University of Nottingham
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Foundation Algebra for Physical Sciences & Engineering CELEN036

Practice Problems SET-4 **Topic:** Trigonometry 1

Type 1: Trigonometric Identities

1. Prove the following trigonometric identities:

   (i) $\dfrac{\cos x}{1-\sin x}+\dfrac{1-\sin x}{\cos x}=2\sec x$

   (ii) $\dfrac{\cos\theta}{1+\cot\theta}=\dfrac{\sin\theta}{1+\tan\theta}$

   (iii) $(1+\tan^2\theta)(1-\sin^2\theta)=1$

   (iv) $(\operatorname{cosec}^2\theta-1)=\cos^2\theta\operatorname{cosec}^2\theta$

   (v) $\cot^2 x-\cos^2 x=\cot^2 x\cos^2 x$

   (vi) $\cos^4\theta+\sin^4\theta=1-2\cos^2\theta\sin^2\theta$

   (vii) $\cos^6\theta+\sin^6\theta=1-3\cos^2\theta\sin^2\theta$

   $(viii)$ $\cos^8\theta-\sin^8\theta=(\cos^2\theta-\sin^2\theta)(1-2\sin^2\theta\cos^2\theta)$

   (ix) $\dfrac{\tan\theta}{\sqrt{1+\tan^2\theta}}=\sin\theta$

   (x) $(\sec\theta+\tan\theta)^2=\dfrac{\operatorname{cosec}\theta+1}{\operatorname{cosec}\theta-1}$

   (xi) $\sqrt{\dfrac{1+\sin^2\theta\sec^2\theta}{1+\cos^2\theta\operatorname{cosec}^2\theta}}=\tan\theta$

2. Given $p=\sec\theta-\tan\theta$ and $q=\sec\theta+\tan\theta$, show that $p=\dfrac{1}{q}$.

3. Derive the identity $1+\cot^2\theta=\operatorname{cosec}^2\theta$ by dividing $x^2+y^2=r^2$ by y^2.

4. Derive the quotient identity $\dfrac{\cos\theta}{\sin\theta}=\cot\theta$.

5. Verify that the following equation is an identity: $\dfrac{\tan t-\cot t}{\sin t\cos t}=\sec^2 t-\operatorname{cosec}^2 t$.

6. Verify that the following equation is an identity (working from both sides):
   $\dfrac{\sec\alpha+\tan\alpha}{\sec\alpha-\tan\alpha}=\dfrac{1+2\sin\alpha+\sin^2\alpha}{\cos^2\alpha}$

7. Prove the following trigonometric identity: $\tan^2 x(1+\cot^2 x)=\dfrac{1}{1-\sin^2 x}$.

8. Write $\cos x$ in terms of $\tan x$.

9. Write $\dfrac{1+\cot^2\theta}{1-\operatorname{cosec}^2\theta}$ in terms of $\sin\theta$ and $\cos\theta$, and then simplify the expression so that no quotients appear.

Type 2: Conversion formulae

10. Convert the following degrees to radians or vice-versa:

(i) -160° (ii) 720° (iii) 255°

(iv) $-\dfrac{16\pi}{7}$ (v) $\dfrac{71\pi}{35}$ (vi) $\dfrac{6\pi}{13}$

Type 3: Range and period of trigonometric functions

11. Find the range and period of the following trigonometric functions:

(i) $3\operatorname{cosec}(3-4x)$ (ii) $2\tan(3x-7)$ (iii) $7\sin\left(-\dfrac{3\pi x}{4}-\dfrac{\pi}{4}\right)+6$

(iv) $-5\cos\left(\dfrac{\pi x}{8}\right)+3$ (v) $-3\cos(\pi x+2)-6$ (vi) $-4\cos(5x-9)-7$

Type 4: Finding values of trigonometric function

12. Given $\cos\theta=-\dfrac{4}{5}$; $\dfrac{\pi}{2}<\theta<\pi$. Find the valueof $\operatorname{cosec}\theta+\cot\theta$.

13. Given $\cot\theta=-\dfrac{15}{8}$; $\dfrac{3\pi}{2}<\theta<2\pi$. Find the value of $\sin\theta+\cos\theta$.

14. Given that $\sec\theta=-3$, and θ is obtuse. Find the values of $\tan\theta$ and $\operatorname{cosec}\theta$.

15. If $\tan\theta=-\frac{5}{3}$ and θ is in quadrant II, find each function value.

(i) $\sec\theta$ (ii) $\sin\theta$ (iii) $\cot(-\theta)$

Type 5: Solving trigonometric equations

16. Solve $\tan^2\theta=\sec 2\theta-1$; $\theta\in[0,180^\circ]$.

17. Solve the equation $\cos^2 x+\cos x=\sin^2 x$ for $0\le x\le\pi$.

18. Find all the solutions of the equation in the given range: $\tan^2 x=2\sec^2 x-3$ for $-\pi\le x\le\pi$.

19. Solve the trigonometric equation: $3\cos^2 x-6\cos x=\sin^2 x-3$ for $-\pi\le x\le\pi$.

20. Solve the following equations for θ in given interval:

(i) $\sin\theta=\dfrac{1}{\sqrt{2}}$; $\theta\in[0,\pi]$ (ii) $\tan\theta=\dfrac{1}{\sqrt{3}}$; $\theta\in[0,2\pi]$

(iii) $\sin 2\theta=0.5$; $\theta\in[0,2\pi]$ (iv) $\cos 3\theta=-0.5$; $\theta\in[0,180^\circ]$

Answers

8 $\dfrac{\pm\sqrt{1+\tan^2 x}}{1+\tan^2 x}$

9 $-\sec^2\theta$

10 (i) $-\dfrac{8}{9}\pi$ (ii) 4π (iii) $\dfrac{17}{12}\pi$ (iv) $-\dfrac{2880^\circ}{7}$ (v) $\dfrac{2556^\circ}{7}$ (vi) $\dfrac{1080^\circ}{13}$

11

	(i)	(ii)	(iii)	(iv)	(v)	(vi)
Range	$\mathbb{R}-(-3,3)$	$\mathbb{R}$	$[-1,13]$	$[-2,8]$	$[-9,-3]$	$[-11,-3]$
Period	$\dfrac{\pi}{2}$	$\dfrac{\pi}{3}$	$\dfrac{8}{3}$	16	2	$\dfrac{2\pi}{5}$

12 $\dfrac{1}{3}$

13 $\dfrac{7}{17}$

14 $-2\sqrt{2}$ and $\dfrac{3\sqrt{2}}{4}$

15 (i) $-\dfrac{\sqrt{34}}{3}$ (ii) $\dfrac{5\sqrt{34}}{\sqrt{34}}$ (iii) $\dfrac{3}{5}$

16 0° or 180°

17 $\dfrac{\pi}{3}$ or π

18 $\pm\dfrac{\pi}{4}$ or $\pm\dfrac{3\pi}{4}$

19 $\pm\dfrac{\pi}{3}$ or 0

20 (i) $\dfrac{\pi}{4}$ or $\dfrac{3\pi}{4}$ (ii) $\dfrac{\pi}{6}$ or $\dfrac{7\pi}{6}$ (iii) $\dfrac{\pi}{12}$ or $\dfrac{5\pi}{12}$ or $\dfrac{13\pi}{12}$ or $\dfrac{17\pi}{12}$ (iv) 40° or 80° or 160°