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Foundation Algebra for Physical Sciences & Engineering CELEN036

Practice Problems SET-2 Sample Solution

Type 1: Quadratic Equations

6. By completing the square, find the range of the following functions for $x \in \mathbb{R}$:

(*i*) $f(x) = x^2 - 2x - 8$

Solution:

$$\begin{aligned} f(x) &= x^2 - 2x - 8 \\ &= x^2 - 2x + 1 - 9 \\ &= (x-1)^2 - 9 \\ \because \quad & (x-1)^2 \geq 0 \\ \therefore \quad & (x-1)^2 - 9 \geq -9 \\ \therefore \quad & y = f(x) \geq -9 \\ \implies \quad & R_f = [-9, +\infty) \end{aligned}$$

Type 1: Quadratic Inequalities

8. Determine the values of x for which the following quadratic inequalities hold:

(*iii*) $b^2 + a^2x^2 > 2abx; \quad a \neq 0$

Solution:

$$\begin{aligned} & b^2 + a^2x^2 > 2abx \\ & a^2x^2 - 2abx + b^2 > 0 \\ & (ax - b)^2 > 0 \\ \because \quad & (ax-b)^2 \geq 0 \\ \therefore \quad & x \neq \frac{b}{a} \\ \implies \quad & x \in \mathbb{R}, \quad x \neq \frac{b}{a}. \end{aligned}$$

Type 3: Exponential Functions

9. Simplify $(1+x^{a-b})^{-1}+(1+x^{b-a})^{-1}$.

Solution:

$$
\begin{aligned}
& (1+x^{a-b})^{-1}+(1+x^{b-a})^{-1} \\
\implies & =\frac{1}{1+x^{a-b}}+\frac{1}{1+x^{b-a}} \\
\implies & =\frac{1+x^{b-a}+1+x^{a-b}}{(1+x^{a-b})(1+x^{b-a})} \\
\implies & x=\frac{2+x^{b-a}+x^{a-b}}{2+x^{b-a}+x^{a-b}} \\
\therefore & =1
\end{aligned}
$$

Type 4: Logarithmic Functions

17. Prove that $\log\left(\frac{a^2}{bc}\right)+\log\left(\frac{b^2}{ac}\right)+\log\left(\frac{c^2}{ab}\right)=0$.

Solution:

$$
\begin{aligned}
\text{LHS} & = 2\log a-\log b-\log c+2\log b-\log a-\log c+2\log c-\log a-\log b \\
& = 2(\log a+\log b+\log c)-2\log a-2\log b-2\log c \\
& = 0 \\
\therefore \text{LHS} & = \text{RHS}
\end{aligned}
$$