University of Nottingham
UK | CHINA | MALAYSIA

Formula Sheet for Foundation Calculus (CELEN037)

- **Differentiation: Useful results**

$$\frac{dy}{dx} = f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$\lim_{h\to 0} \frac{\sin h}{h} = 1 \quad \textbf{and} \quad \lim_{h\to 0} \cos h = 1$$

$$\lim_{m\to 0} (1+m)^{1/m} = e$$

$$\lim_{h\to 0} \frac{e^h - 1}{h} = 1 \quad \textbf{and} \quad \lim_{h\to 0} \frac{a^h-1}{h} = \log_e a$$

If $u = f(x)$, $v = g(x)$, and $w = h(x)$ are differentiable functions of x, then,

$$\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$$

$$\frac{d}{dx}(u \cdot v) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{d}{dx}(u \cdot v \cdot w) = u\,v \cdot \frac{dw}{dx} + v\,w \cdot \frac{du}{dx} + u\,w \cdot \frac{dv}{dx}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dw} \cdot \frac{dw}{dx}$$

$$\frac{dy}{dx} = \frac{\left(\dfrac{dy}{dt}\right)}{\left(\dfrac{dx}{dt}\right)} \quad \textbf{and} \quad \frac{dy}{dx} = \frac{1}{\left(\dfrac{dx}{dy}\right)}$$

- **Derivatives of standard functions**

$$\frac{d}{dx}(x^n) = n\,x^{n-1}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$$

$$\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x\,\cot x$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(a^x) = a^x \log_e a$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\cos^{-1} x\right) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}$$

- **Integration**

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \qquad (n \neq -1)$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \sec^2 x\,dx = \tan x + C$$

$$\int \operatorname{cosec}^2 x\,dx = -\cot x + C$$

$$\int \sec x\,\tan x\,dx = \sec x + C$$

$$\int \operatorname{cosec} x\,\cot x\,dx = -\operatorname{cosec} x + C$$

$$\int e^x\,dx = e^x + C$$

$$\int a^x\,dx = \frac{a^x}{\ln a} + C \qquad (a > 0)$$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left|x + \sqrt{x^2-a^2}\right| + C$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left|x + \sqrt{x^2+a^2}\right| + C$$

$$\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$$

$$\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\ln\left|\frac{x+a}{x-a}\right| + C$$

- **Some useful results**

$$\int [\, f(x) \,]^n \; f'(x) \, dx = \frac{1}{n+1} \; [\, f(x) \,]^{n+1} + C$$

$$\int \frac{f'(x)}{f(x)} \, dx = \ln |f(x)| + C$$

$$\int \frac{f'(x)}{\sqrt{f(x)}} \, dx = 2\sqrt{f(x)} + C$$

- **Integration by Parts**

If u and v are continuous functions of x, then

$$\int u \, dv = u\,v - \int v \, du$$

- **Numerical Integration**

Trapezium Rule:

$$\int_a^b f(x) \;\; dx \approx \frac{h}{2} \; [f_0 + 2\;(f_1 + f_2 + \ldots\ldots + f_{n-1}) + f_n]$$

Simpson's Rule:

$$\int_a^b f(x) \;\; dx \approx \frac{h}{3} \; [\; f_0 + 4\,f_1 + 2\,f_2 + 4\,f_3 + 2\,f_4 + \ldots\ldots + 2\,f_{n-2} + 4\,f_{n-1} + f_n \;]$$

where $h = \dfrac{b-a}{n}$.

- **Maclaurin's series**

$$f(x) = f(0) + x\,f'(0) + \frac{x^2}{2\,!}\; f''(0) + \frac{x^3}{3\,!}\; f'''(0) + \ldots\ldots + \frac{x^n}{n\,!}\; f^{(n)}(0) + \ldots\ldots$$

- **Trigonometry**

$$\begin{cases} \cos^2\theta + \sin^2\theta = 1 \\ \tan^2\theta + 1 = \sec^2\theta \\ \cot^2\theta + 1 = \operatorname{cosec}^2\theta \end{cases}$$

$$\begin{cases} \sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\ \tan(A \pm B) &= \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \end{cases}$$

$$\begin{cases} 2\sin A \cos B &= \sin(A+B) + \sin(A-B) \\ 2\cos A \sin B &= \sin(A+B) - \sin(A-B) \\ 2\cos A \cos B &= \cos(A+B) + \cos(A-B) \\ -2\sin A \sin B &= \cos(A+B) - \cos(A-B) \end{cases}$$

$$\begin{cases} \sin C + \sin D &= 2\,\sin\left(\dfrac{C+D}{2}\right) \cos\left(\dfrac{C-D}{2}\right) \\ \sin C - \sin D &= 2\,\cos\left(\dfrac{C+D}{2}\right) \sin\left(\dfrac{C-D}{2}\right) \\ \cos C + \cos D &= 2\,\cos\left(\dfrac{C+D}{2}\right) \cos\left(\dfrac{C-D}{2}\right) \\ \cos C - \cos D &= -2\,\sin\left(\dfrac{C+D}{2}\right) \sin\left(\dfrac{C-D}{2}\right) \end{cases}$$

$$\begin{cases} \sin 2\theta &= 2\sin\theta\cos\theta \quad ; \quad \sin\theta = 2\,\sin\left(\dfrac{\theta}{2}\right)\cos\left(\dfrac{\theta}{2}\right) \\ \cos 2\theta &= \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta \\ \tan 2\theta &= \dfrac{2\tan\theta}{1-\tan^2\theta} \end{cases}$$

$$\begin{cases} \sin^2\theta &= \dfrac{1}{2}\,(1-\cos 2\theta) \quad ; \quad 1-\cos\theta = 2\,\sin^2\left(\dfrac{\theta}{2}\right) \\ \cos^2\theta &= \dfrac{1}{2}\,(1+\cos 2\theta) \quad ; \quad 1+\cos\theta = 2\,\cos^2\left(\dfrac{\theta}{2}\right) \end{cases}$$

$$\begin{cases} \sin 3\theta = 3\sin\theta - 4\sin^3\theta \\ \cos 3\theta = 4\cos^3\theta - 3\cos\theta \end{cases}$$

$$\begin{cases} \sin\theta &= \dfrac{2t}{1+t^2} \\ \cos\theta &= \dfrac{1-t^2}{1+t^2} \qquad \text{where } \; t = \tan\left(\dfrac{\theta}{2}\right) \\ \tan\theta &= \dfrac{2t}{1-t^2} \end{cases}$$