

Foundation Physics

Lecture 5:

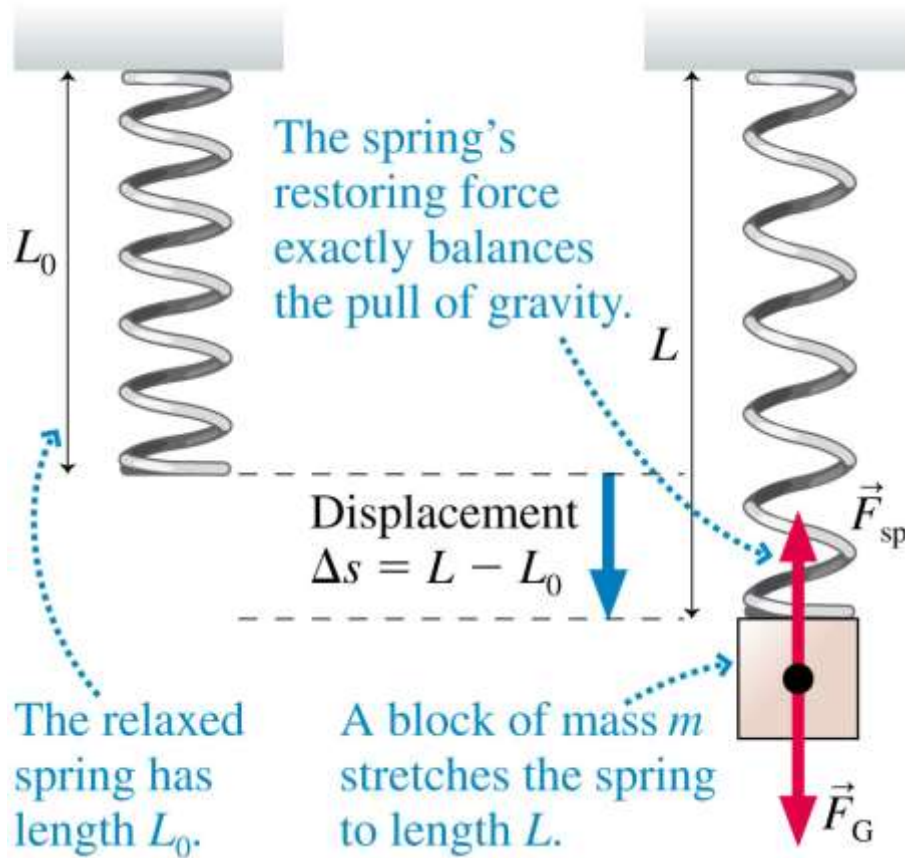
Energy & Momentum

Aims of today's lecture

1. Hooke's Law
2. The Basic Energy Model and Energy Diagrams
3. Conservative and non-conservative systems
4. Power
5. Momentum and Conservation of Momentum
6. Impulse

1. Hooke's Law

Hooke's Law

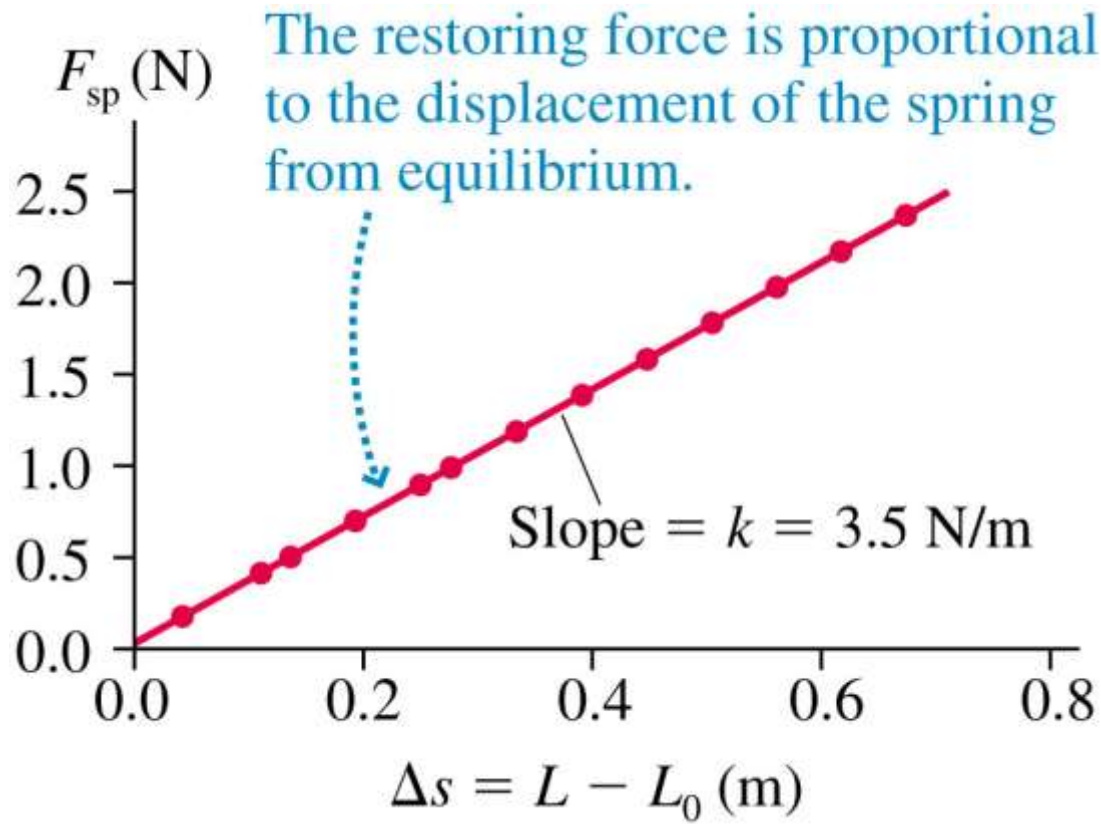


Robert Hooke
1635-1703

- The figure above shows how a hanging mass stretches a spring of equilibrium length L_0 to a new length L .
- The mass hangs in static equilibrium, so the upward spring force balances the downward gravitational force.

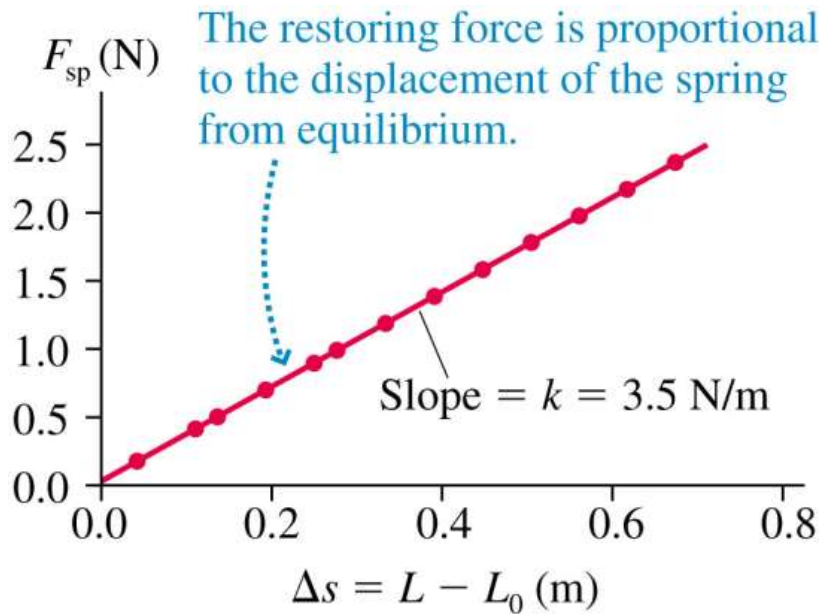
$$F_{sp} = F_G = mg$$

Hooke's Law



- The figure shows measured data for the restoring force of a real spring.
- Δs is the **displacement from equilibrium**. $F_{sp} = k\Delta s$
- The proportionality constant k is called the **spring constant**.
- The units of k are N/m .

Hooke's Law

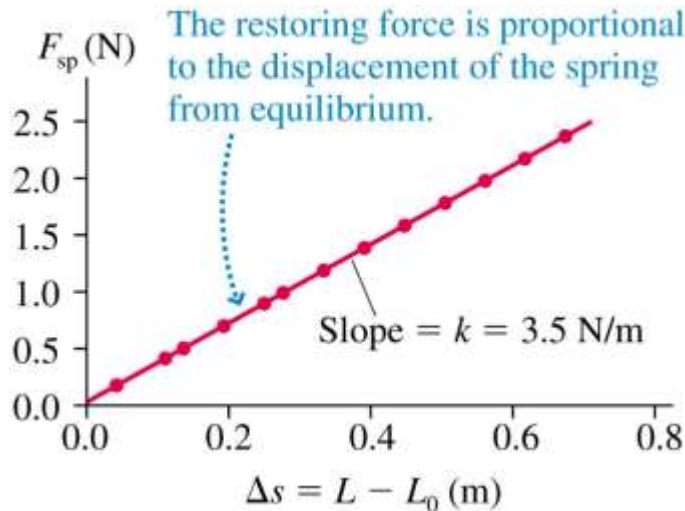


$$(F_{sp})_s = -k\Delta s$$

Q. So what's so special about Hooke's law? What does it imply?

- Well, we can see that the **restoring force** in the spring is proportional to the displacement that the spring undergoes.
- Furthermore, the physical meaning of the integral for the above graph is energy, energy that becomes stored in the spring as we stretch it or compress it, and energy that we refer to as **elastic potential energy, U_s** ; so let's now return to this form of energy.

Elastic Potential Energy, U_s



$$(F_{sp})_s = -k\Delta s$$

- Ignoring the direction of the restoring force, the average restoring force F is given by $F = \frac{ks+0}{2}$
- Thus, the work done in stretching or compressing a spring from rest to a distance s is given by $w = Fs = \frac{ks^2}{2}$
- We can refer to this work, which is now stored in the spring, in the form of a restoring force, as **elastic potential energy**.

Elastic Potential Energy, U_s



- Frequently in physics, we are interested in how energy is transformed from one form to another. In the next couple of slides, we are going to analyse an object attached to a spring as its energy transforms from elastic potential energy into kinetic energy.
- To this, we must briefly look at the **basic energy model** in physics.
- Before we do so, however, we must consider one other form of energy, **thermal energy**.

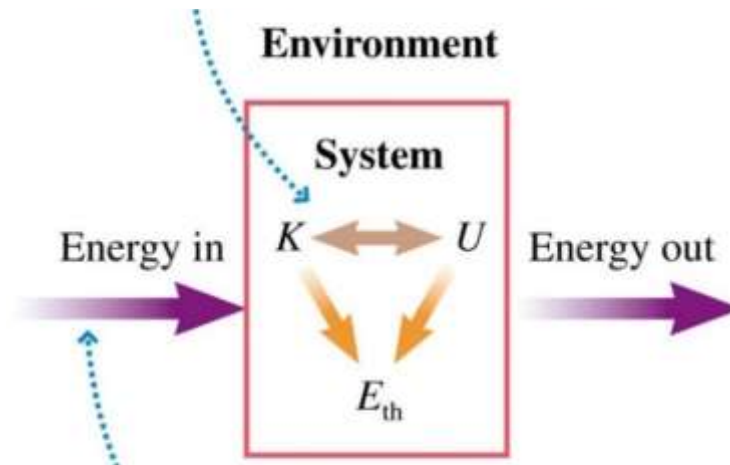
Thermal Energy, E_{th}



- Thermal energy is the sum of the microscopic kinetic and potential energies of all the atoms and bonds that make-up the object.
- This thermal energy occurs as a result of friction, which can be either useful or not useful, depending on the situation.

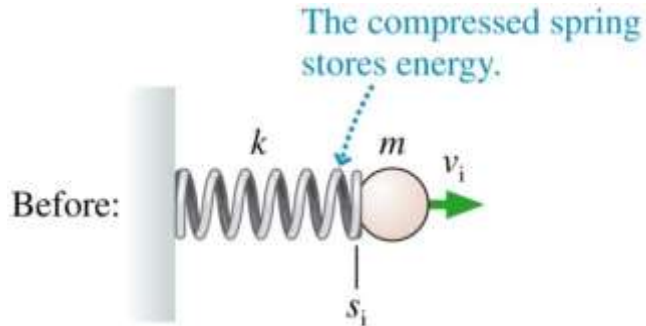
2. The Basic Energy Model

The Basic Energy Model

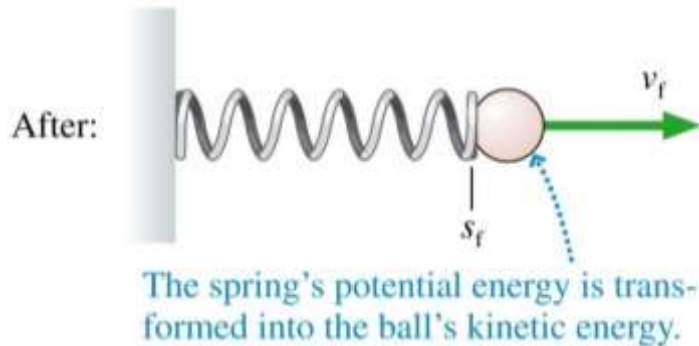


- Within an isolated system, energy can be transformed from one type to another.
- If we just focus on kinetic energy transforming itself into potential energy, and vice-versa, and assume there is no energy lost due to thermal energy, we are looking at the **law of conservation of mechanical energy**.

The Basic Energy Model



$$K_f + U_{gf} = K_i + U_{gi}$$

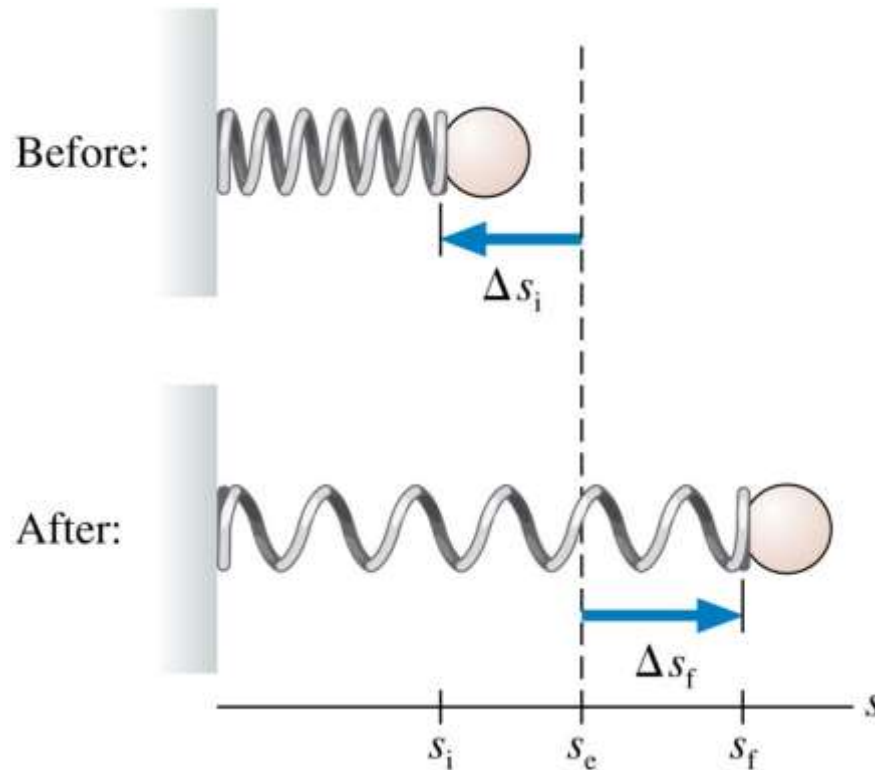


$$\frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i$$

- In the above **conservation-of-mechanical-energy** expression, we can replace **gravitational potential energy (U_{gf})** with elastic potential energy to give us:

$$\frac{1}{2}mv_f^2 + \frac{1}{2}k(\Delta s_f)^2 = \frac{1}{2}mv_i^2 + \frac{1}{2}k(\Delta s_i)^2$$

The Basic Energy Model



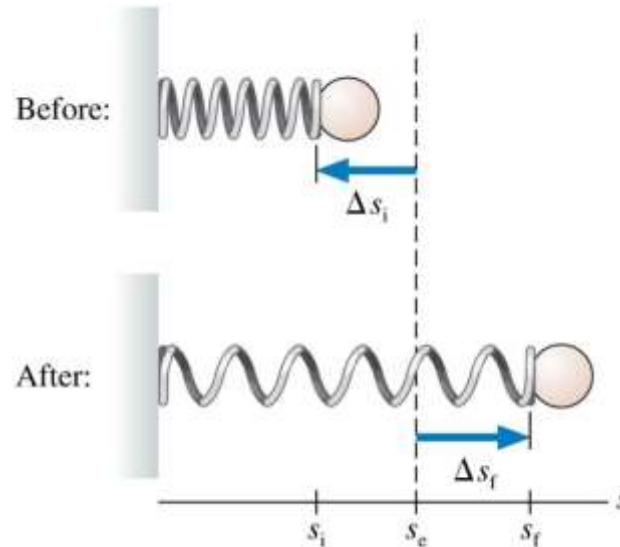
- An object moving without friction on an **ideal spring** obeys:

$$K_f + U_{sf} = K_i + U_{si}$$

where,

$$U_s = \frac{1}{2}k(\Delta s)^2 \quad (\text{elastic potential energy})$$

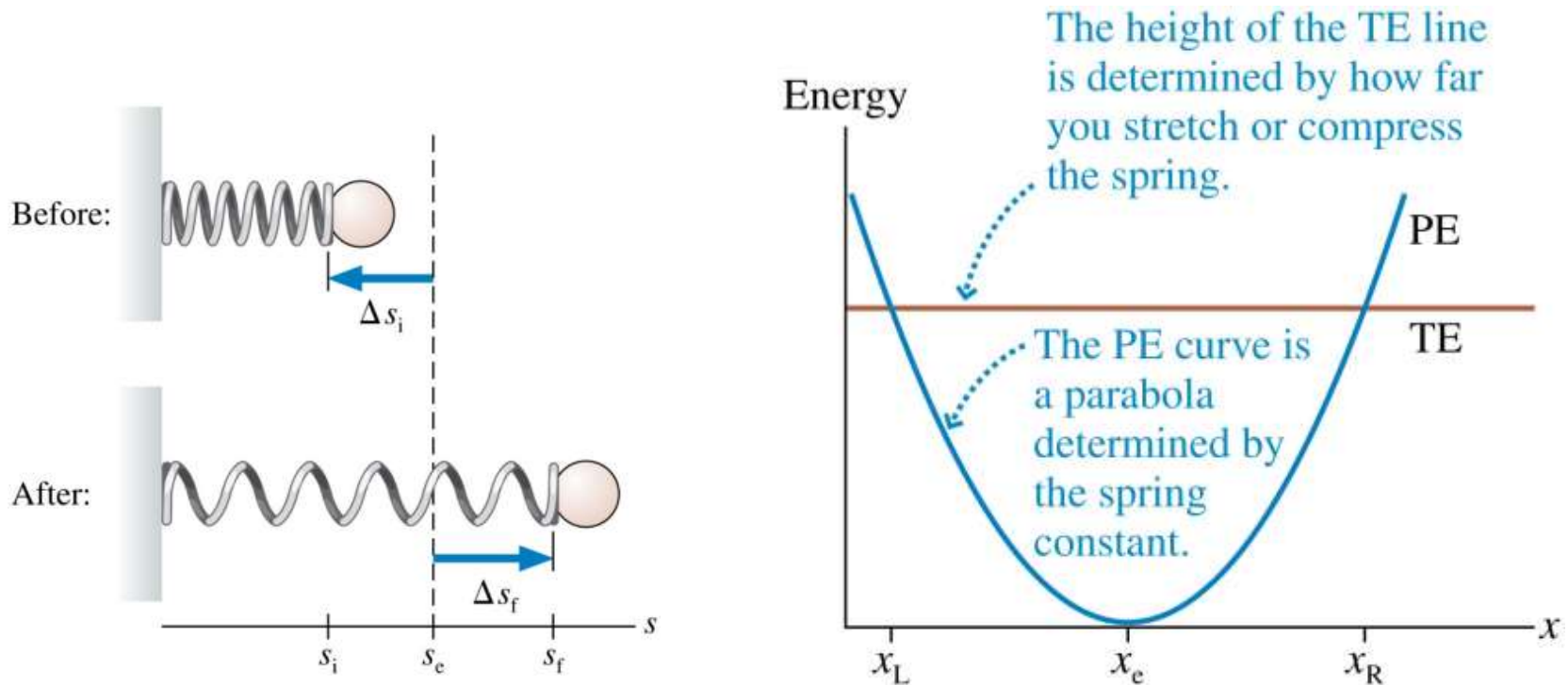
The Basic Energy Model



$$U_s = \frac{1}{2}k(\Delta s)^2 \quad (\text{elastic potential energy})$$

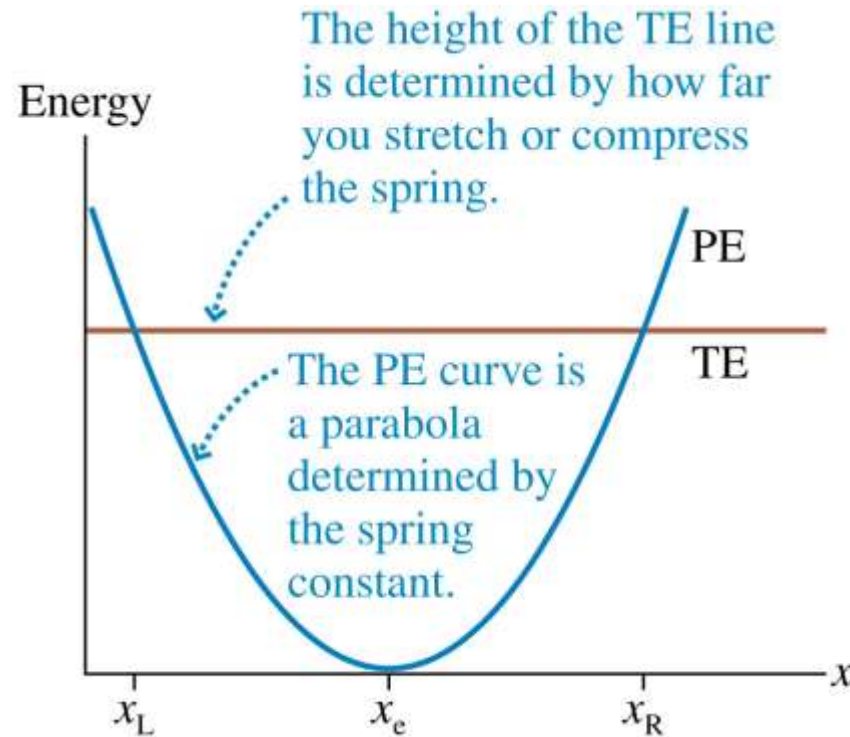
- Because Δs is squared, U_s is positive for a spring that is either stretched or compressed.
- Using a graph and bar charts, let's try to represent the energy changes that take place; we call such representations **energy diagrams**.

Energy Diagrams



- The above energy diagram is for a mass on a horizontal spring.
- You can set the total energy (TE) to any height you wish simply by stretching the spring to the proper length at the beginning of the motion.

Energy Diagrams

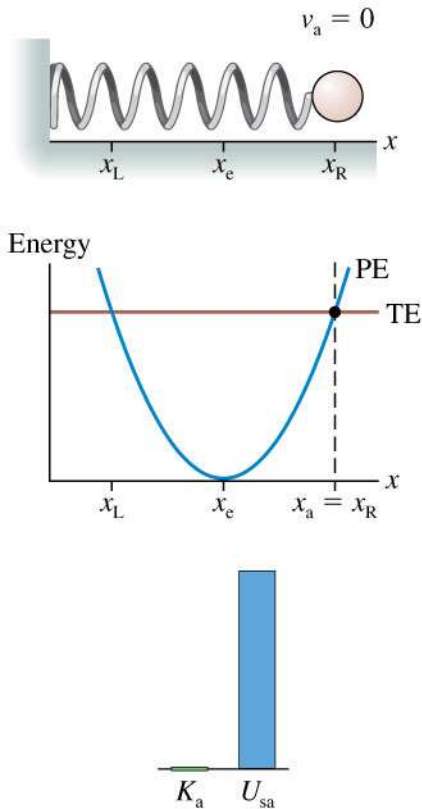


- The elastic potential energy (PE) is given by the parabola:

$$U_s = \frac{1}{2} k(x - x_e)^2$$

- The elastic potential energy (PE) depends on the spring constant k , which is unique for each material.

Energy Diagrams

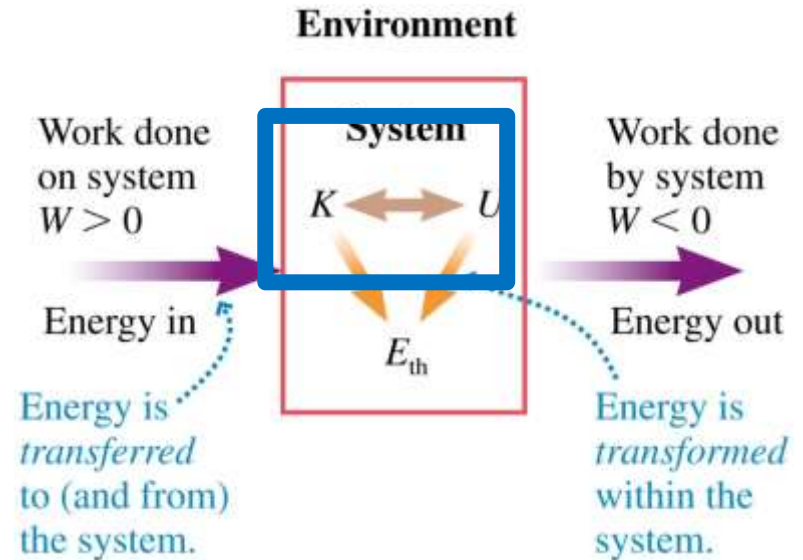


The mass is released from rest. The energy is entirely potential.

- It's important not to forget that we make the assumption that no energy is **lost to friction**; mechanical energy is conserved in other words.

3. Conservative and non-conservative systems

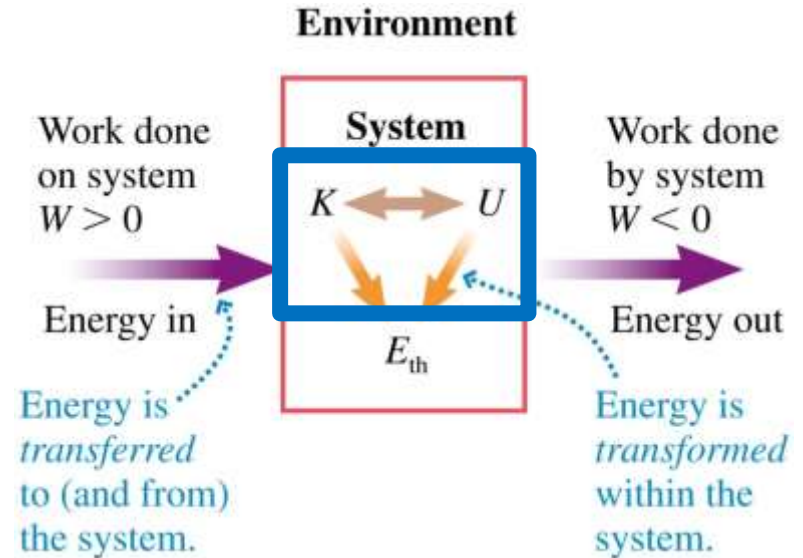
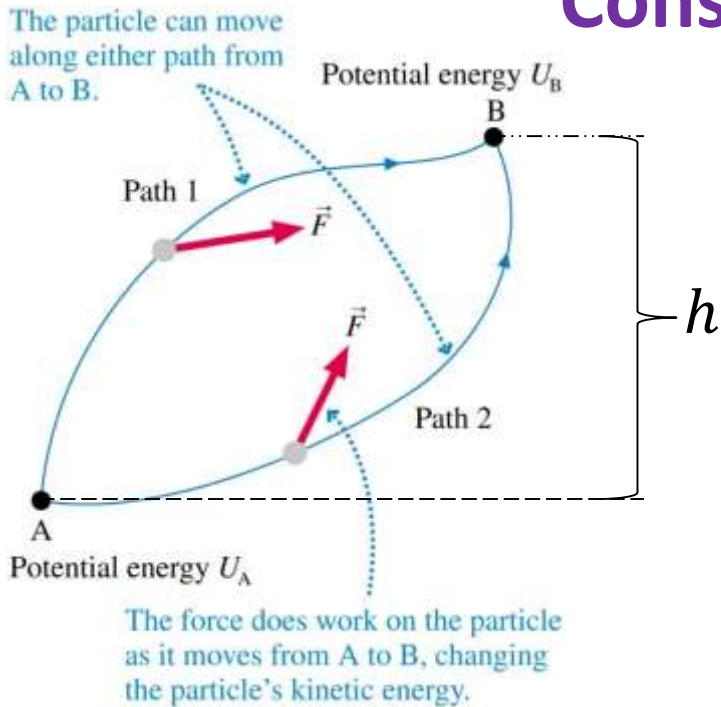
Conservative Systems



- A system in which the total mechanical energy, kinetic and potential, is constant, (assuming other forces such as friction are negligible) is called a **conservative system**.
- The forces that conserve this mechanical energy are called **conservative forces**; in the case of the pendulum, the conservative force is gravity.

Eg. Electric force, Gravitational force, Elastic force

Conservative Forces

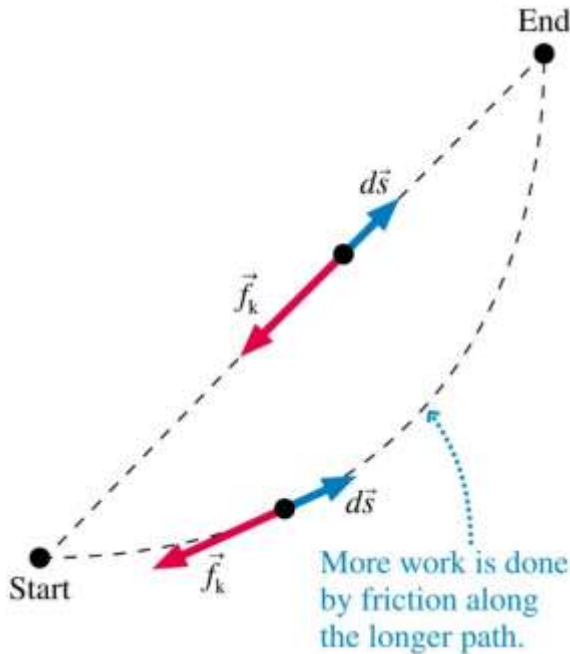


- The figure shows a particle that can move from A to B along either path 1 or path 2 while a force \vec{F} is exerted on it.
- If there is a potential energy associated with the force, this is called a **conservative force**.
- The work done by \vec{F} as the particle moves from A to B **is independent of the path followed**

$$\Delta U = U_f - U_i = -W_c(i \rightarrow f)$$

$$W_c = -mgh = \Delta U = -W_c$$

Non-Conservative Forces

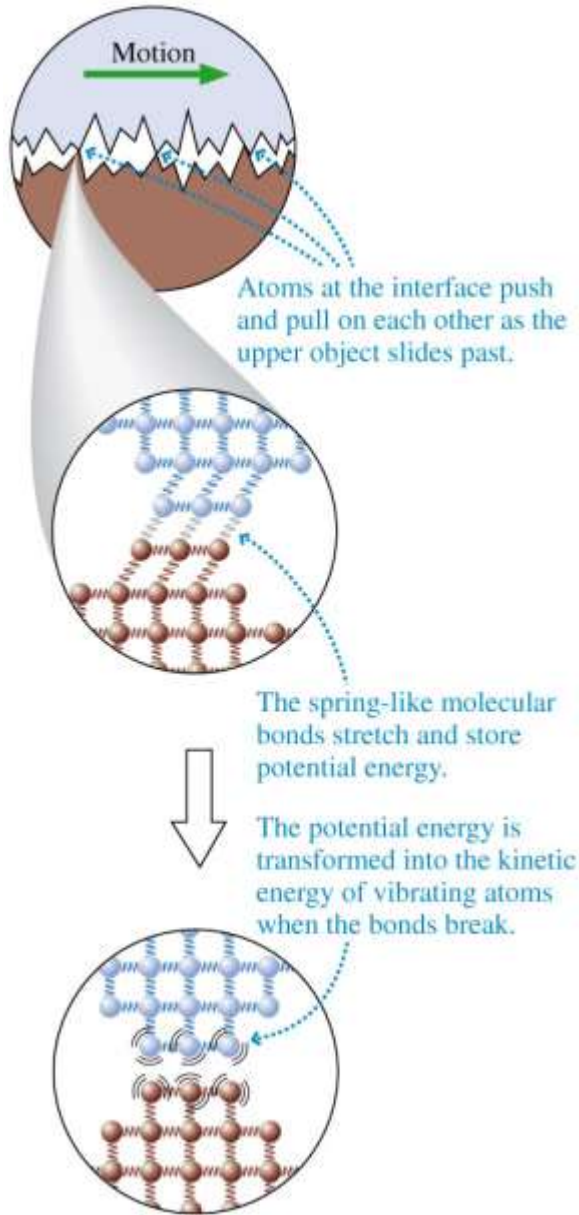


Eg. Frictional force
Applied force
Push and pull action
Tension



- The friction force does negative work on the objects:
 $W_{fric} = -\mu_k mg \Delta s$, work which depends on Δs .
- A force for which the work is not independent of the path is called a **non-conservative force**.
- Taking our pendulum, for example, the non-conservative force, friction due to air resistance, eventually slows the pendulum down. In other words, **it does not conserve the mechanical energy of the system.**

Non-Conservative Forces

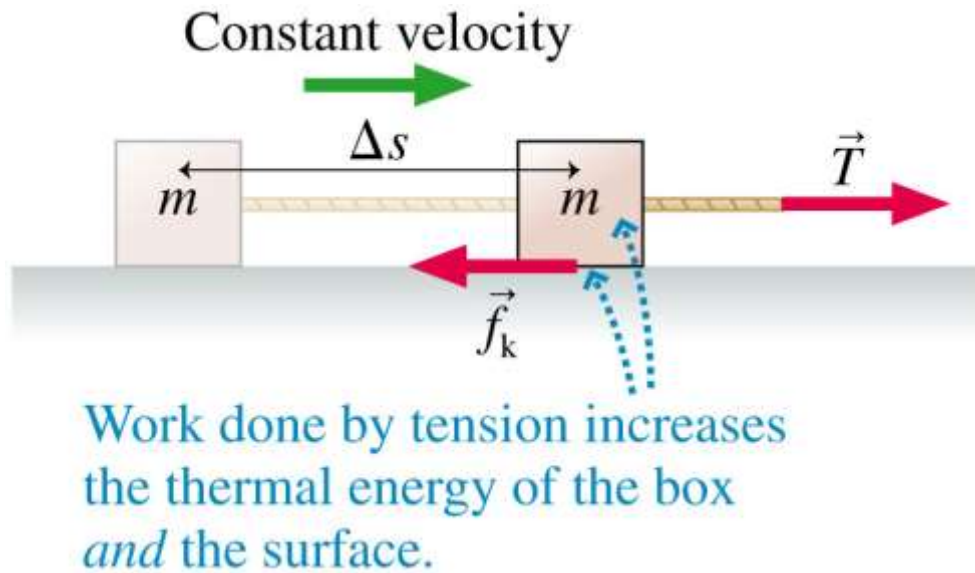


- As two objects slide against each other, atomic interactions at the boundary transform the kinetic energy, K_{macro} , into thermal energy in both objects.

$$K \rightarrow E_{th}$$

- This kinetic friction is a non-conservative force, sometimes referred to as a **dissipative force**.

Dissipative Forces



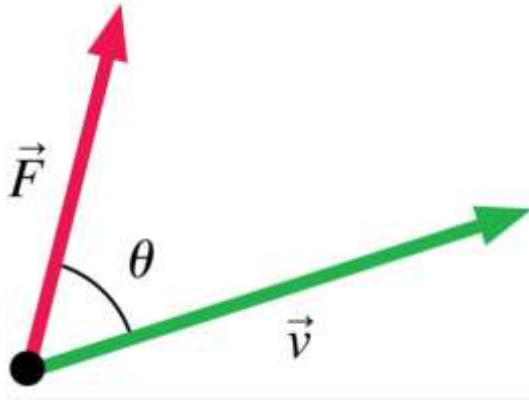
- Dissipative forces always increase the thermal energy; they never decrease it.

$$\Delta E_{th} = f_k \Delta s \text{ (increased thermal energy due to friction)}$$

- Let's now look at the **rate at which work** is transferred to or from a system: such a rate is called **power**.

4. Power

Power



- When energy is transferred by a force doing work, we call the rate at which it is transferred **power**, $P = dW/dt$.
- If an object moves at velocity \vec{v} while acted on by force \vec{F} , the power delivered to the object is:

$$P = \vec{F} \cdot \vec{v} = Fv\cos\theta$$

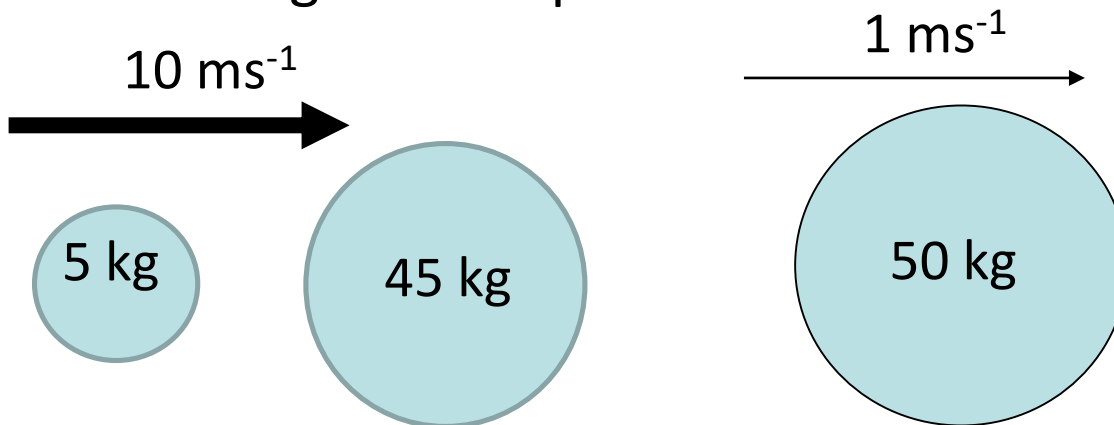
- The SI unit for power is the Watt, which is defined as:

$$1 \text{ Watt} = 1 \text{ W} = 1 \text{ J/s}$$

5. Momentum and Conservation of Momentum

Momentum

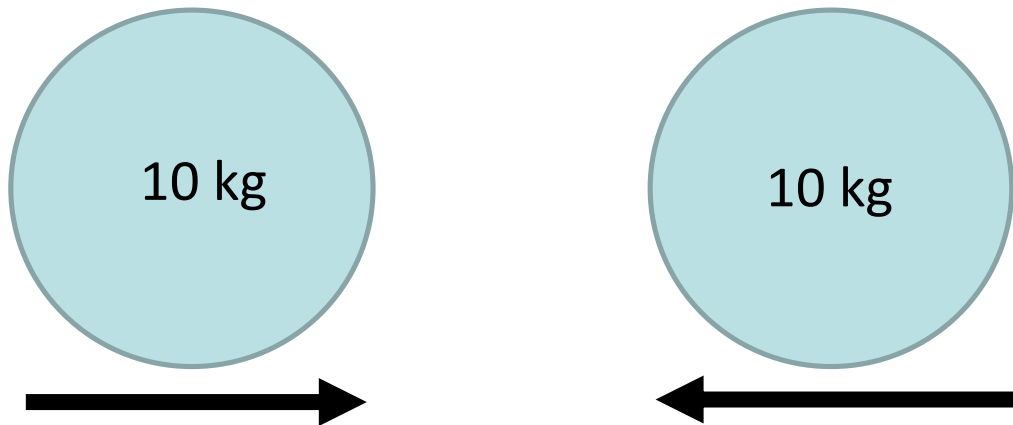
- Momentum is a term that was introduced by the French scientist and philosopher, Descartes, before Newton.
- What Descartes meant by the use of the term momentum is best understood by considering an example:



Rene Descartes
1596-1650

- Think of a ball (with a mass of 45 kg) that is motionless.
- A 5 kg ball is moving towards it, with a speed of 5 m/s, and is quite close to the object, so that we can assume the ball's flight to be close to horizontal. New speed 10 times less.

Momentum



$$\text{momentum} = p = m \times \text{speed}$$



Christian Huygens

1596-16

- There was a problem, however, with Descartes's definition.
- For example, when two objects of equal mass come directly towards each other with equal, but opposite velocities, when they meet, they come to a complete stop. Need to replace speed with velocity.

$$\text{momentum} = p = \text{mass} \times \text{velocity}$$

Momentum and Its Relation to Force

$$\sum \vec{F} = \frac{d\vec{p}}{dt},$$



1642-1726

- A force is required to change the momentum of an object, whether it is to increase the momentum, to decrease it, or to change its direction.
- Newton **originally stated his second law in terms of momentum** (although he called the product mv the 'quantity of motion')

Momentum and Its Relation to Force

- Newton's statement of the second law of motion, translated into modern language, is as follows:

*The rate of change of momentum of an object is equal to the **net force applied to it**, and can be written as an equation,*

$$\sum \vec{F} = \frac{d\vec{p}}{dt},$$

where $\sum \vec{F}$ is the net force applied to the object (the vector sum of all the forces acting on it).

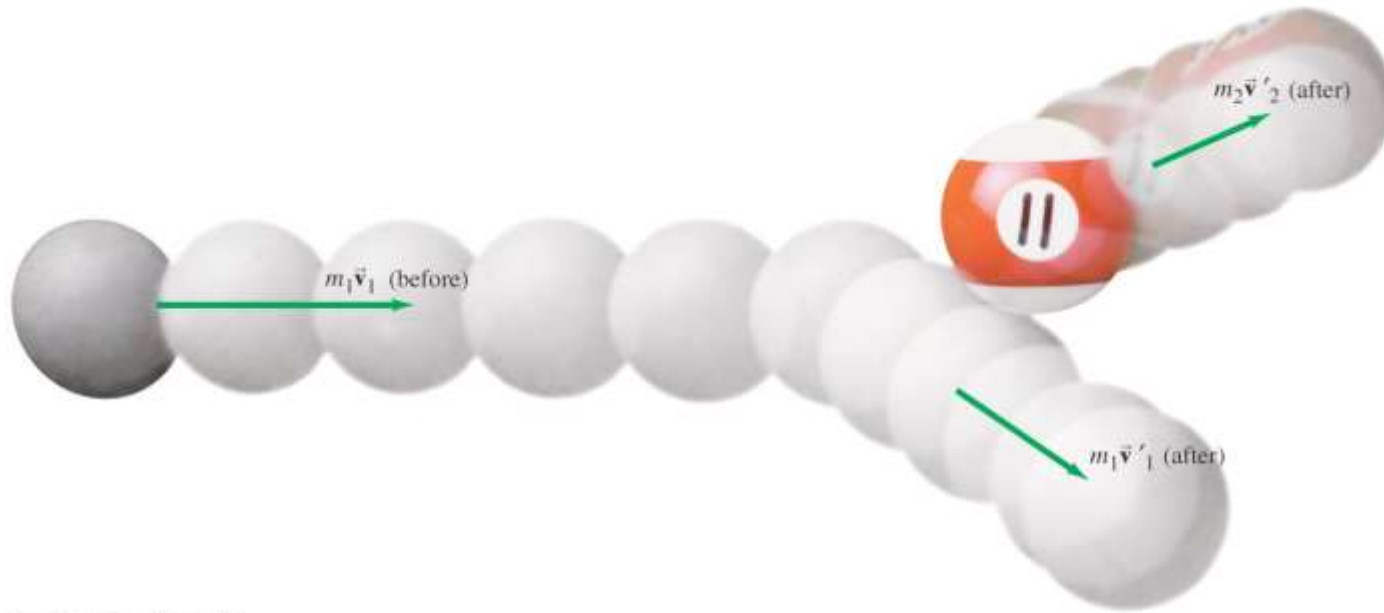
Momentum and Its Relation to Force

$$\sum \vec{F} = \frac{d\vec{p}}{dt},$$

- We can derive the familiar form of the second law, $\sum \vec{F} = m\vec{a}$, from the above equation for the case of **constant mass**.
- If \vec{v} is the velocity of an object at any moment, then the above equation gives

$$\sum \vec{F} = \frac{d\vec{p}}{dt} = \frac{d(m\vec{v})}{dt} = m \frac{d\vec{v}}{dt} = m\vec{a}$$

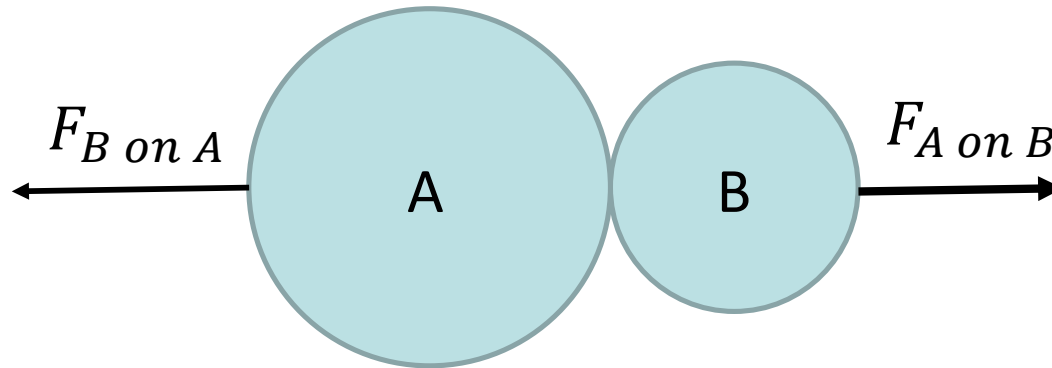
Conservation of Momentum



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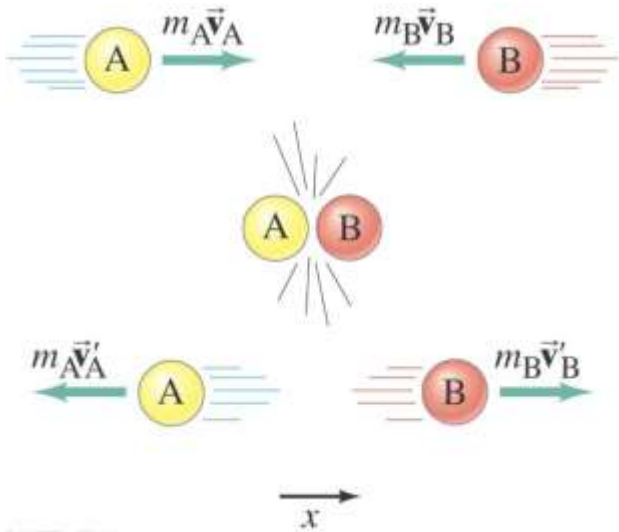
- The law of the conservation of momentum is essentially a reworking of Newton's laws.
- We use the laws of conservation of linear momentum and of energy to analyse collisions.

Conservation of Momentum



- Let's think of any interaction, between two objects, A and B. From Newton's 3rd Law, the force A feels from B is of equal magnitude to the force B feels from A, but in an opposite direction.
- Since force equals rate of change of momentum, it follows that throughout the interaction process, the rate of change of momentum of A is exactly opposite to the rate of change of momentum of B.

Conservation of Momentum

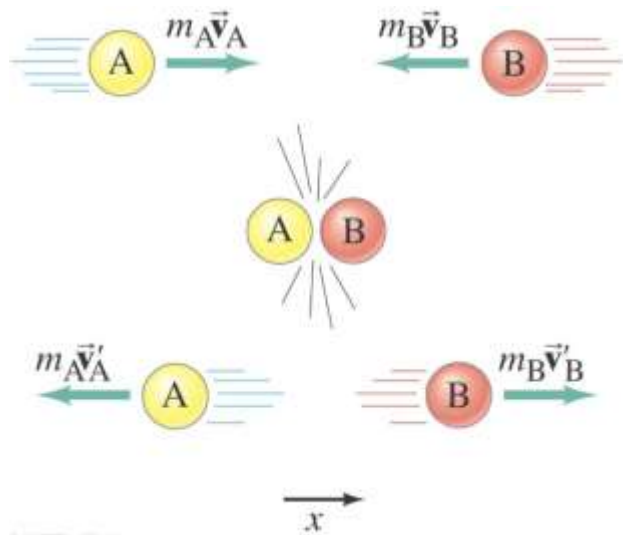


momentum before = momentum after

$$m_A \vec{v}_A + m_B \vec{v}_B = m_A \vec{v}'_A + m_B \vec{v}'_B$$

- If no external force acts on a system, the total momentum of the system is a conserved quantity.
- If we consider, head-on collision of two balls, we can assume that the **net external force on this system of two balls is zero**—that is, the only significant forces during the collision are the forces that each ball exerts on the other.

Conservation of Momentum



momentum before = momentum after

$$m_A \vec{v}_A + m_B \vec{v}_B = m_A \vec{v}'_A + m_B \vec{v}'_B$$

- Our derivation of the conservation of momentum can be extended to include any number of interacting objects.
- Let \vec{P} represent the total momentum of a system of n interacting objects which we number from 1 to n :

$$\vec{P} = m_1 \vec{v}_1 + m_2 \vec{v}_2 + \cdots + m_n \vec{v}_n = \sum \vec{p}_i$$

Conservation of Momentum

- Let \vec{P} represent the total momentum of a system of n interacting objects which we number from 1 to n :

$$\vec{P} = m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots + m_n\vec{v}_n = \sum \vec{p}_i$$

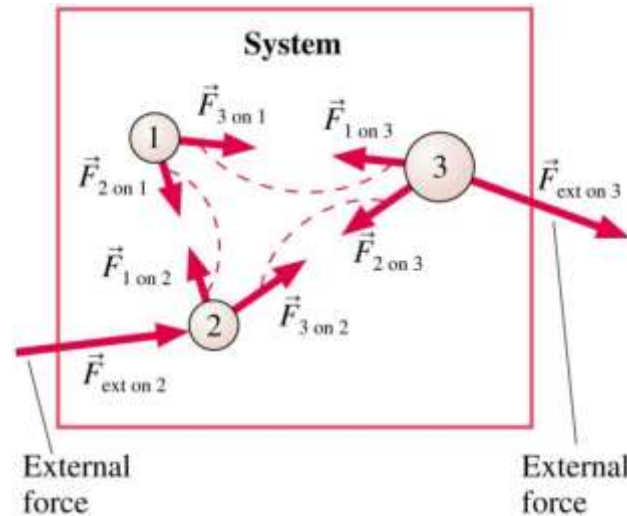
- We can differentiate **with respect to time**:

$$\frac{d\vec{P}}{dt} = \sum \frac{d\vec{p}_i}{dt} = \sum \vec{F}_i$$

where \vec{F}_i represents the net force on the i^{th} object.

Conservation of Momentum

$$\frac{d\vec{P}}{dt} = \sum \frac{d\vec{p}_i}{dt} = \sum \vec{F}_i$$

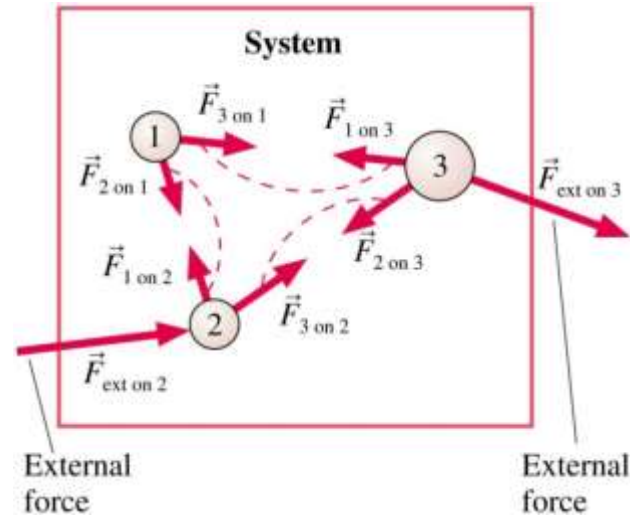


where \vec{F}_i represents the net force on the i^{th} object.

- The forces acting on the objects can be of two types:
 - (1) Internal forces that objects within the system exert on other objects in the system; or
 - (2) External forces on objects of the system, exerted by objects outside the system.

Conservation of Momentum

$$\frac{d\vec{P}}{dt} = \sum \frac{d\vec{p}_i}{dt} = \sum \vec{F}_i$$



- (1) Internal forces that objects within the system exert on other objects in the system.
- By Newton's third law, the internal forces occur in pairs: if one object exerts a force on a second object, the second exerts an equal and opposite force on the first object.
 - Thus, all the internal forces cancel each other in pairs, meaning that **a change in momentum can only occur as a result of external forces.**

Conservation of Momentum

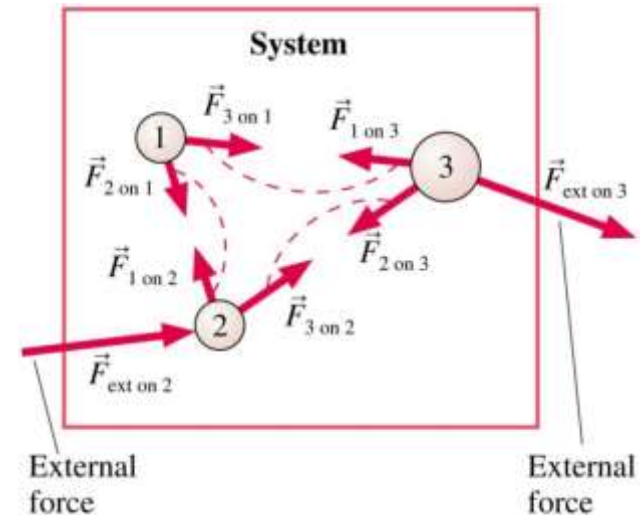
$$\frac{d\vec{P}}{dt} = \sum \frac{d\vec{p}_i}{dt} = \sum \vec{F}_i$$

- Thus, we have

$$\frac{d\vec{P}}{dt} = \sum \vec{F}_{ext},$$

- When the net external force on a system of objects is zero, the total momentum of the system remains constant. **This is the law of conservation of momentum.**

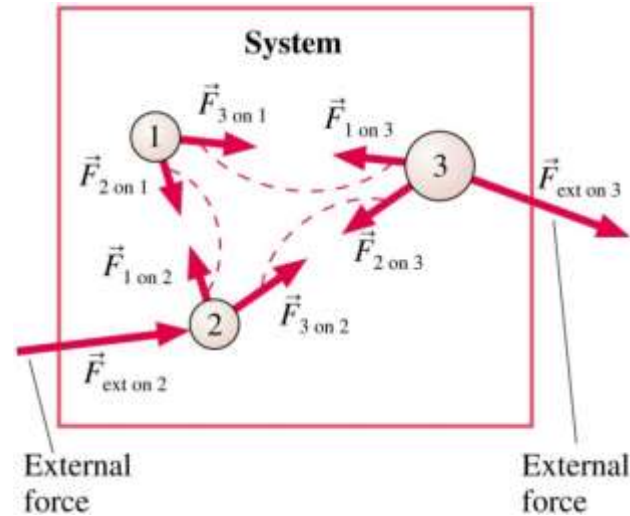
$$\frac{d\vec{P}}{dt} = 0, \quad \vec{P} = \text{constant}, \quad \vec{P} = m\vec{v}$$



Conservation of Momentum

$$\frac{d\vec{P}}{dt} = \sum \vec{F}_{ext},$$

- We can also state the law as

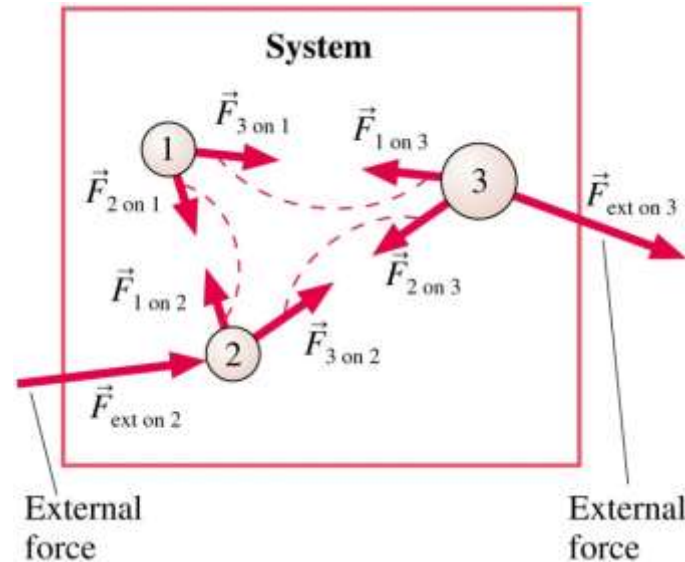


The total momentum of an isolated system of objects remains constant,

where an isolated system means one in which no external forces act—the only forces acting are those between objects of the system.

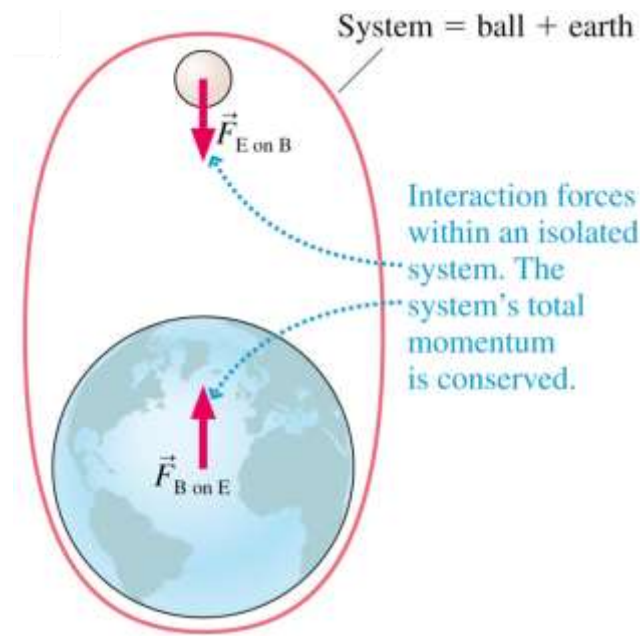
Conservation of Momentum

$$\frac{d\vec{P}}{dt} = \sum \vec{F}_{ext},$$



- If a net external force acts on a system, then the law of conservation of momentum will not apply.
- However, if the 'system' can be **redefined** so as to include the other **objects exerting these forces**, then the conservation of momentum principle can apply. Let's look at an example.

Conservation of Momentum: Defining the System



- A rubber ball is dropped, and falls toward Earth.
- Now let's define the ball + Earth as the system.
- The gravitational forces are interactions within the system.
- This is now an isolated system, so the total momentum is conserved.

$$\vec{p} = \vec{p}_{ball} + \vec{p}_{Earth}$$

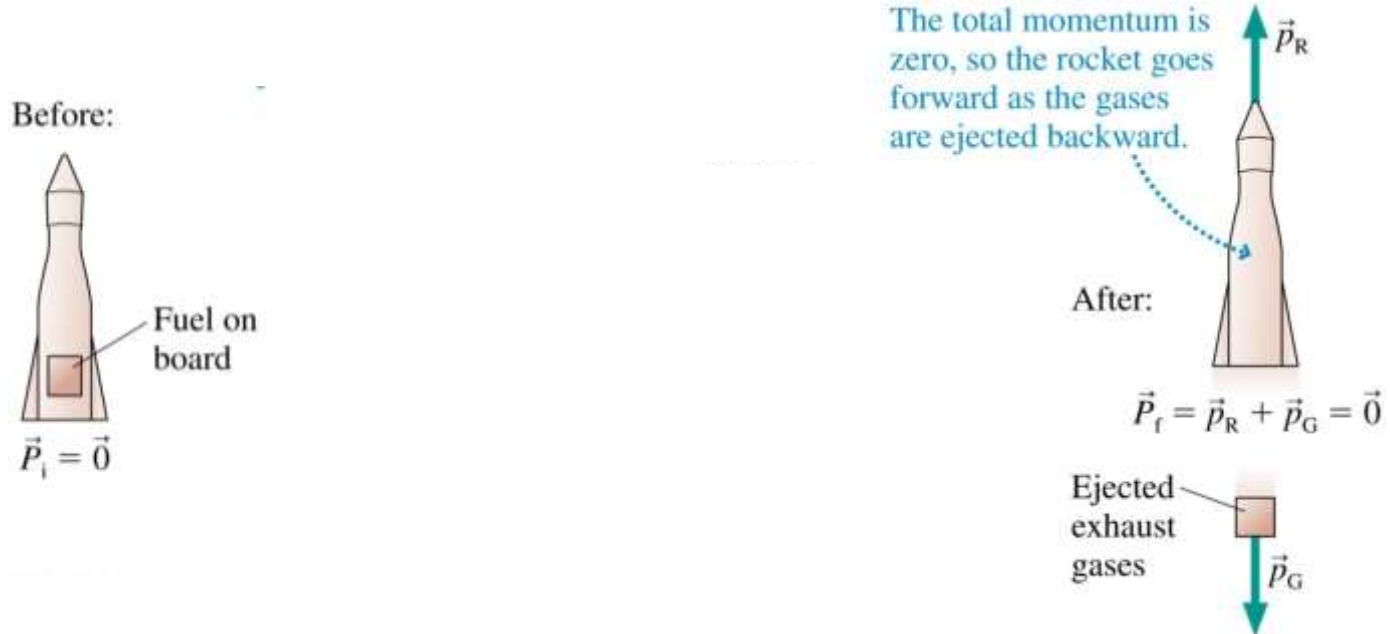
Conservation of Momentum

$$\frac{d\vec{P}}{dt} = \sum \vec{F}_{ext},$$



- The law of conservation of momentum is particularly useful when we are dealing with fairly simple systems such as colliding objects and certain types of explosions.
- For example, rocket propulsion can be understood on the basis of action and reaction, and can also be explained on the basis of the conservation of momentum.
- We can consider the rocket and fuel as an isolated system if it is far out in space (no external forces).

Conservation of Momentum



- The figure shows a rocket with a parcel of fuel on board.
- If we choose the rocket + gases to be the system, the burning and expulsion are both internal forces.
- The exhaust gases gain backward momentum as they are shot-out the back.
- The rocket gains equal momentum, but in the opposite direction.
- Therefore, the total momentum is zero.

Summary Thus Far

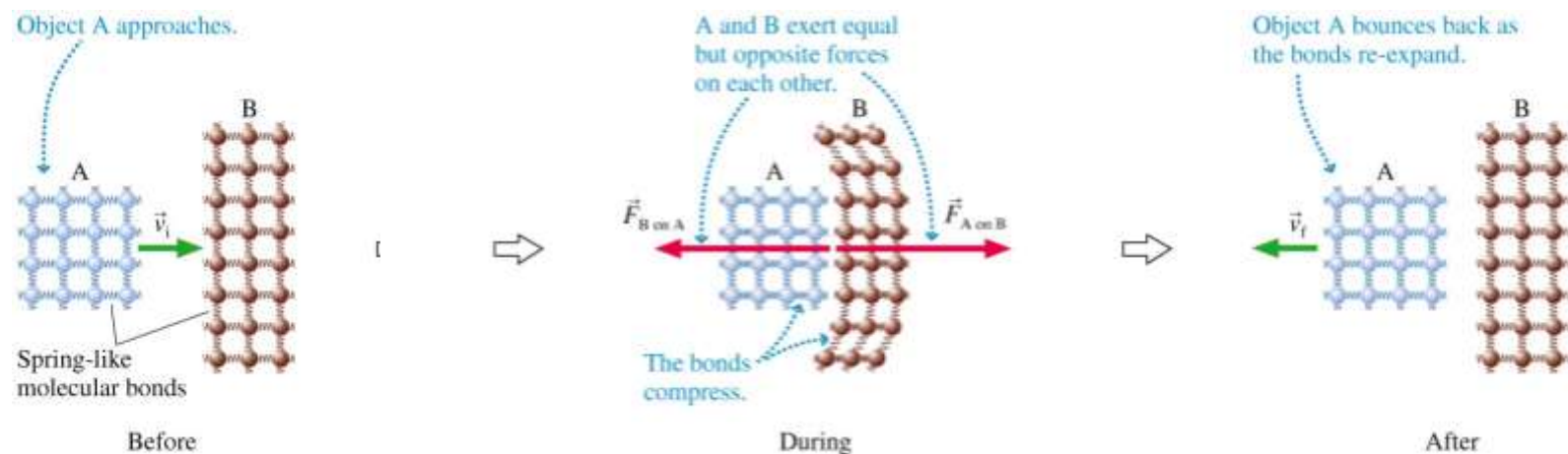
$$\sum \vec{F} = \frac{d\vec{p}}{dt},$$

- Thus far, we have seen that the rate of change of momentum of an object is equal to the net force applied to it, and can be described using the above equation.
- Clearly, this rate of change of momentum occurs when an object interacts with another object, each object exerting an equal and opposite force on each other, and each object affecting each other's rate of change of momentum.
- If, however, we manipulate the above equation into the following,

$$\sum \vec{F} dt = d\vec{p}$$

we see that each object's change in momentum depends on the force as well as the length of time the force acts on each object; we call this **impulse**; let's study it in more detail.

6. Impulse



- During a collision of two ordinary objects, both objects act on each other, causing molecular bonds to compress/deform, if we view these molecular bonds as like springs.
- If the **material** of each **object is different**, then the compression (deformation) for each object will be **different**.
- However, during the collision, **the force that each object exerts on the other will be the same**, but the **acceleration of each object will be different** if they have a different mass and/or are made from different materials.

Impulse

So force multiplied by the change in time is just the change in momentum, **and we can rearrange to find a new formula for force:**

$$\sum F = \frac{\Delta p}{\Delta t} \rightarrow \Delta p = F \Delta t$$

$$\text{Impulse } (I) = F \Delta t = \Delta p = p_{\text{final}} - p_{\text{initial}} = \Delta(mv) = m \Delta v$$

If we break momentum down into
velocity and mass

$$F = \frac{\Delta p}{\Delta t} = \frac{\Delta(mv)}{\Delta t} = \frac{m \Delta v}{\Delta t}$$

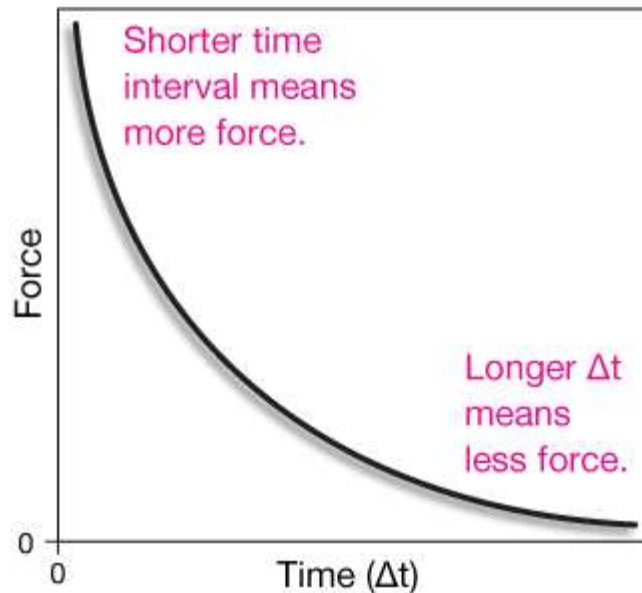
Impulse

For a given velocity change of **some fixed mass** – **like a person**, the force is inversely proportional to the amount of time it takes to undergo that momentum or velocity change

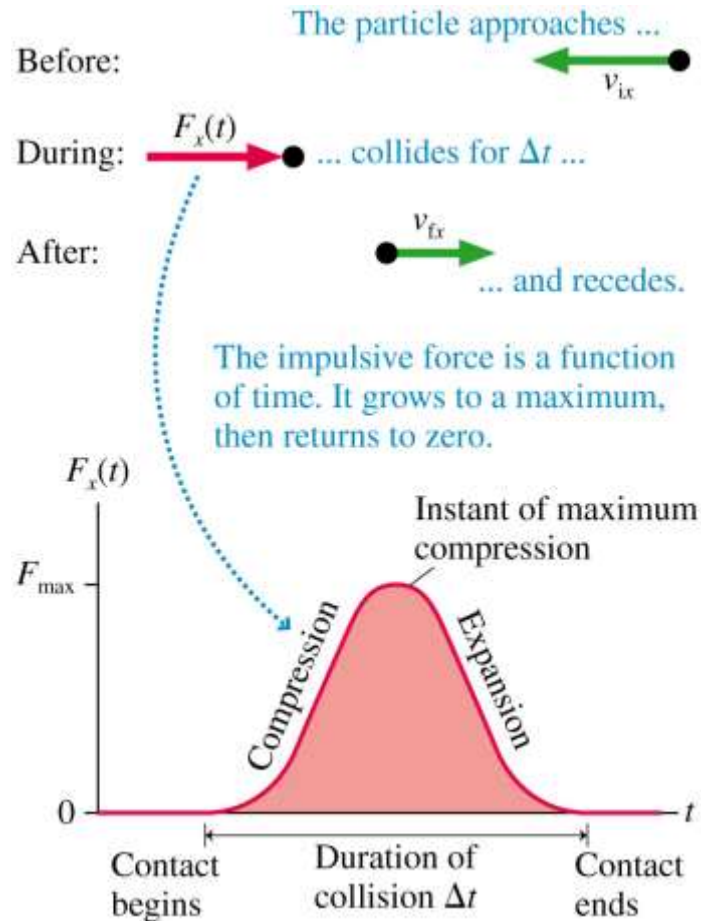
The longer it takes to change momentum, the less force is **applied or experienced**. The shorter the time, the more force. Here's a graph of the situation.

$$F \propto \frac{1}{\Delta t}$$

$$F \propto m \Delta v$$



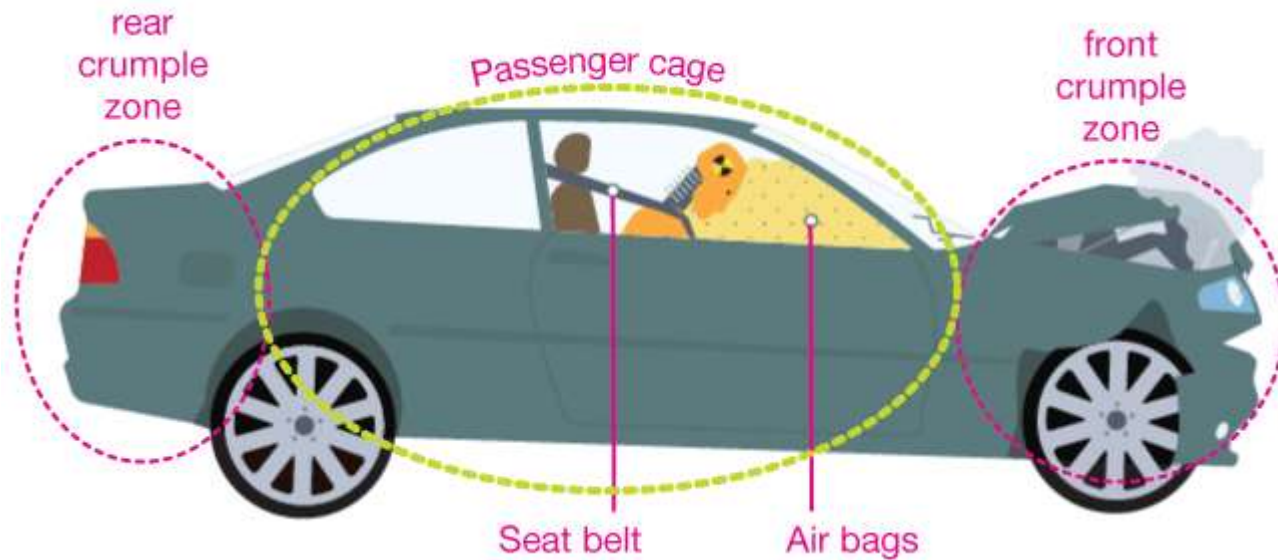
Impulse



- The force usually jumps from zero at the moment of contact to a very large value within a short period of time, and then quickly returns to zero again, something like what is shown above.

Crumple Zones

Cars are full of force-reducing measures

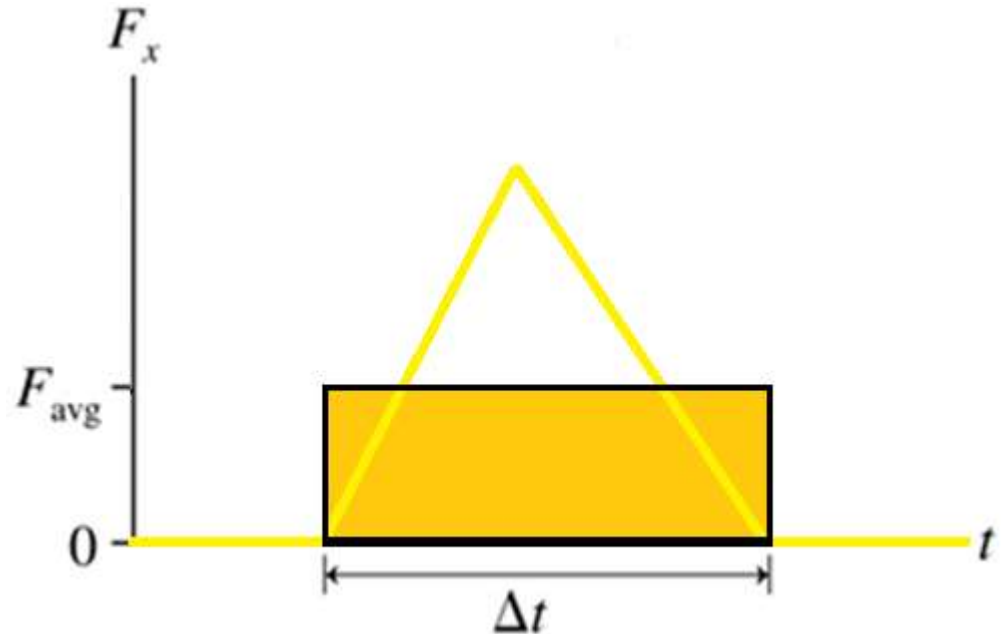


Crumple Zones



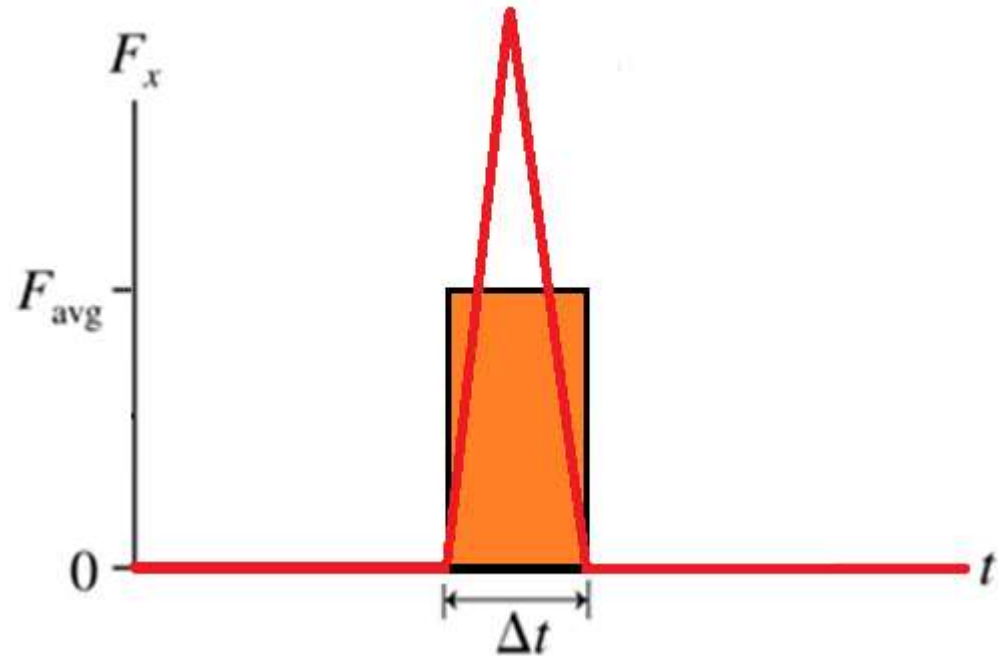
- Crumple zones make use of the idea of impulse. In the above, the yellow metal is weaker than the red metal.

Crumple Zones



- Think of the atoms of metal as being like springs: if the car hits a wall, the yellow metal springs are more easily compressed (small spring constant), taking longer to exert an equal and opposite force. **This lessens the average impact force**, also absorbing some of the energy of the collision.

Crumple Zones



- Think of the atoms of metal as being like springs: on the other hand, for the **red metal**, you can think of the springs as having a higher spring constant, not compressing as much (or absorbing the collision much). This can be a good thing, as you don't want this region of the car to crumple.

Announcement

Seminar 3

1. Students should solve all the seminar **3** questions in a **brand-new BOOK** and bring the solutions to the seminar class.
2. Students should come to the seminar class with their calculators, pens, pencils, and notebooks.
3. Phones, laptops, and tablets will not be used in the seminar classrooms.

Online weekly Quizzes

Do not forget to complete Quiz 1

**The deadline for this assessment is Friday,
18th October 2024, 3pm.**

Lecture 5: Recommended Readings



The University of
Nottingham

UNITED KINGDOM • CHINA • MALAYSIA

- **Ch. 7.1**, Work Done by a Constant Force; p.194-197
- **Ch. 7.2**, Work Done by a Varying Force; p.198-200
- **Ch. 7.3**, Work Done by a Spring Force; p.200-201
- **Ch. 7.4**, Kinetic Energy and the Work-Energy Principle; p.202-206
- **Ch. 8.1**, Conservative and Non-conservative Forces; p.218-219
- **Ch. 8.2**, Potential Energy; p.221-222
- **Ch. 8.4**, Conservation of Mechanical Energy; p.224-229
- **Ch. 8.6**, Energy Conservation with Dissipative Forces; p.230-232
- **Ch. 9.1**, Momentum and Its Relation to Force; p.253-254
- **Ch. 9.2**, Conservation of Momentum; p.255-258
- **Ch. 9.3**, Collisions and Impulse; p.258-259