University of Nottingham
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Foundation Algebra for Physical Sciences & Engineering CELEN036

Practice Problems SET-1 Sample Solution

Topic: Functions

Type 1: Composition of functions

1. Given $f(x) = (x+1)(x-2)$ and $g(x) = 2x$. Find $(f \circ g)(x)$ and $(g \circ f)(x)$.

Solution:

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f(2x) \\ &= (2x+1)(2x-2) \\ &= 4x^2 + 2x - 4x - 2 \\ &= 4x^2 - 2x - 2\end{aligned}$$

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= g((x+1)(x-2)) \\ &= 2(x+1)(x-2) \\ &= 2(x^2 - 2x + x - 2) \\ &= 2x^2 - 2x - 4\end{aligned}$$

Type 2: Inverse functions

4. Given $f(x) = 2x^2 + 7\,;\ x \in \mathbb{R}^+ \cup \{0\}$. Find $f^{-1}(x)$ and its domain.

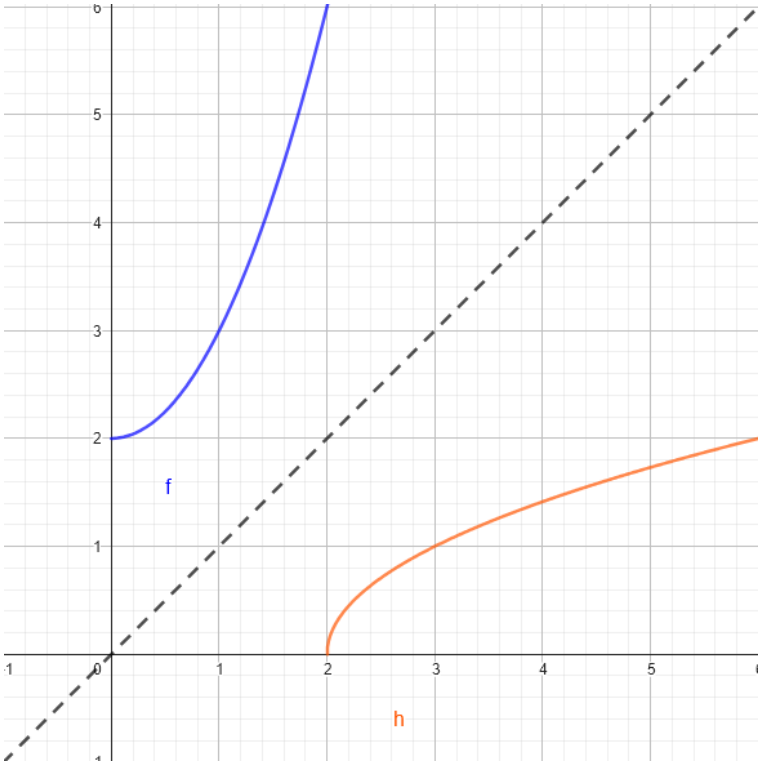
Solution:

$$\begin{aligned}& y = f(x) = 2x^2 + 7 \\ \implies\ & y - 7 = 2x^2 \\ \implies\ & \frac{y-7}{2} = x^2 \\ \implies\ & x = \sqrt{\frac{y-7}{2}} \\ \therefore\ & f^{-1}(x) = \sqrt{\frac{x-7}{2}},\ x \geq 7\end{aligned}$$

Type 3: Sketching graphs of functions

11. Sketch the graph of $f(x) = x^2 + 2\ ;\ \ x \in \mathbb{R},\ x \geq 0$. Use this information to draw the graph of $f^{-1}(x)$ without finding the inverse function $f^{-1}(x)$.

Solution:



Type 4: Modulus inequalities

15. Express the set $\{x \in \mathbb{R} / |2x - 1| < 7\}$ as an interval.

Solution:

$$|2x - 1| < 7$$
$$\implies \quad -7 < 2x - 1 < 7$$
$$\implies \quad -6 < 2x < 8$$
$$\implies \quad -3 < x < 4$$
$$\therefore \quad (-3, 4)$$