking on three dots "..." in the GeoGebra virtual keyboard.



Self-study

- Explore GeoGebra toolboxes and commands by yourself.

- Find related questions from your math modules, and solve them by using the help from GeoGebra.