The University of Nottingham
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Science A Physics

Lecture 15:

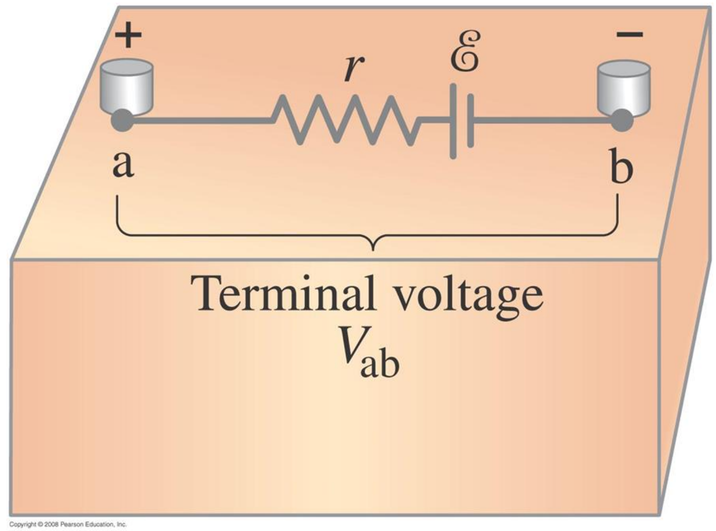
Resistance and RC circuits

Aims of today's lecture

1. EMF and Terminal Voltage
2. Resistors in Series and in Parallel
3. Kirchoff's Rules
4. Electrical Hazards
5. RC Circuits

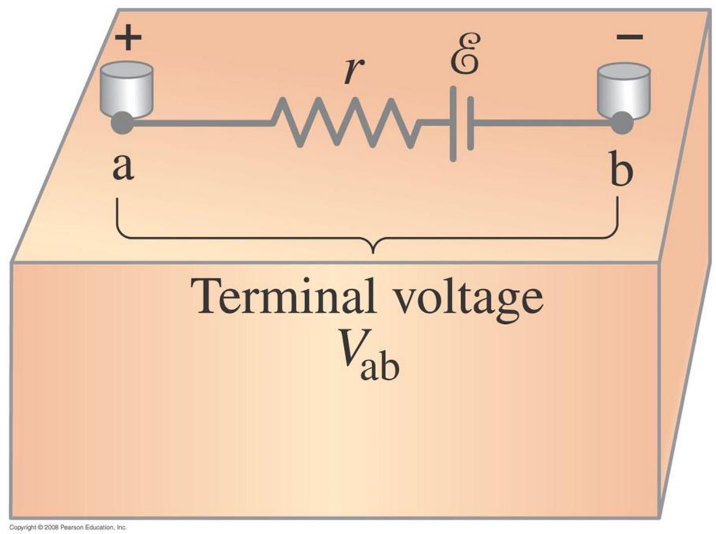
1. EMF and Terminal Voltage

EMF and Terminal Voltage



- As we have already seen, to have a current in an electric circuit, we need a device such as a battery or an electric generator that transforms one type of energy (chemical, mechanical, or light, for example) into electric energy.
- Such a device is called a **source** of **electromotive force** or of **emf**.

EMF and Terminal Voltage

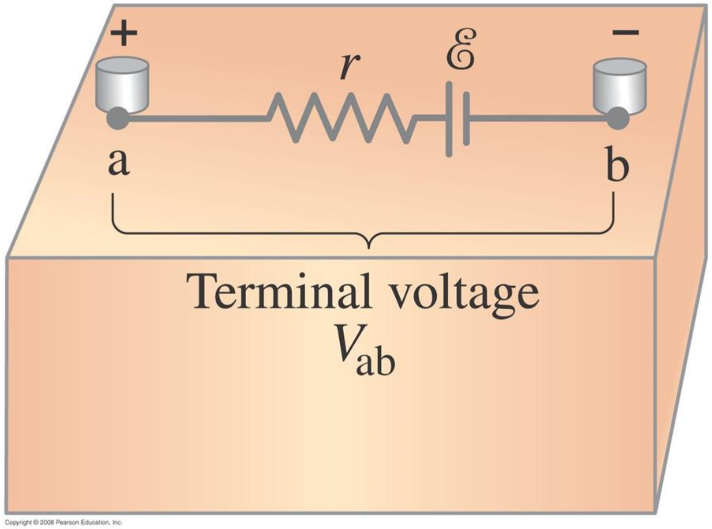


- The potential difference between the terminals of such a source, when no current flows to an external circuit, is called the **emf** of the source.

N.B.

A battery is not a source of constant current—the current out of a battery varies according to the resistance in the circuit. A battery is, however, a nearly constant voltage source, but not perfectly constant.

EMF and Terminal Voltage

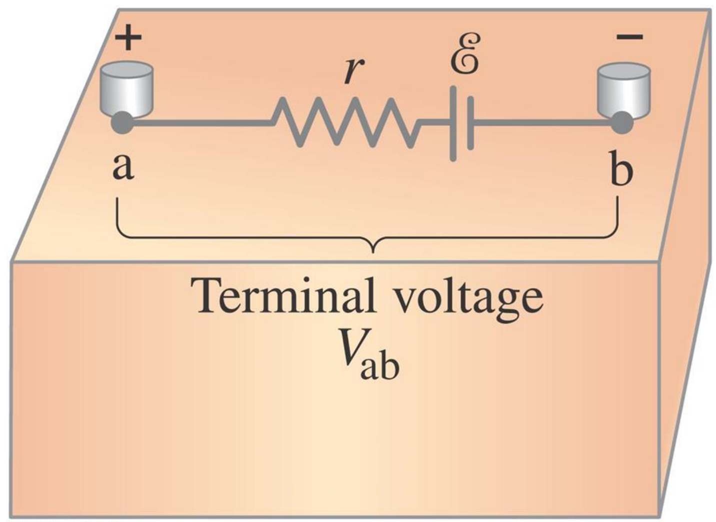


N.B.

The stated (terminal) voltage on the battery and the actual voltage (when the battery forms a circuit) differ.

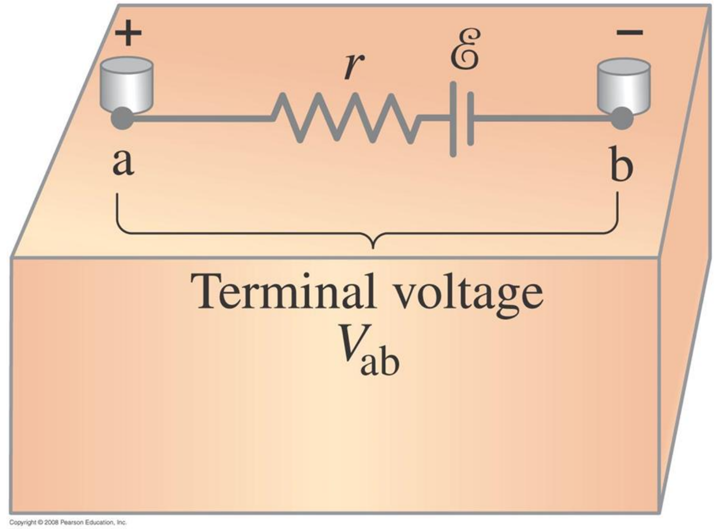
- This happens because the chemical reactions in a battery cannot supply charge fast enough to maintain the full emf (or stated terminal voltage, in other words).

EMF and Terminal Voltage



- The charge must move (within the electrolyte) between the electrodes of the battery, and there is always some hindrance to completely free flow.
- Thus, a battery itself has some resistance, which is called its **internal resistance**; it is usually designated r.
- A real battery is modelled as if it were a perfect emf in series with a resistor r, as shown above.

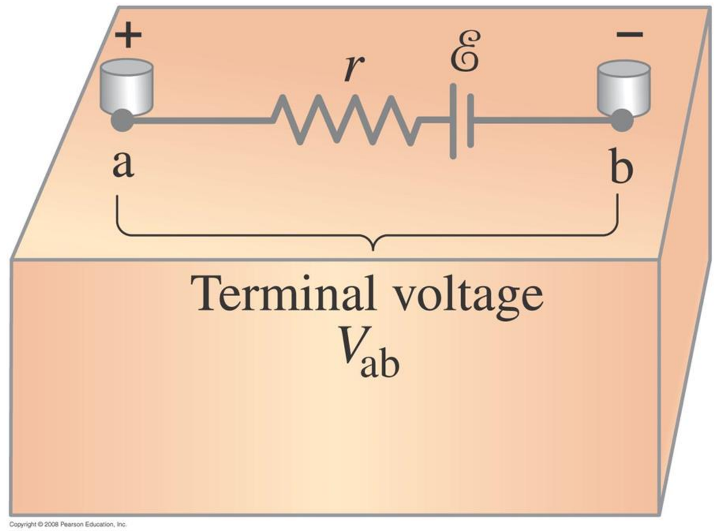
EMF and Terminal Voltage



- When no current is drawn from the battery, the terminal voltage equals the emf, which is determined by the chemical reactions in the battery.
- However, when a current, , I flows naturally from the battery, there is an internal drop in voltage equal to Ir.
- Thus, the actual voltage is

$$V_{ab} = \varepsilon - Ir$$

EMF and Terminal Voltage

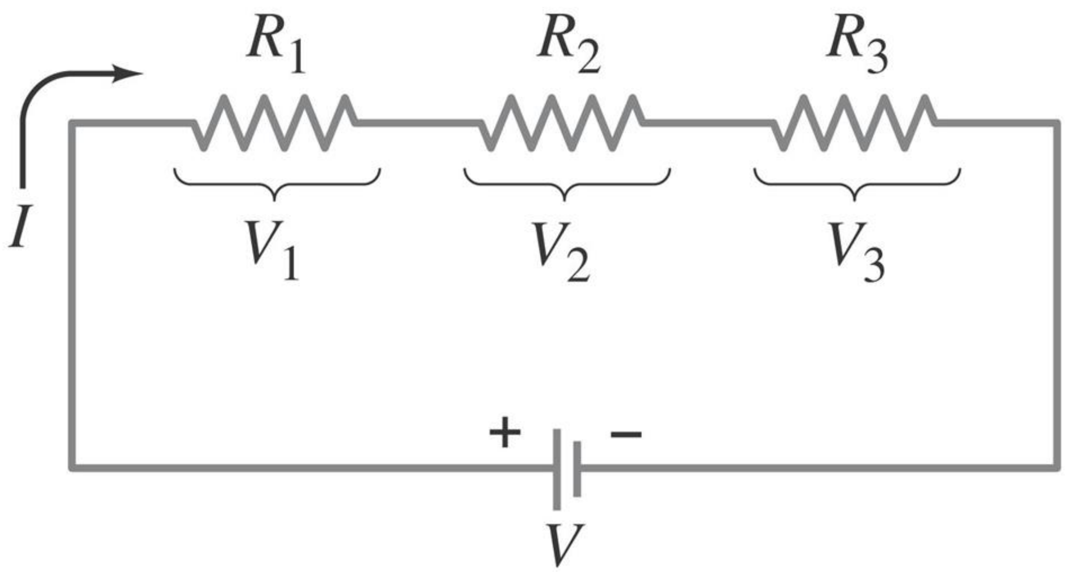


$$V_{ab} = \varepsilon - Ir$$

- The internal resistance of a battery is usually small. For example, an ordinary flashlight battery when fresh may have an internal resistance of perhaps $0.05\ \Omega$.
- However, as it ages and the electrolyte dries out, the internal resistance increases to many ohms.

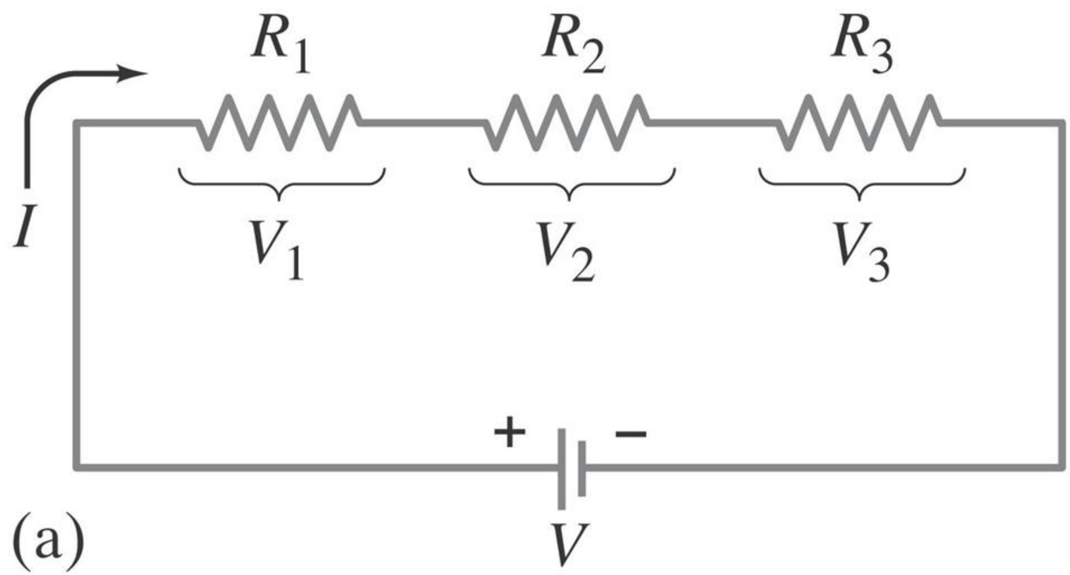
2. Resistors in Series and in Parallel

Resistors in Series and Parallel



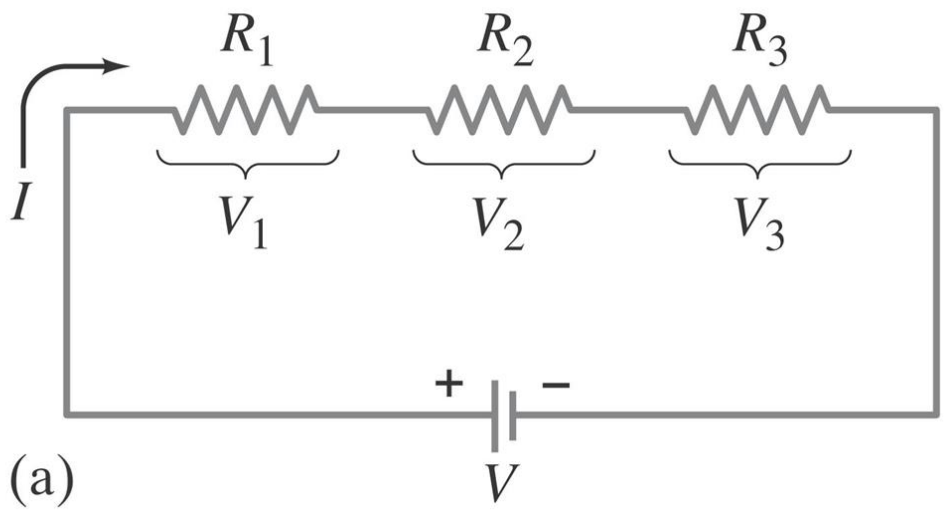
- When two or more resistors are connected end-to-end along a single path as shown above, they are said to be connected in **series**.
- The resistors could be simple resistors, or they could be lightbulbs, or heating elements, or other resistive devices.
- Any charge that passes through R_1 in the above figure will also pass through R_2 and then R_3. Hence, the same current, I, passes through each resistor.

Resistors in Series and Parallel



- Let V represent the potential difference (voltage) across all three resistors in the above figure.
- Assume all other resistance in the circuit can be ignored, so V equals the terminal voltage supplied by the battery.
- Let V_1, V_2, and V_3 be the potential differences across each of the resistors, R_1, R_2, and R_3, respectively.

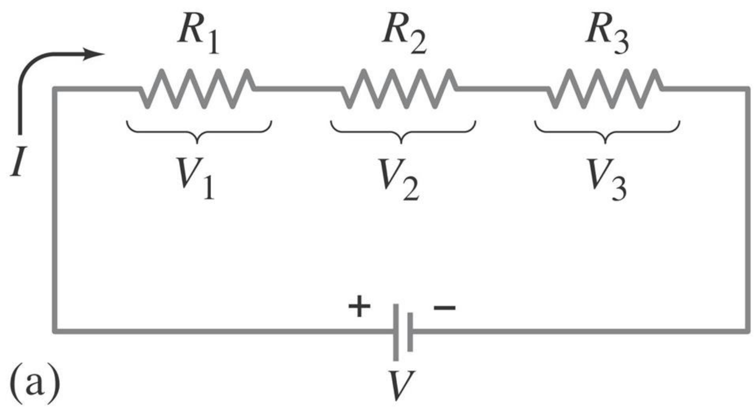
Resistors in Series and Parallel

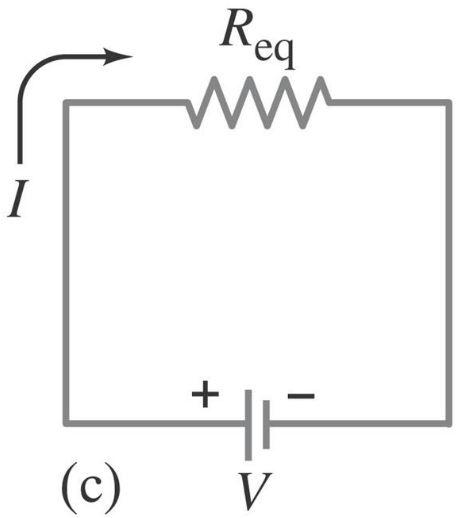


- From Ohm's law, $V = IR$, we can write $V_1 = IR_1$, $V_2 = IR_2$, and $V_3 = IR_3$.
- Because the resistors are connected end-to-end, energy conservation tells us that the total voltage, V, is equal to the sum of the voltages across each resistor:

$$V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3$$

Resistors in Series and Parallel



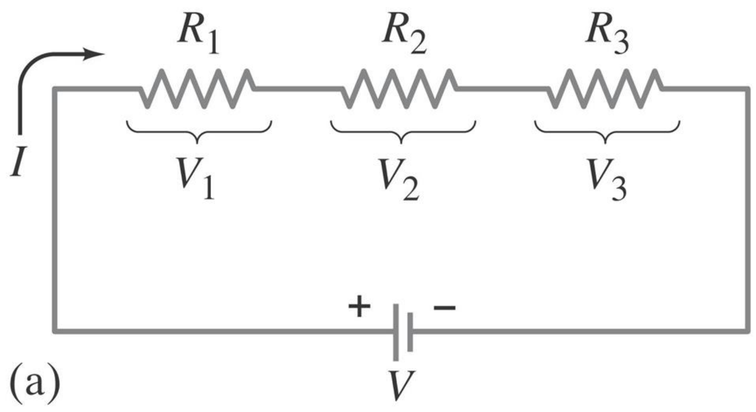


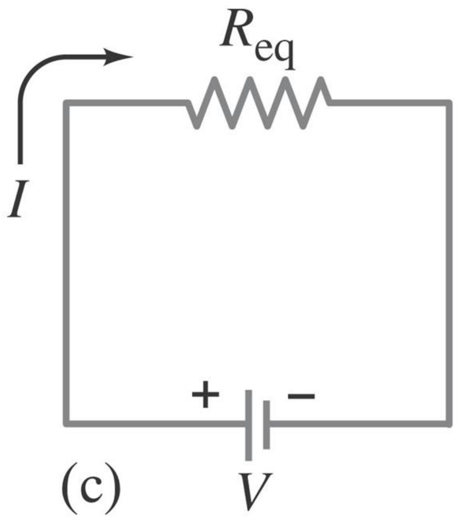
$$V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3$$

- Now let's determine the equivalent single resistance, R_{eq}, that would draw the same current, I, as our combination of three resistors in series, as shown in (c) above.
- Such a single resistance, R_{eq}, would be related to V by

$$V = IR_{eq}$$

Resistors in Series and Parallel





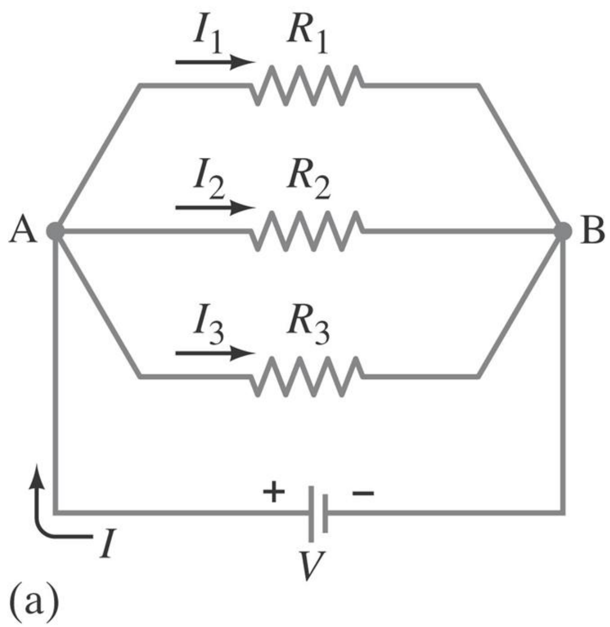
$$V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3 = I(R_1 + R_2 + R_3)$$

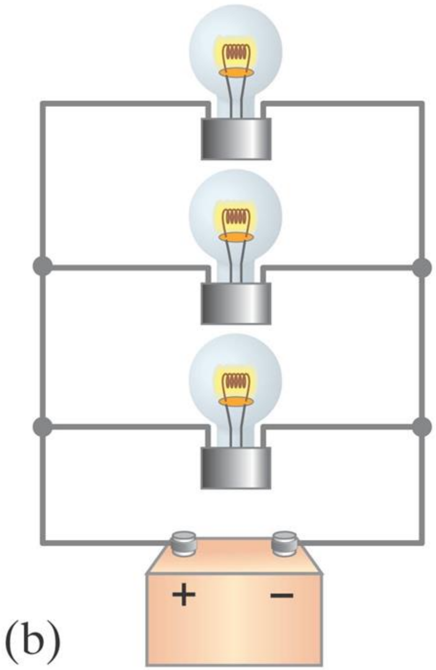
$$V = IR_{eq}$$

- We can equate the above two expressions, and see that

$$R_{eq} = R_1 + R_2 + R_3$$

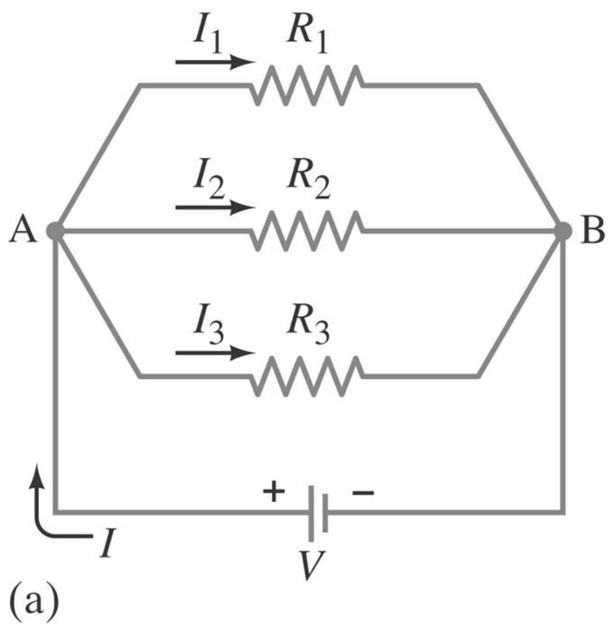
Resistors in Series and Parallel

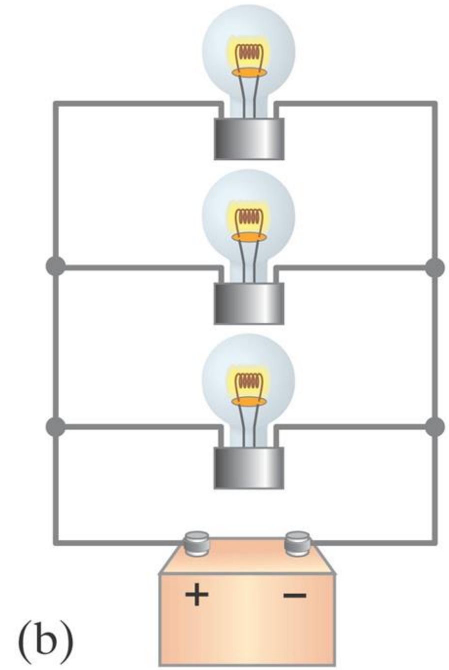




- Another simple way to connect resistors is in parallel so that the current from the source splits into separate branches or paths, as shown above.
- The wiring in houses and buildings is arranged so all electric devices are in parallel.

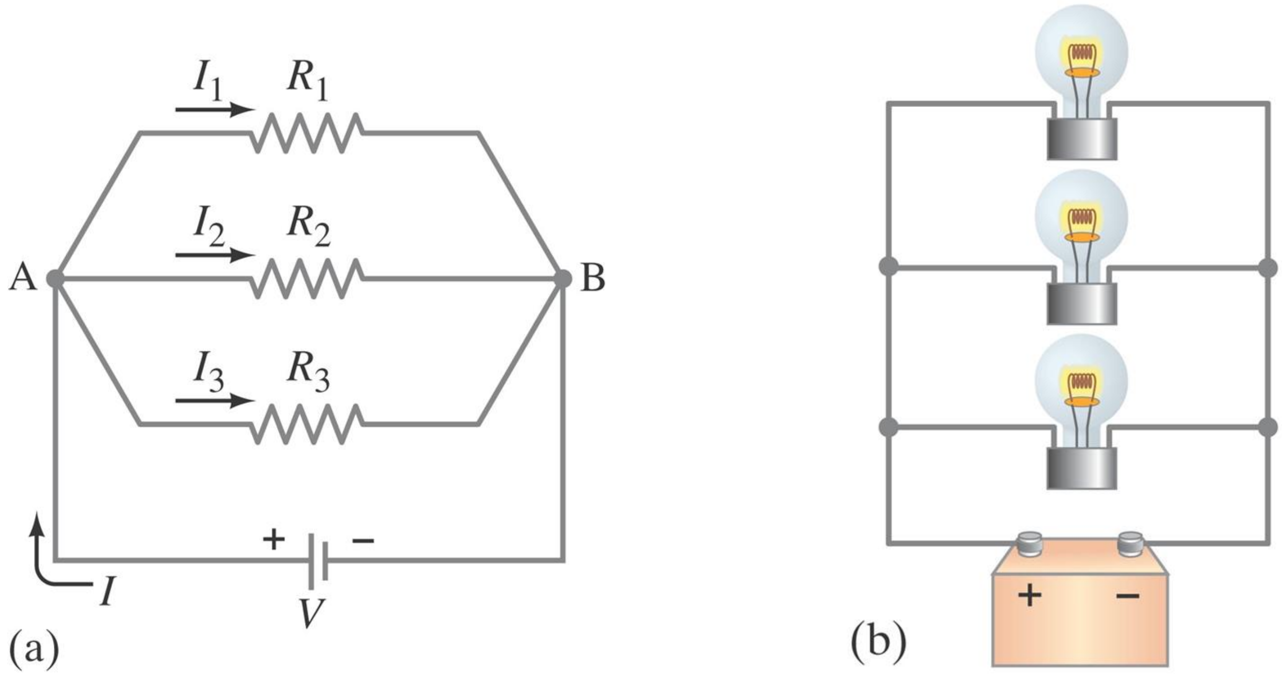
Resistors in Series and Parallel





- In a parallel circuit, as shown in (a) above, the total current that leaves the battery splits into three separate paths.
- We let I_1, I_2, and I_3 be the currents through each of the resistors, R_1, R_2, and R_3, respectively.
- Because *electric charge is conserved*, the current, I, flowing into junction A (where the different wires or conductors meet) must equal the current flowing out of the junction.

Resistors in Series and Parallel

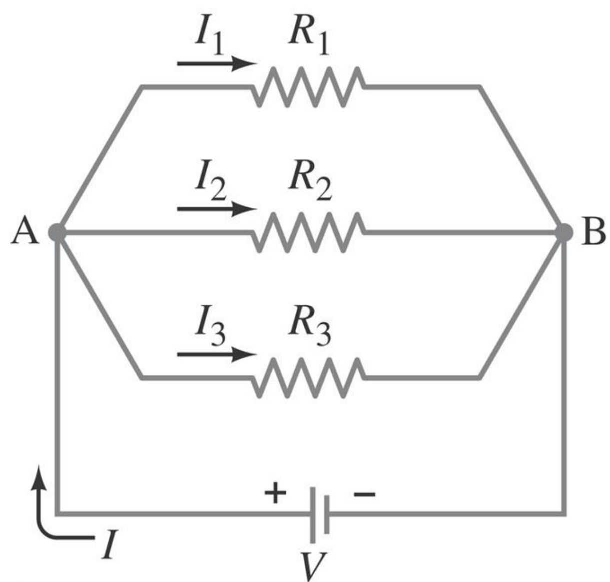


- Thus,

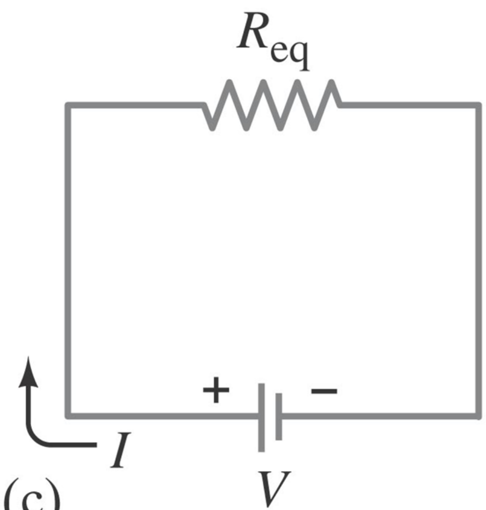
$$I = I_1 + I_2 + I_3$$

- When resistors are connected in parallel, each has the same voltage across it. Hence, the full voltage of the battery is applied to each resistor.

Resistors in Series and Parallel



(a)



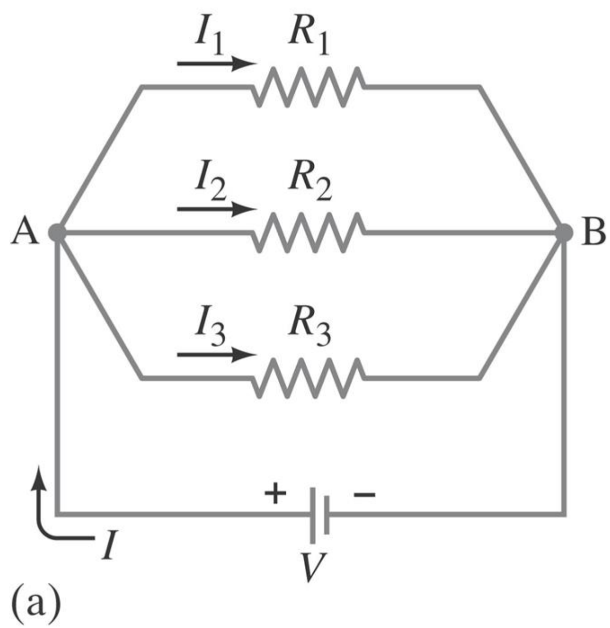
(c)

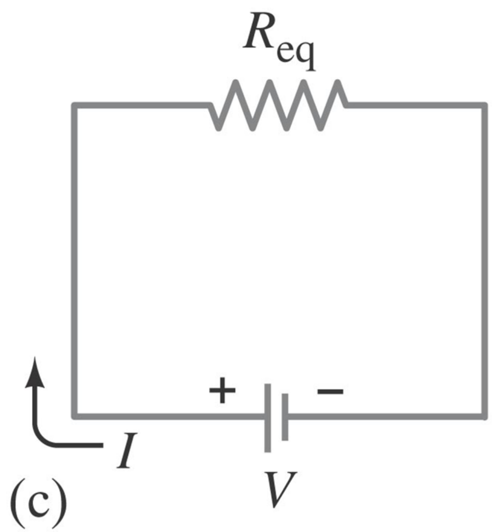
- Applying Ohm’s law to each resistor, we have

$$I_1 = \frac{V}{R_1}, \qquad I_2 = \frac{V}{R_2}, \qquad \text{and} \qquad I_3 = \frac{V}{R_3},$$

- Let us now determine what single resistor R_{eq} (as shown in (c)) will draw the same current I as these three resistances in parallel.

Resistors in Series and Parallel

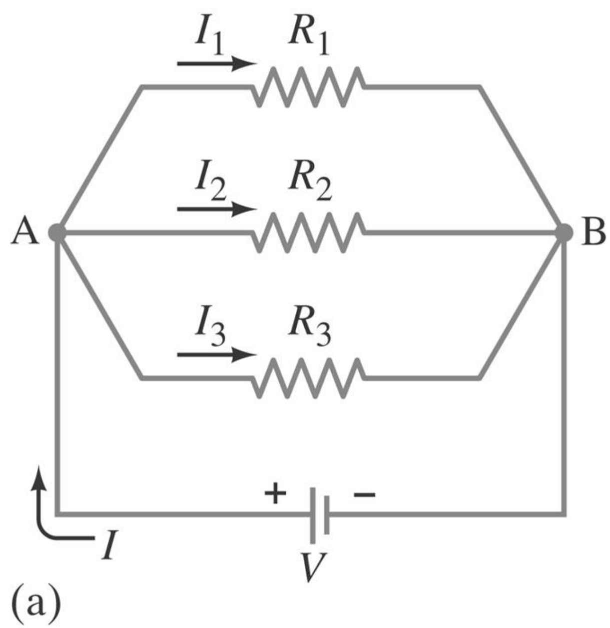


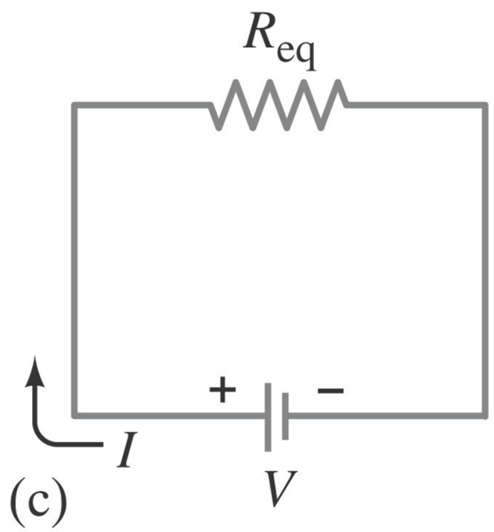


- This equivalent resistance R_{eq} must satisfy Ohm's law too:

$$I = \frac{V}{R_{eq}},$$

Resistors in Series and Parallel





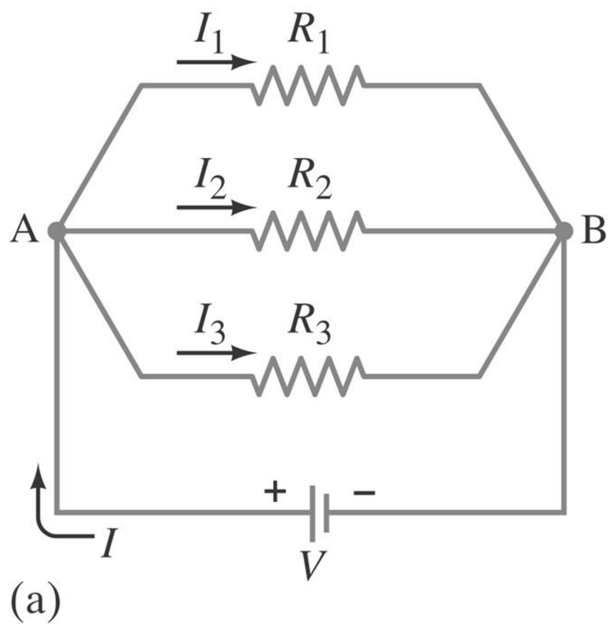
- We can write the following equation

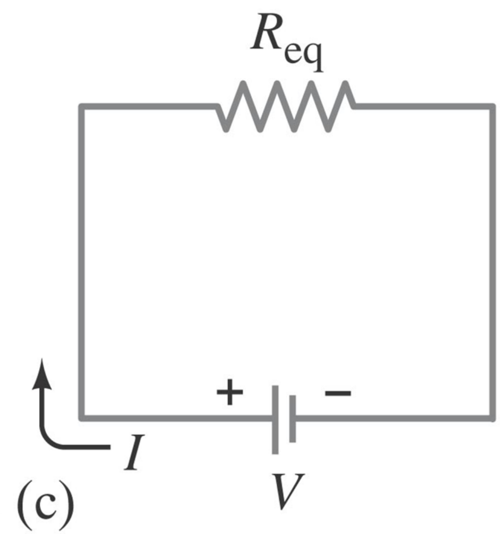
$$I = I_1 + I_2 + I_3$$

as

$$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

Resistors in Series and Parallel



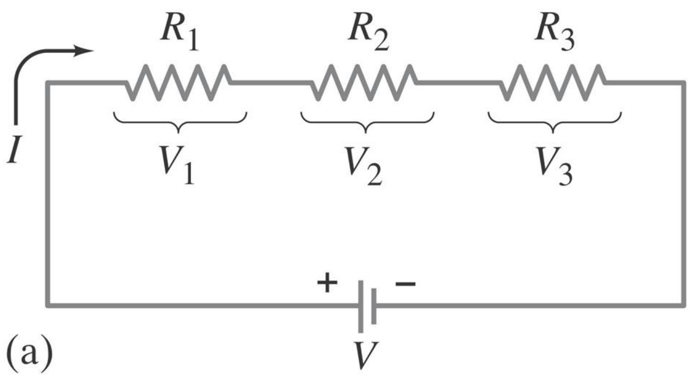


- Dividing out the V from each term, we have

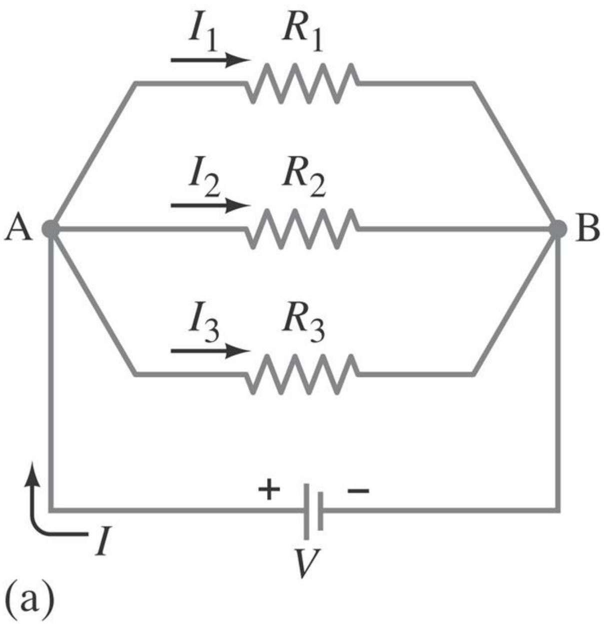
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

N.B. The net (or equivalent) resistance is less than each single resistance. This is because we are giving the current additional paths to follow, so the net resistance will be less.

Resistors in Series and Parallel



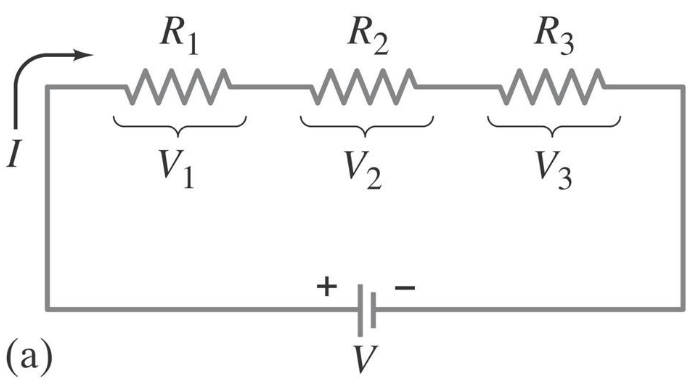
$$R_{eq} = R_1 + R_2 + R_3$$



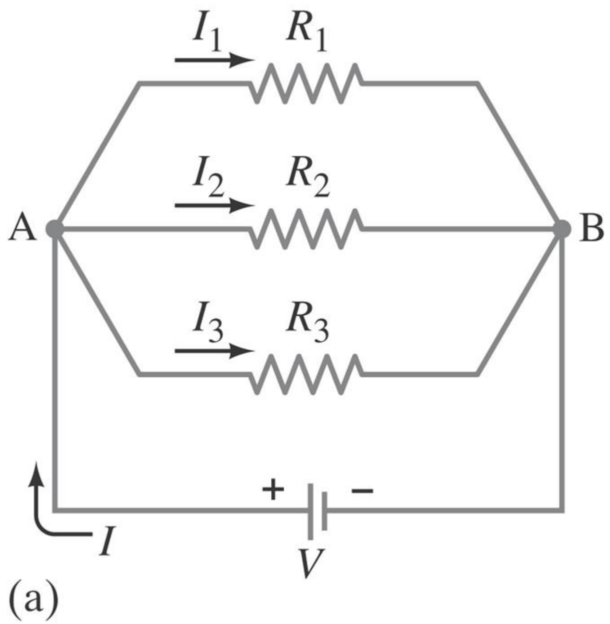
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

- We can also use the equation for resistivity, $R = \rho l/A$, to interpret the above two equations.
- We can see that placing resistors in series increases the length and therefore the resistance.

Resistors in Series and Parallel



$$R_{eq} = R_1 + R_2 + R_3$$



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

- We can also use the equation for resistivity, $R = \rho l / A$, to interpret the above two equations.
- Putting resistors in parallel increases the area through which current flows, thus reducing the overall resistance.

Resistors in Series and Parallel

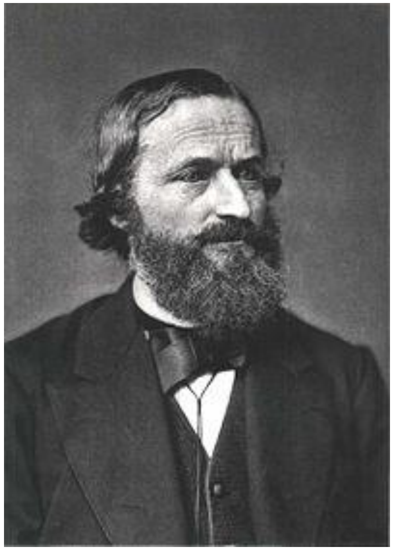
- Consider two identical pipes taking in water near the top of a dam and releasing it below as shown above.
- The gravitational potential difference, proportional to the height h, is the same for both pipes, just as the voltage is the same for parallel resistors.

Resistors in Series and Parallel

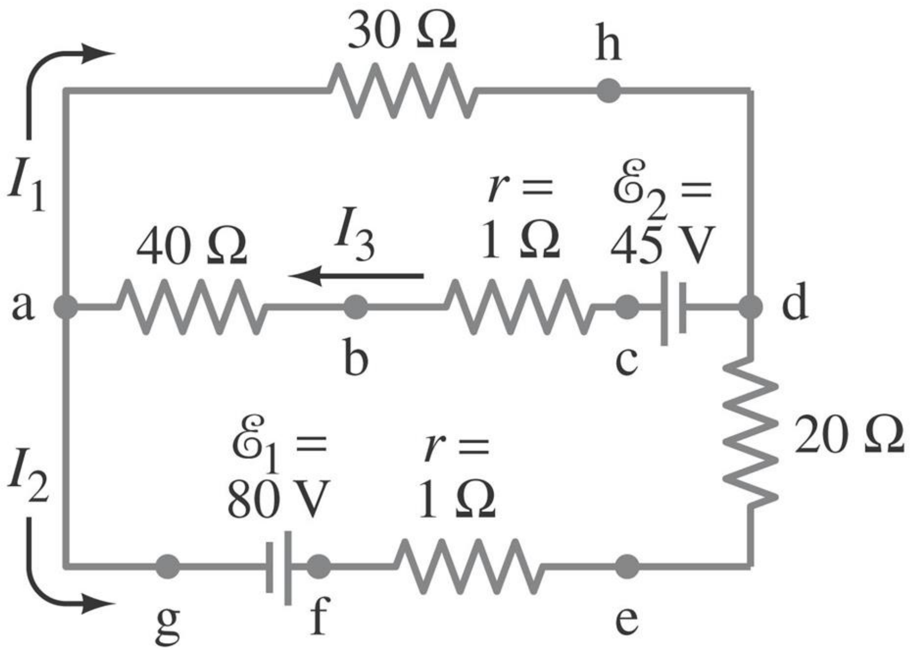
- If both pipes are open, rather than only one, twice as much water will flow through. That is, with two equal pipes open, the net resistance to the flow of water will be reduced, by half, just as for electrical resistors in parallel.

3. Kirchhoff’s Rules

Kirchhoff's Rules

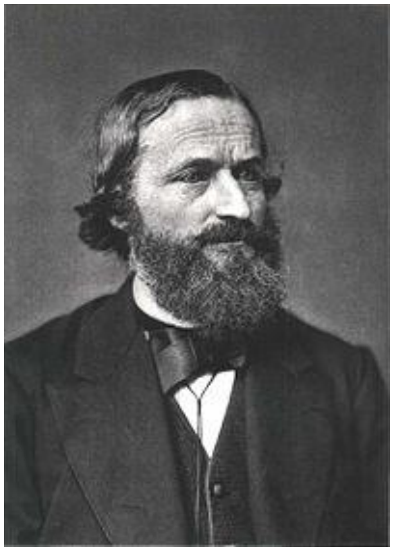


1824-1887



- In the last few examples, we have been able to find the currents in circuits by combining resistances in series and parallel, and using Ohm's law. This technique can be used for many circuits.
- However, some circuits are too complicated for this technique. For example, we cannot find the currents in each part of the circuit shown above simply by combining resistances as we did before.

Kirchhoff's Rules



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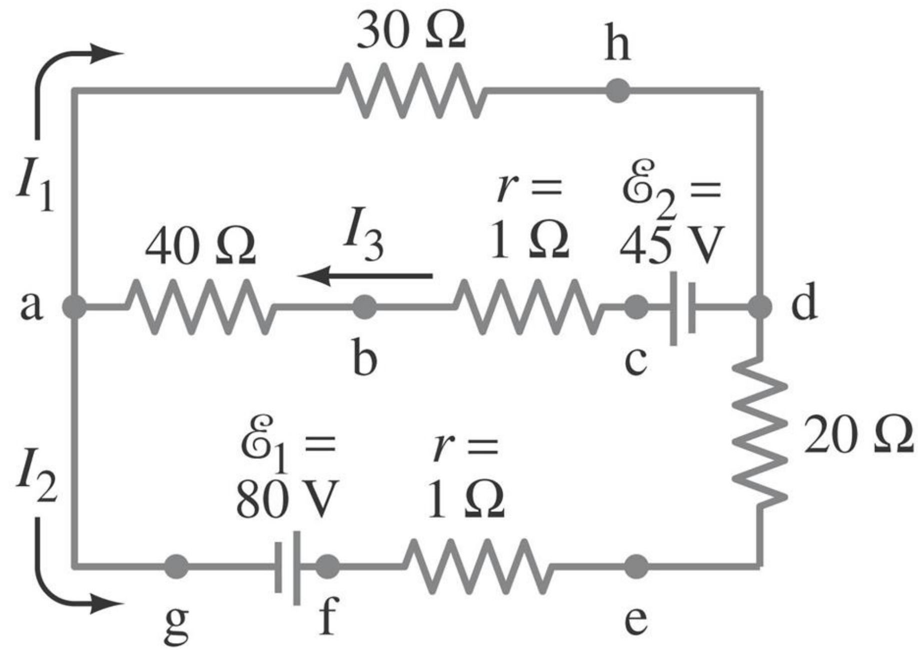
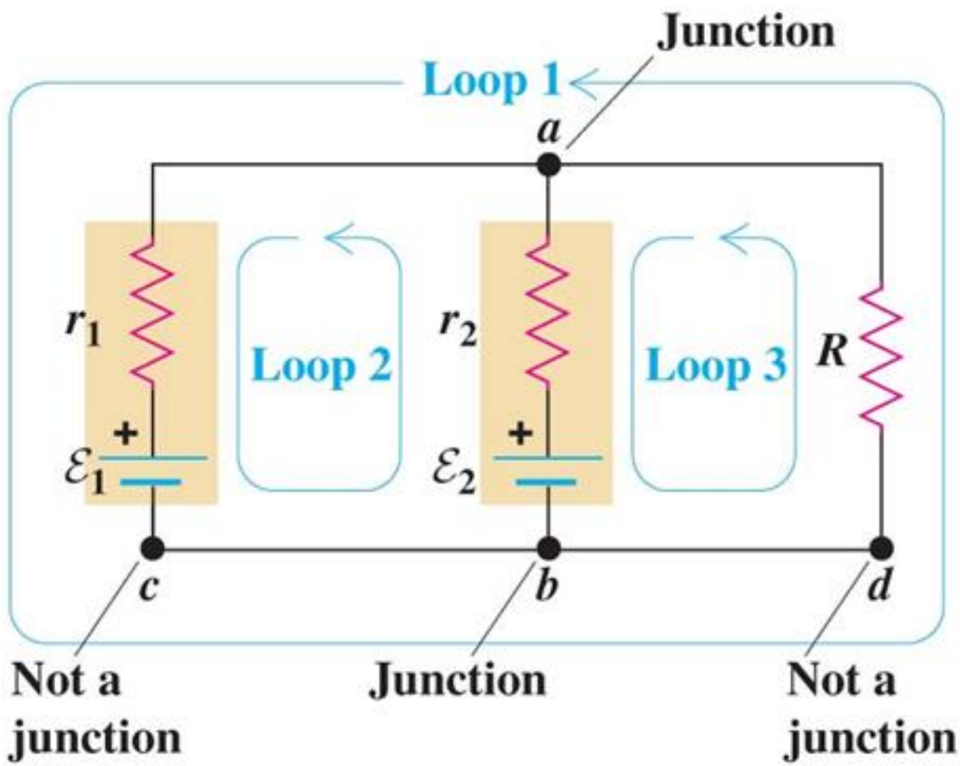


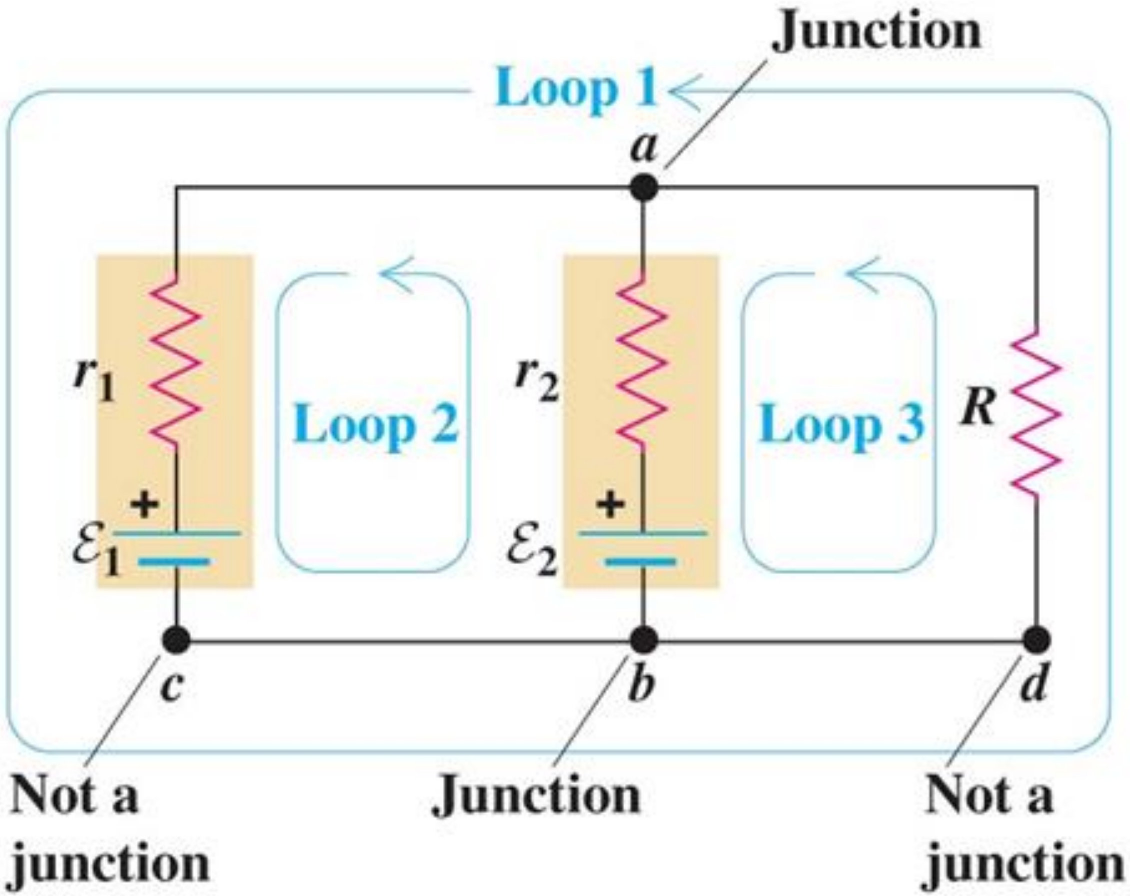
- To deal with such complicated circuits, we use Kirchoff's rules, devised by G.R. Kirchhoff in the mid-nineteenth century.
- There are two rules, and they are simply convenient applications of the laws of conservation of charge and energy. Before we look at each rule though, we need to define two terms 'junction' and 'loop'.

Kirchhoff's Rules – a Junction



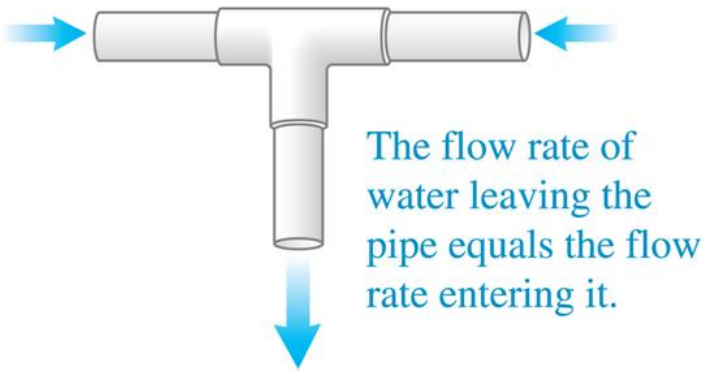
- A junction in a circuit is a point where three or more conductors meet.
- In the circuit above, $\boldsymbol{a}$ and $\boldsymbol{b}$ are junctions, while $\boldsymbol{c}$ and $\boldsymbol{d}$ are not.

Kirchhoff's Rules – a Loop

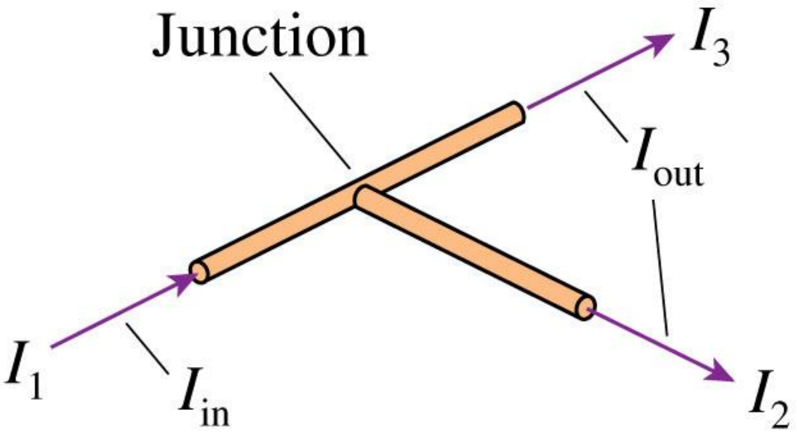


- A loop is any closed conducting path.

Kirchhoff's 1st Rule



Water-pipe analogy for Kirchhoff's junction rule

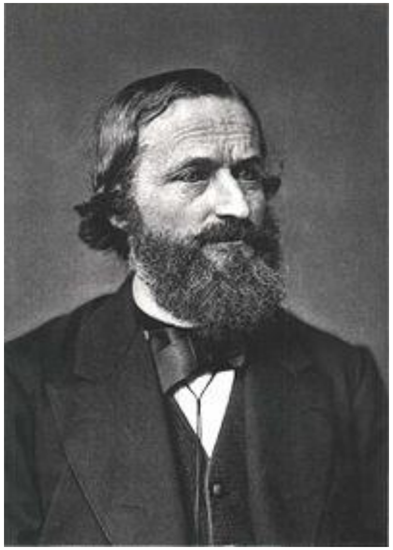


Junction law: $I_1 = I_2 + I_3$

- **Kirchhoff's first rule** or **junction rule** is based on the conservation of electric charge that we already used to derive the rule for parallel resistors. It states that

at any junction point, the sum of all currents entering the junction must equal the sum of all currents leaving the junction.

Kirchhoff's 2nd Rule



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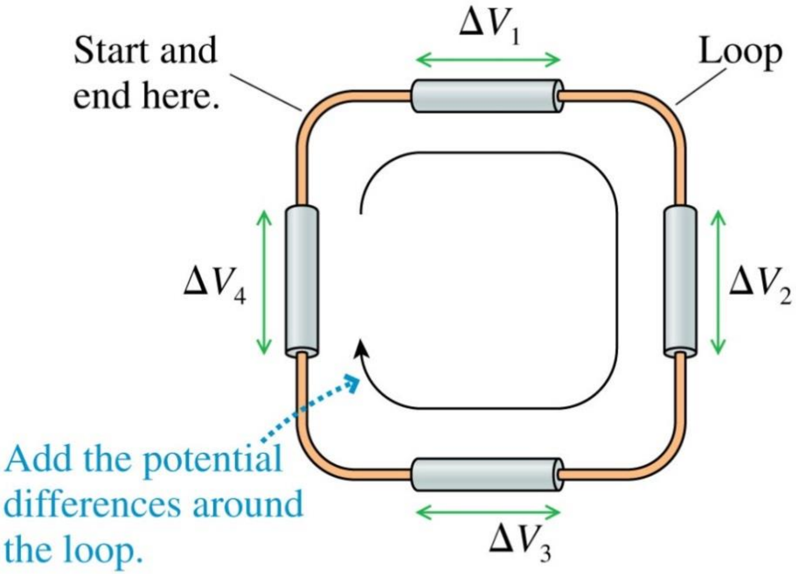
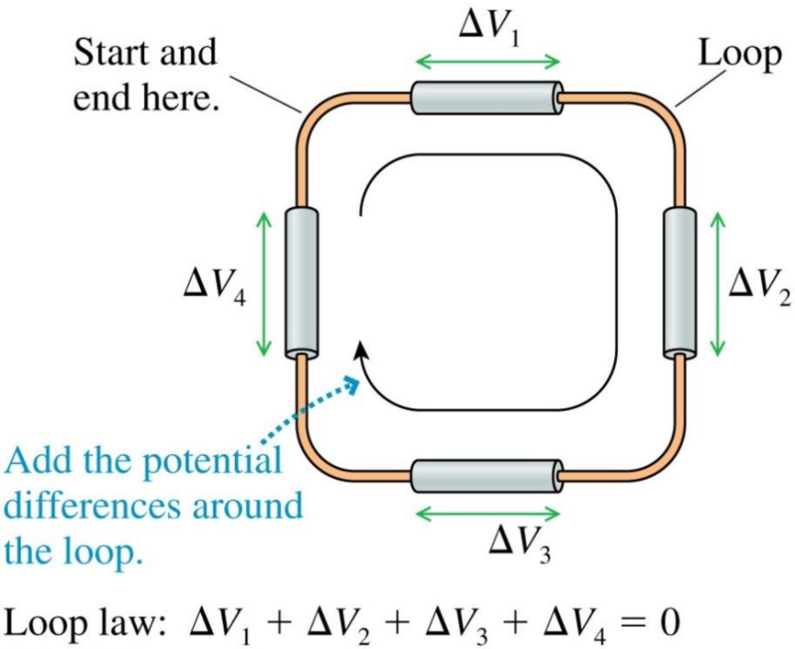


Loop law: $\Delta V_1 + \Delta V_2 + \Delta V_3 + \Delta V_4 = 0$

- **Kirchhoff's second rule** or **loop rule** is based on the conservation of energy. It states that

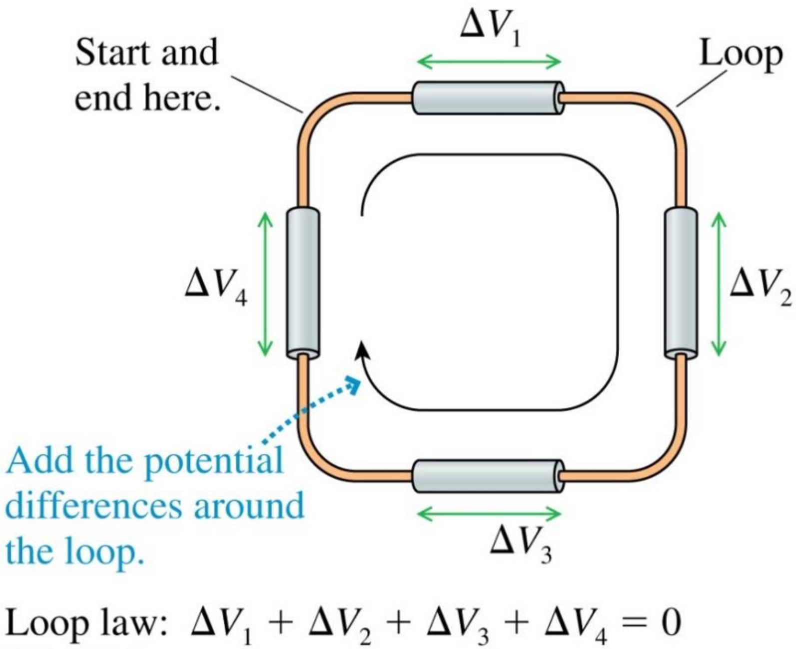
  the sum of the changes in potential around any closed loop of a circuit must be zero.

Kirchhoff's 2nd Rule



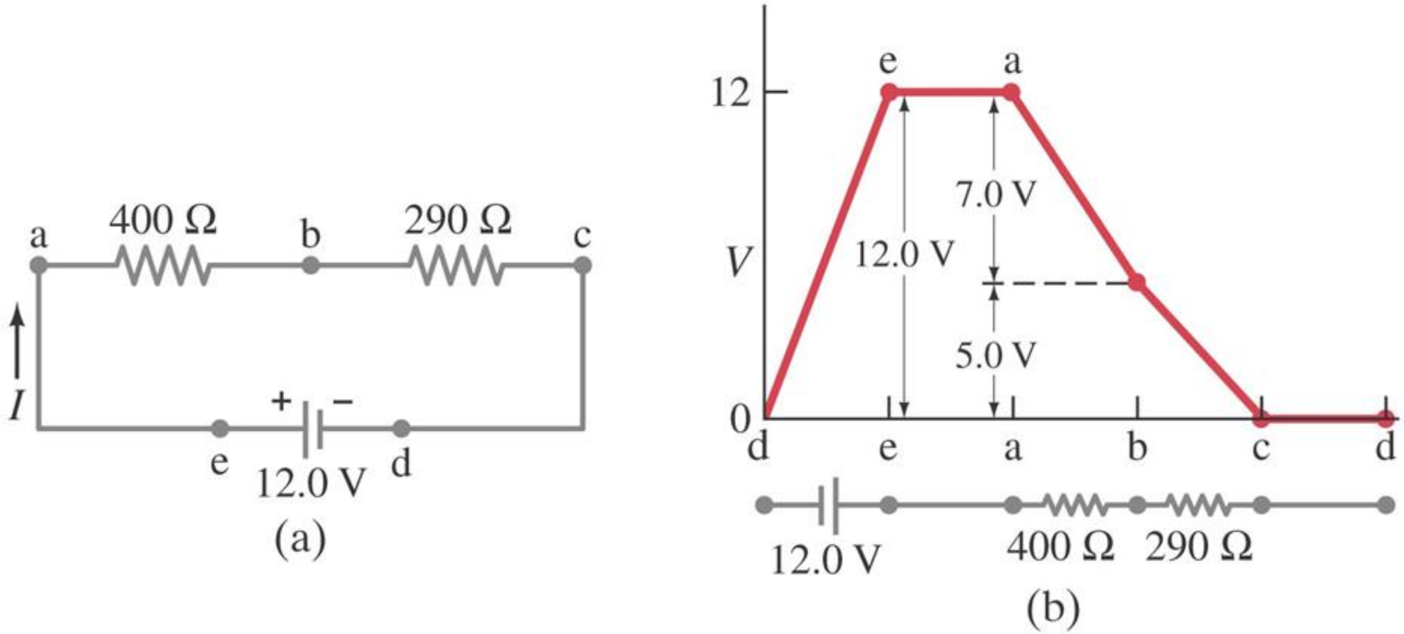
- To see why this rule should hold, consider a rough analogy with the potential energy of a roller coaster on its track.
- When the roller coaster starts from the station, it has a particular potential energy. As it climbs the first hill, its potential energy increases and reaches a maximum at the top.

Kirchhoff's 2nd Rule



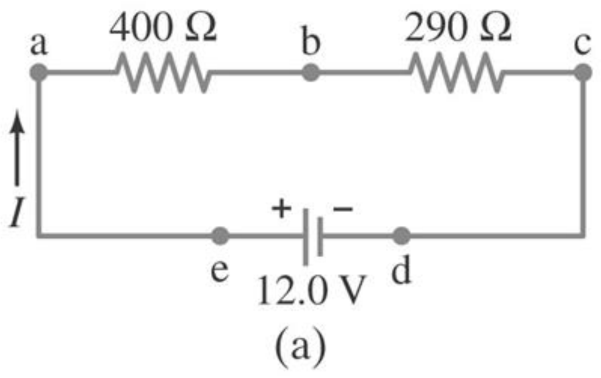
- As it descends the other side, its potential energy decreases, and reaches a local minimum at the bottom of the hill.
- As the roller coaster continues on its path, its potential energy goes through more changes. But when it arrives back at the starting point, it has exactly as much potential energy as it had when it started at this point.

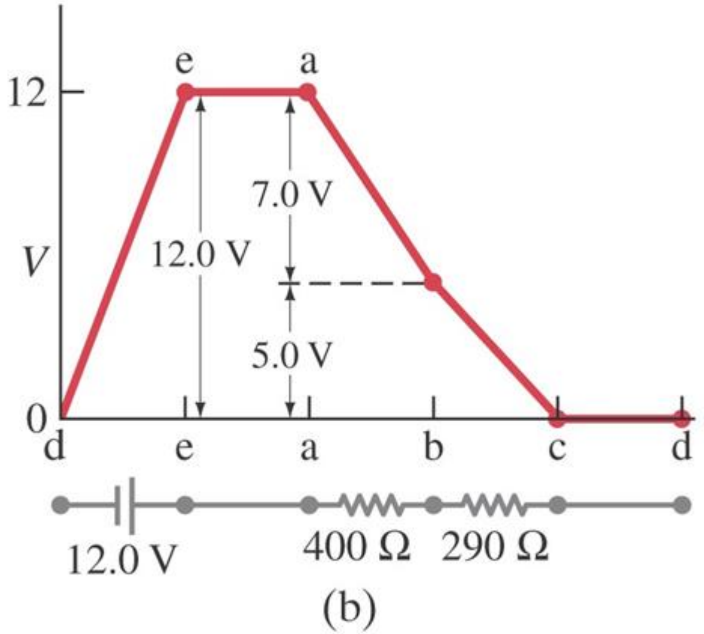
Kirchhoff's 2nd Rule



- We can apply similar reasoning to an electric circuit.
- The current in the above circuit is $I = \frac{12.0\text{V}}{690\ \Omega} = 0.0174\text{A}.$
- The positive side of the battery, point e, is at a high potential compared to point d at the negative side of the battery.
- That is, point e is like the top of a hill for a roller coaster.
- We can follow the current around the circuit starting at any point.

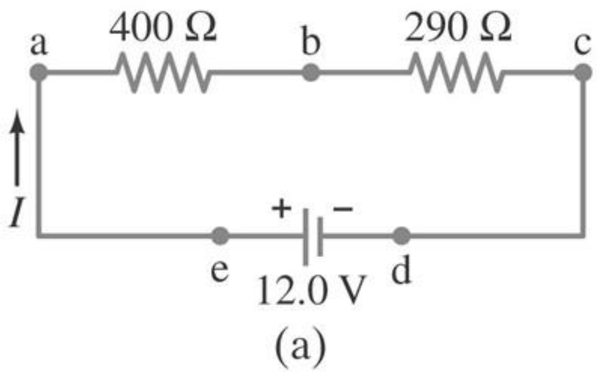
Kirchhoff's 2nd Rule

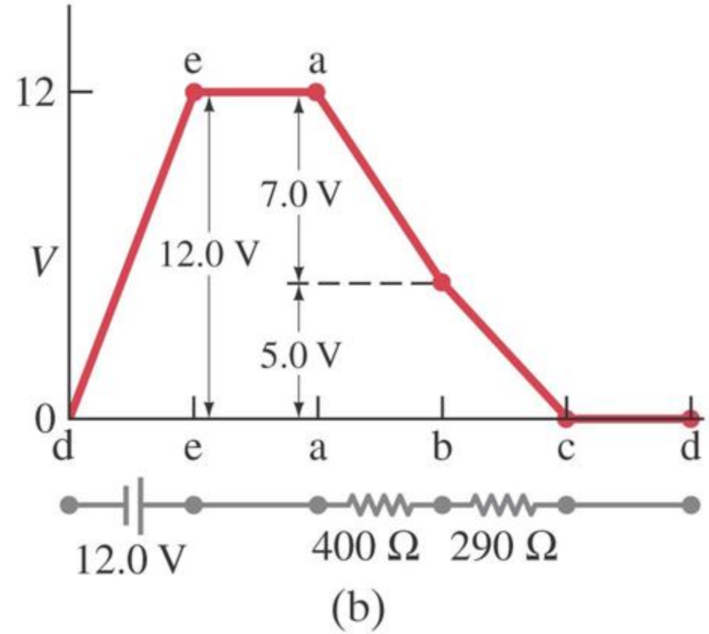




- In our case, we choose to start at point d, and follow a positive test charge completely around the circuit.
- The above graph shows all the changes in potential. When the test charge returns to point d, the potential will be the same as when we started (the total change in potential around the circuit is zero).

Kirchhoff's 2nd Rule





- The sum of all the changes in potential around the circuit is

$$+12.0\ \text{V} - 7.0\ \text{V} - 5.0\ \text{V} = 0$$

For You to Read: Problem Solving with Kirchhoff's Rules

1) **Label the current** in each separate branch of the given circuit with a different subscript, such as I_1, I_2, I_3. Each current refers to a segment between two junctions. Choose the direction of each current, using an arrow. The direction can be chosen randomly: if the current is actually in the opposite direction, it will come out with a minus sign in the solution.

2) **Identify the unknowns.** You will need as many independent equations as there are unknowns. You may write down more equations than this, but you will find that some of the equations will be redundant (that is, not be independent in the sense of providing new information). You may use $V = IR$ for each resistor, which sometimes will reduce the number of unknowns.

For You to Read: Problem Solving with Kirchhoff's Rules

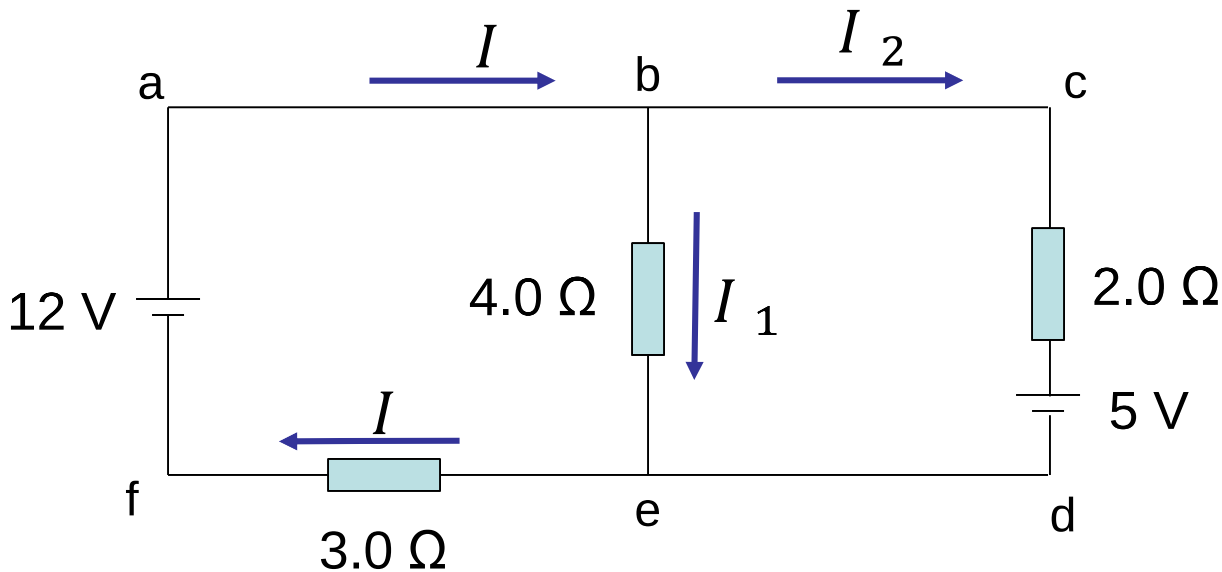
3) **Apply Kirchhoff's junction rule** at one or more junctions.
4) **Apply Kirchhoff's loop rule** for one or more loops: follow each loop in one direction only. Pay careful attention to subscripts, and to signs:

(a) For a resistor, apply Ohm's law; the potential difference is negative (a decrease) if your chosen loop direction is the same as the chosen current direction through that resistor; the potential difference is positive (an increase) if your chosen loop direction is opposite to the chosen current direction.

(b) For a battery, the potential difference is positive if your chosen loop direction is from the negative terminal toward the positive terminal; the potential difference is negative if the loop direction is from the positive terminal toward the negative terminal.

5) **Solve the equations** algebraically for the unknowns.

Example of Using Kirchhoff's Rules



Find the current in each branch:

(i) Apply the junction rule to point *b*:

$$I = I_1 + I_2$$

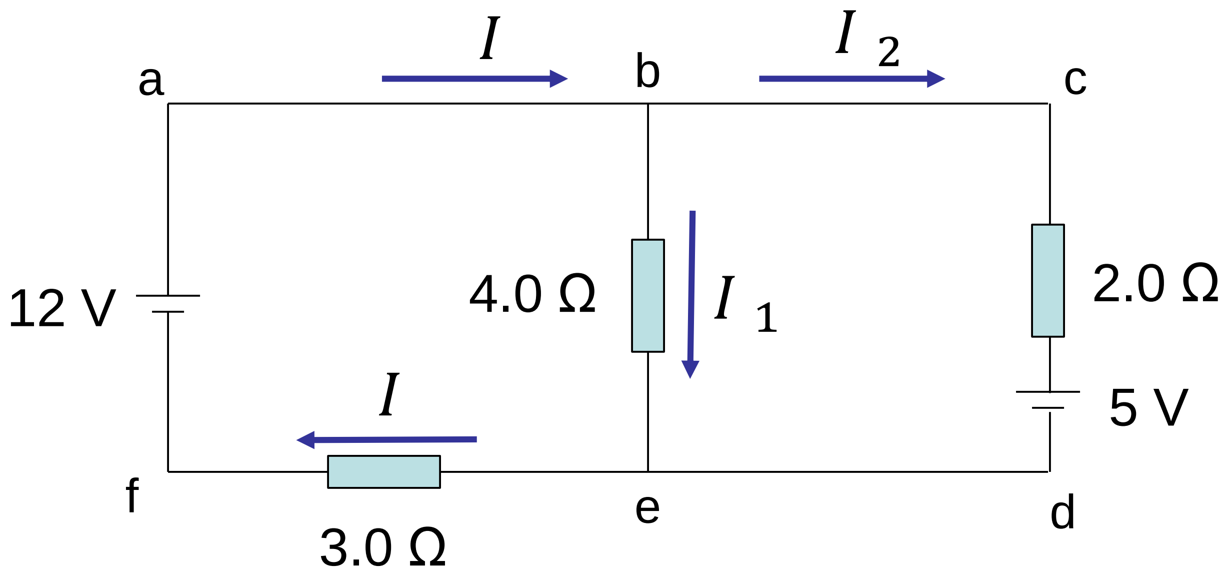
(ii) Apply the loop rule to outer loop $abcdefa$:

$$-(2.0\Omega)I_2 - 5.0\ V\ - (3.0\Omega)(I_1 + I_2)\ +12\ V = 0$$

(iii) Divide (ii) by 1Ω, (as 1 V/1 Ω = 1 A):

$$7.0\ A\ - 3.0I_1\ -\ 5.0I_2 = 0$$

Example of Using Kirchhoff's Rules



Find the current in each branch:

(iv) Apply the loop rule on the right, $bcdeb$:

$$-(2.0\Omega)I_2 - 5.0\ V + (4.0\Omega)I_1 = 0$$

(v) Divide by 1Ω, (1 V/1 Ω) = 1 A:

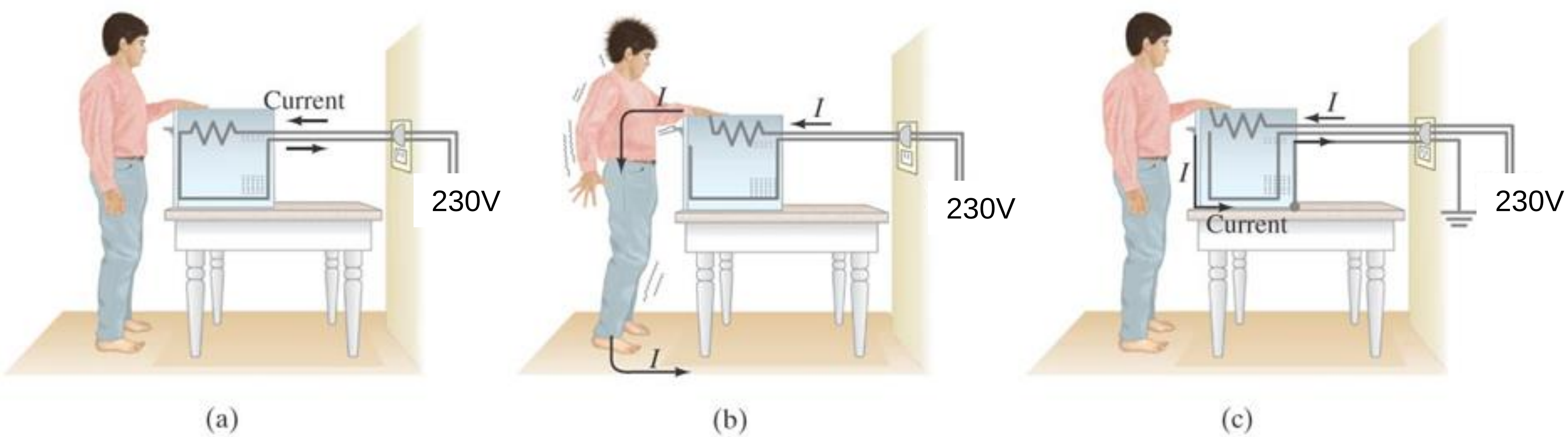
$$-7.0\ A\ - 3.0I_1\ -\ 5.0I_2 = 0$$

(vi) We now have 2 equations with two unknowns.

$$I_1 = 1.5\ A \quad I_2 = 0.5\ A \Rightarrow I = 2.0\ A$$

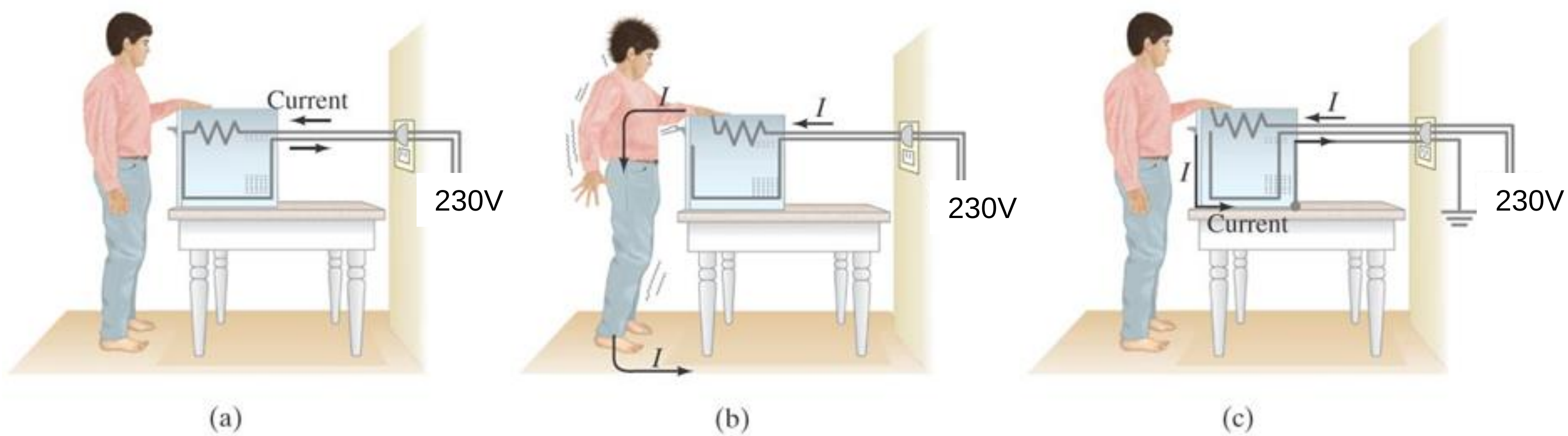
4. Electric Hazards

Electric Hazards



- You can come into contact with a hot wire (a wire at a high potential) by touching a bare wire whose insulation has worn off, or from a bare wire inside an appliance when you’re tinkering with it.
- **ALWAYS UNPLUG AN ELECTRICAL DEVICE BEFORE INVESTIGATING ITS INSIDES!!**
- Another possibility is that a wire inside a device may break or lose its insulation, and come in contact with the case. If the case is metal, it will conduct electricity.

Electric Hazards



- A person could then suffer a severe shock simply by touching the case (as shown in (b) above).
- To prevent an accident, metal cases are supposed to be connected directly to ground by a separate wire.
- Then if a ‘hot’ wire touches the grounded case, a short circuit to ground immediately occurs internally, as shown in (c) above, and most of the current passes through the low resistance ground wire rather than through the person.

Electric Hazards

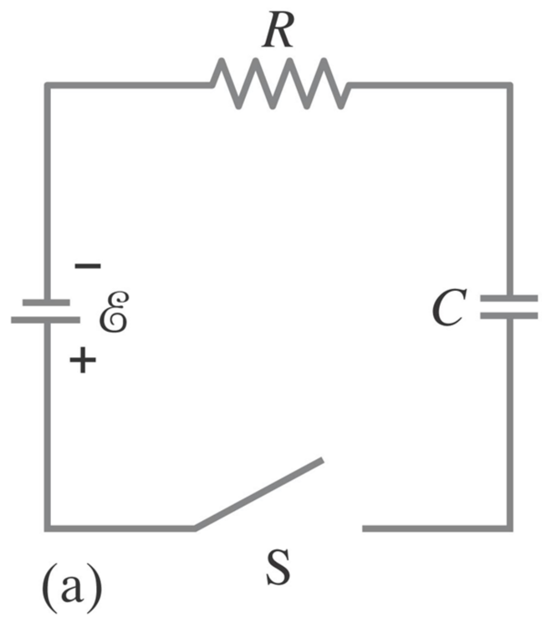
- Furthermore, the high current should open the fuse or **circuit breaker**.
- Grounding a metal case is done by a separate ground wire connected to the third (round) prong of a 3-prong plug.

5. RC Circuits

RC Circuits

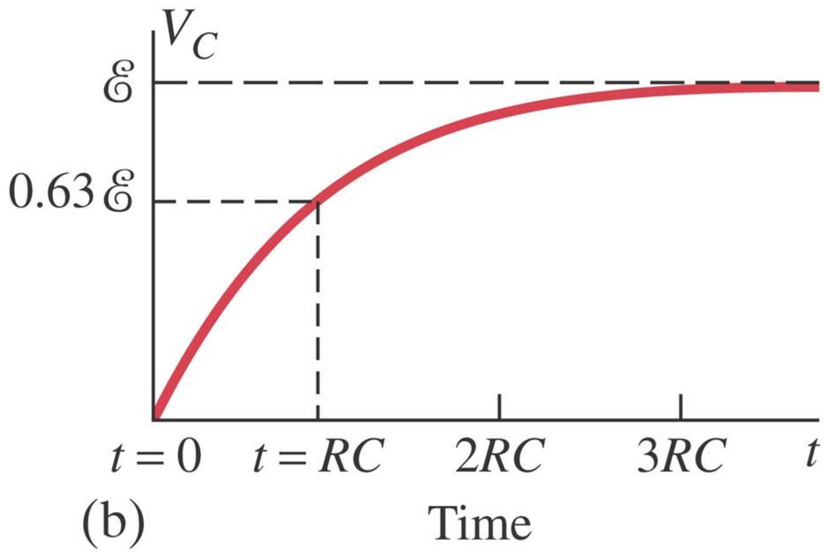
- Our study of circuits so far has dealt with steady currents that do not change in time. We now examine circuits that contain both resistance and capacitance (*RC* circuits).
- RC circuits are common in everyday life: they are used to control the speed of a car's windshield wiper, and the timing of the change of traffic lights. They are also used in camera flashes, in heart pacemakers, and in many other electronic devices.
- In *RC* circuits, we are not so interested in the final 'steady state' voltage and the charge on the capacitor, but rather in how these variables change in time.

RC Circuits



- When the switch is closed, charge accumulates on the capacitor, the potential difference across it increases ($V_c = \frac{Q}{C}$), and the current is reduced until eventually the voltage across the capacitor equals the $emf\,(\varepsilon)$ of the battery.
- There is then no potential difference across the resistor, and no further current flows.

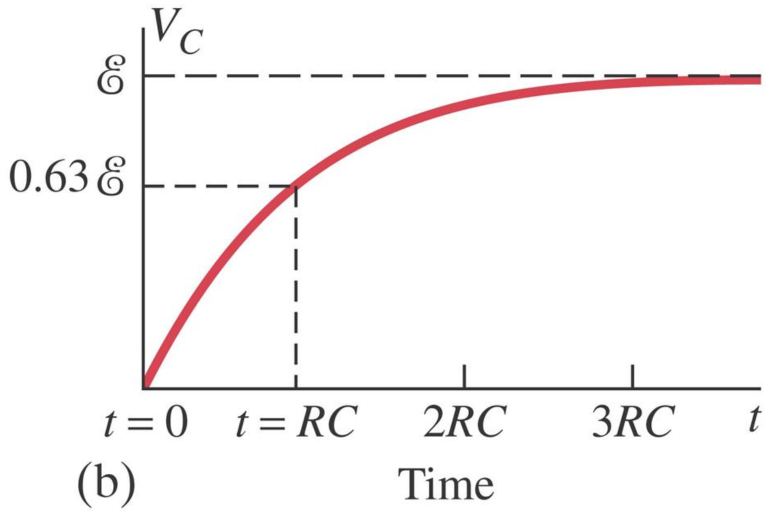
RC Circuits



- The potential difference V_c across the capacitor thus increases in time as shown above.
- The mathematical form of this curve, that is, V_c as a function of time, can be derived using conservation of energy (or Kirchhoff's loop rule).
- The $emf\,(\varepsilon)$ of the battery will equal the sum of the voltage drops across the resistor (IR) and the capacitor (Q/C):

$$\varepsilon = IR + \frac{Q}{C}$$

RC Circuits

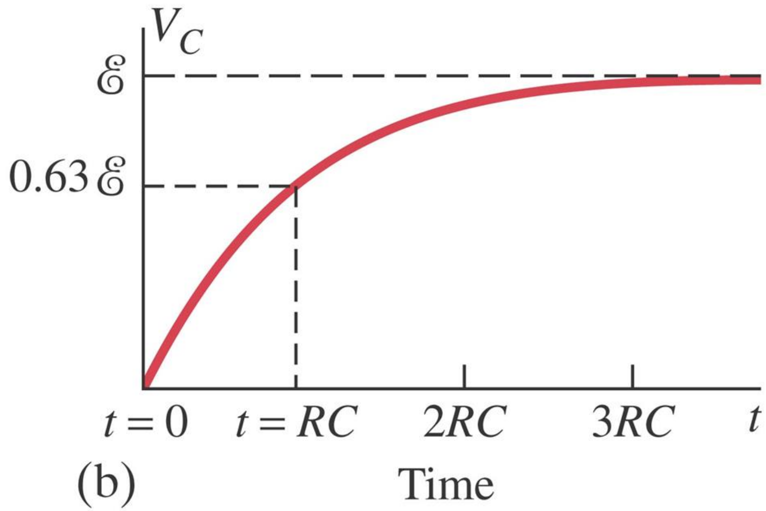


$$\varepsilon = IR + \frac{Q}{C}$$

- The resistance R includes all resistance in the circuit, including the internal resistance of the battery; I is the current in the circuit at any instant, and Q is the charge on the capacitor at that same instant.
- The rate at which charge flows through the resistor ($I = dQ/dt$) is equal to the rate at which charge accumulates on the capacitor. Thus, we can write

$$\varepsilon = R\frac{dQ}{dt} + \frac{1}{C}Q$$

RC Circuits



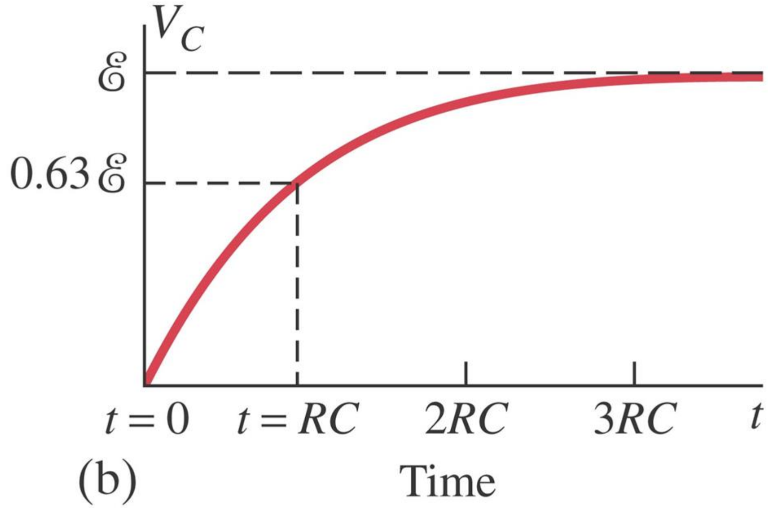
$$\varepsilon = R\frac{dQ}{dt} + \frac{1}{C}Q$$

- The equation can be solved (that, is finding a function for Q as a function of time) by rearranging and integrating:

$$\frac{dQ}{C\varepsilon - Q} = \frac{dt}{RC}$$

- We can now integrate from $t = 0$, when $Q = 0$, to time t when a charge Q is on the capacitor.

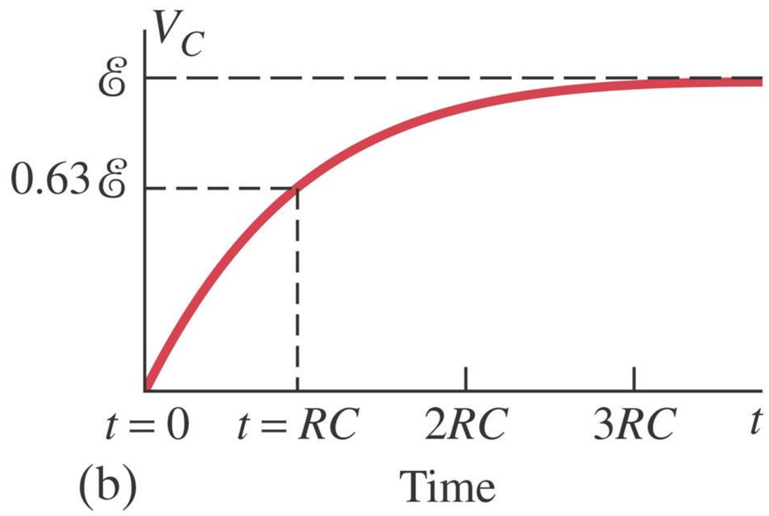
RC Circuits



$$\int_0^Q \frac{dQ}{C\varepsilon - Q} = \frac{1}{RC}\int_0^t dt$$

$$-\ln(C\varepsilon - Q)\,\Big|_0^Q = \frac{t}{RC}\Big|_0^t$$

RC Circuits



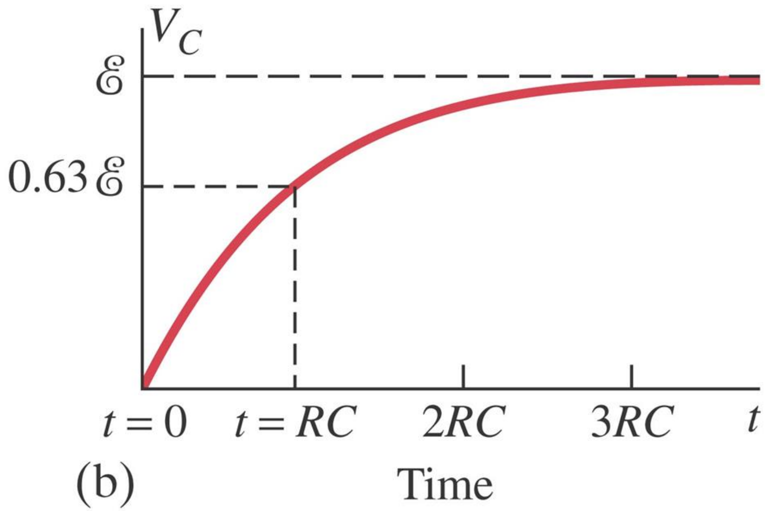
$$ln\left(1 - \frac{Q}{C\varepsilon}\right) = -\frac{t}{RC}$$

- We can take the exponential of both sides

$$1 - \frac{Q}{C\varepsilon} = e^{-t/RC}$$

$$Q = C\varepsilon(1 - e^{-\frac{t}{RC}})$$

RC Circuits



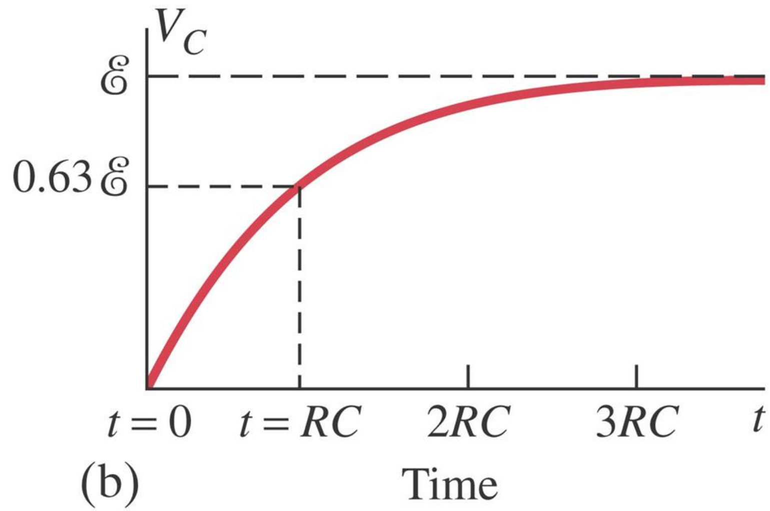
$$Q = C\varepsilon(1 - e^{-\frac{t}{RC}})$$

- The potential difference across the capacitor is $V_c = \frac{Q}{C}$, so

$$V_C = \varepsilon(1 - e^{-\frac{t}{RC}})$$

- From the above equations, we see that the charge, Q, on the capacitor, and the voltage V_c across it, increase from zero at $t = 0$ to maximum values $Q_{max} = C\varepsilon$ and $V_c = \varepsilon$ after a certain time.

RC Circuits



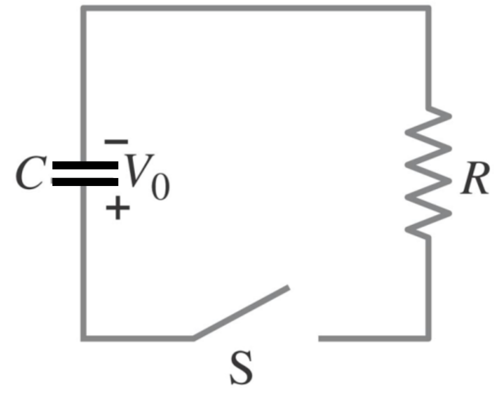
$$Q = C\varepsilon(1 - e^{-\frac{t}{RC}}) \qquad\qquad V_C = \varepsilon(1 - e^{-\frac{t}{RC}})$$

- The quantity RC that appears in the exponent is called the **time constant,** τ, of the circuit:

$$\tau = RC$$

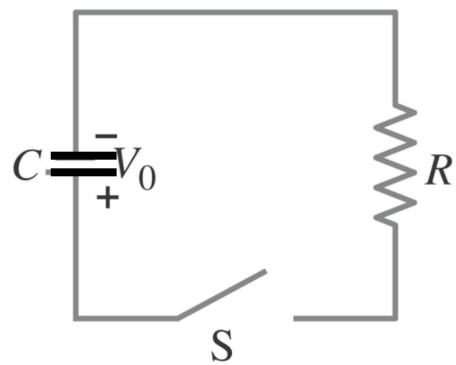
- It represents the time required for the capacitor to reach $(1 - e^{-1}) = 0.63$ or 63% of its full charge and voltage.
- Thus, the product RC is a measure of how quickly the capacitor gets charged.

RC Circuits



- The circuit we just looked at involved the charging of a capacitor by a battery through a resistance.
- Now let us look at another situation: when a capacitor is already charged (say to a voltage V_0), and it is then allowed to ***discharge*** through a resistance R as shown above.
- When the switch S is closed, charge begins to flow through resistor R from one side of the capacitor toward the other side, until the capacitor is fully discharged.

RC Circuits



(a)

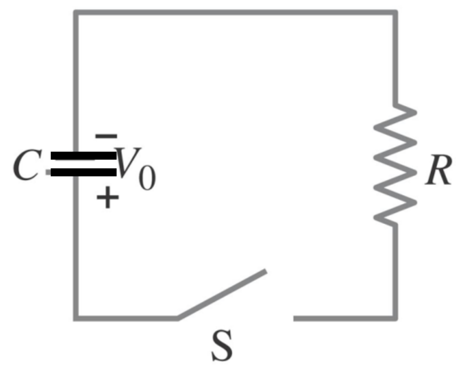
- The voltage across the resistor at any instant equals that across the capacitor:

$$IR = \frac{Q}{C}$$

- The rate at which charge leaves the capacitor equals the negative of the current in the resistor, $I = -dQ/dt$, because the capacitor is discharging (Q is decreasing). So we can write the above equation as

$$-\frac{dQ}{dt}R = \frac{Q}{C}$$

RC Circuits



(a)

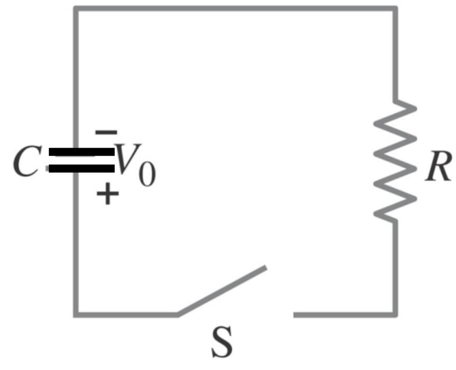
$$-\frac{dQ}{dt}R = \frac{Q}{C}$$

- Rearranging the above,

$$\frac{dQ}{Q} = -\frac{dt}{RC}$$

which can be integrated from $t = 0$ when the charge on the capacitor is Q_0, to some time t later when the charge is Q:

RC Circuits



(a)

$$\frac{dQ}{Q} = -\frac{dt}{RC}$$

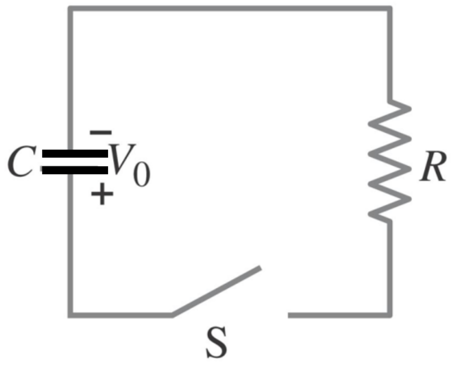
which can be integrated from $t = 0$ when the charge on the capacitor is Q_0, to some time t later when the charge is Q:

$$ln\frac{Q}{Q_0} = -\frac{t}{RC}$$

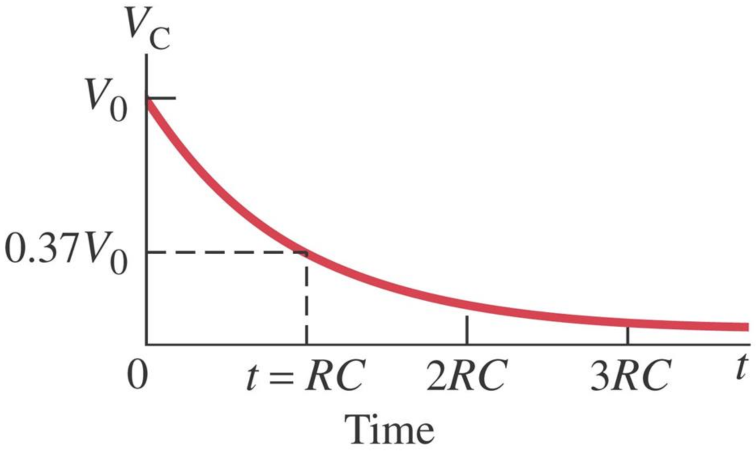
or

$$Q = Q_0 e^{-t/RC}$$

RC Circuits



(a)



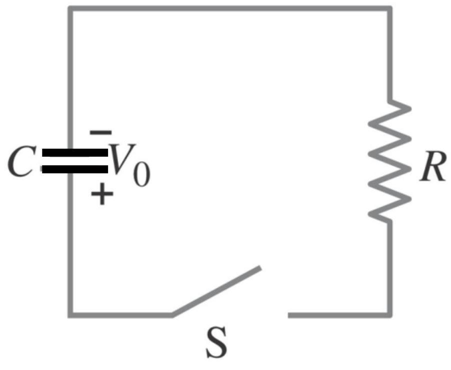
(b)

- The voltage across the capacitor $(V_c = Q/C)$ as a function of time is

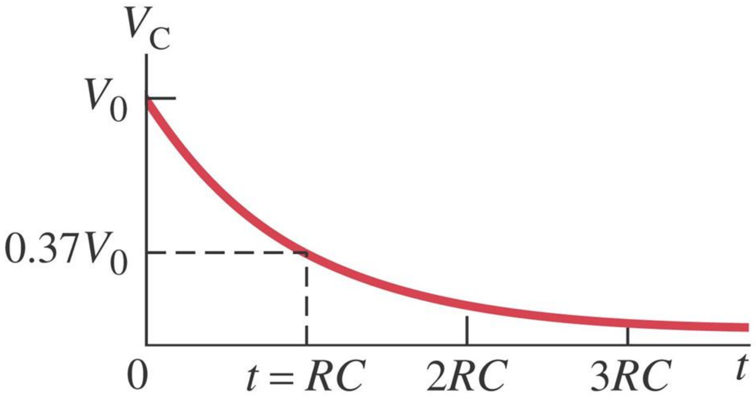
$$V_C = V_0 e^{-t/RC}$$

- Thus, the charge on the capacitor, and the voltage across it, decrease exponentially in time with a time constant RC.

RC Circuits



(a)



(b)

- The current is

$$I = -\frac{dQ}{dt} = \frac{Q_0}{RC} e^{-\frac{t}{RC}} = I_0 e^{-t/RC}$$

- It too is seen to decrease exponentially in time with a time constant RC.

Summary of today's Lecture

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1. EMF and Terminal Voltage
2. Resistors in Series and in Parallel
3. Kirchoff's Rules
4. Electric Hazards
5. RC Circuits

Lecture 22: Optional Reading

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- **Ch. 26.1**, EMF and Terminal Voltage; p.786-787.
- **Ch. 26.2**, Resistors in Series and in Parallel; p.787-791.
- **Ch. 26.3**, Kirchhoff's Rules; p.791-794.
- **Ch. 26.4**, Series and Parallel EMFs; Battery Charging; p.794-795.
- **Ch. 26.5**, RC Circuits; p.795-800.
- **Ch. 26.6**, Electric Hazards; p.800-802.

Home Work

Do not forget to attempt the **Additional Problems** for this lecture before logging in to **Mastering Physics** to complete your assignments.