The University of Nottingham
UNITED KINGDOM · CHINA · MALAYSIA

# Science A Physics

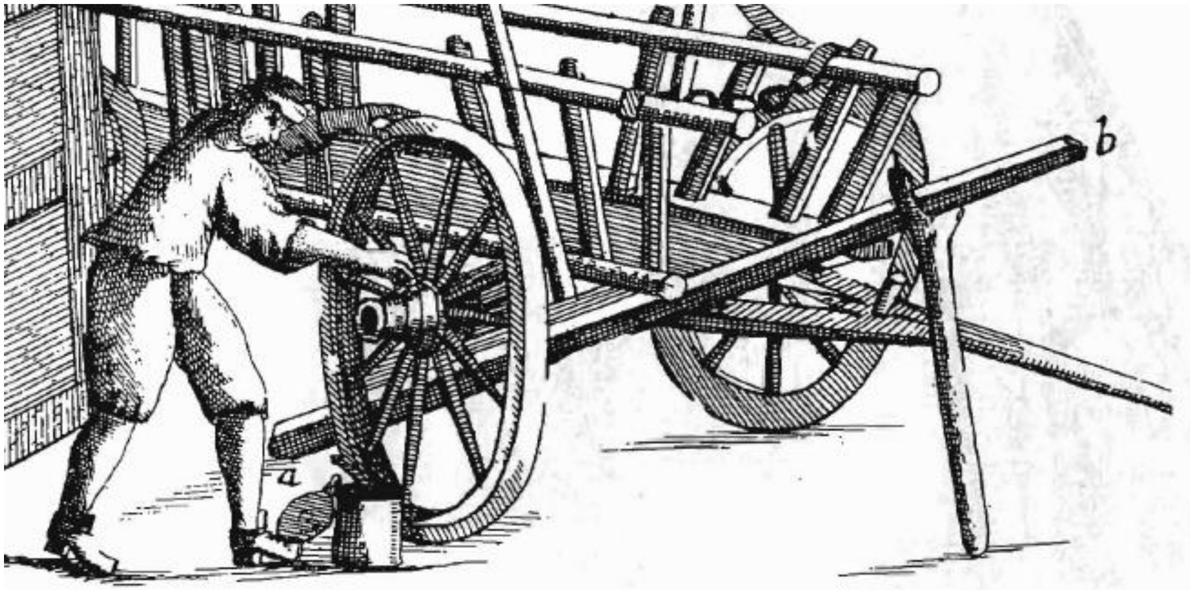
## Lecture 7:

## Static Systems – Simple Machines

# Aims of today's lecture

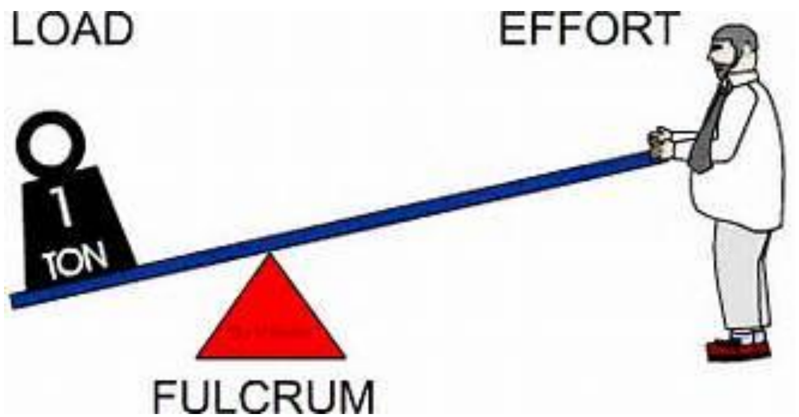
1. Introducing Machines and Structures
2. The Lever
3. The Moment of a Force
4. Simple Machines
5. The Force of Buoyancy
6. Pressure
7. Pascal's Principle and the Hydraulic Machine

# Introducing Machines and Structures



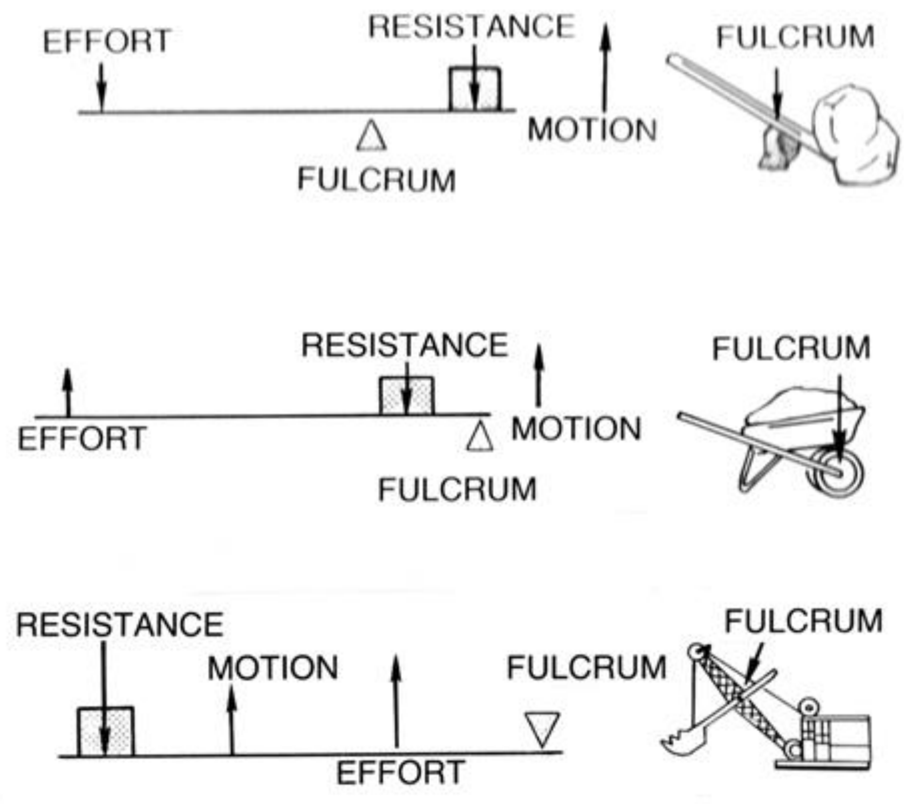

- The man changing the wheel on his wagon is faced with a problem: before he can remove the wheel, he has to raise the heavy vehicle, but muscular strength is not sufficient for the job.
- He solves the problem by means of a familiar device—**a lever**.

# Introducing Machines and Structures



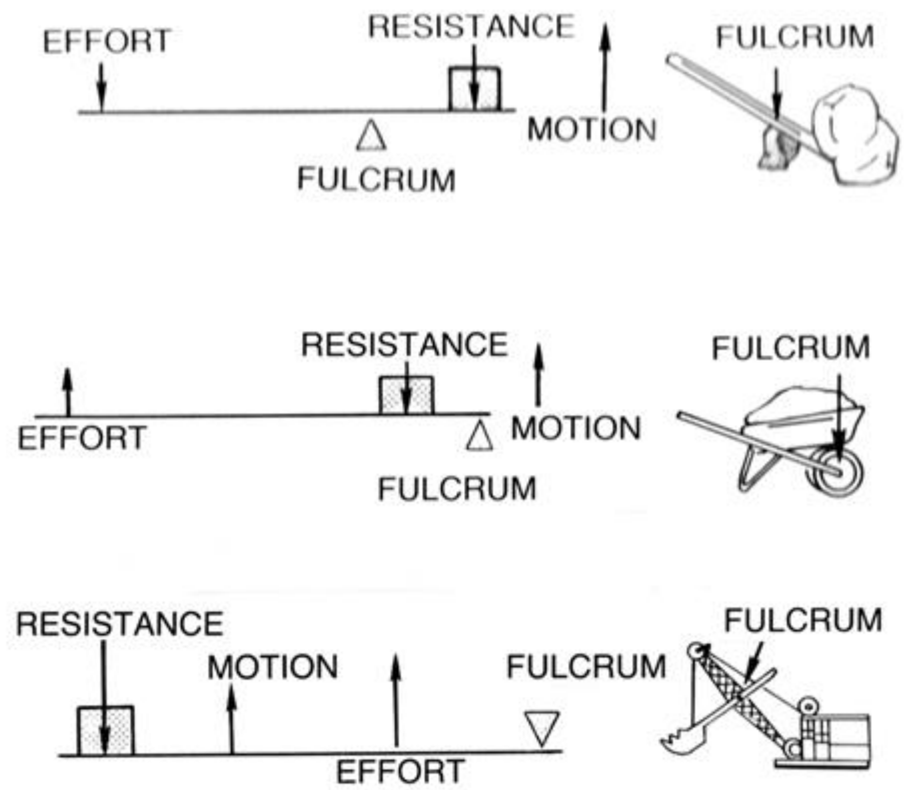

- The modest lever is a so-called simple machine: it multiplies the effort of the human arm to achieve something that, unaided, the arm could not possibly do.
- The advance of human civilization started with devices like this.
- You can see that a lever is a machine consisting of a beam or rigid rod pivoted at a fixed hinge, or fulcrum.

# Introducing Machines and Structures




- On the basis of the location of the fulcrum, load and effort, the lever is divided into **three types/classes**, classified by the relative positions of the fulcrum, effort and resistance (or load).

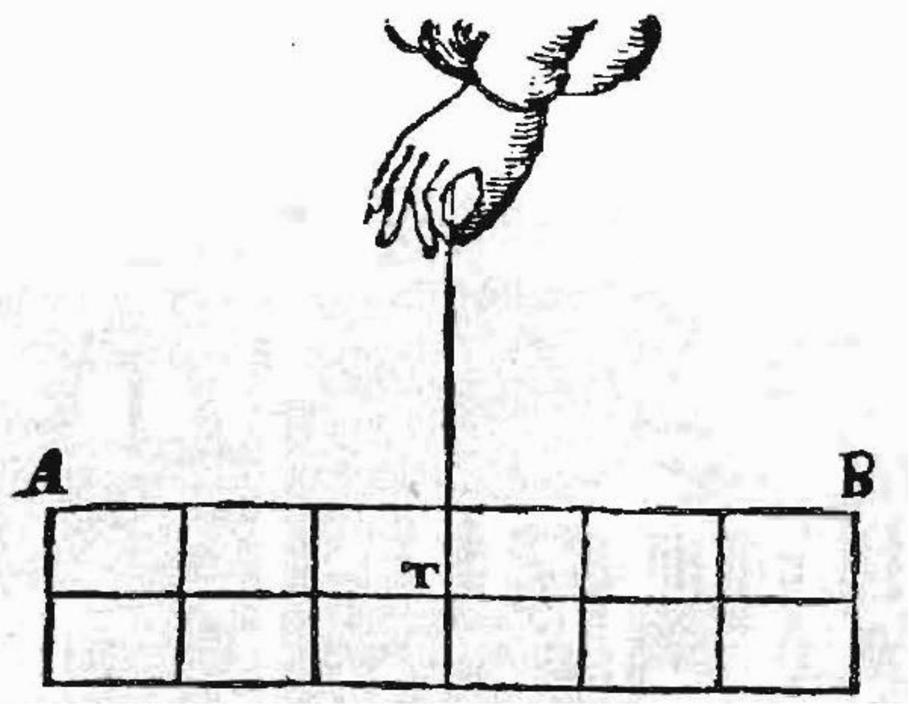
# Introducing Machines and Structures




- It is common to call the **input force *the effort*** and the **output force *the load*** or ***the resistance***. This allows the identification of three classes of levers by the relative locations of the fulcrum, the resistance and the effort.

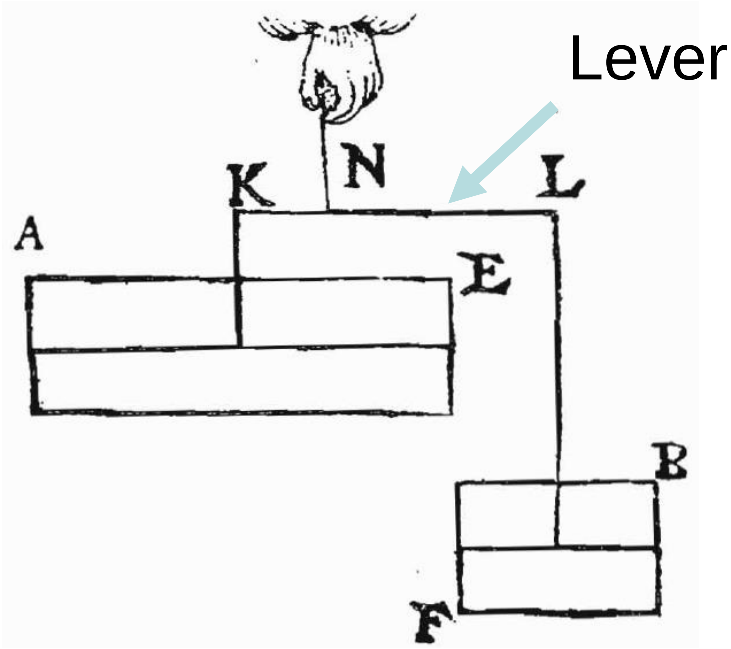
# 2. The Lever

# The Law of the Lever



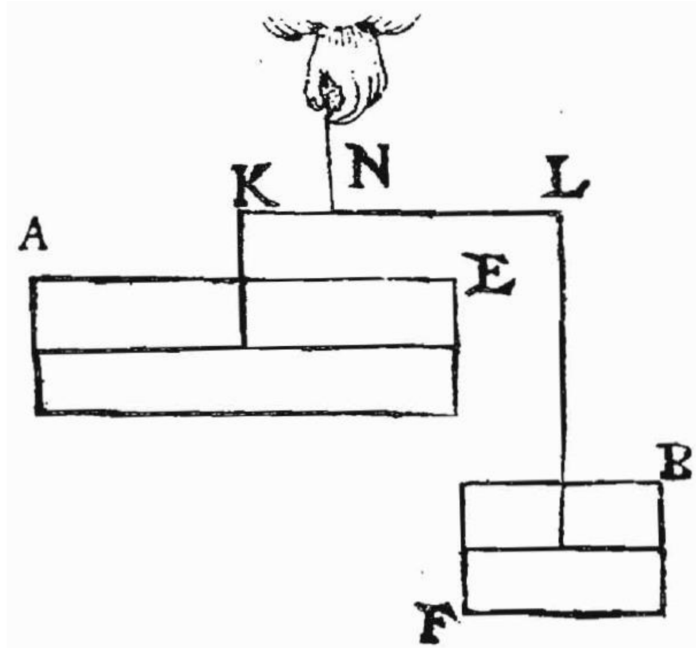

- A uniform rectangular beam $\mathrm{AB}$ (as shown above) will balance if suspended in line with its midpoint $\mathrm{T}$.
- The midpoint is called its **centre of gravity**, or **centre of mass.**

# The Law of the Lever



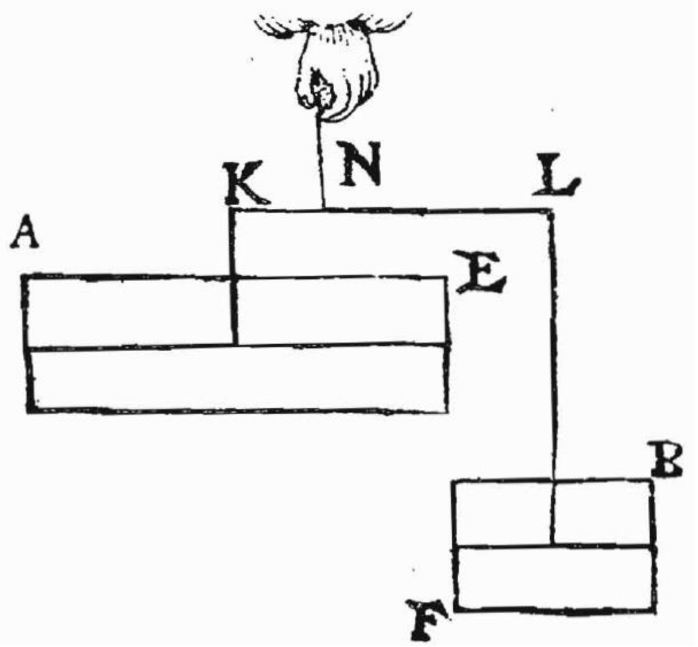

- Consider a more complex system. The body on the left is twice the mass of the body on the right.
- We make the assumption that neither of the threads from which the two parts hang nor the lever, KL, has mass.
- The lever serves only one function: it supports the original beam (now cut in two) in its original position, that is, hanging from a thread in line with its midpoint.

# The Law of the Lever




- The centre of gravity of the body was not displaced sideways by moving the two parts up or down, so the lever with the two portions of the beam hanging from it must balance.
- From the illustration, we see that the distances of the smaller and larger parts from the fulcrum are 1 unit (distance KN) and 2 units (distance NL) respectively.

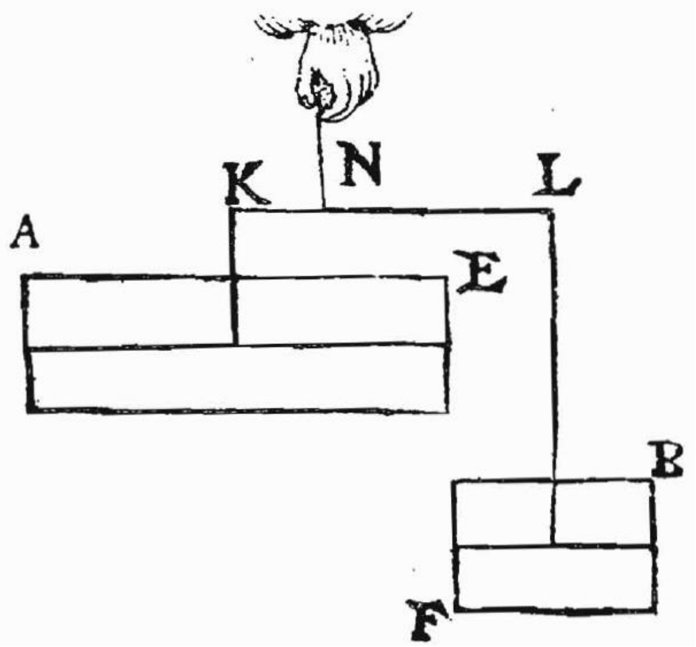
# The Law of the Lever




- Hence, the weight forces on the lever and their lever arms are in the ratio. . .

$4:2$, or $2:1$ in reduced form

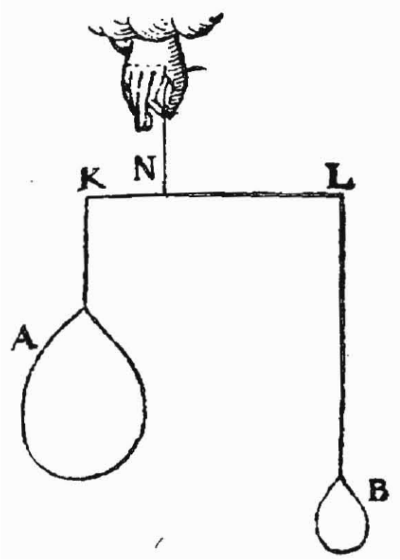
# The Law of the Lever




- . . . it can be said that generally two loads (or forces) on a lever will balance if their magnitudes are inversely proportional to the length of their lever arms.

- This is the **law of the lever**. $\mathrm{A} : \mathrm{B} = \mathrm{NL} : \mathrm{NK}$

# The Law of the Lever



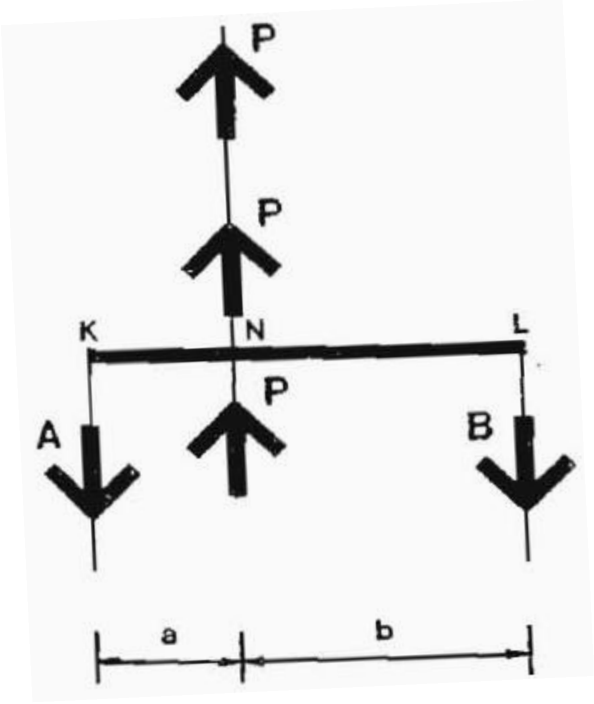


$$A:B = NL:NK$$

- Furthermore, one of the pulls, for example pull $B$, can be exerted by hand. Then the smaller effort $B$ will balance the larger pull of the load $A$.
- The smallest extra effort (force) at $B$ will, in ideal circumstances, be enough to lift the load at $A$; we can call this **mechanical advantage**.

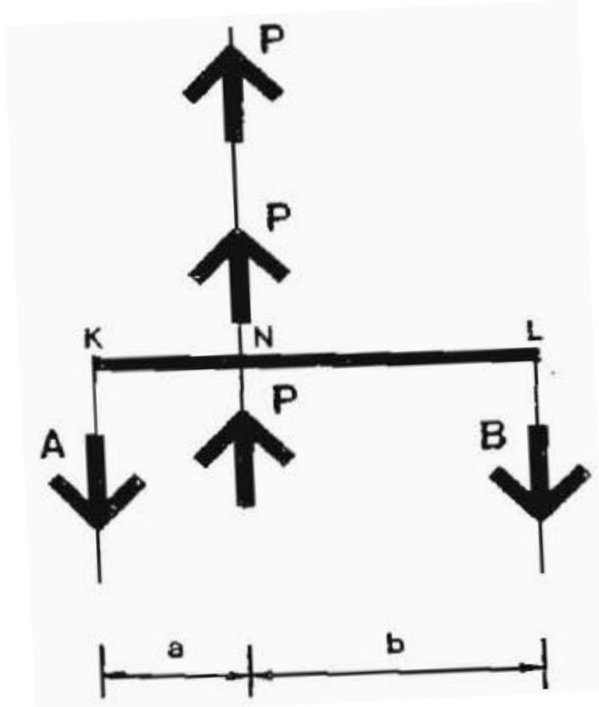
# 3. The Moment of a Force

# The Moment of a Force




- The above figure shows a modernised version of the lever.
- Any push or pull, however caused, is called a force and is represented simply by a line with an arrow on it.
- The line (called the line of action of the force) and the arrow together indicate the direction of the force, thus replacing the string in the previous figures.

# The Moment of a Force

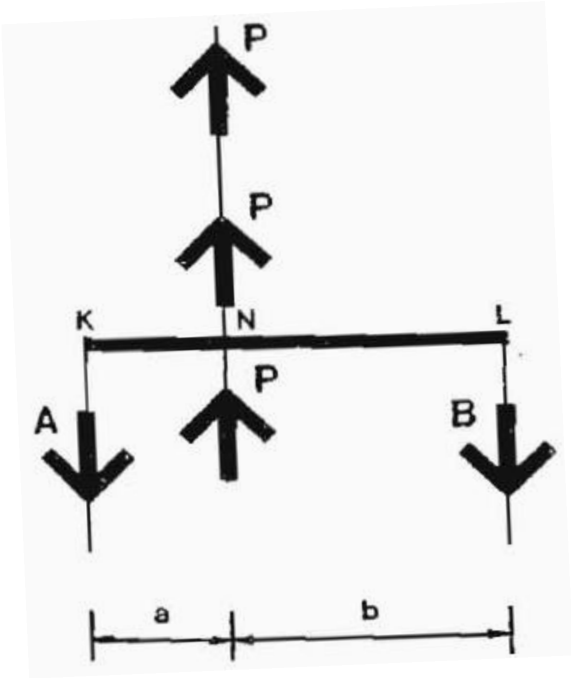



$$A : B = b : a$$

- For thousands of years, people used the law of the lever - inverse proportionality between forces and lever arms, for the simple reason that the branch of mathematics these people were applying was geometry.
- It is only in recent times that we represent the above relationship as follows:

$$A \times a = B \times b$$

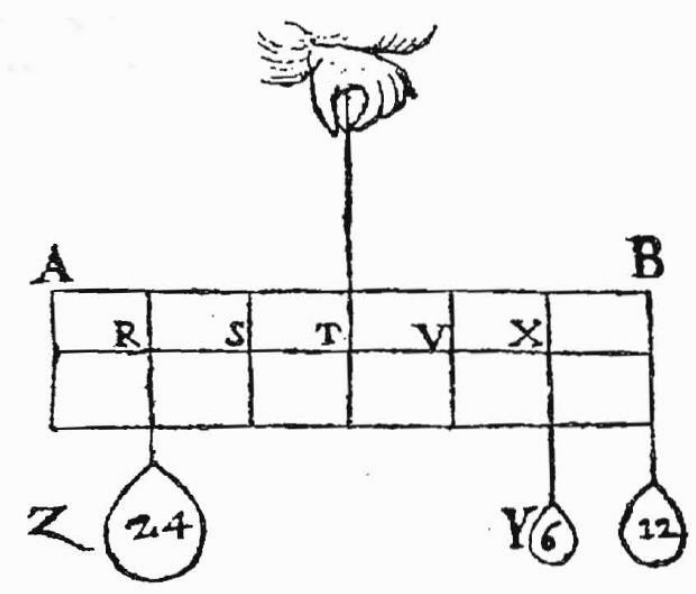
# The Moment of a Force




$$A \times a = B \times b$$

- The product (magnitude of force × lever arm) is called the moment of the force, **or torque**, and is a measure of its turning effect.
- The units of moment are those of force and length combined, namely Newton-metres (N m).
- The lever is balanced on the fulcrum when the moments of the two loads are equal and opposite: the clockwise moment of force B is balanced by the counterclockwise moment of force A.

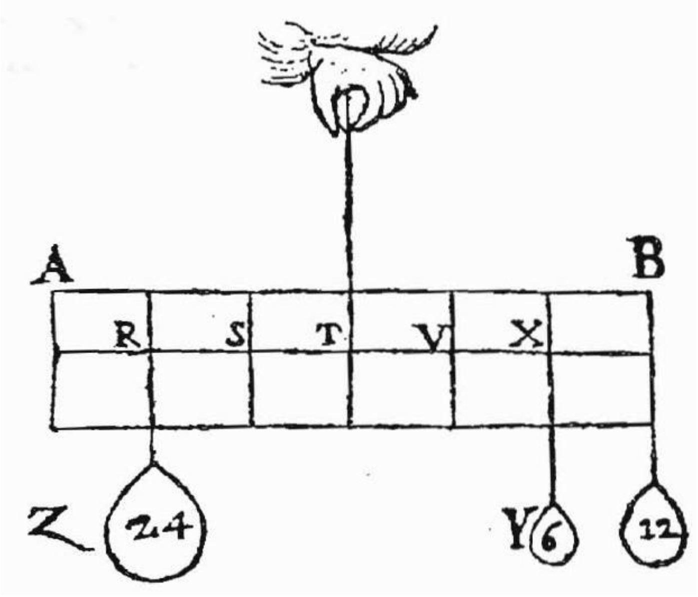
# The Moment of a Force




- The concept of moment freed us from the lever where it originated, and simplifies many things.
- For example, is the above beam in equilibrium?
- There is no need to look for a lever to find out.
- The clockwise moments about the support at point T are

$$6 \times 2 + 12 \times 3 = 48 \text{ Nm}$$

# The Moment of a Force

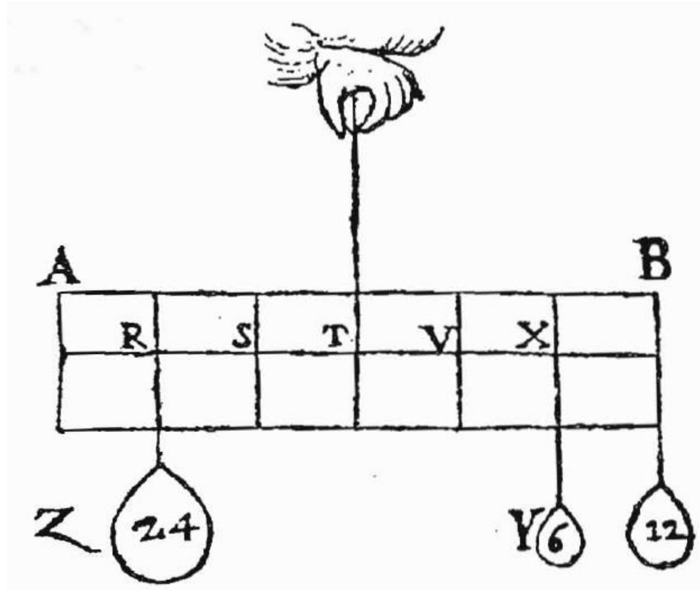



- The counterclockwise moments about the support at point $\mathrm{T}$ is

$$24 \times 2 = 48 \text{ Nm}$$

- Therefore, the beam is in equilibrium.
- The weight force acting on the beam at its centre of gravity was not taken into account because it has no moment about the fulcrum $\mathrm{T}$: the two points coincide and the lever arm of the force is zero.

# The Moment of a Force



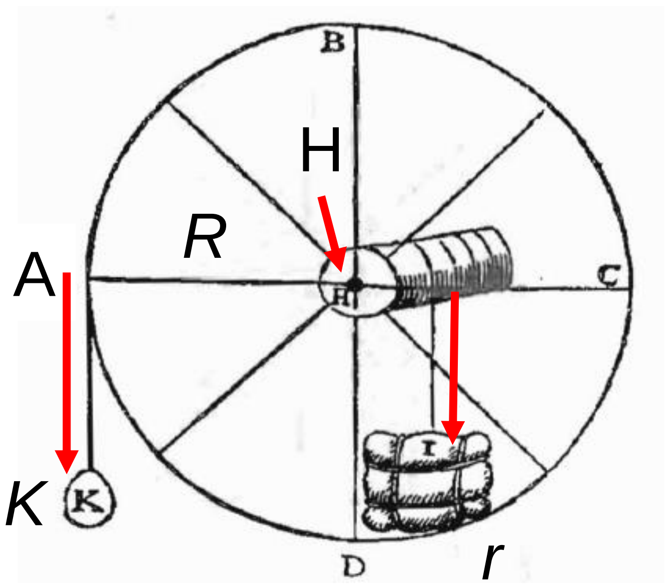

- Clockwise moments are referred to as positive, while counterclockwise moments are referred to as negative.
- The sum total of the moments acting on a body is called the **resultant moment**.

# 4. Some Simple Machines

# Simple Machines

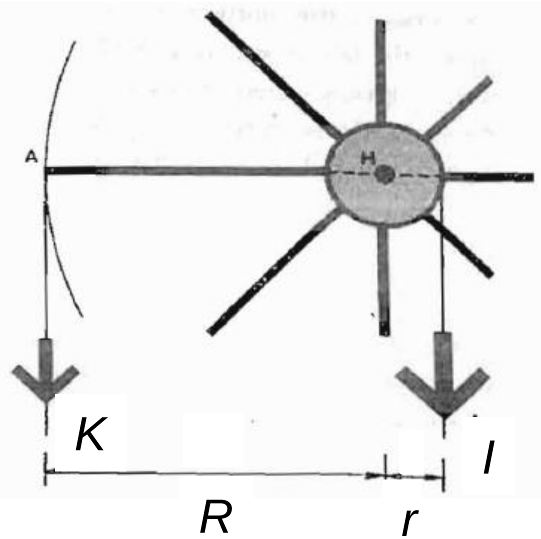

- It should be noted that most modern complex machinery are made-up of simple machines working in tandem with one another.
- Useful though it is, the straight lever working on a fixed fulcrum has a severe limitation.
- It can only be used to raise a load to a height above the fulcrum equal at the most to the length of the short arm. This difficulty is overcome by another simple machine, the wheel-and-axle arrangement.

# The Wheel-and-Axle Machine




- The load, $I$, hangs from a rope coiled on the axle.
- The load is held in balance by a force, $K$, represented by the counterweight hanging from the wheel.
- At any moment, the device functions as a straight lever, the radius $R$ of the large wheel being the long moment arm of the force $K$, and the radius $r$ of the axle, the short arm on which hangs the load $I$.

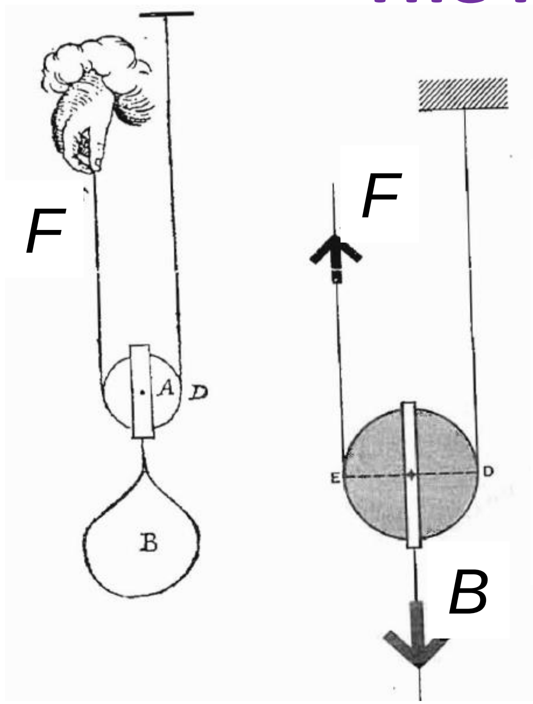
# The Wheel-and-Axle Machine




- As each segment of the axle moves out of position when the wheel is turned, the next takes its place.
- The wheel-and-axle machine can therefore be regarded as a **continuous lever**. Then from the condition of moment equilibrium

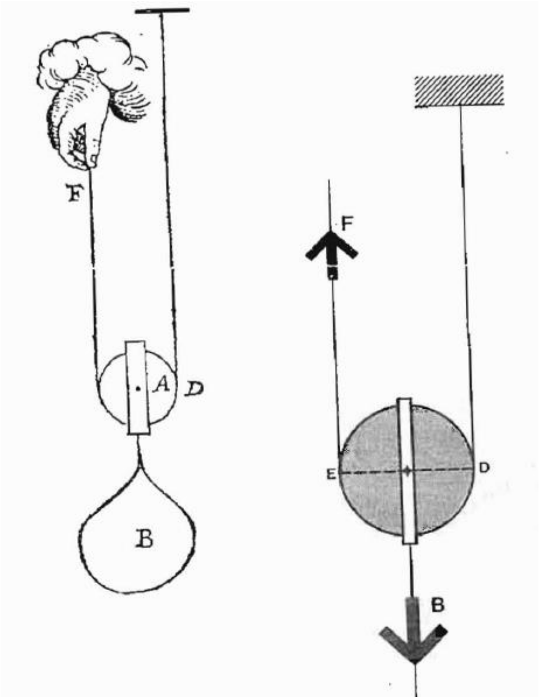
$$K \times R = I \times r$$
$$=> \; K = I \times r/R$$

# The Pulley



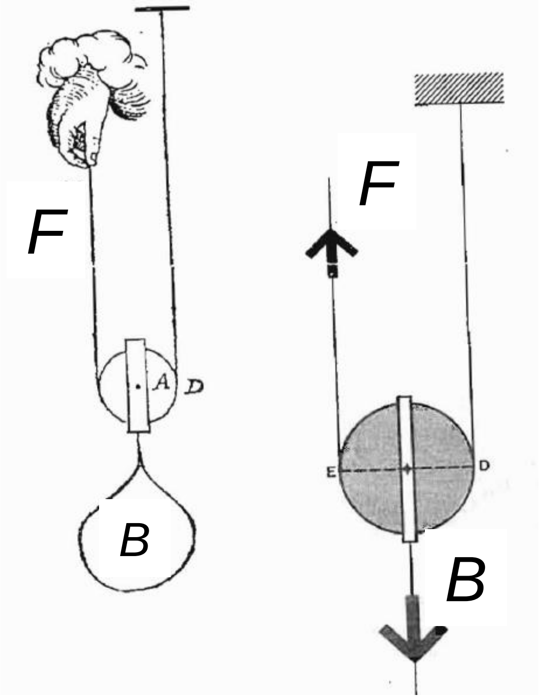

- Another simple machine which works with the moment principle is the pulley.
- In its simplest form, a pulley is just a wheel with a grooved rim around which is passed a rope, as shown by the figure on the right.
- The rope is attached to a support at one end, while the effort, $F$, is applied at the other.

# The Pulley




- The load, $B$, hangs from the axis of the wheel.
- The horizontal line of the pulley $\mathrm{ED}$ acts as a lever.
- Its fulcrum is at point $\mathrm{D}$ where the wheel is supported by the rope; the arm of the load is the radius $\mathrm{r}$ of the wheel while the arm of the effort is the pulley’s diameter.

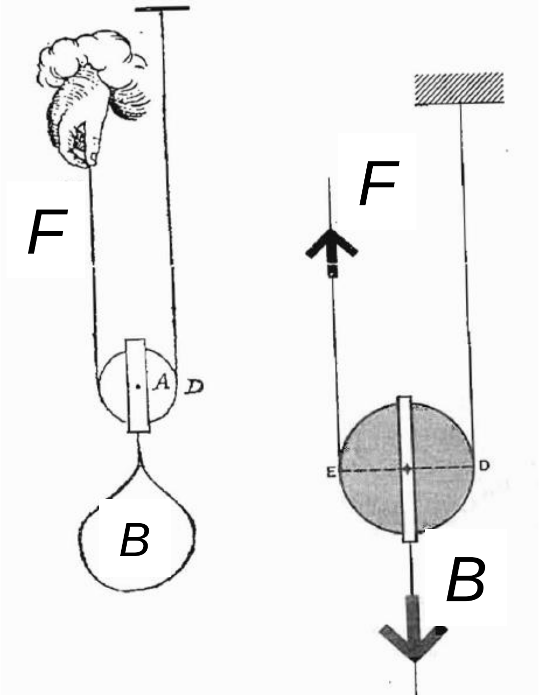
# The Pulley




- Since the lever arm of force $F$ is twice that of the load $B$, we conclude that $F = B/2$.

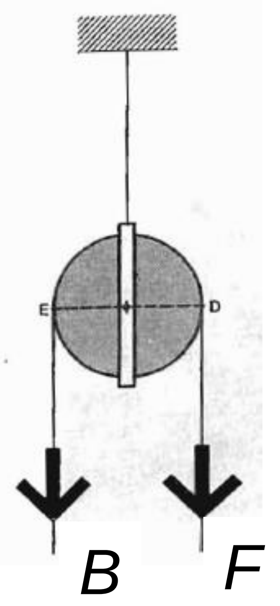
- The weight of the pulley itself must also be lifted and should be added to the load.

# The Pulley



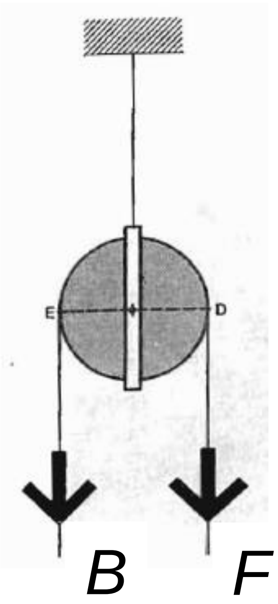

- As the load is raised by pulling on the rope, the ratio of the moment arms is maintained while the machine rises with the load.
- For this reason, the pulley can be called a **travelling lever**.
- It should be noted that only such a movable pulley doubles the lifting capacity of the effort.

# The Pulley




- A different kind of lever is the fixed pulley, so called because the axis of the pulley is attached to the support and acts as the fulcrum.
- The moment arms of the string forces on either side are the same in this case and so the pulls (the load and the effort) are also the same.
- All that this machine does is change the direction of the string force.

# The Pulley



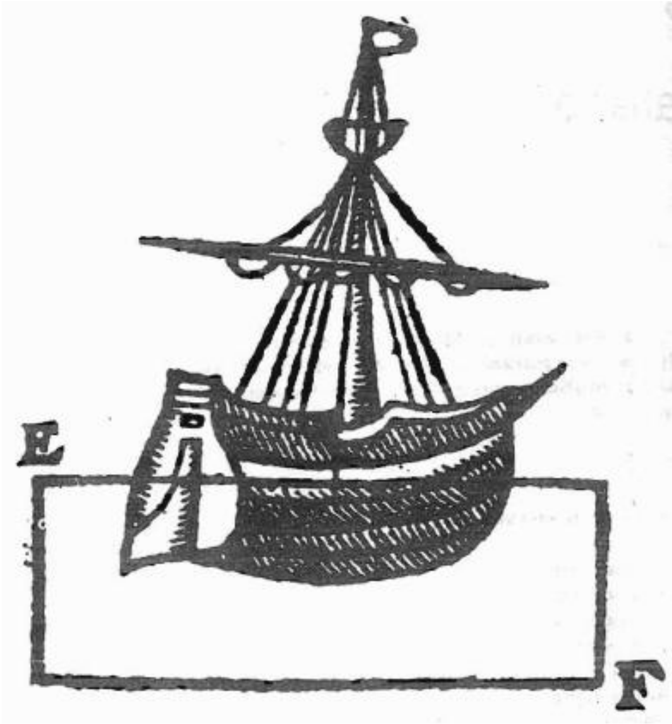

**N.B.**

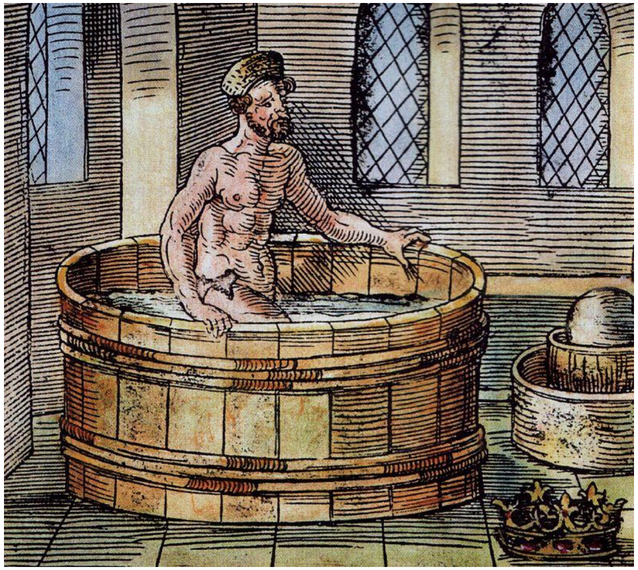
It helps reduce the effort required to lift a load only when combined with one or more movable pulleys.

A real pulley will experience friction, so some effort will be lost pulling the string, reducing the actual mechanical advantage.

# 5. Buoyancy

# The Force of Buoyancy

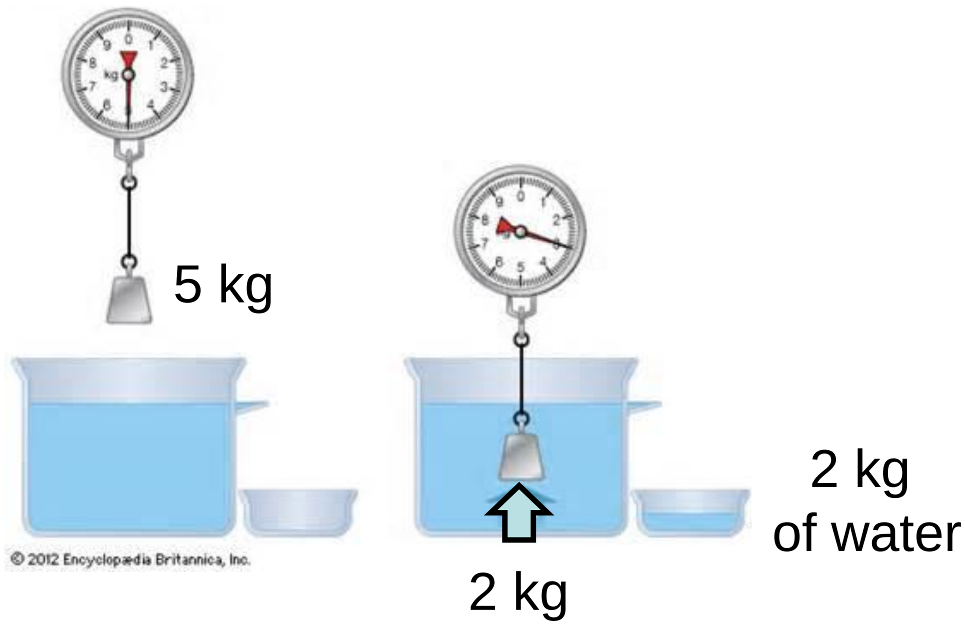


- How can a heavy ship, such as that shown above, complete with equipment and cargo stay afloat?
- The explanation of this is credited to Archimedes, who, according to legend, was so excited when he found the answer while immersed in water that he ran naked from his bath shouting "Eureka, I have found it!"

# The Force of Buoyancy

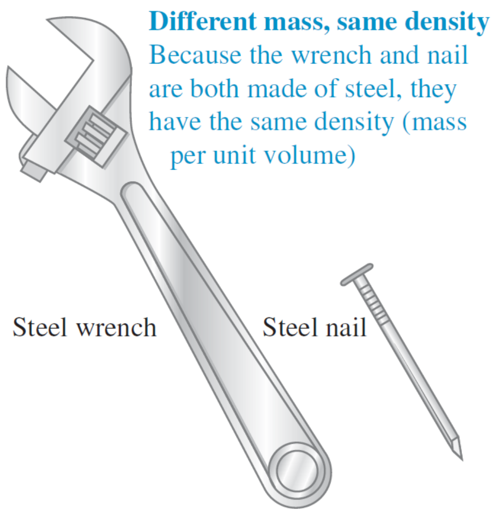
Archimedes' principle




- The **principle of buoyancy**, usually called the principle of Archimedes: a body immersed in water is buoyed up with a force equal to the **weight** of the volume of water displaced by the body.

- We can use this principle in general to determine whether an object floats or sinks when placed in a fluid, because the weight of an object is linked with its **density.**

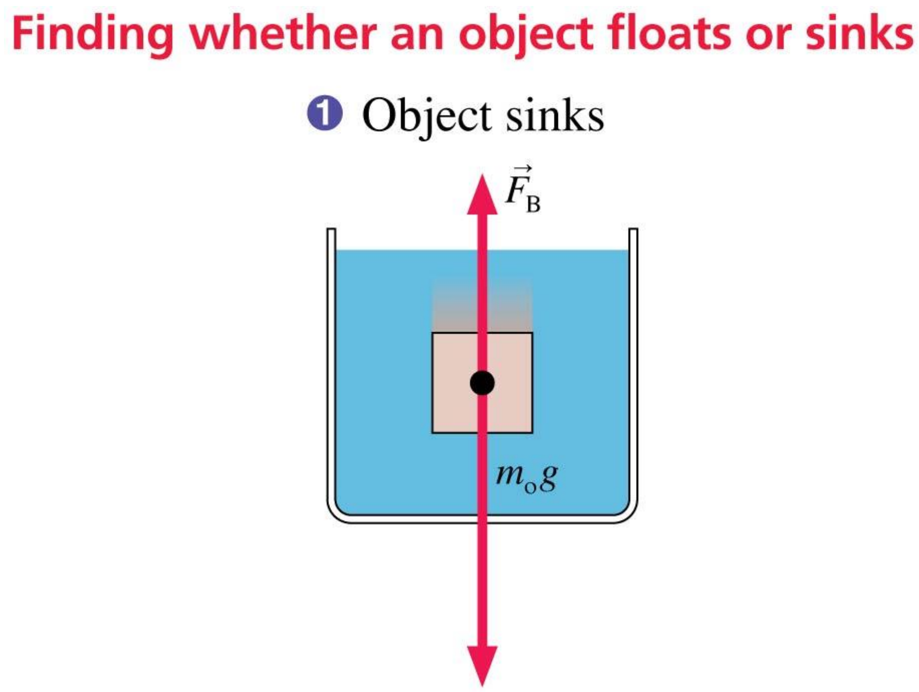
# The Force of Buoyancy




$$\rho = \frac{m}{V};\ \text{units} = \mathrm{kgms}^{-1}$$

- Density is a measure of the **mass per unit volume** of an object.
- It is a constant for objects made of the same material.
- Liquids with lower densities will float on top of liquids with higher densities (provided no mixing occurs).

# The Force of Buoyancy



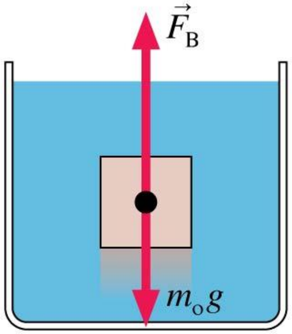

An object sinks if it weighs more than the fluid it displaces—that is, if its average density is greater than the density of the fluid:

$$\rho_{avg} > \rho_f$$

# The Force of Buoyancy

**Finding whether an object floats or sinks**

❷ Object floats

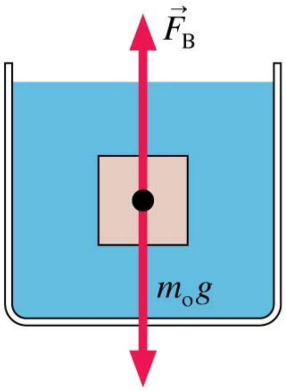



An object floats on the surface if it weighs less than the fluid it displaces—that is, if its average density is less than the density of the fluid:

$$\rho_{avg} < \rho_f$$

**Finding whether an object floats or sinks**

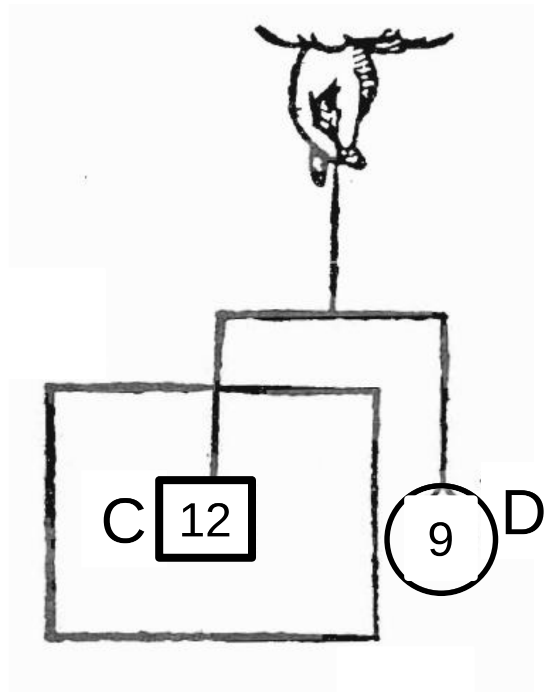
❸ Neutral buoyancy




An object hangs motionless if it weighs exactly the same as the fluid it displaces—that is, if its average density equals the density of the fluid:

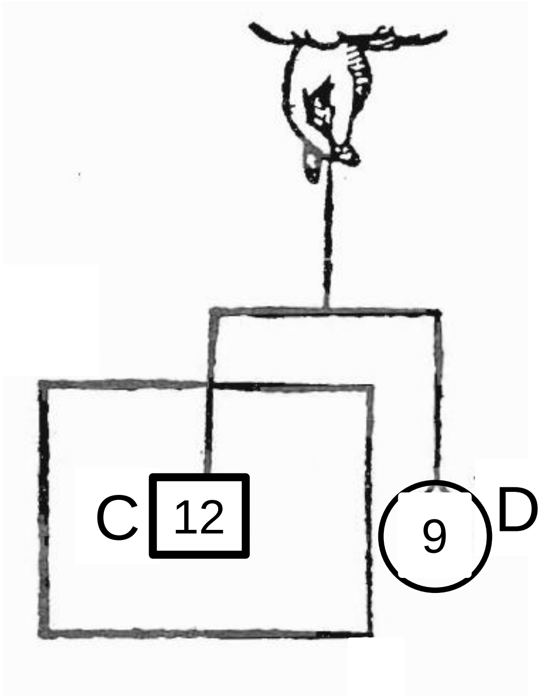
$$\rho_{avg} = \rho_f$$

# The Force of Buoyancy




- In the above figure, a body weighing 12 N is immersed in water.
- The density of the body is four times that of water.
- A force of only 9 N is needed to prevent the body from sinking.

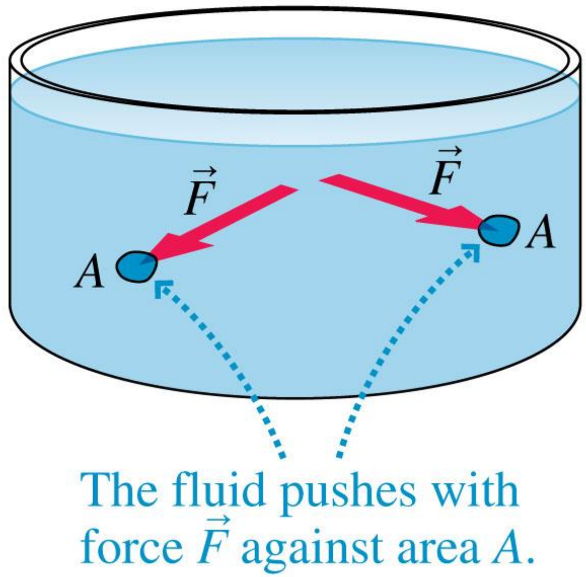
# The Force of Buoyancy




- The same volume of water would weigh only one-quarter as much as the body, or 3 N.
- The weight of the immersed body will therefore be reduced by 3 N and a pull of 9 N will suffice to hold it up.

# 6. Pressure

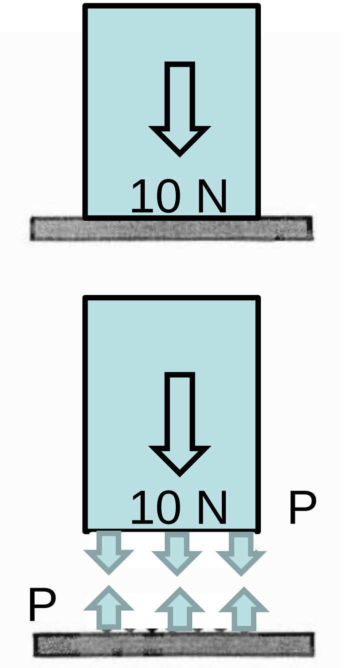
# Pressure




- Simply put, pressure is a measure of the magnitude of force acting over the area on which it acts:

$$P = F/A$$

# Pressure

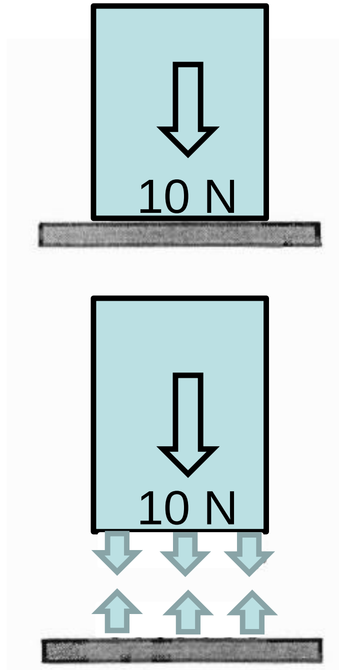



- Suppose we have a box weighing 10 N sitting on a table, as shown above. The area of contact is, let us say $0.2\ \mathrm{m}^2$. The pressure will be equal to

$$P = \frac{F}{A} = \frac{10}{0.2} = 50\ \mathrm{N/m^2}$$

over the entire area of contact. Just like force, pressure always acts between bodies in contact and is exerted on both bodies.

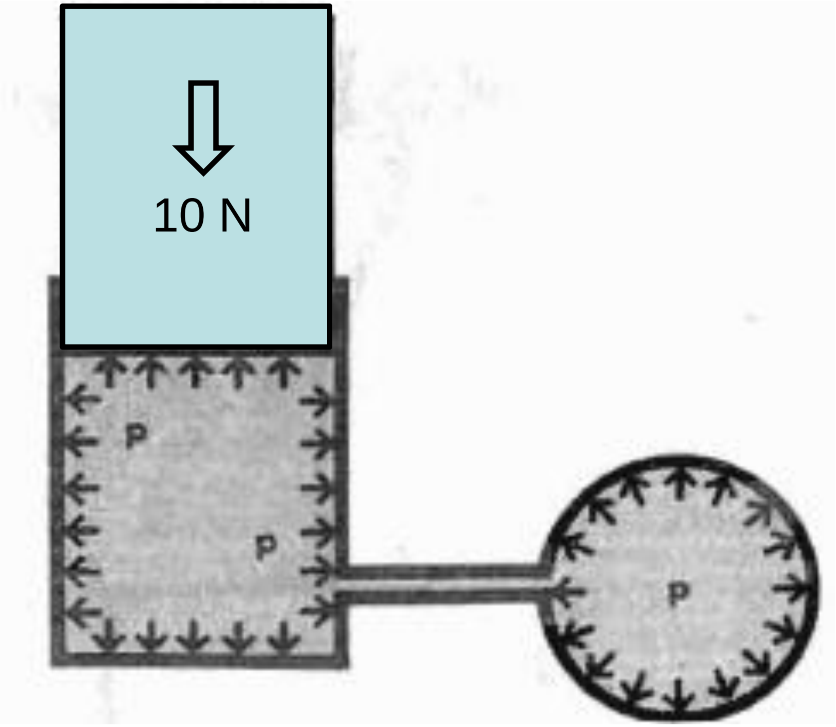
# Pressure




$$P = F/A$$

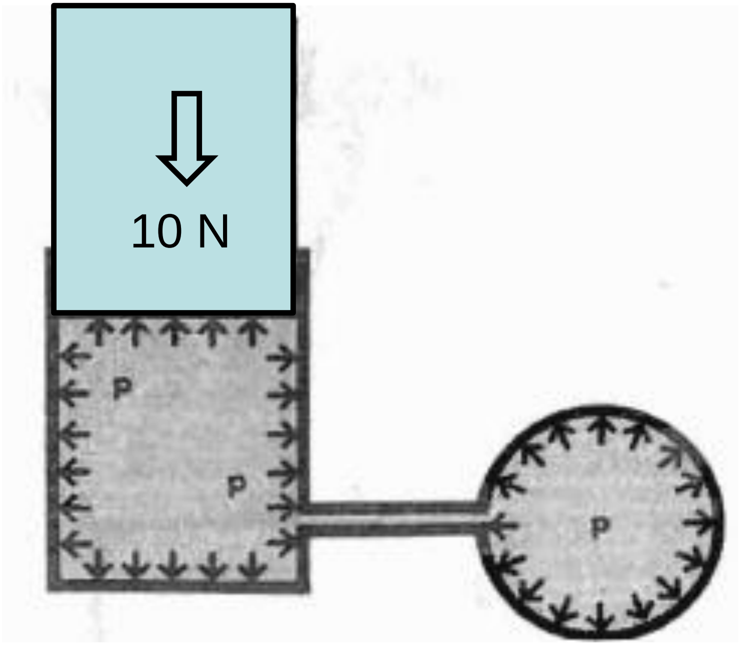
- The box presses on the table, and the table returns an equal and opposite pressure on the box.
- Let us now take our box from the table, and place it in a container filled with water.

# Pressure



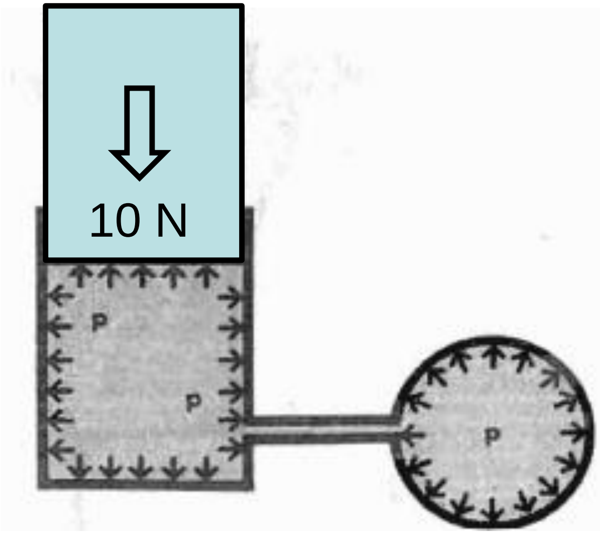

- The container fits tightly around our box, and we assume that there is no friction between the two.
- The water in the container is now under pressure.
- The area of contact is the same as before, so the pressure is also the same.

# Pressure




- Water, like all other fluids, has the special property that it cannot retain its shape.
- The pressure applied to water is distributed undiminished in all directions within the entire volume of water, as shown above.
- The water in a second container (and in the pipe) will be under the same pressure as the water in the first container.

# Pascal's Principle

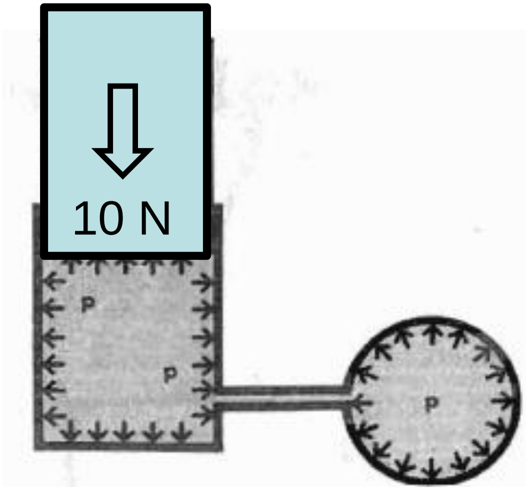


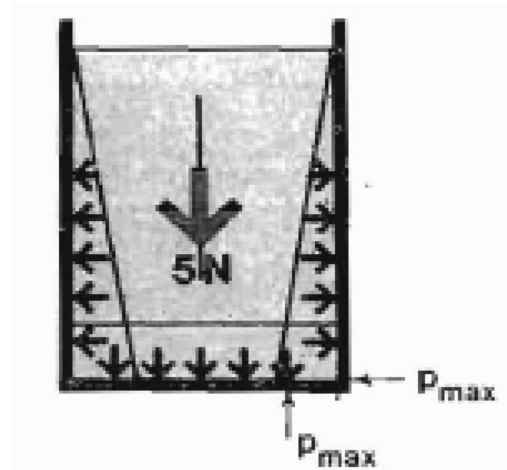

Blaise Pascal

1623-1639

- The pressure of water is always directed at right angles to the surface of the container enclosing it.
- If we drill holes in the sides of the round container, water will squirt out in all directions with equal force..
- Pascal's principle in one sentence is as follows: pressure exerted at any place on a fluid in a closed container is transmitted undiminished throughout the fluid and acts at right angles to all surfaces.

# Pressure

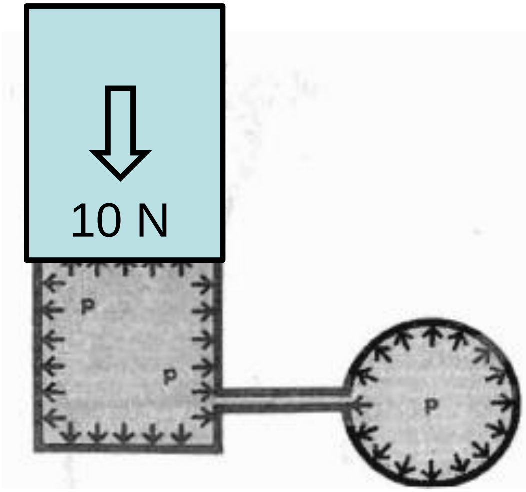


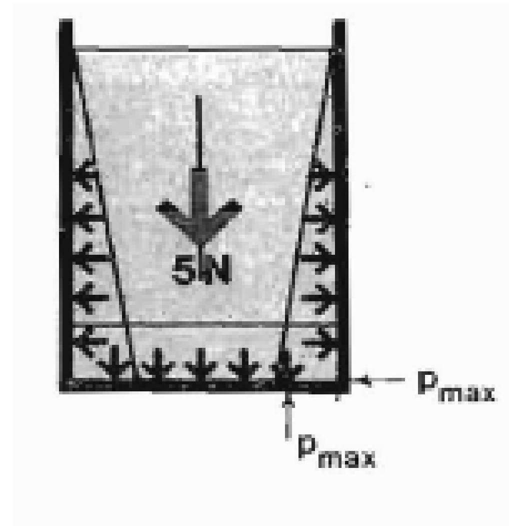




- Thus far, we have neglected to consider the effect the pull of gravity on the water (that is, its weight) has on the pressure inside the container.
- We now remove the box pressing on the water in our container and look at the effect of weight separately, as shown in the figure on the right above.
- Suppose that the weight of the water in the container is 5 N. The effect on the bottom of the container is the same as if a solid box of that weight was sitting on it. The pressure is thus

$$P = \frac{5}{0.2} = 25\ \mathrm{Nm^{-2}}$$

# Pressure

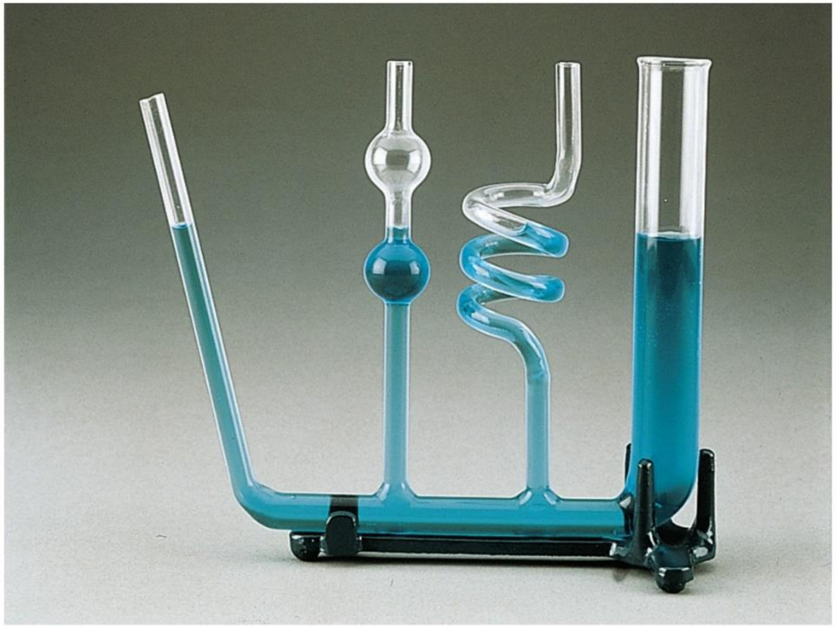



$$P = \frac{5}{0.2} = 25\ \mathrm{Nm}^{-2}$$

- The water at the bottom level is under the same pressure as the container bottom itself.
- Since it transmits the pressure in all directions, the sides of the container at that level are under the same pressure too.
- It pushes down on the top surface and up on the bottom surface, but since the bottom surface is deeper, the pressure on it pushing upward is greater than the downward pressure on the top surface.

# Pascal’s Principle



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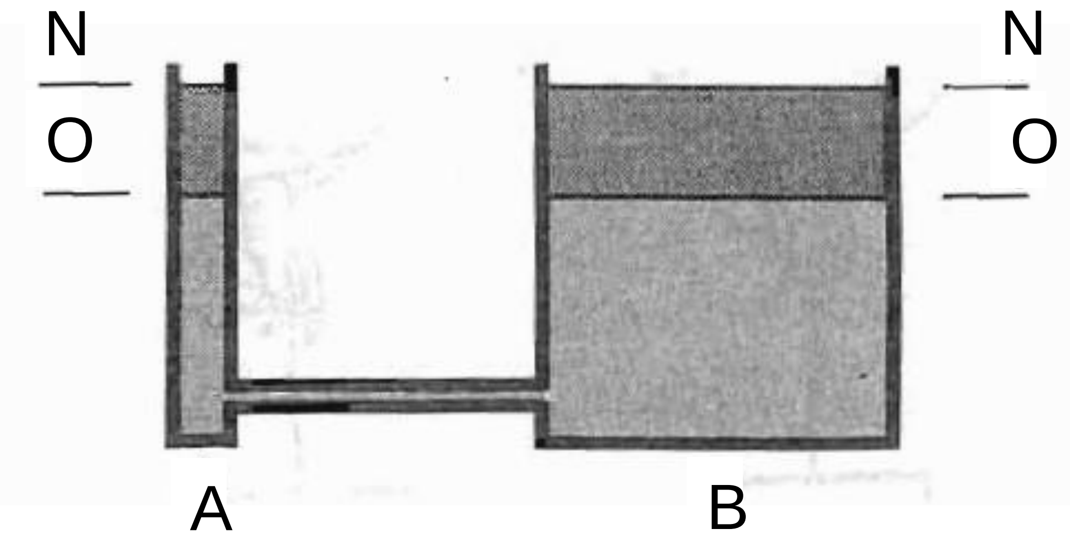
Pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and the walls of the containing vessel. The pressure depends only on depth; the shape of the container does not matter.

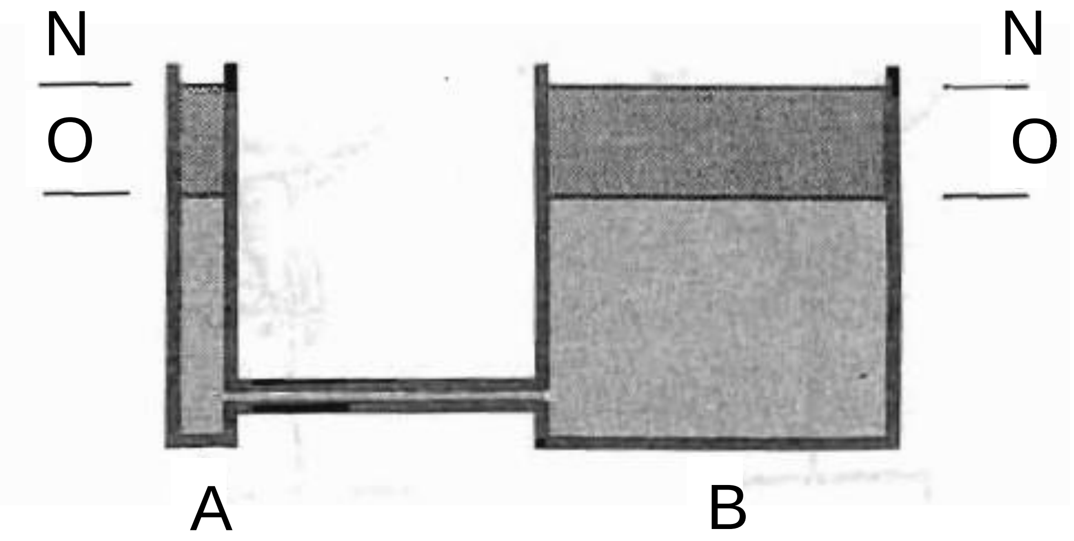
# 4. The Hydraulic Machine

# The Hydraulic Machine



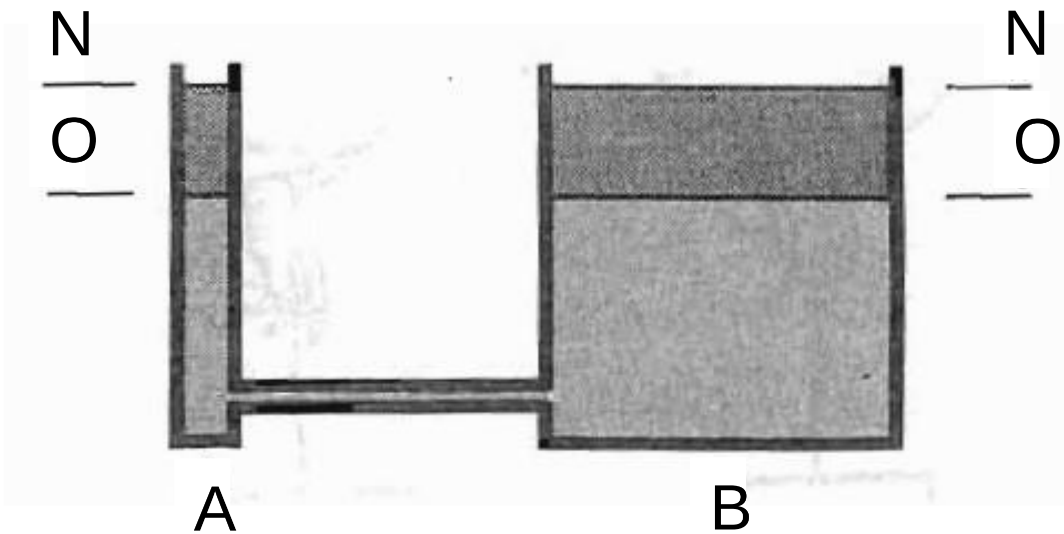

- Pascal took two vessels, A and B, and made the cross section of vessel B 100 times that of vessel A, as shown above.
- The vessels are filled with water up to level N-N.
- Suppose the weight of the water in the narrow vessel between levels O-O and N-N is 10 N; then the weight of the water between the same levels in the wide vessel is 100 times greater, that is, 1,000 N.

# The Hydraulic Machine



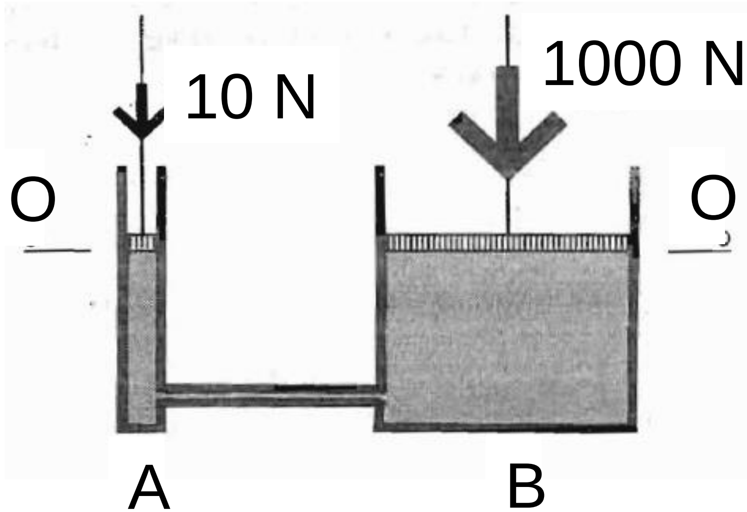

- Imagine this water is solidified: we now have a solid of 10 newtons in the narrow tube holding up a solid weighing 100 times as much in the wide vessel.
- Pascal wrote “if a perfectly fitting piston is adjusted to each tube, then one man pressing on the smaller piston will exert a force equal to that of one hundred men pressing on the larger piston”.

# The Hydraulic Machine




- A push of 10 newtons on the small piston will balance a force of 1,000 newtons on the larger one, just as two forces on a lever are balanced when its arms are in the ratio of 1:100.
- A change in the ratio of the areas of the pistons may also multiply the force exerted on the smaller piston even more.
- Let's consider another example to understand this further.

# The Hydraulic Machine

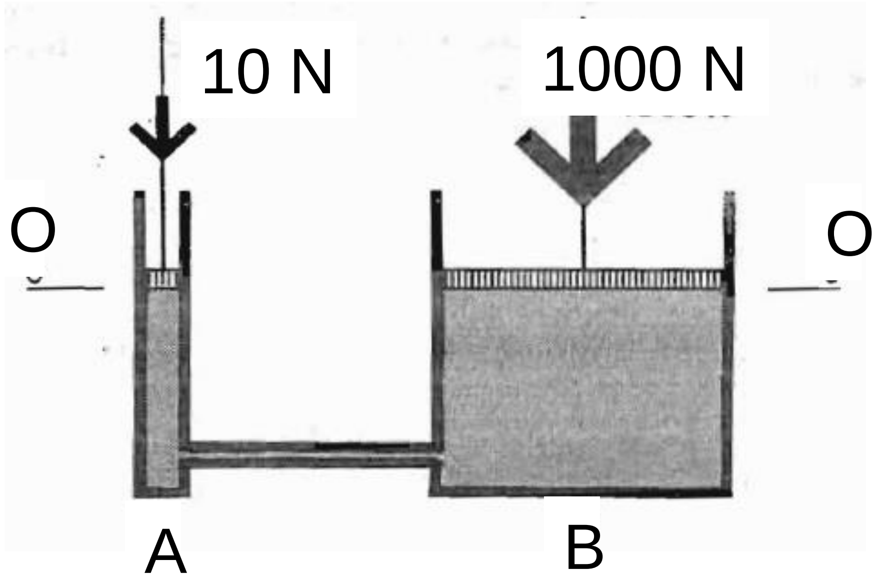



- Suppose the cross section of tube A is $0.01\ \mathrm{m}^2$ and that of tube B is 100 times larger; or $1\ \mathrm{m}^2$.
- A force of 10 N in the first tube will create a pressure in the water

$$P = \frac{F}{A} = 1000\ \mathrm{Nm}^{-2}$$

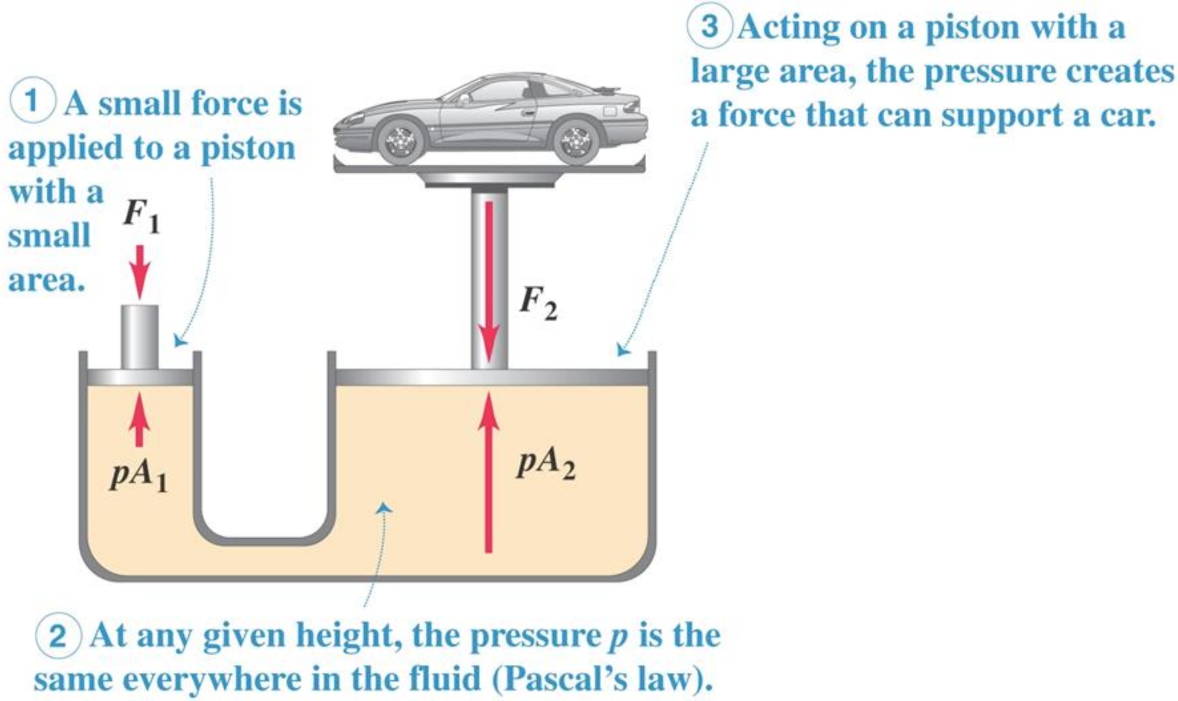
and a force on the larger piston, equal to 1,000 N, but acting on an area of $1\ \mathrm{m}^2$, thus causing the same pressure.

# The Hydraulic Machine




- Thus, the pressure in the water is the same throughout, and the machine is in equilibrium.
- Pascal added a warning though: the pressure is transmitted undiminished in all directions, and therefore all parts of the machine, including the connecting pipe must be strong enough to resist it.

# The Hydraulic Machine




- The modern hydraulic jack is basically a hydraulic machine, and is, in essence, like all machines, a **force-multiplying device** with a multiplication factor equal to the ratio $(A_2/A_1)$ of the areas of the two pistons.

$$F_2 = \frac{A_2}{A_1} F_1$$

# Summary of today's Lecture

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1. Introducing Machines and Structures
2. The Lever
3. The Moment of a Force
4. Simple Machines
5. The Force of Buoyancy
6. Pressure
7. Pascal's Principle
8. The Hydraulic Machine

# Lecture 7: Important Reading

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- **Ch. 10.4**, Torque; p. 298-299
- **Ch. 12.1,** The Conditions for Equilibrium; p.364-365
- **Ch. 12.2**, Solving Statics Problems; p.365-369
- **Ch. 12.3**, Stability and Balance; p.369-370
- **Ch. 13.3**, Pressure in Fluids; p.397-400
- **Ch. 13.4**, Atmospheric Pressure and Gauge Pressure; p.401
- **Ch. 13.5**, Pascal's Principle; p.402
- **Ch. 13.10**, Applications of Bernoulli's Principle: p.412-414

# Lecture 7: Important Videos

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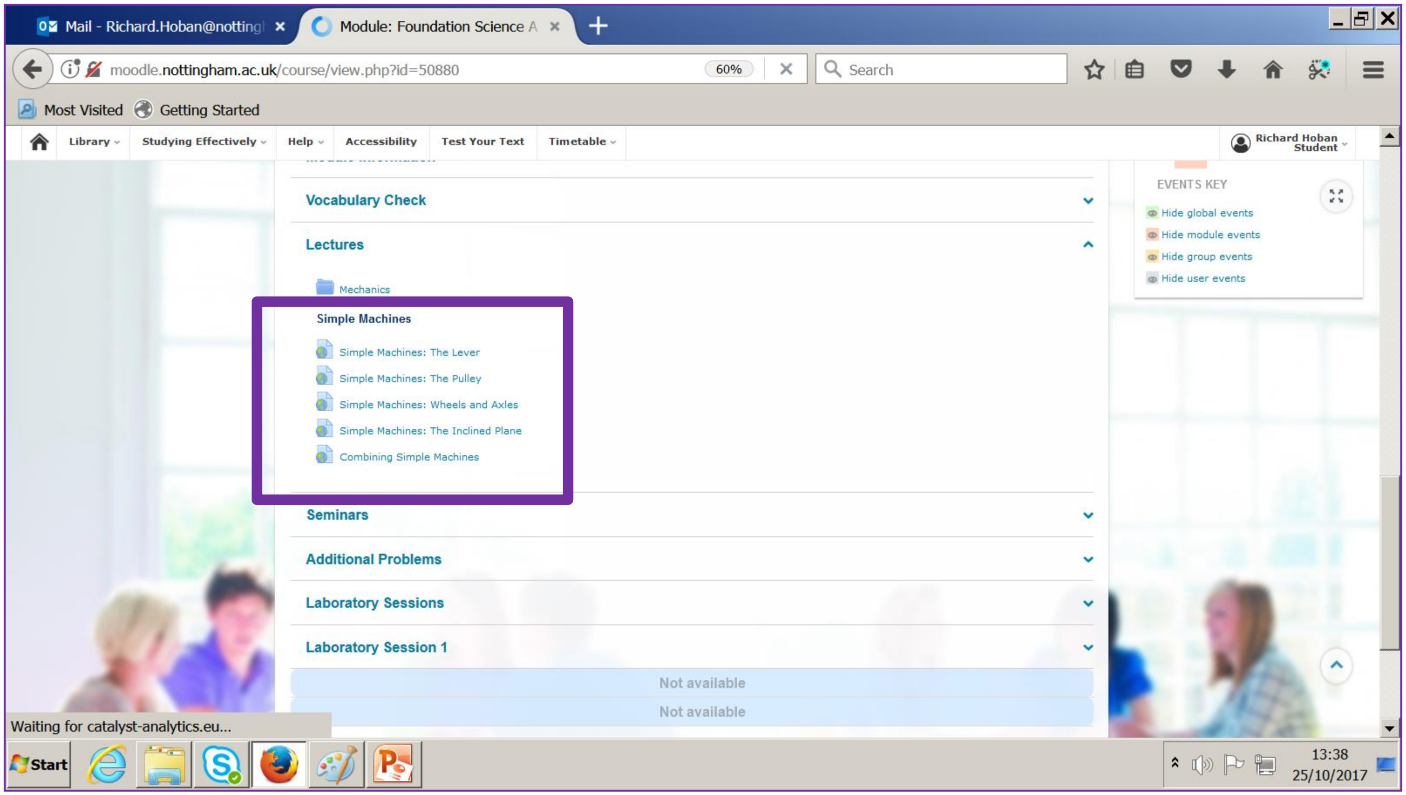


- The above is a screenshot of where you can find important videos on the Science A Moodle Page to help you better understand Lectures 7.

# Home Work

Do not forget to read the **Additional Problems** for this lecture before logging in to **Mastering Physics** to complete your assignments.