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Mathematics Formula Sheet for CELEN036

- **Laws of Indices**

$$a^0 = 1 \qquad (a \neq 0)$$
$$a^m \cdot a^n = a^{m+n}$$
$$\frac{a^m}{a^n} = a^{m-n}$$
$$(ab)^n = a^n \cdot b^n$$
$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$
$$(a^m)^n = a^{mn} = (a^n)^m$$
$$a^{1/n} = \sqrt[n]{a}$$
$$a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$$

- **Laws of Logarithm**

$$\log_a 1 = 0 \qquad (a > 0)$$
$$\log_a a = 1$$
$$\log_a (xy) = \log_a x + \log_a y \qquad \texttt{(Product Rule)}$$
$$\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y \qquad \texttt{(Quotient Rule)}$$
$$\log_a x^n = n \log_a x \qquad \texttt{(Logarithm of Power)}$$
$$\log_y x = \frac{\log_a x}{\log_a y} \qquad \texttt{(Change of base rule)}$$
$$\log_b a = \frac{1}{\log_a b}$$
$$a^{\log_a x} = x$$

Relation between Logarithmic & Exponential Functions

$$a^x = y \quad \Leftrightarrow \quad x = \log_a y$$

- **Rules for Inequalities**

Inequality	Meaning
$a > b$	a is greater than b
$a < b$	a is less than b
$a \geq b$	a is greater than or equal to b
$a \leq b$	a is less than or equal to b

$$a > b \quad \Leftrightarrow \quad a + c > b + c \quad ; \quad c \in \mathbb{R}$$
$$a > b \quad \Leftrightarrow \quad ac > bc \quad ; \quad c > 0$$
$$a > b \quad \Leftrightarrow \quad ac < bc \quad ; \quad c < 0$$
$$|\,x - a\,| < b \quad \Leftrightarrow \quad a - b \; < \; x \; < \; a + b$$

- **Quadratic Equations**

Roots of a quadratic equation $ax^2 + bx + c = 0$ are:

$$x = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad (a \neq 0)$$

Nature of roots:

Discriminant, $\Delta = b^2 - 4ac$	> 0	Roots are **real** and **distinct**
	$= 0$	Roots are **real** and **equal** (i.e. repeated roots)
	< 0	No real roots (i.e. roots are **Complex**)

- **Complex numbers**

Cartesian form : $z = a + i\,b \quad ; \quad a, b \in \mathbb{R}, \quad i = \sqrt{-1}$

Polar form : $z = r\,(\cos\theta + i\,\sin\theta)$

where, $r = \sqrt{a^2 + b^2}, \quad \tan\theta = \dfrac{b}{a}\,;$

$$-\pi < \theta \leq \pi$$

For complex numbers,

$$z_1 = r_1\,(\cos\theta_1 + i\,\sin\theta_1)$$

and

$$z_2 = r_2\,(\cos\theta_2 + i\,\sin\theta_2),$$

$$z_1\,z_2 = r_1\,r_2\,\left(\cos(\theta_1 + \theta_2) + i\,\sin(\theta_1 + \theta_2)\right)$$
$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\,\left(\cos(\theta_1 - \theta_2) + i\,\sin(\theta_1 - \theta_2)\right)$$

- **Matrices**

Product of matrices

$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $B = \begin{pmatrix} x & y \\ z & w \end{pmatrix}$ is

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x & y \\ z & w \end{pmatrix} = \begin{pmatrix} ax + bz & ay + bw \\ cx + dz & cy + dw \end{pmatrix}$$

Inverse of a 2×2 matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is

$$A^{-1} = \frac{1}{(ad - bc)} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \quad ; \quad ad - bc \neq 0.$$

- **Sequence and Series**

Arithmetic Progression:

$a,\ a+d,\ a+2d,\ a+3d,\ .....$

where,

$a =$ first term, $d =$ common difference.

n^{th} term is $\quad a_n = a + (n-1)\,d$

Sum of first n terms is

$$S_n = \frac{n}{2}\,[\,2a + (n-1)\,d\,]$$

Geometric Progression:

$a,\ ar,\ ar^2,\ ar^3,\ .....$

where,

$a =$ first term, $r =$ common ratio.

n^{th} term is $\quad a_n = a\,r^{n-1}$

Sum of first n terms is

$$S_n = \begin{cases} \dfrac{a\,(1-r^n)}{1-r} & ; \quad r \neq 1 \\ n\,a & ; \quad r = 1 \end{cases}$$

Sum of *Infinite Geometric Series*:

$$S = \frac{a}{1-r} \quad ; \quad |r| < 1$$

Harmonic Progression:

n^{th} term is $\quad a_n = \dfrac{1}{n}\ ;\ n = 1, 2, 3, ...$

Fibonacci Sequence:

$f(1) = f(2) = 1,\ f(n+2) = f(n) + f(n+1)\ ;$

$n = 1, 2, 3, ...$

Power Series:

$$\sum n = \frac{n(n+1)}{2}$$

$$\sum n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum n^3 = \frac{n^2(n+1)^2}{4}$$

- **Trigonometry**

$$\begin{cases} \cos^2\theta + \sin^2\theta = 1 \\ \tan^2\theta + 1 = \sec^2\theta \\ \cot^2\theta + 1 = \operatorname{cosec}^2\theta \end{cases}$$

$$\begin{cases} \sin(A \pm B) &= \sin A\cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A\cos B \mp \sin A \sin B \\ \tan(A \pm B) &= \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \end{cases}$$

$$\begin{cases} 2\sin A\cos B &= \sin(A+B) + \sin(A-B) \\ 2\cos A\sin B &= \sin(A+B) - \sin(A-B) \\ 2\cos A\cos B &= \cos(A+B) + \cos(A-B) \\ -2\sin A\sin B &= \cos(A+B) - \cos(A-B) \end{cases}$$

$$\begin{cases} \sin C + \sin D &= 2\,\sin\left(\dfrac{C+D}{2}\right)\,\cos\left(\dfrac{C-D}{2}\right) \\ \sin C - \sin D &= 2\,\cos\left(\dfrac{C+D}{2}\right)\,\sin\left(\dfrac{C-D}{2}\right) \\ \cos C + \cos D &= 2\,\cos\left(\dfrac{C+D}{2}\right)\,\cos\left(\dfrac{C-D}{2}\right) \\ \cos C - \cos D &= -2\,\sin\left(\dfrac{C+D}{2}\right)\,\sin\left(\dfrac{C-D}{2}\right) \end{cases}$$

$$\begin{cases} \sin 2A &= 2\sin A\cos A \\ \cos 2A &= \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A \\ \tan 2A &= \dfrac{2\tan A}{1-\tan^2 A} \end{cases}$$

$$\begin{cases} \sin^2\theta &= \dfrac{1}{2}\,(1-\cos 2\theta) \\ \cos^2\theta &= \dfrac{1}{2}\,(1+\cos 2\theta) \end{cases}$$

$$\begin{cases} \sin 3\theta = 3\sin\theta - 4\sin^3\theta \\ \cos 3\theta = 4\cos^3\theta - 3\cos\theta \end{cases}$$

$$\begin{cases} \sin\theta &= \dfrac{2t}{1+t^2} \\ \cos\theta &= \dfrac{1-t^2}{1+t^2} \qquad \text{where } t = \tan\left(\dfrac{\theta}{2}\right) \\ \tan\theta &= \dfrac{2t}{1-t^2} \end{cases}$$

- **Binomial Series**

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + ... + \binom{n}{r}a^{n-r}b^r + ... + b^n \qquad \text{if } n \in \mathbb{N}$$

$$\text{where } \binom{n}{r} = {}^nC_r = \frac{n\,!}{r\,!\,(n-r)\,!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2\,!}\,x^2 + ... + \frac{n(n-1).....(n-r+1)}{r\,!}\,x^r + ... \qquad |\,x\,| < 1,\ n \in \mathbb{R}$$