University of Nottingham
UK | CHINA | MALAYSIA

Foundation Calculus and Mathematical Techniques CELEN037

Practice Problems SET-1 Sample Solution

Type 1: Derivatives using First Principles

2. Use First Principles to find the derivative of the following functions: (iii) $y = \cos 2x$

Proof:

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{\cos 2(x+h) - cos2x}{h} \\
f'(x) &= \lim_{h\to 0} -2\frac{\sin\dfrac{2(x+h)+2x}{2}\sin\dfrac{2(x+h)-2x}{2}}{h} \\
f'(x) &= \lim_{h\to 0} \ -2\frac{\sin(2x+h)\sin h}{h}
\end{aligned}
$$

As $\displaystyle \lim_{h\to 0} \frac{\sin h}{h} = 1$

$$f'(x) = -2\sin 2x$$

Type 2: The Sum and Difference Rules

9. Consider the hyperbolic functions $\cosh x = \frac{1}{2}\left(e^x + e^{-x}\right)$ and $\sinh x = \frac{1}{2}\left(e^x - e^{-x}\right)$. Find $\frac{d}{dx}(\cosh x) + \frac{d}{dx}(\sinh x)$.

Solution:

$$
\begin{aligned}
\frac{d}{dx}(\cosh x) + \frac{d}{dx}(\sinh x) &= \frac{d}{dx}\left(\frac{1}{2}\left(e^x + e^{-x}\right)\right) + \frac{d}{dx}\left(\frac{1}{2}\left(e^x - e^{-x}\right)\right) \\
&= \frac{1}{2}\left(\frac{d}{dx}(e^x) + \frac{d}{dx}(e^{-x}) + \frac{d}{dx}(e^x) - \frac{d}{dx}(e^{-x})\right) \\
&= \frac{1}{2}\left(2\cdot\frac{d}{dx}(e^x)\right) \\
&= \frac{d}{dx}(e^x) \\
&= e^x
\end{aligned}
$$

Type 3: Product Rule

11. Use the Product Rule to find the derivative of the following functions: $y = 2x^6(1+x)^5$.

Solution:

$$\begin{aligned}
\frac{d}{dx}\left(2x^6(1+x)^5\right) &= 2\frac{d}{dx}\left(x^6 \cdot (1+x)^5\right) \\
&= 2\left(\frac{d}{dx}\left(x^6\right) \cdot (1+x)^5 + \frac{d}{dx}\left((1+x)^5\right) \cdot x^6\right) \\
&= 2\left(6x^5 \cdot (1+x)^5 + x^6 \cdot 5(1+x)^4\right) \\
&= 2x^5(x+1)^4(11x+6)
\end{aligned}$$

Type 4: The Quotient Rule

18. Use the Quotient Rule to find the derivative of the following functions:

(iv) $y = 1 + x + x^2 + x^3 + \cdots, \quad (|x| < 1)$

Solution:

Based on infinite geometric series:

first term $a = 1,$ common ratio $r = x.$

therefore the sum of infinite geometric progression $S = \dfrac{a}{1-r} = \dfrac{1}{1-x}$ as $|r| = |x| < 1.$

$$\therefore \frac{d}{dx}(1 + x + x^2 + x^3 + \cdots) = \frac{d}{dx}\left(\frac{1}{1-x}\right)$$

$$= \frac{(1-x)\frac{d}{dx}(1) - 1 \cdot \frac{d}{dx}(1-x))}{(1-x)^2}$$

$$= \frac{0-(-1)}{(1-x)^2}$$

$$= \frac{1}{(1-x)^2}$$