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Foundation Algebra for Physical Sciences & Engineering CELEN036

Practice Problems SET-6 **Topic:** Complex numbers

Type 1: Real and imaginary parts of complex numbers

1. Solve the following equations for x and y, where $x, y \in \mathbb{R}$:

   (i) $x + iy = (3 + i)(2 - 3i)$ (ii) $x + iy = 3$

   (iii) $x + iy = 2i$ (iv) $2 + 3i = (x + iy)(1 - i)$

2. Find the real and imaginary parts of w defined by $w = \dfrac{1+z}{1-z}$, where $z = x + iy$ for some $x, y \in \mathbb{R}$.

Type 2: Expressing complex numbers in the form $a + i\,b$

3. If $z_1 = 4 - i$ and $z_2 = 3 + 2i$, express the following in the form $a + i\,b$, where $a, b \in \mathbb{R}$:

   (i) $z_1 + z_2$ (ii) $z_1 - z_2$ (iii) $z_1 \cdot z_2$ (iv) $\dfrac{z_1}{z_2}$

4. Express the following complex numbers in the form $a + b\,i$, where $a, b \in \mathbb{R}$:

   (i) $(5 + 6i) + (7 + 3i)$ (ii) $(2 - 3i) - (5 + 2i)$ (iii) $(3 + 2i)(2 - i)$

5. Write in the form $a + b\,i$, where $a, b \in \mathbb{R}$:

   (i) $\dfrac{2+3i}{1+i}$ (ii) $\dfrac{-4+3i}{-2-i}$ (iii) $\dfrac{4i}{2-i}$

6. Simplify $(1 + i)^6 - (1 - i)^3$.

7. Simplify the following complex numbers:

   (i) $\dfrac{(2+i)(3-2i)}{1+i}$ (ii) $\dfrac{(1-i)^3}{(2+i)^2}$ (iii) $\dfrac{1}{3+i} - \dfrac{1}{3-i}$

Type 3: Solving equations

8. Solve for $z \in \mathbb{C}$ in the following equations.

   (i) $z(2+i) = 3-2i$ (ii) $(z+i)(1-i) = 2+3i$ (iii) $\dfrac{1}{z} + \dfrac{1}{2-i} = \dfrac{3}{1+i}$

9. Solve the following polynomial equations for $z \in \mathbb{C}$:

   (i) $z^2 + 6z + 10 = 0$

   (ii) $4z^2 + 25 = 0$

   (iii) If $z = 2+3i$, rove that $z^3 - 5z^2 + 17z - 13 = 0$

   (iv) If $z = -2$, prove that $z^3 + 6z^2 + 13z + 10 = 0$

Type 4: Argand diagram

10. Plot the following complex numbers on the Argand diagram:

    (i) $3-4i$ (ii) $2+5i$ (iii) $-4+2i$ (iv) $1-6i$

11. For the given complex numbers z_1 and z_2, plot $z_1 + z_2$ and $z_1 - z_2$ on the Argand diagram:

    (i) $z_1 = 3+4i, \quad z_2 = 2-3i$ (ii) $z_1 = 4-3i, \quad z_2 = 1+2i$

Type 5: Modulus and argument

12. Given $z_1 = 3-2i$, $z_2 = 1+4i$, and $z_3 = 4+5i$, find the following values:

    (i) $\left|\dfrac{z_1\, z_3}{z_2}\right|$ (ii) $\left|\dfrac{z_1\, z_2 + z_3}{z_3}\right|$

13. Find the modulus and the principal value of the argument θ $(-\pi < \theta \leq \pi)$ of the following complex numbers:

    (i) $z_1 = 1+i$ (ii) $z_2 = 1+\sqrt{3}\,i$ (iii) $z_3 = -1+\sqrt{3}\,i$

Type 6: Polar form of complex numbers

14. Express the following complex numbers in the polar form $r(\cos\theta + i\sin\theta)$ $(-\pi < \theta \leq \pi)$:

(i) $\sqrt{3}+i$ (ii) $1-\sqrt{3}\,i$ (iii) $\sqrt{3}+\sqrt{3}\,i$

(iv) $-1+\sqrt{3}\,i$ (v) $-\sqrt{3}-i$

15. Find the polar form of the following complex numbers:

(i) $z_1 = 2+2i$ (ii) $z_2 = 2-2i$ (iii) $z_3 = i$

Hence find the modulus r and principal argument ($\theta \in (-\pi, \pi]$) of the complex numbers:

(iv) $z_1 \cdot z_2$ (v) ${z_1}^2$ (vi) $z_1 \cdot z_3$

Answers

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1 (i) $x=9,\ y=-7$ (ii) $x=3,\ y=0$ (iii) $x=0,\ y=2$ (iv) $x=-0.5,\ y=2.5$

2 $Re(w) = \dfrac{1-x^2-y^2}{x^2+y^2-2x+1}$ $Im(w) = \dfrac{2y}{x^2+y^2-2x+1}$

3 (i) $7+i$ (ii) $1-3i$ (iii) $14+5i$ (iv) $\dfrac{10}{13}-\dfrac{11}{13}i$

4 (i) $12+9i$ (ii) $-3-5i$ (iii) $8+i$

5 (i) $\dfrac{5}{2}+\dfrac{1}{2}i$ (ii) $1-2i$ (iii) $-\dfrac{4}{5}+\dfrac{8}{5}i$

6 $2-6i$

7 (i) $\dfrac{7}{2}-\dfrac{9}{2}i$ (ii) $-\dfrac{14}{25}+\dfrac{2}{25}i$ (iii) $-\dfrac{1}{5}i$

8 (i) $\dfrac{4}{5}-\dfrac{7}{5}i$ (ii) $-\dfrac{1}{2}+\dfrac{3}{2}i$ (iii) $\dfrac{11}{31}-\dfrac{17}{41}i$

9 (i) $-3\pm i$ (ii) $\pm\dfrac{5}{2}i$ (iii) $2\pm 3i,\quad 1$ (iv) $-2\pm i,\quad -2$

12 (i) $\sqrt{\dfrac{533}{17}}$ (ii) $15\sqrt{\dfrac{2}{41}}$

13 (i) $|z_1|=\sqrt{2},\quad \arg(z_1)=\dfrac{\pi}{4}$ (ii) $|z_2|=2,\quad \arg(z_2)=\dfrac{\pi}{3}$ (iii) $|z_3|=2,\quad \arg(z_3)=\dfrac{2\pi}{3}$

14 (i) $2\left[\cos\left(\dfrac{\pi}{6}\right)+i\sin\left(\dfrac{\pi}{6}\right)\right]$ (ii) $2\left[\cos\left(-\dfrac{\pi}{3}\right)+i\sin\left(-\dfrac{\pi}{3}\right)\right]$ (iii) $\sqrt{6}\left[\cos\left(\dfrac{\pi}{4}\right)+i\sin\left(\dfrac{\pi}{4}\right)\right]$

(iv) $2\left[\cos\left(\dfrac{2\pi}{3}\right)+i\sin\left(\dfrac{2\pi}{3}\right)\right]$ (v) $2\left[\cos\left(-\dfrac{5\pi}{6}\right)+i\sin\left(-\dfrac{5\pi}{6}\right)\right]$

15 (i) $2\sqrt{2}\left[\cos\left(\dfrac{\pi}{4}\right)+i\sin\left(\dfrac{\pi}{4}\right)\right]$ (ii) $2\sqrt{2}\left[\cos\left(-\dfrac{\pi}{4}\right)+i\sin\left(-\dfrac{\pi}{4}\right)\right]$ (iii) $\cos\left(\dfrac{\pi}{2}\right)+i\sin\left(\dfrac{\pi}{2}\right)$

(iv) $r=8,\quad \theta=0$ (v) $r=8,\quad \theta=\dfrac{\pi}{2}$ (vi) $r=2\sqrt{2},\quad \theta=\dfrac{3\pi}{4}$