# Seminar 10

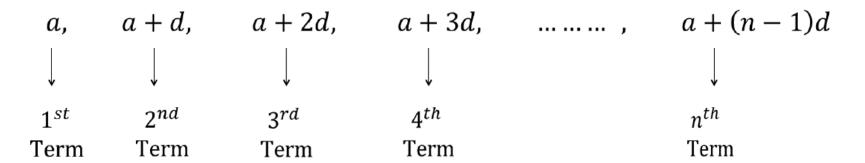
# In this seminar you will study:

- Arithmetic/Geometric progressions
- Arithmetic/Geometric series
- Sum of Powers
- Method of difference



# Arithmetic progressions (AP)

An arithmetic sequence (arithmetic progression) is given by



: the  $n^{th}$  term of an A.P. is  $a_n = a + (n-1)d$  and  $a_n = a + (n-1)d$ 

d is the common difference

(1)



# Arithmetic progressions (AP)

**Example:** For an AP, the third term is 8 and the sixteenth term is 47.

Find the first term a and the common difference d. Hence, write the first seven terms of the AP.

#### **Solution:**

Third term is 
$$8 \Rightarrow a + 2d = 8$$

Sixteenth term is 
$$47 \Rightarrow a + 15d = 47$$
 (2)

$$(2) - (1)$$
 gives  $39 = 13d$  :  $d = 3$ 

From (1) 
$$a + 2 \times 3 = 8$$
 :  $a = 2$ 

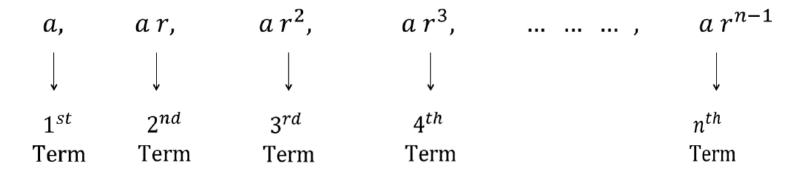
Thus, a = 2, and d = 3.

 $\Rightarrow$  The first seven terms of the AP are: 2, 5, 8, 11, 14, 17, 20...

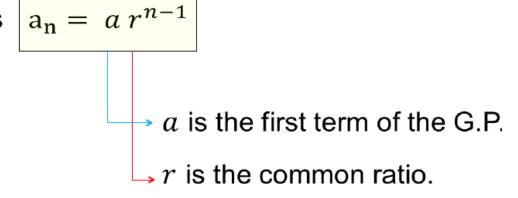


# Geometric progressions (GP)

A geometric sequence (Geometric progression) is given by



 $\therefore$  the  $n^{th}$  term of the G.P. is  $a_n = a r^{n-1}$ 



# Geometric progressions (GP)

**Example:** For a GP, the third term is 400 and the seventh term is 250,000.

Find the first term a and the common ratio r. Hence, write the first seven terms of the GP.

#### **Solution:**

Third term is 
$$400 \Rightarrow ar^2 = 400$$
 (1)

Seventh term is 
$$250,000 \implies ar^6 = 250,000$$
 (2)

$$(2) \div (1)$$
 gives  $r^4 = 625 \implies (r^2)^2 = (25)^2$ 

$$\Rightarrow r^2 = 25$$

$$\therefore r = \pm 5$$

From (1)  $ar^2 = 400 \implies a \times 25 = 400$ 

$$\therefore a = 16$$

Thus, a = 16, and  $r = \pm 5$ .



# Geometric progressions (GP)

**Example:** For a GP, the third term is 400 and the seventh term is 250,000.

Find the first term a and the common ratio r. Hence, write the first seven terms of the GP.

- $\Rightarrow$  with r=5 the first seven terms of the GP are:
  - 16, 80, 400, 2000, 10,000, 50,000, 250,000...
- $\Rightarrow$  with r=-5 the first seven terms of the GP are:

$$16, -80, 400, -2000, 10,000, -50,000, 250,000...$$

## Arithmetic series

The sum of the first n terms of an A.P. is:

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

where a is the first term and d is the common difference.

The sum of the first n terms of an A.P. is also given by

$$S_n = \frac{n}{2} [a+l]$$

where l = a + (n - 1)d =last term given in A.P.



## Arithmetic series

**Example:** The sixth term of an A.P. is 23 and its twenty-second term is

39. Find the sixteenth term of the sequence and sum of its first 19 terms.

Sixth term is 
$$23 \Rightarrow a + 5d = 23$$
 (1)

Twenty-second term is 
$$39 \Rightarrow a + 21d = 39$$
 (2)

$$(2) - (1)$$
 gives  $16d = 16$  :  $d = 1$ 

From (1) 
$$a + 5 \times 1 = 23$$
 :  $a = 18$ 

$$\Rightarrow$$
 Sixteenth term is :  $a_{16} = a + 15d$ 

$$\therefore a_{16} = 18 + 15 \times 1 = 33$$

Sum of first nineteen terms is: 
$$S_{19} = \frac{19}{2} [2 \times 18 + (18) \times 1]$$
  
=  $19[27] = 513$ 

## Geometric series

The sum of the first n terms of a G.P. is:

$$S_n = \begin{cases} na & ; r = 1 \\ a\left(\frac{1-r^n}{1-r}\right) & ; r \neq 1 \end{cases}$$

where a is the first term and r is the common ratio.

The sum of infinite terms of a G.P. is  $S = \frac{a}{1-r}$  (if -1 < r < 1)

## Geometric series

**Example 1:** For a G.P. the second term is 6 and the fifth term is 48, find the sum of the first 10 terms.

#### **Solution:**

Second term is  $6 \Rightarrow ar = 6$ 

(1)

Fifth term is  $48 \Rightarrow ar^4 = 48$ 

(2)

$$(2) \div (1)$$
 gives  $r^3 = 8 \implies r = 2$ 

From (1)  $a \times 2 = 6 \Rightarrow a = 3$ 

Sum of first ten terms is:  $S_{10} = \frac{3(2^{10} - 1)}{2 - 1}$ 

$$\therefore S_n = \frac{a(r^n - 1)}{r - 1}$$

$$=3(1023)=3069$$



## Geometric series

**Example 2:** Express  $3.123123\overline{123}$  as a vulgar fraction.

#### **Solution:**

$$3.123123\overline{123} = 3 + \frac{123}{10^3} + \frac{123}{10^6} + \frac{123}{10^9} + \dots$$

$$3.123123\overline{123} = 3 + \underbrace{\frac{123}{10^3} + \frac{123}{10^6} + \frac{123}{10^9} + \dots}_{}$$

sum of infinite terms of a GP

The first term of the GP is:  $a = \frac{123}{10^3}$ 

The common ratio of the GP is: 
$$r = \left(\frac{123}{10^6}\right) \div \left(\frac{123}{10^3}\right) = \frac{1}{10^3}$$

$$\therefore \frac{123}{10^3} + \frac{123}{10^6} + \frac{123}{10^9} + \dots = \frac{\frac{123}{10^3}}{1 - \frac{1}{10^3}} = \frac{41}{333} \qquad \therefore S = \frac{a}{1 - r} \text{ if } |r| < 1$$

$$\Rightarrow 3.123123\overline{123} = 3 + \frac{41}{333} = \frac{1040}{333}$$

## The formulae for the sum of power series

$$\underbrace{1+1+1+\ldots+1}_{n \text{ times}} = \sum_{1}^{n} 1 = n$$

$$1+2+3+\ldots+n = \sum_{1}^{n} k = \frac{n(n+1)}{2}$$

$$1^{2}+2^{2}+3^{2}+\ldots n^{2} = \sum_{1}^{n} k^{2} = \frac{n(n+1)(2n+1)}{6}$$

$$1^{3}+2^{3}+3^{3}+\ldots n^{3} = \sum_{1}^{n} k^{3} = \frac{n^{2}(n+1)^{2}}{4}$$



## The formulae for the sum of power series

**Example:** Prove that 
$$\sum_{1}^{n} (6n^2 + 4n - 1) = n(n + 2)(2n + 1)$$

Solution: 
$$\sum_{1}^{n} (6n^{2} + 4n - 1) = 6 \sum_{1}^{n} n^{2} + 4 \sum_{1}^{n} n - \sum_{1}^{n} 1$$

$$= 6 \cdot \frac{n(n+1)(2n+1)}{6} + 4 \cdot \frac{n(n+1)}{2} - n$$

$$= n(n+1)(2n+1) + 2n(n+1) - n$$

$$= n [(n+1)(2n+1) + 2(n+1) - 1]$$

$$= n [(n+1)(2n+1) + (2n+1)]$$

$$= n(2n+1)[(n+1) + 1]$$

$$= n(2n+1)(n+2)$$



## The Method of differences

### **Example:**

(i). Use the method of partial fractions to show that  $\frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}$ 

(ii). Hence use the method of differences to show that 
$$\sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{n}{n+1}$$

(i). Let 
$$\frac{1}{k(k+1)} = \frac{A}{k} + \frac{B}{k+1}$$

$$\Rightarrow 1 = A(k+1) + B(k)$$

$$\text{Put } k = 0 \Rightarrow 1 = A(1)$$

$$\therefore A = 1$$

$$\text{Put } k = -1 \Rightarrow 1 = B(-1)$$

$$\therefore B = -1$$

$$\Rightarrow \frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}$$

## The Method of differences

(ii). 
$$\sum_{k=1}^{n} \frac{1}{k(k+1)} = \sum_{k=1}^{n} \left(\frac{1}{k} - \frac{1}{k+1}\right)$$
$$= \left(1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \frac{1}{n-1} - \frac{1}{n} + \frac{1}{n} - \frac{1}{n+1}\right)$$
$$= 1 - \frac{1}{n+1}$$
$$= \frac{n}{n+1}$$



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