Science A Physics

Lecture 12:

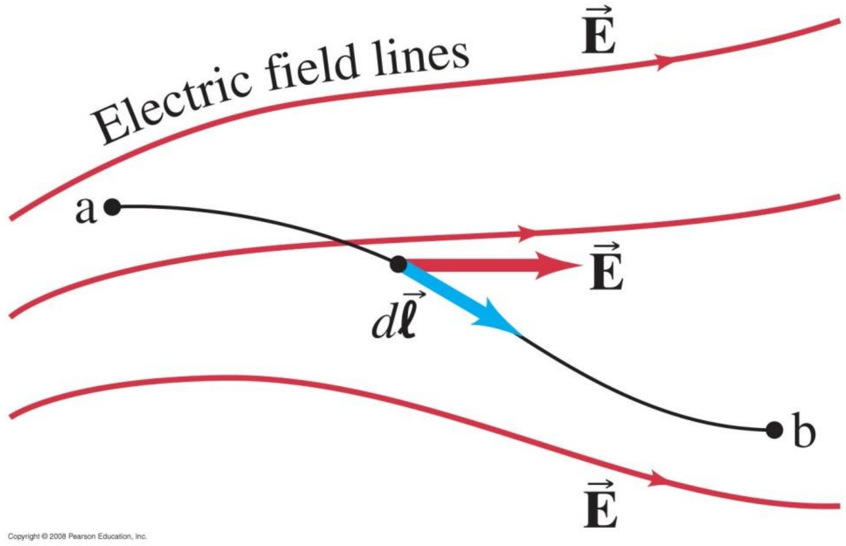
Electric Potential; Part 2

Aims of today’s lecture

1. The Relation between Electric Potential and Electric Field
2. Electric Potential Due to Point Charges
3. Equipotential Surfaces

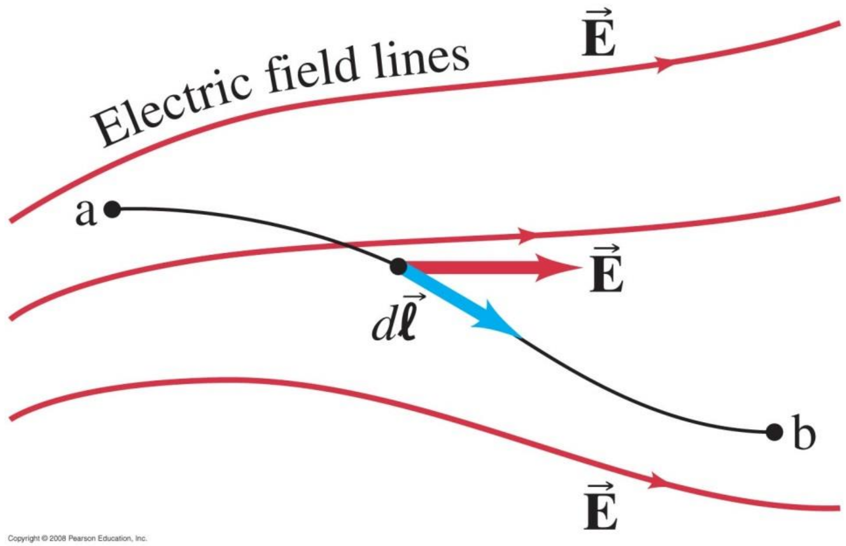
1. Relation between Electric Potential and Electric Field

Relation between Electric Potential and Electric Field



- The effects of any charge distribution can be described either in terms of electric field or in terms of electric potential.
- Electric potential is often easier to use because it is a scalar, as compared to electric field which is a vector.
- There is a connection between the electric potential produced by a given arrangement of charges and the electric field due to those charges.

Relation between Electric Potential and Electric Field

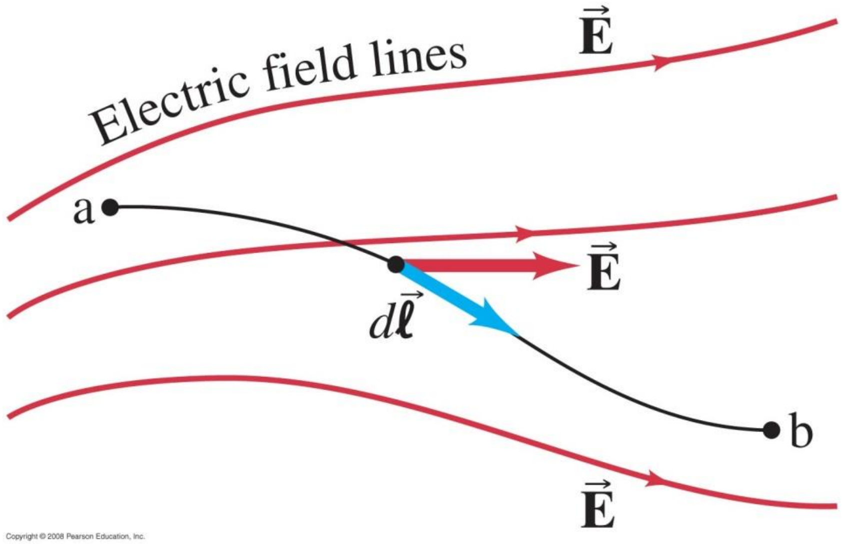


- The difference in potential energy, always for a positive test charge, between any two points in space, a and b, is given by:

$$U_b - U_a = -\int_a^b \vec{F} \cdot d\vec{l}$$

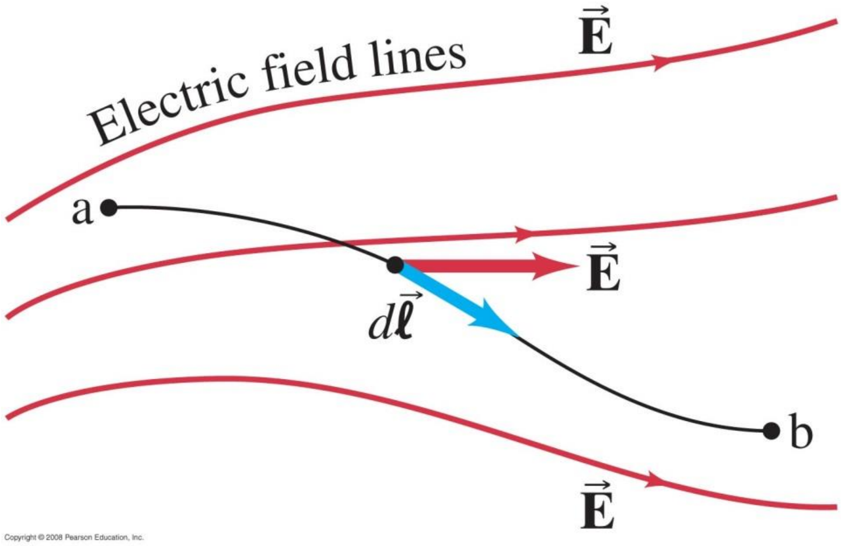
Where $d\vec{l}$ is an infinitesimal increment of displacement, and the integral is taken along any path in space from point a to point b.

Relation between Electric Potential and Electric Field



- For the electrical case, we are more interested in the potential difference, $V_{ba} = V_b - V_a = (U_b - U_a)/q$, rather than in the potential energy itself.
- Also, the electric field $\vec{E}$ at any point in space is defined as the force per unit charge: $\vec{E} = \vec{F}/q$.
- Combining these two relations together, we get . . .

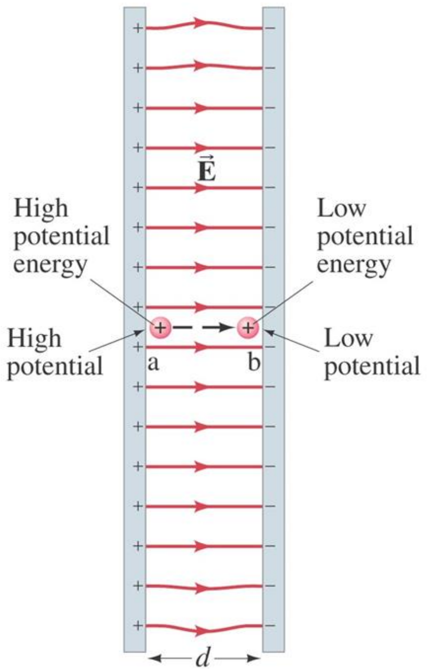
Relation between Electric Potential and Electric Field



$$V_{ba} = V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l}$$

- If we are given the electric field due to some arrangement of electric charge, we can use the above equation to determine V_{ba}.

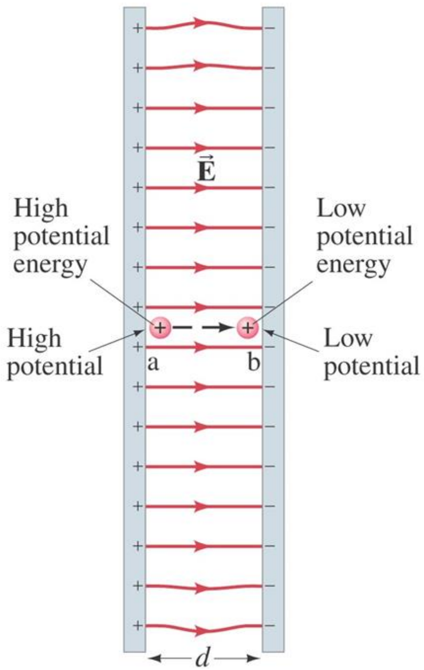
Relation between Electric Potential and Electric Field



- A simple special case is a uniform field. In the figure above, for example, a path parallel to the electric field lines from point a at the positive plate to point b at the negative plate gives (since $\vec{E}$ and $d\vec{l}$ are in the same direction at each point),

$$V_{ba} = V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l} = -\mathrm{E} \int_a^b dl = -Ed$$

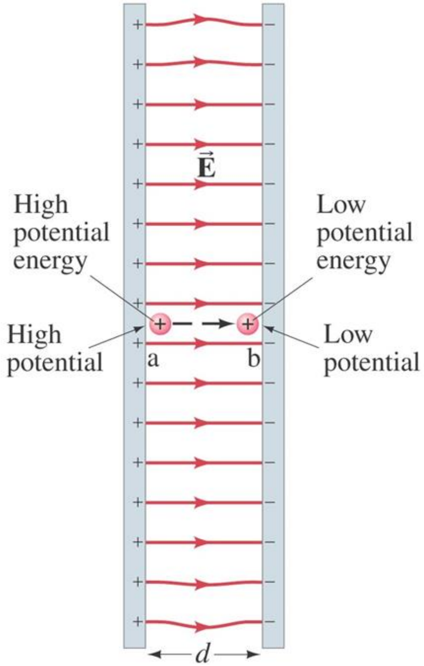
Relation between Electric Potential and Electric Field



$$V_{ba} = V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l} = -\mathrm{E}\int_a^b dl = -Ed$$

where d is the distance, parallel to the field lines, between points a and b, and where we only use $-Ed$ if we are sure that the electric field is uniform.

Relation between Electric Potential and Electric Field

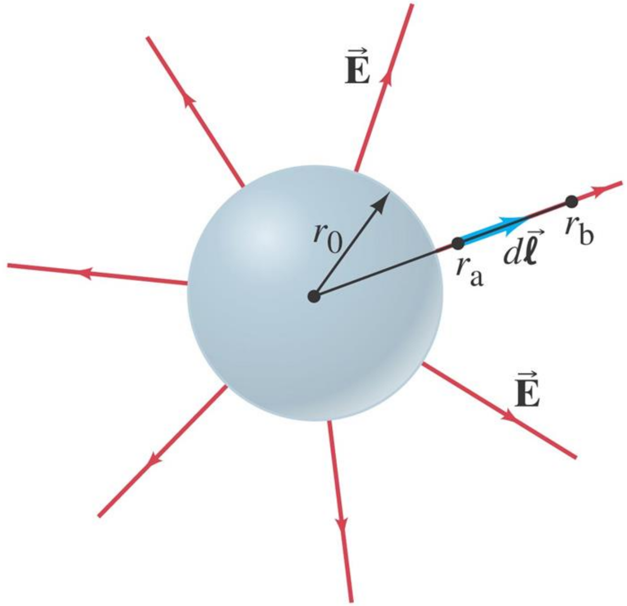


$$V_{ba} = V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l} = -\mathrm{E}\int_a^b dl = -Ed$$

$$V_{ba} = -Ed$$

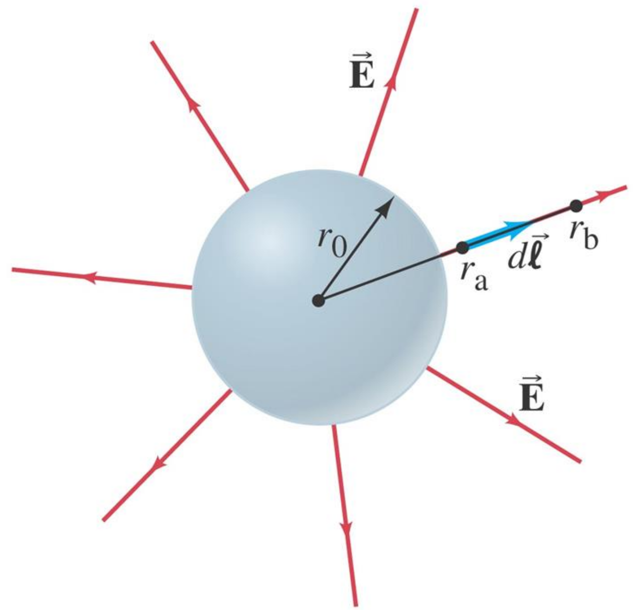
- From the above equations, we can see that the units for electric field can be written as volts per metre (V/m) as well as newtons per coulomb (N/C); these are equivalent in general.

Relation between Electric Potential and Electric Field



Q. Determine the potential at a distance r from the centre of a charged conducting sphere of radius r_0 for (a) $r > r_0$, (b) $r = r_0$, (c) $r < r_0$. The total charge on the sphere is Q.

Relation between Electric Potential and Electric Field

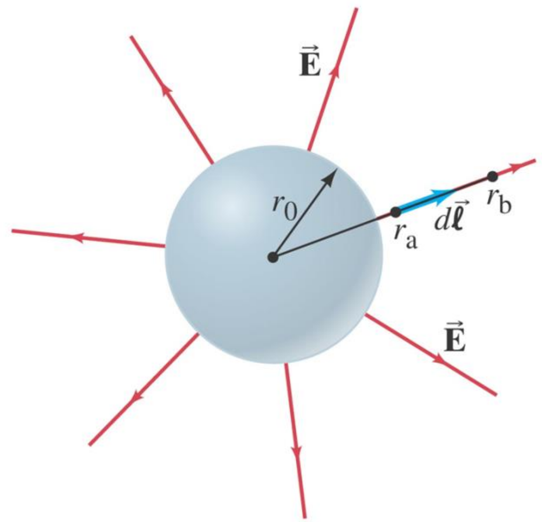


Approach: The charge Q is distributed over the surface of the sphere since it is a conductor. The electric field outside a conducting sphere is

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$

and points radially outward (inward if $Q < 0$). Since we know $\vec{E}$, we can start by using $V_{ba} = V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l}$

Relation between Electric Potential and Electric Field

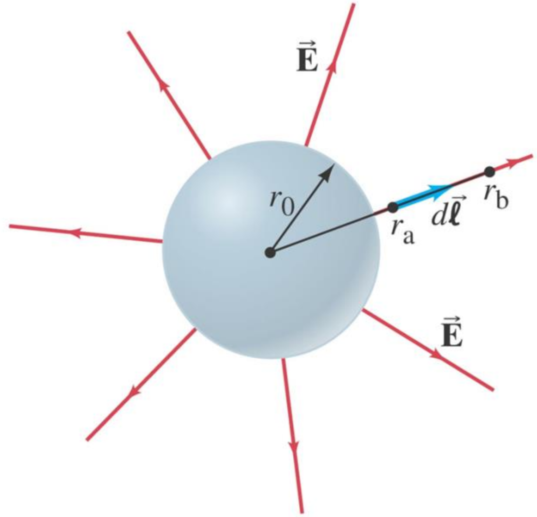


Solution: we integrate along a radial line with $d\vec{l}$ parallel to $\vec{E}$, as shown above, between two points which are distances r_a and r_b from the sphere's centre:

$$V_{ba} = V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l} = -\frac{Q}{4\pi\epsilon_0}\int_{r_a}^{r_b} \frac{dr}{r^2} = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

and we set $dl = dr$.

Relation between Electric Potential and Electric Field

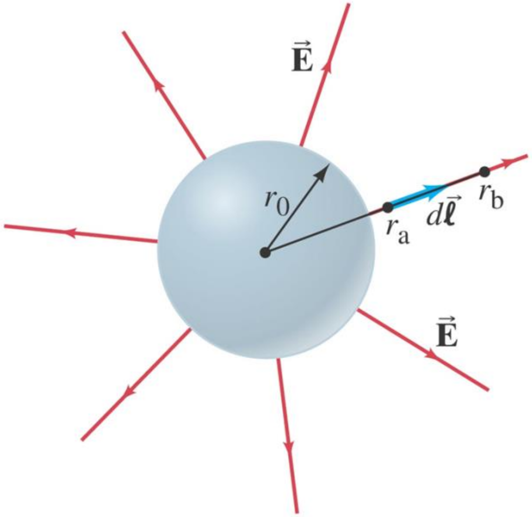


(a) If we let $V = 0$ for $r = \infty$ (let's choose $V_b = 0$ at $r_b = \infty$), then at any other point r (for $r > r_0$), we have

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$$

Thus, the electric potential outside a spherical conductor with a uniformly distributed charge is the same as if all the charge were at its centre.

Relation between Electric Potential and Electric Field

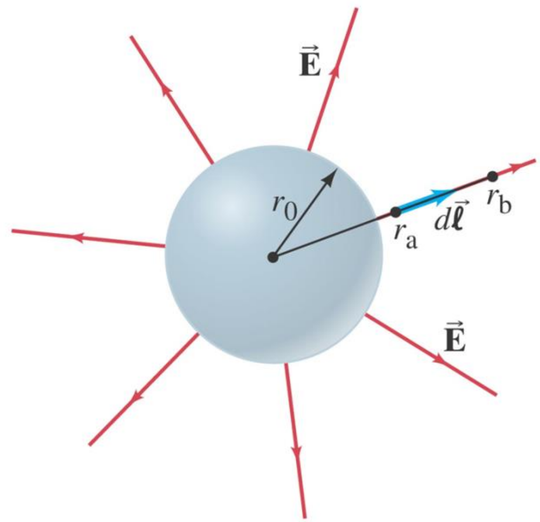


(b) As r approaches r_0, we see that

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r_0}$$

at the surface of the conductor.

Relation between Electric Potential and Electric Field

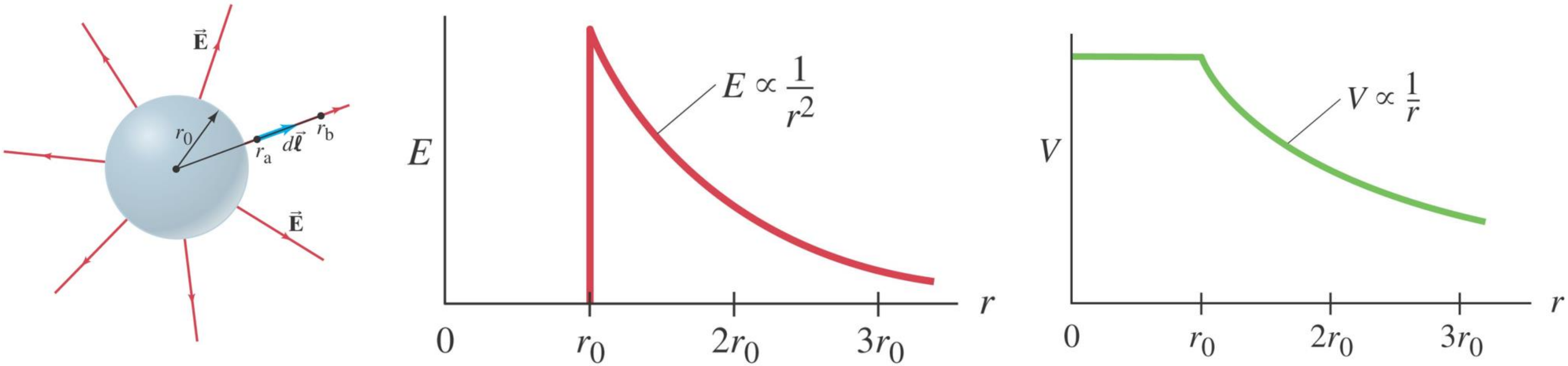


(c) For the points within the conductor, $E = 0$. Thus, the integral, $\int \vec{E} \cdot d\vec{l}$, between $r = r_0$ and any point within the conductor gives zero change in V. Hence, V is constant within the conductor:

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r_0}$$

The whole conductor, not just its surface, is at this same potential.

Relation between Electric Potential and Electric Field

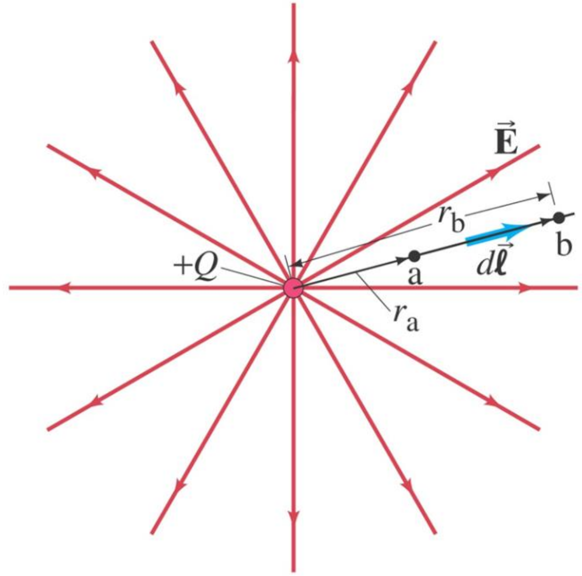


$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r_0}$$

- Plots of both E and V as a function of r are shown in the figures above for a positively charged conducting sphere.

2. Electric Potential Due to Point Charges

Electric Potential Due to Point Charges

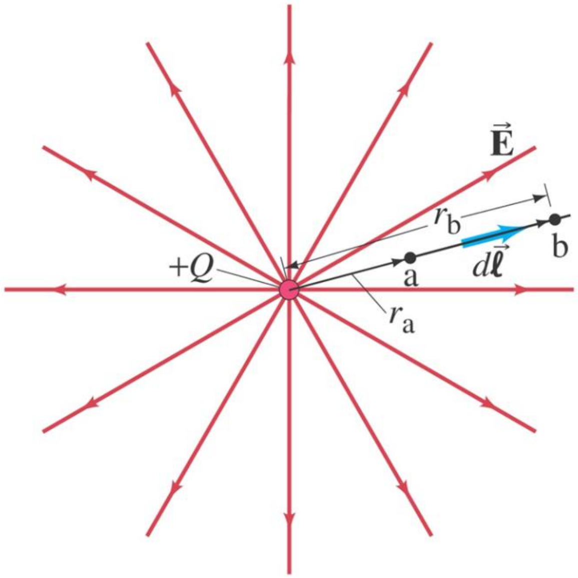


- The electric potential at a distance r from a single point charge Q can be derived directly from $V_b - V_a = -\int \vec{E} \cdot d\vec{l}$. The electric field due to a single point charge has magnitude

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} \qquad \text{or} \qquad E = k\frac{Q}{r^2}$$

(where $k = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 N \cdot m^2/C^2$)

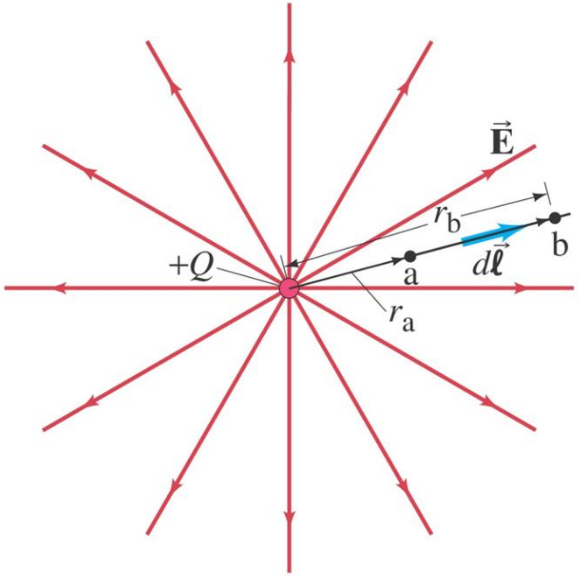
Electric Potential Due to Point Charges



- Taking the integral from point a to point b, as shown above, where $dl = dr$, we get the following

$$V_b - V_a = -\int_{r_a}^{r_b} \vec{E} \cdot d\vec{l} = -\frac{Q}{4\pi\epsilon_0}\int_{r_a}^{r_b} \frac{1}{r^2}dr = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{r_b} - \frac{Q}{r_a}\right)$$

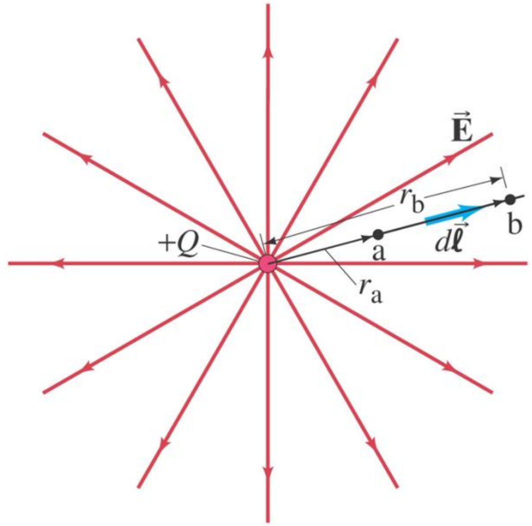
Electric Potential Due to Point Charges



$$V_b - V_a = -\int_{r_a}^{r_b} \vec{E} \cdot d\vec{l} = -\frac{Q}{4\pi\epsilon_0}\int_{r_a}^{r_b} \frac{1}{r^2} dr = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{r_b} - \frac{Q}{r_a}\right)$$

- As mentioned earlier, only differences in potential have physical meaning. We are free, therefore, to choose the value of the potential at some one point to be whatever we please.

Electric Potential Due to Point Charges

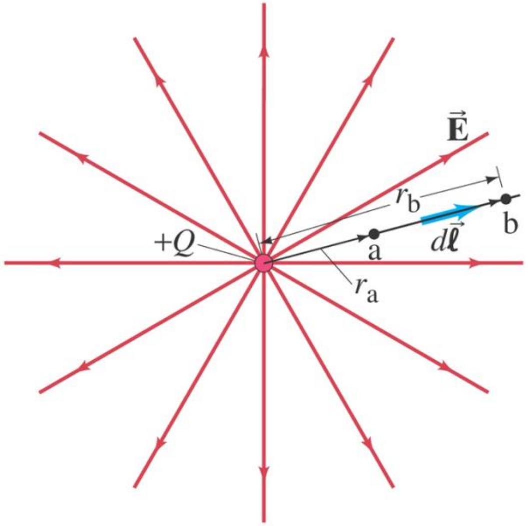


$$V_b - V_a = -\int_{r_a}^{r_b} \vec{E} \cdot d\vec{l} = -\frac{Q}{4\pi\epsilon_0}\int_{r_a}^{r_b} \frac{1}{r^2} dr = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{r_b} - \frac{Q}{r_a}\right)$$

- It is common to choose the potential to be zero at infinity (let $V_b = 0$ at $r_b = \infty$). Then the electric potential, V, at a distance r from a single point charge is

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$$

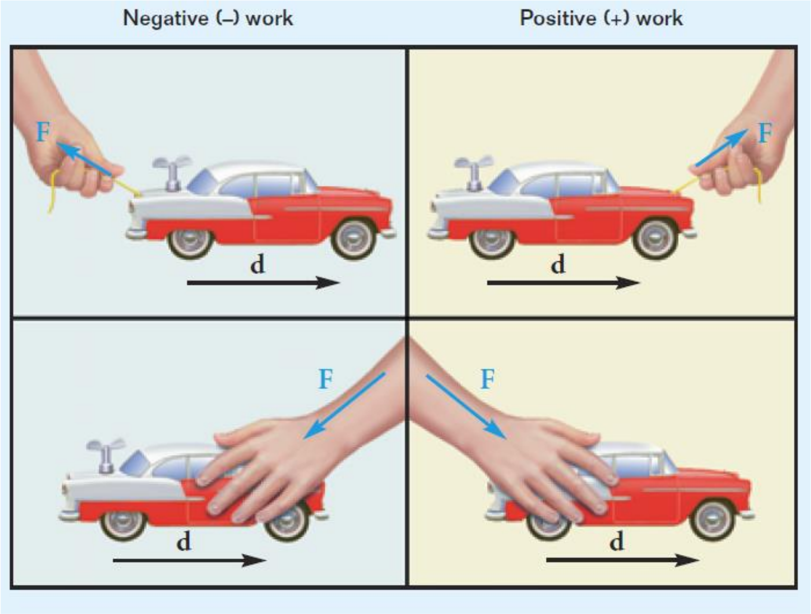
Electric Potential Due to Point Charges



$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$$

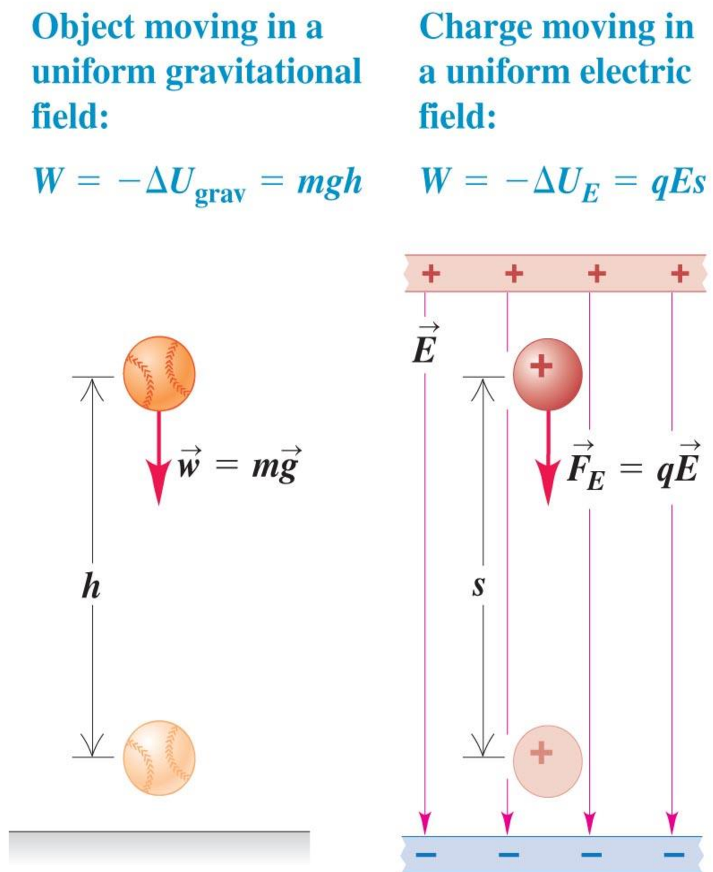
- We can think of V as representing the absolute potential, where $V = 0$ at $r = \infty$, or we can think of V as the potential difference between r and infinity.
- The potential V decreases with the first power of the distance, whereas the electric field decreases as the *square* of the distance.

Previously: the Sign for Work



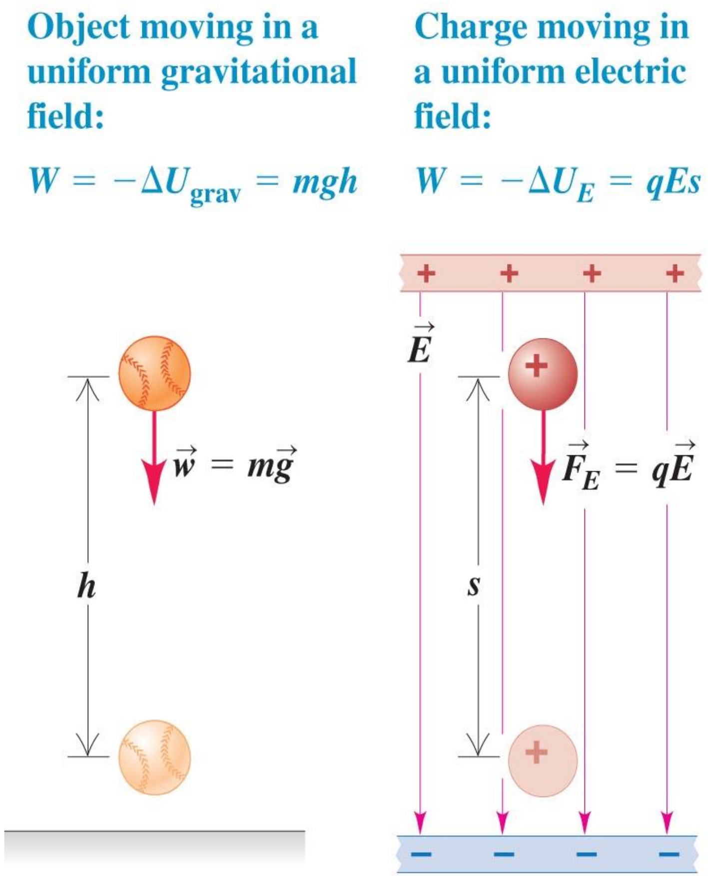
- If a force acts on an object in the direction in which the object moves, then we say that positive work is being done on the object.
- If a force acts on an object against the direction in which the object moves, then we say that negative work is being done on the object.

Electrical and Gravitational Forces



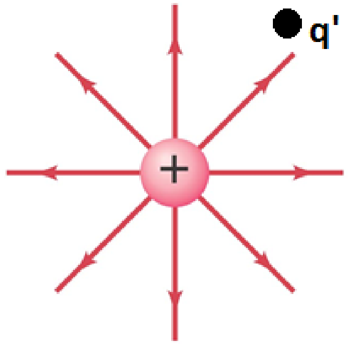
- The forces are similar and conservative.
- For the ball, the conservative force is gravity, while for the positive charge, the conservative force is the electrostatic force.

Electrical and Gravitational Forces



- By a conservative force, we mean that the energy in both objects (the ball and the positive charge respectively) is completely conserved when it is transformed from potential energy to kinetic energy.

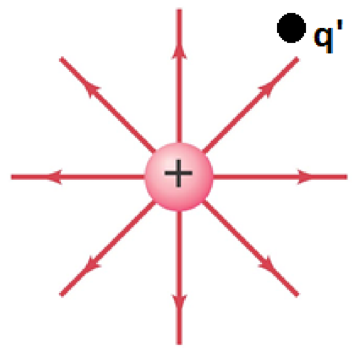
Electric Potential Due to Point Charges



- In the above figure, our object of interest is the positive test charge q'.

- For a positive test charge q' placed near a positive charge, the test charge has the potential to be repelled by the positive charge, due to the electrostatic force (electrical field lines) coming from this positive charge.

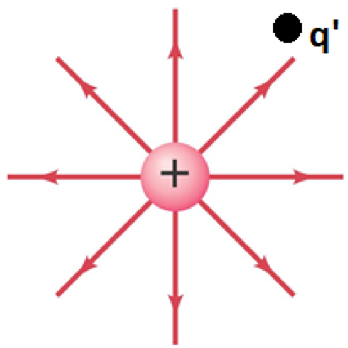
Electric Potential Due to Point Charges

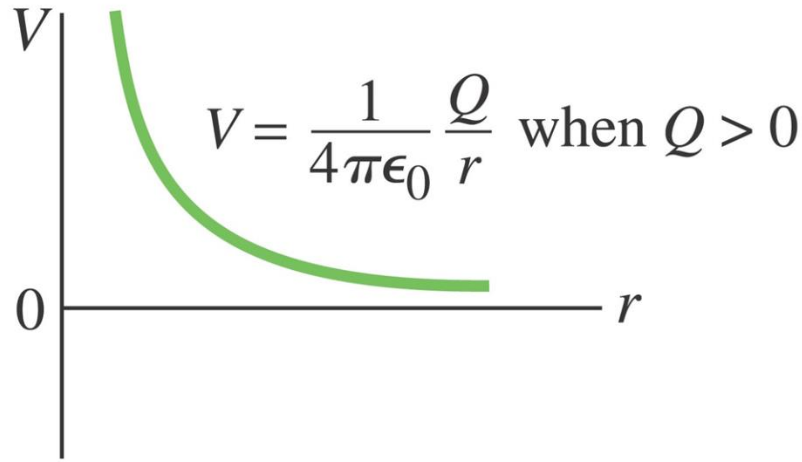


- In other words, the test charge has potential energy, the potential to move in the direction of the electrostatic force if released.

- The electrostatic force coming from the positive charge is a conservative force because it can act on the test charge, and transform its potential energy into kinetic energy.

Electric Potential Due to Point Charges

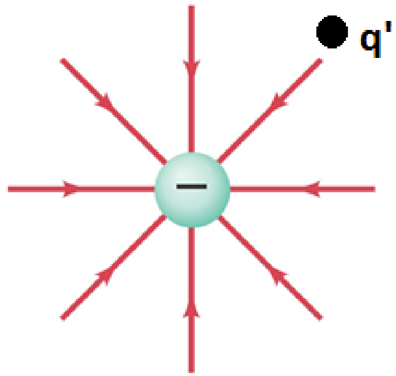




- The closer the positive test charge q' is placed initially to the positive charge, the more potential it has. This is what the above graph shows.

- We define the potential to be $0\ \mathrm{V}$ when the test charge is an infinite distance away from the positive charge.

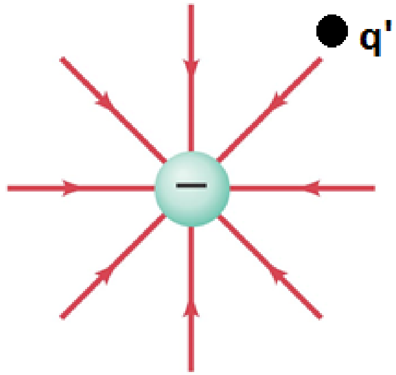
Electric Potential Due to Point Charges

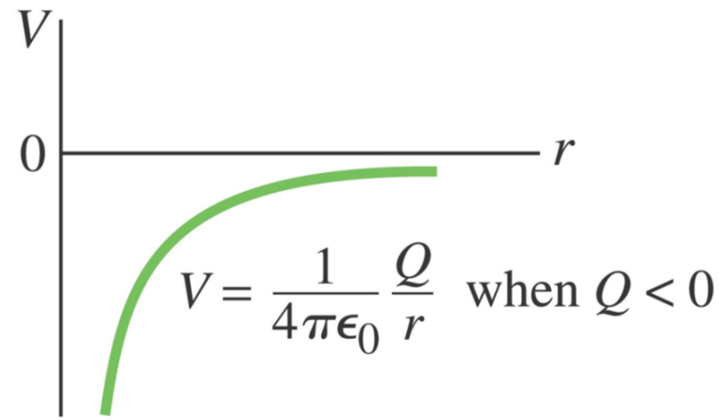


- In the above figure, again our object of interest is the positive test charge q'.

- For a positive test charge q' placed near a negative charge, the test charge has the potential to be attracted by the negative charge, due to the electrostatic force (electrical field lines) converging on the negative charge. Again, the test charge has potential energy.

Electric Potential Due to Point Charges

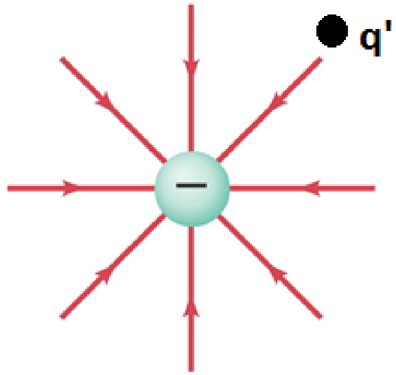


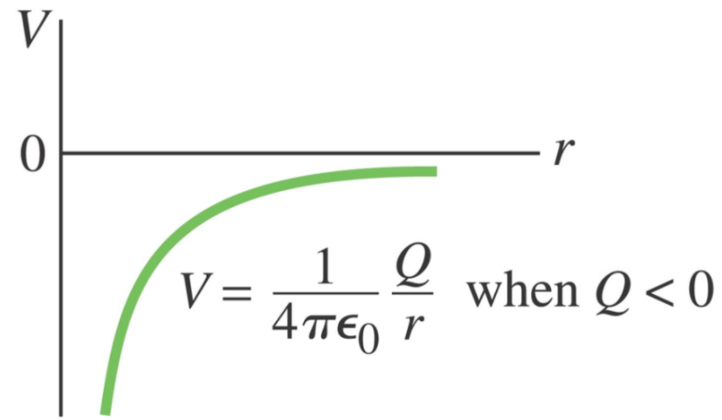


- The electrostatic force (electrical field lines) converging on the negative charge is a conservative force because it can act on the test charge, and transform its potential energy into kinetic energy.

- The closer the positive test charge q' is placed initially to the negative charge, the more potential it has. However, instead of using the same graph as in slide 29 to depict this, we use the above graph, which shows that the potential is referred to as '**negative potential energy**'. **Why?**

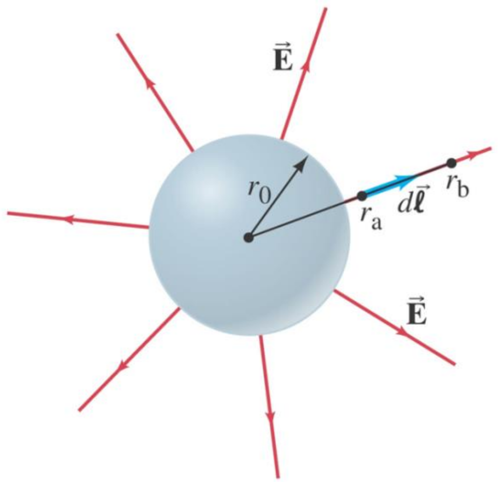
Electric Potential Due to Point Charges

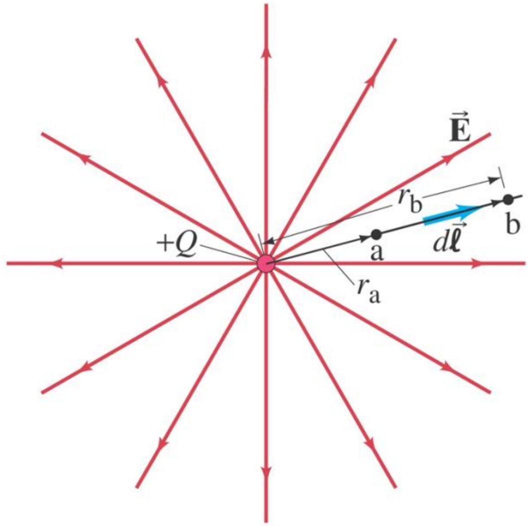




- The reason we use the above graph is due to our definition of $0\ \mathrm{V}$, the potential that a test charge q' has when it is an infinite distance from either a positive charge or negative charge.
- To make the test charge move away from the negative charge, a non-conservative force would have to act on the test charge, against the natural electrostatic force (conservative force) if we just release the test charge. Thus, we can say the potential for the test charge to move a distance **from** the negative charge is negative.

Electric Potential Due to Point Charges



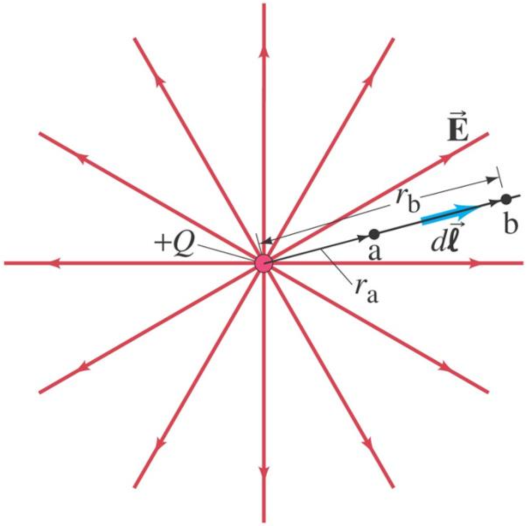


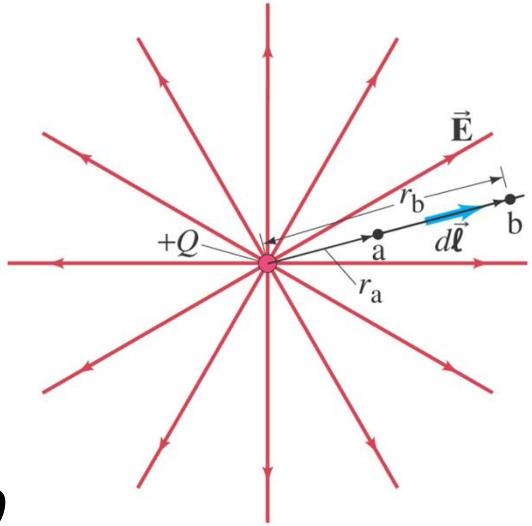
$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$$

- Previously, we've seen that the potential due to a uniformly charged sphere is given by the same relation, as above.

- Thus, we see that the potential outside a uniformly charged sphere is the same as if all the charge were concentrated at its centre.

Electric Potential Due to Point Charges

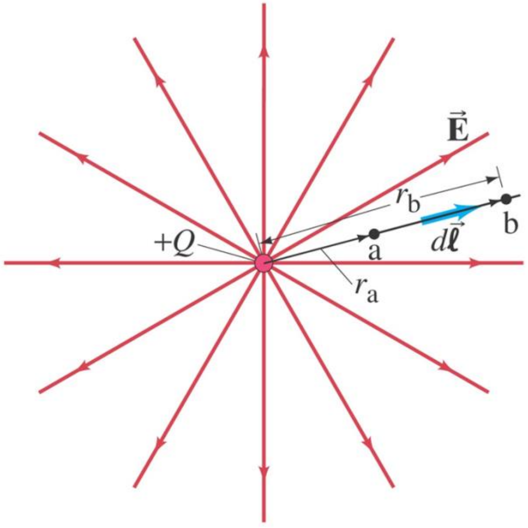


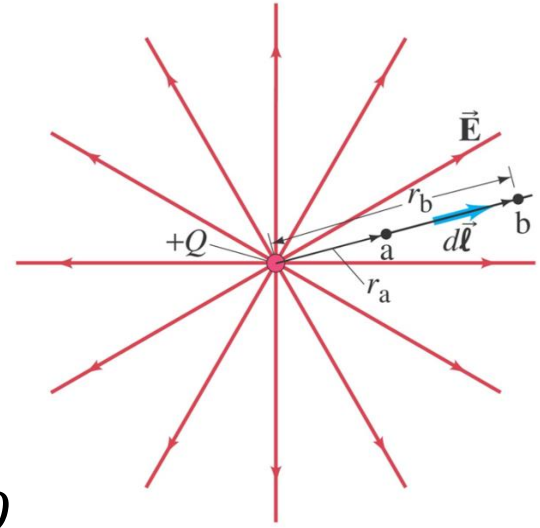


$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$$

- To determine the electric field at points near a collection of two or more point charges requires adding up the electric fields due to each charge.
- Since the electric field is a vector, this can be time consuming or complicated.

Electric Potential Due to Point Charges



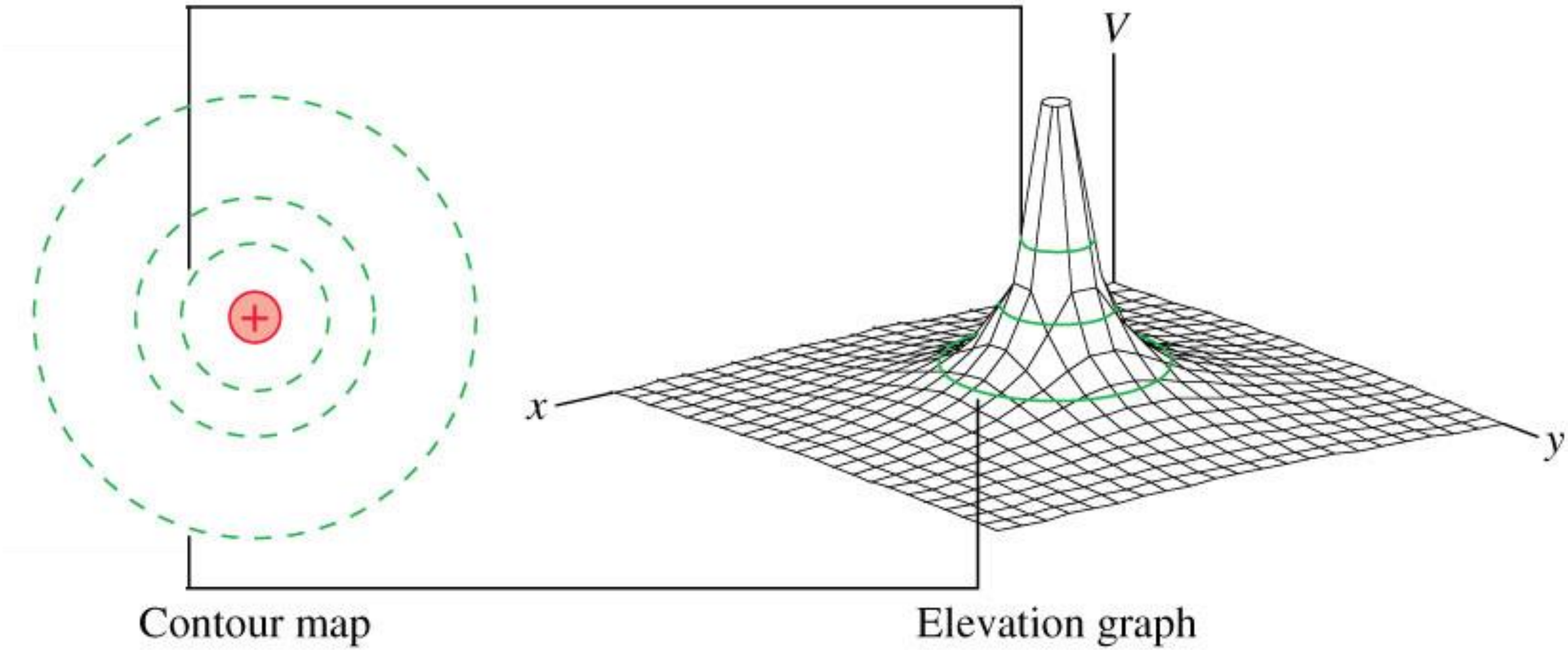


$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$$

- To find the electric potential at a point due to a collection of point charges is far easier, since the electric potential is a scalar, and you only need to add numbers without concern for direction.
- This a major advantage in using electric potential for solving problems.

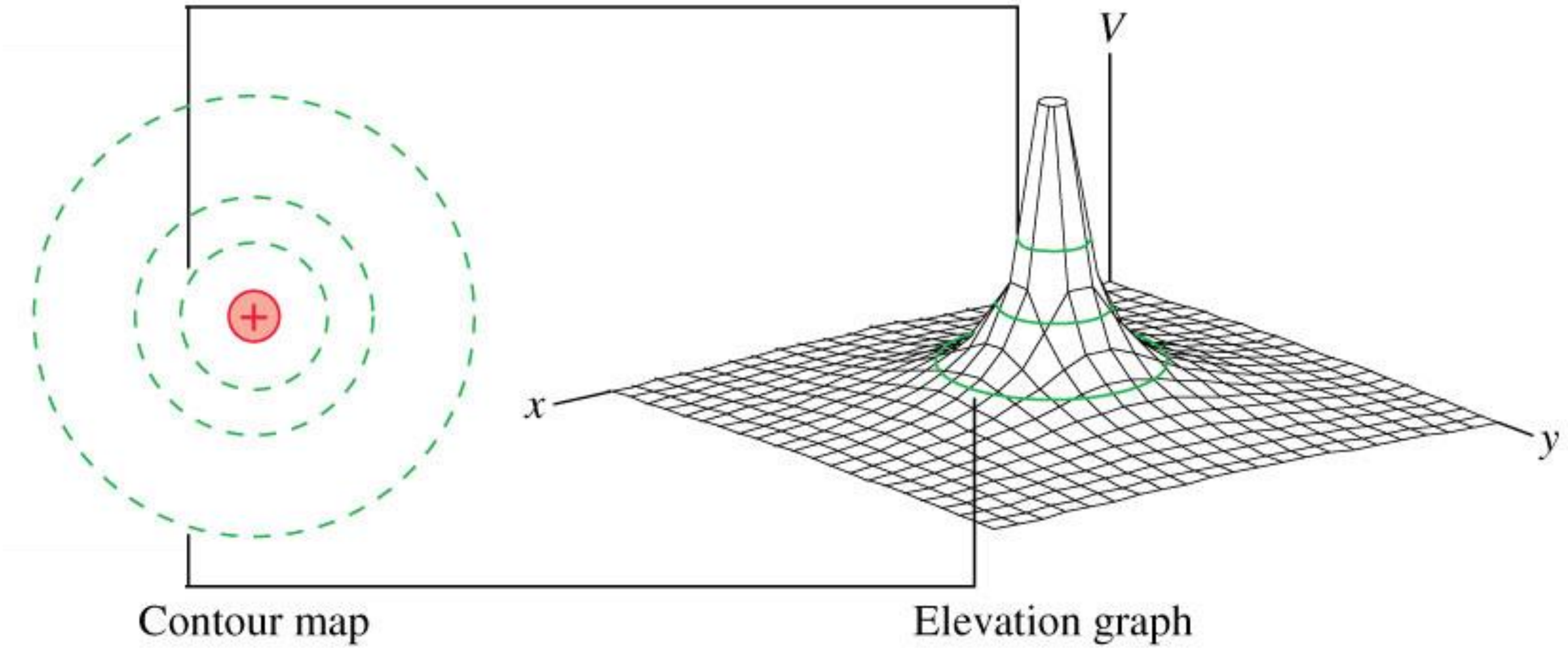
3. Equipotential Surfaces

Equipotential Surfaces



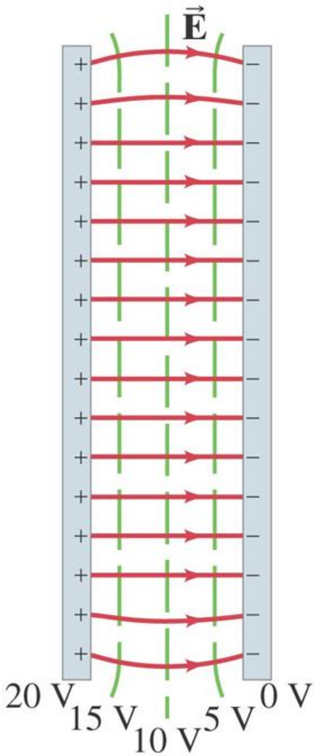
- The electric potential can be represented graphically by drawing **equipotential lines**, or, in three dimensions, **equipotential surfaces**.
- An equipotential surface is one on which all points are at the same potential.

Equipotential Surfaces



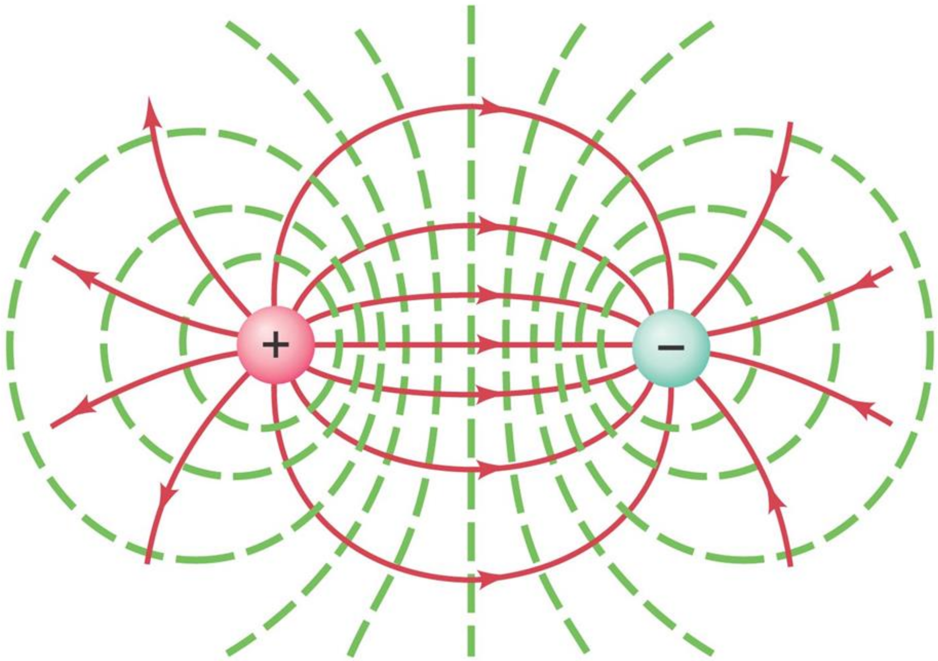
- An equipotential surface must be perpendicular to the electric field at any point.
- In a normal two-dimensional drawing, we show equipotential lines, which are the intersections of equipotential surfaces with the plane of the drawing.

Equipotential Surfaces



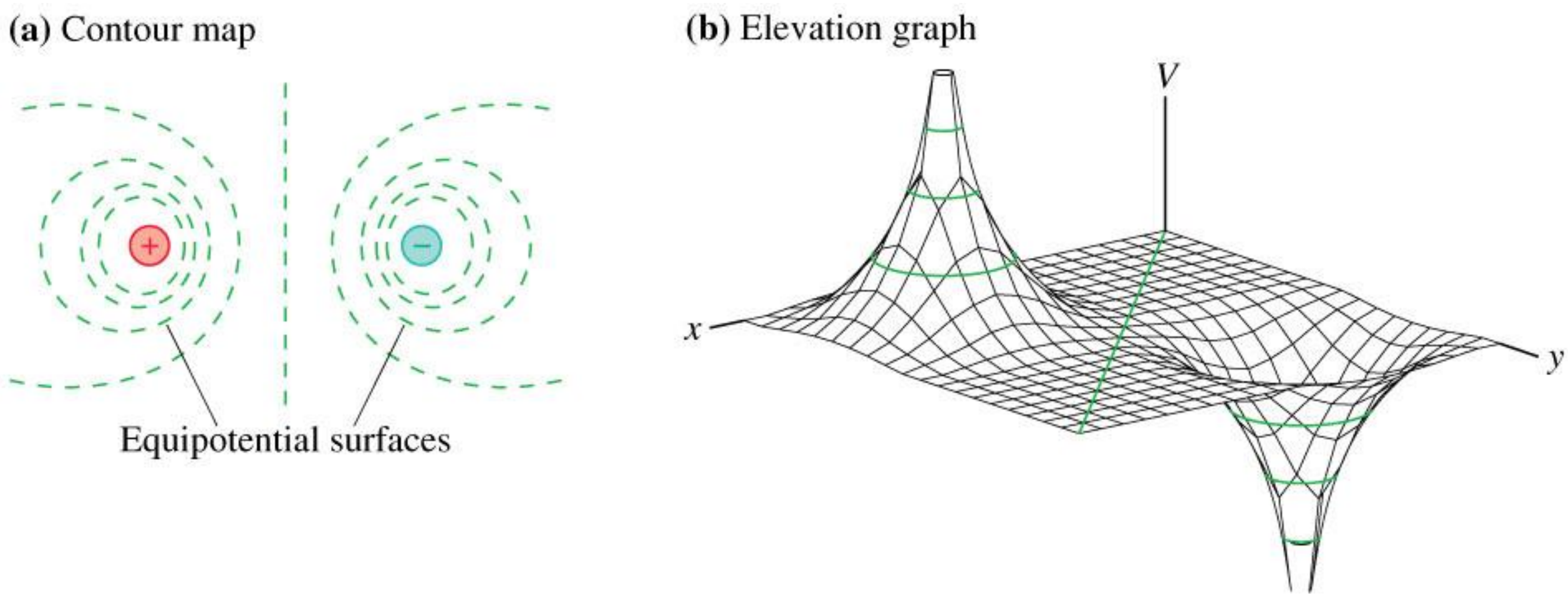
- In the above figure, a few of the equipotential lines are drawn (dashed green lines) for the electric field (red lines) between two parallel plates at a potential difference of 20 V.

Equipotential Surfaces



- The equipotential lines for the case of two equal but oppositely charged particles are shown above as green dashed lines.

Equipotential Surfaces



- The equipotential lines for the case of two equal but oppositely charged particles are shown above as green dashed lines.

Summary of today's Lecture

1. Relation between Electric Potential and Electric Field
2. Electric Potential Due to Point Charges
3. Equipotential Surfaces

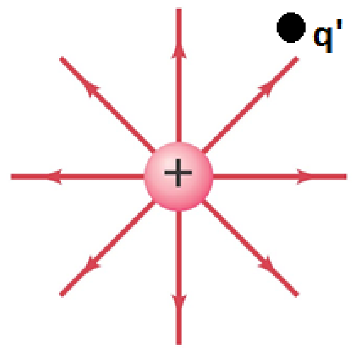
Lecture 18: Optional Reading

- **Ch. 23.2**, Relation between Electric Potential and Electric Field; p.706-708.
- **Ch. 23.3**, Electric Potential due to Point Charges; p.708-710.
- **Ch. 23.5**, Equipotential Surfaces; p.712.

Home Work

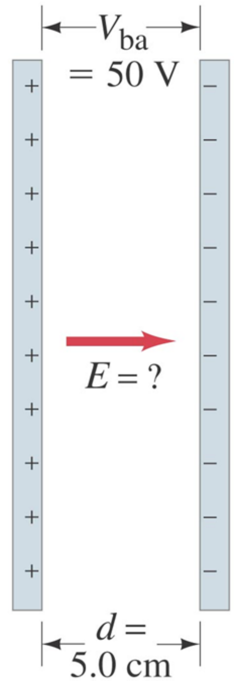
Do not forget to attempt the **Additional Problems** for this lecture before logging in to **Mastering Physics** to complete your assignments.

Possible Exam Question: Have a Read (p.709)



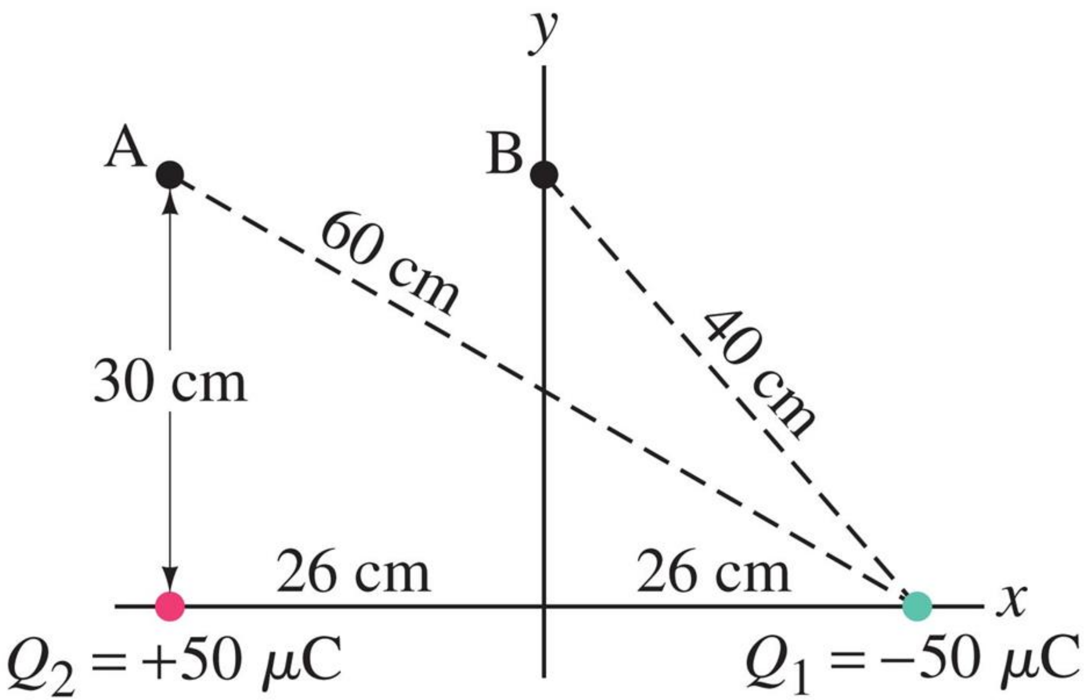
Q.1 What minimum work must be done by an external force to bring a charge $q = 3.00\mu C$ from a great distance away (take $r = \infty$) to a point 0.500 m from a charge $Q = 20.0\mu C$?

Possible Exam Question: Have a Read (p.707)



Q.2 Two parallel plates are charged to produce a potential difference of $50\ \mathrm{V}$, as shown above. If the separation between the plates is $0.050\ \mathrm{m}$, calculate the magnitude of the electric field in the space between the plates.

Possible Exam Question: Have a Read (p.710)



Q.3 Calculate the electric potential (a) at point A in the above figure due to the two charges shown, and (b) at point B.

Possible Exam Question: Have a Read (p.712)

Q.4 Point charge equipotential surfaces. For a single point charge with $Q = 4.0 \times 10^{-9} C$, sketch the equipotential surfaces (or lines in a plane containing the charge) corresponding to $\mathrm{V}_1 = 10\ \mathrm{V}$, $\mathrm{V}_2 = 20\ \mathrm{V}$, and $\mathrm{V}_3 = 30\ \mathrm{V}$.