The University of Nottingham
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Science A Physics

Lecture 14:

Electric Current, and Resistance; Part 2

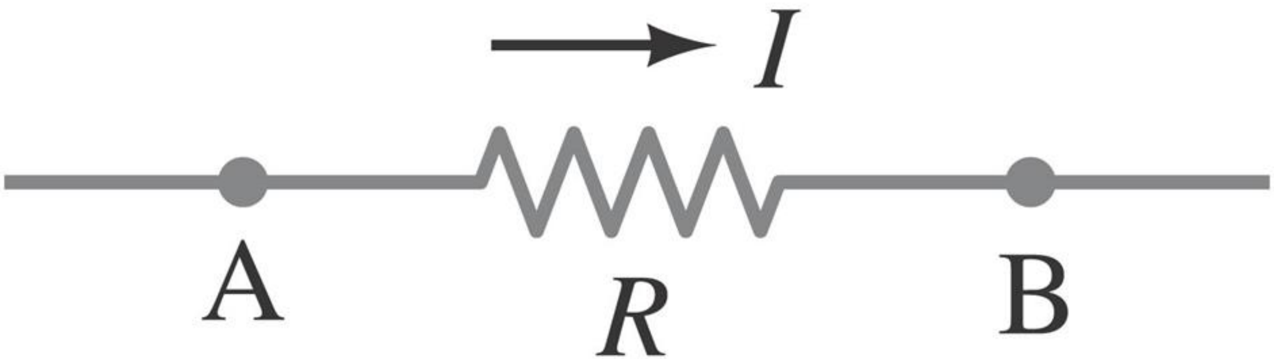
Capacitance

Aims of today's lecture

1. Resistivity
2. Electric Power
3. Alternating Current
4. Capacitors
5. Determination of Capacitance
6. Capacitors in Series and Parallel
7. Electric Energy Storage

1. Resistivity

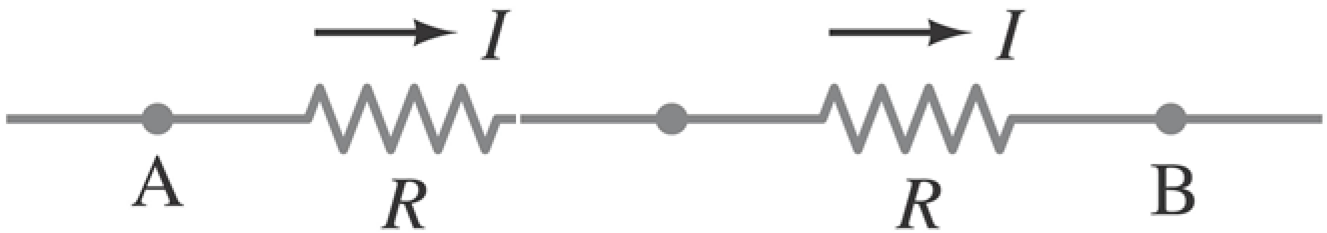
What does Resistance Depend on?



- So far, we've said that the ratio of the voltage drop (V) between two points, to the current (I) that flows between those two points, for a wire (or resistor), is a measure of the resistance (R), for that particular wire, which is made from a particular material.

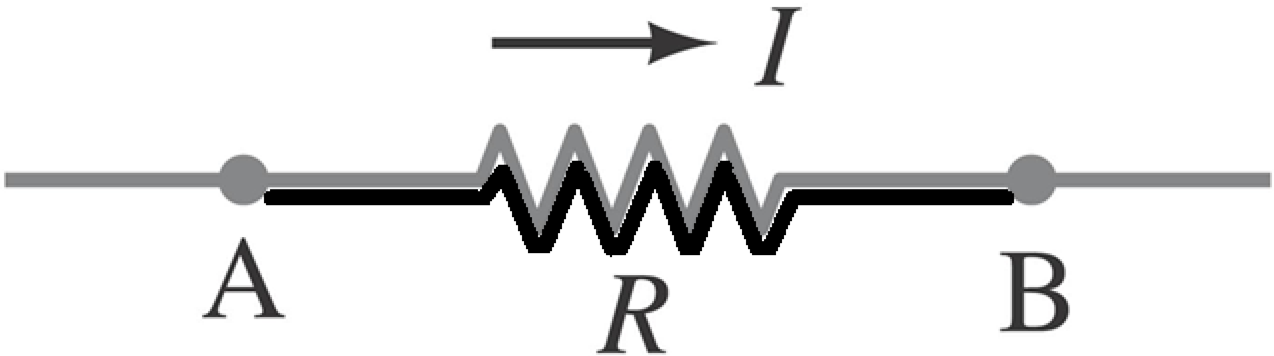
- A natural question to ask, however, is 'does the resistance of a wire depend on anything else'?

What does Resistance Depend on?



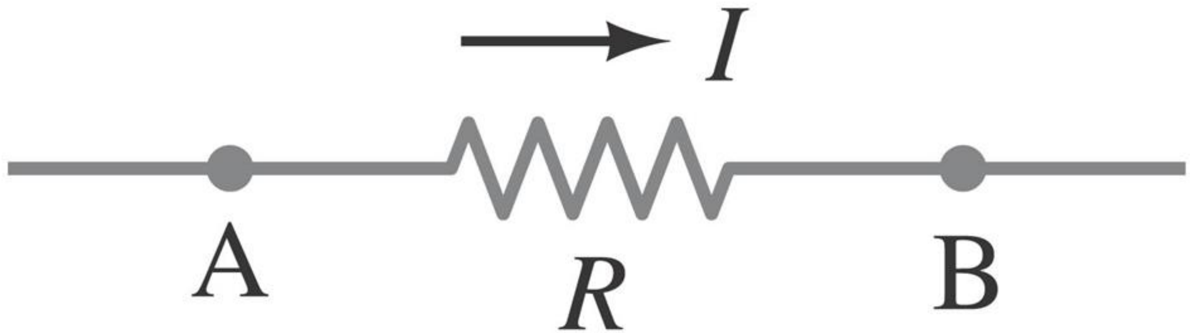
- Well, if we double the length of the wire/resistor, the voltage drop (V) between points A and B will double, but the current (I) will stay the same; therefore, the resistance (R) will double.
- Thus, $R \propto l$,
  where l is the length of the wire/resistor.

What does Resistance Depend on?



- If, on the other hand, we only double the cross-sectional area (A) of the wire, then the voltage (V) will stay the same, but the current (I) will double; therefore, the resistance (R) will be halved.
- Thus, $R \propto \frac{1}{A}$,
  where A is the cross-sectional area of the wire/resistor.

What does Resistance Depend on?



- In summary, it is found experimentally that the resistance (R) of any wire is directly proportional to its length (l) and inversely proportional to its cross-sectional area A. That is,

$$R = \rho \frac{l}{A}$$

where ρ, the constant of proportionality, is called the **resistivity** and depends on the material used. Its unit is the $\Omega \cdot \mathrm{m}$.

Resistivity

TABLE 25–1 Resistivity and Temperature Coefficients (at 20°C)

Material	Resistivity, ρ ($\Omega \cdot$ m)	Temperature Coefficient, α $(\mathrm{C}^\circ)^{-1}$
Conductors		
Silver	1.59×10^{-8}	0.0061
Copper	1.68×10^{-8}	0.0068
Gold	2.44×10^{-8}	0.0034
Aluminum	2.65×10^{-8}	0.00429
Tungsten	5.60×10^{-8}	0.0045
Iron	9.71×10^{-8}	0.00651
Platinum	10.60×10^{-8}	0.003927
Mercury	98.00×10^{-8}	0.0009
Nichrome (Ni, Fe, Cr alloy)	100.00×10^{-8}	0.0004
Semiconductors†		
Carbon (graphite)	$(3-60) \times 10^{-5}$	−0.0005
Germanium	$(1-500) \times 10^{-3}$	−0.05
Silicon	0.1 – 60	−0.07
Insulators		
Glass	$10^{9} - 10^{12}$	
Hard rubber	$10^{13} - 10^{15}$	

† Values depend strongly on the presence of even slight amounts of impurities.

- A low resistivity value means that a material is a good conductor. We can see that silver is the best conductor, but yet, we use copper for most wiring; we do so because it is cheaper.

Resistivity

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- Aluminum, although it has a higher resistivity than copper, is much less dense than it. It is thus preferable to copper in some situations, such as for transmission lines, because its resistance for the same weight is less than that for copper.

Temperature Dependence of Resistivity

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- The resistivity of a material depends somewhat on temperature.
- The resistance of metals generally increases with temperature. If the temperature change is not too great, the resistivity of metals usually increases nearly linearly with temperature. That is,

$$\rho_T = \rho_0[1 + \alpha(T - T_0)]$$

Temperature Dependence of Resistivity

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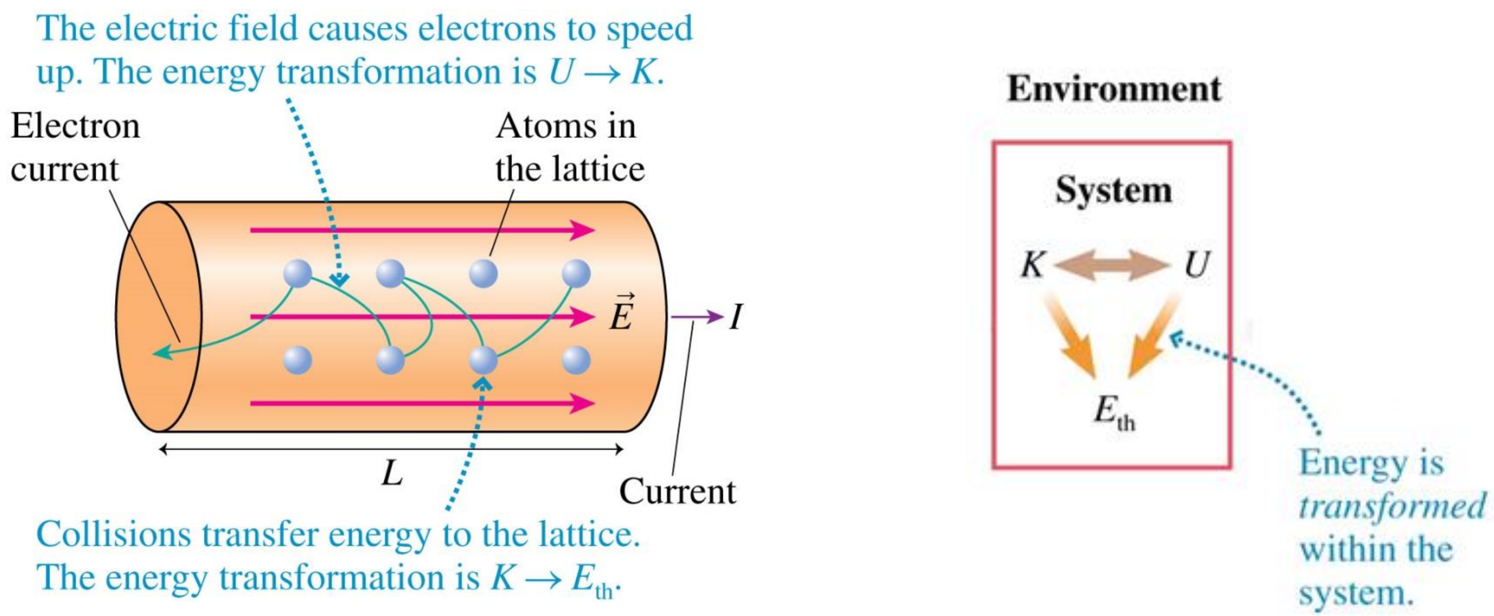
$$\rho_T = \rho_0[1 + \alpha(T - T_0)]$$

- Where ρ_0 is the resistivity at some reference temperature T_0 (such as 0°C or 20°C), ρ_T is the resistivity at a temperature T, and α is the **temperature coefficient of resistivity**.

2. Electric Power

Electric Power



- Electrical energy is useful to us because it can be easily transformed into other forms of energy.
- In devices such as electric heaters, stoves, toasters and hair dryers, electric energy is transformed into thermal energy in a wire resistance known as a 'heating element'.

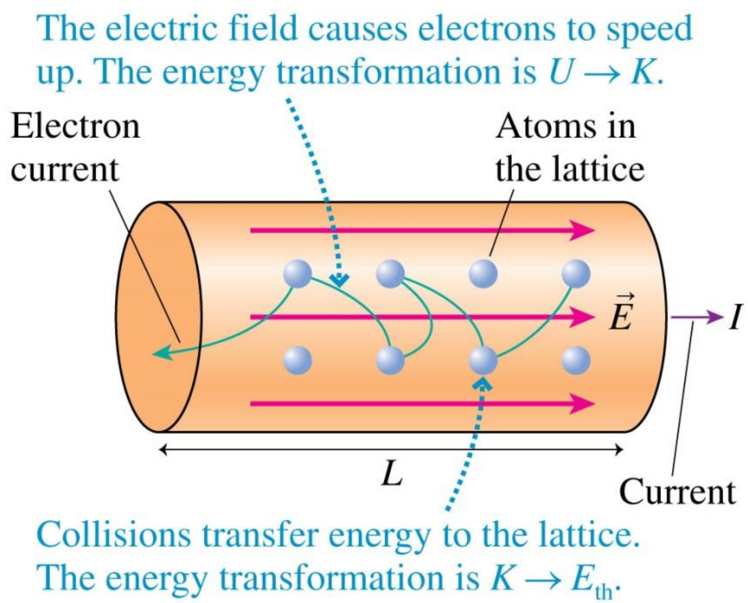
Electric Power

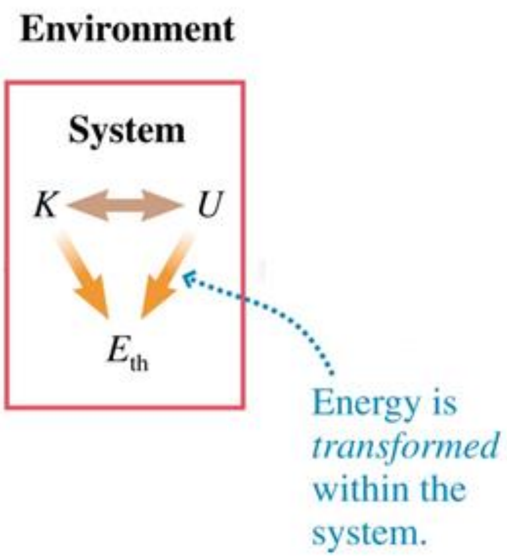
- In an ordinary lightbulb, the tiny wire filament becomes so hot it glows; only a few percent of the energy is transformed into visible light, and the rest, over 90%, into thermal energy.
- Electric energy is transformed into thermal energy or light in such devices due to many collisions between the moving electrons and the atoms of the wire.

Electric Power

- In each collision, part of the electron's kinetic energy is transferred to the atom with which it collides.
- As a result of this, the kinetic energy of the wire's atoms increases and hence the temperature of the wire element increases.
- The increased thermal energy can be transferred as heat by conduction and convection to the air in a heater or to food in a pan, by radiation to bread in a toaster, or radiated as light.

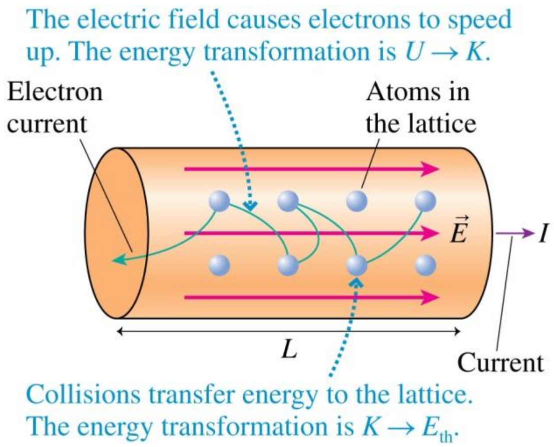
Electric Power

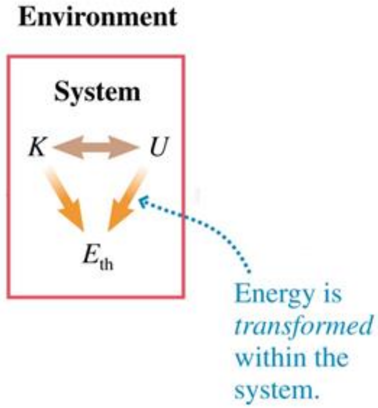




- To find the power transformed by an electric device, we recall that the energy transformed when an infinitesimal charge dq moves through a potential difference V is $dU = Vdq$.
- Let dt be the time required for an amount of charge dq to move through a potential difference V.

Electric Power





- Then the power P, which is the rate energy is transformed, is

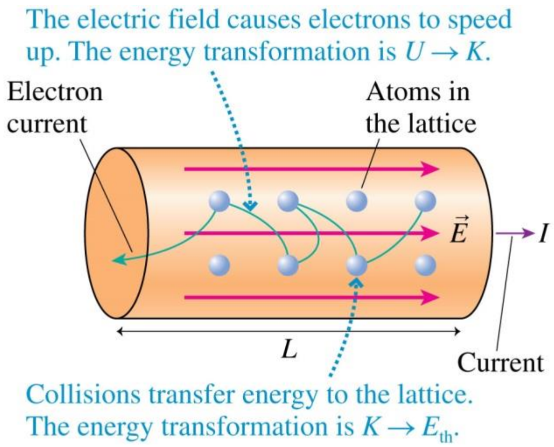
$$P = \frac{dU}{dt} = \frac{dq}{dt} V$$

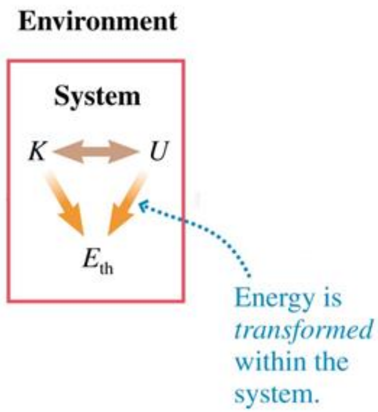
- The charge that flows per second, dq/dt, is the electric current I.

- Thus, we have $P = IV$

- The SI unit of electric power is the same as for any kind of power, the watt ($1\text{W} = 1\ \text{J/s}$)

Electric Power





- The rate of energy transformation in a resistance R can be written in two other ways, starting with the general relation $P = IV$ and substituting in $V = IR$:

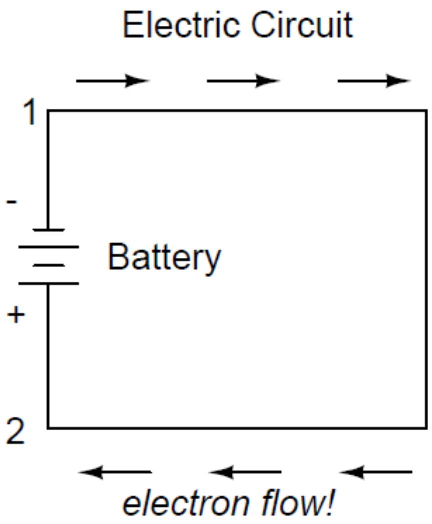
$$P = IV = I(IR) = I^2R$$

$$or$$

$$P = IV = \frac{V}{R} = \frac{V^2}{R}$$

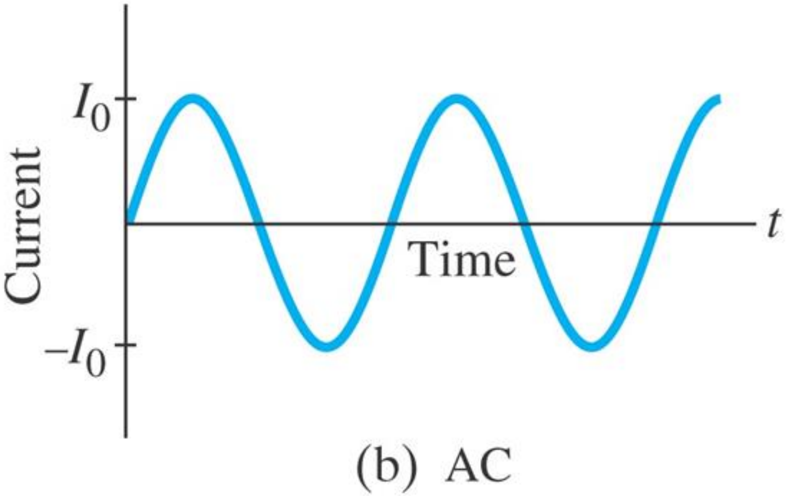
3. Alternating Current

Direct Current



- If we were to monitor a **cross-section of the wire** in a single circuit, counting the electrons flowing by, we would **notice the exact same quantity per unit of time** as in any other part of the circuit, regardless of conductor length or conductor diameter.
- Current moving steadily in one direction is called **direct current**, or **dc**.

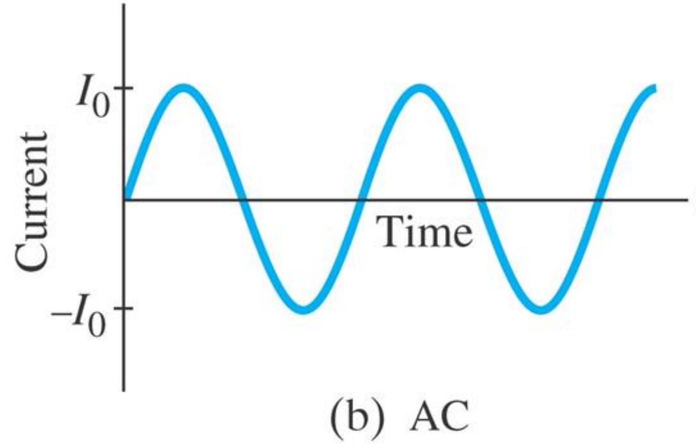
Alternating Current



(b) AC

- Electric generators (which we will study in later lectures) at electric power plants, however, produce **alternating current**, or **ac**.
- An alternating current reverses direction many times per second; the movement of the current is **approximately simple harmonic** motion.
- Thus, we can represent this movement with the above graph.

Alternating Current

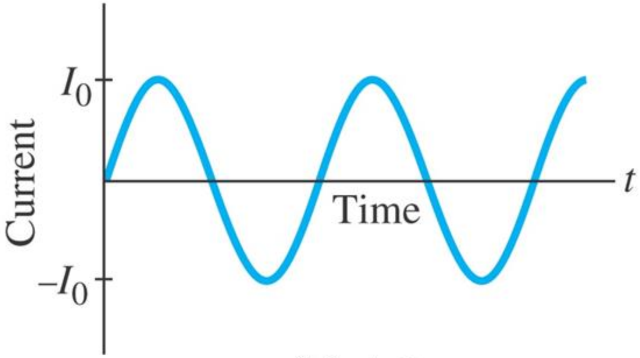


- The voltage produced by an ac electric generator is sinusoidal, as we will see in later lectures. The current this voltage produces is thus sinusoidal. We can write the voltage as a function of time as

$$V = V_0 \sin 2\pi f t = V_0 sin\omega t$$

- The potential V oscillates between $+V_0$ and $-V_0$, and V_0 is referred to as the **peak voltage**.
- The frequency f is the number of complete oscillations made per second, and $\omega = 2\pi f$.

Alternating Current



(b) AC

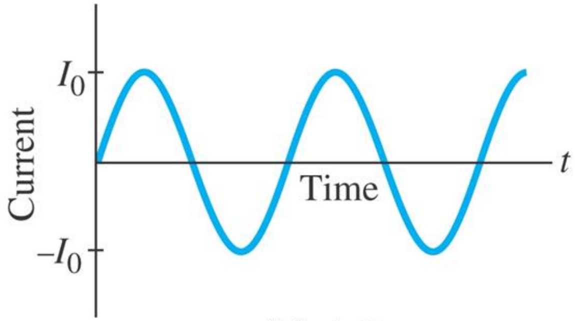
$$V = V_0 \sin 2\pi f t = V_0 sin\omega t$$

- In most countries, $f\ is\ 50\text{Hz}$.
- We can use $V = IR$ for ac. If a voltage exists across a resistance R, then the current I through the resistance is

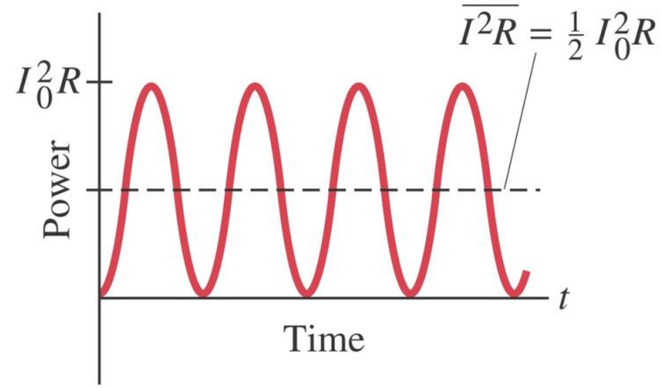
$$I = \frac{V}{R} = \frac{V_0}{R} sin\omega t = I_0 sin\omega t$$

- The quantity $I_0 = V_0/R$ is the **peak current**.
- The current is considered positive when the electrons flow in one direction and negative when they flow in the opposite direction.

Alternating Current



(b) AC



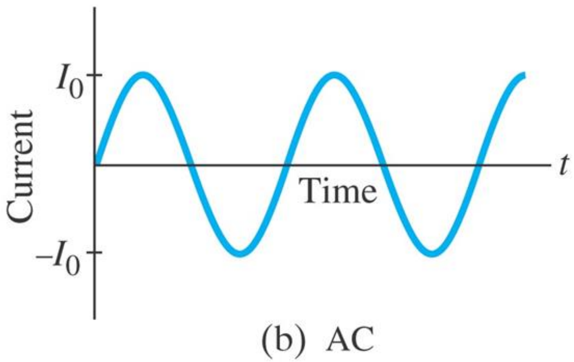
- It is clear from the above figure, that an alternating current is as often positive as it is negative. Thus, the average current is zero.
- This does not mean, however, that no power is produced, or that no heat is produced in a resistor.
- Electrons do move back and forth, and do produce heat. Thus, the power transformed in a resistance R at any instant is

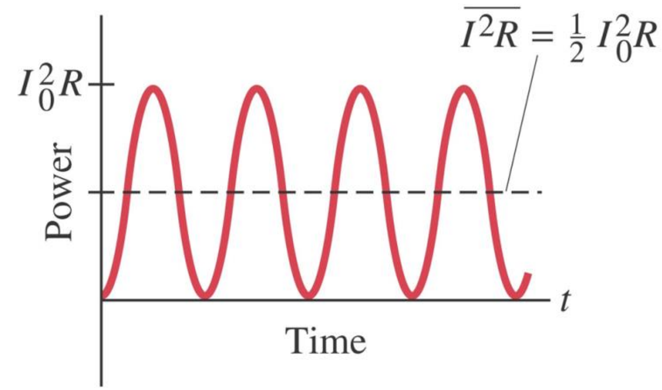
$$P = I^2R = I^2{}_0 R sin^2 \omega t$$

- Because the current is squared, we see that the power is always positive. The average power transformed, $\bar{P}$, is

$$\bar{P} = \frac{1}{2} I^2{}_0 \mathrm{R}$$

Alternating Current





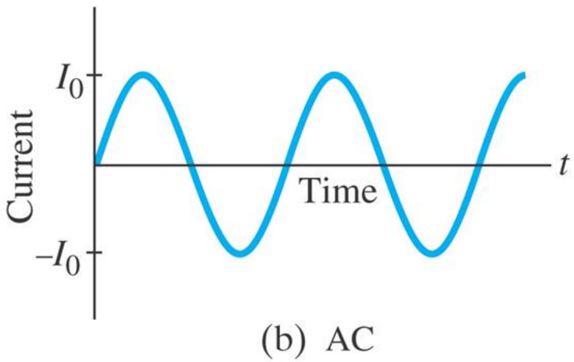
- Since power can also be written $P = \frac{V^2}{R} = \frac{V^2{}_0}{R} sin^2\omega t$, we also have that the average power is

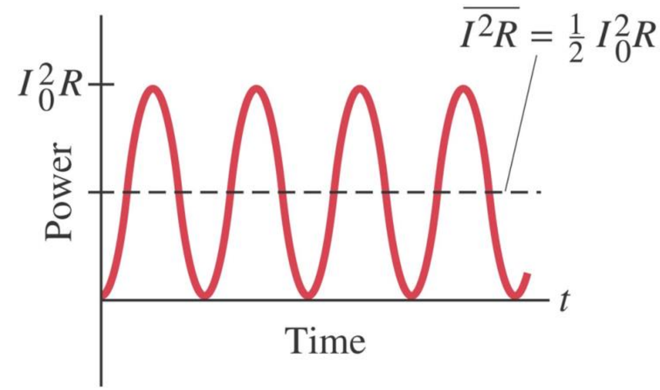
$$\bar{P} = \frac{1}{2}\frac{V^2{}_0}{R}$$

- The **average or mean value of the square** of the current or voltage is thus what is important for calculating average power: $\overline{I^2} = \frac{1}{2}{I_0}^2$ and $\overline{V^2} = \frac{1}{2}{V_0}^2$.
- The square root of each of these is the **rms** (root-mean-square) value of the current or voltage:

$$I_{rms} = \sqrt{\overline{I^2}} = \frac{I_0}{\sqrt{2}} = 0.707 I_0$$

Alternating Current





$$\overline{I^2} = \frac{1}{2} {I_0}^2 \text{ and } \overline{V^2} = \frac{1}{2} {V_0}^2.$$

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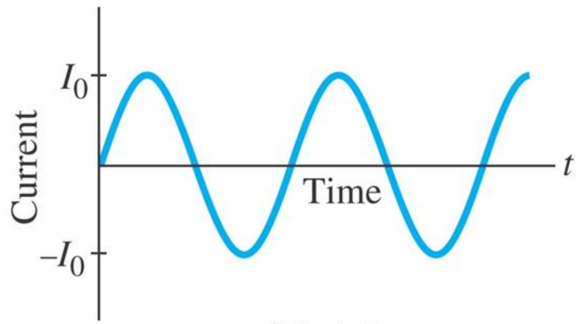
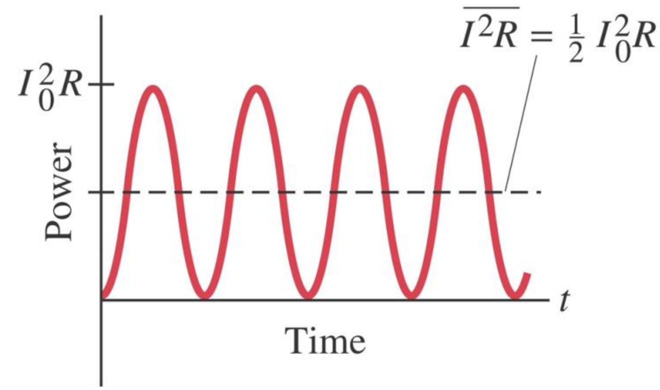
Alternating Current



(b) AC



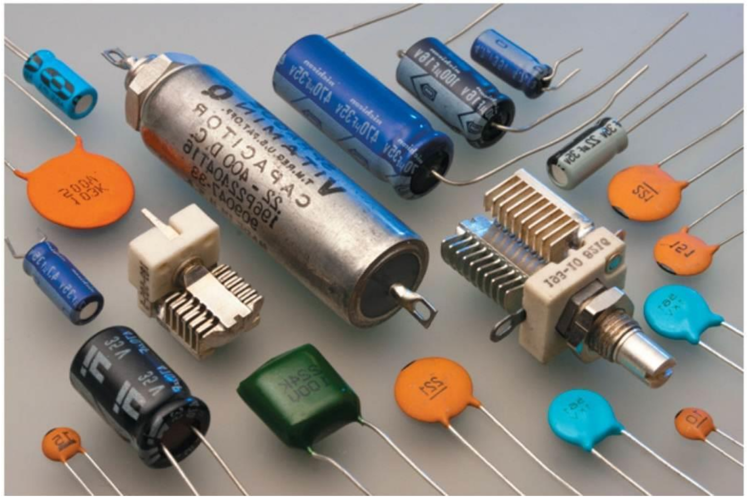
$$V_{rms} = \sqrt{\overline{V^2}} = \frac{V_0}{\sqrt{2}} = 0.707V_0$$

$$I_{rms} = \sqrt{\overline{I^2}} = \frac{I_0}{\sqrt{2}} = 0.707I_0$$

- The above formulae also imply that a direct current whose values of I and V equal the rms values of I and V for an alternating current will produce the same power.

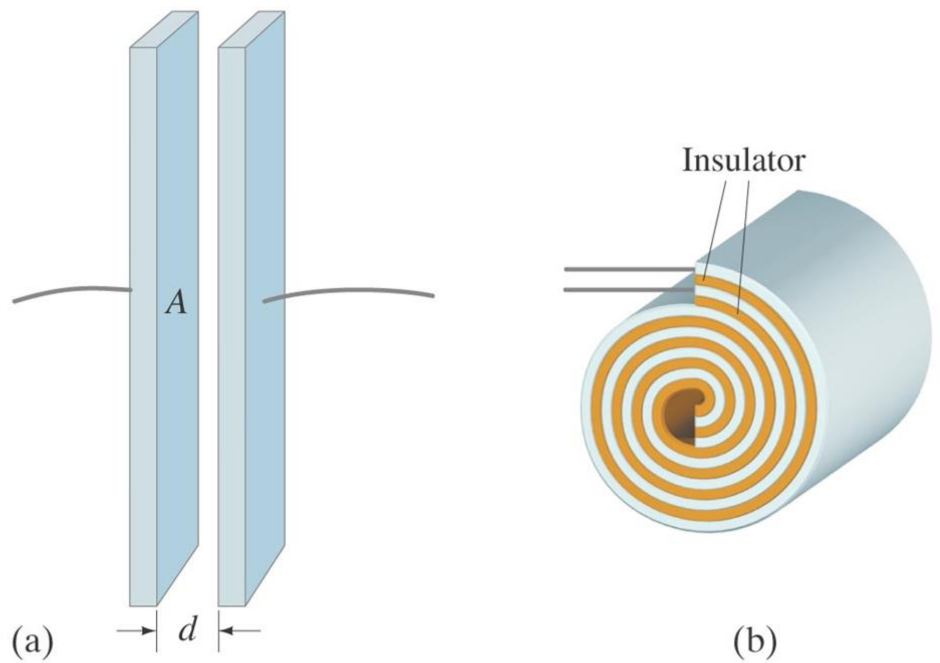
4. Capacitors

Capacitors



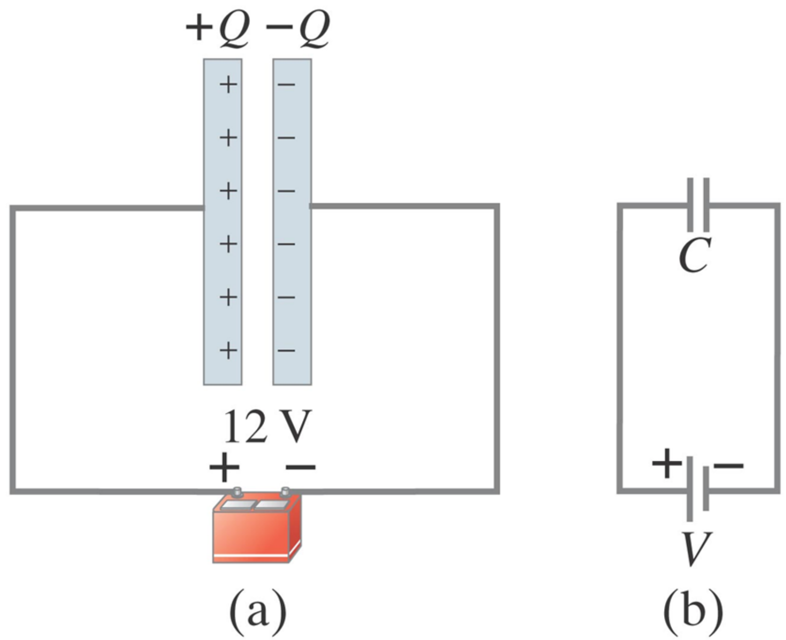
- Capacitors are important elements in electric circuits. They come in a variety of sizes and shapes.
- They allow us to temporarily store charge in an electric circuit as well as control the current in different parts of a circuit.
- They have the advantage of not producing heat, or dissipating energy, unlike resistors. They are widely used in electronic circuits for blocking direct current while allowing alternating current to pass.
- They can be also be used to tune radios to particular frequencies.
- Let’s now see how capacitors work.

Capacitors



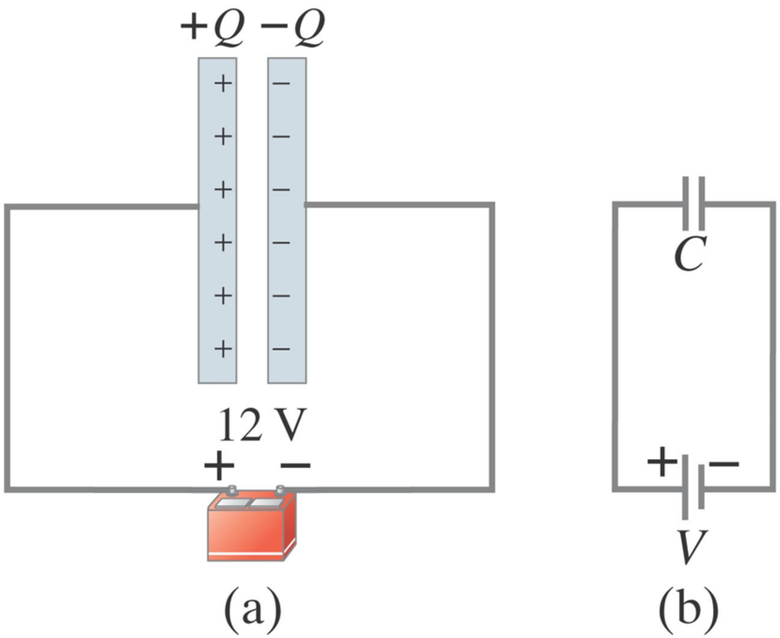
- A simple capacitor consists of a pair of parallel plates/electrodes of area A separated by a small distance d, as shown above in figure (a). Often the two plates are rolled into the form of a cylinder with plastic, paper, or other insulator separating the plates, as shown in figure (b).

Capacitors



- If a voltage is applied across a capacitor by connecting the capacitor to a battery (a source of voltage) with conducting wires as shown above, the two plates quickly become charged: one plate acquires a negative charge, and the other an equal amount of positive charge.
- Each battery terminal and the plate of the capacitor connected to it are at the same potential; hence the full battery voltage appears across the capacitor.

Capacitors

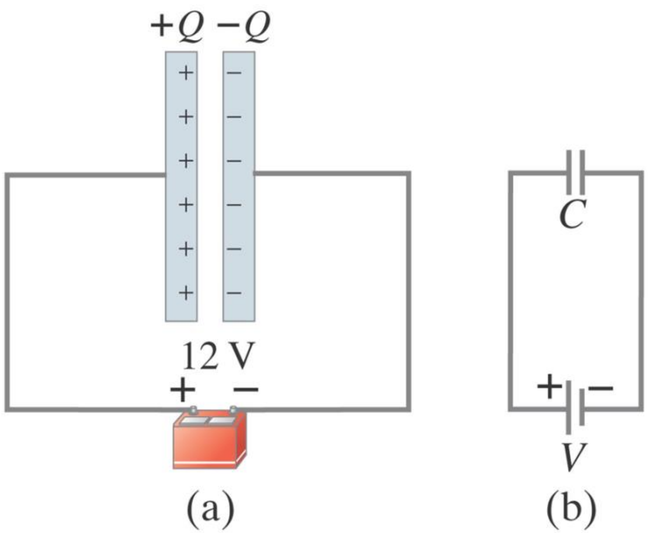


- For a given capacitor, it is found that the amount of charge Q acquired by each plate is proportional to the magnitude of the potential difference V between them:

$$Q = CV$$

- The constant of proportionality, C, in the above relation is called the capacitance of the capacitor.
- The unit of the capacitance is coulombs per volt and this unit is called a **farad** (F)

Capacitors

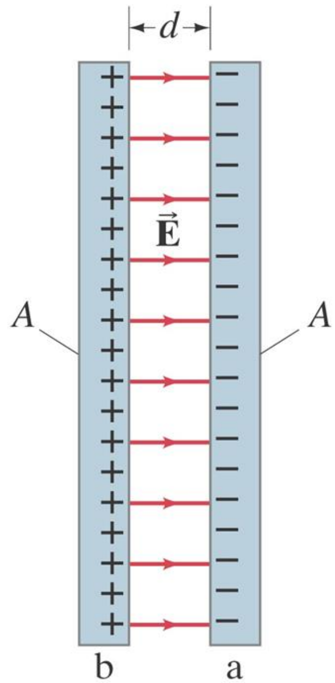


$$Q = CV$$

- Common capacitors have capacitance in the range of $1p$F ($1 \text{ picofarad} = 10^{-12}$ F) to $10^3\ \mu$F ($1 \text{ microfarad} = 10^{-6}$F).
- The capacitance C does not depend on Q or V. Its value depends only on the size, shape and relative position of the two conductors, and also on the material that separates them.

5. Determination of Capacitance

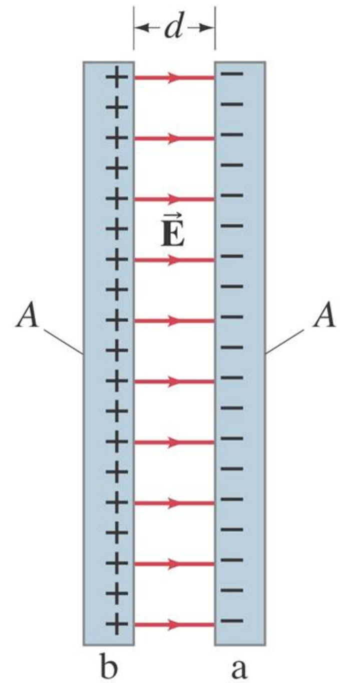
Determination of Capacitance



$$Q = CV$$

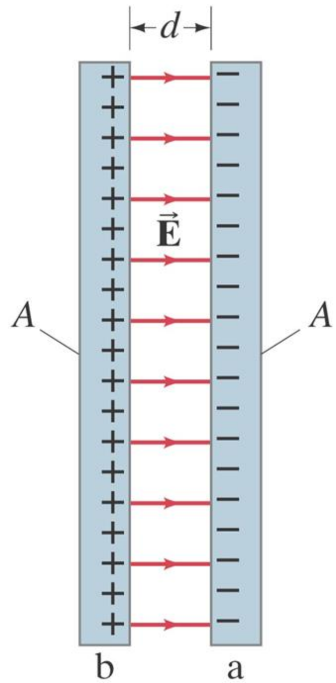
- The capacitance of a given capacitor can be determined **experimentally** directly from the above equation, by measuring the charge Q on either conductor for a given potential difference V.
- For capacitors whose geometry is simple, we can determine C **analytically**, and in this section, we assume the conductors are separated by a vacuum or air.

Determination of Capacitance



- For the capacitor above, each plate has area A, and the two plates are separated by a distance d.
- We assume d is small compared to the dimensions of each plate so that the electric field $\vec{E}$ is uniform between them and we can ignore fringing (lines of $\vec{E}$ not straight) at the edges.

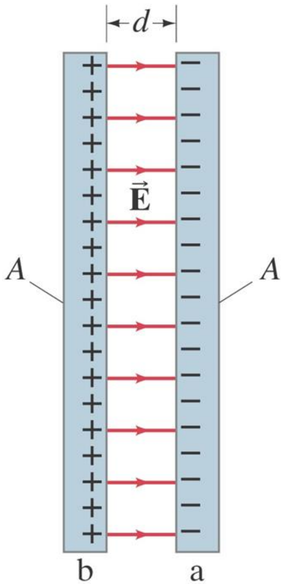
Determination of Capacitance



- As we seen in previous lectures, the electric field between two closely spaced parallel plates has magnitude $E = \sigma/\epsilon_0$ and its direction is perpendicular to the plates.
- Since σ is the charge per unit area, $\sigma = \frac{Q}{A}$, then the field between the plates is

$$E = \frac{Q}{\epsilon_0 A}$$

Determination of Capacitance



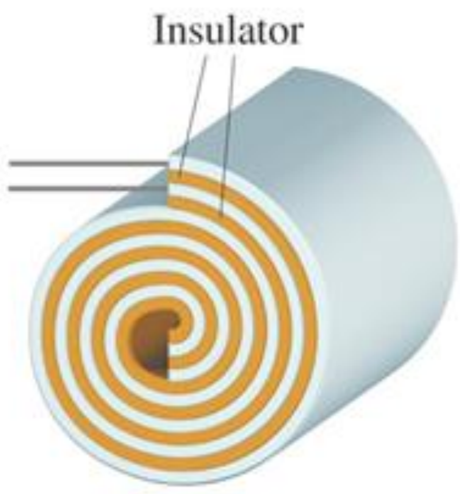
- The relation between electric field and electric potential is

$$V = V_{ba} = V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l} = \frac{Qd}{\epsilon_0 A}$$

- This relates Q to V, and from it we can get the capacitance C in terms of the geometry of the plates:

$$C = \frac{Q}{V} = \epsilon_0 \frac{A}{d}$$

Determination of Capacitance

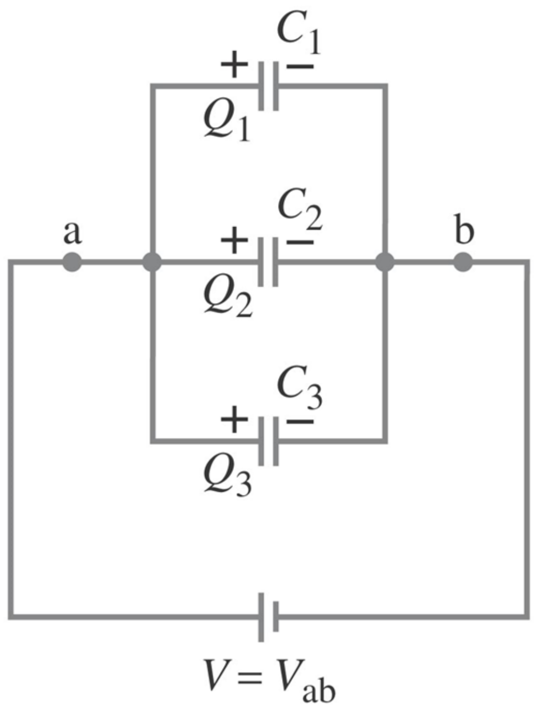


$$C = \frac{Q}{V} = \epsilon_0 \frac{A}{d}$$

- The proportionality, $C \propto A/d$, is valid also for a parallel-plate capacitor that is rolled up into a spiral cylinder, as shown above.
- However, the constant factor, ϵ_0, must be replaced if an insulator such as paper separates the plates, as is usual; if you want to read more about this, you can do so in pages 738-741 of your textbook.

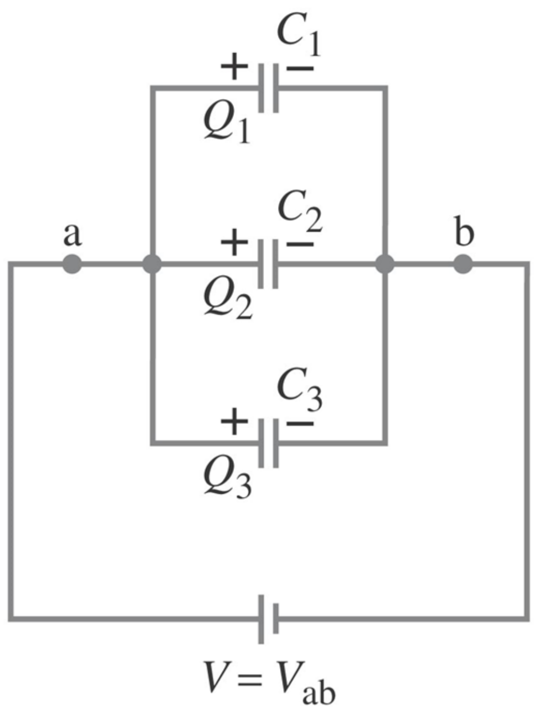
6. Capacitors in Series and Parallel

Capacitors in Series and Parallel



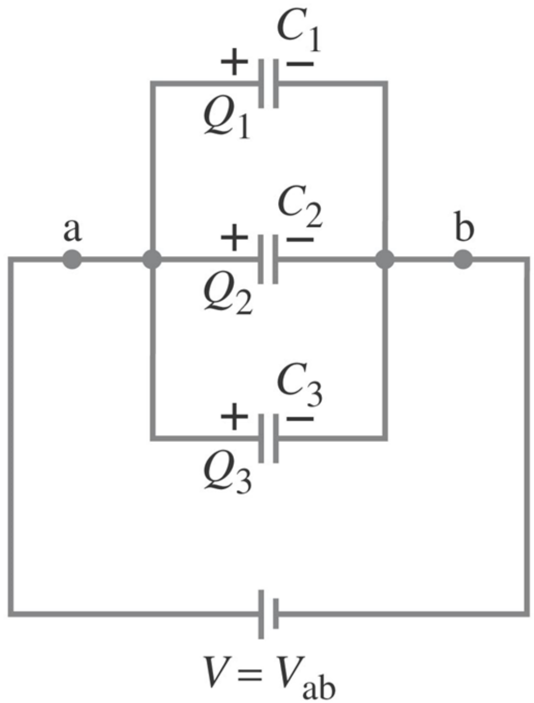
- Capacitors can be connected together in various ways. Two common ways are in **series**, or in **parallel**; let's first look at capacitors in parallel.
- The battery voltage is usually given the symbol V, which means that V represents a potential difference.

Capacitors in Parallel



- A circuit containing three capacitors connected in parallel is shown in the figure above. They are in ‘parallel’ because when a battery of voltage V is connected to points a and b, this voltage $V = V_{ab}$ exists across each of the capacitors.

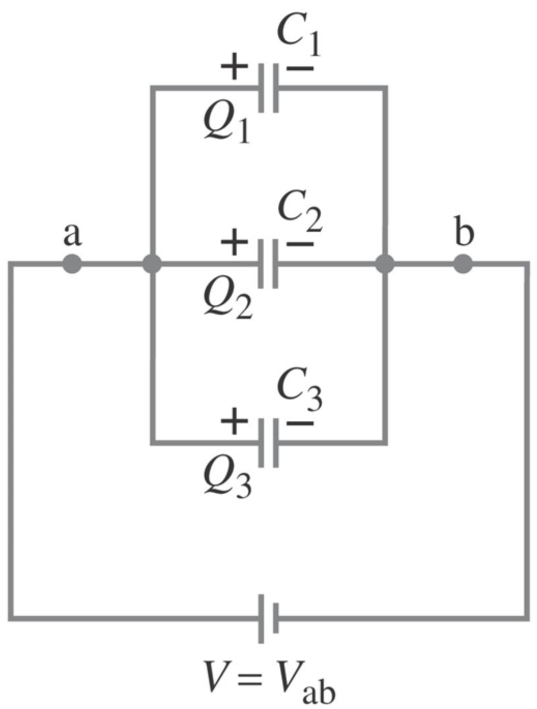
Capacitors in Parallel



- Since the left-hand plates of all the capacitors are connected by conductors, they all reach the same potential V_a when connected to the battery; and the right-hand plates each reach potential V_b.
- Each capacitor plate acquires a charge given by

$$Q_1 = C_1 V, \qquad Q_2 = C_2 V, \qquad and \qquad Q_3 = C_3 V$$

Capacitors in Parallel



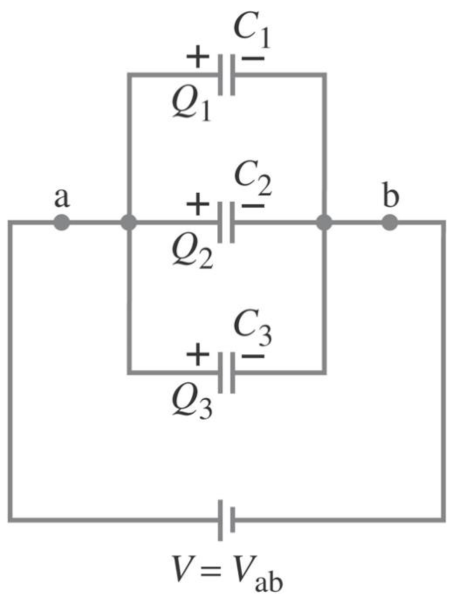
- Each capacitor plate acquires a charge given by

$$Q_1 = C_1V, \qquad Q_2 = C_2V, \qquad and \qquad Q_3 = C_3V$$

- The total charge Q that must leave the battery is then

$$Q = Q_1 + Q_2 + Q_3 = C_1V + C_2V + C_3V$$

Capacitors in Parallel



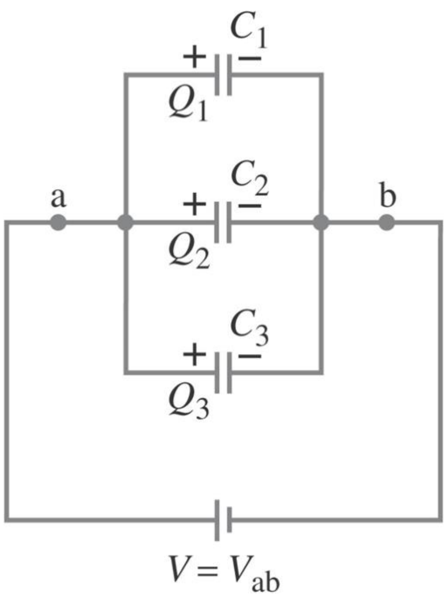
- The total charge Q that must leave the battery is then

$$Q = Q_1 + Q_2 + Q_3 = C_1V + C_2V + C_3V$$

- Let us try to find a single equivalent capacitor that will hold the same charge Q at the same voltage $V = V_{ab}$. It will then have a capacitance C_{eq} given by

$$Q = C_{eq}V$$

Capacitors in Parallel



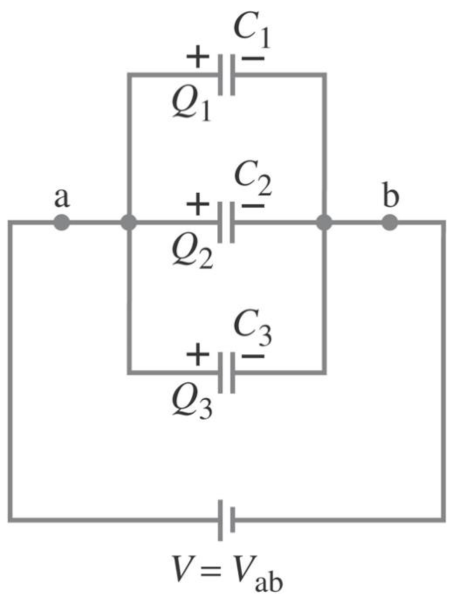
$$Q = Q_1 + Q_2 + Q_3 = C_1V + C_2V + C_3V$$

$$Q = C_{eq}V$$

- Combing the above two equations, we get

$$C_{eq}V = C_1V + C_2V + C_3V = (C_1 + C_2 + C_3)V$$

Capacitors in Parallel

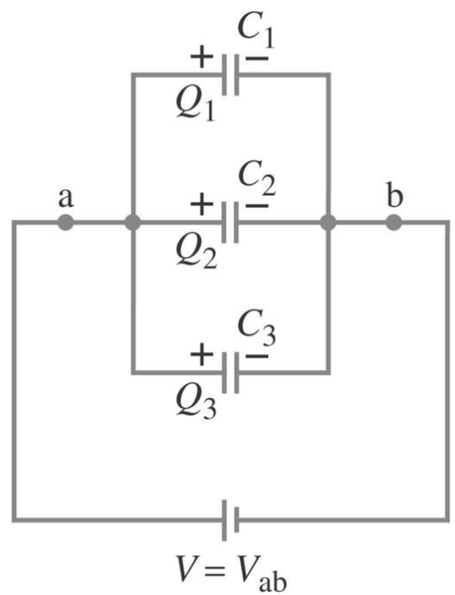


$$C_{eq}V = C_1V + C_2V + C_3V = (C_1 + C_2 + C_3)V$$

or

$$C_{eq} = C_1 + C_2 + C_3$$

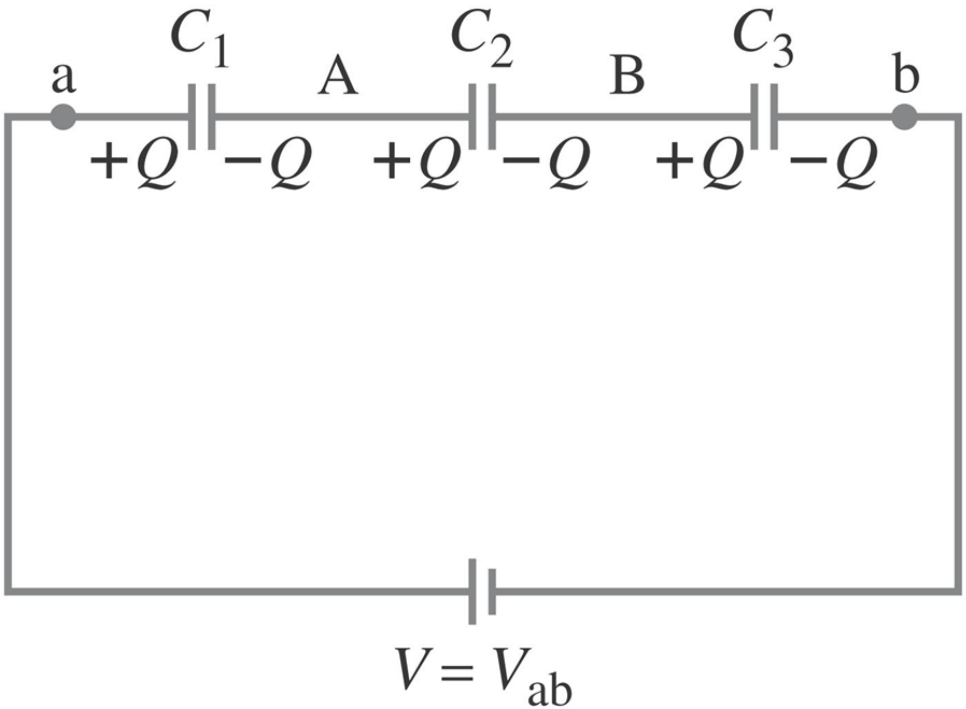
Capacitors in Parallel



$$C_{eq} = C_1 + C_2 + C_3$$

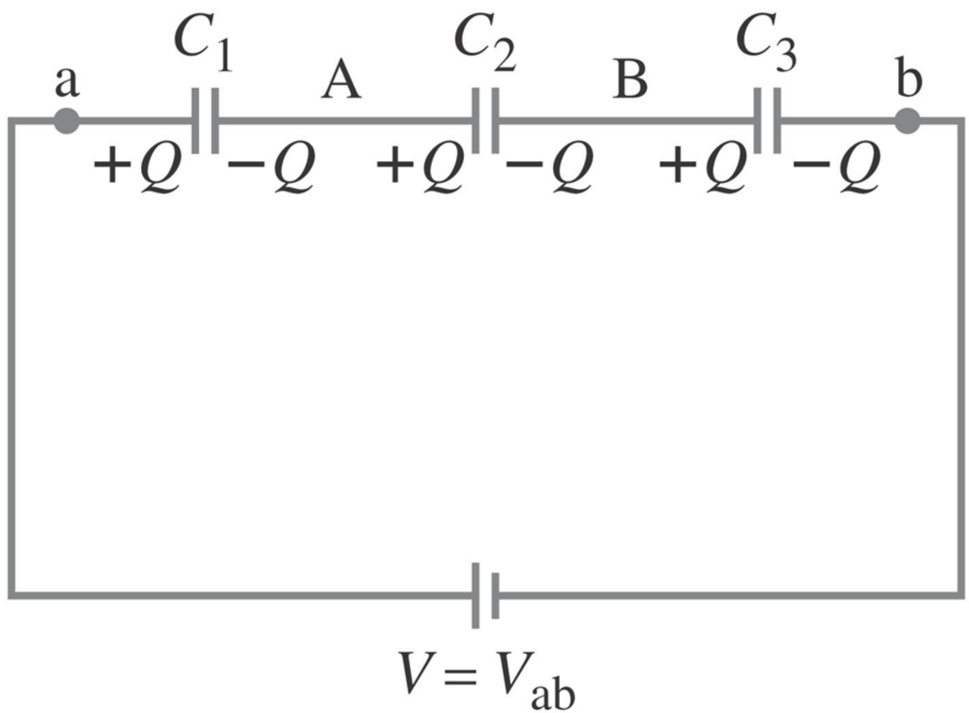
- The net effect of connecting capacitors in parallel is to increase the capacitance. This makes sense because we are essentially increasing the area of the plates where charge can accumulate.
- Let's now look at capacitors in series.

Capacitors in Series



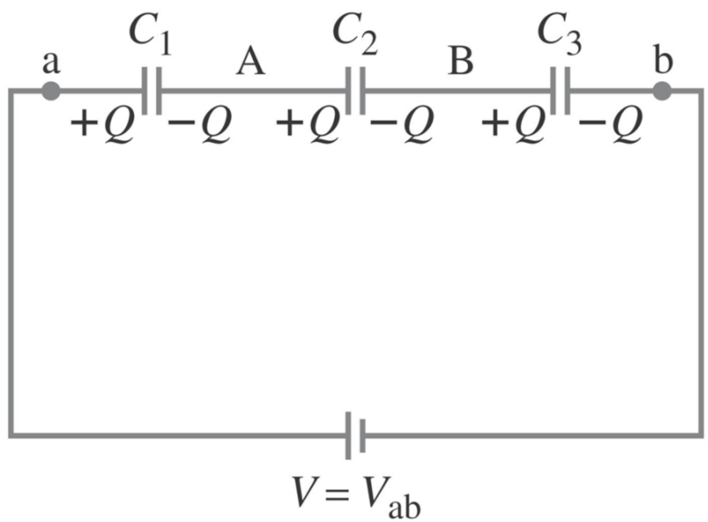
- A charge of $+Q$ is created on one plate of C_1, as a charge of $-Q$ flows to one plate of C_3.
- The regions A and B between the capacitors were originally neutral; so the net charge there must still be zero.

Capacitors in Series



- The $+Q$ on the left plate of C_1 attracts a charge of $-Q$ on the opposite plate.
- Because region A must have a zero net charge, there is thus $+Q$ on the left plate of C_2. The same considerations apply to the other capacitors, so we see the charge on each capacitor is the same value Q.

Capacitors in Series



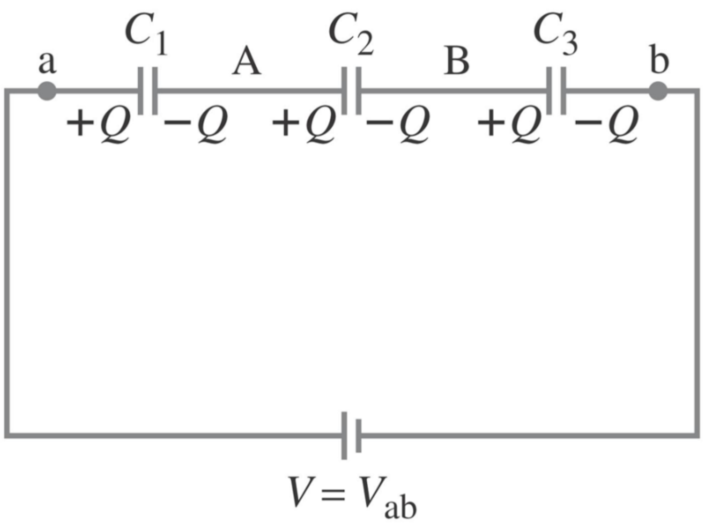
- A single capacitor that could replace these three in series without affecting the circuit (that is, Q and V the same) would have a capacitance C_{eq} where

$$Q = C_{eq}V.$$

- The total voltage V across the three capacitors in series must equal the sum of the voltages across each capacitor:

$$V = V_1 + V_2 + V_3$$

Capacitors in Series

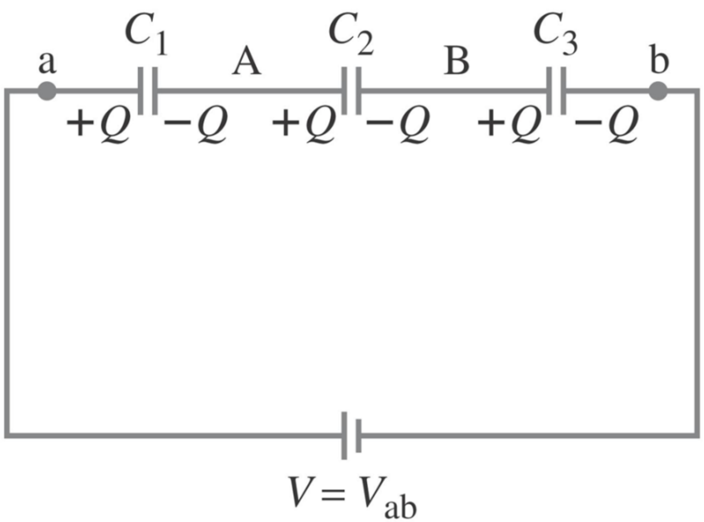


$$V = V_1 + V_2 + V_3$$

- We also have for each capacitor $Q = C_1 V_1$, $Q = C_2 V_2$, and $Q = C_3 V_3$, so we can substitute for V_1, V_2, and V_3 into the above equation, and get

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

Capacitors in Series



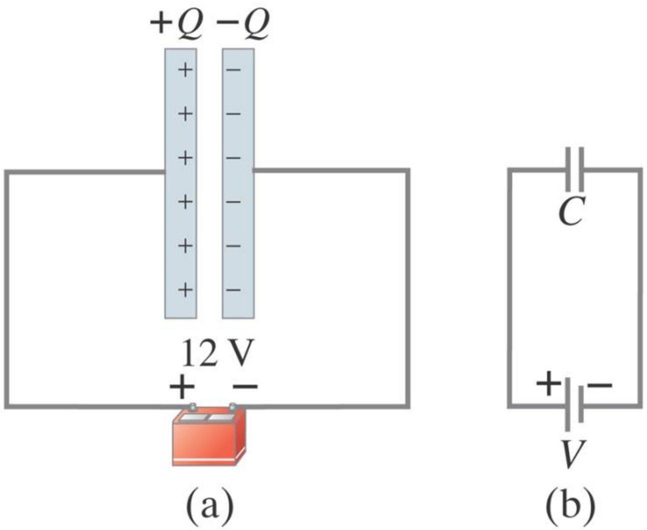
$$V = V_1 + V_2 + V_3$$

- We also have for each capacitor $Q = C_1 V_1,\ Q = C_2 V_2$, and $Q = C_3 V_3$, so we can substitute for V_1, V_2, and V_3 into the above equation, and get

or $$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

7. Electric Energy Storage

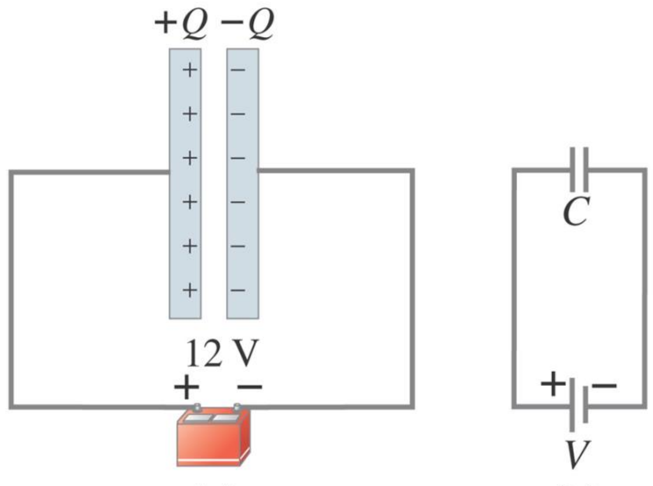
Electric Energy Storage



- A charged capacitor stores electrical energy. The energy stored in a capacitor will be equal to the work done to charge it.

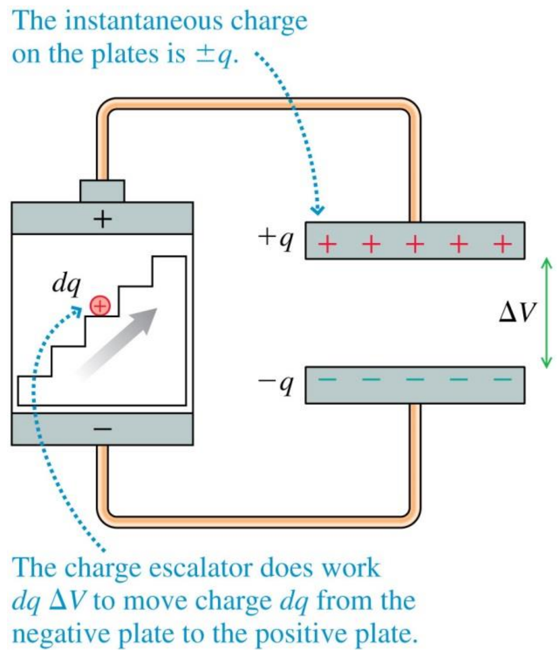
- The net effect of charging a capacitor is to remove charge from one plate and add it to the other plate. This is what a battery does when it is connected to a capacitor.

Electric Energy Storage



- A capacitor does not become charged instantly; it takes time.
- Initially, when the capacitor is uncharged, it requires no work to move the first bit of charge over.
- When some charge is on each plate, it requires work to add more charge of the same sign because of the electric repulsion.
- The more charge already on a plate, the more work required to add additional charge.

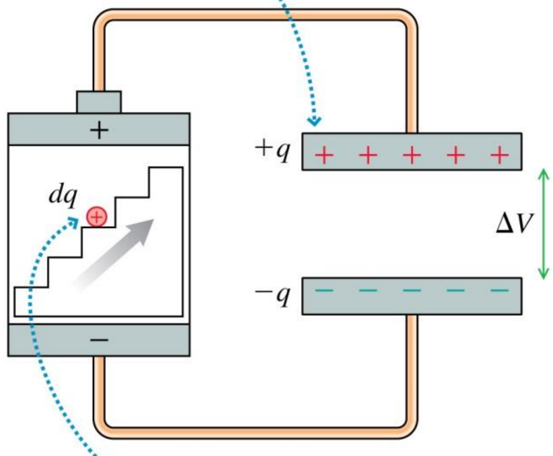
Electric Energy Storage



- The work needed to add a small amount of charge dq, when a potential difference V is across the plates, is $dW = Vdq$.
- Since $V = q/C$ at any moment, where C is the capacitance, the work needed to store a total charge Q is

$$W = \int_0^Q V dq = \frac{1}{C}\int_0^Q q dq = \frac{1}{2}\frac{Q^2}{C}$$

Electric Energy Storage

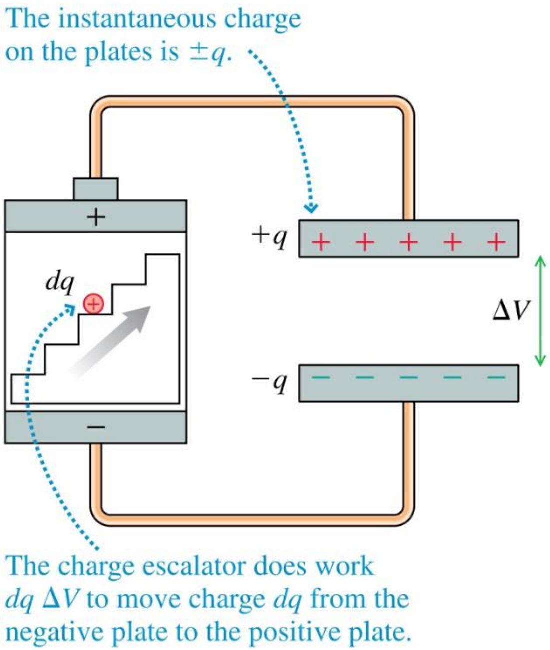


- Thus, we can say that the energy ‘stored’ in a capacitor is

$$U = \frac{1}{2}\frac{Q^2}{C}$$

when the capacitor C carries charges $+Q$ and $-Q$ on its two conductors.

Electric Energy Storage



- Since $Q = CV$, where V is the potential difference across the capacitor, we can also write

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

Summary of today's Lecture

1. Resistivity
2. Electric Power
3. Alternating Current
4. Capacitors
5. Determination of Capacitance
6. Capacitors in Series and Parallel
7. Electric Energy Storage

Lecture 21: Optional Reading

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- **Ch. 25.4**, Resistivity; p.762-763.
- **Ch. 25.5**, Electric Power; p.764-766.
- **Ch. 25.7**, Alternating Current; p.768-769.
- **Ch. 24.1**, Capacitors; p.728-729.
- **Ch. 24.2**, Determination of Capacitance; p.730-733
- **Ch. 24.3**, Capacitors in Series and Parallel; p.733-735
- **Ch. 24.4**, Electric Energy Storage; p.736-738

Home Work

Do not forget to attempt the **Additional Problems** for this lecture before logging in to **Mastering Physics** to complete your assignments.