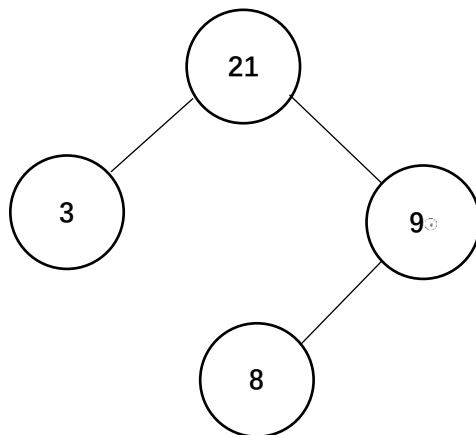
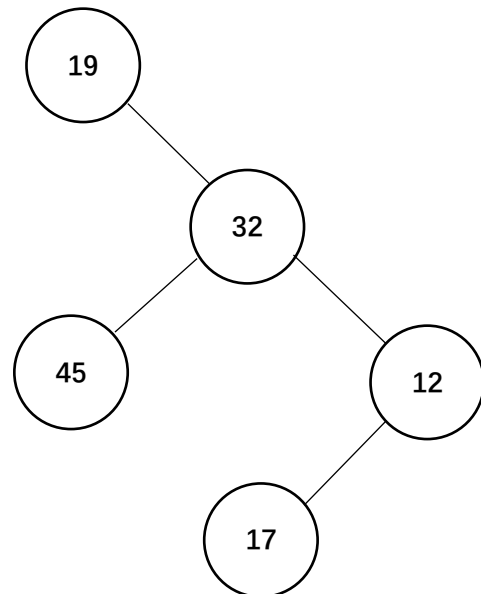


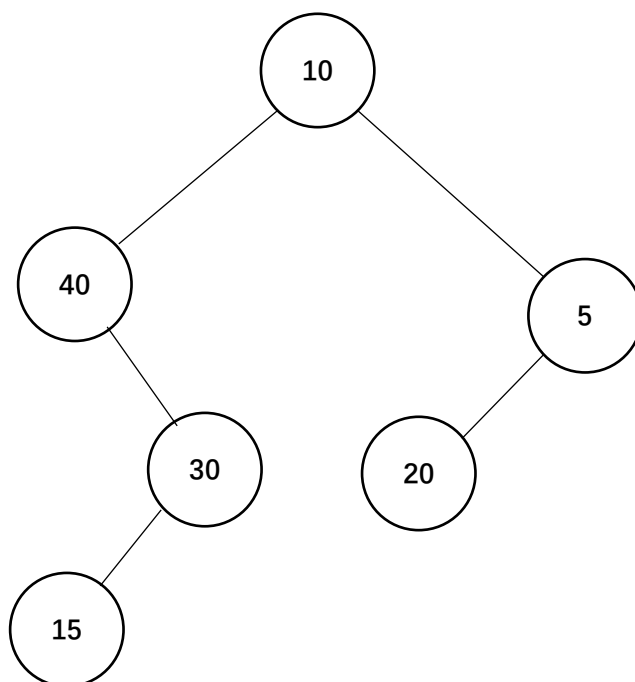
1. (i)



(ii)



2. (i)



2. (ii): depth of node 30 is 2; height of node 30 is 1.

2. (iii): 30

2. (iv): 15, 20

3.

Algorithm: isSingle(T) Requires: a binary tree T Returns: True if it has a single node; False otherwise
<pre> 1:  if isLeaf(T) 2:      return False 3:  elseif isLeaf(left(T)) &amp;&amp; isLeaf(right(T)) 4:      return True 5:  else 6:      return False 7:  endif         </pre>

4.

Algorithm: sum(T) Requires: a binary tree T Returns: sum of all node values
1: if isLeaf(T) 2:     return 0 3: else 4:     return root(T)+sum(left(T))+sum(right(T)) 5: endif

5.

Algorithm: min(T) Requires: a non-empty binary tree Returns: minimum value in the tree
1: if isLeaf(left(T)) && isLeaf(right(T)) 2:     return root(T) 3: elseif isLeaf(left(T)) 4:     return min2(root(T),min(right(T))) 5: elseif isLeaf(right(T)) 6:     return min2(root(T),min(left(T))) 7: else 8:     return min2(root(T),min2(min(left(T)),min(right(T)))) 9: endif

The following sub-algorithm min2(x,y) is used:

Algorithm: min2(x,y) Requires: two numbers x and y Returns: smaller value of two numbers
1: if x<y 2:     return x 3: else 4:     return y 5: endif

6.

Algorithm: add(x,T) Requires: a non-empty binary tree and a number x Returns: a new binary tree with updated values, each added by x
1: if isLeaf(left(T)) && isLeaf(right(T)) 2:     return node(leaf,x+root(T),leaf) 3: elseif isLeaf(left(T)) 4:     return node(leaf,x+root(T),add(x,right(T))) 5: elseif isLeaf(right(T)) 6:     return node(add(x,left(T)),x+root(T),leaf) 7: else 8:     return node(add(x,left(T)),x+root(T),add(x,right(T)))

9: endif
----------

7. Refer to Seminar 7 slides.

8.

Algorithm: depth(x,T) Requires: a number x and a binary tree T Returns: depth of node with value x, or -1.
1: if !search(x,T) 2:     return -1 3: elseif x==root(T)    isLeaf(T) 4:     return 0 5: else 6:     return 1+max2(depth(x,left(T)), depth(x,right(T))) 7: endif

The following two sub-algorithms are used:

Algorithm: max2(x,y) Requires: two numbers x and y Returns: larger value of two numbers
1: if x>y 2:     return x 3: else 4:     return y 5: endif

Algorithm: search(x,T). Refer to Lecture 7 slides.

9.

Algorithm: leafnodeNum(T) Requires: a binary tree T Returns: total number of leaf nodes
1: if isLeaf(T) 2:     return 0 3: elseif isLeaf(left(T)) && isLeaf(right(T)) 4:     return 1 5: else 6:     return leafnodeNum(left(T))+leafnodeNum(right(T)) 7: endif

10.

Algorithm: bisection(a,b,error)

Requires: three numbers a,b ( $a < b$ ) and error

Returns: approximate root of  $f(x)$

```
1:  if  $f(a)*f(b) > 0$ 
2:      return 'Choose a different interval'
3:  else
4:      let  $c = 0.5*(a+b)$  // find middle point
5:      if  $\text{abs}(f(c)) < \text{error}$  // approximate solution is accurate enough
6:          return c
7:      elseif  $f(a)*f(c) < 0$  // root exists on interval (a, c)
8:          return bisection(a,c,error) // replace b by c
9:      else // take the other half interval
10:         return bisection(c,b,error) // replace a by c
11:     endif
12: endif
```