The University of Nottingham
UNITED KINGDOM · CHINA · MALAYSIA

Foundation Physics

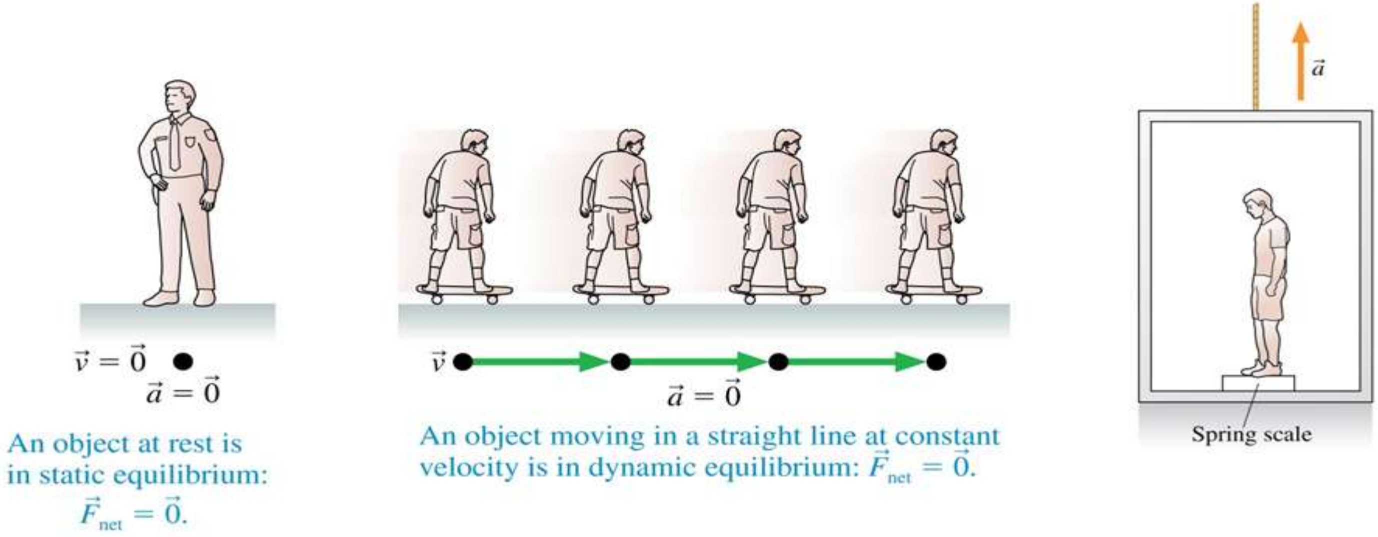
Lecture 4:

Free-body Diagrams

Aims of today's lecture

1. Free-body diagrams
2. Weight
3. Static friction
4. Kinetic friction
5. Modelling joined objects
6. Forms of Energy

Analysing the Forces Acting on an Object



- In our first three lectures, we seen how we can describe motion using terms such as '**displacement**', '**velocity**' and '**acceleration**'.

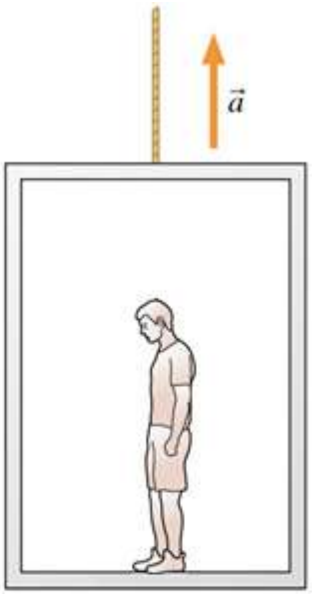
- We also have seen how we can describe this motion in terms of **equations (kinematics),** and **what it is that causes motion, namely force**.

- In this lecture, we look at how we analyse the forces acting on an object. To do so, we use a technique called **free-body diagrams**.

1. Free-body diagrams

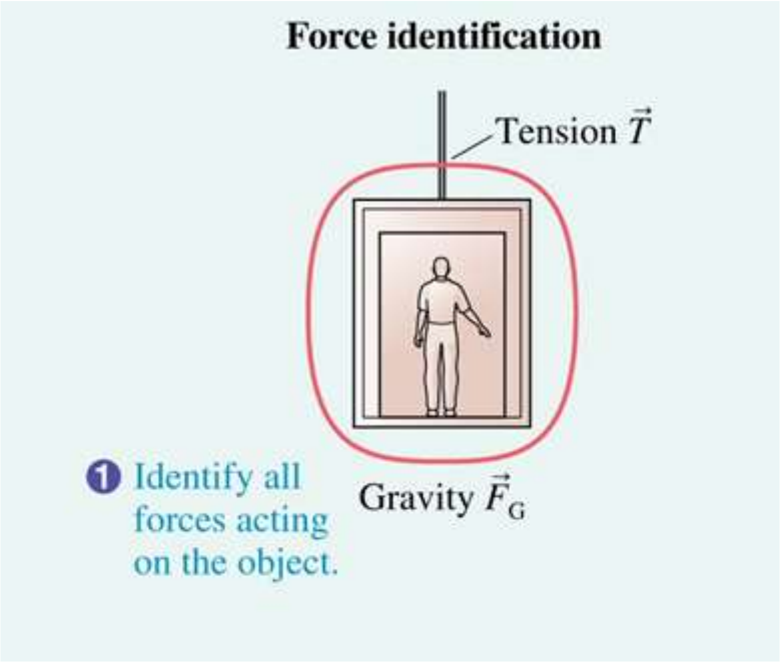
Free-body diagrams

- Consider an elevator suspended by a cable, speeding up as it moves upward from the ground floor.



- How do we identify the forces and draw a free-body diagram of the elevator?

Free-body diagrams

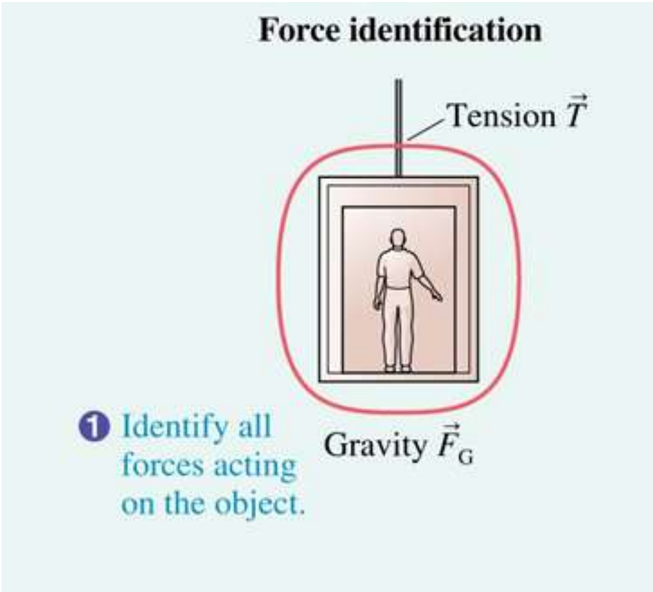


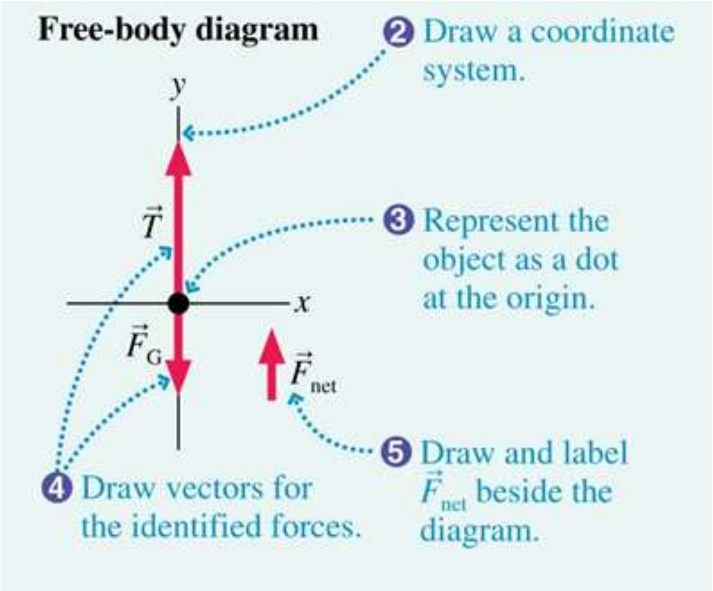
MODEL:

Treat the elevator as a particle.

Free-body diagrams

Free-body diagram of an elevator accelerating upward





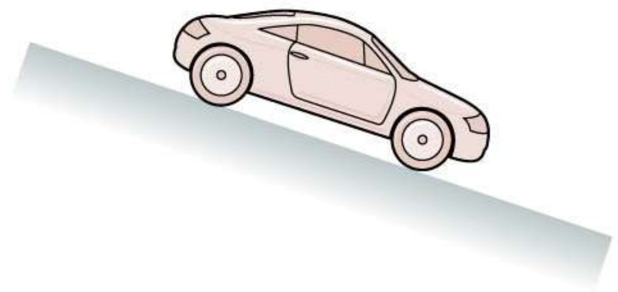
ASSESS:

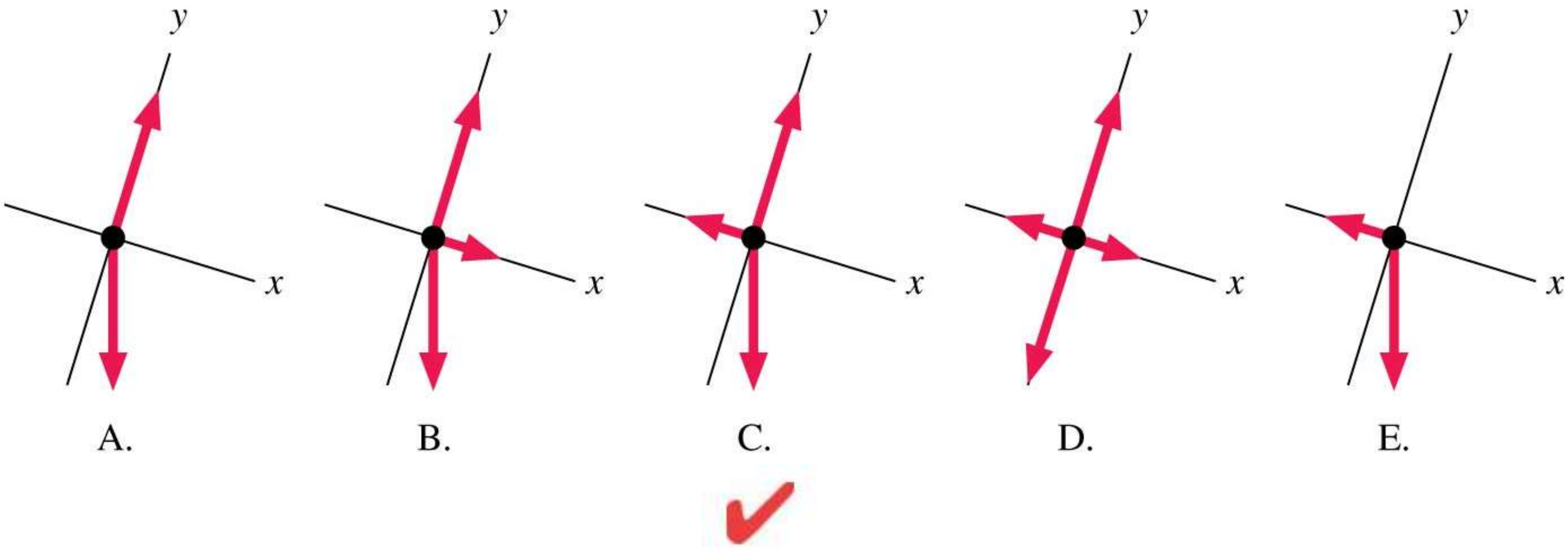
The coordinate axes, with a vertical y-axis, are the ones we would use in a pictorial representation of the motion.

The elevator is accelerating upward, so $\vec{F}_{net}$ must point **upward**. For this to be true, the magnitude of $\vec{T}$ must be **larger than the magnitude** of $\vec{F}_G$.

Have a Think: A Static Equilibrium Problem

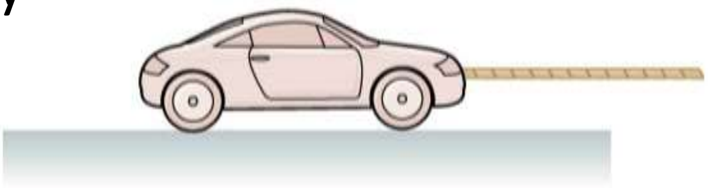
Q.1 A car is parked on a hill. Which is the correct free-body diagram?

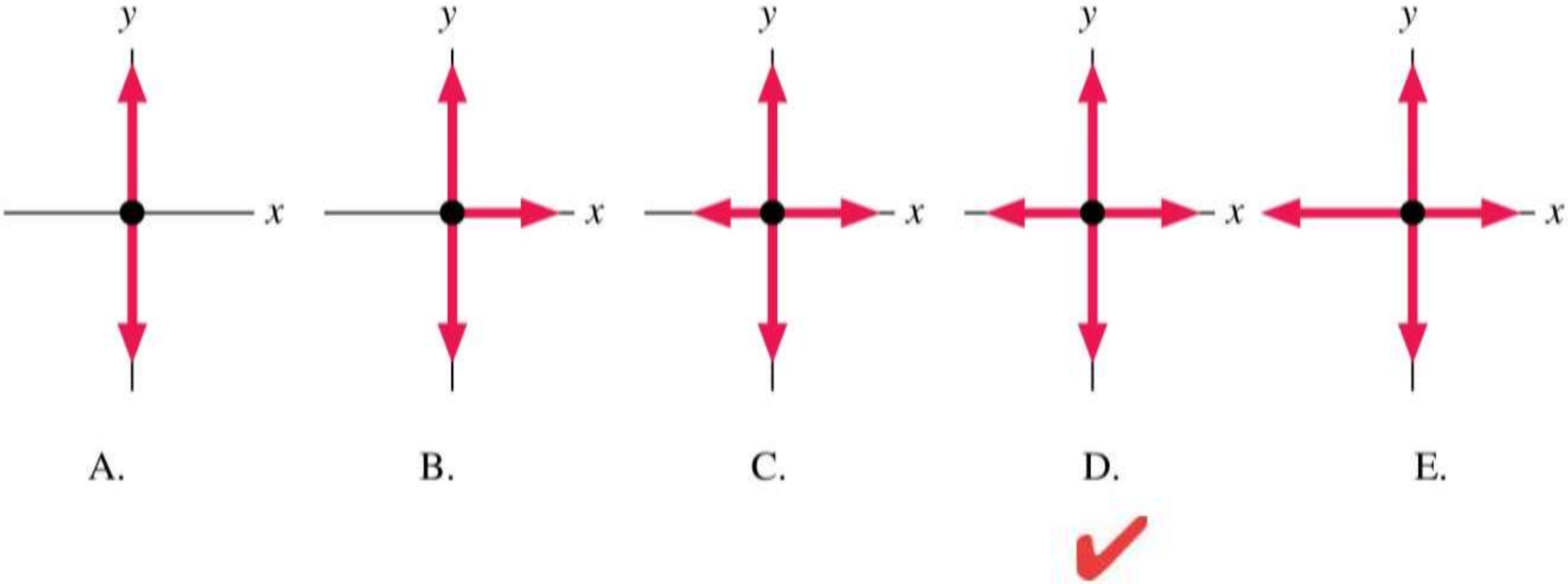




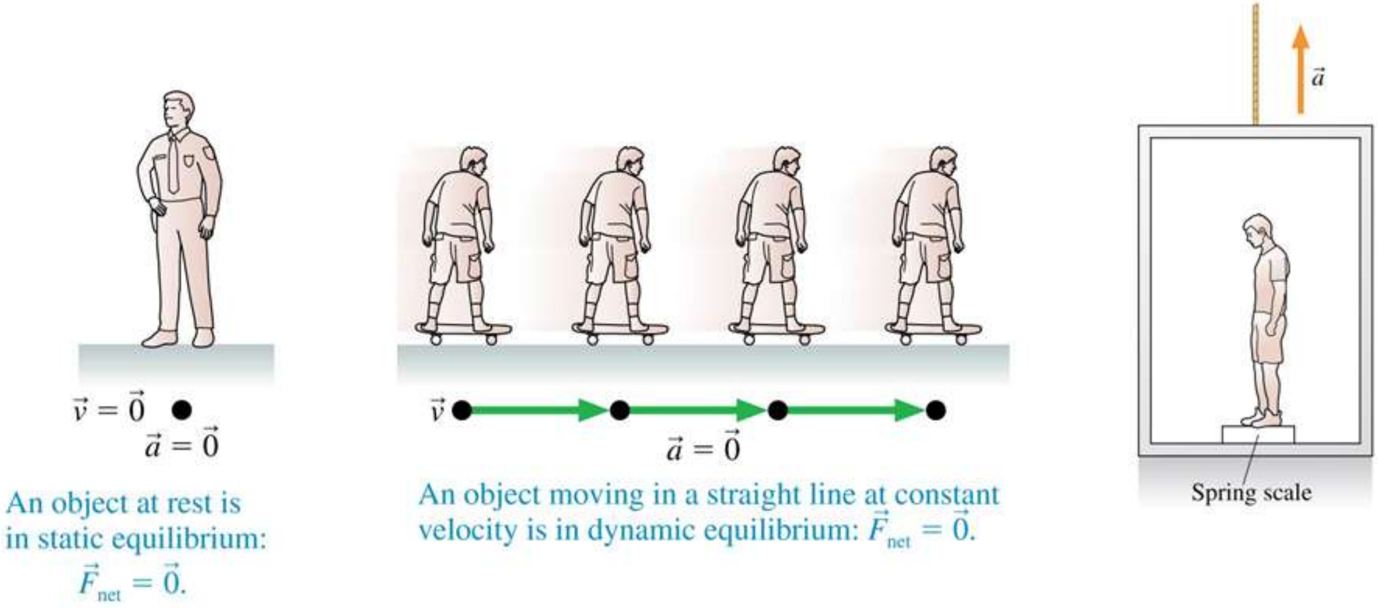
Have a Think: A Dynamics Equilibrium Problem

Q.2 A car is towed to the right at constant speed. Which is the correct free-body diagram?



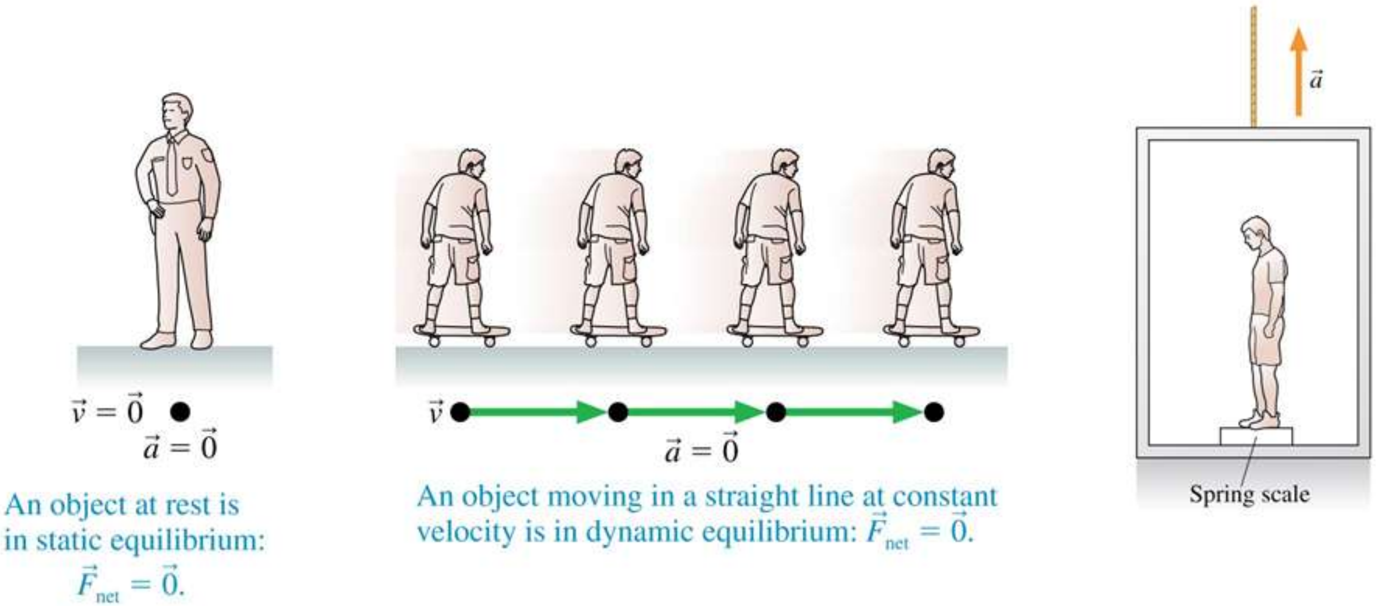


Analysing the Forces Acting on an Object



- Let us now look at how we 'free' an object from its surroundings to analyze the forces acting on it, **in the context of free-body diagrams.**

Analysing the Forces Acting on an Object



- Many of these contexts involve the idea of **weight,** so let's look at weight in a bit more detail.

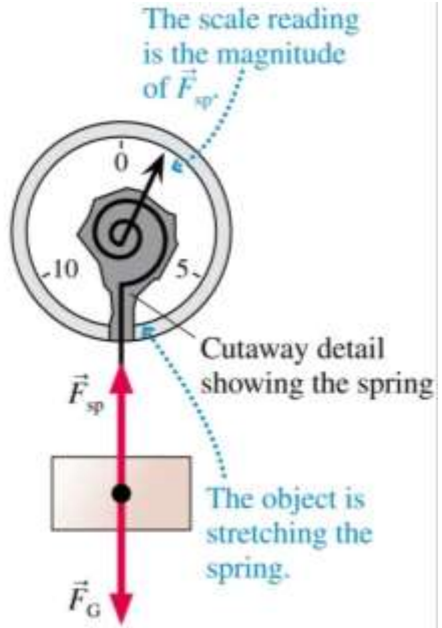
- We will see that when we talk about **weighing an object**, we are really determining the **force that gravity** exerts on the object.

2. Weight

Weight

- You can weigh apples in a grocery store by placing them on a pan which **stretches a spring**.

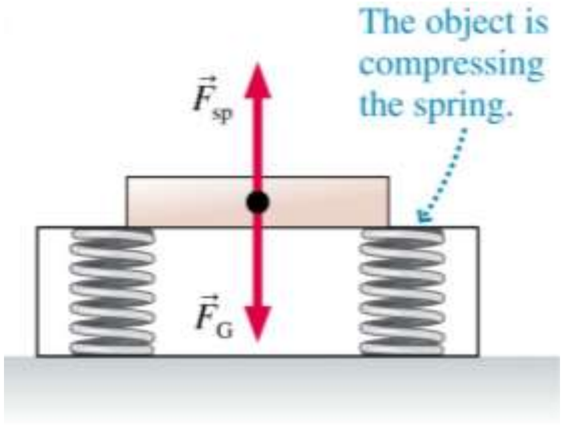
- The reading on the **spring scale** is the magnitude of F_{sp}.



- We define the **weight** of an object as the reading for F_{sp} when this force (measured in Newtons) is balanced by the **force of gravity**; in other words, when the object is **stationary**.

Weight

- Similarly, a bathroom scale **uses compressed springs** attached to a calibrated scale to give us a measurement for the **weight** of an object.



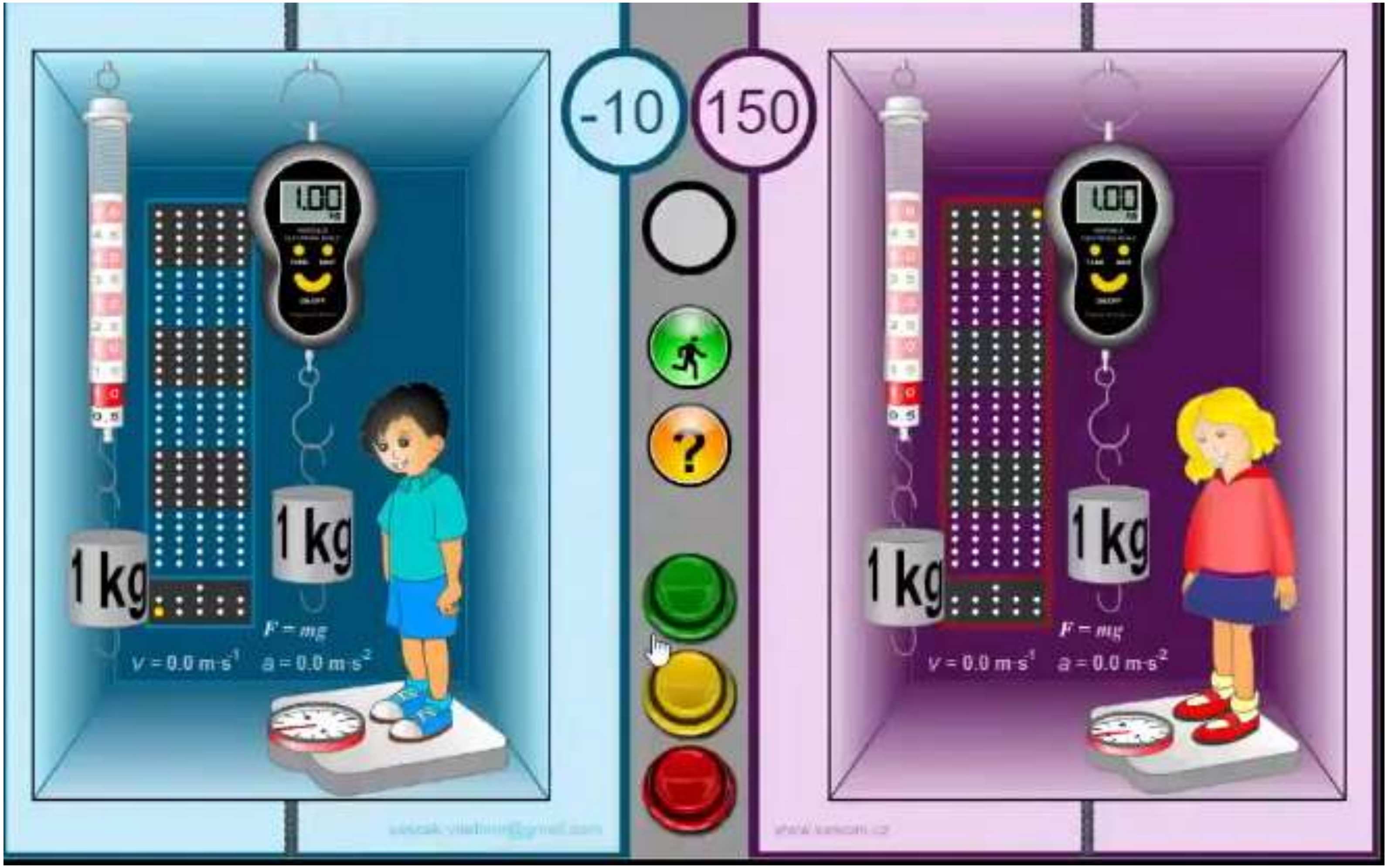
- When the object is **stationary**, the upward **spring force** exactly balances the downward **gravitational force** of magnitude mg:

$$F_{sp} = F_G = mg$$

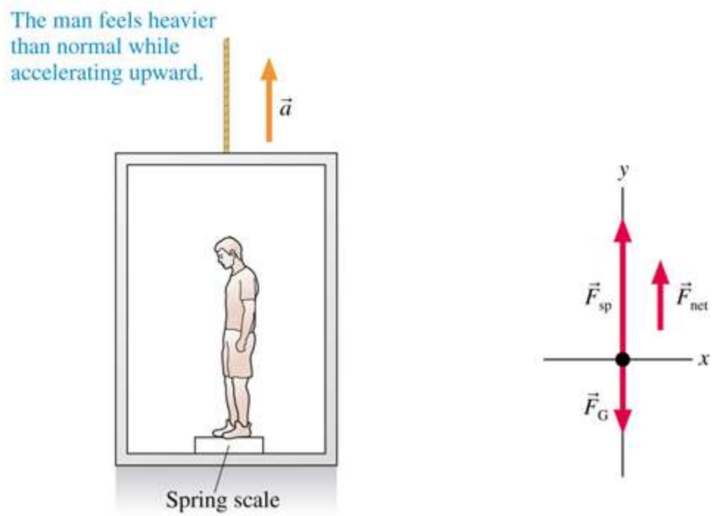
- Let's now consider putting a weighing scale inside an elevator.

Weight

- The video shows a boy and a girl weighing themselves in an accelerating elevator.



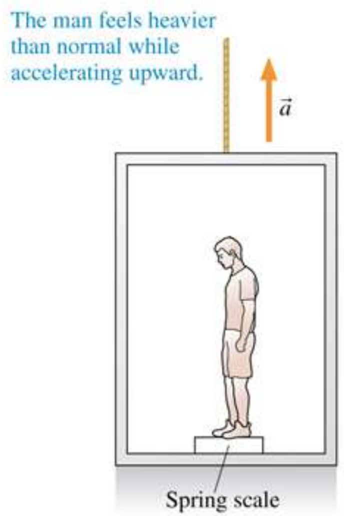
Weight

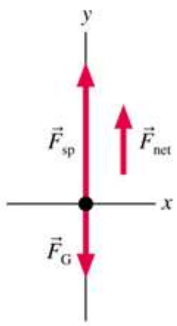


- The figure shows a man weighing himself in an accelerating elevator.
- Looking at the free-body diagram, the y-component of Newton's second law is:

$$(F_{net})_y = (F_{sp})_y + (F_G)_y = F_{sp} - mg = ma_y$$

Weight





- The man's weight as he accelerates vertically is:

$$F_{sp} - mg = ma_y \rightarrow F_{sp} = mg + ma_y \rightarrow F_{sp} = \mathrm{m}(g + a_y)$$

$$w = \text{scale reading for } F_{sp} = mg + ma_y = mg\left(1 + \frac{a_y}{g}\right)$$

=> You weigh **more** as an elevator **accelerates upward**!

Weightlessness

Astronauts while orbiting the earth are also weightless

Does this mean that they are in free fall?

- If an object is accelerating downward with $a_y = -g$.

$$F_{sp} - mg = ma_y \rightarrow F_{sp} = mg + ma_y \rightarrow F_{sp} = \mathrm{m}(g + a_y)$$

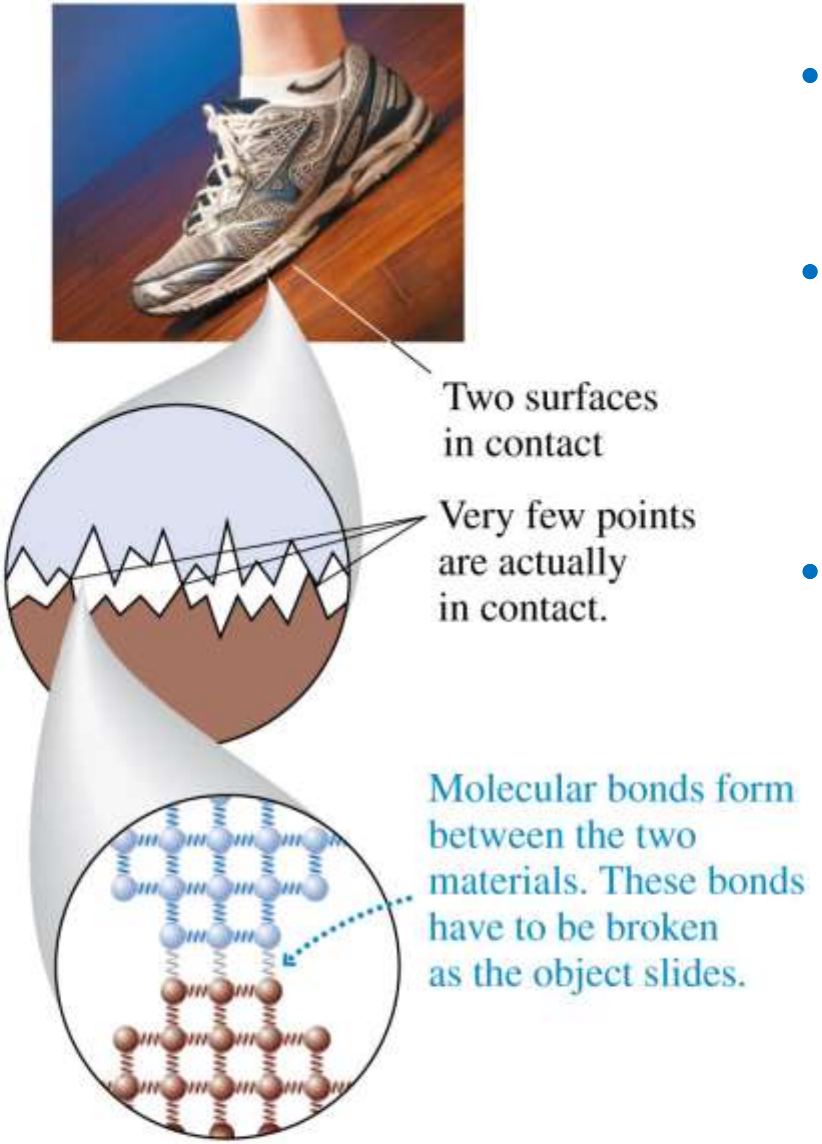
$$w = \text{scale reading for } F_{sp} = mg + ma_y = mg\left(1 + \frac{a_y}{g}\right)$$

then $w = 0 =>$ the object is in free-fall. **An object in free-fall has no weight!**

- The next context that we consider applying free-body diagrams to is that of **static friction**.

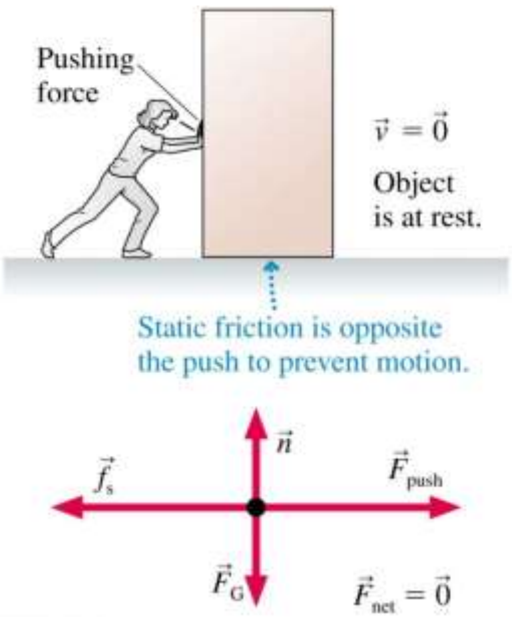
3. Static Friction

Static Friction



- All surfaces are very rough on a microscopic scale.
- When two surfaces are pressed together, the high points on each side come into contact and form molecular bonds.
- These bonds can produce a force **tangent** to the surface, called the **static friction** force.

Static Friction



- The figure shows a person pushing on a box that, due to static friction, isn't moving.
- Because of Newton's first law, the static friction force must exactly balance the pushing force:

$$f_s = F_{push}$$

- $\vec{f}_s$ points in the direction opposite to the way the object would move if there was no static friction.

Static friction acts in response to an applied force.

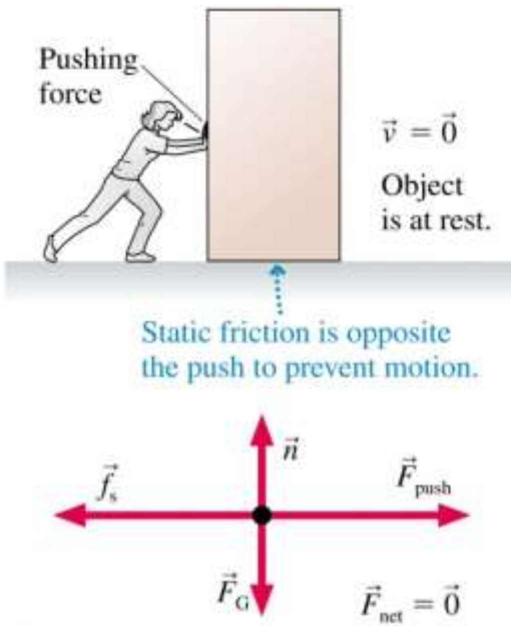
Static Friction

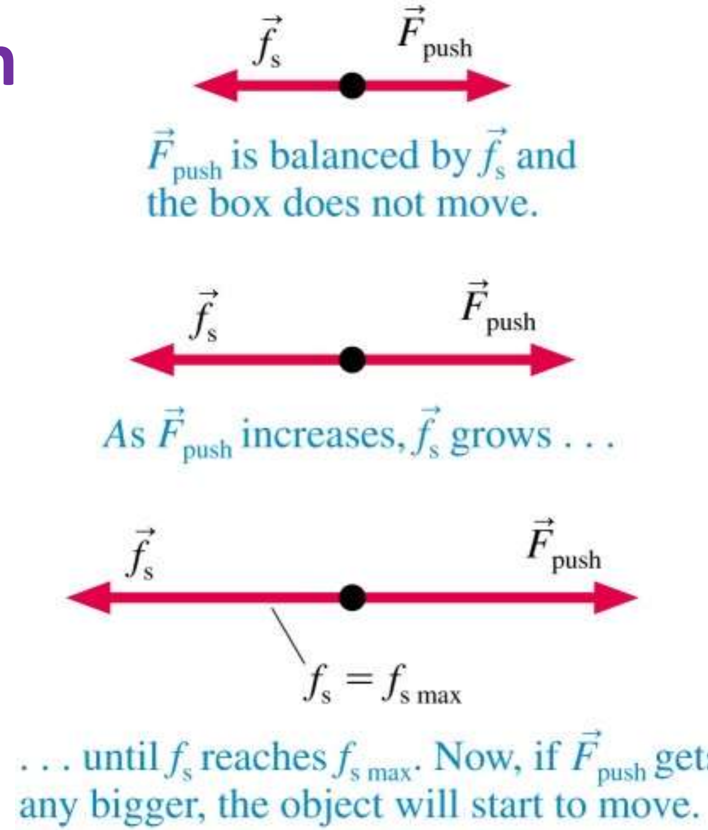
- Static friction force has a maximum possible size $f_{s\,\max}$.
- An object remains at rest as long as $f_s < fs_{,\,\max}$.
- The object just begins to slip when $f_s = f_{s,\,max}$.
- A static friction force $f_s > f_{s,\,max}$ is not physically possible.

$$f_{s,\,max} = \mu_s N$$

where the proportionality constant μ_s is called the **coefficient of static friction**.

Static Friction

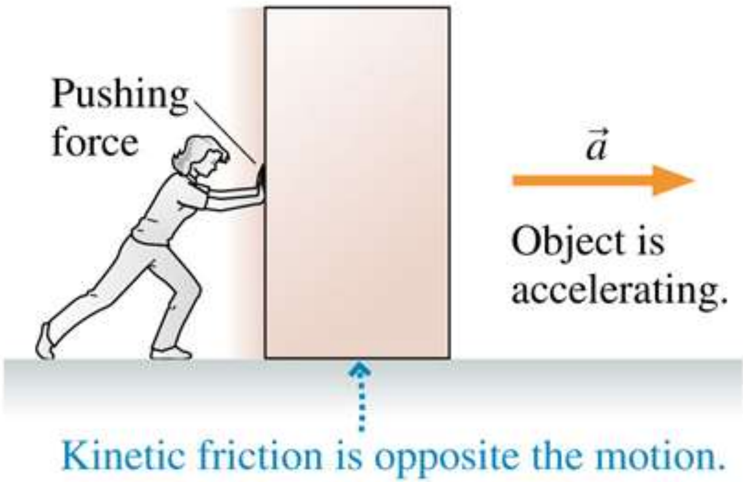


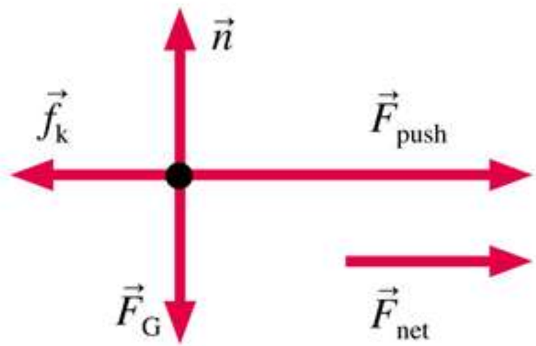


- Once we overcome the static friction, the object will begin to move, but does the friction between the object and the ground disappear? Not quite.
- There is still friction, albeit it is less; we call such friction **kinetic friction**.

4. Kinetic Friction

Kinetic Friction



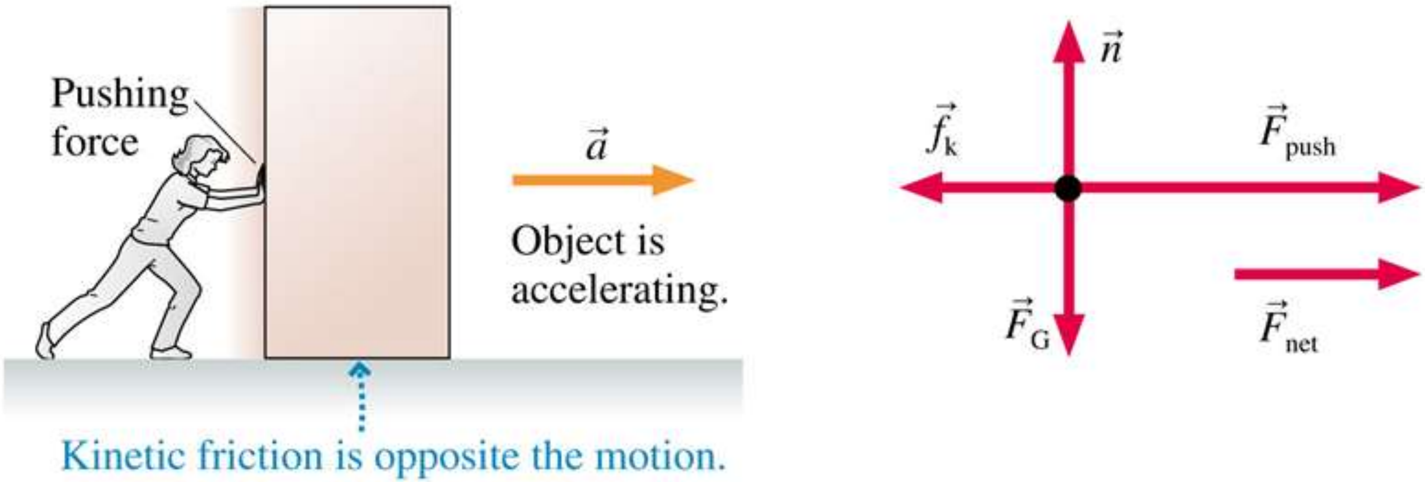


- The **kinetic friction** force is proportional to the magnitude of the normal force:

$$f_k = \mu_k N$$

where the proportionality constant μ_k is called the **coefficient of kinetic friction**.

Kinetic Friction



- The kinetic **friction direction** is opposite to the velocity of the object relative to the surface.

- For any particular pair of surfaces, $\mu_k < \mu_s$.

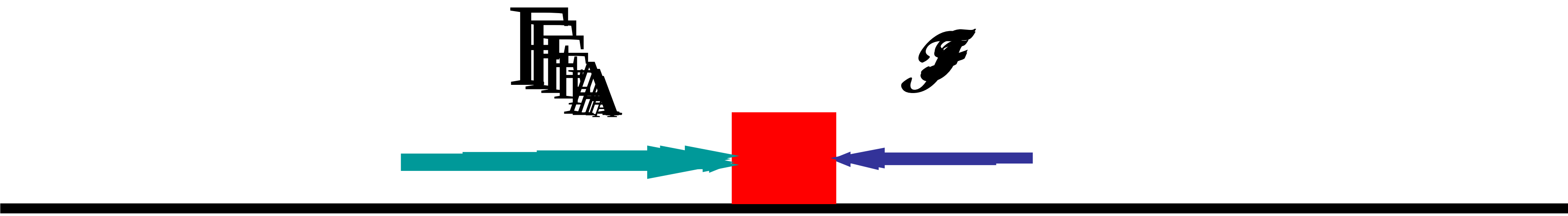
Coefficients of Static friction vs Coefficients of Kinetic Friction

Materials	Static μ_s	Kinetic μ_k
Rubber on concrete	1.00	0.80
Steel on steel (dry)	0.80	0.60
Steel on steel (lubricated)	0.10	0.05
Wood on wood	0.50	0.20
Wood on snow	0.12	0.06
Ice on ice	0.10	0.03

- As you can see for different surfaces, the **coefficient of kinetic friction** is always less than the **coefficient of static friction**.

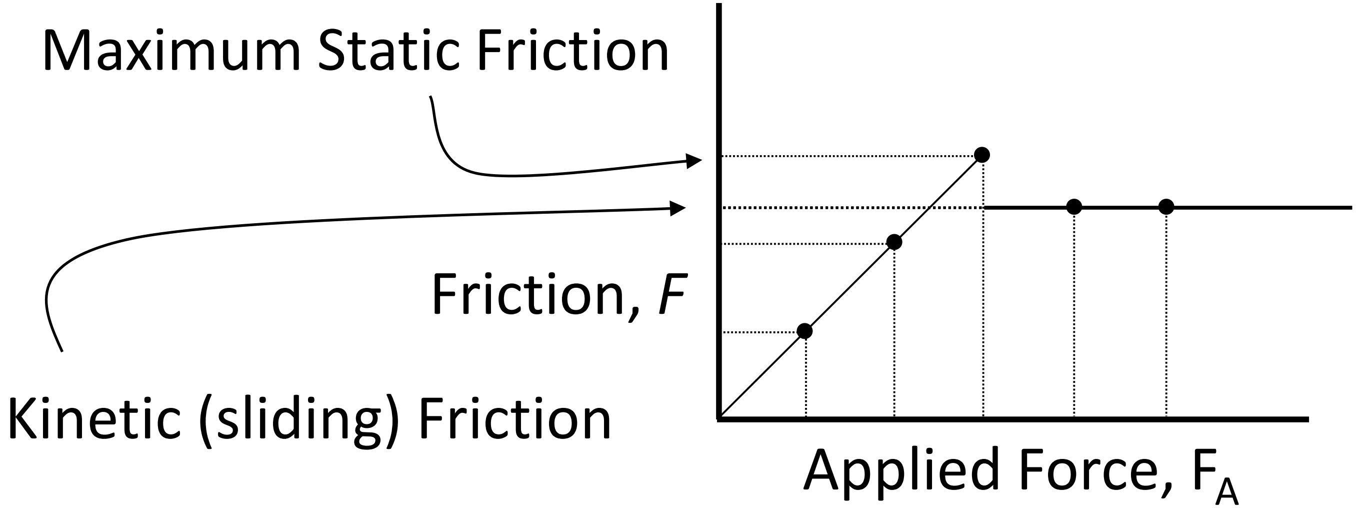
- As you can see for different surfaces, the coefficient of kinetic Let's now summarise, by way of a graph, the friction force response to an increasing applied force on an object.

The Friction Force Response to an Increasing Applied Force

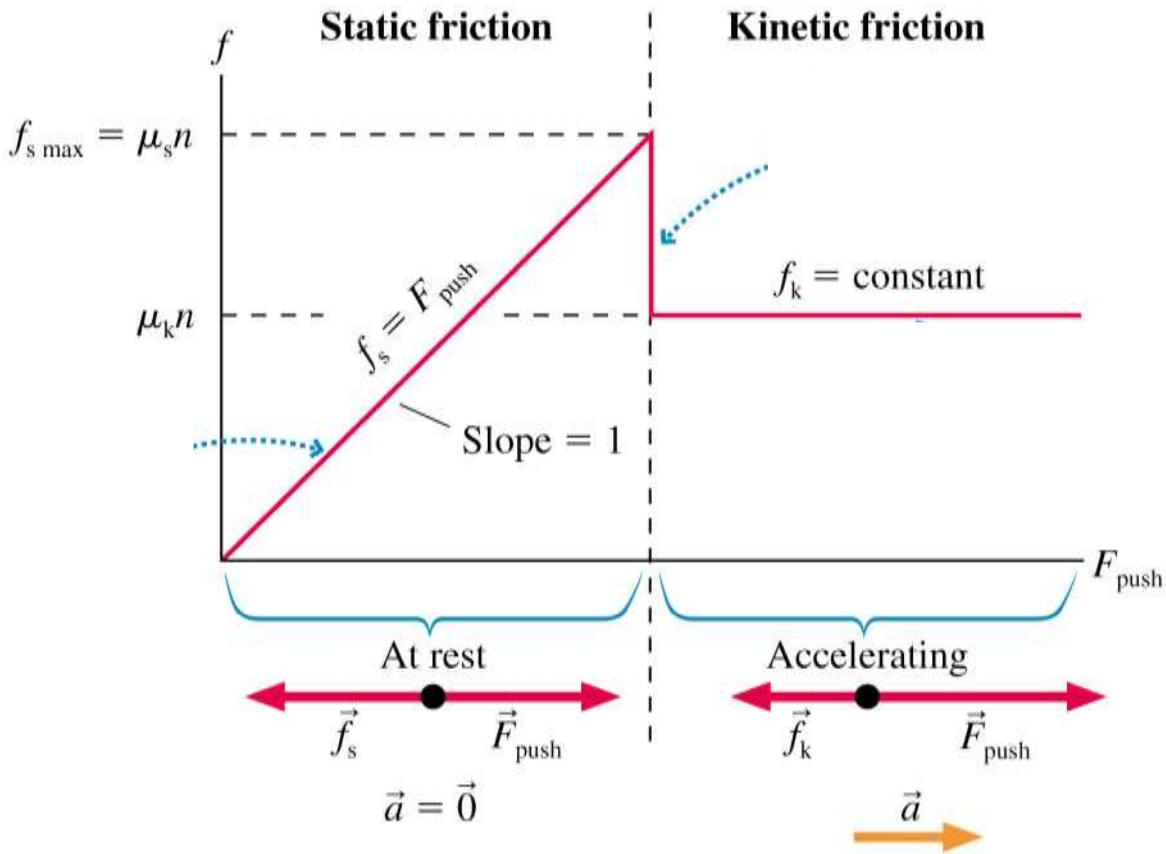


On the verge of slipping

Sliding



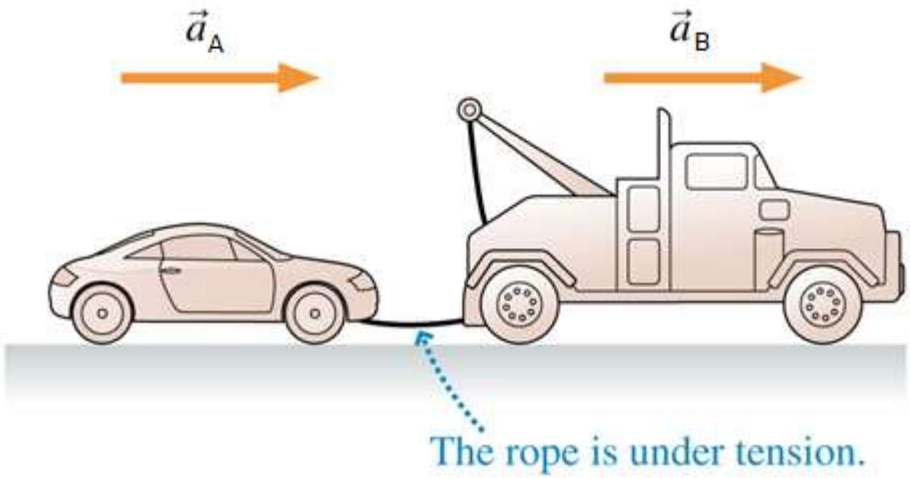
The Friction Force Response to an Increasing Applied Force



- We are now going to talk about how we model (**using free-body diagrams**) objects that are joined together (via ropes or strings) and are moving.

5. Modelling Joined Objects

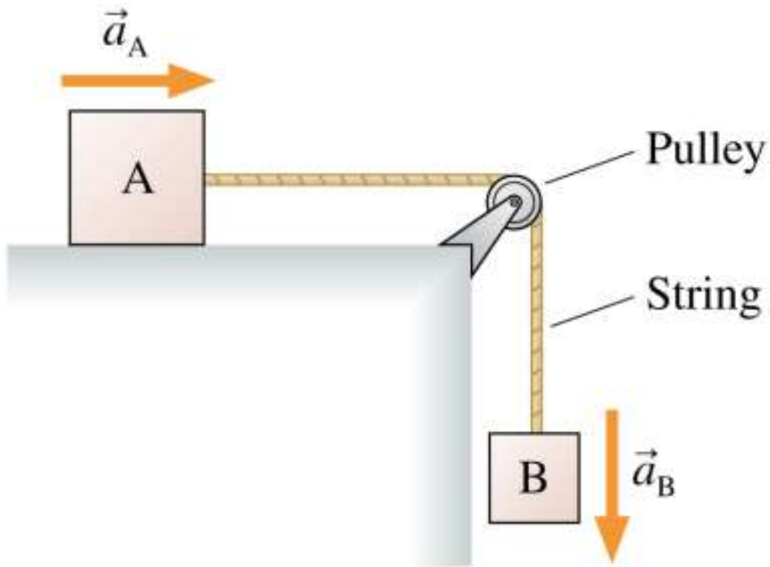
Modelling Joined Objects



- If two objects, such as a car (A) and a truck (B) are joined to each other and accelerate together, we say that both objects have the same acceleration: $\boldsymbol{a_A = a_B}$.

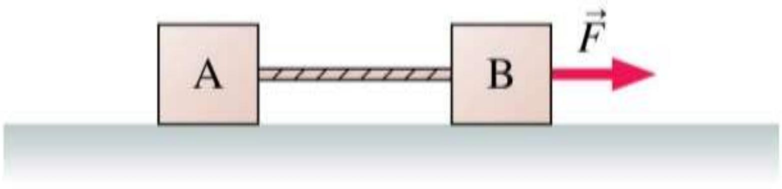
- Because the accelerations of both objects are equal, we can drop the subscripts A and B, and call both of them $\boldsymbol{a_x}$.

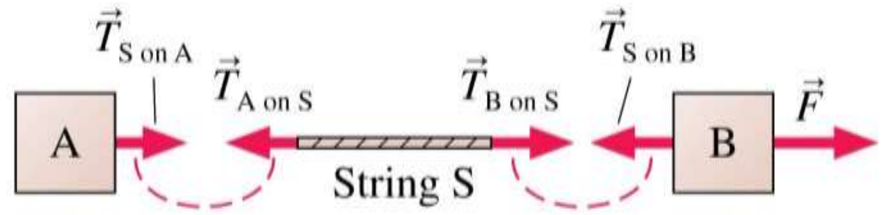
Modelling Joined Objects



- Sometimes the acceleration of A and B may have different signs.
- For the two blocks A and B, because they are joined by a string, they both experience **the same acceleration** when acted on by **gravity**.
- But, as A moves to the right in the $+x$ **direction**, B moves down in the $-\,y$ direction.
- In this case, the acceleration is $\boldsymbol{a_{Ax} = -a_{By}}$.

Modelling Joined Objects

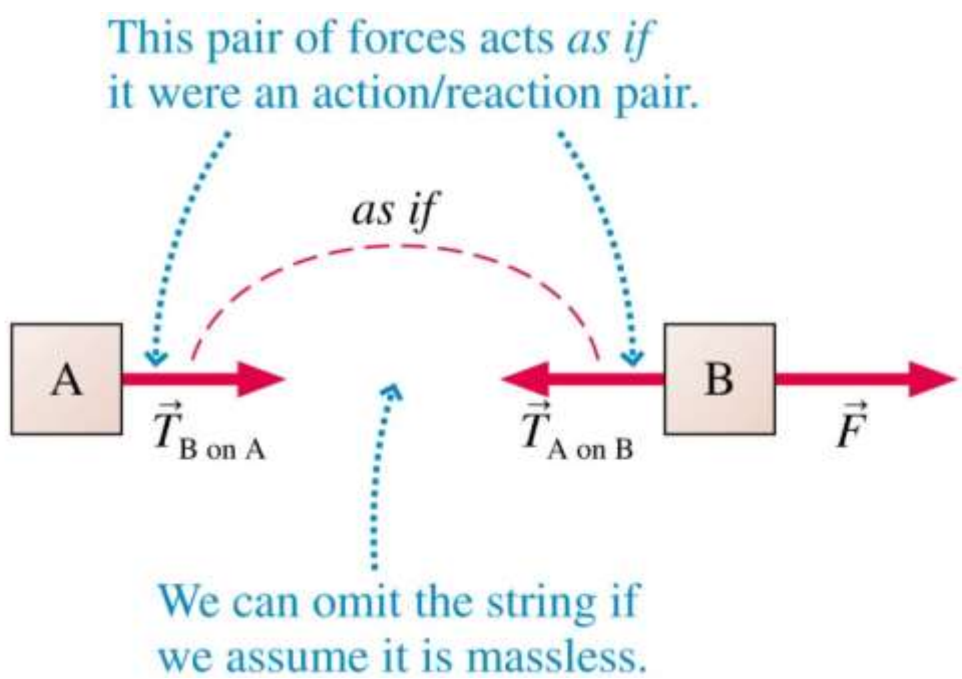




- Often in problems, the mass of the string or rope is much less than the masses of the objects that it connects.
- In such cases, we can adopt the following **massless string approximation**:

$$T_{\text{B on S}} = T_{\text{A on S}} \qquad \text{(massless string approximation)}$$

Modelling Joined Objects

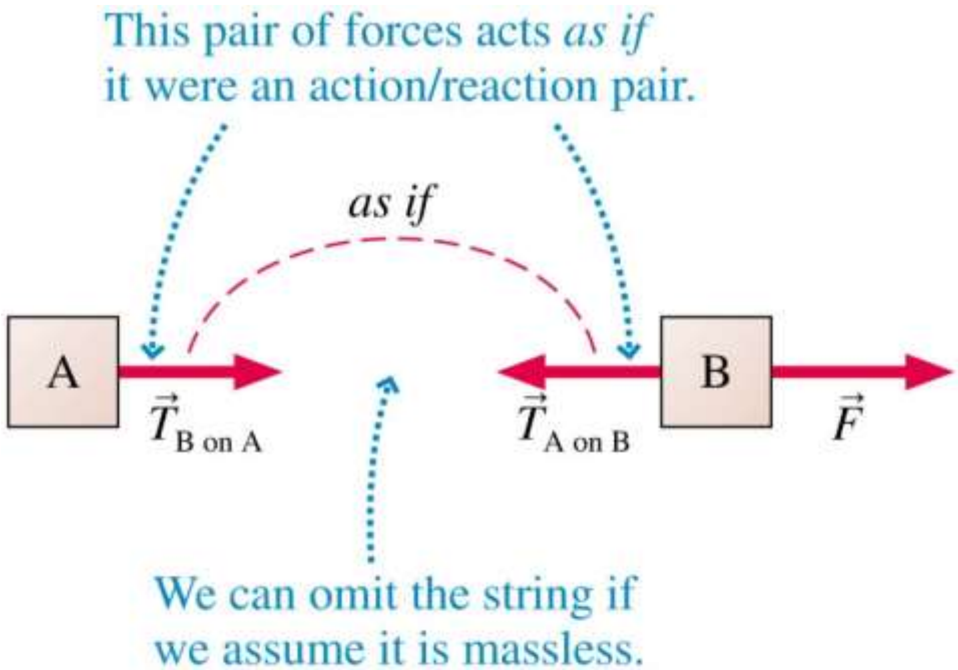


- Two blocks are connected by a massless string, as block B is pulled to the right.

- Forces $\vec{T}_{S\ on\ A}$ and $\vec{T}_{S\ on\ B}$ act ***as if*** they are an action/reaction pair:

$$\vec{T}_{S\ on\ A} = -\vec{T}_{S\ on\ B}$$

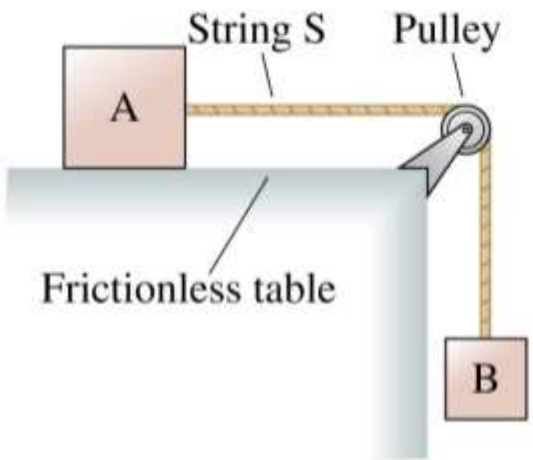
Modelling Joined Objects

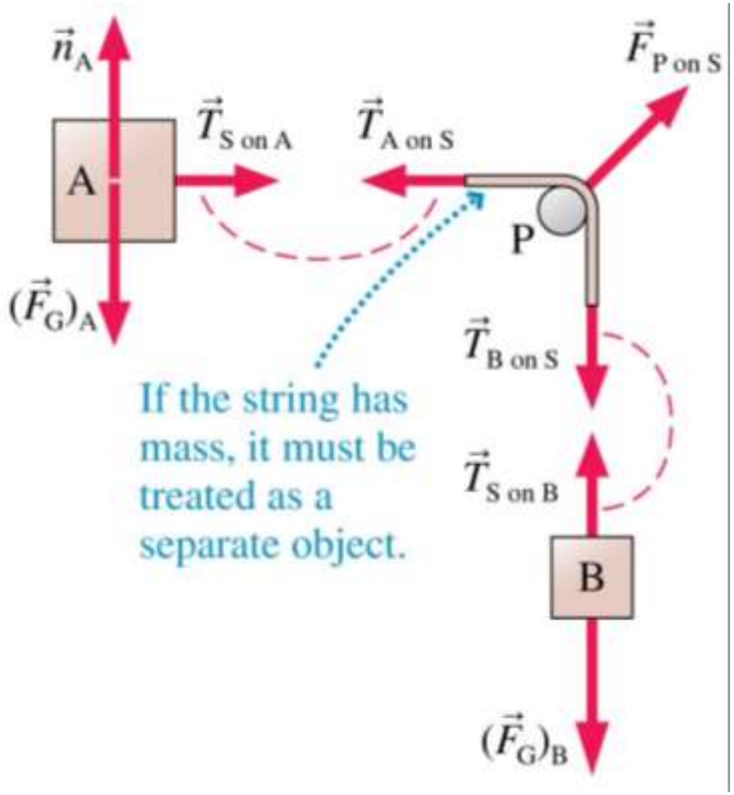


$$\vec{T}_{S\ on\ A} = -\vec{T}_{S\ on\ B}$$

- All the massless string **does** is transmit a force from A to B without **changing the magnitude of that force**.
- For problems in this module, you can assume that any strings or ropes are massless unless it explicitly states otherwise.

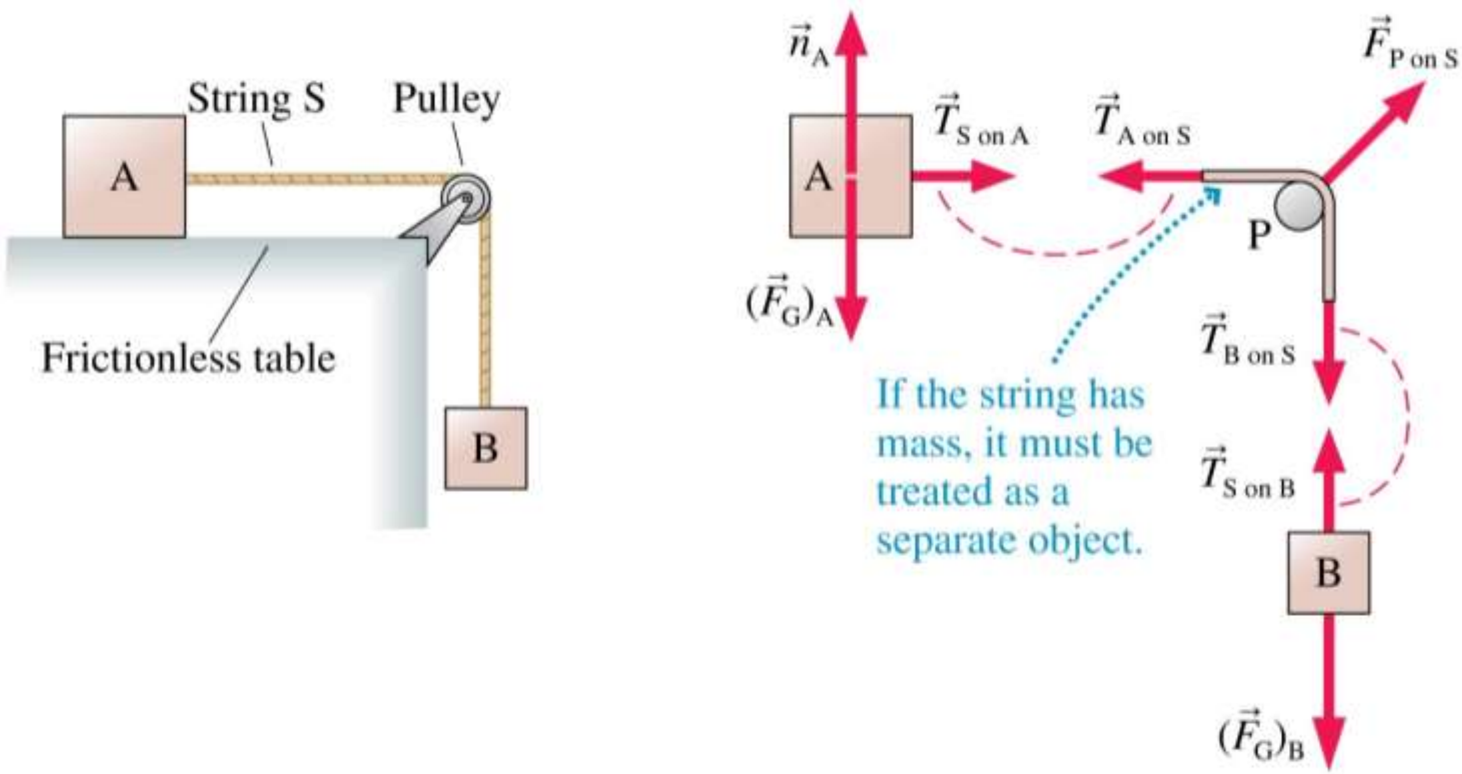
Modelling Joined Objects





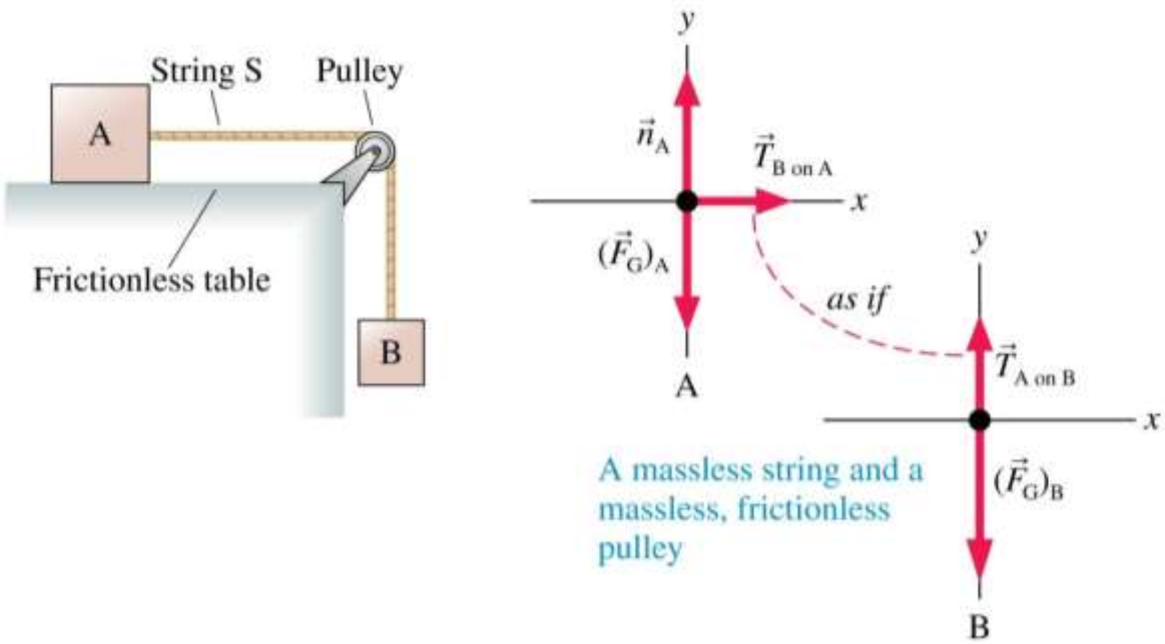
- Block B drags block A across a frictionless table as it falls.
- We assume that the string and the pulley are both massless.

Modelling Joined Objects



- We also assume that there is no friction where the pulley turns on its axle.
- Therefore, $\vec{T}_{A\ on\ S} = \vec{T}_{B\ on\ S}$.

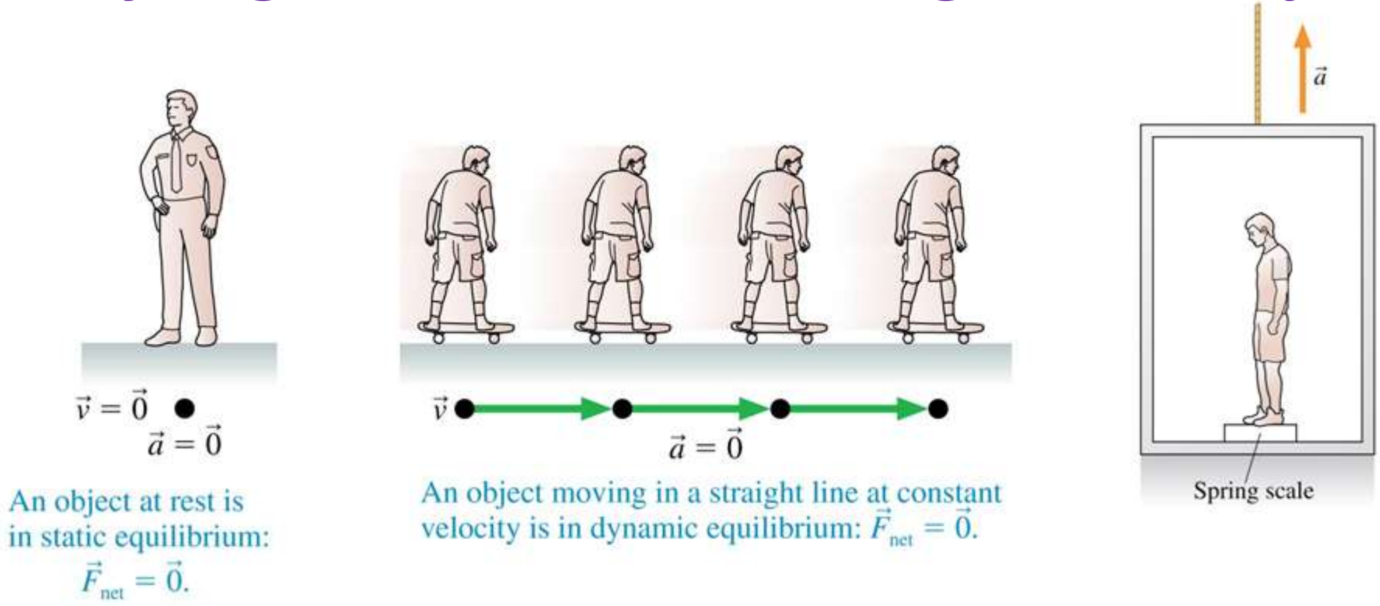
Modelling Joined Objects



- Since $\vec{T}_{A\ on\ B} = \vec{T}_{B\ on\ A}$, we can draw the simplified free-body diagram as shown above.
- Forces $\vec{T}_{A\ on\ B}$ and $\vec{T}_{B\ on\ A}$ act ***as if*** they are in an action/reaction pair, even though they are not opposite in direction because the tension force gets 'turned' by the pulley.

6. Forms of Energy

Analysing the net Force Acting on an Object



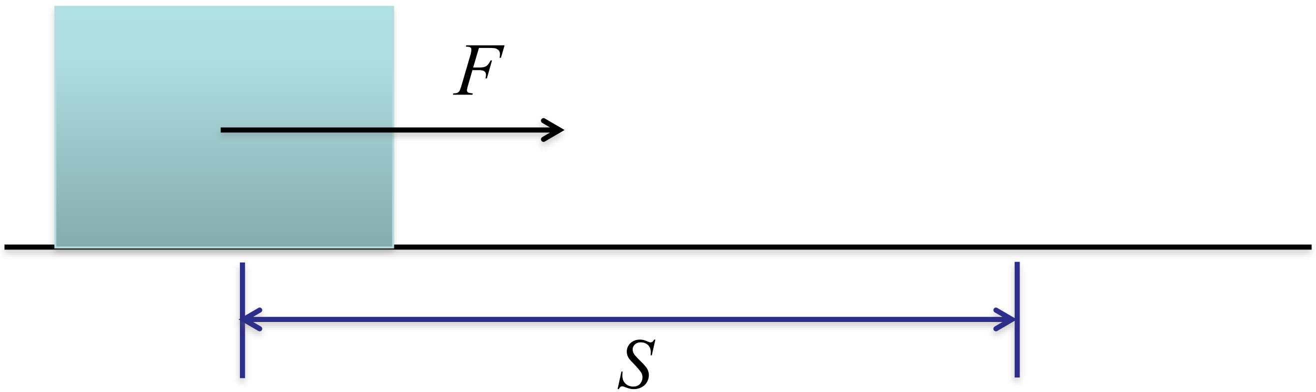
- Let **look at a relatively new idea used to describe an object which is moving**, or which has the potential to move, over a certain distance because of a **net force** acting on the object.

- In general, we call this idea **'energy'**, or **'work'**, depending on the context.

Energy

- Consider the following situation. We have an object at rest on a smooth horizontal surface.

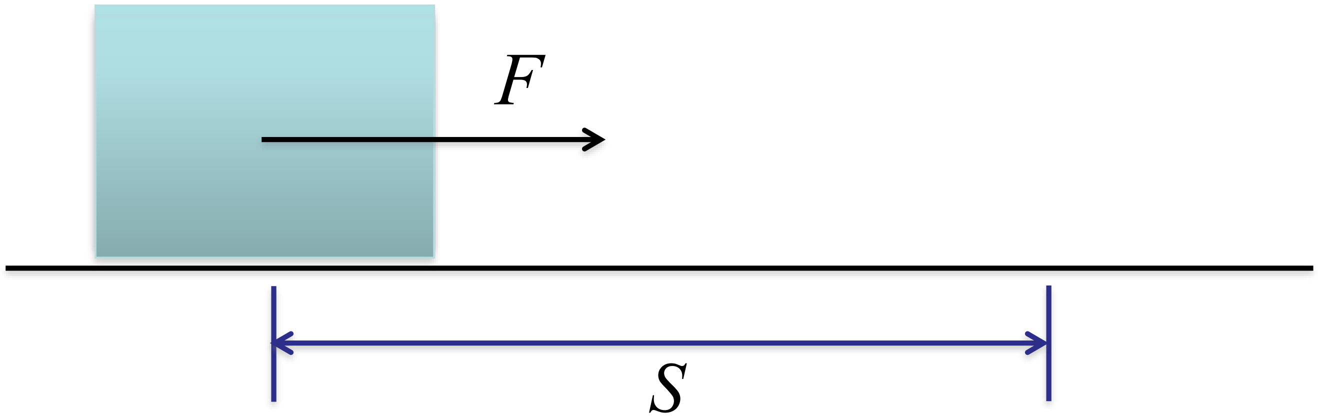
- We then apply a constant net force F, and the object travels a displacement S.



- Clearly, the **net force acting parallel** to the object's displacement causes it to accelerate over this displacement.

- We say that **'work has been done on the object',** or we have given the object . . . something . . . something we call **energy**.

Energy



- The work done on the object, or the energy given to the object is defined as:

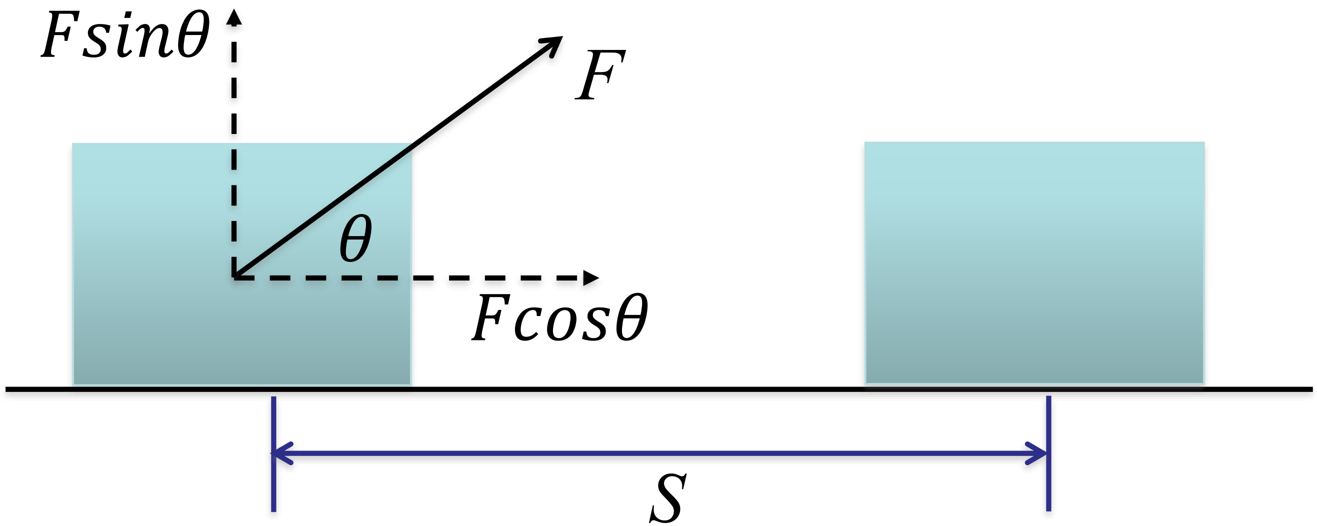
$$W = FS$$

- The unit for work is the **Joule (J)**, 1 Joule being equal to 1 N m; this is also the unit for energy.

- It is worth noting that although work is the product of two vectors, it is still a **scalar quantity**.

Energy

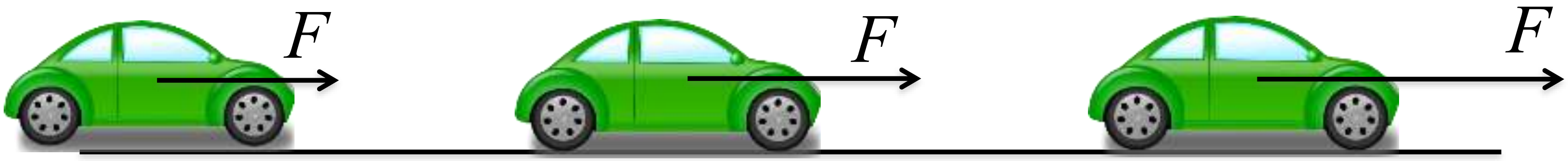
- Consider the same situation, except now the **net force F** is at an angle θ to the displacement.



- The work done will be given by the horizontal component of F multiplied by S.

$$w = FScos\theta$$

The Work Done by a Varying Force

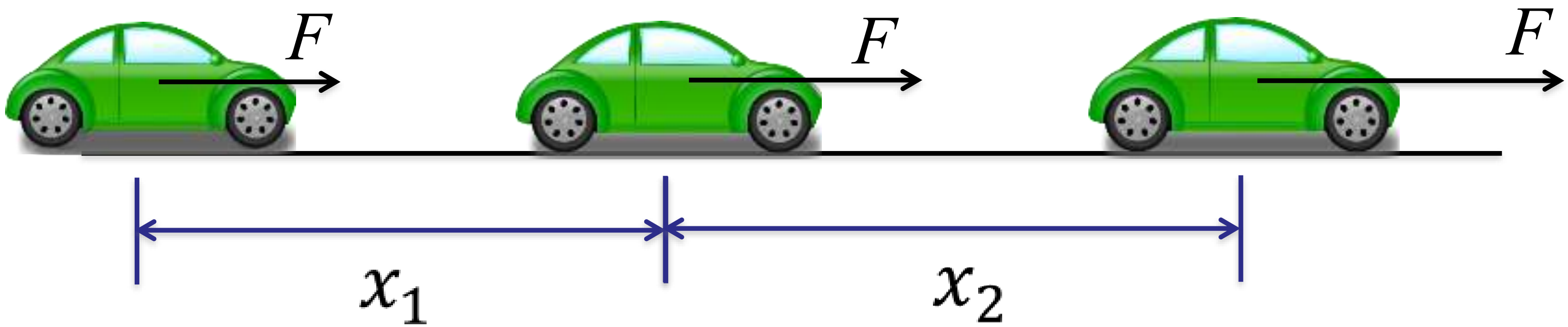


- Consider a car where the driver is constantly increasing the driving force of the vehicle as it travels along a straight road.

- Clearly the **net force acting on the object is not constant**, but varies; in this case, it is increasing.

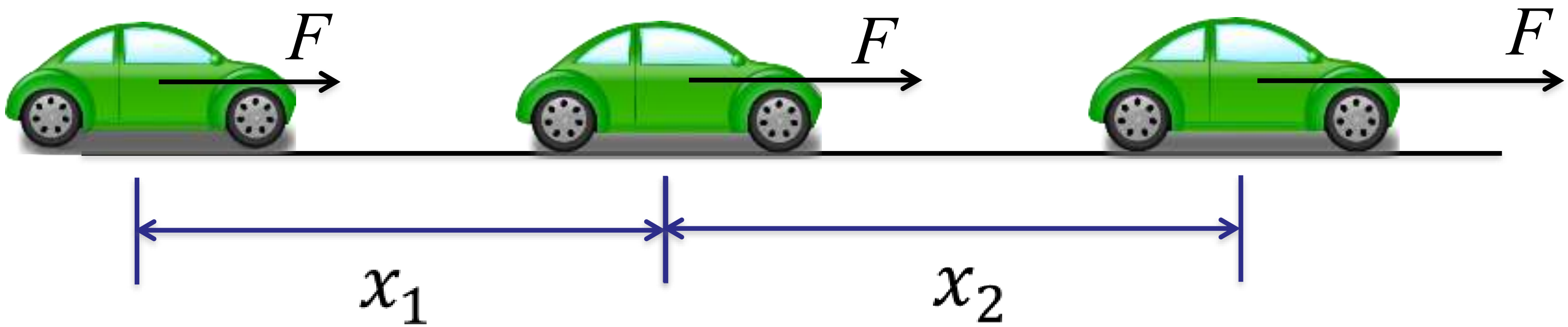
Q. How do we then calculate the work done on the object?; or the energy that is given to the object, in other words?

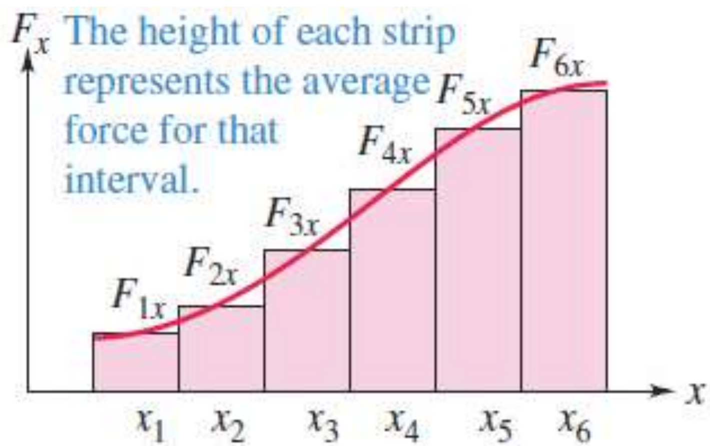
The Work Done by a Varying Force



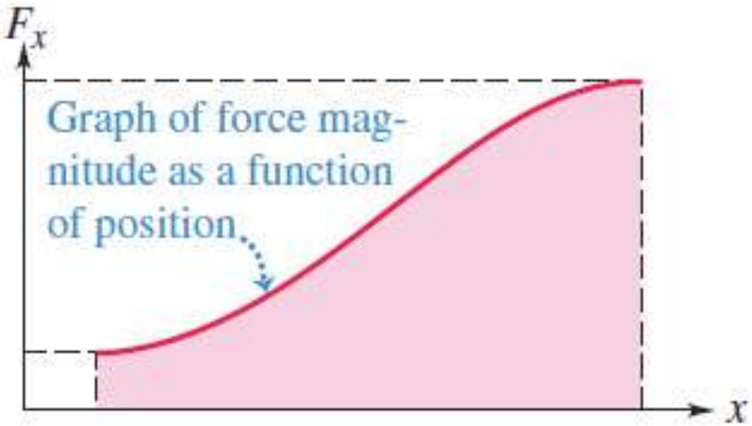
- Well, let's split the road into a number of distinct separate lengths.
- We can now approximate the work done over the distance x_1 by taking the average force over x_1, and the same for x_2, etc.

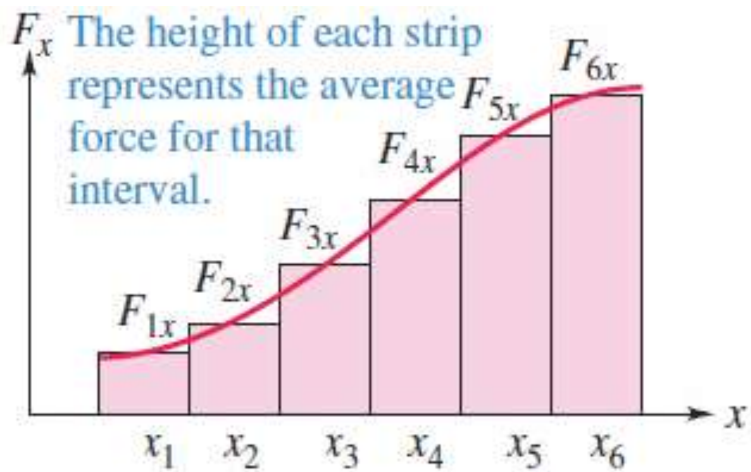
The Work Done by a Varying Force





The Work Done by a Varying Force



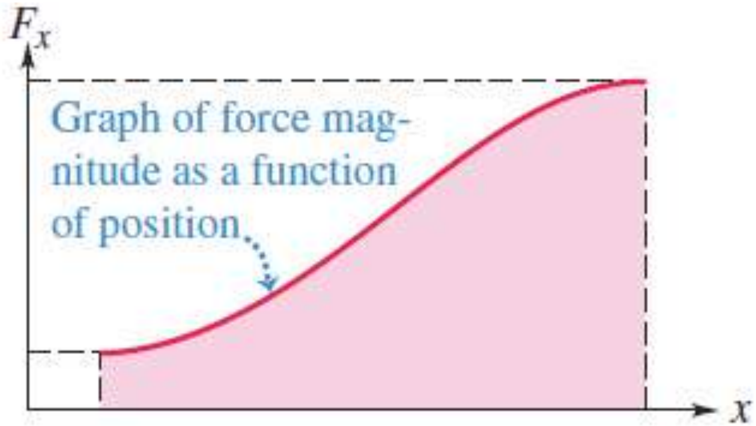


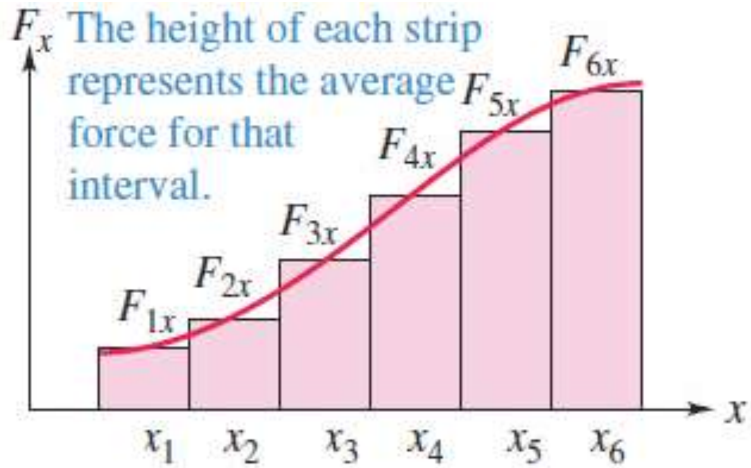
- We can see that the area under the graph actually gives us the work done on the object, or the energy given to the object, in other words. This area is another example of using **integration** in physics.

$$Area = F_{1x}x_1 + F_{2x}x_2 + \cdots + F_{6x}\,x_6 \quad or \quad W = \sum_{n=1}^{n=6}(F_n x_n)$$

$$W = \lim_{x \to 0} \sum_{n=1}^{n=\infty} F_n x_n \qquad W = \int_i^f F dx \qquad W = \int_i^f (F cos\theta) dx$$

The Work Done by a Varying Force



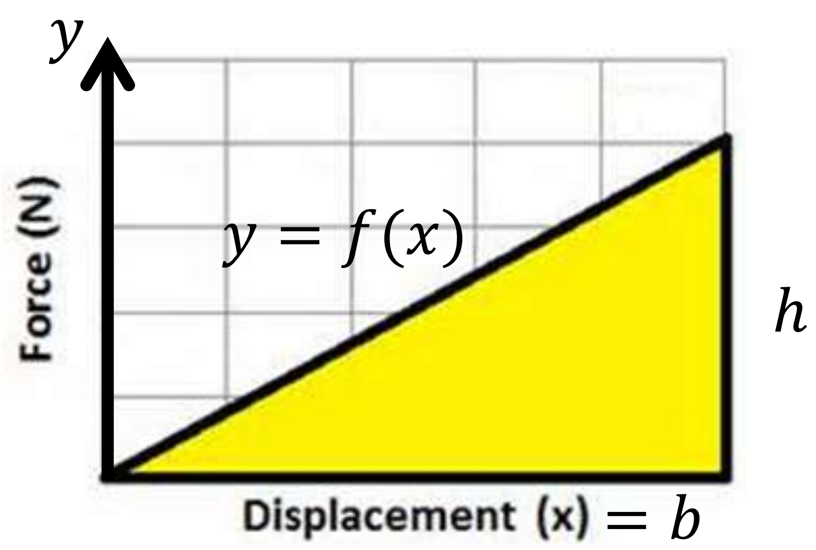


- Having now defined 'work'/'energy', as well as considered it when the force acting on the object is constant or varies, let's now consider how **energy can take different forms**, depending on the context.

Kinetic Energy, K

- All moving objects have kinetic energy.
- The more massive an object or the faster it moves, the larger its kinetic energy.
- **Kinetic energy** is the work required to bring an object from rest to some final speed over a certain displacement.

Kinetic Energy, K



$$y = mx + c \quad m = \frac{h}{b} \quad c = 0$$

$$y = \frac{h}{b}x \qquad f(x) = \frac{h}{b}x$$

$$A = \int f(x)dx \quad A = \int_0^b \left(\frac{h}{b}x\right)dx$$

- **Kinetic energy** is the work required to bring an object from rest to some final speed over a certain displacement.

- Looking at the above figure, the **integral** of this graph is

$$\frac{1}{2}N.x = \frac{1}{2}m \cdot a \cdot x = \frac{1}{2}m \cdot \frac{v}{t} \cdot x = \frac{1}{2}m \cdot v \cdot \frac{x}{t} = \mathbf{\frac{1}{2}m.v^2 = \text{kinetic energy}}$$

Potential Energy, *U*

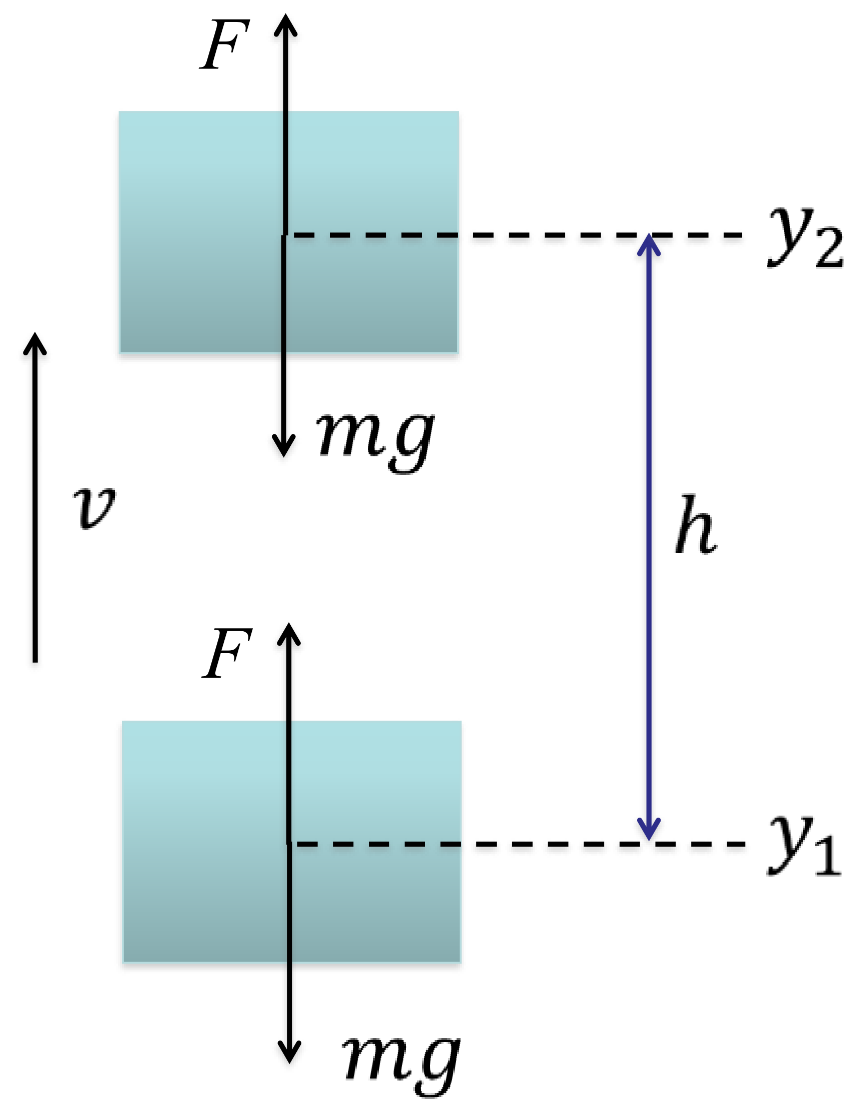
- Potential energy is stored energy associated with an object's position.

- In the above figure, we talk about the roller-coaster having a **gravitational potential energy**, depending on its height above the ground; let's define this type of energy more precisely.

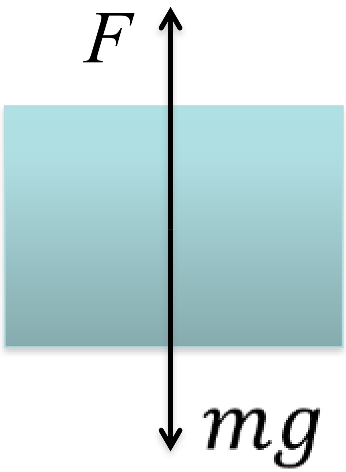
Gravitational Potential Energy, U_{grav}

- Consider an object with mass m which is raised by a constant force F at a constant velocity from a point y_1 to a point y_2.

- Let's label the distance from y_1 to y_2 as h.



Gravitational Potential Energy, U_{grav}



- Since the object is raised at a constant velocity, the forces must be in equilibrium. Therefore,

$$F = mg$$

Gravitational Potential Energy, U_{grav}

- Therefore, the work done in raising an object with a mass of $\boldsymbol{m}$ a distance of $\boldsymbol{h}$ is given by

$$w = mgh$$

- This work has transformed one form of energy (whichever type was used to provide the force $\boldsymbol{F}$) into gravitational potential energy.

$$U_{grav} = mgh$$

Elastic Potential Energy, U_s

- In physics, we tend to model objects in terms of springs (which can also be thought of as rubber bands).

- We tend to think of these springs as being able to store a potential energy that we refer to as **elastic potential energy, U_s**.
- To understand how springs, or rubber bands, store this elastic potential energy, we must look at an important idea called **Hooke's law**.

Summary of today's Lecture

1. Free-body diagrams
2. Weight
3. Static friction
4. Kinetic friction
5. Modelling joined objects

Announcement

Seminar 1

1. Students should solve all the seminar **2** questions on a presentable piece of paper and bring the solutions to the seminar class.
2. Students should come to the seminar class with their calculators, pens, pencils, and notebooks.
3. Phones, laptops, and tablets will not be used in the seminar classrooms.

Lecture 4: Recommended Readings

- **Ch. 4.6**, Free-body diagrams; p.154-158.
- **Ch. 5.3**, Contact force and friction; p.175-182.
- **Ch. 7.1**, Energy; p.224-227.
- **Ch. 7.2**, Work; p.228-231.
- **Ch. 7.3**, Work and kinetic energy; p.232-236.
- **Ch. 7.4**, Work done by varying force; p.236-238.

Online weekly Quizzes

Do not forget to complete Quiz 2

The deadline for this assessment is Friday, 25th October 2024, 3pm.