University of Nottingham
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Foundation Algebra for Physical Sciences & Engineering CELEN036

Practice Problems SET-3 **Topic:** Polynomial Factorisation

Type 1: Remainder and Factor theorems

1. Find all values of k for which $(x-1)$ is a factor of the polynomial $p(x) = k^2x^3 - 7kx + 10$.
2. Find the value(s) of k such that $\left(x + \frac{k}{2}\right)$ and $(x+2k)$ are factors of $x^2 + \left(\frac{15}{2}\right)x + 9$.
3. Find constants a and b such that $ax^3 - bx^2 + 45x + 54 = 0$ has a root 3, and yields a remainder of 12 when divided by $(x+1)$.
4. Find the polynomial $p(x)$ with roots of $x = \pm 1, \pm 2$ and leading term of $117x^4$.
5. Two cubic polynomials are defined by $f(x) = x^3 + (a-3)x + 2b$, $g(x) = 3x^3 + x^2 + 5ax + 4b$, where a and b are constants. Given that $f(x)$ and $g(x)$ have a common factor of $(x-2)$. Find the value of a and b.

Type 2: Methods of long division and synthetic division

6. Find the quotient $q(x)$ and the remainder $r(x)$ that result when $p(x)$ is divided by $s(x)$.

   (i) $p(x) = x^4 + 3x^3 - 5x + 10$; $s(x) = x^2 - x + 2$

   (ii) $p(x) = 6x^4 + 10x^2 + 5$; $s(x) = 3x^2 - 1$

   (iii) $p(x) = 5x^4 - 3x^3 + 2x^2 - 1$; $s(x) = x^2 + 4$

   (iv) $p(x) = -x^5 + 7x^3 - x$; $s(x) = x^3 - x^2 + 1$

7. Use the method of synthetic division to find the quotient $q(x)$ and the remainder $r(x)$ that result when $p(x)$ is divided by $s(x)$.

   (i) $p(x) = 2x^4 + 3x^3 - 17x^2 - 27x - 9$; $s(x) = x + 4$

   (ii) $p(x) = 3x^3 - 4x - 1$; $s(x) = x - 2$

   (iii) $p(x) = x^4 - 5x^2 + 4$; $s(x) = x + 5$

   (iv) $p(x) = 18x^2 - 15x - 25$; $s(x) = x - \frac{5}{3}$

   (v) $p(x) = x^4 - 6x^2 + 9$; $s(x) = x - \sqrt{3}$

   (vi) $p(x) = x^6 - 6x^4 + 12x^2 - 8$; $s(x) = x + \sqrt{2}$

   (vii) $p(x) = x^3 - 3x^2 - 3x + 3$; $s(x) = x + \sqrt{3} - 2$

8. Given $p_1(x) = x^3 + 4x^2 + x - 6$. Find a polynomial $q(x)$ and a constant r such that:

   (i) $p(x) = (x-2)q(x) + r$ (ii) $p(x) = (x+1)q(x) + r$

9. Use synthetic division method to divide $4 - 8x - 12x^2$ by $2x - 3$. Hint: factor $2x - 3$ as $2(x - \frac{3}{2})$.

10. Use synthetic division to divide $f(x) = x^3 + x^2 - 26x + 24$ by $x + 6$. Use the result to find all zeros.

Type 3: Polynomial factorisation and solving

11. Factorize the following polynomials completely:

    (i) $p(x) = x^3 - 2x^2 - x + 2$ (ii) $p(x) = 3x^3 + x^2 - 12x - 4$

12. Factorize the following polynomials and solve $p(x) = 0$ for $x \in \mathbb{R}$ in each case.

    (i) $p(x) = x^3 - x^2 - 10x - 8$ (ii) $p(x) = x^3 - x^2 - 16x - 20$

    (iii) $p(x) = x^3 + 4x^2 - 8$ (iv) $p(x) = 2x^3 - 3x^2 - 11x + 6$

13. If $(x + 3)$ and $(x - 4)$ are both factors of $2x^3 + 3x^2 - 29x - 60$, find the third factor.

14. Let $p(x) = 4x^4 - 4x^3 - 11x^2 + 12x - 3$. Given that $x = \frac{1}{2}$ is a zero of multiplicity 2, find all of the real zeros of p.

15. Let $\cos\theta = x$ to find the all the solution for θ of the equation $4\cos^3\theta - 7\cos\theta - 3 = 0$ for $0 \le \theta \le 2\pi$.

Type 4: Partial fractions

$$\boxed{\frac{1}{(x+a)(x+b)} = \frac{A}{(x+a)} + \frac{B}{(x+b)}}$$

16. Express the following as the sum of partial fractions:

    (i) $\dfrac{3x}{(x-1)(x+2)}$ (ii) $\dfrac{3x-2}{(x+1)(x-1)}$ (iii) $\dfrac{1}{x^2+5x+6}$

    (iv) $\dfrac{2x+3}{x^2+3x+2}$ (v) $\dfrac{3}{x(x-1)(x+1)}$ (vi) $\dfrac{5x+1}{(x-1)(x-2)(x-3)}$

$$\boxed{\frac{1}{(x+a)(x^2+b)} = \frac{A}{(x+a)} + \frac{Bx+C}{(x^2+b)}}$$

17. Express the following as the sum of partial fractions:

    (i) $\dfrac{13}{(x^2+1)(2x+3)}$ (ii) $\dfrac{3x+1}{(x^2+x+10)(x-1)}$ (iii) $\dfrac{x}{(x^2-x+1)(3x-2)}$

$$\boxed{\frac{1}{(x+a)^2(x+b)} = \frac{A}{(x+a)} + \frac{B}{(x+a)^2} + \frac{C}{(x+b)}}$$

18. Express the following as the sum of partial fractions:

    (i) $\dfrac{5}{(x+1)^2(x-1)}$ (ii) $\dfrac{3x}{(x+2)^2(x-1)}$ (iii) $\dfrac{x^2+1}{(x-1)^2(x+1)}$

Answers

1 $k = 2,\ 5$

2 $k = 3$

3 $a = -6,\ b = 3$

4 $p(x) = 117(x+1)(x-1)(x+2)(x-2)$

5 $a = -4,\ b = 3$

6 (i) $q(x) = x^2 + 4x + 2,\ r(x) = -11x + 6$ (ii) $q(x) = 2x^2 + 4,\ r(x) = 9$

(iii) $q(x) = 5x^2 - 3x - 18,\ r(x) = 12x + 71$ (iv) $q(x) = -x^2 - x + 6,\ r(x) = 7x^2 - 6$

7 (i) $q(x) = 2x^3 - 5x^2 + 3x - 39,\ r(x) = 147$ (ii) $q(x) = 3x^2 + 6x + 8,\ r(x) = 15$

(iii) $q(x) = x^3 - 5x^2 + 20x - 100,\ r(x) = 504$ (iv) $q(x) = 18x + 15,\ r(x) = 0$

(v) $q(x) = x^3 + \sqrt{3}x^2 - 3x - 3\sqrt{3},\ r(x) = 0$ (vi) $q(x) = x^5 - \sqrt{2}x^4 - 4x^3 + 4\sqrt{2}x^2 + 4x - 4\sqrt{2},\ r(x) = 0$

(vii) $q(x) = x^2 - (1+\sqrt{3})x - \sqrt{3} - 2,\ r(x) = 2$

8 (i) $q(x) = x^2 + 6x + 13,\ r = 20$ (ii) $q(x) = x^2 + 3x - 2,\ r = -4$

9 $(2x-3)(-6x-13) - 35$

10 $-6,\ 4,\ 1$

11 (i) $(x-1)(x+1)(x-2)$ (ii) $(x-2)(x+2)(3x+1)$

12 (i) $-1,\ -2,\ 4$ (ii) $-2,\ 5$

(iii) $-2,\ -1 \pm \sqrt{5}$ (iv) $-2,\ \frac{1}{2},\ 3$

13 $(2x+5)$

14 $\frac{1}{2},\quad \sqrt{3},\quad -\sqrt{3}$

15 $\frac{2\pi}{3},\ \pi,\ \frac{4\pi}{3}$

16 (i) $\frac{1}{x-1} + \frac{2}{x+2}$ (ii) $\frac{5/2}{x+1} + \frac{1/2}{x-1}$ (iii) $\frac{1}{x+2} + \frac{-1}{x+3}$

(iv) $\frac{1}{x+1} + \frac{1}{x+2}$ (v) $\frac{3/2}{x+1} + \frac{3/2}{x-1} + \frac{-3}{x}$ (vi) $\frac{3}{x-1} + \frac{-11}{x-2} + \frac{8}{x-3}$

17 (i) $\frac{4}{2x+3} + \frac{-2x+3}{x^2+1}$ (ii) $\frac{1/3}{x-1} + \frac{-x/3+7/3}{x^2+x+10}$ (iii) $\frac{6/7}{3x-2} + \frac{-2x/7+3/7}{x^2-x+1}$

18 (i) $\frac{-5/4}{x+1} + \frac{-5/2}{(x+1)^2} + \frac{5/4}{x-1}$ (ii) $\frac{-1/3}{x+2} + \frac{2}{(x+2)^2} + \frac{1/3}{x-1}$ (iii) $\frac{1/2}{x-1} + \frac{1}{(x-1)^2} + \frac{1/2}{x+1}$