University of Nottingham
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Foundation Algebra for Physical Sciences & Engineering CELEN036

Practice Problems SET-9 **Topic:** Matrices

Type 1: Algebra of matrices

1. Find the values of $x, y, z,$ and w if the equation holds: $\begin{pmatrix} x+1 & 2y+3 \\ 3z+4 & 4w+1 \end{pmatrix} = \begin{pmatrix} 1 & 9 \\ 10 & 5 \end{pmatrix}$.

2. Given matrices $A = \begin{pmatrix} 1 & a & -3 \\ x+z & 0 & y \\ 9 & b & 4 \end{pmatrix}$ and $B = \begin{pmatrix} a+b & 4 & -3 \\ 7 & x & 2 \\ b+c & -3 & w \end{pmatrix}$, if $A = B$, find the values of $a, b, c, x, y, z,$ and w.

3. Given matrices $A = \begin{pmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 8 & 9 & 10 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 & 2 \\ 5 & 6 & 7 \\ 6 & 7 & 8 \end{pmatrix}$,

   Find (i) $A + B$ (ii) $B - A$ (iii) $2A - 3B$.

4. Given $A = \begin{pmatrix} 1 & 5 & 7 \\ k^2 & 4 & 0 \\ 0 & 2 & 6 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 7 & 1 \\ 3 & 1 & -k \\ 8 & k+4 & -1 \end{pmatrix}$ and $C = A + B$.

   Find the value of k if C is symmetric.

5. Given matrices $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $B = \begin{pmatrix} w & x \\ y & z \end{pmatrix}$, find AB and BA.

6. Given matrices $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$, show that $AB = O$.

7. Define the square of a matrix as: $A^2 = A \cdot A$, given matrices $A = \begin{pmatrix} 2 & 3 \\ 1 & -4 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$,

   show that $(A + B)^2 = \begin{pmatrix} 121 & 99 \\ 88 & 88 \end{pmatrix}$.

8. Given matrices $A = \begin{pmatrix} 2 & 3 \\ 1 & -4 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$, verify that $(AB)^T = B^T A^T$.

9. Given matrices $A = \begin{pmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$,

   show that $AB = I$, where I is the identity matrix of the same order as A and B.

10. Given $A = \begin{pmatrix} 3 & 5 \\ 1 & -4 \\ 2 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 0 & 1 \end{pmatrix}$. Find the following matrices (if they exist):

   (i) AB (ii) BA (iii) $A^2 - B^2$

Type 2: Inverse matrices

11. Evaluate the following determinants:

   (i) $\begin{vmatrix} 2 & 4 \\ 6 & 7 \end{vmatrix}$ (ii) $\begin{vmatrix} -2 & 1 \\ 5 & -3 \end{vmatrix}$ (iii) $\begin{vmatrix} -2 & -3 \\ -4 & -5 \end{vmatrix}$ (iv) $\begin{vmatrix} 6 & 45 \\ -2 & 15 \end{vmatrix}$

12. Find the inverse of the following matrices, if they exists:

   (i) $\begin{pmatrix} 2 & 4 \\ 6 & 7 \end{pmatrix}$ (ii) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (iii) $\begin{pmatrix} -1 & 3 \\ 2 & 2 \end{pmatrix}$ (iv) $\begin{pmatrix} 8 & 4 \\ 2 & 1 \end{pmatrix}$

13. Show that the inverse matrix of $A = \begin{pmatrix} -1 & 1 \\ -2 & 0 \end{pmatrix}$ is $B = \begin{pmatrix} 0 & -0.5 \\ 1 & -0.5 \end{pmatrix}$.

Type 3: Solving 2×2 systems of equations using matrix method

14. Solve the following systems of linear equations using matrix method:

   (i) $\begin{cases} x + 2y = 13 \\ 2x - 5y = 8 \end{cases}$ (ii) $\begin{cases} 3x + 2y = -3 \\ 5x + 3y = -4 \end{cases}$ (iii) $\begin{cases} x + y = 17 \\ 2x - y = 10 \end{cases}$

15. Express the following system of equations into the matrix form

$$\begin{cases} 3x + 2y = 4 \\ 6x + my = 8 \end{cases}$$

   where m is a constant.

   (i) Determine the value of m for which the system does not have a unique solution.

   (ii) Use the matrix method to find the solution to the system when $m = 1$.

Answers

1 $x = 0,\ y = 3,\ z = 2,\ w = 1$

2 $a = 4,\ b = -3,\ c = 12,\ x = 0,\ y = 2,\ z = 7,\ w = 4$

3 (i) $\begin{pmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \\ 14 & 16 & 18 \end{pmatrix}$ (ii) $\begin{pmatrix} 4 & 3 & 2 \\ -9 & -10 & -11 \\ -2 & -3 & -4 \end{pmatrix}$

4 $k = -3$

5 $AB = \begin{pmatrix} aw + by & ax + bz \\ cw + dy & cx + dz \end{pmatrix}, \quad BA = \begin{pmatrix} aw + cx & bw + dx \\ ay + cz & by + dz \end{pmatrix}$

10 (i) $\begin{pmatrix} 13 & 0 & 8 \\ -7 & 0 & -3 \\ -2 & 0 & 0 \end{pmatrix}$ (ii) $\begin{pmatrix} 5 & 3 \\ 8 & 8 \end{pmatrix}$ (iii) Not defined

11 (i) -10 (ii) 1 (iii) -2 (iv) 180

12 (i) $\begin{pmatrix} -\frac{7}{10} & \frac{2}{5} \\ \frac{3}{5} & -\frac{1}{5} \end{pmatrix}$ (ii) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (iii) $\begin{pmatrix} -\frac{1}{4} & \frac{3}{8} \\ \frac{1}{4} & \frac{1}{8} \end{pmatrix}$ (iv) A^{-1}does not exist

14 (i) $x = 9,\ y = 2$ (ii) $x = 1,\ y = -3$ (iii) $x = 9,\ y = 8$

15 (i) $m = 4$ (ii) $x = \frac{4}{3},\ y = 0$