The University of Nottingham

UNITED KINGDOM · CHINA · MALAYSIA

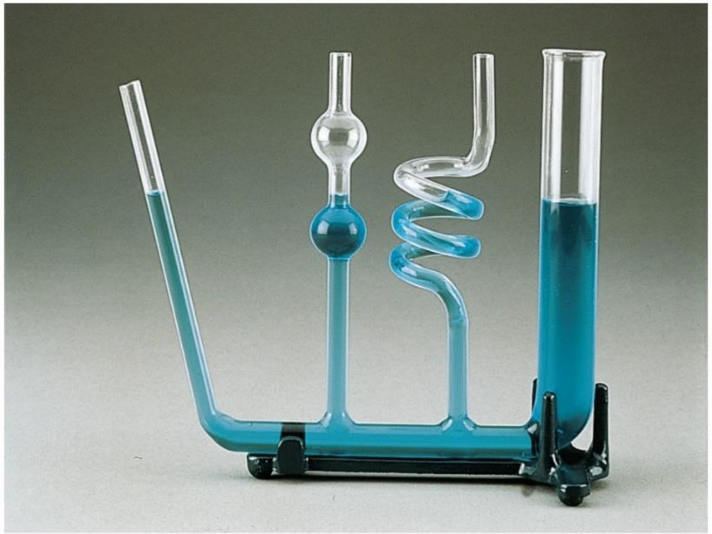
# Science A Physics

## Lecture 8: Fluid Dynamics and Light

# Aims of today's lecture

1. The Continuity Equation
2. Bernoulli's Equation
3. The Venturi Effect
4. The Ray Model of Light
5. Reflection
6. Refraction
7. Total Internal Reflection

# Fluid Mechanics



- In the previous, when looking at the hydraulic machine, it was necessary to look at a branch of fluid mechanics called hydrostatics; that is, we considered fluids which are stationary or in equilibrium.
- In this lecture, we are going to look at the other branch of fluid mechanics called fluid dynamics; that is, fluids which are moving.
- We are going to look at several important ideas, the first of which is called the **continuity equation**.

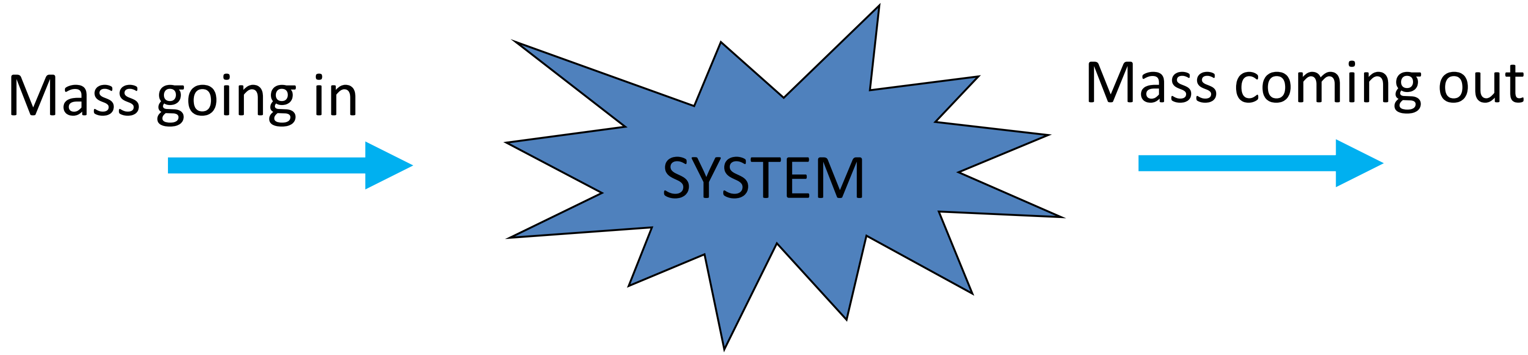
# 1. The Continuity Equation

# The Continuity Equation

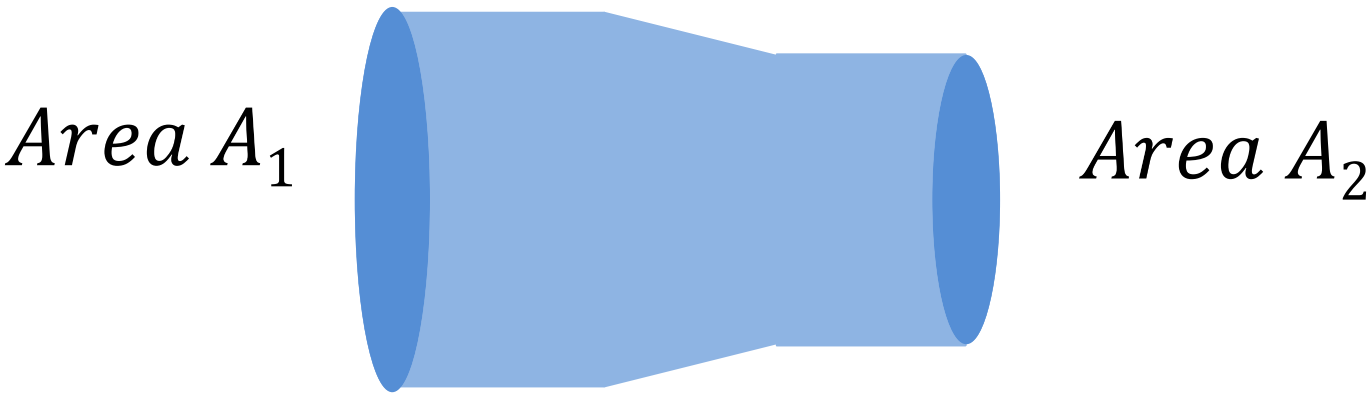
- You may have noticed that the speed, $v$, of water from a garden hose, or tap, depends on the cross-sectional area $A$.
- As gravity accelerates the flow, the cross-sectional area must decrease. Let's see why.

# The Continuity Equation

- If there is no accumulation of mass, the rate of what goes in must also be the rate of what comes out.



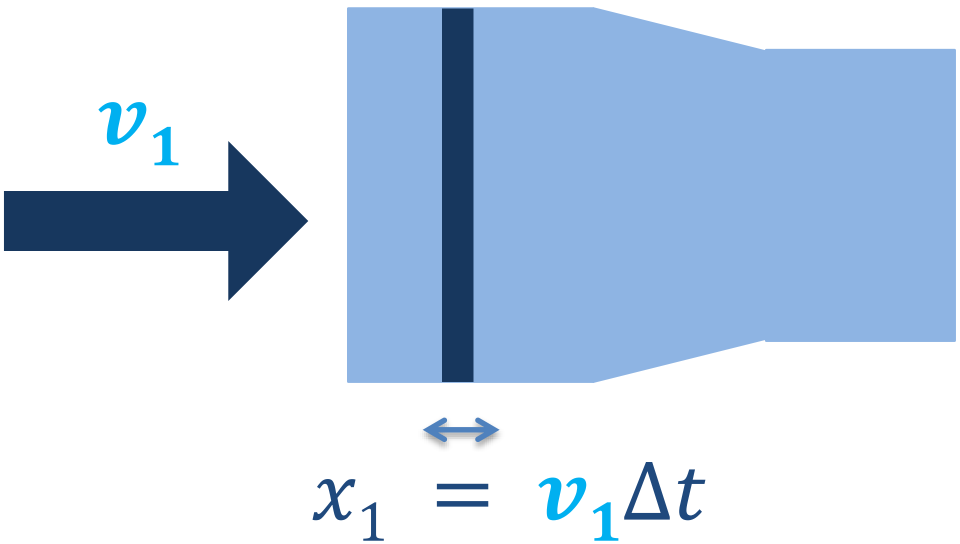
# The Continuity Equation




- Consider the flow of a fluid through a pipe with varying cross-section.
- Assume that the fluid is incompressible

# The Continuity Equation

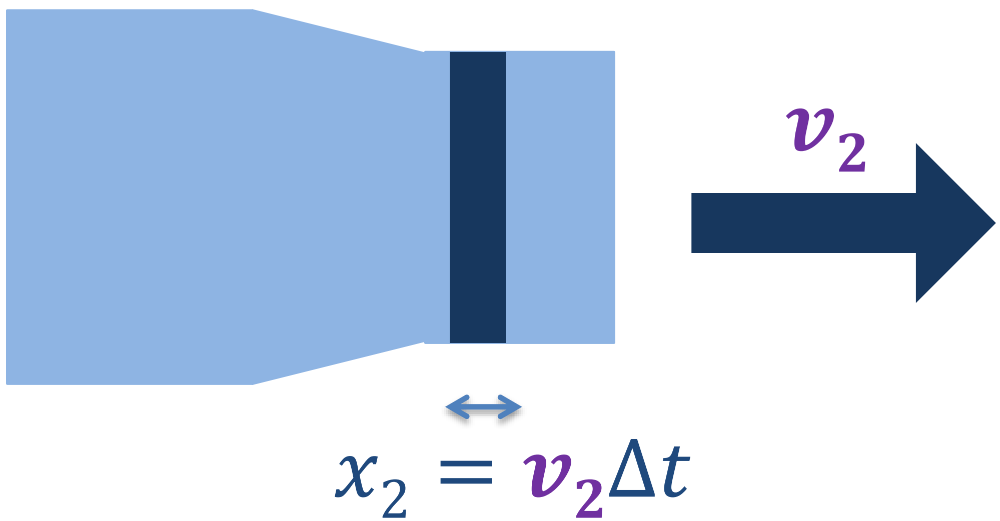
- In a short time $\Delta t$, the mass travels a distance $x_1 = \boldsymbol{v_1}\Delta t$.




- So the volume $\Delta V$ passing through $A_1$ during $\Delta t$ is $area \times length = \boldsymbol{A_1 v_1}\Delta t$

# The Continuity Equation

- In the same time, the distance travelled through the narrower section is $x_2 = \boldsymbol{v_2}\Delta t$,




so the volume $\Delta V$ passing through $\boldsymbol{A_2}$ during $\Delta t$ is
$area \times length = \boldsymbol{A_2 v_2}\Delta t$

# The Continuity Equation

- Conservation of mass now relates the two terms:

$$\begin{aligned}\Delta m &= density \times volume \\ &= \rho \Delta V \\ &= \rho \boldsymbol{A_1 v_1} \Delta t = \rho \boldsymbol{A_2 v_2} \Delta t\end{aligned}$$

This leads to the continuity equation

$$\boldsymbol{A_1 v_1} = \boldsymbol{A_2 v_2}$$

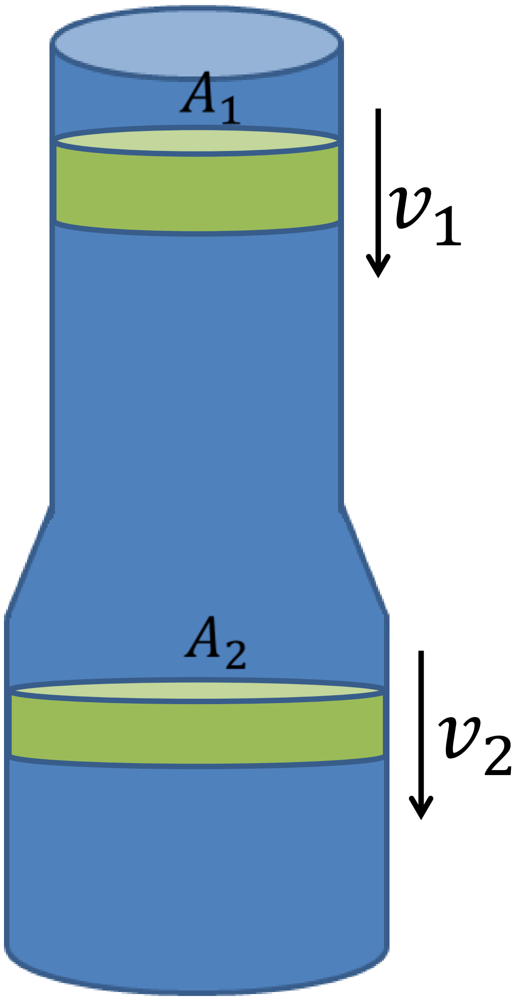
(for an **incompressible** fluid).

# The Continuity Equation

- The product of **speed, $v$, and area $A$** is known as the **volume flow rate,** as it represents the **volume of water flowing** through a section of the tube **per second**

$$\frac{\Delta V}{\Delta t} = vA$$

- The volume flow rate will be **constant at all points.**



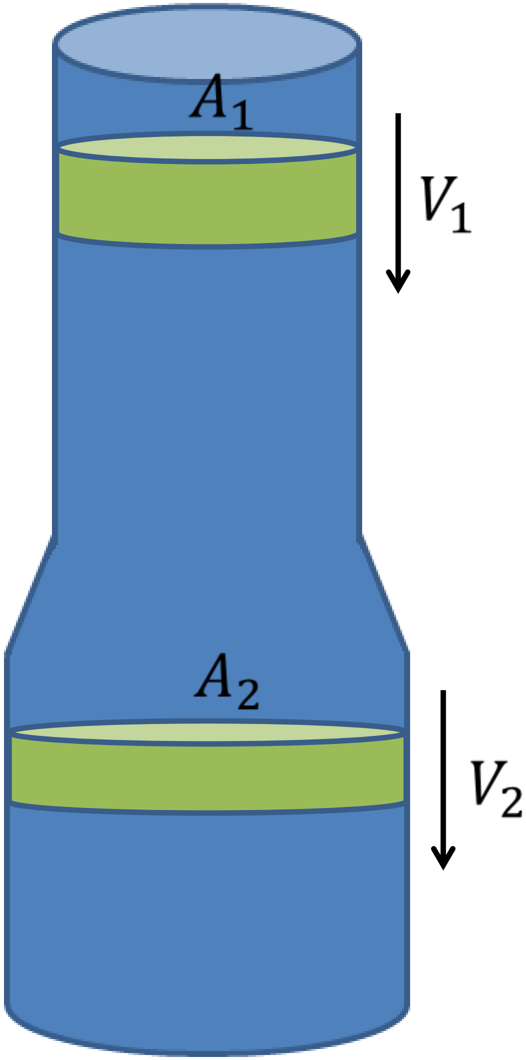
# The Continuity Equation

- We can also calculate the **mass flow rate** using the relationship between mass, volume and density;

$$m = \rho V$$

$$\frac{\Delta m}{\Delta t} = \rho \frac{\Delta V}{\Delta t} = \rho v A$$

- The mass flow rate will be **constant at all points.**



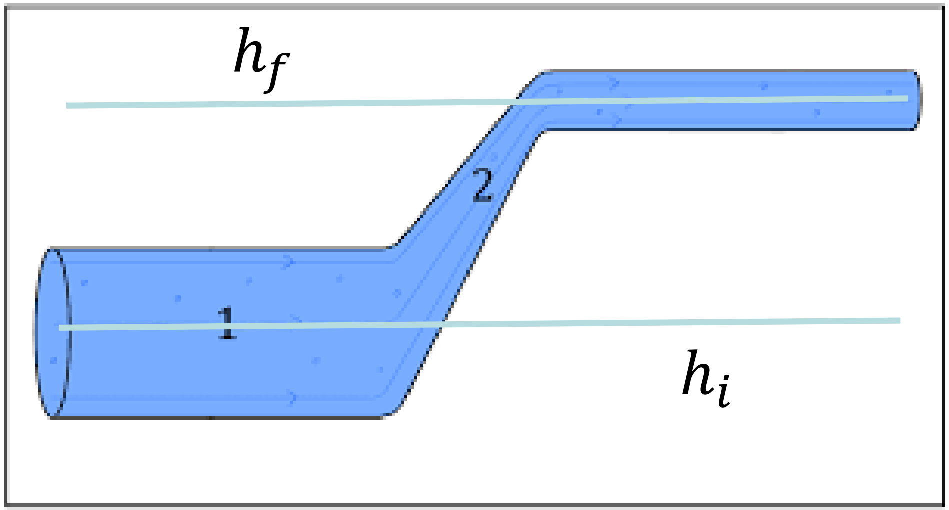
Daniel Bernoulli

1700-1782

# 2. The Bernoulli Equation

# Bernoulli Equation's

Consider water 'flowing up hill'. Can we determine the new parameters?




Final flow

$P_f, v_f, h_f, r_f$

Initial flow

$P_i, v_i, h_i, r_i$

Assume incompressible fluid.

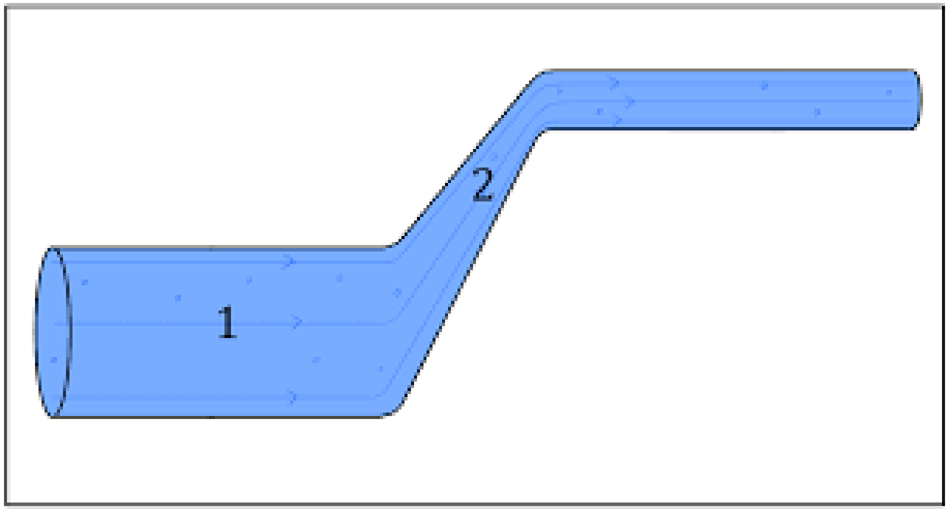
Conservation of mass.

Conservation of energy.

# Bernoulli Equation's

Initial flow

$F_i, P_i, v_i, h_i, r_i$




Final flow

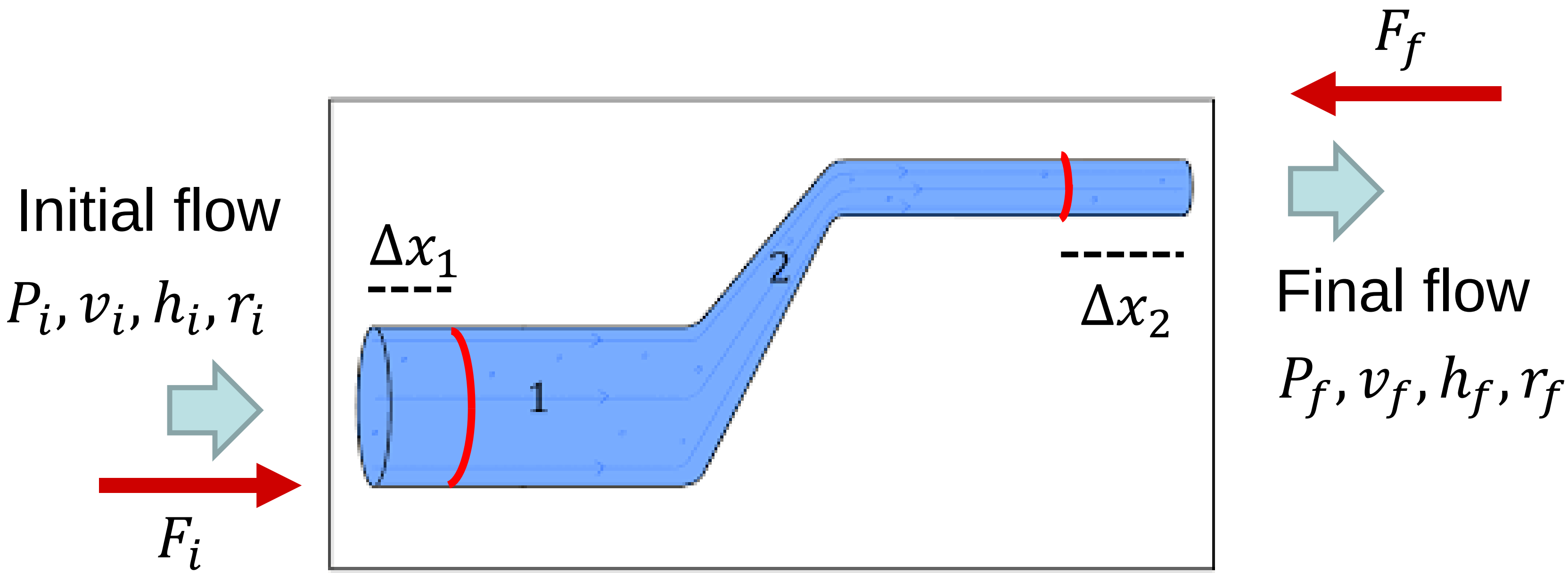
$F_{f,} P_f, v_f, h_f, r_f$

## The Work-Energy Theorem

$$\Delta K = W_{net}$$

- $\Delta K$ is the change in kinetic energy of the volume of fluid we consider during the motion we're considering,
- $W_{net}$ is the net work done by the forces acting on our system.

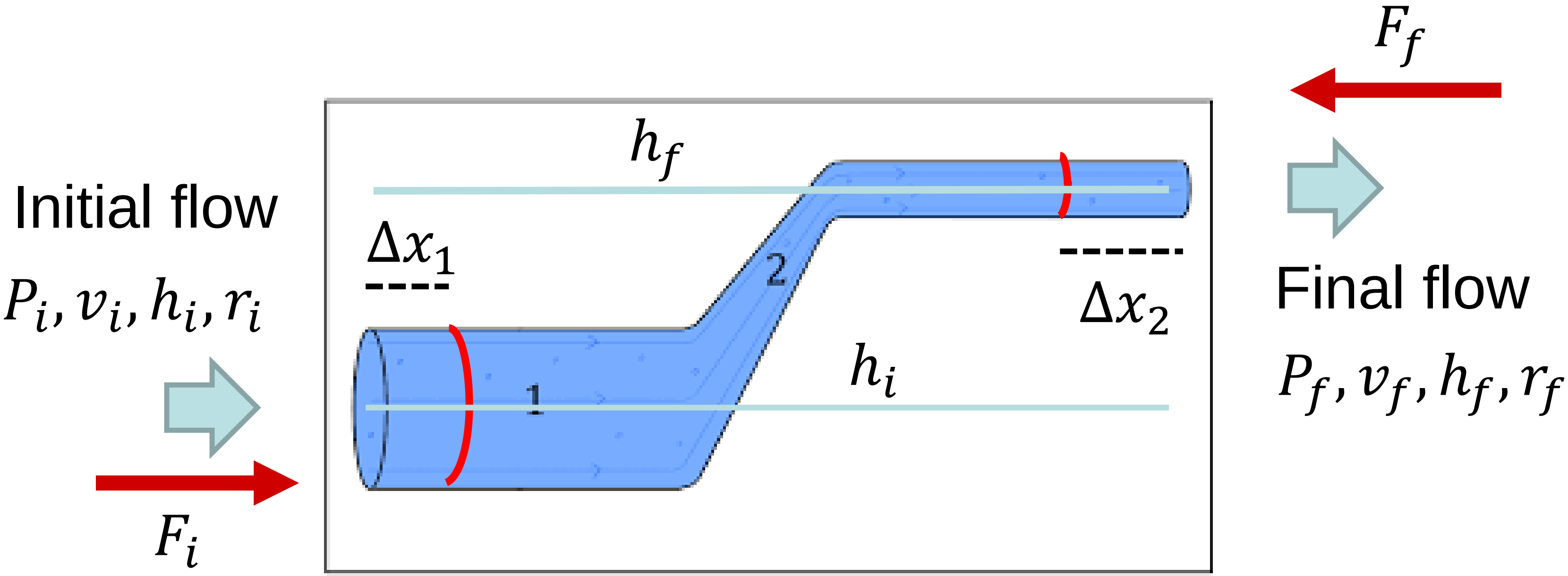
# Bernoulli Equation’s




- At each end, there is a force $F_{ext} = pA$ due to external pressure.
- The forces at the two ends are different, and in opposite directions.
- As $work = force \times distance$, the work is

$$W = p_1 A_1 \Delta x_1 - p_2 A_2 \Delta x_2$$

# Bernoulli Equation's



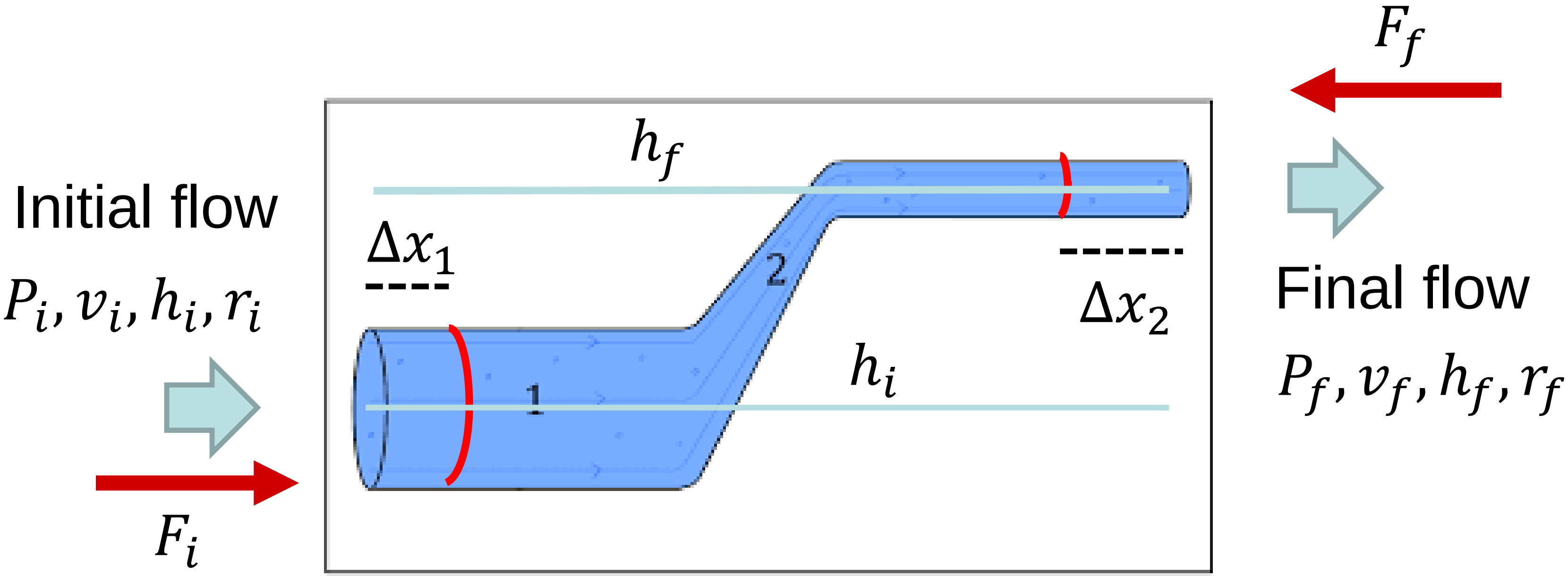

Note that $A_1 \Delta r_1 = A_2 \Delta r_2 = \Delta V$,

so $W = (p_1 - p_2)\,\Delta V$.

Work ($W_{grav}$) is also done by gravity in raising the mass of fluid through a height $h_f - h_i$.

$$W_{grav} = -\,mg(h_f - h_i).$$

# Bernoulli Equation's




The total work done is then

$$W_{net} = W + W_{grav}$$

$$= (p_1 - p_2)\, \Delta V - mg(h_f - h_i).$$

As energy is conserved, $W_{net}$ must be equal to the difference in kinetic energy of the system:

$$W_{net} = \Delta KE = ½ m{v_2}^2 - ½ m{v_1}^2.$$

# Bernoulli's Equation — Energy Conservation

## Collecting everything together:

$$W_{net} = \Delta KE$$

$$(p_1 - p_2)\,\Delta V \;-\; mg(h_f - h_i) \;=\; ½mv_2{}^2 \;-\; ½mv_1{}^2$$

- For liquids, density is usually more convenient than mass:
  $m = \rho\,\Delta V$.

Substituting for $m$ in the equation so far gives

$$(p_1 - p_2)\,\cancel{\Delta V} - \rho\,\cancel{\Delta V}\,g(h_f - h_i) = ½\rho\,\cancel{\Delta V}\,v_2{}^2 - ½\rho\,\cancel{\Delta V}\,v_1{}^2.$$

$$(p_1 - p_2) - \rho g(h_f - h_i) \;=\; ½\rho\,v_2{}^2 \;-\; ½\rho\,v_1{}^2$$

# Bernoulli's Equation

$$(p_1 - p_2) - \rho g(h_f - h_i) = ½\rho\, v_2{}^2 - ½\rho\, v_1{}^2$$

$$\boxed{\frac{1}{2}\rho v_1{}^2 + \rho g h_i + p_1 = \frac{1}{2}\rho v_2{}^2 + \rho g h_f + p_2}$$

# Bernoulli's Equation

$$p_1 + \frac{1}{2}\rho {v_1}^2 + \rho g h_i = p_2 + \frac{1}{2}\rho {v_2}^2 + \rho g h_f = constant$$

- Consider what happens if the velocity increases whilst the height remains constant.

- As the flow rate increases, the pressure must decrease; therefore:
  - where the flow speed is high, the pressure is low.
  - where flow speed is low, pressure is high.

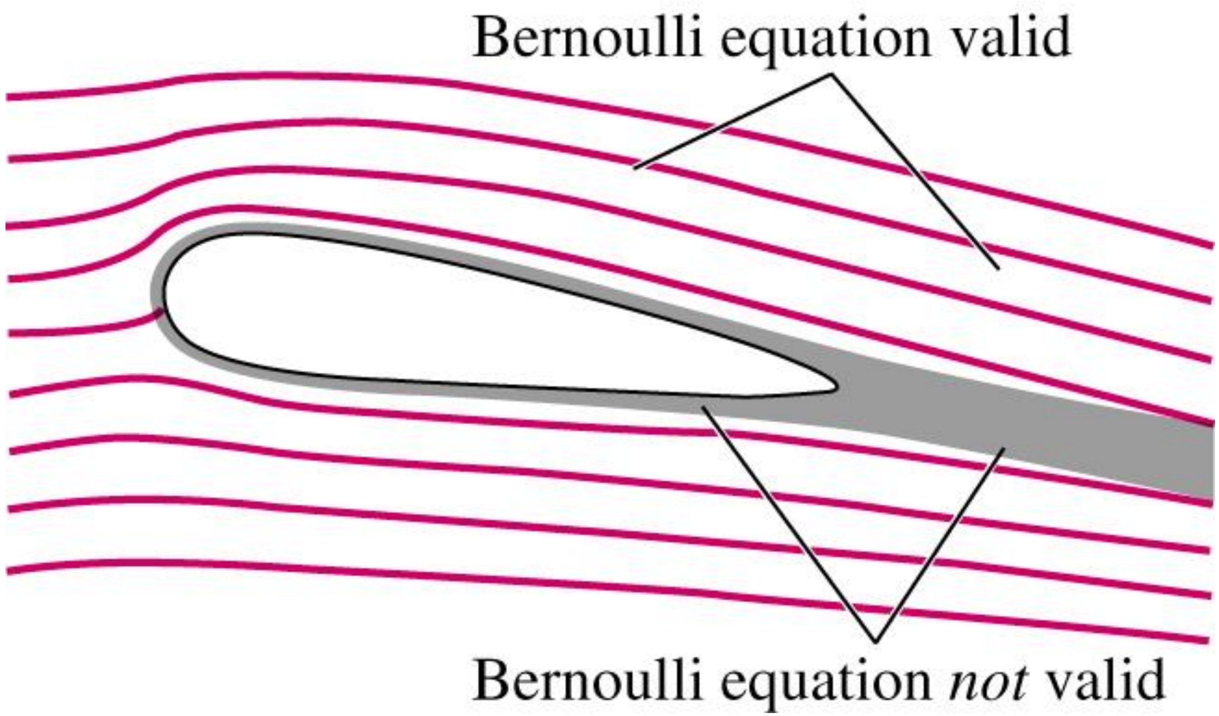
# Bernoulli's Equation—Its Limitations

- Assumes an **incompressible** fluid, or at least that the variations in density are small.
  - Not used for gases when the pressure changes are significant.

- Requires **steady flow** with negligible friction.

- Only valid along a streamline.

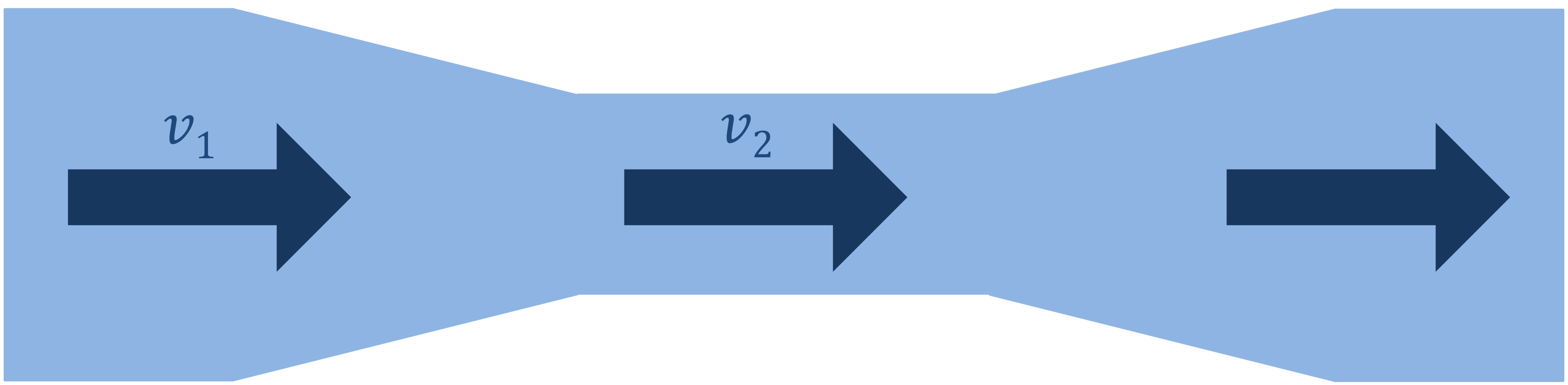
# Bernoulli's Equation—Its Limitations




- Streamlines are tangent to the fluid's velocity vector at each point.
- Let's now consider some applications for the Bernoulli equation, one of which is the **Venturi effect**, named after an Italian.

# 3. The Venturi Effect

# The Venturi effect

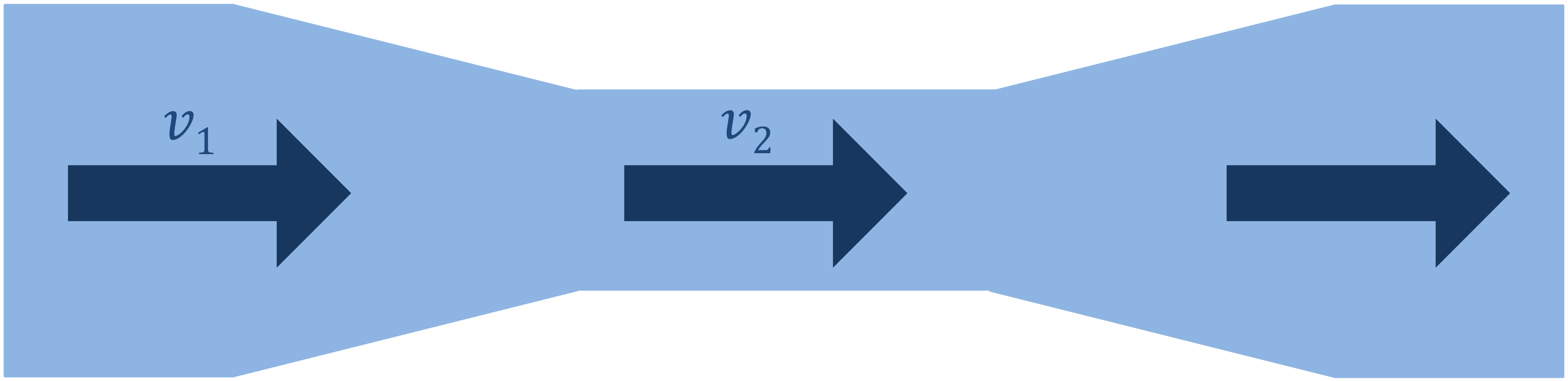



**Q.** Is the pressure in the middle section of the pipe higher or lower than at the edges?

The continuity equation says $A_1 v_1 = A_2 v_{2.}$

$A_1 > A_2$, so it follows that $v_1 < v_2$, as before.

# The Venturi effect

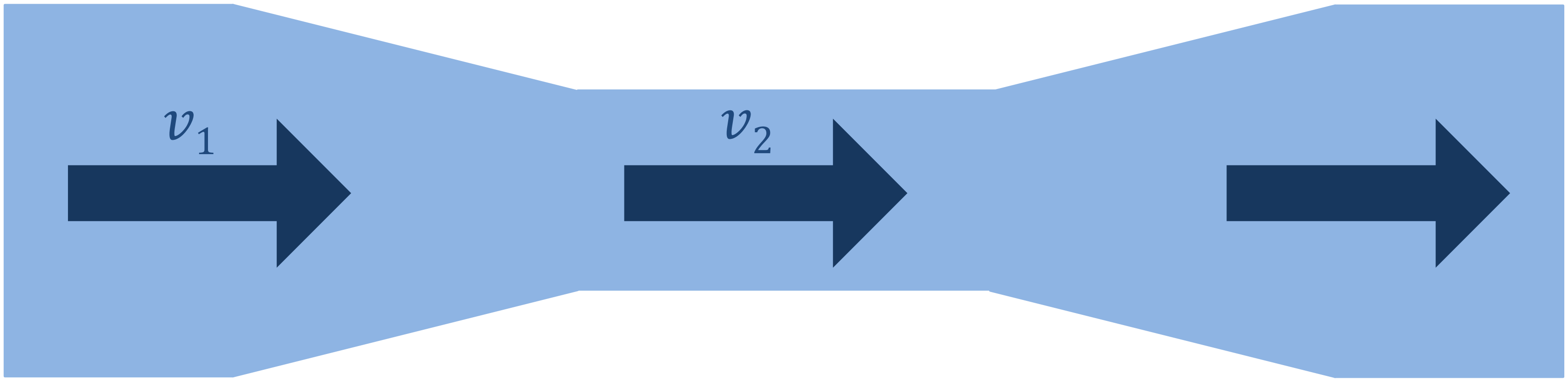



**Q.** Is the pressure in the middle section of the pipe higher or lower than at the edges?

Now use **Bernoulli's equation**:

$$p_1 + \frac{1}{2}\rho {v_1}^2 + \cancel{\rho g h_i} = p_2 + \frac{1}{2}\rho {v_2}^2 + \cancel{\rho g h_f}$$

# The Venturi effect



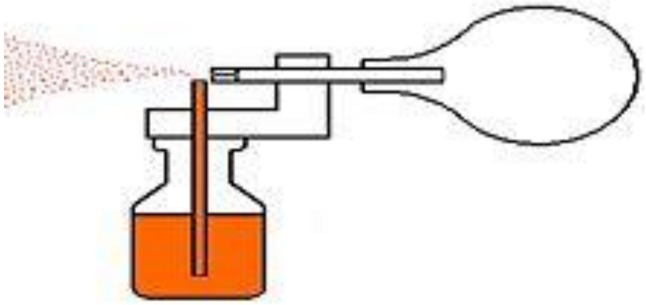

**Q.** Is the pressure in the middle section of the pipe higher or lower than at the edges?

$$p_1 + \frac{1}{2}\rho {v_1}^2 = p_2 + \frac{1}{2}\rho {v_2}^2$$

Because $v_1 < v_2$, it follows that $p_1 > p_{2.}$

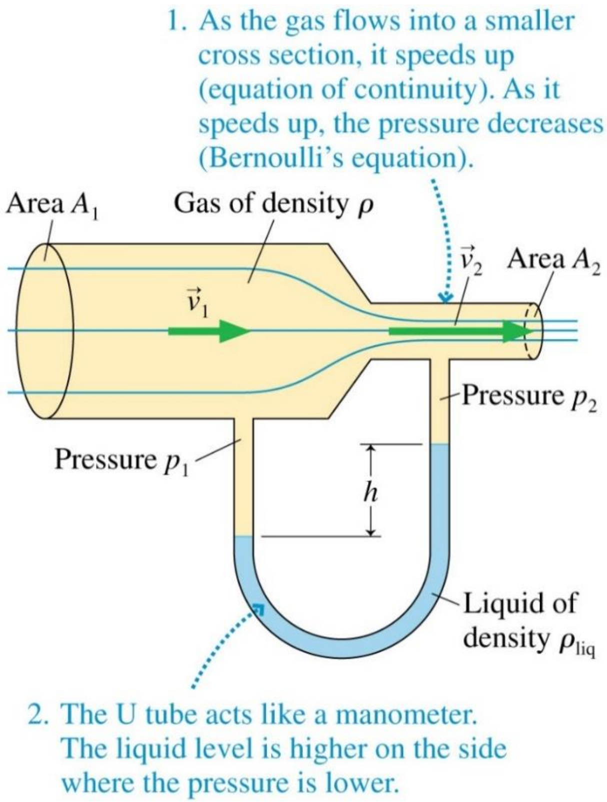
Pressure is **lower** in the central section.

# The Venturi effect



- When a fast gas stream is injected into the atmosphere and across the top of the vertical tube, it is forced to follow a curved path up, over and downward on the other side of the tube.

# The Venturi effect




Giovanni Venturi

1746-1822

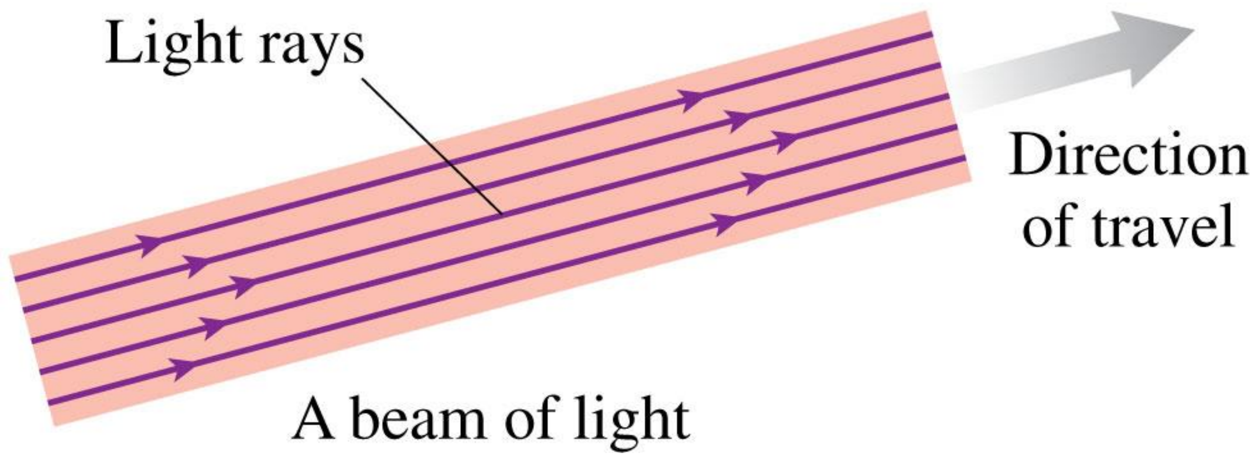
- Venturi tubes measure gas speeds in environments such as chemistry laboratories, wind tunnels, and jet engines.
- The gas-flow speed can be determined from the liquid height $h$.

# 4. The Ray Model of Light

# Our Progress thus Far

- We are going to briefly look at the idea of light, not in great detail though, as light really is quite mysterious, if you think long enough about what it actually is.
- Instead, we are going to consider briefly the **ray model of light** (which is quite useful), and which can be used to understand a very interesting application of light that is relevant to electrical engineering, namely fibre optic cables.

# The Ray Model of Light



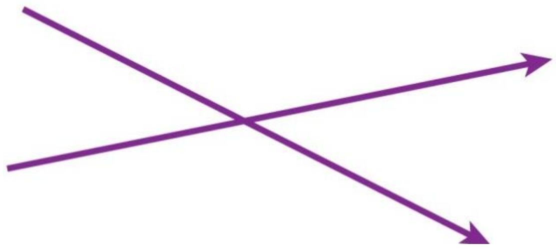

- Let us define a **light ray** as a line in the direction along which light energy is flowing.
- Any narrow beam of light, such as a laser beam, is actually a bundle of many parallel light rays.

# The Ray Model of Light

- You can think of a single light ray as the limiting case of a laser beam whose diameter approaches zero.

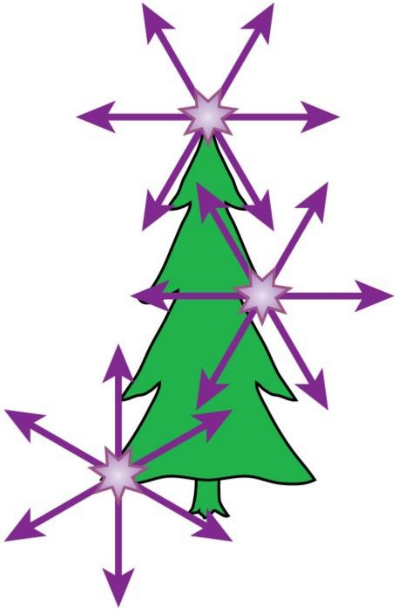
# The Ray Model of Light

- Light travels through a transparent material in straight lines called light rays.
- The speed of light is $v = c/n$, where $n$ is the index of refraction of the material.

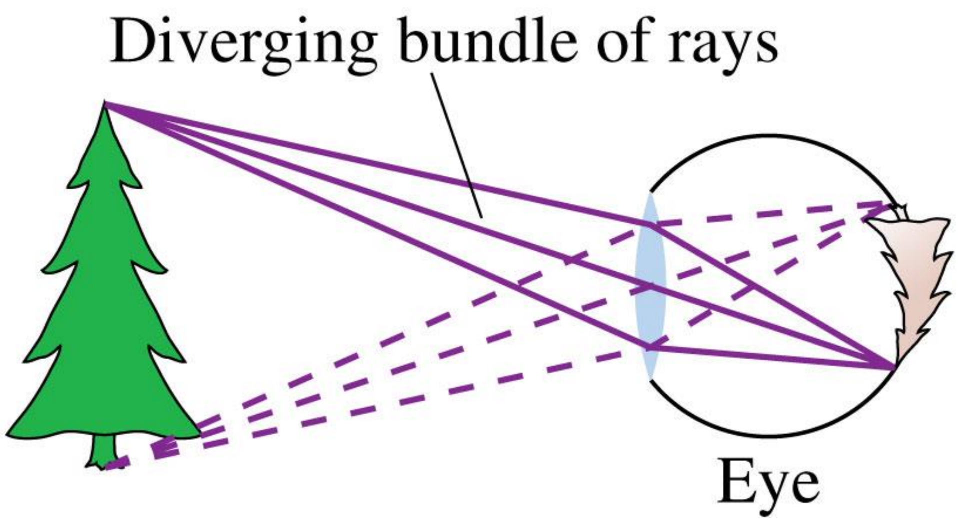


- Light rays do not interact with each other.
- Two rays can cross without either being affected in any way.

# The Ray Model of Light

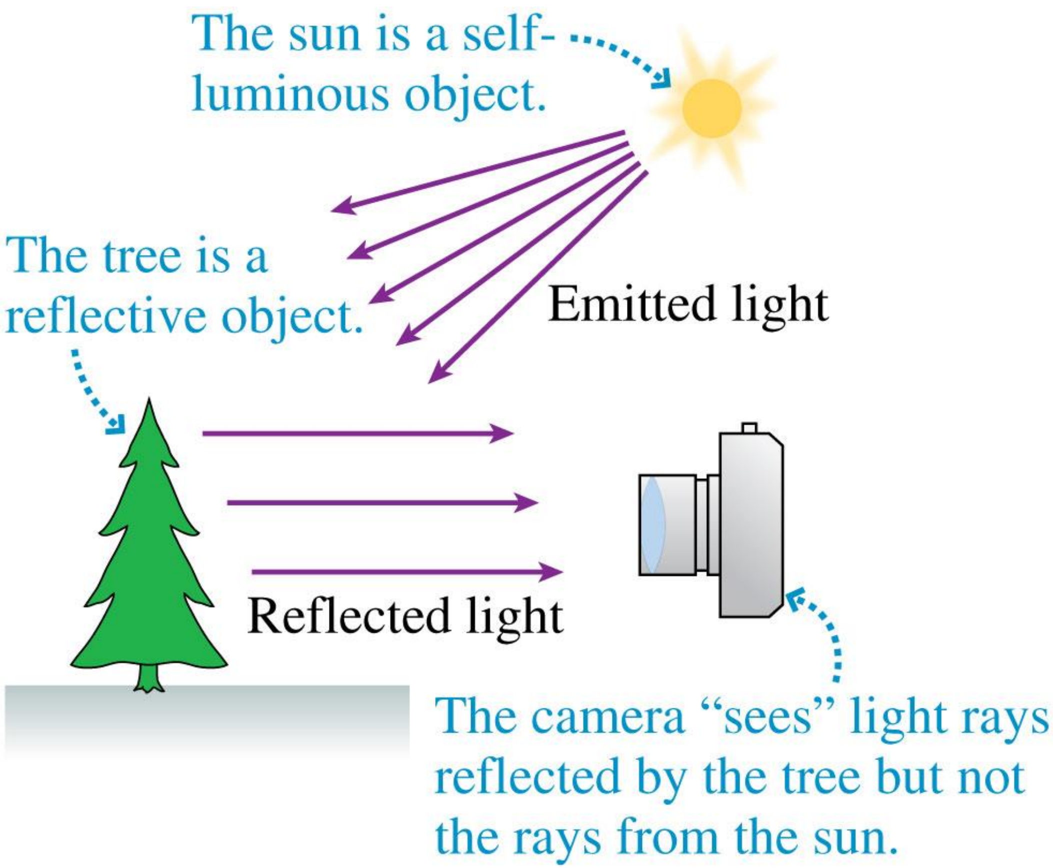


- An **object** is a source of light rays.
- Rays originate from every point on the object, and each point sends rays in all directions.



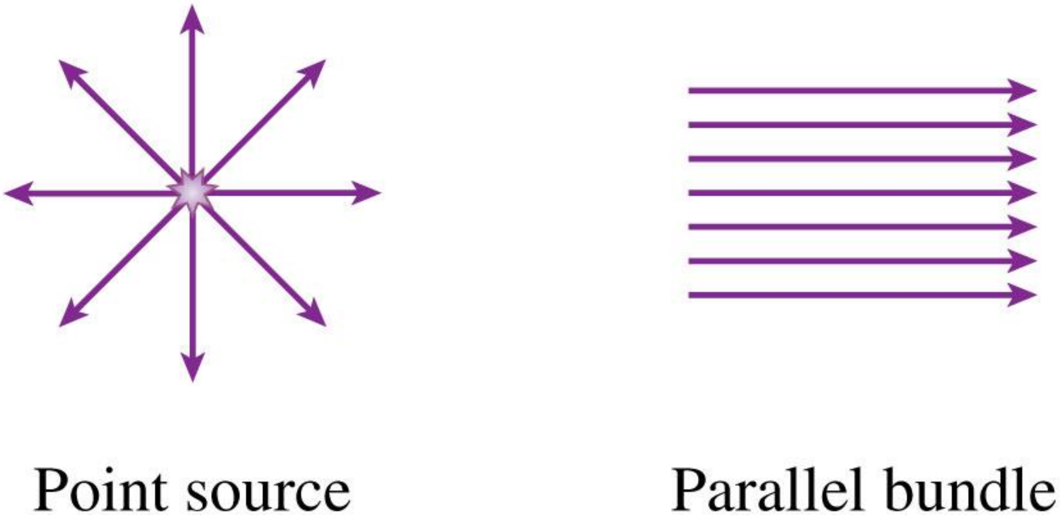

- The eye 'sees' an object when diverging bundles of rays from each point on the object enter the pupil and are focused to an image on the retina.

# The Ray Model of Light



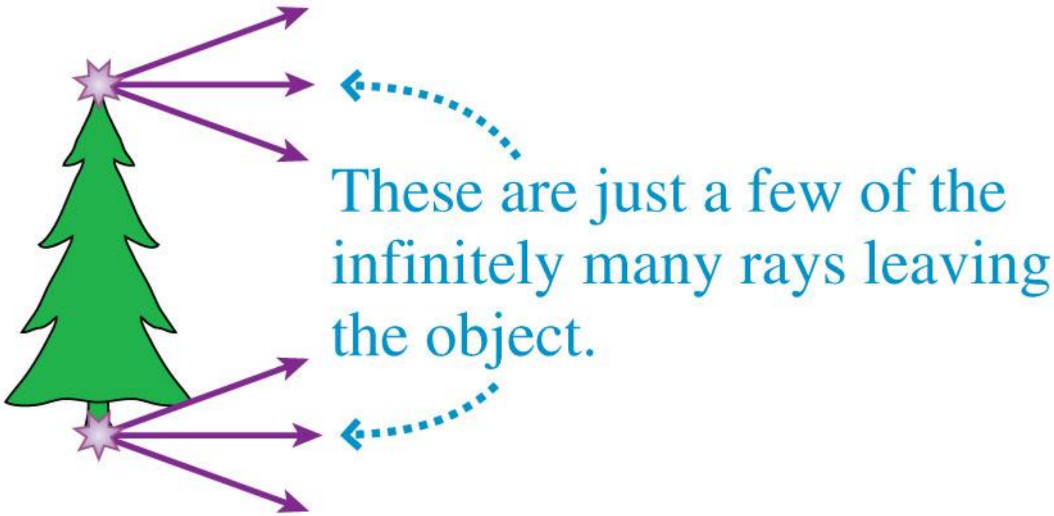

- Objects can be either self-luminous, such as the sun, flames, and lightbulbs, or reflective.
- Most objects are reflective.

# The Ray Model of Light



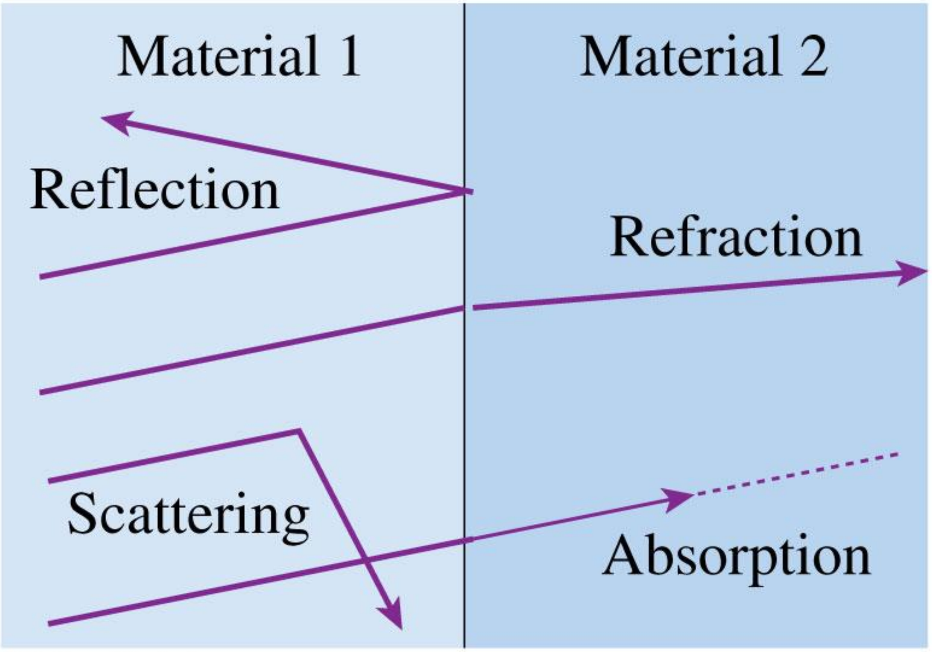

- The diverging rays from a **point source** are emitted in all directions.
- Each point on an object is a point source of light rays.
- A **parallel bundle** of rays could be a laser beam, or light from a distant object.

# The Ray Model of Light




- Rays originate from every point on an object and travel outward in all directions, but a diagram trying to show all these rays would be messy and confusing.
- To simplify the picture, we use a **ray diagram** showing only a few rays.

# The Ray Model of Light

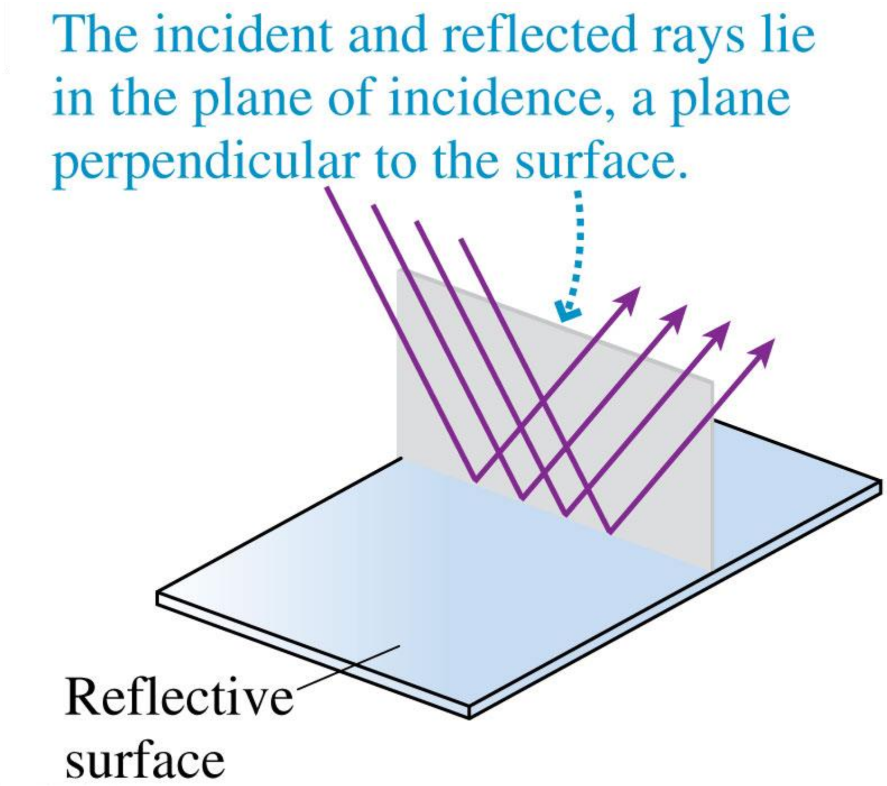



Light interacts with matter in four different ways:

- At an interface between two materials, light can be either **reflected** or **refracted**.
- Within a material, light can be either **scattered** or **absorbed**.
- Let's look at reflection and refraction in more detail.

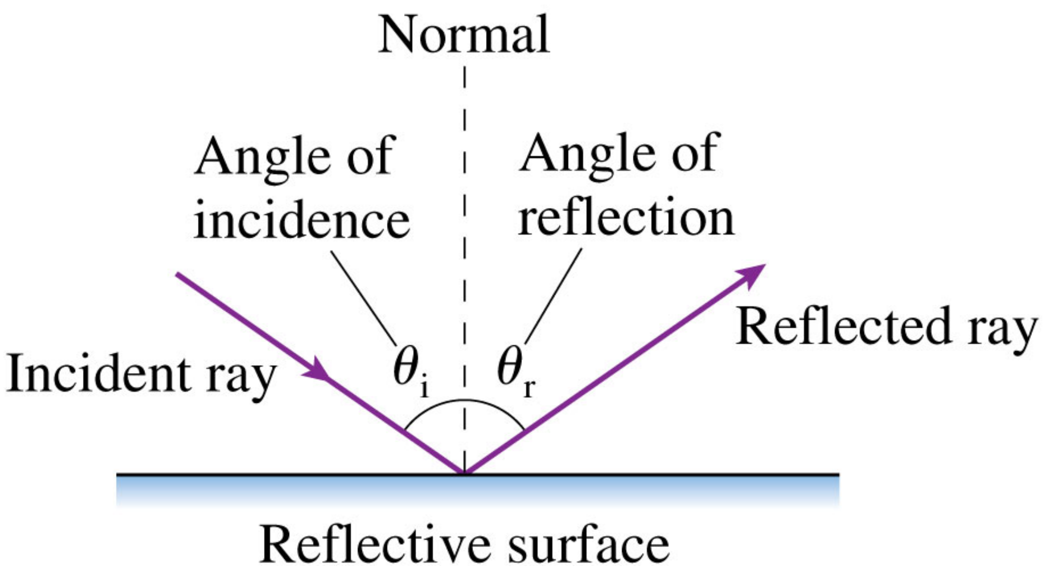
# 5. Reflection

# Reflection




- Reflection occurs when light interacts with a smooth surface, such as a mirror or a piece of polished metal, as shown above.

# Reflection



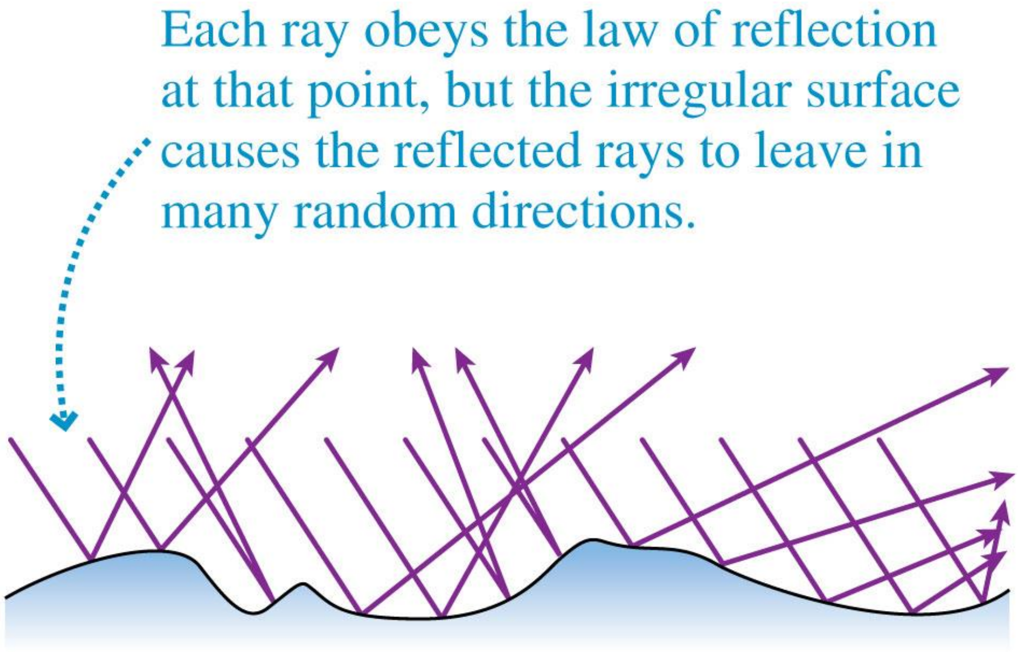

The **law of reflection** states that:

1. The incident ray and the reflected ray are in the same plane normal to the surface, and
2. The angle of reflection equals the angle of incidence:

$$\theta_r = \theta_I$$

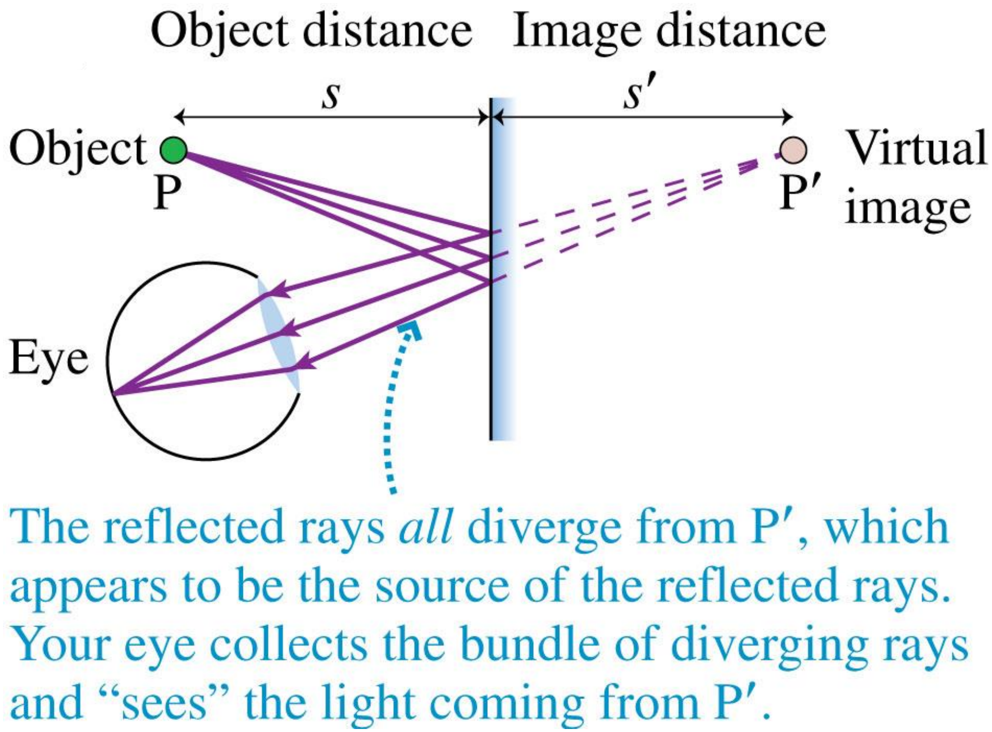
# Diffuse Reflection

- Most objects are seen because of their reflected light.
- For a ‘rough surface’, the law of reflection is obeyed at each point, but the irregularities of the surface cause the reflected rays to leave in many random directions.
- This situation is called **diffuse reflection**.
- It is how you see this slide, the wall, your hand, your friend, and so on.



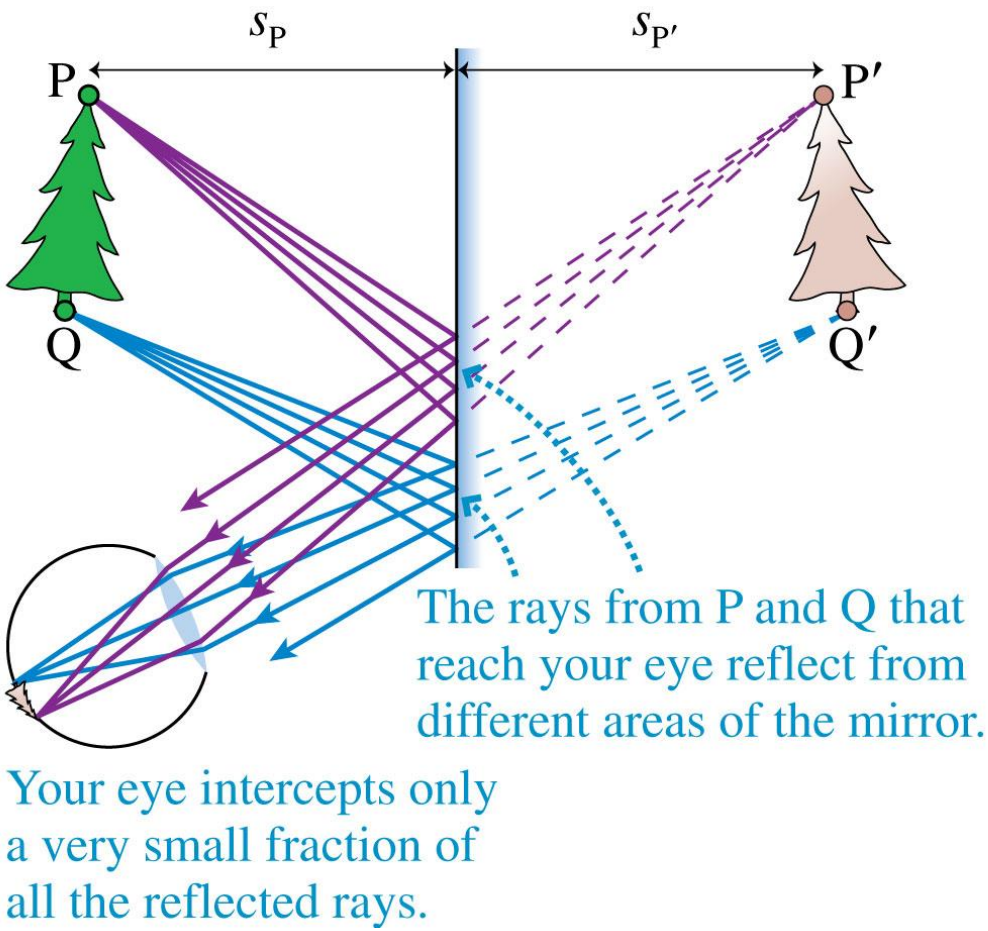

Magnified view of surface

# Reflection, and The Plane Mirror




- Consider $P$, a source of rays which reflect from a mirror.
- The reflected rays appear to come from $P'$, the same distance behind the mirror as $P$ is in front of the mirror.
- That is, $s' = s$.

# Reflection, and The Plane Mirror



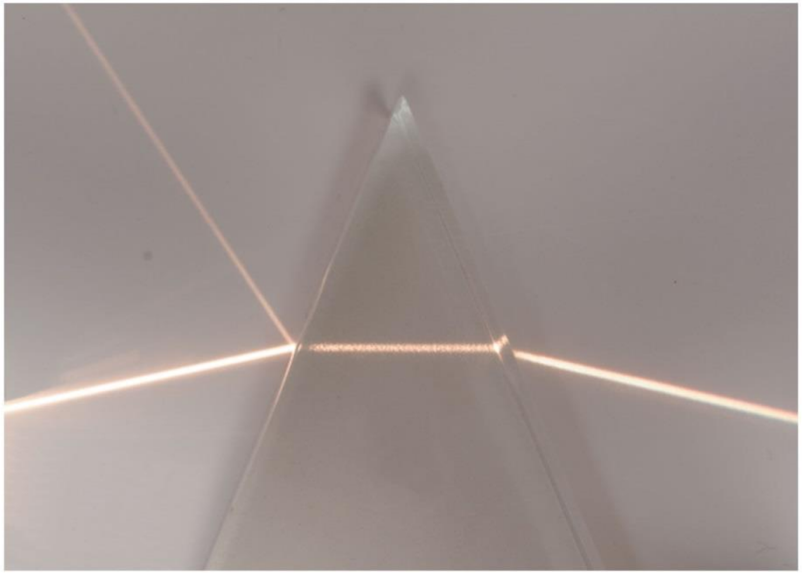

- Let's now consider the next type of interaction that light can undergo with matter, namely **refraction**.

# 6. Refraction

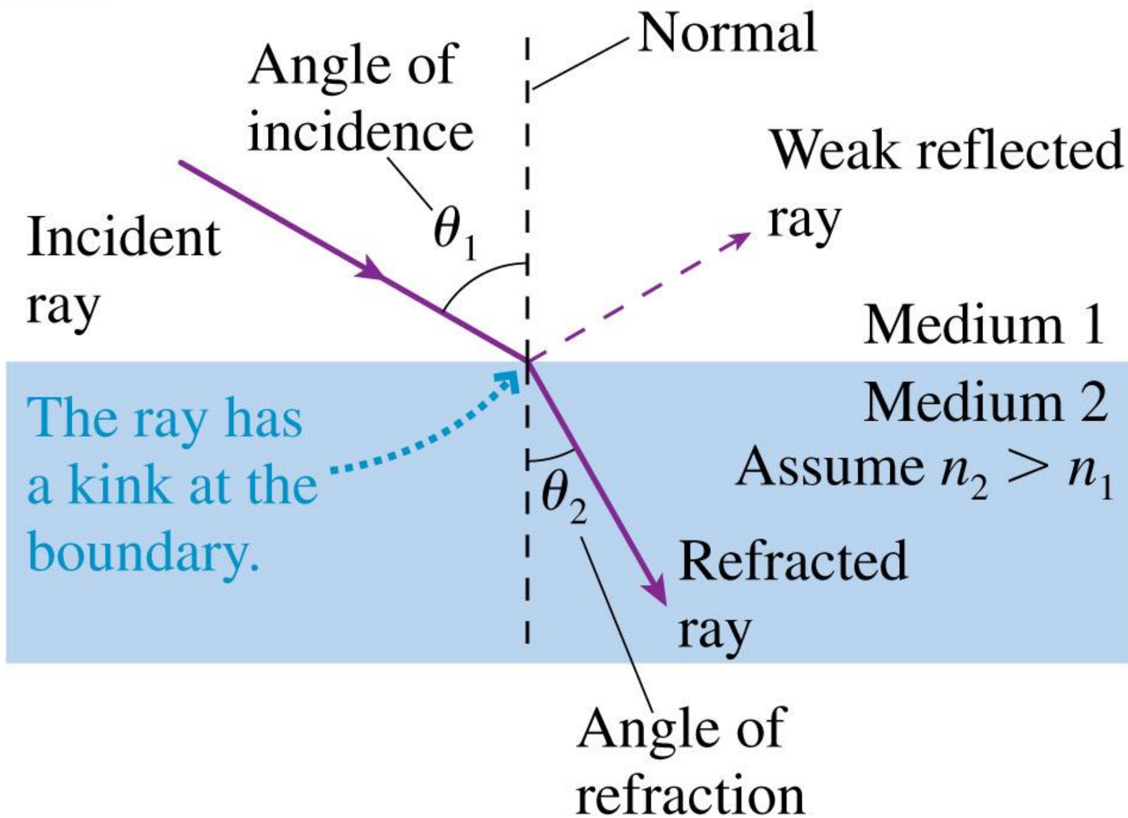
# Refraction

Two things happen when a light ray is incident on a smooth boundary between two transparent materials:

1) Part of the light reflects from the boundary, obeying the law of reflection.
2) Part of the light continues into the second medium. The transmission of light from one medium to another, but with a change in direction, is called **refraction**.

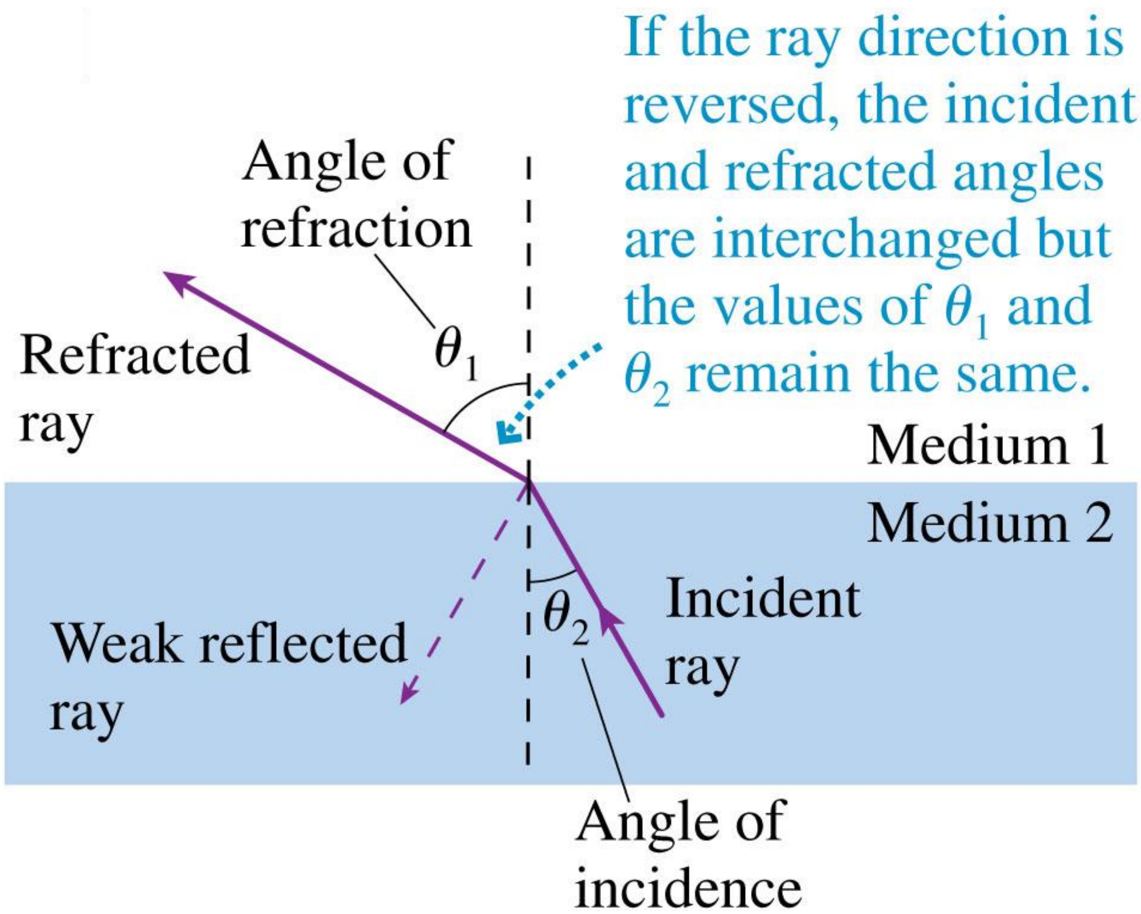


# Refraction




$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \qquad \text{(Snell's law of refraction)}$$

# Refraction




$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \qquad \text{(Snell's law of refraction)}$$

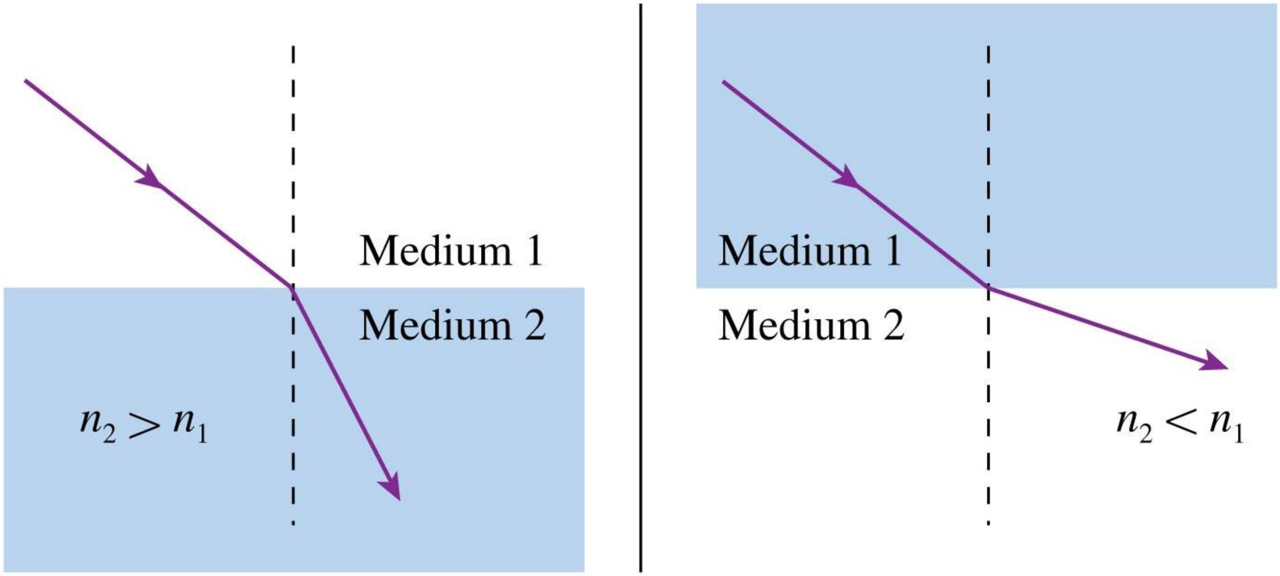
# Refraction

**TABLE 23.1** Indices of refraction

| Medium | $n$ |
|---|---|
| Vacuum | 1.00 exactly |
| Air (actual) | 1.0003 |
| Air (accepted) | 1.00 |
| Water | 1.33 |
| Ethyl alcohol | 1.36 |
| Oil | 1.46 |
| Glass (typical) | 1.50 |
| Polystyrene plastic | 1.59 |
| Cubic zirconia | 2.18 |
| Diamond | 2.41 |
| Silicon (infrared) | 3.50 |

$$n = \frac{c}{v_{\text{medium}}}$$

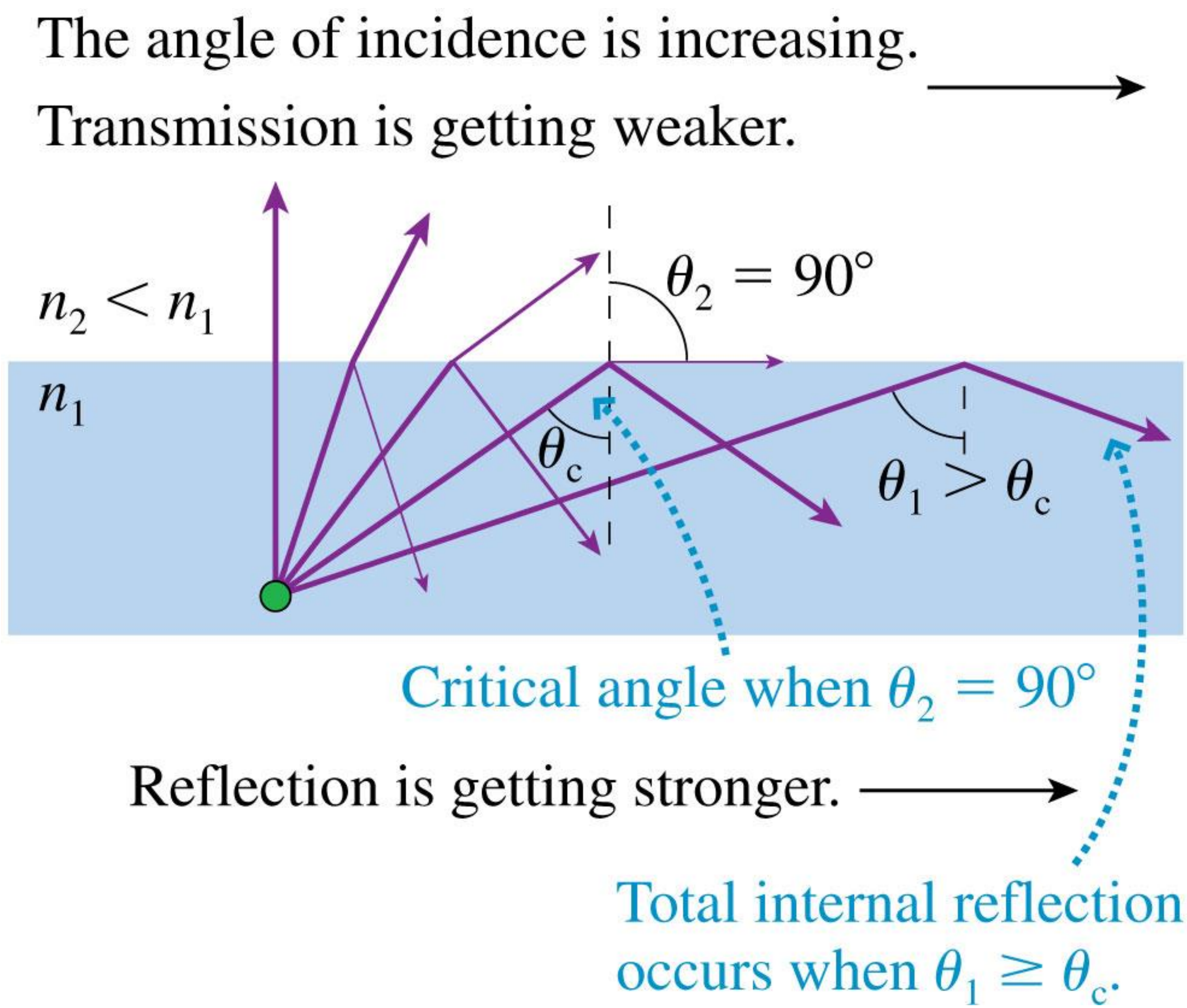
# Refraction




- When a ray is transmitted into a material with a higher index of refraction, it bends toward the normal.
- When a ray is transmitted into a material with a lower index of refraction, it bends away from the normal.
- As light rays move from a material with a high refractive index to a low refractive index, as the angle of incidence gets bigger, the transmission of the rays get weaker, reaching a point where **total internal reflection** occurs.

# 7. Total Internal Reflection

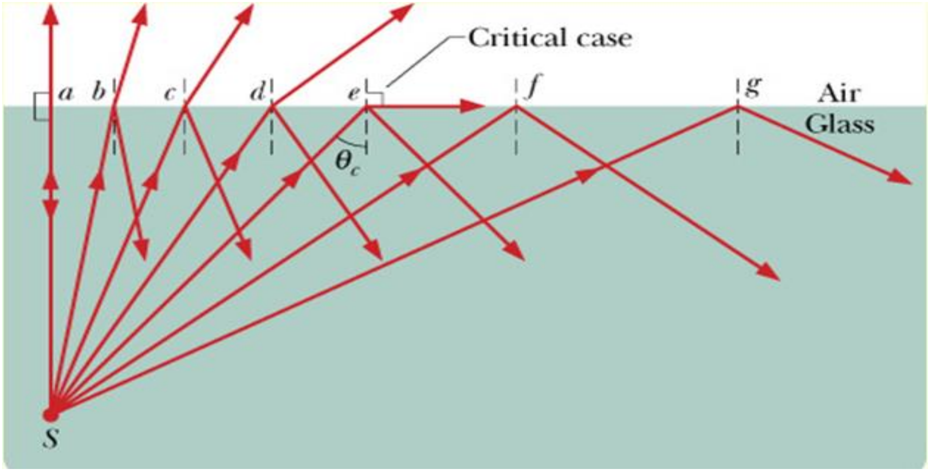
# Total Internal Reflection



# Total Internal Reflection

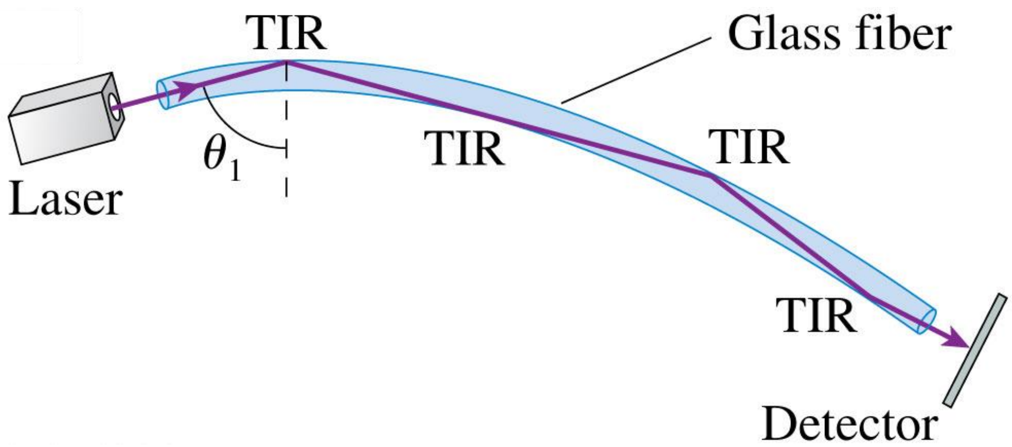
- When a ray crosses a boundary into a material with a lower index of refraction, it bends away from the normal.
- As the angle $\theta_1$ increases, the refraction angle $\theta_2$ approaches 90°, and the fraction of the light energy transmitted decreases while the fraction reflected increases.
- The critical angle of incidence occurs when $\theta_2 = 90°$:

$$\theta_c = \sin^{-1}\frac{n_2}{n_1}$$



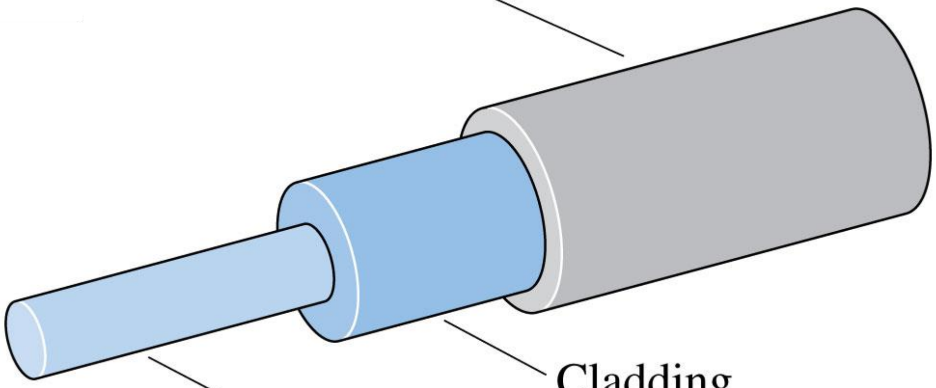

- The refracted light vanishes at the critical angle and the reflection becomes 100% for any angle $\theta_1 > \theta_c$.

# Total Internal Reflection




- The most important modern application of **total internal reflection** (TIR) is optical fibres.
- The light 'bounces its way down' the tube, as if it were inside a pipe.
- We can use the light to carry 'information'.

# Total Internal Reflection




- In a practical optical fibre, a small-diameter glass core is surrounded by a layer of glass cladding.
- The glasses used for the core and the cladding have:

$$n_{core} > n_{cladding}$$

# Summary of today's Lecture

1. The Continuity Equation
2. Bernoulli's Equation
3. The Venturi Effect
4. The Ray Model of Light
5. Reflection
6. Refraction
7. Total Internal Reflection

# Lecture 12: Optional Reading

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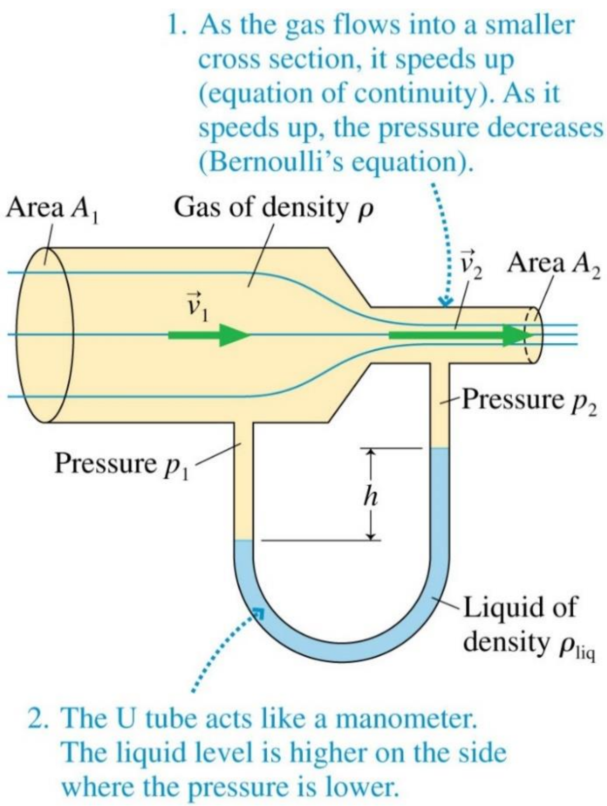
- **Ch. 13.3**, Pressure in Fluids; p.397-400
- **Ch. 13.4**, Atmospheric Pressure and Gauge Pressure; p.401
- **Ch. 13.5**, Pascal's Principle; p.402
- **Ch. 13.8**, Fluids in Motion; Flow Rate and the Eqn. of Continuity.
- **Ch. 13.9**, Bernoulli's Equation.
- **Ch. 13.10**, Applications of Bernoulli's Principle; p.412-414
- **Ch. 34.2**, Huygens Principle and the Law of Refraction; p.1,040-1,041

# Home Work

Do not forget to attempt the **Additional Problems** for this lecture before logging in to **Mastering Physics** to complete your assignments.

# The Venturi effect

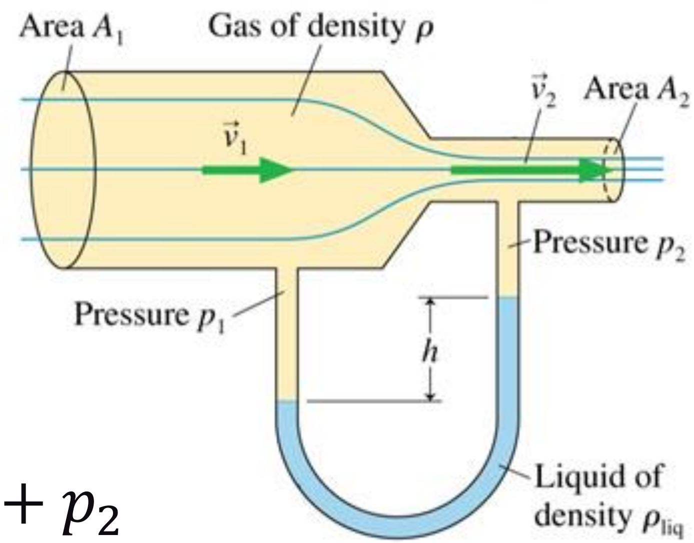



1746-1822

- Venturi tubes measure gas speeds in environments such as chemistry laboratories, wind tunnels, and jet engines.
- The gas-flow speed can be determined from the liquid height $h$.

# The Venturi effect

- First, we need to create an equation relating the speeds and pressures using Bernoulli's equation:



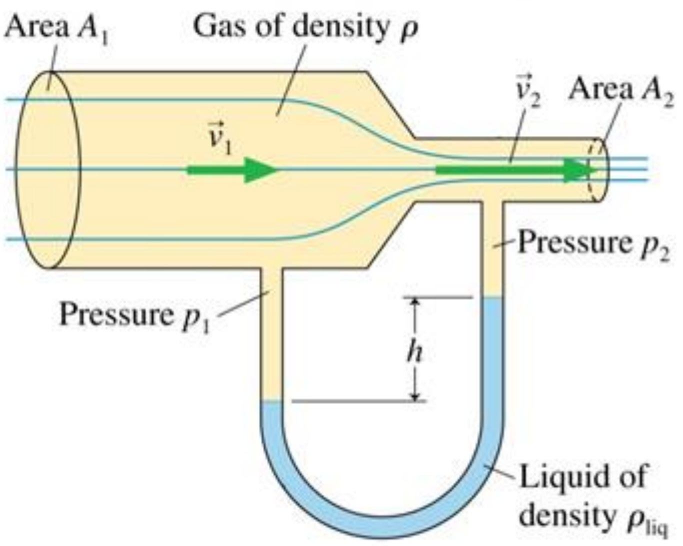

$$\frac{1}{2}\rho {v_1}^2 + \rho g y_1 + p_1 = \frac{1}{2}\rho {v_2}^2 + \rho g y_2 + p_2$$

- Since $y_1 = y_2$

$$\frac{1}{2}\rho {v_1}^2 + p_1 = \frac{1}{2}\rho {v_2}^2 + p_2$$

# The Venturi effect

- Consider the liquid. Since it is stationary (after the initial movement), we can apply Pascal's principle/law:




$$p_1 = p_2 + \rho_{liquid} g h$$

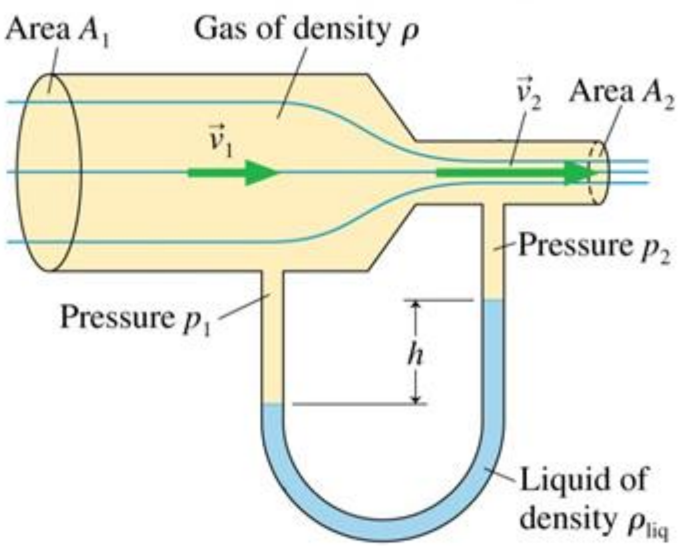
$$p_1 - p_2 = \Delta P = \rho_{liquid} g h$$

- Now we can return to Bernoulli's equation and rearrange to get

$$p_1 - p_2 = \frac{1}{2}\rho {v_2}^2 - \frac{1}{2}\rho {v_1}^2 = \Delta p = \rho_{liquid} g h$$

# The Venturi effect

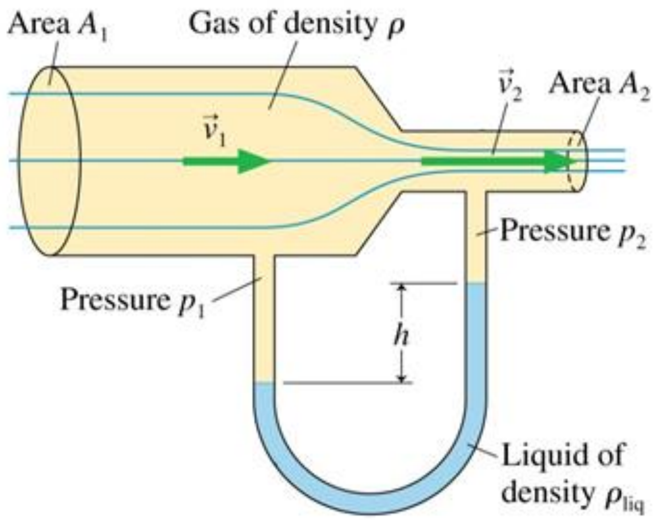
$$v_1 A_1 = v_2 A_2$$




$$v_1 = \frac{v_2}{A_1} A_2$$

$$\frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho\left(\frac{v_2^2 A_2^2}{A_1^2}\right) = \rho_{liquid} g h$$

# The Venturi effect

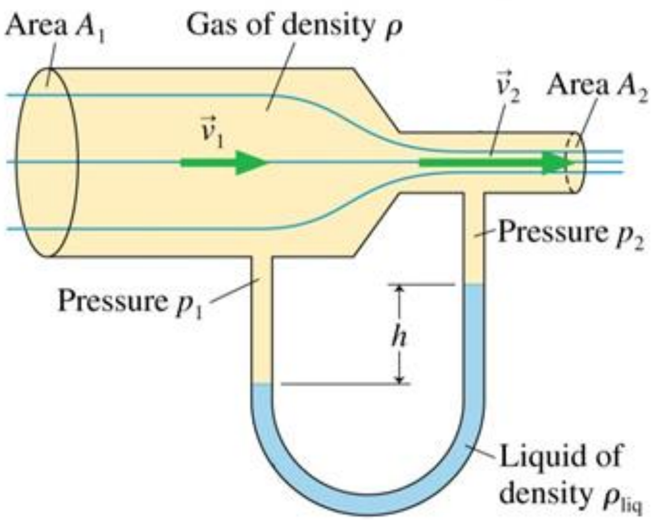



$$\rho v_2^2 - \rho\left(\frac{v_2^2 A_2^2}{A_1^2}\right) = 2\rho_{liquid} g h$$

$$A_1^2 \rho v_2^2 - \rho v_2^2 A_2^2 = 2A_1^2 \rho_{liquid} g h$$

$$v_2^2 (A_1^2 \rho - \rho A_2^2) = 2A_1^2 \rho_{liquid} g h$$

# The Venturi effect




$$\rho v_2^2 - \rho\left(\frac{v_2^2 A_2^2}{A_1^2}\right) = 2\rho_{liquid} g h$$

$$A_1^2 \rho v_2^2 - \rho v_2^2 A_2^2 = 2A_1^2 \rho_{liquid} g h$$

$$v_2^2 (A_1^2 \rho - \rho A_2^2) = 2A_1^2 \rho_{liquid} g h$$

$$v_1 = A_2 \sqrt{\frac{2\rho_{\text{liq}} g h}{\rho(A_1{}^2 - A_2{}^2)}}$$

$$v_2 = A_1 \sqrt{\frac{2\rho_{\text{liq}} g h}{\rho(A_1{}^2 - A_2{}^2)}}$$