The University of Nottingham
UNITED KINGDOM · CHINA · MALAYSIA

Science A Physics

Lecture 20:

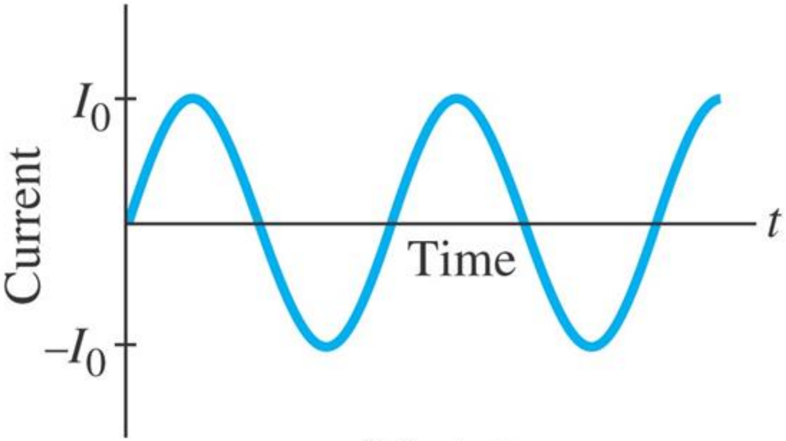
LRC Series AC Circuits and Resonance

Aims of today’s lecture

1. AC Circuits with AC Source
2. LRC Series AC Circuit
3. Resonance in AC Circuits
4. Three-Phase AC

1. AC Circuits with AC Source

AC Circuits with AC Source



(b) AC

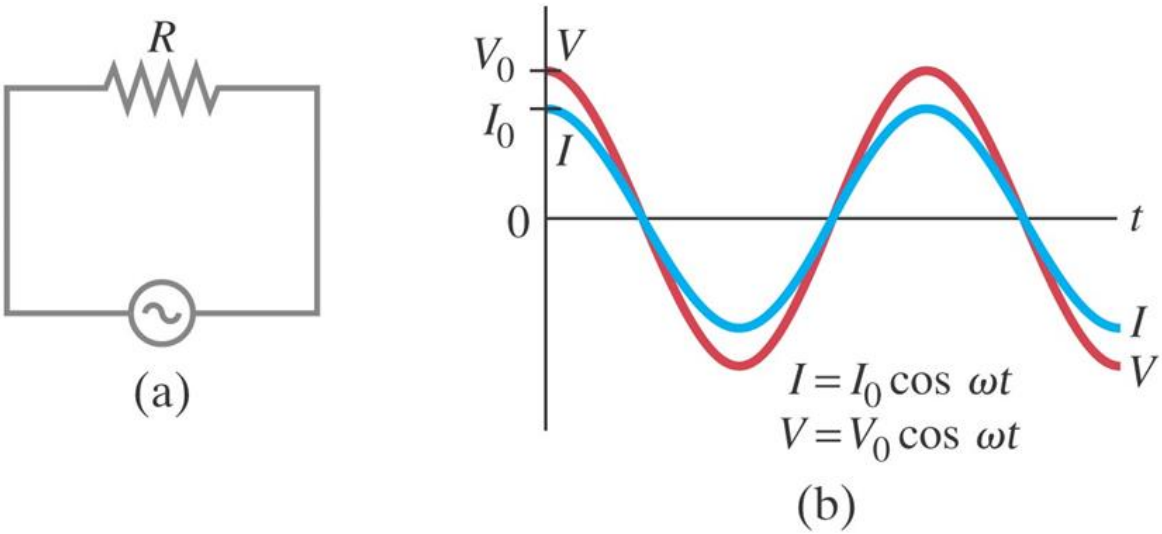
- We have discussed circuits of resistors, capacitors, and inductors, connected to a dc source of emf or to no source.
- Now consider circuit elements connected to a source of alternating voltage, which produces a sinusoidal voltage of frequency f.
- We assume in each case that the emf gives rise to a current

$$I = I_0 \cos 2\pi f t = I_0 cos \omega t$$

where t is time and I_0 is the peak current.

- Recall that $V_{rms} = V_0/\sqrt{2}$ and $I_{rms} = I_0/\sqrt{2}$

AC Source Connected to a Resistor

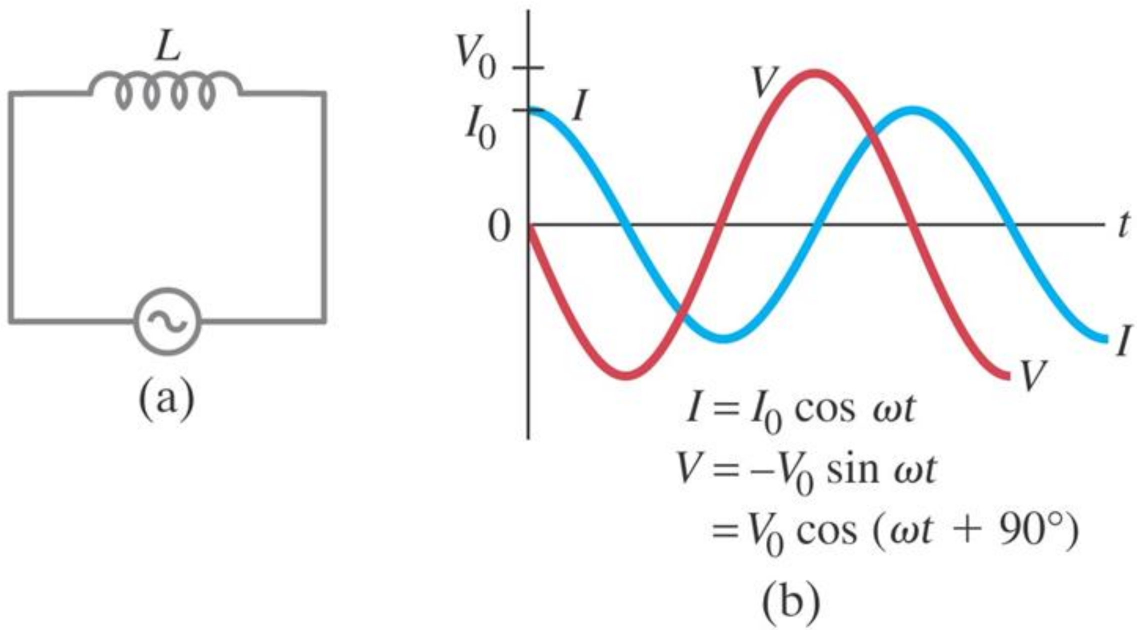


- When an ac source is connected to a resistor, as shown in (a), the current increases and decreases with the alternating voltage according to Ohm's law:

$$V = IR = I_0 R cos\omega t = V_0 cos\omega t$$

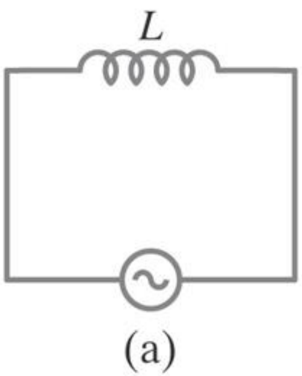
- Figure (b) shows the voltage (red curve) and the current (blue curve).
- Because the current is zero when the voltage is zero and the current reaches a peak when the voltage does, we say that the current and voltage are **in phase**.

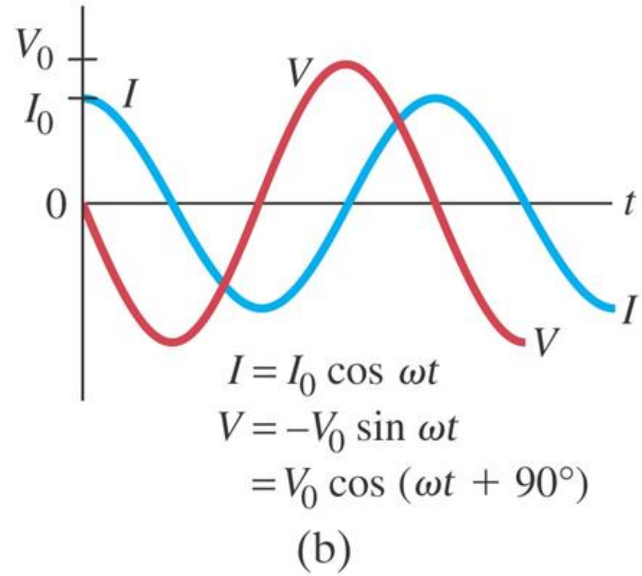
AC Source Connected to an Inductor



- We ignore any resistance that the inductor might have (it is usually small).
- The voltage applied to the inductor will be equal to the 'back' emf generated in the inductor by the changing current.
- This is because the sum of the electric potential changes around any closed circuit must add up to zero, according to Kirchoff's rule.

AC Source Connected to an Inductor



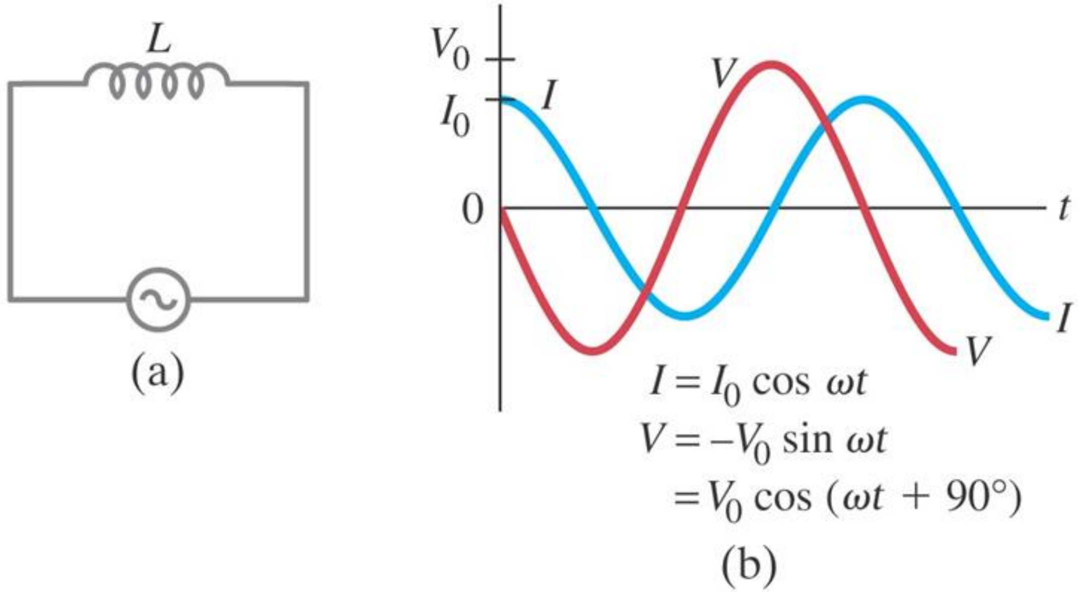


- Thus, $$V - L\frac{dI}{dt} = 0$$

or
$$V = L\frac{dI}{dt} = -\omega L I_0 sin\omega t$$

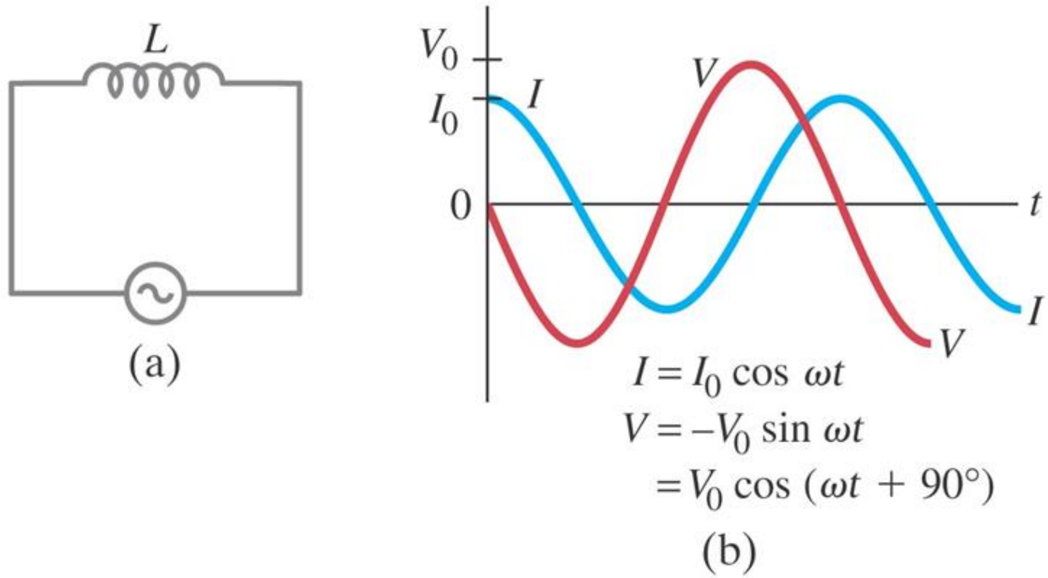
- Using the identity $\sin(\theta) = -\cos\left(\theta + \frac{\pi}{2}\right)$ we can write
$$V = \omega L I_0 \cos(\omega t + 90°) = V_0 \cos(\omega t + 90°)$$
- Where $V_0 = I_0 \omega L =$ the peak or maximum inductor voltage.

AC Source Connected to an Inductor



- From the graph (in (b)), it is clear that the current and voltage are out of phase by a quarter cycle, which is equivalent to $\pi/2$ radians or $90°$.
- We can see from the graph that **the current lags the voltage by 90° in an inductor.**
- In other words, the current in an inductor reaches its peak a quarter cycle later than the voltage does, or alternatively, we can say that the voltage leads the current by $90°$.

AC Source Connected to an Inductor

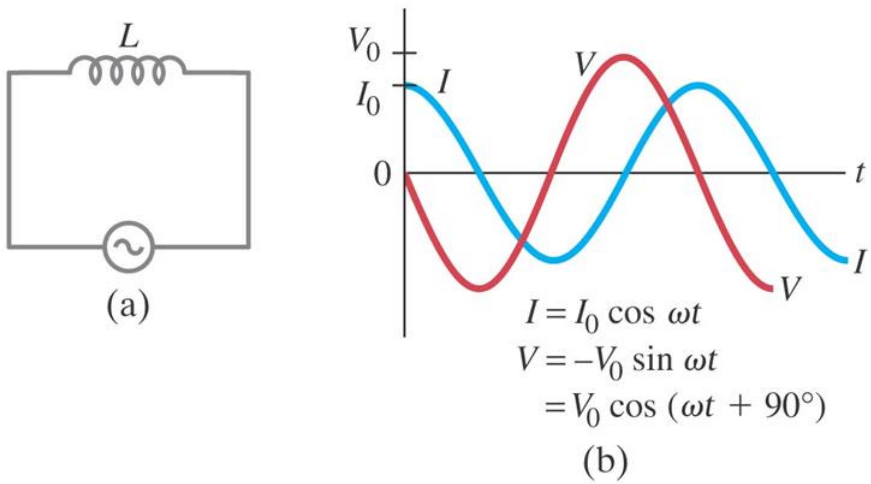


N.B.

Because the current and voltage in an inductor are out of phase by 90°, the product IV $(= \text{power})$ is as often positive as it is negative. So no energy is transformed in an inductor on the average; and no energy is dissipated as thermal energy.

Just as a resistor impedes the flow of charge, so too an inductor impedes the flow of charge in an alternating current due to the back emf produced.

Inductive Reactance



$$V = I_0 \omega L cos(\omega t + 90°)$$

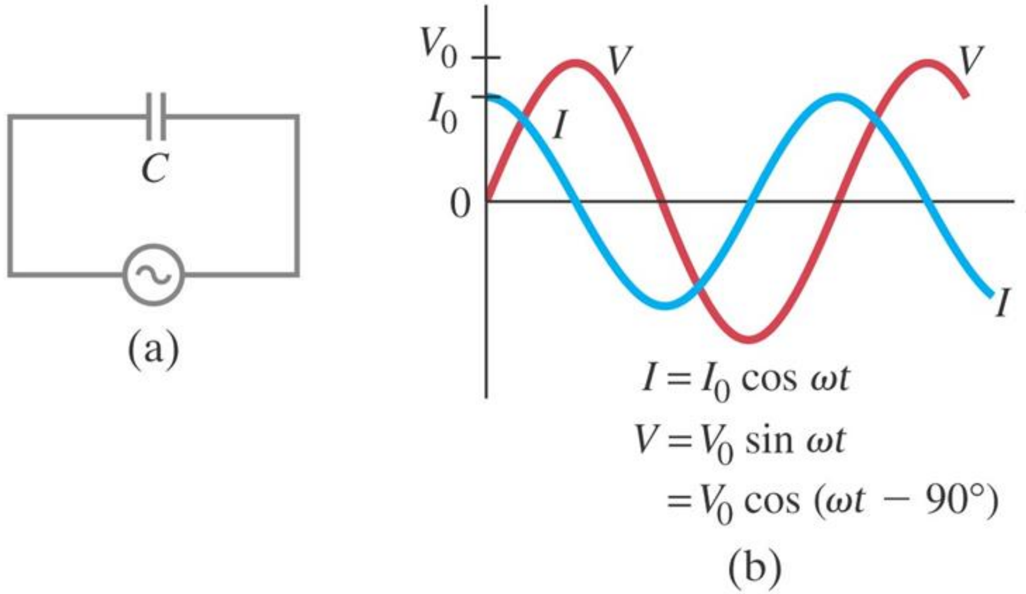
$$V = I_0 \omega L$$

$$\Rightarrow \frac{V}{I_0} = X_L$$

$$\Rightarrow \frac{V}{I_0} = \omega L = inductive\ reactance = X_L$$

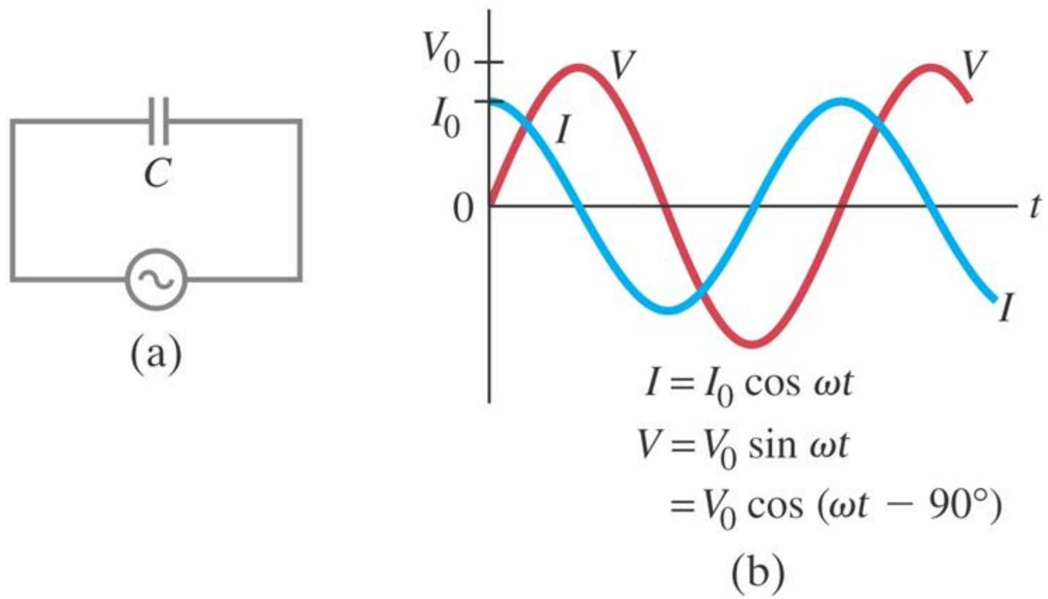
- The above equation (**in blue**) is valid for peak values of I and V; it is also valid for rms values, $V_{rms} = I_{rms} X_L$.
- Because the peak values of current and voltage are not reached at the same time, the above equation is not valid at a particular instant, as is the case for a resistor ($V = IR$).

AC Source Connected to a Capacitor



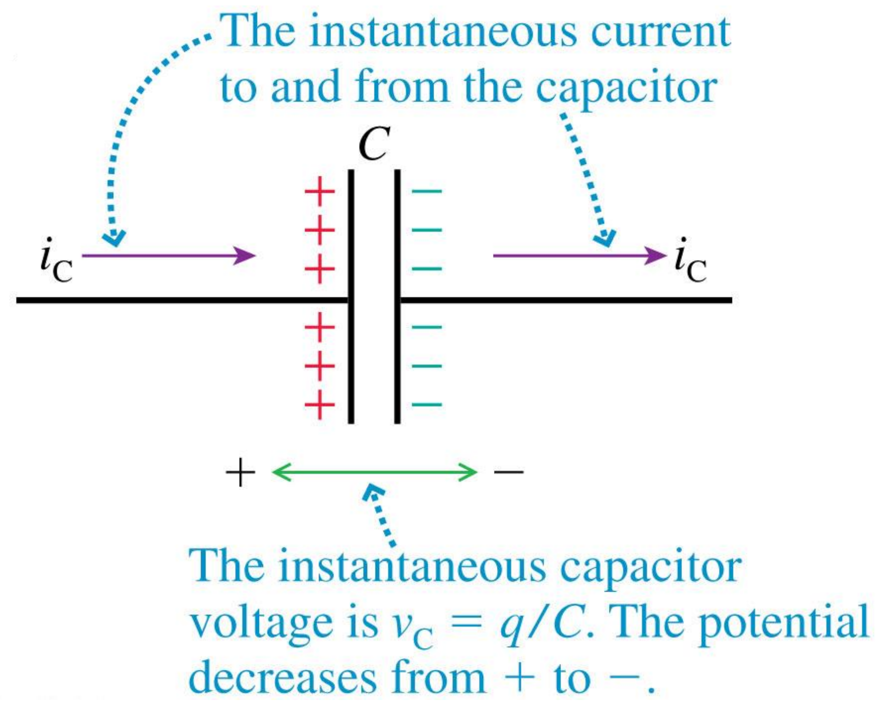
- When a capacitor is connected to a battery, the capacitor plates quickly acquire equal and opposite charges; but no steady current flows in the circuit.
- A capacitor prevents the flow of a dc current.
- But if a capacitor is connected to an alternating source of voltage, as in (a) above, an alternating current will flow continuously.

AC Source Connected to a Capacitor



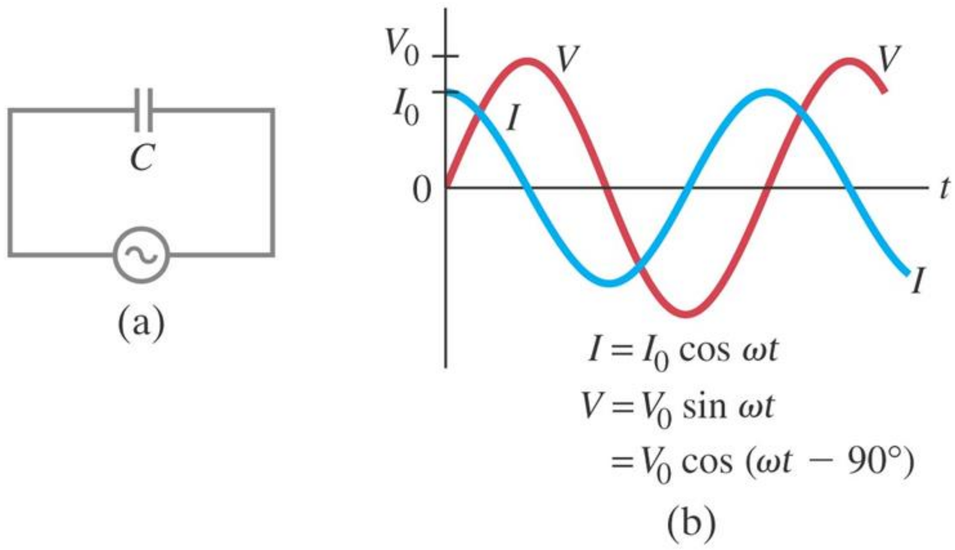
- This can happen because when the ac voltage is first turned on, charge begins to flow and one plate acquires a negative charge and the other a positive charge.
- But when the voltage reverses itself, the charges flow in the opposite direction.
- Thus, for an alternating applied voltage, an ac current is present in the circuit continuously.

A Capacitor in an AC Circuit



- As the capacitor charges and discharges, we have a current $\boldsymbol{i_C}$ into one plate and an equal current out of the other plate, just as though the charge were being conducted through the capacitor.

AC Source Connected to a Capacitor



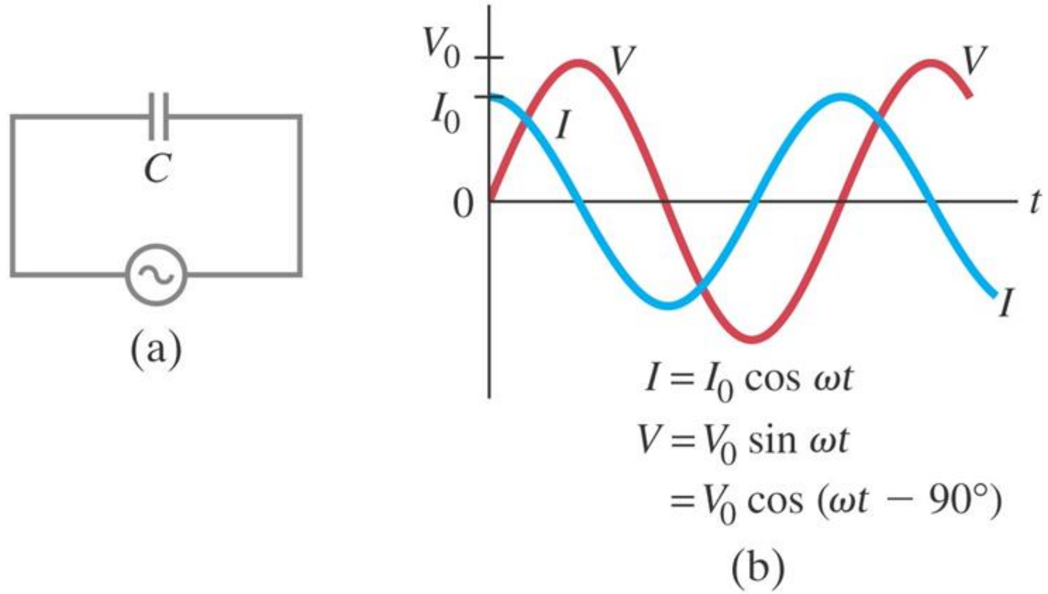
- By Kirchhoff's loop rule, the applied source voltage must equal the voltage V across the capacitor at any moment:

$$V = \frac{Q}{C}$$

- The current I at any instant (given as $I = I_0 cos\omega t$) is

$$I = \frac{dQ}{dt} = I_0 cos\omega t$$

AC Source Connected to a Capacitor



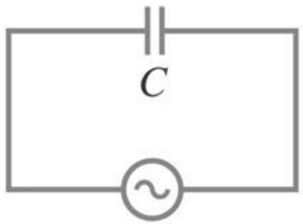
- The charge, Q, on the plates at any instant is given by

$$Q = \int_0^t dQ = \int_0^t I_0 \cos(\omega t)\, dt = \frac{I_0}{\omega} sin\omega t$$

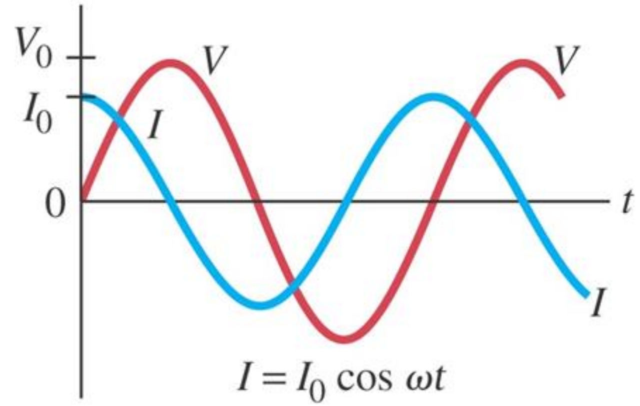
- Then the voltage across the capacitor is

$$V = \frac{Q}{C} = I_0 \left(\frac{1}{\omega C}\right) sin\omega t$$

AC Source Connected to a Capacitor



(a)



$I = I_0 \cos \omega t$
$V = V_0 \sin \omega t$
$= V_0 \cos(\omega t - 90°)$
(b)

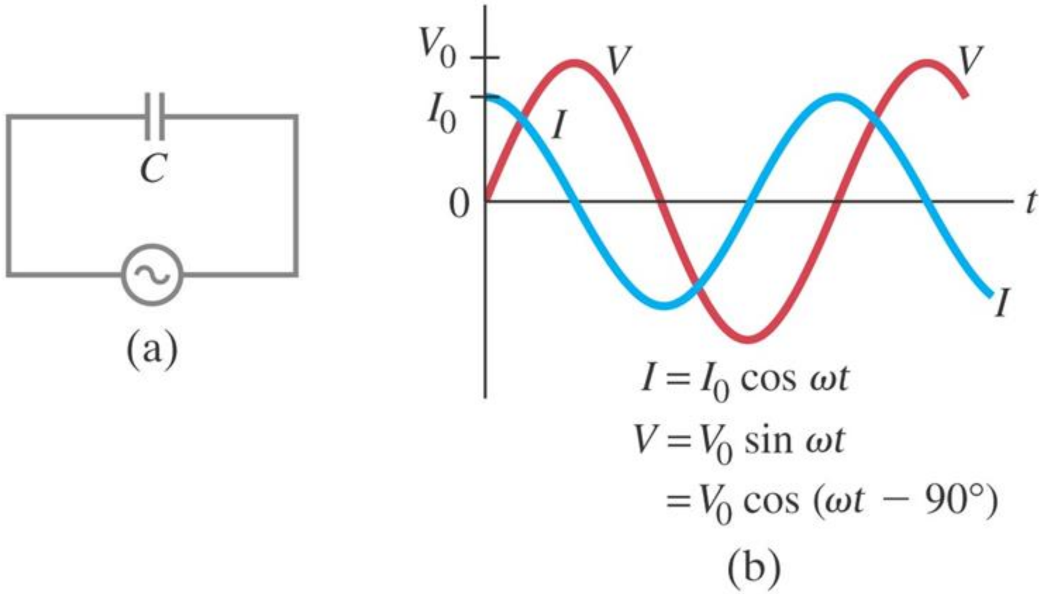
$$V = \frac{Q}{C} = I_0 \left(\frac{1}{\omega C} \right) sin\omega t$$

- Using the identity $\sin(\theta) = \cos(90° - \theta) = \cos(\theta - 90°)$ we can re-write the above as

$$V = \frac{I_0}{\omega C} \cos(\omega t - 90°) = V_0 \cos(\omega t - 90°)$$

- Where $V_0 = I_0 (\frac{1}{\omega C})$ = the peak or maximum capacitor voltage

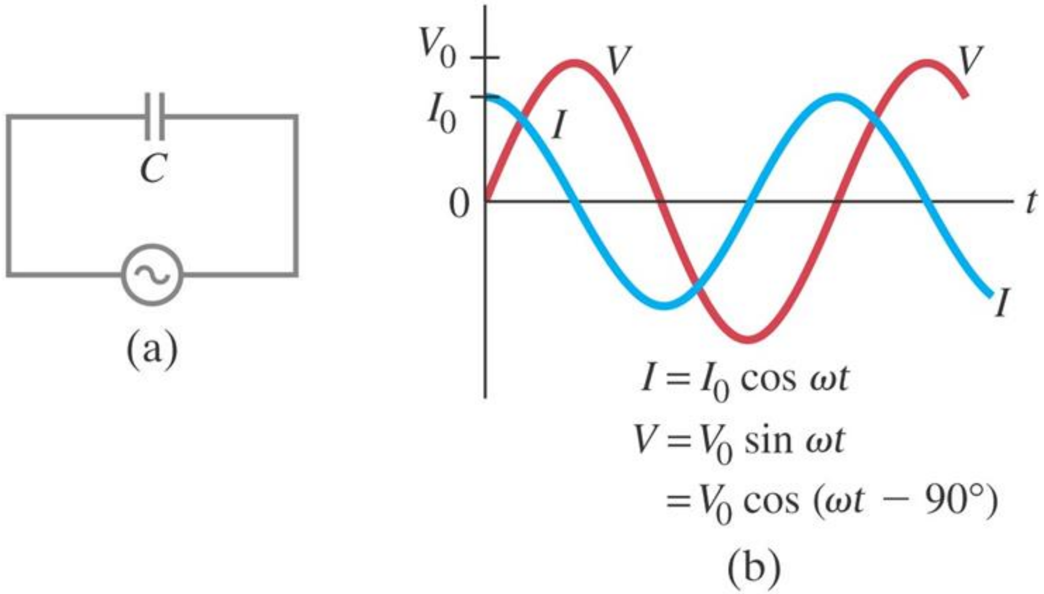
AC Source Connected to a Capacitor



- Looking at the graph in (b), it is clear that the current and voltage are out of phase by quarter cycle or $90°$ ($\pi/2$ radians).

- **The current leads the voltage across a capacitor by $90°$.**

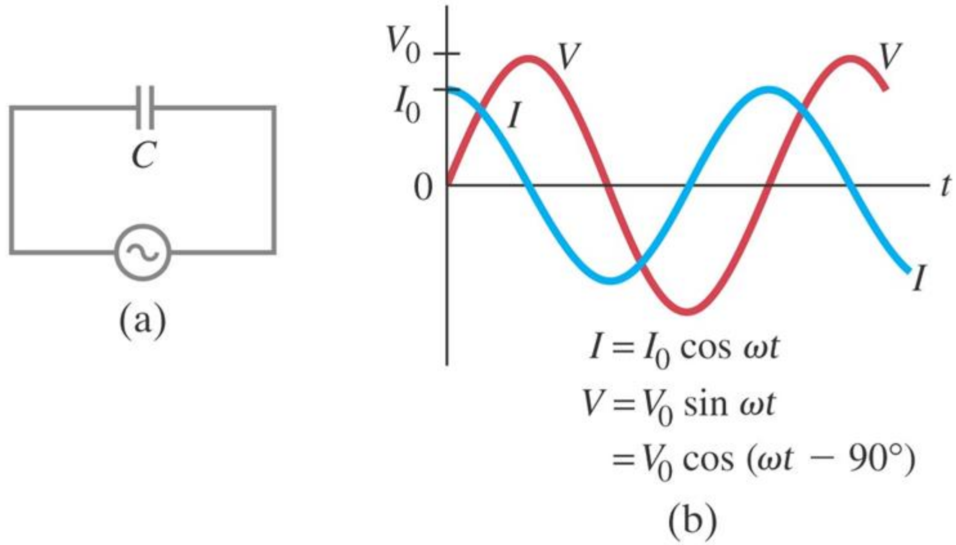
AC Source Connected to a Capacitor



N.B.

- Because the current and voltage are out of phase by 90°, the average power dissipated is zero, just as for an inductor.
- Energy from the source is fed to the capacitor, where it is stored in the electric field between its plates.
- As the field decreases, the energy returns to the source.
- Thus, only a resistance will dissipate energy as thermal energy in an ac circuit.

AC Source Connected to a Capacitor



$$V = \frac{I_0}{\omega C} \cos(\omega t - 90°) = V_0 \cos(\omega t - 90°)$$

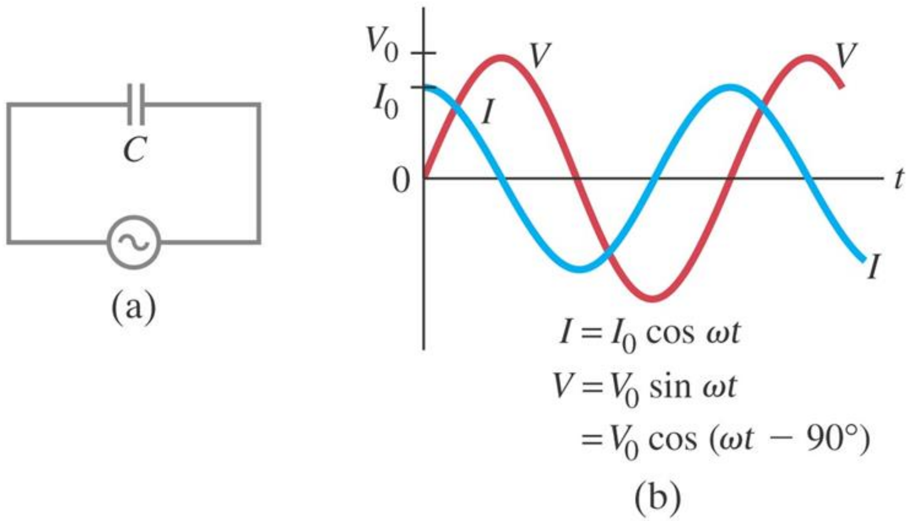
- A relationship between the applied voltage and the current in a capacitor can be written just as for an inductance:

$$V_C = I_0 X_c$$

- Where X_C is the **capacitive reactance** of the capacitor, and has units of ohms;

$$X_C = \frac{V_C}{I_o} = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

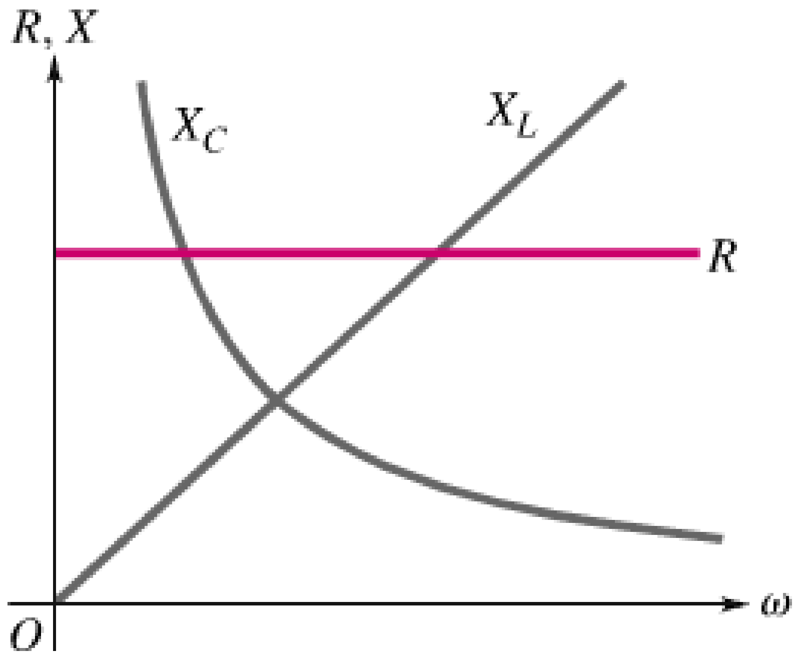
Capacitive Reactance



$$V_C = I_0 X_c$$

- Like for an inductor in an ac circuit, the above equation relates the peak values of V and I, or the rms values ($V_{rms} = I_{rms} X_C$).
- But it is not valid at a particular instant because I and V are not in phase.

Summary of Circuit Elements



Circuit element	Circuit quantity	Amplitude relation	Phase of v
Resistor	R	$V_R = IR$	In phase with i
Inductor	$X_L = \omega L$	$V_L = IX_L$	Leads i by 90°
Capacitor	$X_C = 1/\omega C$	$V_C = IX_C$	Lags i by 90°

The Loudspeaker

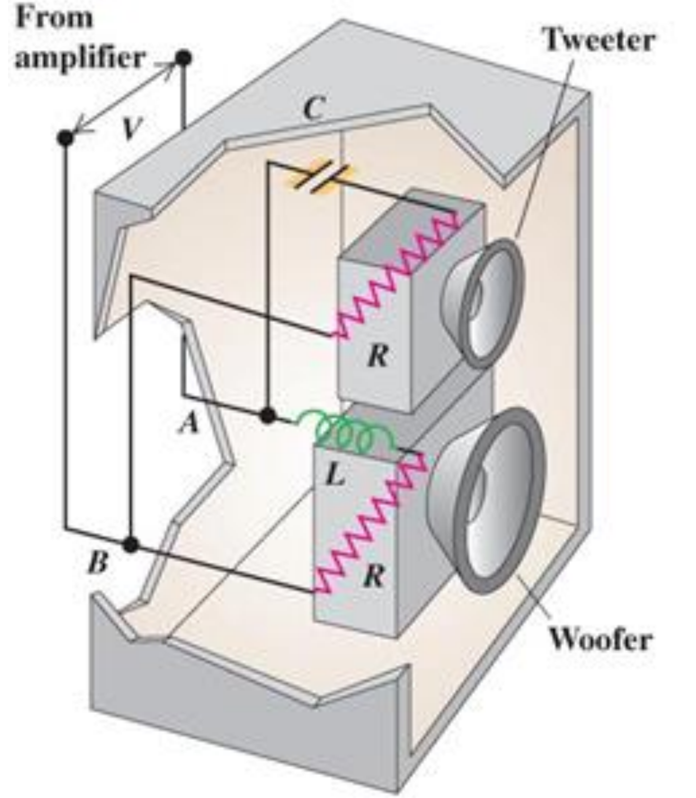
$$X_L = \frac{V_L}{I_0} = \omega L$$

$$X_C = \frac{V_C}{I_0} = \frac{1}{\omega C}$$

- In loudspeaker systems, inductors can be used to block high frequencies, while permitting lower frequencies or dc to pass through . . .

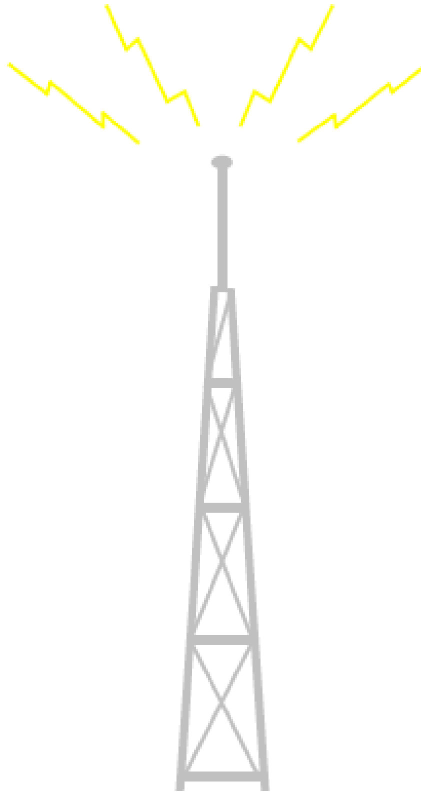
- . . . capacitors can be used to block low-frequency current and dc current, while permitting high-frequency current, just the opposite of inductors.

The inductor and capacitor feed low frequencies mainly to the woofer and high frequencies mainly to the tweeter.



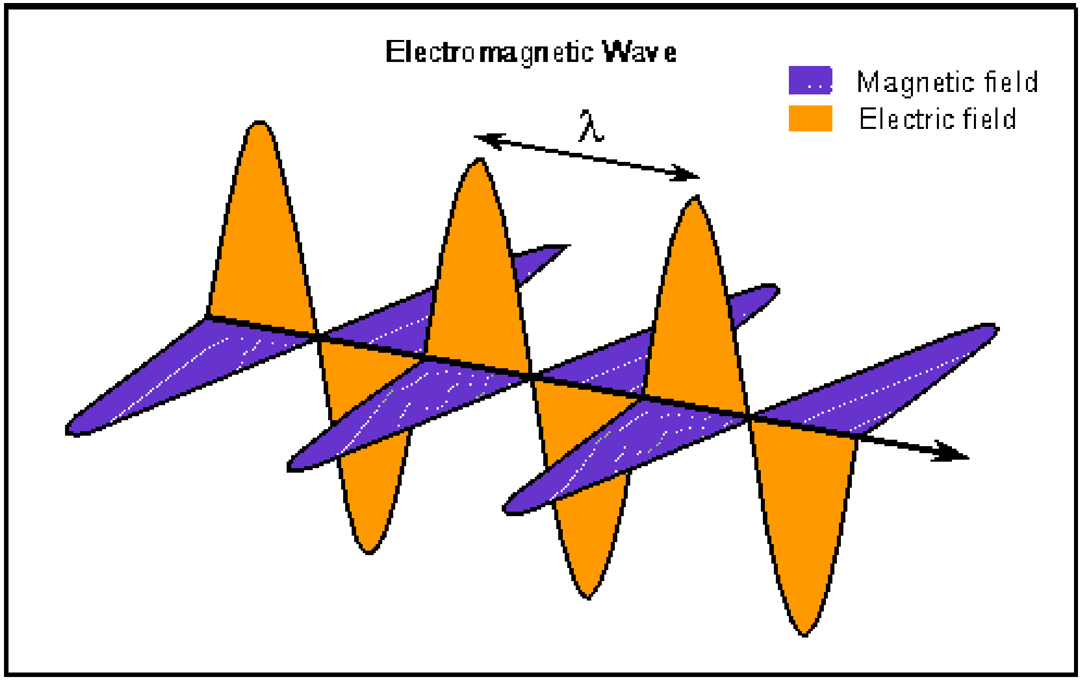
A crossover network in a loudspeaker system

LRC Circuits – An Important Part of Radios



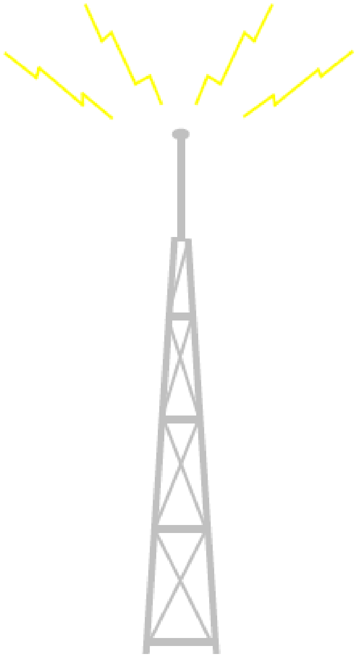
- In this lecture, we are going to analyse how an LRC (inductor-resistor-capacitor) circuit can be used to detect radio waves, or electromagnetic waves in other words.

LRC Circuits – An Important Part of Radios



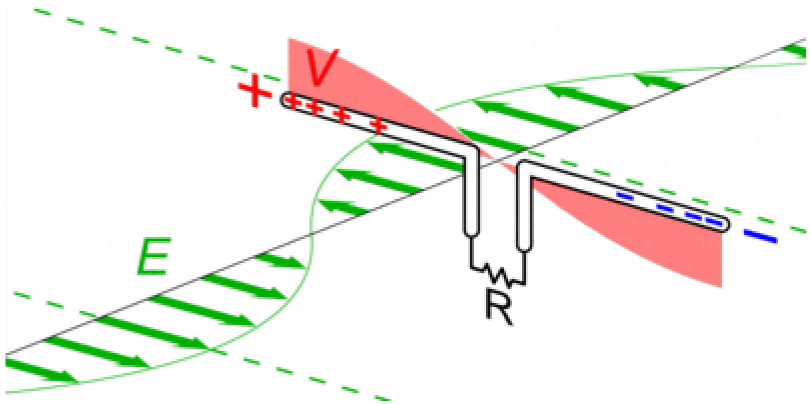
- Electromagnetic radiation consists of interconnected electric and magnetic fields. One way to produce such waves is to wiggle something that is electrically charged such as an electron; in other words create an alternating current.
- It should be noted that the current must alternate very fast.

LRC Circuits – An Important Part of Radios



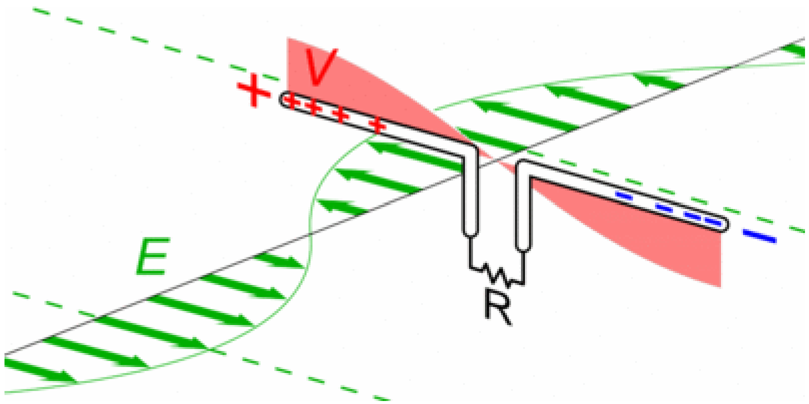
- A radio transmitter periodically moves electric charges up and down its antenna to propagate a signal in the form of electromagnetic waves.
- These waves are then received by the antennae on your radio or television set. When the waves encounter the antennae of your radio, they can push electric charges up and down on that antenna.

LRC Circuits – An Important Part of Radios



- A half-wave dipole antenna receiving power from a radio wave. The electric field of the wave **(E, green arrows)** pushes the electrons in the antenna elements back-and-forth **(black arrows)**, charging the ends of the antenna alternately positive and negative.
- These oscillating currents flow down the transmission line into the radio receiver (represented by the resistor, *R*). The action is shown slowed down in this animation.

LRC Circuits – An Important Part of Radios



- When you switch-on your radio, if the current in your radio receiver oscillates at the same frequency as the radio waves, we say that it is tuned.
- This makes it easier to detect the radio waves, undergoing electrical resonance when it does so, which is similar to mechanical resonance.

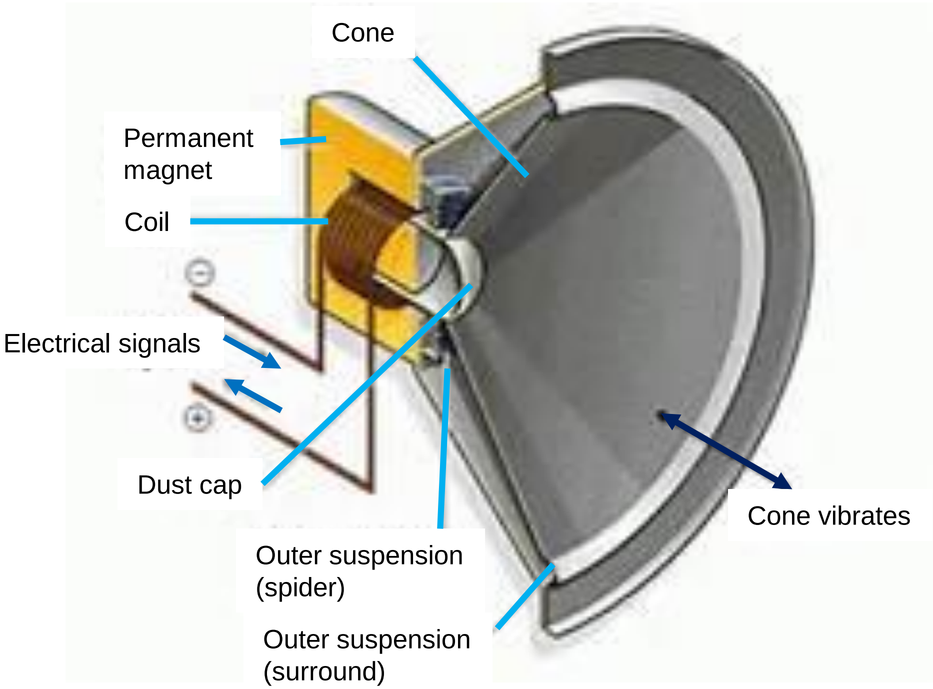
Driven Oscillations and Resonance

- The child on the swing is an oscillator. If she moves back-and-forth with a certain frequency, she will eventually stop moving.
- However, if her mother pushes her with a frequency close to the child's frequency, the child will continue oscillating.
- As the mother keeps pushing her with a frequency close to or equal to the child's frequency of movement, she will make the child oscillate with a bigger amplitude: the mother is effectively adding more energy to the oscillator (the child).

Driven Oscillations and Resonance

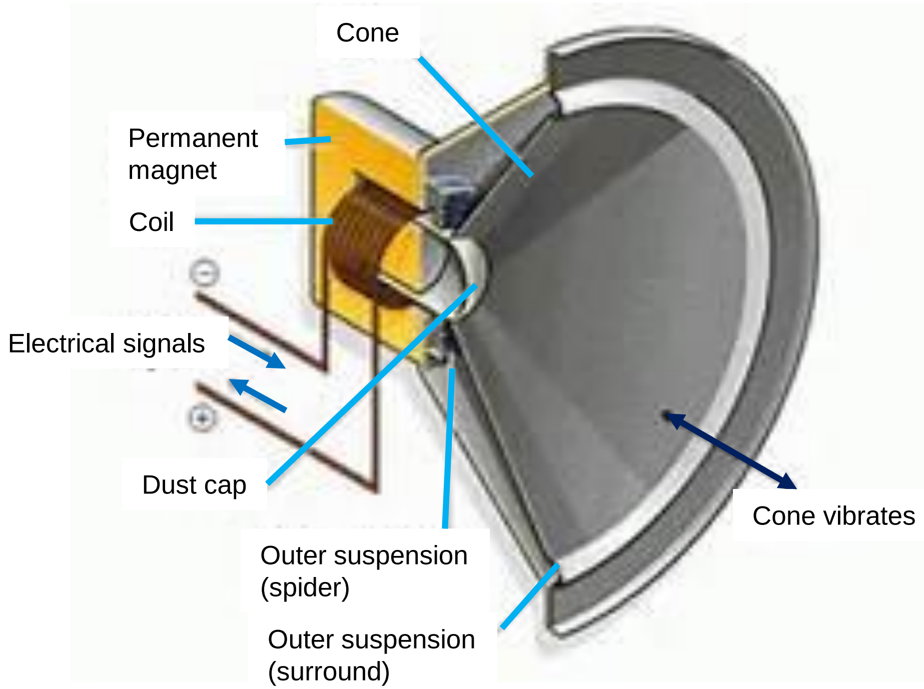
- We call this a **driven oscillation**. If the mother's pushing frequency is exactly equal to the child's oscillation frequency, we call it **resonance**.
- You can substitute the child for the circuit in your radio. If the electromagnetic waves match the resonant frequency of the circuit, it will undergo what we call **electrical resonance**.
- To keep things simple, this electrical resonance can be transformed into sound waves by using speakers.

LRC Circuits – An Important Part of Radios



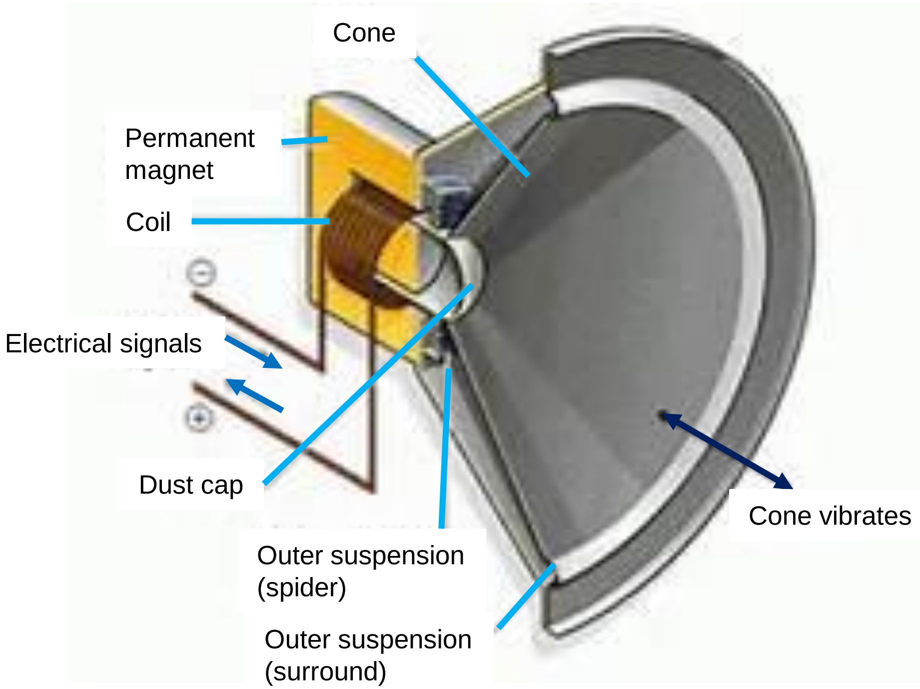
- In order to translate an electrical signal into an audible sound, speakers contain an electromagnet: a metal coil which creates a magnetic field when an electric current flows through it.
- This coil behaves much like a normal (permanent) magnet, with one particularly handy property: reversing the direction of the current in the coil flips the poles of the magnet.

LRC Circuits – An Important Part of Radios



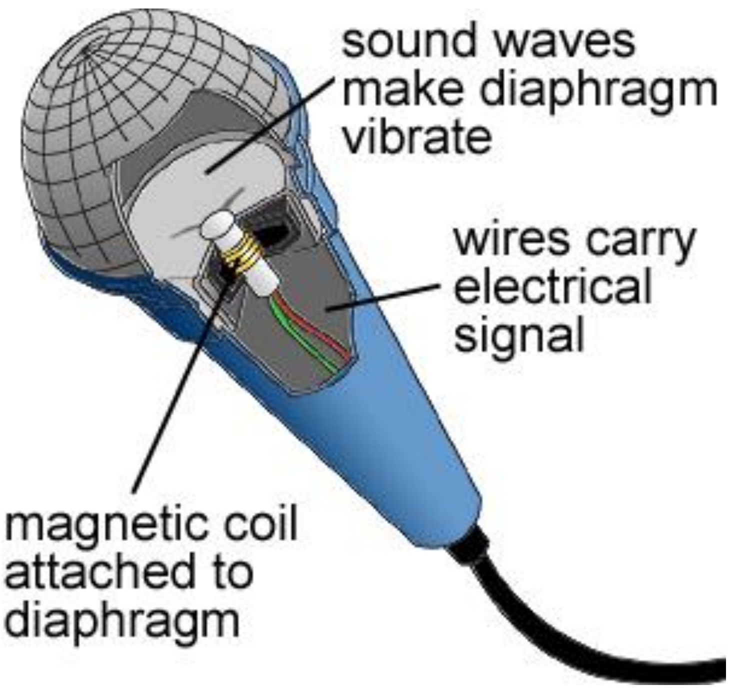
- Inside a speaker, an electromagnet is placed in front of a permanent magnet. The permanent magnet is fixed firmly into position whereas the electromagnet is mobile. As pulses of electricity pass through the coil of the electromagnet, the direction of its magnetic field is rapidly changed. This means that it is in turn attracted to and repelled from the permanent magnet, vibrating back-and-forth.

LRC Circuits – An Important Part of Radios



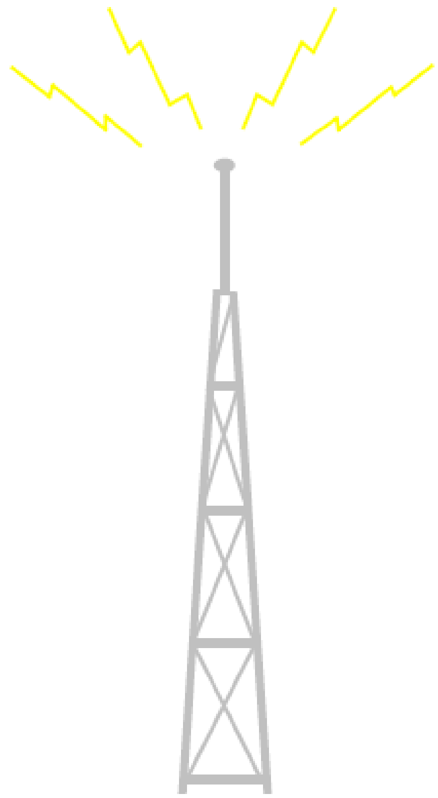
- The electromagnet is attached to a cone made of a flexible material such as paper or plastic which amplifies these vibrations, pumping sound waves into the surrounding air and towards your ears.

LRC Circuits – An Important Part of Radios



- A microphone uses the same mechanism as a speaker in reverse to convert sound into an electrical signal.

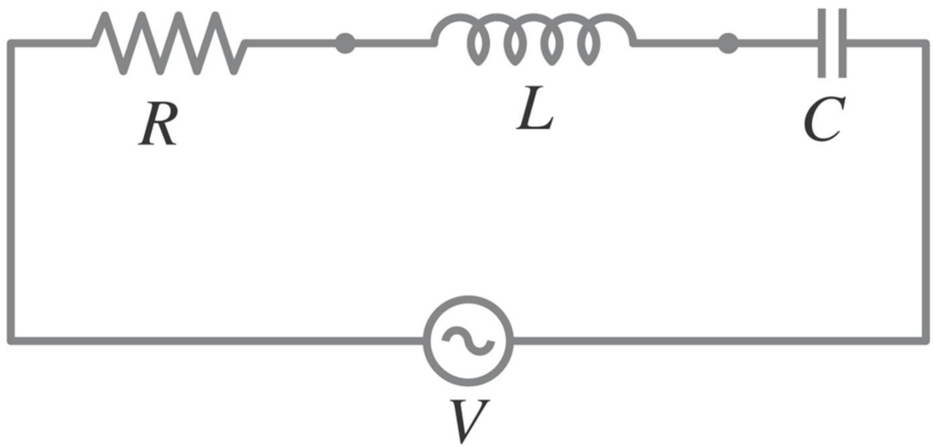
LRC Circuits – An Important Part of Radios



- In this lecture, we are going to analyse how an LRC (inductor-resistor-capacitor) circuit can be used to detect radio waves, electromagnetic waves in other words. So let's see what these **LRC circuits** do.

2. LRC Series AC Circuit

LRC Series AC Circuit

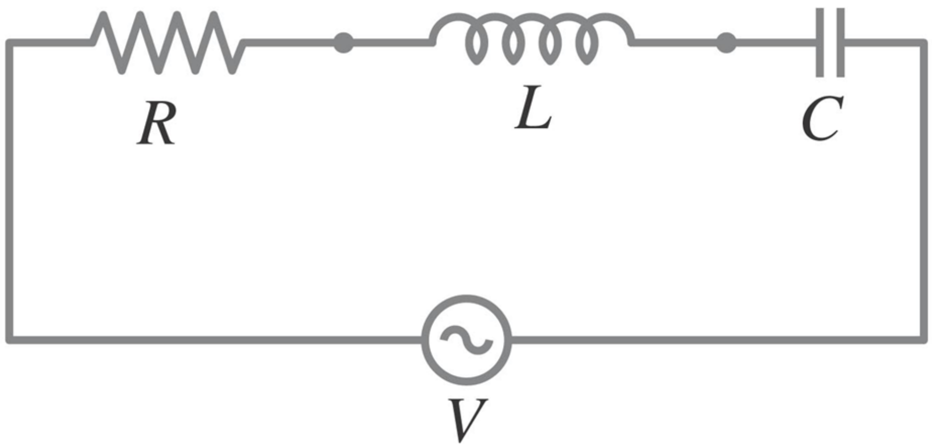


- We are now going to examine a circuit containing all three elements in series: a resistor R, an inductor L, and a capacitor C.

- If a given circuit contains only two of these elements, we can still use the results of this Section, by setting $R = 0$, $X_L = 0$, or $X_C = 0$.

- We let V_R, V_L, and V_C represent the voltage across each element at a given instant in time; and V_{R0}, V_{L0}, and V_{C0}, represent the maximum (peak) values of these voltages.

LRC Series AC Circuit

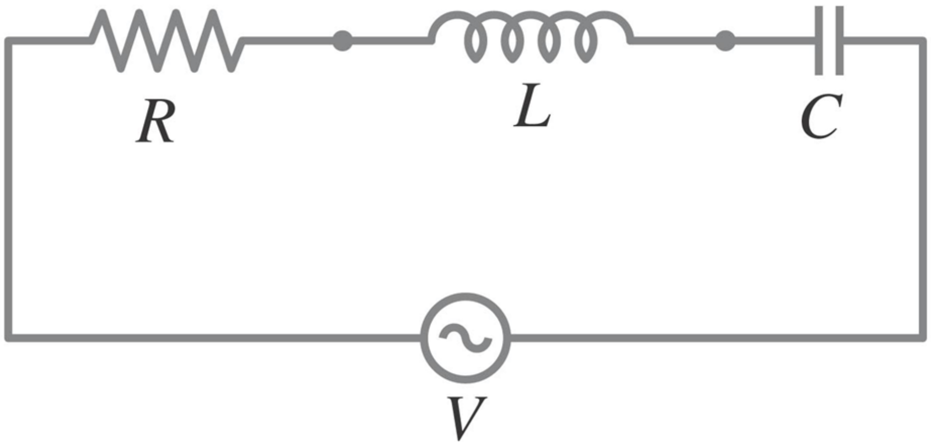


- At any instant, the voltage V supplied by the source will be, by Kirchhoff's loop rule,

$$V = V_R + V_L + V_C$$

- Because the various voltages are not in phase, they do not reach their peak values at the same time, so the peak voltage of the source V_0 will not equal $V_{R0} + V_{L0} + V_{C0}$

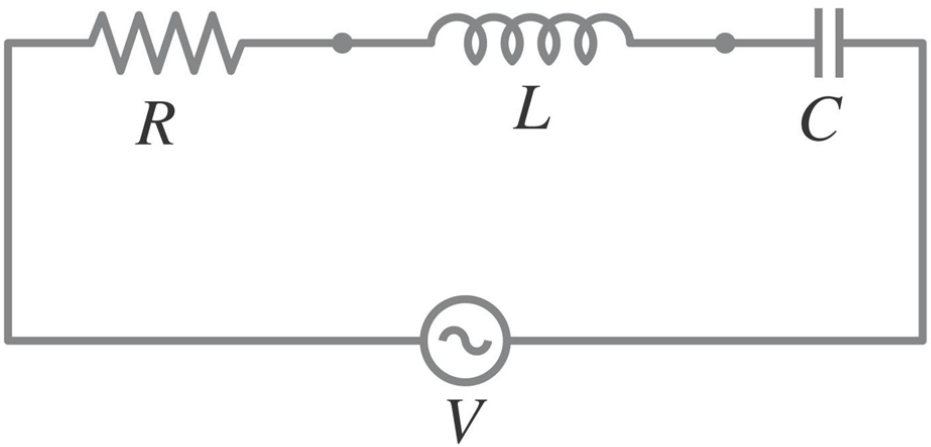
LRC Series AC Circuit



- Let us now find the impedance of an LRC circuit as a whole (the effect of R, X_C, and X_L), as well as the peak current I_0, and the phase relation between V and I.

- The current at any instant must be the same at all points in the circuit. Thus the currents in each element are in phase with each other, even though the voltages are not.

LRC Series AC Circuit

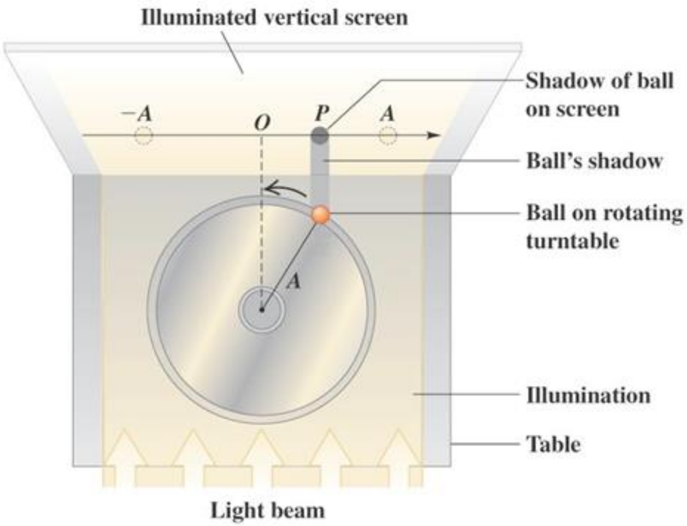


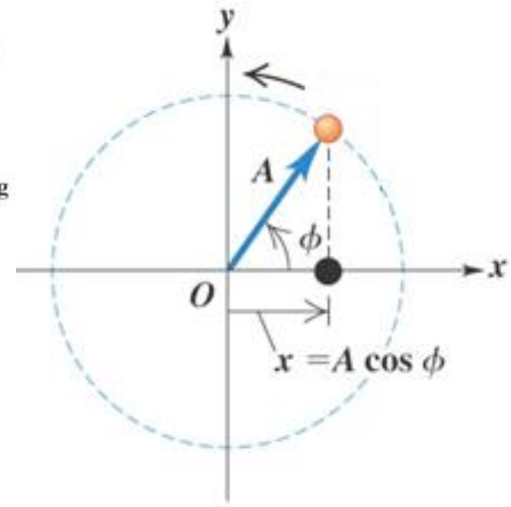
- We choose our origin in time ($t = 0$) so that the current I at any time t is

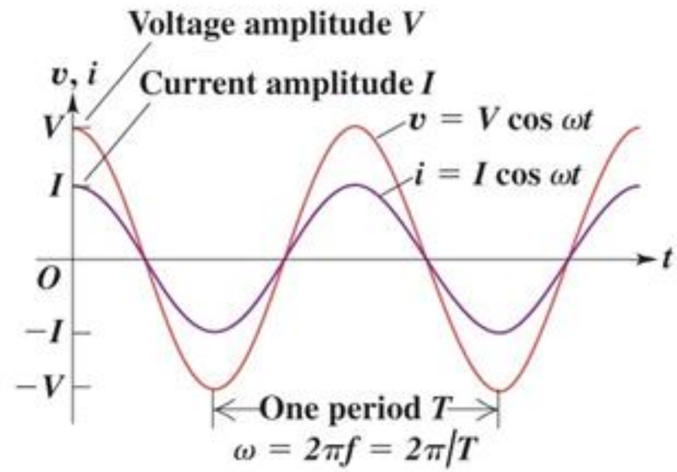
$$I = I_0 cos\omega t$$

- We analyse an LRC circuit using a **phasor diagram**.

Previously



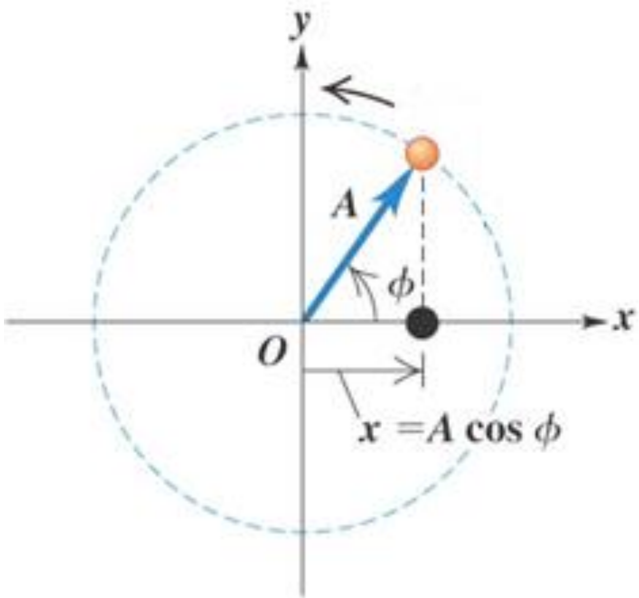




N.B.

We have shown that an electric generator can produce a sinusoidal voltage. Consequently, the accompanying current is also sinusoidal. This is somewhat similar to the graph of a body undergoing simple harmonic motion, but bear in mind that the **above graph** for voltage and current only represents the **direction and size** of the two quantities **relative to an equilibrium position**, not the direction and distance of the two quantities relative to an equilibrium position.

Representing Sinusoidally Varying Voltages & Currents

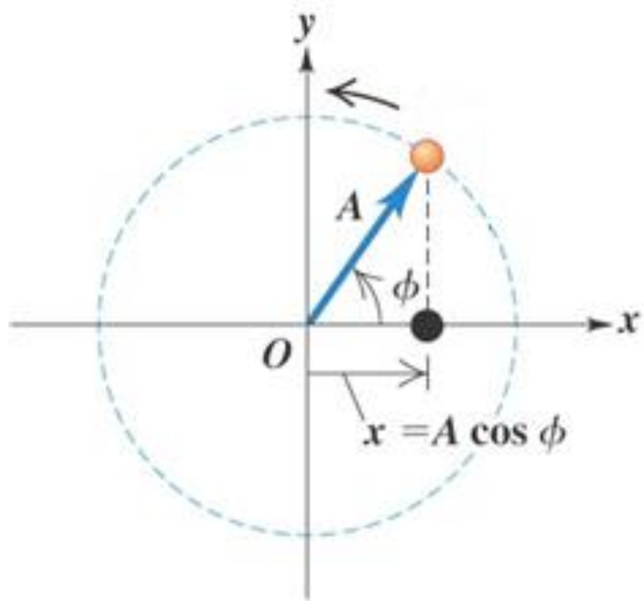


N.B.

Another way to view the arrow in the circle is to view it like a vector which rotates anti-clockwise.

- The length of the vector/arrow represents the maximum value (amplitude) of the quantity as it undergoes simple harmonic motion.

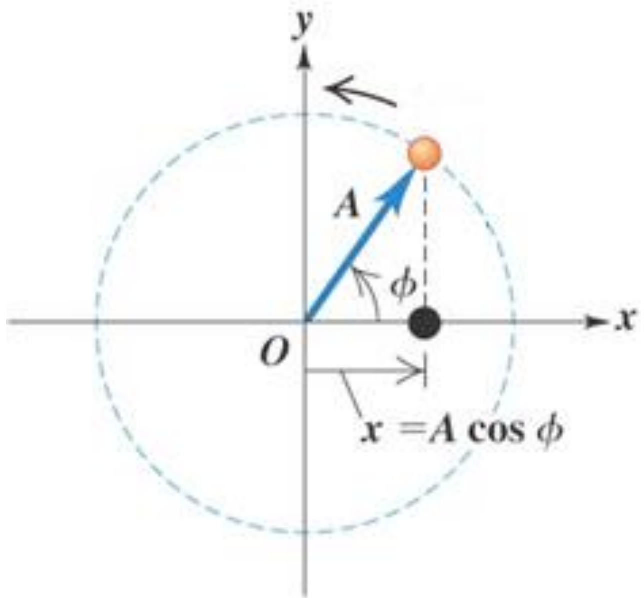
Representing Sinusoidally Varying Voltages & Currents

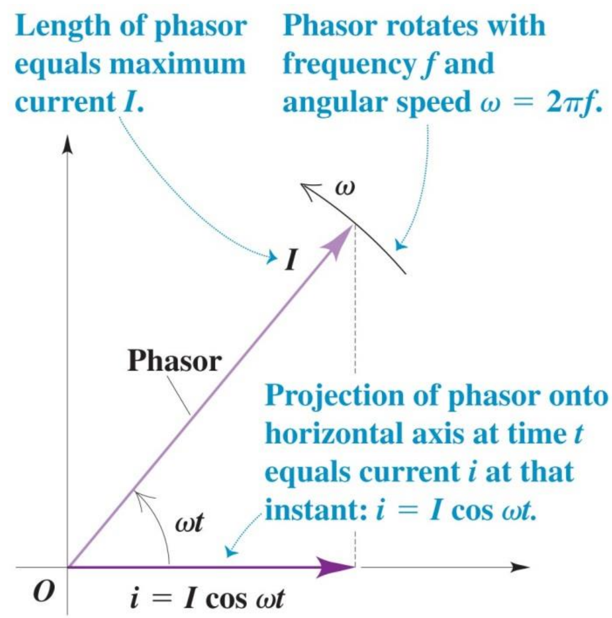


- Again, the instantaneous value of the quantity that varies in a sinusoidal way with time is represented by the projection of the vector onto a horizontal axis.

- In an electricity context, the instantaneous value of the voltage or current that varies in a sinusoidal way with time is represented by the projection of the corresponding rotating vector onto a horizontal axis.

Phasors

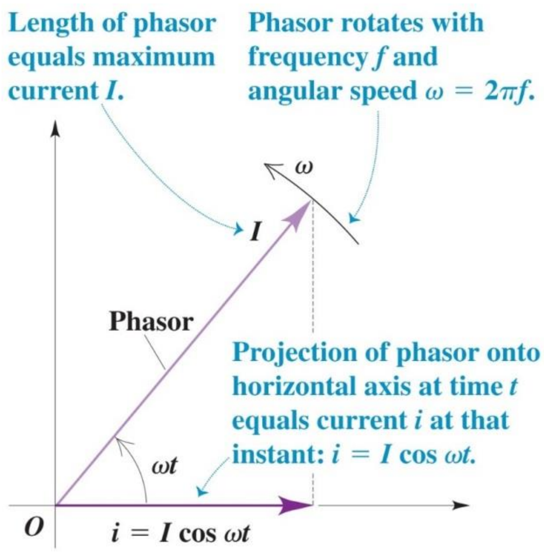




We call these rotating vectors, **phasors**.

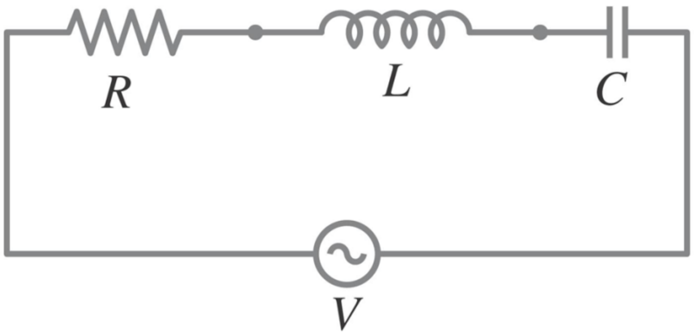
- Diagrams containing these **phasors** do not contain the circle which the phasor traces out as it rotates anti-clockwise.
- The existence of this circle is implied.
- Instead, **phasors** are represented as shown above.

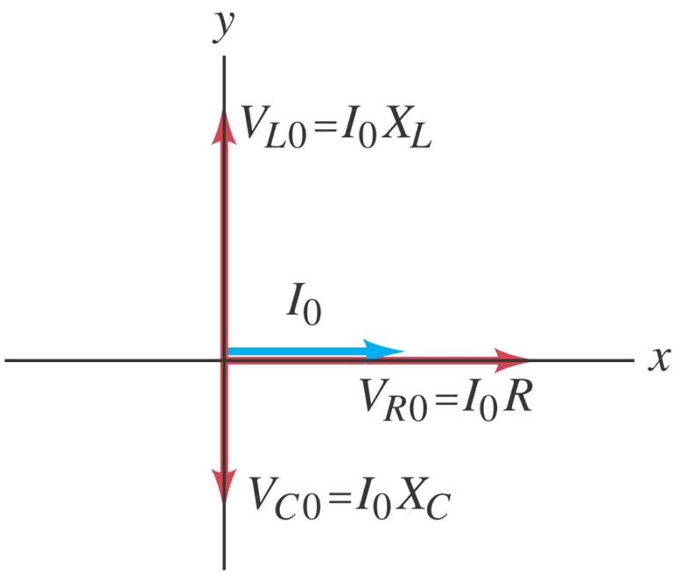
Phasor Diagrams



- A phasor is not a real physical quantity with a direction in space, like velocity or electric field, for example.
- A phasor is a geometric entity that provides a language for describing and analysing physical quantities that undergo simple harmonic motion.
- We can also use phasors to ***add*** sinusoidal voltages together; a matter of vector addition.

LRC Series AC Circuit

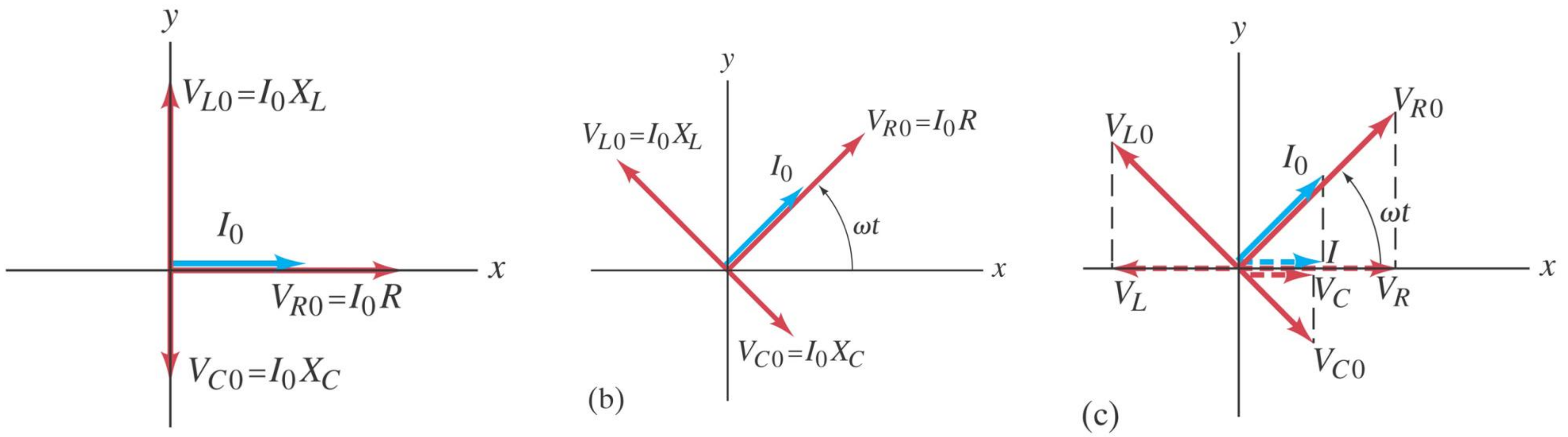




- Arrows (treated like vectors) are drawn in an xy coordinate system to represent each voltage.
- The length of each arrow represents the magnitude of the peak voltage across each element:

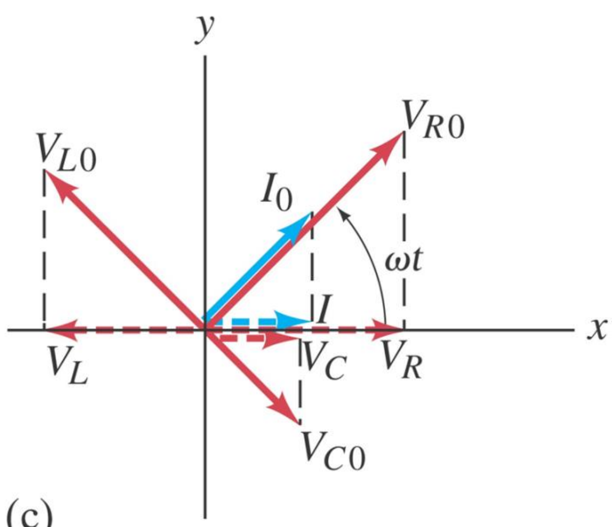
$$V_{R0} = I_0 R, \qquad V_{L0} = I_0 X_L, \qquad V_{C0} = I_0 X_C,$$

LRC Series AC Circuit

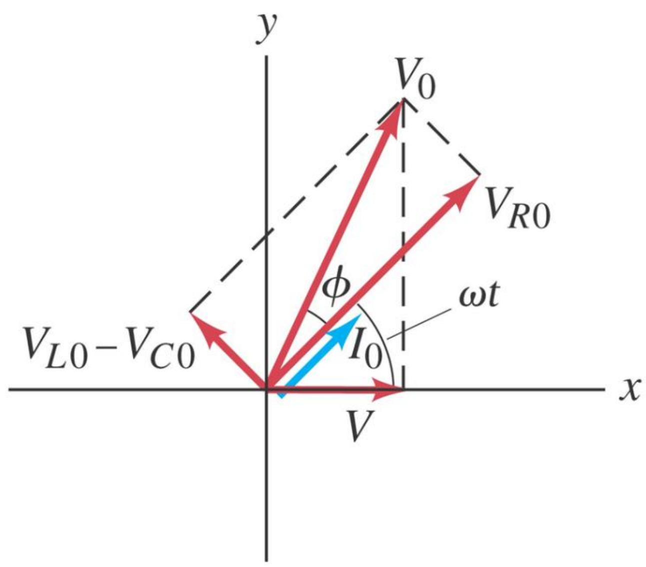


- If we let the vector diagram rotate counterclockwise at frequency F, we get the diagram shown in (b) above; after a time, t, each arrow has rotated through an angle ωt.
- Then, the projections of each arrow on the x axis represent the voltages across each element at the instant t. For example, $I = I_0 cos\omega t$, as shown in (c) above.

LRC Series AC Circuit

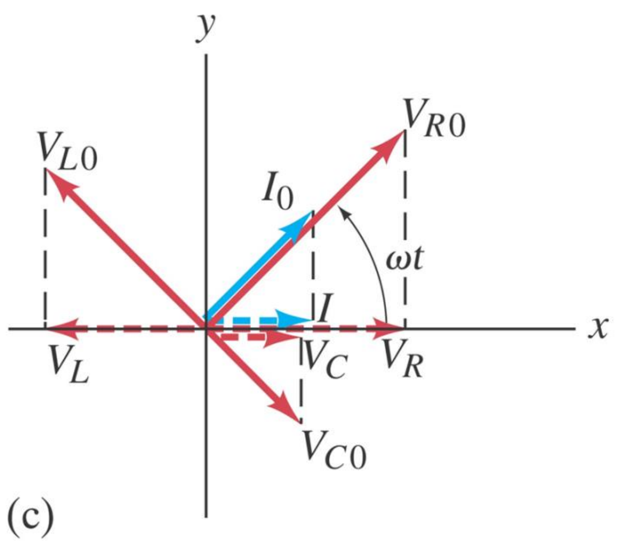


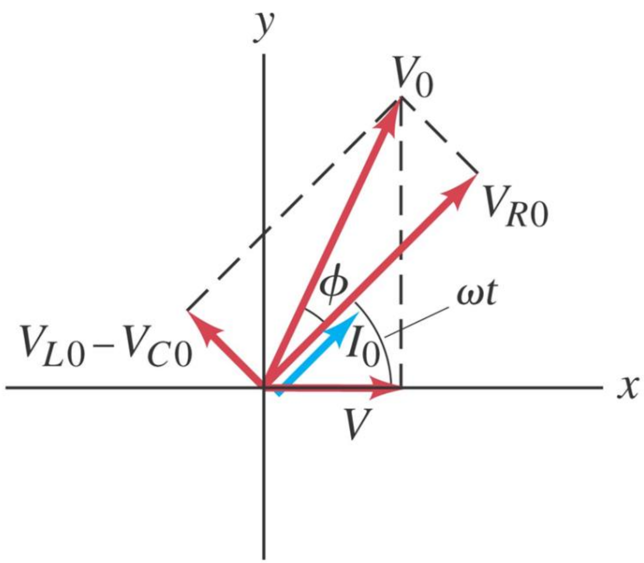
(c)



- The sum of the projections of the three voltage vectors represents the instantaneous voltage across the whole circuit, V.
- Therefore, the vector sum of these vectors will be the vector that represents the peak source voltage, V_0, as shown in the above figure to the right where it is seen that V_0 makes an angle ϕ with I_0 and V_{R0}.

LRC Series AC Circuit



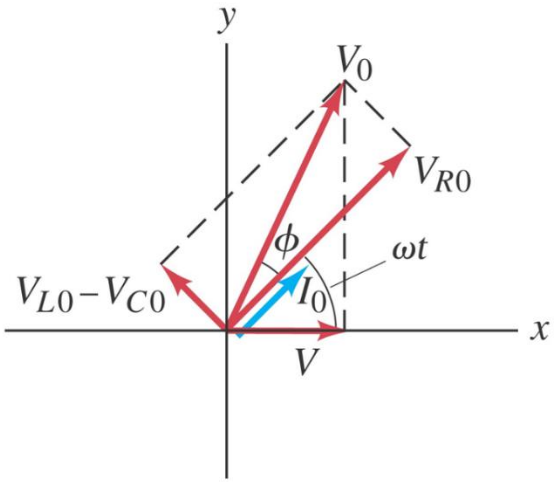


- As time passes, V_0 rotates with the other vectors, so the instantaneous voltage V (projection of V_0 on the x axis) is

$$V = V_0 \cos(\omega t + \phi)$$

- The voltage V across the whole circuit must equal the source voltage. Thus the voltage from the source is out of phase with the current by an angle ϕ.

LRC Series AC Circuit



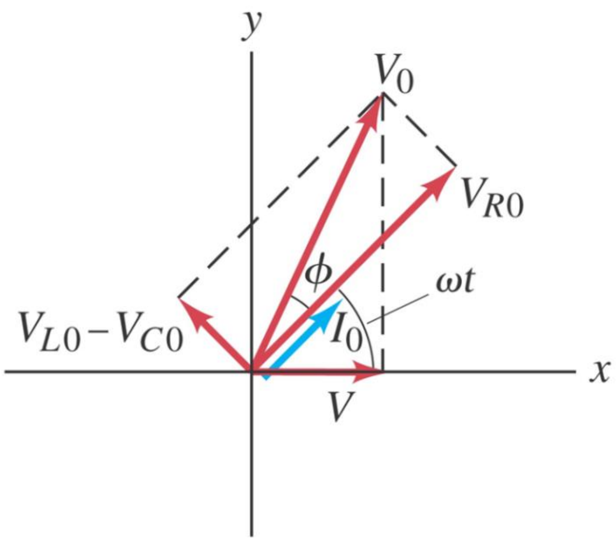
- We can now determine the total **impedance Z** of the circuit.
- Because the source voltage V_0 is a function of R, L, and C, as well as the angular frequency ω, the impedance of the circuit depends on these quantities.

$$V_0 = I_0 Z$$

$$V_0 = \sqrt{V_{R0}^2 + (V_{L0} - V_{C0})^2}$$

$$= I_0\sqrt{R^2 + (X_L - X_C)^2}$$

LRC Series AC Circuit



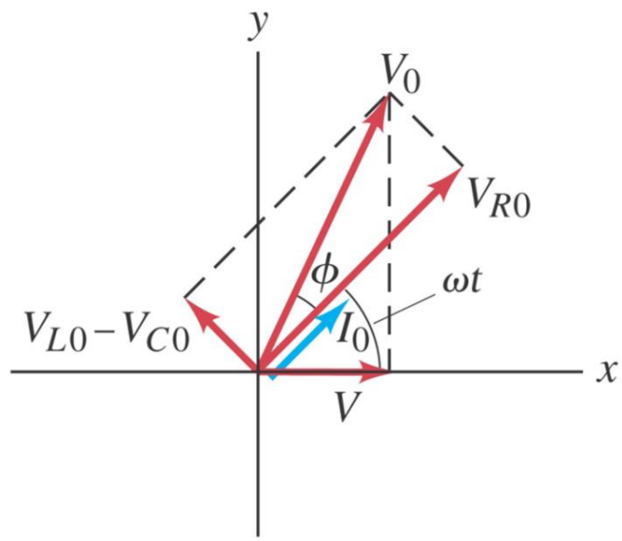
- The total impedance Z is

$$Z = \sqrt{R^2 + X^2}$$

$$= \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{R^2 + [\omega L - \left(\frac{1}{\omega C}\right)]^2}$$

LRC Series AC Circuit

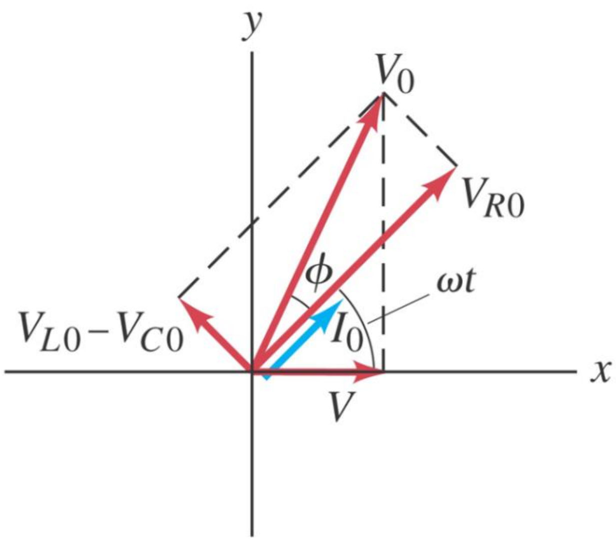


- We can find the phase angle ϕ between the voltage and current:

$$\tan\phi = \frac{V_{L0} - V_{C0}}{V_R} = \frac{I_0\,(X_L - X_C)}{I_0 R} = \frac{X_L - X_C}{R} = \frac{X}{R}$$

$$\phi = \tan^{-1}\left(\frac{\omega L - 1/\omega C}{R}\right)$$

LRC Series AC Circuit



- The above figure is drawn for the case $X_L > X_C$, and the current lags the source voltage by ϕ.

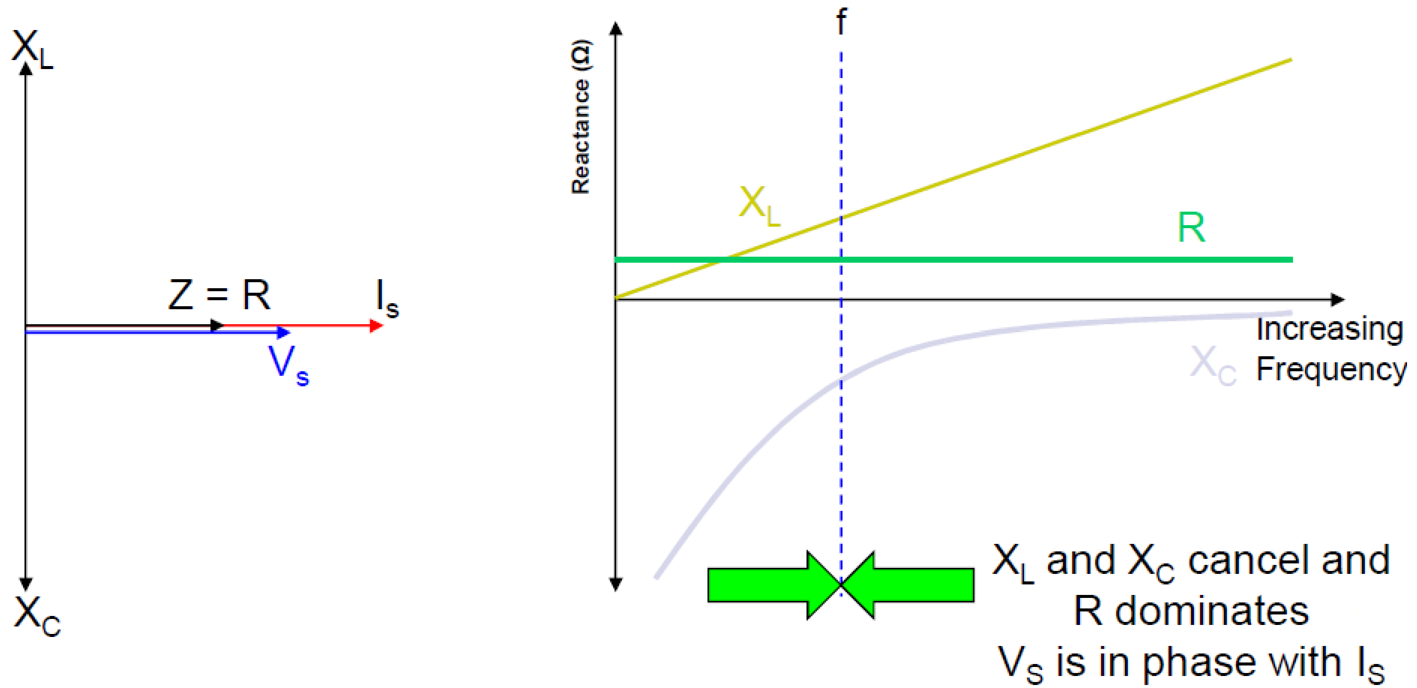
- When the reverse is true, $X_L < X_C$, then ϕ is less than zero, and the current leads the source voltage.

3. Resonance in AC Circuits

Electrical Resonance

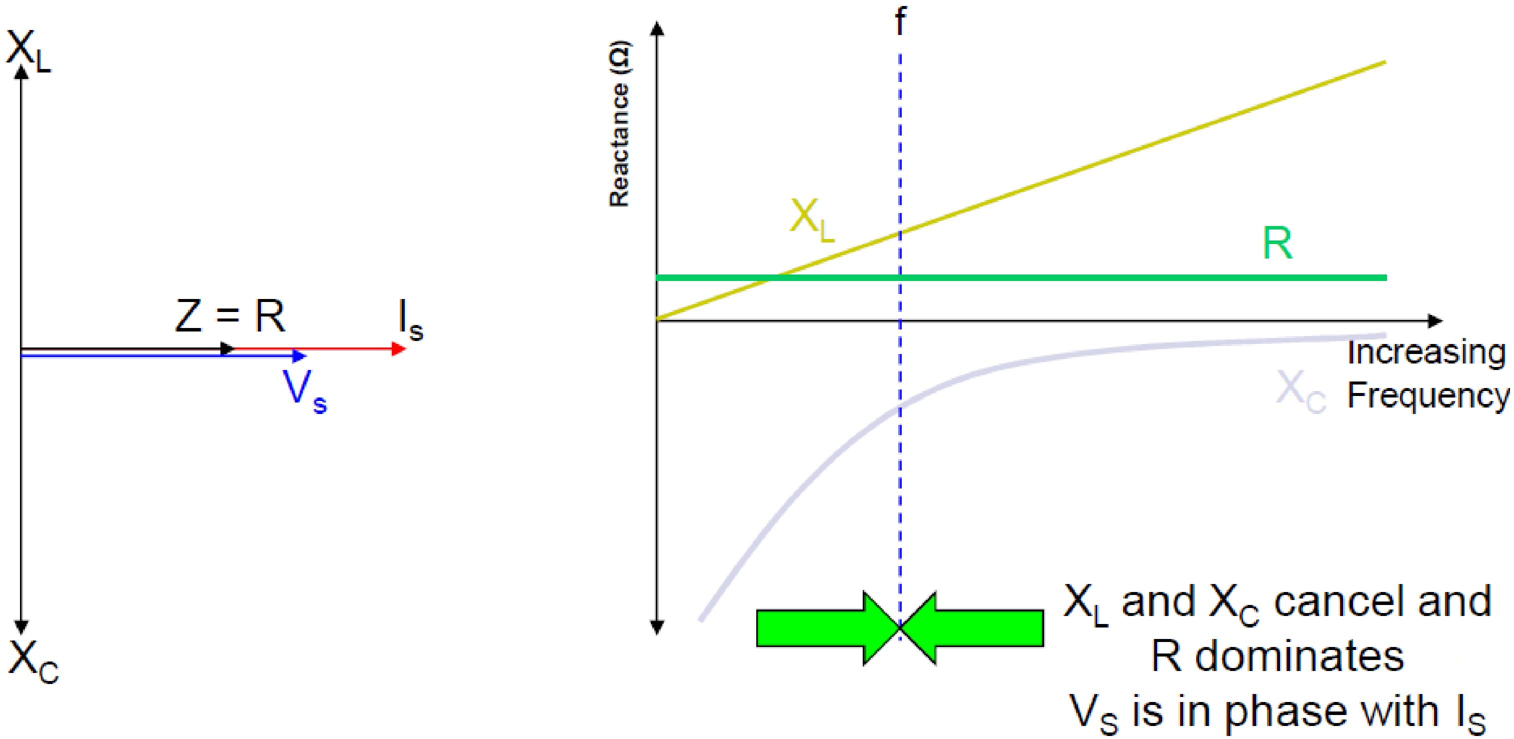
- Electrical resonance (similar to mechanical resonance) is used in many circuits. Radio and TV sets, for example, use resonant circuits (some form of an LRC circuit) for tuning into a station.
- Many frequencies reach the circuit (via antennae), but a significant current will flow only for those at or near what we call a **resonance frequency**.
- So what is this thing that we call a resonance frequency?

Resonance in AC Circuits



- For our LRC circuit, because X_L increases and X_C decreases with increasing frequency, there is always one particular frequency at which X_L and X_C are equal, and thus $X_L - X_C$ is zero.
- The particular frequency at which X_L and X_C are equal, and $X_L - X_C$ is zero, is called the **resonance angular frequency**.

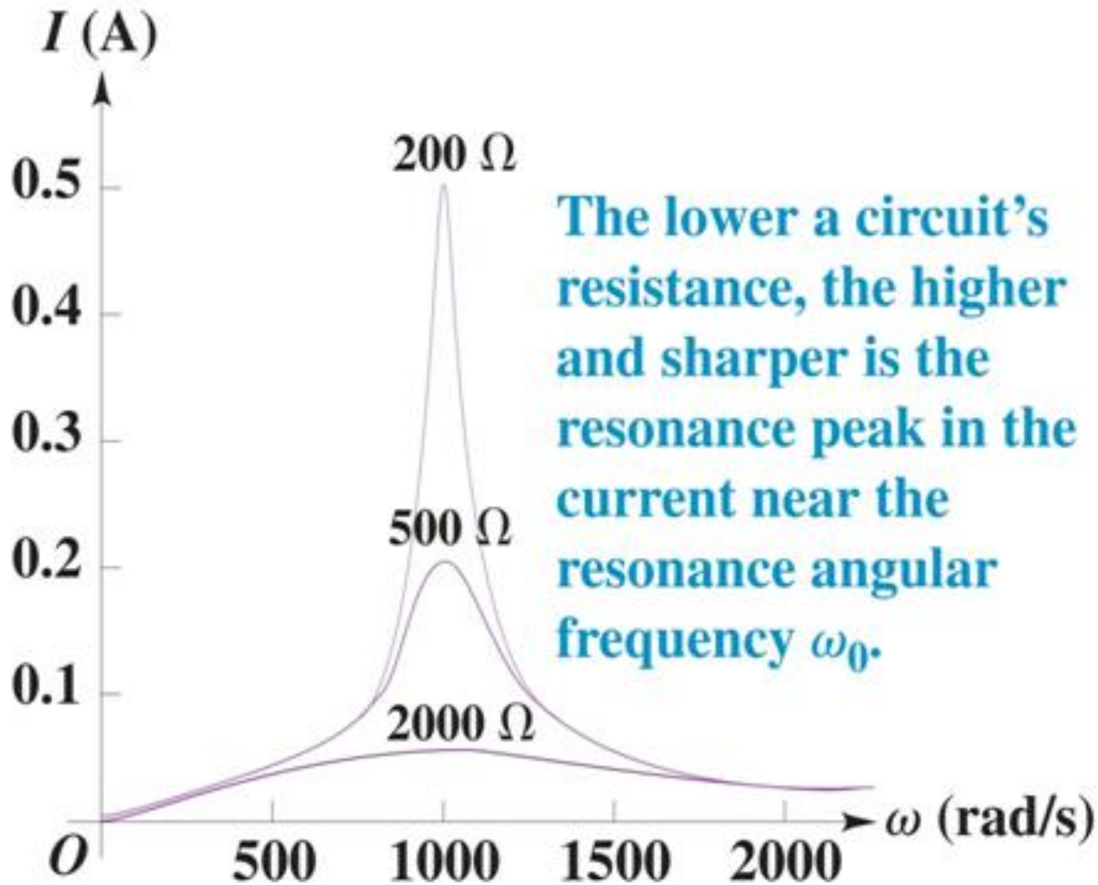
Resonance in AC Circuits



- At the **resonance angular frequency**, the impedance $\boldsymbol{Z}$ has its smallest value, and is just equal to the resistance $\boldsymbol{R}$ of the circuit.

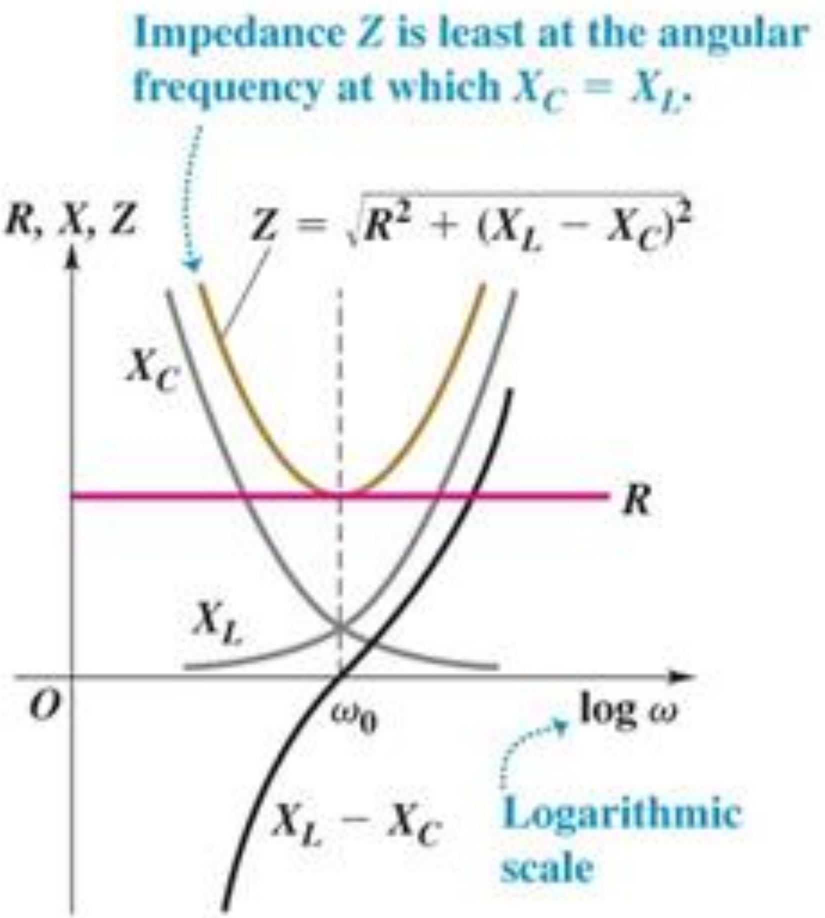
$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2}$$

Resonance in AC Circuits



- At the **resonance angular frequency**, the impedance Z has its smallest value, and therefore the current I has its largest value.

The Resonance Angular Frequency



(a) Reactance, resistance, and impedance as functions of angular frequency

$$X_L = X_C$$

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

The Resonance Frequency in Our Radio Antennae

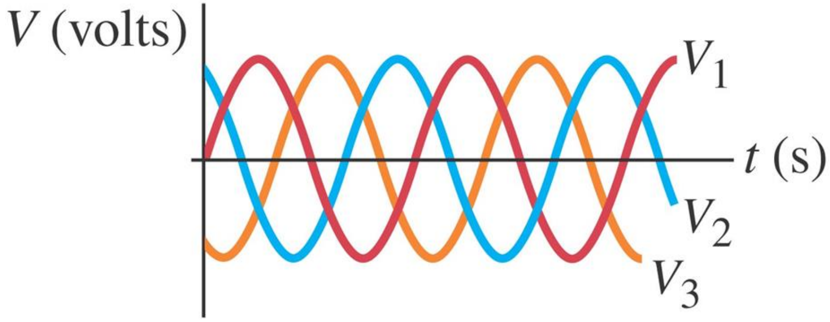
$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$f_0 = \frac{\omega_0}{2\pi}$$

- Because of these formulae, we can control (in old radios and TV sets, by turning knobs, thus varying inductor values, or capacitor values) what the resonance frequency in our radio antennae or TV antennae will be.
- This allows us to tune-in to different radio stations.

4. Three-Phase AC

Three-Phase AC



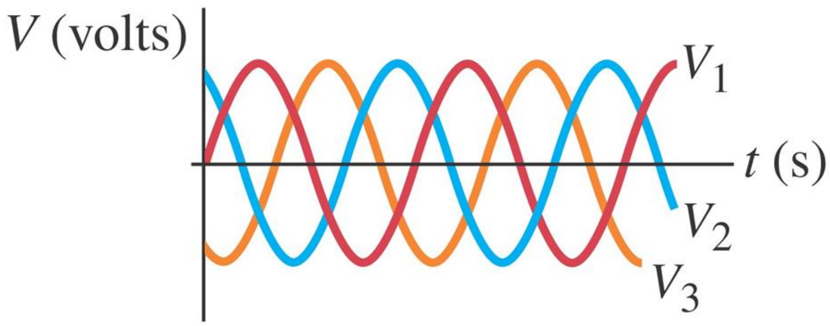
- Transmission lines typically consist of four wires, rather than two. One of these wires is connected to the ground; the remaining three are used to transmit three-phase ac power which is a superposition of three ac voltages 120° out of phase with each other:

$$V_1 = V_0 sin\omega t$$

$$V_2 = V_0 \sin(\omega t + \frac{2\pi}{3})$$

$$V_3 = V_0 \sin(\omega t + \frac{4\pi}{3})$$

Three-Phase AC

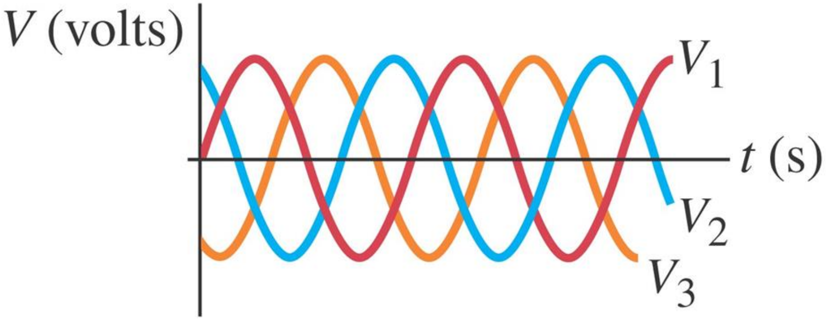


$$V_1 = V_0 sin\omega t \qquad V_2 = V_0 \sin(\omega t + \frac{2\pi}{3}) \qquad V_3 = V_0 \sin(\omega t + \frac{4\pi}{3})$$

- We use three-phase power because single-phase ac delivers power to the load in pulses.
- A much smoother flow of power can be delivered using three-phase power.
- Suppose that each of the three voltages making up the three-phase source is hooked up to a resistor R. Then the power delivered is

$$P = \frac{1}{R}({V_1}^2 + {V_2}^2 + {V_3}^2)$$

Three-Phase AC



$$P = \frac{1}{R}\left({V_1}^2 + {V_2}^2 + {V_3}^2\right)$$

- You can show that power is a constant equal to $\frac{3{V_0}^2}{2R}$, which is three times the rms power delivered by a single-phase source.
- This smooth flow of power makes electrical equipment run smoothly.
- Although houses use single-phase ac power, most industrial-grade machinery is wired for three-phase power.

Summary of today's Lecture

1. LRC Series AC Circuit
2. Resonance in AC Circuits
3. Three-Phase AC

Lecture 30: Optional Reading

- **Ch. 30.7**, AC circuits with AC Source;p.918-921.
- **Ch. 30.8**, LRC Series AC Circuit;p.921-923.
- **Ch. 30.9**, Resonance in AC Circuits;p.924.
- **Ch. 30.11**, Three-Phase AC;p.925.

Home Work

Do not forget to attempt the **Additional Problems** for this lecture before logging in to **Mastering Physics** to complete your assignments.