1 a). $y = \sqrt{x-2}$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{\sqrt{(x+h)-2} - \sqrt{x-2}}{h}$$

$$= \lim_{h \to \infty} \frac{\sqrt{x+h-2} - \sqrt{x-2}}{h} \cdot \frac{\sqrt{x+h-2} - \sqrt{x-2}}{\sqrt{x+h-2} - \sqrt{x-2}}$$

$$= \lim_{h \to \infty} \frac{(x+h-2) - (x-2)}{h(\sqrt{x+h-2} - \sqrt{x-2})}$$

$$= \lim_{h \to \infty} \frac{1}{\sqrt{x+h-2} + \sqrt{x-2}} = \frac{1}{2\sqrt{x-2}}$$

1 b.) (i) $y = \ln(2x-1) \cdot \cos 3x$

$$\frac{dy}{dx} = \frac{2}{2x-1} \cdot \cos 3x + \ln(2x-1) \cdot (-\sin 3x \cdot 3)$$

$$= \frac{2\cos 3x}{2x-1} - 3\sin 3x \ln(2x-1)$$

1 b. ii) $y = \frac{e^{-x}}{x^3+8}$

$$\frac{dy}{dx} = \frac{-e^{-x}(x^3+8) - e^{-x} \cdot 3x^2}{(x^3+8)^2}$$

$$= \frac{-e^{-x}(x^3+3x^2+8)}{(x^3+6)^2}$$

1c i). $\sin xy = x - y$.

$\cos xy \cdot (y + x\frac{dy}{dx}) = 1 - \frac{dy}{dx}$

$y\cos xy + x\cos xy \frac{dy}{dx} = 1 - \frac{dy}{dx}$

$(x\cos xy + 1)\frac{dy}{dx} = 1 - y\cos xy$

$\frac{dy}{dx} = \frac{1 - y\cos xy}{1 + x\cos xy}$

1c ii). $\frac{dy}{dx} = \frac{-1}{\sqrt{1-(\sqrt{x})^2}} \cdot \frac{1}{2} \cdot x^{-\frac{1}{2}}$

$= -\frac{1}{2\sqrt{1-x} \cdot \sqrt{x}}$

2a. $\ln y = \ln(\sin x)^{2x} = 2x\ln(\sin x)$

$\frac{1}{y}\frac{dy}{dx} = 2\ln(\sin x) + 2x\frac{\cos x}{\sin x}$

$\frac{dy}{dx} = 2(\sin x)^{2x}[\ln(\sin x) + x\cot x]$

2b (i) $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2}{-t^{-2}} = -2t^2$

when $t=-0.5$. $\left.\frac{dy}{dx}\right|_{t=-0.5} = -2 \cdot 0.5^2 = -\frac{1}{2}$

(ii) $y-0=-0.5(x-(-2)) \Rightarrow y=-\frac{1}{2}x-1$, tangent line

$y-0=2(x+4) \Rightarrow y=2x+4$, normal line

2c (i) $\frac{dV}{dt} = 50\pi$, $V=\frac{4}{3}\pi r^3$

$\therefore \frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \cdot \frac{dr}{dt} = 4\pi r^2 \frac{dr}{dt} = 50\pi$

$\therefore \frac{dr}{dt} = \frac{50\pi}{4\pi \cdot 3^2} = \frac{25}{18}$ m/min

(ii) $f(x) = -x^3+2x^2+23$

$f'(x) = -3x^2+4x > 0$

$\Rightarrow -x(3x-4) > 0$

$\Rightarrow x(3x-4) < 0$

$\therefore 0 < x < \frac{4}{3}$

3a. (i) $f(x) = x^3 - 4x^2 + 2$

$f'(x) = 3x^2 - 8x = 0 \Rightarrow x = 0$ or $\frac{8}{3}$

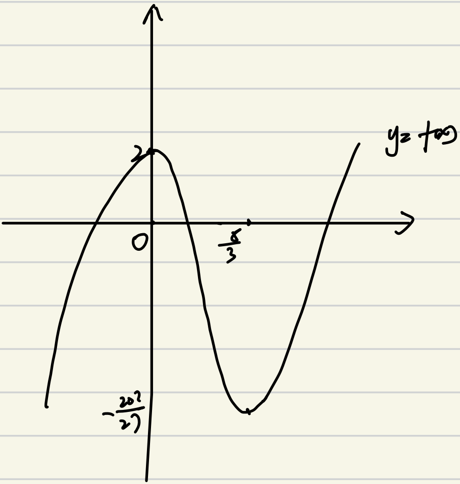
$f(0) = 2$, $f(\frac{8}{3}) = -\frac{202}{27}$

(ii) $f''(x) = 6x - 8$

$f''(0) = -8 < 0$, f has a maximum at $(0, 2)$

$f''(\frac{8}{3}) = 8 > 0$, f has a minimum at $(\frac{8}{3}, \frac{-202}{27})$

(iii)



3b (i) $f(x) = x^3 - 4x^2 + 2$

$f'(x) = 3x^2 - 8x$

$\therefore \quad x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)} = x_n - \dfrac{x_n^3 - 4x_n^2 + 2}{3x_n^2 - 8x_n}$

$= \dfrac{3x_n^3 - 8x_n^2 - x_n^3 + 4x^2 - 2}{3x_n^2 - 8x_n}$

$= \dfrac{2x_n^3 - 4x_n^2 - 2}{3x_n^2 - 8x_n}$

(ii)

n	x_n
0	-0.5
1	-0.6842
2	-0.6562
3	-0.6554
4	-0.6554

$\therefore \ x^* \approx -0.6554$

3c $y = e^{2x} + \sin 3x$

$\frac{dy}{dx} = 2e^{2x} + 3\cos 3x$

$\frac{d^2y}{dx^2} = 4e^{2x} - 9\sin 3x$

when $x = \frac{\pi}{2}$

$\left.\frac{d^2y}{dx^2}\right|_{x=\frac{\pi}{2}} = 4e^{2\cdot\frac{\pi}{2}} - 9\sin\left(3\cdot\frac{\pi}{2}\right)$

$= 4e^{\pi} - 9\sin\left(\frac{3\pi}{2}\right)$

$= 4e^{\pi} + 9$

4a (i)

$f(x) = \sqrt[3]{1-x}$, $f(0) = 1$

$f'(x) = -\frac{1}{3(1-x)^{\frac{2}{3}}}$, $f'(0) = -\frac{1}{3}$

$f''(x) = -\frac{2}{9(1-x)^{\frac{5}{3}}}$, $f''(0) = -\frac{2}{9}$

$$\therefore f(x) = f(0) + f'(0)\cdot x + \frac{f''(0)\cdot x^2}{2!} + \cdots$$

$$= 1 + (-\tfrac{1}{3})x + \frac{-\frac{2}{9}x^2}{2!} + \cdots$$

$$\approx 1 - \tfrac{1}{3}x - \tfrac{1}{9}x^2$$

(ii)

$$\sqrt[3]{0.9} = \sqrt[3]{1-0.1}$$

$$\approx 1 - \tfrac{1}{3}\cdot 0.1 - \tfrac{1}{9}\cdot (0.1)^2$$

$$= 0.966$$

4b (i) $I = \int 3x^{10}\,dx - \int x^{-\frac{3}{2}}\,dx + \int 4\,dx$

$= \frac{3}{11}x^{11} + 2x^{-\frac{1}{2}} + 4x + c$

(ii) $4 + x^2 = t$

$\frac{dt}{dx} = 2x \Rightarrow x\,dx = \frac{1}{2}dt$

$\therefore I = \int \frac{\frac{1}{2}}{\sqrt{t}}\,dt$

$= \sqrt{t} + c$

$= \sqrt{4+x^2} + c$

(iii) $\ln(x) = t$

$\frac{dt}{dx} = \frac{1}{x} \Rightarrow \frac{1}{x}dx = dt$

$\therefore I = \int t^3\,dt$

$= \frac{t^4}{4} + c$

$= \frac{(\ln x)^4}{4} + c$

5a (i) Let $-3\cos x = t \Rightarrow \frac{dt}{dx} = 3\sin x$ $\therefore \frac{1}{3} dt = \sin x \, dx$

$\therefore I = \frac{1}{3}\int e^t \, dt$

$= \frac{1}{3} e^t + c$

$= \frac{1}{3} e^{-3\cos x} + c$

(ii) Let $\ln x = f(x)$ $\therefore f'(x) = \frac{1}{x}$

$\therefore I = \int \frac{f'(x)}{\sqrt{f(x)}} dx = 2\sqrt{f(x)} + c$

$= 2\sqrt{\ln x} + c$

5b $I = \int \frac{2}{x^2 - 4x + 4 + 4} dx$

$= 2\int \frac{1}{(x-2)^2 + 2^2} dx$

$= 2 \cdot \frac{1}{2} \tan^{-1}\left(\frac{x-2}{2}\right) + c$

$= \tan^{-1}\left(\frac{x-2}{2}\right) + c$

5c (i) $I=\frac{1}{2}\int \sin(4x+5x)-\sin(4x-5x)\,dx$

$=\frac{1}{2}\int \sin 9x+\sin x\,dx$

$=\frac{1}{2}\left(-\frac{\cos 9x}{9}-\cos x\right)+C$

$=-\frac{\cos 9x}{18}-\frac{\cos x}{2}+C$

(ii) Let $\tan\left(\frac{x}{2}\right)=t$ $\quad dx=\frac{2}{1+t^2}dt$

$\cos x=\frac{1-t^2}{1+t^2}$

$\therefore I=\int \frac{1}{2+\frac{1-t^2}{1+t^2}}\cdot\frac{2}{1+t^2}\,dt$

$=\int \frac{2}{2+2t^2+1-t^2}\,dt$

$=\int \frac{2}{3+t^2}\,dt$

$=\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{t}{\sqrt{3}}\right)+C$

$=\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{\tan\frac{x}{2}}{\sqrt{3}}\right)+C$

6a Let $\frac{x+8}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}$

$\therefore x+8 = A(x+2) + B(x-1)$

Let $x=1 \quad \therefore \quad A=3$

Let $x=-2 \quad \therefore \quad B=-2$

$\therefore I = \int \frac{3}{x-1} - \frac{2}{x+2}\, dx$

$= 3\ln|x-1| - 2\ln|x+2| + C$

6b (i) $I = \int_{-4}^{-1} 3x^2 - 4x^3\, dx$

$= \left[x^3 - x^4\right]_{-4}^{-1}$

$= \left[(-1)^3 - (-1)^4\right] - \left[(-4)^3 - (-4)^4\right]$

$= -2 - (-64 - 256)$

$= 318$

6b (ii) $I = 7\int_0^{\frac{\pi}{2}} \sin x\,dx - 2\int_0^{\frac{\pi}{2}} \cos x\,dx$

$= 7\left[-\cos x\right]_0^{\frac{\pi}{2}} - 2\left[\sin x\right]_0^{\frac{\pi}{2}}$

$= 7(0-(-1)) - 2(1-0)$

$= 7-2 = 5$

6c Let $\ln x = u$, $\frac{dv}{dx} = 1$

$\therefore \frac{du}{dx} = \frac{1}{x}$, $v = x$

$\therefore I = \left[x\ln x\right]_{\frac{1}{e}}^{1} - \int_{\frac{1}{e}}^{1} 1\,dx$

$= \left[0-(-\frac{1}{e})\right] - \left[x\right]_{\frac{1}{e}}^{1}$

$= \frac{1}{e} - (1-\frac{1}{e})$

$= \frac{2}{e} - 1$

6d $(x-1)^2+1 = x+2$

$\therefore x^2 - 2x + 2 = x+2$

$x^2 - 3x = 0$

$\therefore x(x-3) = 0$

$\therefore x = 0 \text{ or } x = 3$

$$\therefore A = \left| \int_0^3 (x-1)^2+1 - x - 2 \, dx \right|$$

$$= \left| \int_0^3 x^2 - 3x \, dx \right|$$

$$= \left| \left[\frac{x^3}{3} - \frac{3x^2}{2} \right]_0^3 \right|$$

$$= \left| -\frac{9}{2} \right|$$

$$= \frac{9}{2}$$

7a (i) Eqn $\Rightarrow$ $\frac{dy}{dx} - 3\frac{d^3y}{dx^3} = \left(\frac{d^2y}{dx^2}\right)^4$

$\therefore$ degree $= 1$, order $= 3$.

(ii) $\frac{dy}{dx} = 6e^{3x} - 2$

$\therefore$ LHS $= 6e^{3x} - 2 - 3(2e^{3x} - 2x - 2)$

$= 6e^{3x} - 2 - 6e^{3x} + 6x + 6$

$= 6x + 4 =$ RHS.

7b (i) $\frac{1}{y^2-4}\,dy = (2x+3)\,dx$

$\int \frac{1}{y^2-2^2}\,dy = \int (2x+3)\,dx$

$\therefore \frac{1}{4}\ln\left|\frac{y-2}{y+2}\right| = x^2 + 3x + C$

(ii) $\frac{dy}{dx} = x + 2xy = x(1+2y)$

$\int \frac{1}{1+2y}\,dy = \int x\,dx$

$\frac{1}{2}\ln|1+2y| = \frac{1}{2}x^2 + c$

when $x=0$, $y=0$

$\therefore \frac{1}{2}\ln|1| = 0 + c$

$\therefore c = 0$

$\therefore \ln|1+2y| = x^2$

or $y = \frac{e^{x^2}}{2} - \frac{1}{2}$

7c (i) $\frac{dM}{dt} = -kM$

$\frac{1}{M} dM = -k\,dt$

$\therefore \ln M = -kt + c$

$M = e^{-kt+c}$

$M = e^{c} \cdot e^{-kt}$

when $t=0$, $M = M_0$

$\therefore M_0 = e^{c}$

$M = M_0 e^{-kt}$

(ii) when $M = \frac{1}{2}M_0$, $t = 10$

$\therefore \frac{1}{2}M_0 = M_0 e^{-k \cdot 10}$

$k = -\frac{\ln \frac{1}{2}}{10}$

$\therefore$ when $t = 8$, $M_0 = 25$

$M = 25 e^{\frac{\ln \frac{1}{2}}{10} \cdot 8} \approx 14.3587$ mg