# Foundation Algebra for Physical Sciences & Engineering CELEN036

**Practice Problems SET-7** **Topic:** Numerical Methods

### Type 1: Intermediate value theorem

1. Show that the following equation has a root in the given intervals.

   $(i)$ $x^3 - x + 5 = 0$ ; $-2 < x < -1$

   $(ii)$ $\sqrt[3]{x} - \cos x = 0$ ; $0.5 < x < 0.6$

   $(iii)$ $e^{-x} = x^2$ ; $0.7 < x < 0.71$

   $(iv)$ $\dfrac{x-1}{x^2+2} = \dfrac{3-x}{x+1}$ ; $0 < x < 3$

2. Given that the function $f(x) = x + \sin x - 1$ is continuous at $(-\infty, +\infty)$, prove that there exist at least one real root of the equation $x + \sin x - 1 = 0$.

### Type 2: Bisection method

3. Given that a root of the equation $x^3 - 3x^2 - 2x + 5 = 0$ lies between $x = 3$ and $x = 4$. Use the Bisection method to approximate this root, correct to 2 decimal places. (write 3 rows)

4. Find the root of $f(x) = e^{-x}(3.2\sin x - 0.5\cos x)$ on the interval $[3, 4]$ in 3 decimal places. (write 3 rows)

5. Use the Bisection method to find solutions accurate to within $10^{-4}$ for $x^3 - 7x^2 + 14x - 6 = 0$ on each interval. (write 3 rows)

   $(i)$ $[0, 1]$ $(ii)$ $[1, 3.2]$ $(iii)$ $[3.2, 4]$

### Type 3: Iteration method

6. Given the equation of $2x^3 - 2x - 5 = 0$, find the estimated root within 4 d.p. using the iterative formula $x_{n+1} = \left(\dfrac{2x_n + 5}{2}\right)^{\frac{1}{3}}$ and $x_0 = 1.5$.

7. Find the root of $\cos x - xe^x = 0$ using the iteration formula $x_{n+1} = \cos x_n - x_n e^{x_n} + x_n$ and $x_0 = 2$ within 3 d.p..

8. Find the root of $e^{-x}(x^2 + 5x + 2) + 1 = 0$ using the iteration formula:

   $x_{n+1} = \dfrac{e^{x_n} + x_n^3 + 4x_n^2 + 2x_n + 2}{x_n^2 + 3x_n - 3}$ and $x_0 = -2$ within 3 d.p..

9. Calculate the approximation of $\sqrt{12}$ using the iterative formula $x_{n+1} = \frac{1}{2}\left(x_n + \frac{12}{x_n}\right)$ with $x_0 = 2$, correct to 4 decimal places.

10. The equation $x^3 - 5x - 2 = 0$ has a root between 2 and 3. Use the iterative formula: $x_{n+1} = \dfrac{2x_n^3 + 2}{3x_n^2 - 5}$ and $x_0 = 2$ to find the root correct to 5 decimal places.

11. Use the iterative formula: $x_{n+1} = 2\sin x_n$ with $x_0 = 1$, to find the root of the equation $x = 2\sin x$, correct to 3 decimal places.

12. Show that the two possible arrangements of $x^3 - 4x + 1 = 0$ leads to the iterative formulae $x_{n+1} = \dfrac{1}{4}\left(x_n^3 + 1\right)$ and $x_{n+1} = \sqrt[3]{4x_n - 1}$.
    - $(i)$ $x_0 = 1,$ use $x_{n+1} = \dfrac{1}{4}\left(x_n^3 + 1\right)$ to calculate the positive root, correct to 3 d.p..
    - $(ii)$ $x_0 = 2,$ use $x_{n+1} = \sqrt[3]{4x_n - 1}$ to calculate the root, correct to 3 d.p..

13. Show that $x^3 - 3x^2 - 2x + 5 = 0$ has a root in the interval $3 < x < 4$.
    - $(i)$ Use the iteration formula $x_{n+1} = \sqrt{\dfrac{x_n^3 - 2x_n + 5}{3}}$ to find an approximation for the root, correct to 4 d.p. by taking $x_0 = 3$.
    - $(ii)$ What happens if you take starting value as $x_0 = 3.5$? (Compared with (i))

14. Find the approximation within 5 d.p. of $7^{\frac{1}{5}}$ by proposing an equation of $x^5 - 7 = 0$. Use the iteration formula of $x_{n+1} = x_n - \dfrac{x_n^5 - 7}{5x_n^4}$ and $x_0 = 1$.

15. The following for methods are proposed to compute $21^{\frac{1}{3}}$. Rank them in order based on the number of iteration steps to achieve the same accuracy of 4 d.p., assuming $p_0 = 1$.
    - $(i)$ $p_{n+1} = \dfrac{20p_n + \frac{21}{p_n^2}}{21}$
    - $(ii)$ $p_{n+1} = p_n - \dfrac{p_n^3 - 21}{3p_n^2}$
    - $(iii)$ $p_{n+1} = p_n - \dfrac{p_n^4 - 21p_n}{p_n^2 - 21}$
    - $(iv)$ $p_{n+1} = \left(\dfrac{21}{p_n}\right)^{\frac{1}{2}}$

## Answers

**3** 3.13

**4** 3.375

**5** $(i)$ 0.6250 $(ii)$ 2.9250 $(iii)$ 3.5000

**6** 1.6006

**7** $-10.995$

**8** $-0.579$

**9** 3.4641

**10** 2.41421

**11** 1.895

**12** $(i)$ 0.254 $(ii)$ 1.861

**13** $(i)$ 1.2017 $(ii)$ The sequence of approximations is divergent

**14** 1.47577

**15** (i), (ii), (iv), the (iii) one done not converge.