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Foundation Calculus and Mathematical Techniques CELEN037

Practice Problems SET-3 Sample Solution

Type 1: Increasing and Decreasing Functions

1. Find the intervals where $f(x) = x^3 + 4x^2 + 4x + 9$ is increasing and decreasing. Solution:

$f(x) = x^3 + 4x^2 + 4x + 9$

$f'(x) = 3x^2 + 8x + 4$

Let $f'(x) = 0$

$x = -2$ or $-\dfrac{2}{3}$

In intervals $(-\infty, -2)$ and $(-\dfrac{2}{3}, \infty)$, $f'(x) > 0$ the function is increasing

In interval $(-2, -\dfrac{2}{3})$, $f'(x) < 0$ the function is decreasing

Type 2: Classification of Stationary Points

5. Find and classify the stationary points for the following functions: (i) $f(x) = \dfrac{1}{3}x^3 + \dfrac{3}{2}x^2 + 2x$.

Solution:

$f(x) = \dfrac{1}{3}x^3 + \dfrac{3}{2}x^2 + 2x$

$f'(x) = x^2 + 3x + 2$

Let $f'(x) = 0$

$x = -2$ or -1

$x = -2, y = f(-2) = -\dfrac{2}{3}$

$x = -1, y = f(-1) = -\dfrac{5}{6}$

$f''(x) = 2x + 3$

$f''(-2) = -1, < 0$ $(-2, -\dfrac{2}{3})$ is a local maximum point

$f''(-1) = 1, > 0$ $(-1, -\dfrac{5}{6})$ is a local minimum point.

Type 3: Newton-Raphson Method

10. Use the Newton-Raphson method to approximate the value of $\sqrt[3]{3}$, correct to 8 d.p., by starting with $x_0 = 1.5$.

Solution:

Form an equation with root of $\sqrt[3]{3}$: $x^3 - 3 = 0$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^3 - 3}{3x_n^2}$$

take $x_0 = 1.5$

n	x_n
0	1.5
1	1.444444444
2	1.44225290
3	1.44224957
4	1.44224957

$\therefore$ the desired root $x^* = 1.44224957$

Type 4: Optimisation Problems

11. Suppose that $r(x) = 9x$ is the revenue function and $c(x) = x^3 - 6x^2 + 15x$ is the cost function, where x represents millions of MP4 players produced. Is there a production level that maximizes profit? If so, what is it? (Hint: Profit = Revenue − Cost.)

Solution:

Let profit function $p(x) = r(x) - c(x)$

$p(x) = 9x - (x^3 - 6x^2 + 15x) = -x^3 + 6x^2 - 6x$

$p'(x) = -3x^2 + 12x - 6$

Let $p'(x) = 0$

$x = \sqrt{2} \pm 2$

As $x > 0,$ therefore $x = \sqrt{2} + 2$

$p''(x) = -6x + 12$

$p''(\sqrt{2} + 2) = -6\sqrt{2} \; < 0$

$\therefore$ When $x = \sqrt{2} + 2$ the profit is maximized.