The University of Nottingham Ningbo China

Centre for English Language Education

AUTUMN SEMESTER SAMPLE EXAM 2017-2018 ANSWERS

SCIENCE A – Physics

Time allowed: **TWO HOURS**

Candidates may complete the front cover of the answer book and sign the attendance card.

Candidates must NOT start writing their answers until told to do so.

There are 6 questions. ATTEMPT ANY 4 QUESTIONS. Each question is worth 25 marks.

Only silent, self-contained calculators with a Single-Line Display or Dual-Line Display are permitted in this examination.

Dictionaries are not allowed with one exception: those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination.
Subject-specific translation dictionaries are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries and mobile phones, may be used.

DO NOT turn the examination paper over until instructed to do so.

INFORMATION FOR INVIGILATORS:
Please collect the examination paper and the answer booklets at the end of the exam.
A 15-minute warning should be announced before the end of the exam.

Constants:

$$k = 8.99 \times 10^{9}\mathrm{N \cdot m^2/C^2}$$
$$\varepsilon_0 = 8.85 \times 10^{-12}\mathrm{C^2/N \cdot m^2}$$
$$\mathrm{g} = 9.80\ \mathrm{m}/s^2$$
$$\mu_0 = 4\pi \times 10^{-7}\mathrm{T \cdot m/A}$$

Q.1 (25 Marks)

Answer each of the following questions.

A. A stone is thrown vertically upward with a speed of 24.0 m/s.
(i) How fast is it moving when it reaches a height of 13.0 m?
(ii) How much time is required to reach this height?
(iii) Why are there two answers to (ii)?

Answer:
Choose upward to be the positive direction, and $y_0 = 0$ to be the height from which the stone is thrown. We have $v_0 = 24.0$ m/s, $a = -9.8$ m/s^2, and $y - y^0 = 13.0$ m.

(*i*) The velocity can be found from Eq, 2-12c, with *x* replaced by *y*.

$$v^2 = v_0^2 + 2a(y - y_0) = 0 \quad \rightarrow$$

$$v = \pm\sqrt{v_0^2 + 2ay} = \pm\sqrt{(24.0\,\mathrm{m/s})^2 + 2(-9.80\,\mathrm{m/s^2})(13.0\ \mathrm{m})} = \pm 17.9\,\mathrm{m/s}$$

Thus the speed is $\boxed{|v| = 17.9\,\mathrm{m/s}}$.

(*ii*) The time to reach that height can be found from Eq. 2-12b.

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 \quad \rightarrow \quad t^2 + \frac{2(24.0\,\mathrm{m/s})}{-9.80\,\mathrm{m/s^2}}t + \frac{2(-13.0\ \mathrm{m})}{-9.80\,\mathrm{m/s^2}} = 0 \quad \rightarrow$$

$$t^2 - 4.898t + 2.653 = 0 \quad \rightarrow \quad \boxed{t = 4.28\ \mathrm{s}\ ,\ 0.620\ \mathrm{s}}$$

(*iii*) There are two times at which the object reaches that height – once on the way up (t = 0.620 s), and once on the way down (t = 4.28 s).

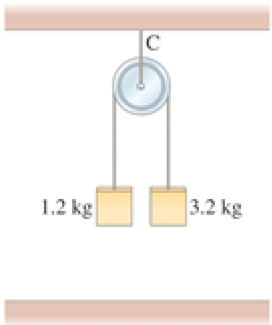
B. Suppose you adjust your garden hose nozzle for a hard stream of water. You point the nozzle vertically upward at a height of 1.5 m above the ground (shown in the figure below). When you quickly turn off the nozzle, you hear the water striking the ground next to you for another 2.0 s. What is the water speed as it leaves the nozzle?

Answer:

Choose upward to be the positive direction, and y_0 = 0 to be the location of the nozzle. The initial velocity is v_0 ,the acceleration is a = -9.8 m/s^2, the final location is y = -1.5 m, and the time of flight is t = 2.0 s. Using the equation below, and substituting y for x gives the following.

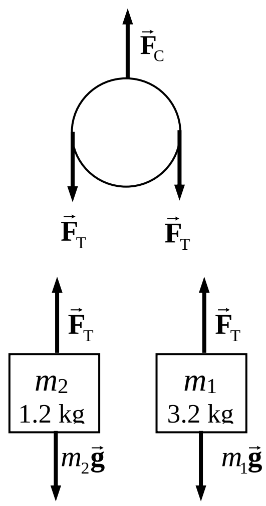
$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 \quad \rightarrow \quad v_0 = \frac{y - \frac{1}{2}at^2}{t} = \frac{-1.5\text{ m} - \frac{1}{2}\left(-9.80\,\text{m/s}^2\right)\left(2.0\text{ s}\right)^2}{2.0\text{ s}} = \boxed{9.1\,\text{m/s}}$$

C. Suppose the pulley in the figure below is suspended by a cord C. Determine the tension in this cord after the masses are released and before one hits the ground. Ignore the mass of pulley and cords.



Answer:

We draw a free-body diagram for each mass. We choose UP to be the positive direction. The tension force in the cord is found from analyzing the two hanging masses. Notice that the same tension force is applied to each mass. Write Newton's second law for each of the masses.



$$F_T - m_1 g = m_1 a_1 \qquad F_T - m_2 g = m_2 a_2$$

Since the masses are joined together by the cord, their accelerations will have the same magnitude but opposite directions.Thus $a_1 = -a_2$. Substitute this into the force expressions and solve for the tension force.

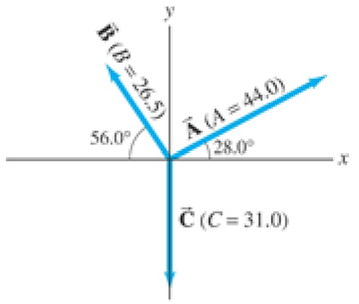
$$F_T - m_1 g = -m_1 a_2 \quad \rightarrow \quad F_T = m_1 g - m_1 a_2 \quad \rightarrow \quad a_2 = \frac{m_1 g - F_T}{m_1}$$

$$F_T - m_2 g = m_2 a_2 = m_2\left(\frac{m_1 g - F_T}{m_1}\right) \quad \rightarrow \quad F_T = \frac{2m_1 m_2 g}{m_1 + m_2}$$

Apply Newton's second law to the stationary pulley.

$$F_C - 2F_T = 0 \quad \rightarrow \quad F_C = 2F_T = \frac{4m_1 m_2 g}{m_1 + m_2} = \frac{4(3.2\,\text{kg})(1.2\,\text{kg})\left(9.80\,\text{m/s}^2\right)}{4.4\,\text{kg}} = \boxed{34\,\text{N}}$$

D. Three vectors are shown in the figure below. Their magnitudes are given in arbitrary units. Determine the sum of the three vectors. Give the resultant in terms of
(i) components, and
(ii) magnitude and angle with the *x* axis.



Answer:

$A_x = 44.0\cos 28.0° = 38.85 \qquad A_y = 44.0\sin 28.0° = 20.66$

$B_x = -26.5\cos 56.0° = -14.82 \qquad B_y = 26.5\sin 56.0° = 21.97$

$C_x = 31.0\cos 270° = 0.0 \qquad C_y = 31.0\sin 270° = -31.0$

(*i*) $\left(\vec{\mathbf{A}}+\vec{\mathbf{B}}+\vec{\mathbf{C}}\right)_x = 38.85+(-14.82)+0.0 = 24.03 = \boxed{24.0}$

$\left(\vec{\mathbf{A}}+\vec{\mathbf{B}}+\vec{\mathbf{C}}\right)_y = 20.66+21.97+(-31.0) = 11.63 = \boxed{11.6}$

(*ii*) $\left|\vec{\mathbf{A}}+\vec{\mathbf{B}}+\vec{\mathbf{C}}\right| = \sqrt{(24.03)^2+(11.63)^2} = \boxed{26.7} \qquad \theta = \tan^{-1}\frac{11.63}{24.03} = \boxed{25.8°}$

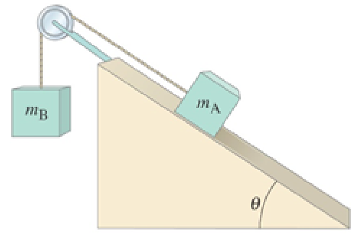
Q.2 (25 Marks)

Answer each of the following questions.

A. Suppose the coefficient of kinetic friction between m_A and the plane in the figure below is μ_k = 0.15, and that $m_A = m_B$ = 2.7 kg. As m_B moves down, determine

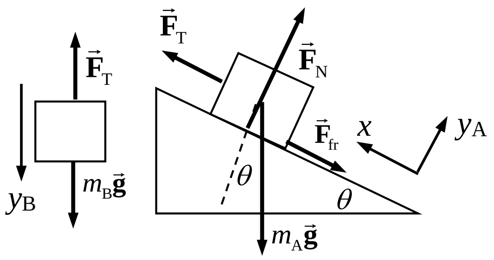
(i) the magnitude of the acceleration of m_A and m_B, given θ = 34°.

(ii) What smallest value of μ_k will keep the system from accelerating?



Answer:

(*i*) Given that m_B is moving down, m_A must be moving up the incline, and so the force of kinetic friction on m_A will be directed down the incline. Since the blocks are tied together, they will both have the same acceleration, and so $a_{yB} = a_{xA} = a$.



Write Newton's second law for each mass.

$$\sum F_{yB} = m_B g - F_T = m_B a \quad \rightarrow \quad F_T = m_B g - m_B a$$

$$\sum F_{xA} = F_T - m_A g \sin\theta - F_{fr} = m_A a$$

$$\sum F_{yA} = F_N - m_A g \cos\theta = 0 \quad \rightarrow \quad F_N = m_A g \cos\theta$$

Take the information from the two *y* equations and substitute into the *x* equation to solve for the acceleration.

$$m_B g - m_B a - m_A g \sin\theta - \mu_k m_A g \cos\theta = m_A a \quad \rightarrow$$

$$a = \frac{m_B g - m_A g \sin\theta - m_A g \mu_k g \cos\theta}{(m_A + m_B)} = \tfrac{1}{2} g \left(1 - \sin\theta - \mu_k g \cos\theta\right)$$

$$= \tfrac{1}{2}\left(9.80\,\text{m/s}^2\right)\left(1 - \sin 34° - 0.15\cos 34°\right) = \boxed{1.6\,\text{m/s}^2}$$

(ii) To have an acceleration of zero, the expression for the acceleration must be zero.

$$a = \tfrac{1}{2} g \left(1 - \sin\theta - \mu_k \cos\theta\right) = 0 \quad \rightarrow \quad 1 - \sin\theta - \mu_k \cos\theta = 0 \quad \rightarrow$$

$$\mu_k = \frac{1 - \sin\theta}{\cos\theta} = \frac{1 - \sin 34°}{\cos 34°} = \boxed{0.53}$$

B. A 46.0 kg crate, starting from rest, is pulled across a floor with a constant horizontal force of 225 N. For the first 11.0 m the floor is frictionless, and for the next 10.0 m the coefficient of friction is 0.20. What is the final speed of the crate after being pulled these 21.0 m?

Answer:

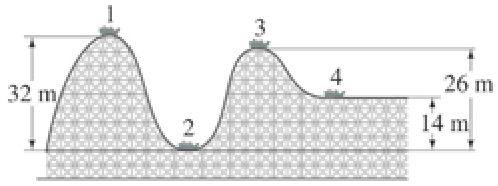
For the first part of the motion, the net force doing work is the 225 N force. For the second part of the motion, both the 225 N force and the force of friction do work. The friction force is the coefficient of friction times the normal force, and the normal force is equal to the weight.The work-energy theorem is then used to find the final speed.

$$W_{\text{total}} = W_1 + W_2 = F_{\text{pull}} d_1 \cos 0° + F_{\text{pull}} d_2 \cos 0° + F_f d_2 \cos 180° = \Delta K = \tfrac{1}{2} m\left(v_f^2 - v_i^2\right) \rightarrow$$

$$v_f = \sqrt{\frac{2\left[F_{\text{pull}}\left(d_1 + d_2\right) - \mu_k mgd_2\right]}{m}}$$

$$= \sqrt{\frac{2\left[(225\,\text{N})(21.0\,\text{m}) - (0.20)(46.0\,\text{kg})\left(9.80\,\text{m/s}^2\right)(10.0\,\text{m})\right]}{(46.0\,\text{kg})}} = \boxed{13\,\text{m/s}}$$

C. A roller-coaster car shown in the figure below is pulled up to point 1 where it is released from rest. Assuming no friction, calculate the speed at point 2, 3 and 4.



Answer:

Since there are no dissipative forces present, the mechanical energy of the roller coaster will be conserved .Subscript 1 represents the coaster at point 1, etc. The height of point 2 is the zero location for gravitational potential energy. We have $v_1 = 0$ and $y_1 = 32\,\text{m}$.

Point 2: $\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \;\; ; \; y_2 = 0 \rightarrow \;\; mgy_1 = \tfrac{1}{2}mv_2^2 \;\; \rightarrow$

$$v_2 = \sqrt{2gy_1} = \sqrt{2\left(9.80\,\text{m/s}^2\right)(32\,\text{m})} = \boxed{25\,\text{m/s}}$$

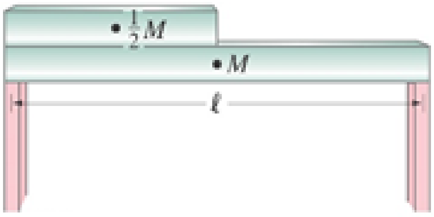
Point 3: $\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_3^2 + mgy_3 \;\; ; \; y_3 = 26\,\text{m} \rightarrow \;\; mgy_1 = \tfrac{1}{2}mv_3^2 + mgy_3 \;\; \rightarrow$

$$v_3 = \sqrt{2g\left(y_1 - y_3\right)} = \sqrt{2\left(9.80\,\text{m/s}^2\right)(6\,\text{m})} = \boxed{11\,\text{m/s}}$$

Point 4: $\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_4^2 + mgy_4 \;\; ; \; y_4 = 14\,\text{m} \rightarrow \;\; mgy_1 = \tfrac{1}{2}mv_4^2 + mgy_1 \;\; \rightarrow$

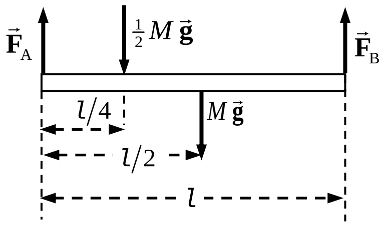
$$v_4 = \sqrt{2g\left(y_1 - y_4\right)} = \sqrt{2\left(9.80\,\text{m/s}^2\right)(18\,\text{m})} = \boxed{19\,\text{m/s}}$$

D. A uniform steel beam has a mass of 940 kg. On it is resting half of an identical beam, as shown in Figure below. What is the vertical support force at each end?



Answer:

The centre of gravity of each beam is at its geometric centre. Calculate torques about the left end of the beam, and take counterclockwise torques to be positive. The conditions of equilibrium for the beam are used to find the forces that the support exerts on the beam.



$$\sum \tau = F_B l - Mg(l/2) - \tfrac{1}{2}Mg(l/4) = 0 \quad \rightarrow$$

$$F_B = \tfrac{5}{8}Mg = \tfrac{5}{8}(940\,\text{kg})(9.80\,\text{m/s}^2) = 5758\,\text{N} \approx \boxed{5800\,\text{N}}$$

$$\sum F_y = F_A + F_B - Mg - \tfrac{1}{2}Mg = 0 \quad \rightarrow$$

$$F_A = \tfrac{3}{2}Mg - F_B = \tfrac{7}{8}Mg = \tfrac{7}{8}(940\text{ kg})(9.80\,\text{m/s}^2) = 8061\,\text{N} \approx \boxed{8100\,\text{N}}$$

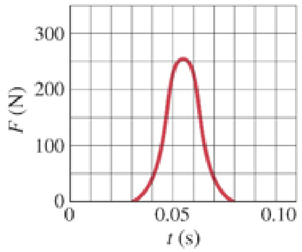
Q.3 (25 Marks)

Answer each of the following questions.

A. Suppose the force acting on a tennis ball (mass 0.060 kg) points in the +*x* direction and is given by the graph shown in the figure below as a function of time. Use graphical methods to estimate

(i) the total impulse given the ball, and

(ii) the velocity of the ball after being struck, assuming the ball is being served so it is nearly at rest initially.



Answer:

(*i*) The impulse given the ball is the area under the *F* vs. *t* graph. Approximate the area as a triangle of “height” 250 N, and “width” 0.04 sec. A width slightly smaller than the base was chosen to compensate for the “inward” concavity of the force graph.

$$\Delta p = \tfrac{1}{2}(250\ \mathrm{N})(0.04\ \mathrm{s}) = \boxed{5\,\mathrm{N{\cdot}s}}$$

(*ii*) The velocity can be found from the change in momentum. Call the positive direction the direction of the ball’s travel after being served.

$$\Delta p = m\Delta v = m\left(v_f - v_i\right) \ \rightarrow\ v_f = v_i + \frac{\Delta p}{m} = 0 + \frac{5\,\mathrm{N{\cdot}s}}{6.0\times10^{-2}\,\mathrm{kg}} = \boxed{80\,\mathrm{m/s}}$$

B. A 0.35 kg mass at the end of a spring oscillates 2.5 times per second with an amplitude of 0.15 m. Determine

(i) the velocity when it passes the equilibrium point,

(ii) the velocity when it is 0.10 m from equilibrium,

(iii) the total energy of the system, and

(iv) the equation describing the motion of the mass, assuming that at *t* = 0, *x* was a maximum.

Answer:

(*i*) At equilibrium, the velocity is its maximum. Use the equation below, and realize that the object can be moving in either direction.

$$v_{\max} = \omega A = 2\pi fA = 2\pi(2.5\,\mathrm{Hz})(0.15\,\mathrm{m}) = 2.356\,\mathrm{m/s} \ \rightarrow\ v_{\mathrm{equib}} \approx \boxed{\pm 2.4\,\mathrm{m/s}}$$

(*ii*) From Eq. 14-11b, we find the velocity at any position.

$$v = \pm v_{\max}\sqrt{1-\frac{x^2}{A^2}} = \pm(2.356\,\mathrm{m/s})\sqrt{1-\frac{(0.10\,\mathrm{m})^2}{(0.15\,\mathrm{m})^2}} = \pm 1.756\,\mathrm{m/s} \approx \boxed{\pm 1.8\,\mathrm{m/s}}$$

(*III*) $$E_{\mathrm{total}} = \tfrac{1}{2}mv_{\max}^2 = \tfrac{1}{2}(0.35\,\mathrm{kg})(2.356\,\mathrm{m/s})^2 = 0.9714\,\mathrm{J} \approx \boxed{0.97\,\mathrm{J}}$$

(*Iv*) Since the object has a maximum displacement at t = 0, the position will be described by the cosine function.

$$x = (0.15\,\text{m})\cos\left(2\pi(2.5\text{ Hz})t\right) \quad \rightarrow \quad \boxed{x = (0.15\,\text{m})\cos(5.0\pi t)}$$

C. A fire hose exerts a force on the person holding it. This is because the water accelerates as it goes from the hose through the nozzle. How much force is required to hold a 7.0 cm diameter hose delivering 450 L/min through a 0.75 cm diameter nozzle?

Answer:

There is a forward force on the exiting water, and so by Newton's third law there is an equal force pushing backwards on the hose. To keep the hose stationary, you push forward on the hose, and so the hose pushes backwards on you. So the force on the exiting water is the same magnitude as the force on the person holding the hose. Use Newton's second law and the equation of continuity to find the force. Note that the 450 L/min flow rate is the volume of water being accelerated per unit time. Also, the flow rate is the product of the cross-sectional area of the moving fluid, times the speed of the fluid, and so

$$\frac{V}{t} = A_1 v_1 = A_2 v_2 .$$

$$F = m\frac{\Delta v}{\Delta t} = m\frac{v_2 - v_1}{t} = \rho\left(\frac{V}{t}\right)(v_2 - v_1) = \rho\left(\frac{V}{t}\right)\left(\frac{A_2 v_2}{A_2} - \frac{A_1 v_1}{A_1}\right) = \rho\left(\frac{V}{t}\right)^2\left(\frac{1}{A_2} - \frac{1}{A_1}\right)$$

$$= \rho\left(\frac{V}{t}\right)^2\left(\frac{1}{\pi r_2^2} - \frac{1}{\pi r_1^2}\right)$$

$$= \left(1.00\times10^3\,\text{kg/m}^3\right)\left(\frac{450\,\text{L}}{\text{min}}\times\frac{1\,\text{min}}{60\text{ s}}\times\frac{1\,\text{m}^3}{1000\,\text{L}}\right)^2\left(\frac{1}{\pi\frac{1}{2}\left(0.75\times10^{-2}\,\text{m}\right)^2} - \frac{1}{\pi\frac{1}{2}\left(7.0\times10^{-2}\,\text{m}\right)^2}\right)$$

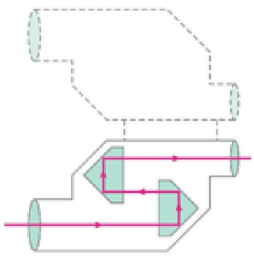
$$= 1259\text{ N} \approx \boxed{1300\text{ N}}$$

D. Given the binoculars shown in the figure below,

(i) What is the minimum index of refraction for a glass or plastic prism to be used in so that total internal reflection occurs at 45°?

(ii) Will binoculars work if their prisms (assume n = 1.58) are immersed in water?

(iii) What minimum n is needed if the prisms are immersed in water?



Answer:

(*i*) The ray enters normal to the first surface, so there is no deviation there. The angle of incidence is 45° at the second surface. When there is air outside the surface, we have the following.

$$n_1\sin\theta_1 = n_2\sin\theta_2 \quad \rightarrow \quad n_1\sin 45° = (1.00)\sin\theta_2$$

For total internal reflection to occur, $\sin\theta_2 \geq 1$, and so $n_1 \geq \dfrac{1}{\sin 45°} = \boxed{1.41}$.

(*ii*) When there is water outside the surface, we have the following.

$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad \rightarrow \quad (1.58)\sin 45° = (1.33)\sin\theta_2 \quad \rightarrow \quad \sin\theta_2 = 0.84$

Because $\sin\theta_2 < 1$, $\boxed{\text{the prism will not be totally reflecting.}}$

(*iii*) For total reflection when there is water outside the surface, we have the following.

$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad \rightarrow \quad n_1 \sin 45° = (1.33)\sin\theta_2$

$n_1 \sin 45° = (1.33)\sin\theta_2$.

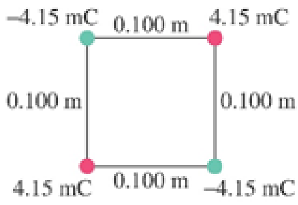
For total internal reflection to occur, $\sin\theta_2 \geq 1$.

$$n_1 \geq \frac{1.33}{\sin 45°} = \boxed{1.88}$$

Q.4 (25 Marks)

Answer each of the following questions.

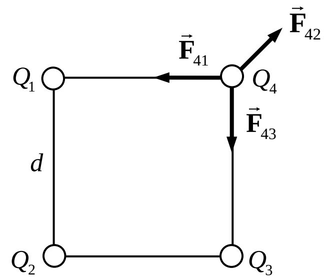
A. Two negative and two positive point charges (magnitude Q = 415 mC) are placed on opposite corners of a square as shown in the figure below. Determine the magnitude and direction of the force on each charge.



Answer:

Determine the force on the upper right charge, and then use the symmetry of the configuration to determine the force on the other charges.

The force at the upper right corner of the square is the vector sum of the forces due to the other three charges. Let the variable d represent the 0.100 m length of a side of the square, and let the variable Q represent the 4.15 mC charge at each corner.



$$F_{41} = k\frac{Q^2}{d^2} \rightarrow F_{41x} = -k\frac{Q^2}{d^2},\ F_{41y} = 0$$

$$F_{42} = k\frac{Q^2}{2d^2} \rightarrow F_{42x} = k\frac{Q^2}{2d^2}\cos 45^\circ = k\frac{\sqrt{2}Q^2}{4d^2},\ F_{42y} = k\frac{\sqrt{2}Q^2}{4d^2}$$

$$F_{43} = k\frac{Q^2}{d^2} \rightarrow F_{43x} = 0,\ F_{43y} = -k\frac{Q^2}{d^2}$$

Add the *x* and *y* components together to find the total force, noting that $F_{4x} = F_{4y}$.

$$F_{4x} = F_{41x} + F_{42x} + F_{43x} = -k\frac{Q^2}{d^2} + k\frac{\sqrt{2}Q^2}{4d^2} + 0 = k\frac{Q^2}{d^2}\left(-1 + \frac{\sqrt{2}}{4}\right) = -0.64645k\frac{Q^2}{d^2} = F_{4y}$$

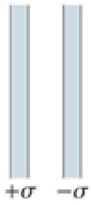
$$F_4 = \sqrt{F_{4x}^2 + F_{4y}^2} = k\frac{Q^2}{d^2}(0.64645)\sqrt{2} = k\frac{Q^2}{d^2}(0.9142)$$

$$= \left(8.988\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\frac{\left(4.15\times10^{-3}\mathrm{C}\right)^2}{\left(0.100\,\mathrm{m}\right)^2}(0.9142) = \boxed{1.42\times10^7\ \mathrm{N}}$$

$\theta = \tan^{-1}\dfrac{F_{4y}}{F_{4x}} = \boxed{225^\circ}$ from the *x*-direction, or exactly towards the center of the square.

For each charge, there are two forces that point towards the adjacent corners, and one force that points away from the center of the square.Thus for each charge, the net force will be the magnitude of $\boxed{1.42\times10^7\ \mathrm{N}}$ and will lie along the line from the charge inwards towards the centre of the square.

B. The electric field between two square metal plates is 160 N/C. The plates are 1.0 m on a side and are separated by 3.0 cm, as shown in the figure below. What is the charge on each plate? Neglect edge effects.



Answer:

Because 3.0 cm << 1.0 m, we can consider the plates to be infinite in size, and ignore any edge effects. We use the following equation:

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q/A}{\varepsilon_0} \quad \rightarrow \quad Q = EA\varepsilon_0 = (160\,\mathrm{N/C})(1.0\,\mathrm{m})^2 \left(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}\right) = \boxed{1.4\times10^{-9}\,\mathrm{C}}$$

C. A 32 cm diameter conducting sphere is charged to 680 V relative to $V = 0$ at $r = \infty$.

(i) What is the surface charge density σ?

(ii) At what distance will the potential due to the sphere be only 25 V?

Answer:

(i) The potential at the surface of a charged sphere is

$$V_0 = \frac{Q}{4\pi\varepsilon_0 r_0} \quad \rightarrow \quad Q = 4\pi\varepsilon_0 r_0 V_0 \quad \rightarrow$$

$$\sigma = \frac{Q}{\mathrm{Area}} = \frac{Q}{4\pi r_0^2} = \frac{4\pi\varepsilon_0 r_0 V_0}{4\pi r_0^2} = \frac{V_0\varepsilon_0}{r_0} = \frac{(680\,\mathrm{V})\left(8.85\times10^{-12}\ \mathrm{C^2/Nm^2}\right)}{(0.16\,\mathrm{m})} = 3.761\times10^{-8}\,\mathrm{C/m^2}$$

$$\approx \boxed{3.8\times10^{-8}\,\mathrm{C/m^2}}$$

(ii) The potential away from the surface of a charged sphere is

$$V = \frac{Q}{4\pi\varepsilon_0 r} = \frac{4\pi\varepsilon_0 r_0 V_0}{4\pi\varepsilon_0 r} = \frac{r_0 V_0}{r} \quad \rightarrow \quad r = \frac{r_0 V_0}{V} = \frac{(0.16\,\mathrm{m})(680\,\mathrm{V})}{(25\,\mathrm{V})} = 4.352\,\mathrm{m} \approx \boxed{4.4\,\mathrm{m}}$$

D. The work done by an external force to move a -9.10 µC charge from point a to point b is 7.00×10^{-4} J. If the charge was started from rest and had 2.10×10^{-4} J of kinetic energy when it reached point b, what must be the potential difference between a and b?

Answer:

By the work-energy theorem, the total work done, by the external force and the electric field together, is the change in kinetic energy. The work done by the electric field is given by

$$W_{\mathrm{external}} + W_{\mathrm{electric}} = \mathrm{KE}_{\mathrm{final}} - \mathrm{KE}_{\mathrm{initial}} \quad \rightarrow \quad W_{\mathrm{external}} - q(V_b - V_a) = \mathrm{KE}_{\mathrm{final}} \quad \rightarrow$$

$$(V_b - V_a) = \frac{W_{\mathrm{external}} - \mathrm{KE}_{\mathrm{final}}}{q} = \frac{7.00\times10^{-4}\,\mathrm{J} - 2.10\times10^{-4}\,\mathrm{J}}{-9.10\times10^{-6}\,\mathrm{C}} = \boxed{-53.8\,\mathrm{V}}$$

Since the potential difference is negative, we see that $V_a > V_b$.

E. A parallel-plate capacitor has fixed charges *+Q* and -*Q*. The separation of the plates is then tripled.

(i) By what factor does the energy stored in the electric field change?

(ii) How much work must be done to increase the separation of the plates from *d* to 3.0*d?* The area of each plate is *A.*

Answer:

(*i*) The charge is constant, and the tripling of separation reduces the capacitance by a factor of 3.

$$\frac{U_2}{U_1} = \frac{\dfrac{Q^2}{2C_2}}{\dfrac{Q^2}{2C_1}} = \frac{C_1}{C_2} = \frac{\varepsilon_0 \dfrac{A}{d}}{\varepsilon_0 \dfrac{A}{3d}} = \boxed{3}$$

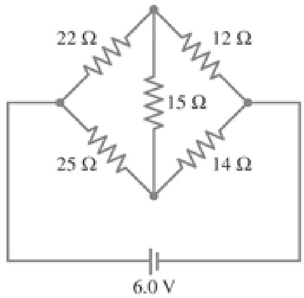
(*ii*) The work done is the change in energy stored in the capacitor.

$$U_2 - U_1 = 3U_1 - U_1 = 2U_1 = 2\frac{Q^2}{2C_1} = \frac{Q^2}{\varepsilon_0 \dfrac{A}{d}} = \boxed{\frac{Q^2 d}{\varepsilon_0 A}}$$

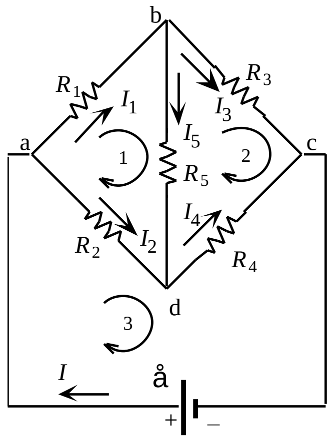
Q.5 (25 Marks)

Answer each of the following questions.

A. Determine the current through each of the resistors in the figure below.



Answer:



The circuit diagram has been labeled with six different currents. We apply the junction rule to junctions a, b, and c. We apply the loop rule to the three loops labeled in the diagram.

$$1)\ I = I_1 + I_2 \ ;\ 2)\ I_1 = I_3 + I_5 \ ;\ 3)\ I_3 + I_4 = I$$
$$4)\ -I_1R_1 - I_5R_5 + I_2R_2 = 0 \ ;\ 5)\ -I_3R_3 + I_4R_4 + I_5R_5 = 0$$
$$6)\ \mathcal{E} - I_2R_2 - I_4R_4 = 0$$

Eliminate *I* using equations 1) and 3).

$$1)\ I_3 + I_4 = I_1 + I_2 \ ;\ 2)\ I_1 = I_3 + I_5$$
$$4)\ -I_1R_1 - I_5R_5 + I_2R_2 = 0 \ ;\ 5)\ -I_3R_3 + I_4R_4 + I_5R_5 = 0$$
$$6)\ \mathcal{E} - I_2R_2 - I_4R_4 = 0$$

Eliminate I_1 using equation 2.

$$1)\ I_3 + I_4 = I_3 + I_5 + I_2 \ \rightarrow\ I_4 = I_5 + I_2$$

$$4)\ -(I_3 + I_5)R_1 - I_5R_5 + I_2R_2 = 0 \ \rightarrow\ -I_3R_1 - I_5(R_1 + R_5) + I_2R_2 = 0$$
$$5)\ -I_3R_3 + I_4R_4 + I_5R_5 = 0$$
$$6)\ \mathcal{E} - I_2R_2 - I_4R_4 = 0$$

Eliminate I_4 using equation 1.

$$4)\ -I_3R_1 - I_5(R_1 + R_5) + I_2R_2 = 0$$
$$5)\ -I_3R_3 + (I_5 + I_2)R_4 + I_5R_5 = 0 \ \rightarrow\ -I_3R_3 + I_5(R_4 + R_5) + I_2R_4 = 0$$
$$6)\ \mathcal{E} - I_2R_2 - (I_5 + I_2)R_4 = 0 \ \rightarrow\ \mathcal{E} - I_2(R_2 + R_4) - I_5R_4 = 0$$

Eliminate I_2 using equation 4: $I_2 = \dfrac{1}{R_2}\left[I_3R_1 + I_5(R_1 + R_5)\right]$.

5) $-I_3R_3 + I_5(R_4 + R_5) + \frac{1}{R_2}\left[I_3R_1 + I_5(R_1 + R_5)\right]R_4 = 0 \quad \rightarrow$

$$I_3(R_1R_4 - R_2R_3) + I_5(R_2R_4 + R_2R_5 + R_1R_4 + R_5R_4) = 0$$

6) $\mathcal{E} - \frac{1}{R_2}\left[I_3R_1 + I_5(R_1 + R_5)\right](R_2 + R_4) - I_5R_4 = 0 \quad \rightarrow$

$$\mathcal{E}R_2 - I_3R_1(R_2 + R_4) - I_5(R_1R_2 + R_1R_4 + R_5R_2 + R_5R_4 + R_2R_4) = 0$$

Eliminate I_3 using equation 5: $I_3 = -I_5\frac{(R_2R_4 + R_2R_5 + R_1R_4 + R_5R_4)}{(R_1R_4 - R_2R_3)}$

$$\mathcal{E}R_2 + \left[I_5\frac{(R_2R_4 + R_2R_5 + R_1R_4 + R_5R_4)}{(R_1R_4 - R_2R_3)}\right]R_1(R_2 + R_4) - I_5(R_1R_2 + R_1R_4 + R_5R_2 + R_5R_4 + R_2R_4) = 0$$

$$\mathcal{E} = -\frac{I_5}{R_2}\left\{\left[\frac{(R_2R_4 + R_2R_5 + R_1R_4 + R_5R_4)}{(R_1R_4 - R_2R_3)}\right]R_1(R_2 + R_4) - (R_1R_2 + R_1R_4 + R_5R_2 + R_5R_4 + R_2R_4)\right\}$$

$$= -\frac{I_5}{25\Omega}\left\{\begin{array}{l}\left[\frac{(25\Omega)(14\Omega) + (25\Omega)(15\Omega) + (22\Omega)(14\Omega) + (15\Omega)(14\Omega)}{(22\Omega)(14\Omega) - (25\Omega)(12\Omega)}\right](22\Omega)(25\Omega + 14\Omega) \\ -\left[(22\Omega)(25\Omega) + (22)(14) + (15\Omega)(25\Omega) + (15\Omega)(14\Omega) + (25\Omega)(14\Omega)\right]\end{array}\right\}$$

$$= -I_5(5261\Omega) \quad \rightarrow \quad I_5 = -\frac{6.0\,\text{V}}{5261\Omega} = -1.140\,\text{mA}\ (\text{upwards})$$

$$I_3 = -I_5\frac{(R_2R_4 + R_2R_5 + R_1R_4 + R_5R_4)}{(R_1R_4 - R_2R_3)}$$

$$= -(-1.140\,\text{mA})\frac{(25\Omega)(14\Omega) + (25\Omega)(15\Omega) + (22\Omega)(14\Omega) + (15\Omega)(14\Omega)}{(22\Omega)(14\Omega) - (25\Omega)(12\Omega)} = 0.1771\,\text{A}$$

$$I_2 = \frac{1}{R_2}\left[I_3R_1 + I_5(R_1 + R_5)\right] = \frac{1}{25\Omega}\left[(0.1771\,\text{A})(22\Omega) + (-0.00114\,\text{A})(37\Omega)\right] = 0.1542\,\text{A}$$

$$I_4 = I_5 + I_2 = -0.00114\,\text{A} + 0.1542\,\text{A} = 0.1531\,\text{A}$$
$$I_1 = I_3 + I_5 = 0.1771\,\text{A} - 0.00114\,\text{A} = 0.1760\,\text{A}$$

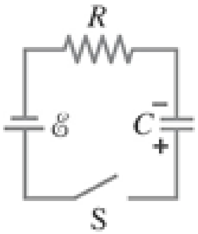
We keep an extra significant figure to show the slight difference in the currents.

$$\boxed{I_{22\Omega} = 0.176\text{A} \quad I_{25\Omega} = 0.154\,\text{A} \quad I_{12\Omega} = 0.177\,\text{A} \quad I_{14\Omega} = 0.153\,\text{A} \quad I_{15\Omega} = 0.001\,\text{A, upwards}}$$

B. In the figure below, the total resistance is 15.0 kΩ, and the battery's emf is 24.0 V. If the time constant is measured to be 24.0 μs, calculate

(i) the total capacitance of the circuit and

(ii) the time it takes for the voltage across the resistor to reach 16.0 V after the switch is closed.



Answer:

(*i*) From the equation below, the product *RC* is equal to the time constant.

$$\tau = RC \quad \rightarrow \quad C = \frac{\tau}{R} = \frac{24.0\times10^{-6}\,\mathrm{s}}{15.0\times10^{3}\,\Omega} = \boxed{1.60\times10^{-9}\,\mathrm{F}}$$

(*ii*) Since the battery has an EMF of 24.0 V, if the voltage across the resistor is 16.0 V, the voltage across the capacitor will be 8.0 V as it charges. Use the expression for the voltage across a charging capacitor.

$$V_C = \mathrm{e}\left(1-e^{-t/\tau}\right) \quad \rightarrow \quad e^{-t/\tau} = \left(1-\frac{V_C}{\mathrm{e}}\right) \quad \rightarrow \quad -\frac{t}{\tau} = \ln\left(1-\frac{V_C}{\mathrm{e}}\right) \quad \rightarrow$$

$$t = -\tau\ln\left(1-\frac{V_C}{\mathrm{e}}\right) = -\left(24.0\times10^{-6}\,\mathrm{s}\right)\ln\left(1-\frac{8.0\,\mathrm{V}}{24.0\,\mathrm{V}}\right) = \boxed{9.73\times10^{-6}\,\mathrm{s}}$$

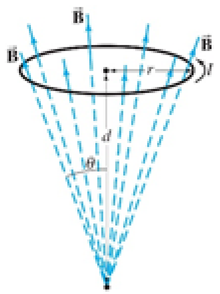
C. An electron is projected vertically upward with a speed of 1.70 × 10^6 m/s into a uniform magnetic field of 0.480 T that is directed horizontally away from the observer. What is the radius of motion? The mass of an electron is 9.11 × 10^{-31}kg, and the charge on an electron is 1.66 × 10^{-19} C.

Answer:

The magnetic force will cause centripetal motion, and the electron will move in a clockwise circular path if viewed in the direction of the magnetic field. The radius of the motion can be determined.

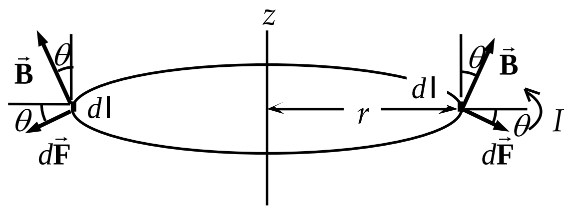
$$F_{\max} = qvB = m\frac{v^2}{r} \quad \rightarrow \quad r = \frac{mv}{qB} = \frac{\left(9.11\times10^{-31}\,\mathrm{kg}\right)\left(1.70\times10^{6}\,\mathrm{m/s}\right)}{\left(1.60\times10^{-19}\,\mathrm{C}\right)\left(0.480\,\mathrm{T}\right)} = \boxed{2.02\times10^{-5}\,\mathrm{m}}$$

D. A circular loop of wire, of radius *r,* carries current *I*. It is placed in a magnetic field whose straight lines seem to diverge from a point a distance *d* below the loop on its axis. In other words, the field makes an angle θ with the loop at all points, as shown in the figure below, where $\tan\theta = r/d$. Determine the force on the loop.



Answer:

The net force on the current loop is the sum of the infinitesimal forces obtained from each current element. From the figure, we see that at each current segment, the magnetic field is perpendicular to the current. This results in a force with only radial and vertical components. By symmetry, we find that the radial force components from segments on opposite sides of the loop cancel. The net force then is purely vertical. Symmetry also shows us that each current element contributes the same magnitude of force.

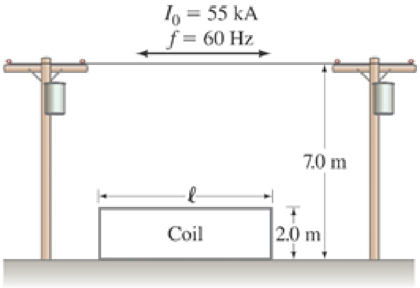


$$\vec{\mathbf{F}} = \int I d\vec{\mathrm{L}} \times B = -IB_r \hat{\mathbf{k}} \int d\mathrm{l} = -I\left(B\sin\theta\right)\hat{\mathbf{k}}\left(2\pi r\right) = \boxed{-2\pi IB\frac{r^2}{\sqrt{r^2+d^2}}\hat{\mathbf{k}}}$$

Q.6 (25 Marks)

Answer each of the following questions.

A. A power line carrying a sinusoidally varying current with frequency f = 60 Hz and peak value I_0 = 55 kA runs at a height of 7.0 m across a farmer's land (as shown in the figure below). The farmer constructs a vertically oriented 2.0 m high 10 turn rectangular wire coil below the power line. The farmer hopes to use the induced voltage in this coil to power 120 volt electrical equipment, which requires a sinusoidally varying voltage with frequency f = 60 Hz andpeak value V_0 = 170 V. What should the length l of the coil be?



Answer:

The sinusoidal varying current in the power line creates a sinusoidal varying magnetic field encircling the power line, given by Eq. 28-1. Using Eq. 29-1b we integrate this field over the area of the rectangle to determine the flux through it. Differentiating the flux as in Eq. 29-2b gives the emf around the rectangle. Finally, by setting the maximum emf equal to 170 V we can solve for the necessary length of the rectangle.

$$B(t) = \frac{\mu_0 I_0}{2\pi r}\cos(2\pi f t) \ ;$$

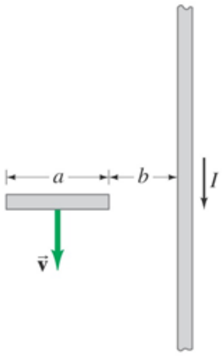
$$\Phi_B(t) = \int B dA = \int_{5.0\text{ m}}^{7.0\text{ m}} \frac{\mu_0 I_0}{2\pi r}\cos(2\pi f t) l\, dr = \frac{\mu_0 I_0}{2\pi} l \cos(2\pi f t)\int_{5.0\text{ m}}^{7.0\text{ m}} \frac{dr}{r} = \frac{\mu_0 I_0}{2\pi}\ln(1.4) l \cos(2\pi f t)$$

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -\frac{N\mu_0 I_0}{2\pi}\ln(1.4) l\left[\frac{d}{dt}\cos(2\pi f t)\right] = N\mu_0 I_0 f \ln(1.4) l \sin(2\pi f t) \ ;$$

$$\mathcal{E}_0 = N\mu_0 I_0 f \ln(1.4) l \ \rightarrow$$

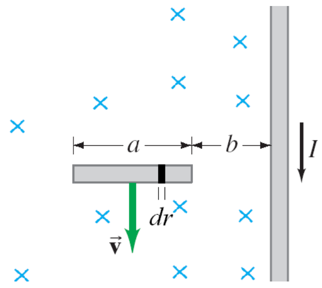
$$l = \frac{\mathcal{E}_0}{N\mu_0 I_0 f \ln(1.4)} = \frac{170\text{ V}}{10(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(55{,}000\text{ A})(60\text{ Hz})\ln(1.4)} = \boxed{12\text{ m}}$$

B. A short section of wire, of length *a,* is moving with velocity **v,** parallel to a very long wire carrying a current *I* as shown in the figure below. The near end of the wire section is a distance *b* from the long wire. Assuming the vertical wire is very long compared to *a* + *b,* determine the emf between the ends of the short section. Do this for when **v** is
(i) in the same direction as *I,* and
(ii) in the opposite direction to *I*.



Answer:

(*i*) The magnetic field is perpendicular to the rod, with the magnetic field decreasing with distance from the rod, as in Eq. 28-1. The emf, $d\varepsilon$, across a short segment, *dr*, of the rod is given by the differential version of Eq. 29-3. Integrating this emf across the length of the wire gives the total emf.



$$d\varepsilon = Bvdr \rightarrow$$

$$\varepsilon = \int d\varepsilon = \int_b^{b+a} \frac{\mu_0 I}{2\pi r} vdr = \boxed{\frac{\mu_0 I v}{2\pi} \ln\left(\frac{b+a}{b}\right)}$$

This emf points $\boxed{\text{toward the wire}}$, as positive charges are attracted toward the current.

(*ii*) The only change is the direction of the current, so the magnitude of the emf remains the same, but points $\boxed{\text{away from the wire}}$, since positive charges are repelled from the current.

C. A 250 loop circular armature coil with a diameter of 10.0 cm rotates at 120 rev/s in a uniform magnetic field of strength 0.45 T. What is the rms voltage output of the generator? What would you do to the rotation frequency in order to double the rms voltage output?

Answer:

Rms voltage is found from the peak induced emf. Peak induced emf is calculated from below

$$\varepsilon_{\text{peak}} = NB\omega A \rightarrow$$

$$V_{\text{rms}} = \frac{\varepsilon_{\text{peak}}}{\sqrt{2}} = \frac{NB\omega A}{\sqrt{2}} = \frac{(250)(0.45\,\text{T})(2\pi\,\text{rad/rev})(120\,\text{rev/s})\pi(0.050\,\text{m})^2}{\sqrt{2}}$$

$$= 471.1\,\text{V} \approx \boxed{470\,\text{V}}$$

To double the output voltage, you must $\boxed{\text{double the rotation frequency}}$ to 240 rev/s

D. A motor has an armature resistance of 3.05 Ω. If it draws 7.20 A when running at full speed and connected to a 120-V line, how large is the back emf?

Answer:
When the motor is running at full speed, the back emf opposes the applied emf, to give the net voltage across the motor.

$$\mathcal{E}_{\text{applied}} - \mathcal{E}_{\text{back}} = IR \quad \rightarrow \quad \mathcal{E}_{\text{back}} = \mathcal{E}_{\text{applied}} - IR = 120\,\text{V} - (7.20\,\text{A})(3.05\,\Omega) = \boxed{98\,\text{V}}$$

E. A 75 W lightbulb is designed to operate with an applied ac voltage of 120 V rms. The bulb is placed in series with an inductor *L,* and this series combination is then connected to a 60 Hz 240 V rms voltage source. For the bulb to operate properly, determine the required value for *L*. Assume the bulb has resistance *R* and negligible inductance.

Answer:

The light bulb acts like a resistor in series with the inductor. Using the desired rms voltage across the resistor and the power dissipated by the light bulb we calculate the rms current in the circuit and the resistance. Then using this current and the rms voltage of the circuit we calculate the impedance of the circuit (Eq. 30-27) and the required inductance (Eq. 30-28b).

$$I_{\text{rms}} = \frac{P}{V_{R,\text{rms}}} = \frac{75\,\text{W}}{120\,\text{V}} = 0.625\,\text{A} \qquad R = \frac{V_{R,\text{rms}}}{I_{\text{rms}}} = \frac{120\,\text{V}}{0.625\,\text{A}} = 192\,\Omega$$

$$Z = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \sqrt{R^2 + (2\pi fL)^2}$$

$$L = \frac{1}{2\pi f}\sqrt{\left(\frac{V_{\text{rms}}}{I_{\text{rms}}}\right)^2 - R^2} = \frac{1}{2\pi(60\text{ Hz})}\sqrt{\left(\frac{240\text{ V}}{0.625\text{ A}}\right)^2 - (192\ \Omega)^2} = \boxed{0.88\text{ H}}$$

END