# The University of Nottingham Ningbo China

Centre for English Language Education

Semester One, SAMPLE PAPER

**FOUNDATION ALGEBRA FOR PHYSICAL SCIENCES & ENGINEERING**

Time allowed: 1 hour 30 minutes

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*Candidates may complete the front cover of their answer book and sign their attendance card but must NOT write anything else until the start of the examination period is announced.*

**This paper contains SEVEN questions which carry equal marks.**

**Answer all questions with neccesary steps.**

*An indication is given of the weighting of each subsection of a question by means of a figure enclosed by square brackets, e.g. [3], immediately following that subsection.*

**Only CELE approved calculator (fx-82 SERIES) is allowed during this exam.**

*Dictionaries are not allowed with one exception.Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject specific translation dictionaries are not permitted.*

*No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.*

***Do not turn examination paper over until instructed to do so.***

**ADDITIONAL MATERIAL:** *Formula Sheet.*

**INFORMATION FOR INVIGILATORS:**

1. *Please give a 15-minute warning before the end of the exam.*
2. *Please collect Answer Booklets, Question Papers and Formula Sheet at the end of the exam.*

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1. (a) Given $f(x) = 2x^2 + 3x - 2$ and $g(x) = x - 2$.

   ($i$) Find $h(x) = (f \circ g)(x)$.

   ($ii$) Use the method of completing the square to express $h(x)$ in the form $a\,(x - b)^2 - c$, where $a,\ b,\ c \in \mathbb{R}$ are to be determined.

   ($iii$) Hence, find the range of $h(x)$. [4]

   (b) ($i$) Given the function $f : [0, +\infty) \rightarrow [0, +\infty)$ defined by $f(x) = \sqrt{x+8} - 4$, find the formula for its inverse function, $f^{-1}(x)$.

   ($ii$) Solve the modulus inequality $|\, x - 2 \,| \geq 5$. [3]

   (c) ($i$) Solve the exponential equation $e^{2x} - 5e^{x} - 24 = 0$ for $x \in \mathbb{R}$.

   ($ii$) Solve the logarithmic equation $\log{(6x)} - \log(4 - x) = \log(3)$. [3]

2. (a) Prove the identity $2\,\tan\theta \sec\theta = \dfrac{2\sin\theta}{1 - \sin^2\theta}$. [1]

   (b) Use appropriate trigonometric identity to solve the equation $2\cos^2\theta + \cos\theta - 1 = 0$, where $\theta \in (0, \pi)$. [3]

   (c) Given $f(x) = \sin x + \sqrt{3}\cos x$.

   ($i$) Express $f(x)$ in the form $f(x) = R\,\cos(x - \theta)$, where $R > 0$ and $\theta \in \left(0, \dfrac{\pi}{2}\right)$. Find $R$ and $\theta$ (in radians).

   ($ii$) Hence, plot the graph of $y = f(x)$ [4]

   (d) ($i$) Prove the trigonometric identity: $\sin 65^\circ + \cos 65^\circ = \sqrt{2}\,\sin 70^\circ$.

   ($ii$) By showing necessary steps (i.e. without direct use of a calculator), evaluate $\sin^{-1}\left[\sin\left(\dfrac{2\pi}{3}\right)\right]$. [2]

3. (a) Given $p(x) = 6\,x^3 + 5\,x^2 - 12\,x + 4$.

   ($i$) Use the method of Synthetic Division to show that $(x+2)$ is a factor of $p(x)$.

   ($ii$) Hence express $p(x)$ completely as a product of linear factors.

   ($iii$) Use the expression obtained in question 3(a)($ii$) to solve the polynomial equation $p(x) = 0$. [4]

   (b) Given a polynomial function $f(x) = x^2 + a\,x + b \ \left(a, b \in \mathbb{Z}\right)$.

   The remainder when $f(x)$ is divided by $(x-2)$ is $-4$, and the remainder when $f(x)$ is divided by $(x+4)$ is $-4$.

   Use the Remainder Theorem to find the constants $a$ and $b$. [4]

   (c) Find the coefficient of $x^{10}$ in the expansion of $\left(x^2 - \dfrac{1}{2}\right)^9$. [2]

4. (a) Use the generalised Binomial Theorem to expand $\dfrac{1}{\sqrt[4]{1-3x}}$, $\text{where}\,|\,x\,| < \dfrac{1}{3}$

   up to and including the terms with $x^3$. [2]

   (b) The volume of a cylinder with radius $r$ and height $h$ is given by $V = \pi\,r^2 h$.

   If there is an error of $2\%$ in the measured value of $r$, use the approximation

$$(1+x)^n \;\approx\; 1 + n\,x + \frac{n(n-1)}{2\,!}\,x^2$$

   to find the resulting error, $\delta V$, in the calculated volume. [3]

   (c) Consider finding numerically, the only positive root of the polynomial equation:

$$f(x) = x^3 - 7x + 2 = 0. \tag{4.1}$$

   ($i$) Apply the Intermediate Value Theorem to show that a positive root $x^*$ of the equation $(4.1)$ lies in the interval $(0, 1)$.

   ($ii$) Show that equation $(4.1)$ can be rearranged to obtain the iterative formula

$$x_{n+1} = \frac{x_n^3 + 2}{7} \quad ; \quad n = 0, 1, 2, \ldots. \tag{4.2}$$

   ($iii$) Starting with $x_0 = 0.5$, use the iterative formula $(4.2)$ to find the root $x^*$ of the equation $(4.1)$, correct to $5$ decimal places (d.p.). Write the approximations $x_i$ $(i = 0,\,1,\,2,\,3,\ldots)$, obtained in the process, and also state $x^*$, correct to $5$ d.p. [5]

5. (a) Given matrices $A = \begin{pmatrix} 2 & 5 \\ 3 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, and $C = \begin{pmatrix} 11 & 16 \\ k & 6 \end{pmatrix}$, find the constant $k \in \mathbb{Z}$ such that $C = A^T B$. [2]

(b) Given a system of two linear equations:

$$\begin{cases} x - 7\,y & = & -\,11 \\ 5\,x + 2\,y & = & -\,18. \end{cases} \qquad (6.1)$$

($i$) Write the system of equations $(6.1)$ in a matrix form:

$$AX = B. \qquad (6.2)$$

($ii$) Find the inverse matrix, $A^{-1}$.

($iii$) By rewriting $(6.2)$ as $X = A^{-1}B$ for an invertible matrix $A$, solve the system of equations $(6.1)$.

[4]

(c) Find the constant $k$ if the matrix $C = \begin{pmatrix} 9 & 1-k \\ 2 & 2\,k \end{pmatrix}$ is a singular matrix. [1]

(d) Express $\dfrac{3x}{(2x+1)\,(x^2+2)}$ as a sum of partial fractions. [3]

6. (a) Express the complex number $z = (i+3)^2 - 4i$ in the form $z = a + i\,b$, where $a, b \in \mathbb{R}$, and $i^2 = -\,1$. [1]

(b) ($i$) Solve for $x, y \in \mathbb{R}$, the complex equation $(2+3i)\,x - (4+2i)\,y - 2 = 0$.

($ii$) For the values of $x$ and $y$ obtained in question 6(b)($i$), evaluate $|\,x + i\,y\,|$. [3]

(c) Given complex number $z = \dfrac{i}{1+i}$.

($i$) Express $z$ in the Cartesian form $x + iy$ where $x, y \in \mathbb{R}$.

($ii$) Find $r = |\,z\,| > 0$ and $\theta = \arg(z)$ such that $-\,\pi < \theta \leq \pi$.

($iii$) Hence, express $z$ in the polar form $r\Big(\cos\theta + i\sin\theta\Big)$. [4]

(d) Given complex numbers $z_1 = 3 + 4\,i$, $z_2 = 6 + 3\,i$, and $z_3 = \overline{z_1} \cdot z_2$, use the properties of modulus to find the value of $\left|\dfrac{z_1^2 \cdot z_2}{\overline{z_2} \cdot z_3}\right|$. [2]

7. (a) The fourth and the eighth terms of an arithmetic progression (A.P.) are $-20$ and $-10$, respectively.

   ($i$) Find the twelfth term of this A.P.

   ($ii$) Find the sum of the first 100 terms of this A.P. [3]

   (b) Consider the geometric progression (G.P.) $1, -\frac{1}{8}, \frac{1}{64}, -\frac{1}{512}, \ldots\ldots$

   ($i$) Find the sixth term of this G.P.

   ($ii$) Find the sum $\sum_{1}^{\infty} a_n$ for this G.P. [2]

   (c) ($i$) Prove that $\sum_{1}^{n} n\,(n-10) = \frac{n\,(n+1)\,(2n-29)}{6}$.

   ($ii$) Use the result in 8(c)(i) to find the sum: $\sum_{n=15}^{30} n\,(n-10)$. [3]

   (d) For $f(n) = \frac{1}{n(n+1)}$, given that $f(n) - f(n+1) = \frac{2}{n\,(n+1)(n+2)}$.

   Use the method of differences to show that

$$\frac{1}{1\times 2\times 3} + \frac{1}{2\times 3\times 4} + \frac{1}{3\times 4\times 5} + \ldots\ldots \text{ (up to } n \text{ terms)} = \frac{n\,(n+3)}{4(\,n+1)\,(n+2)}.$$ [2]