Seminar 1

In this seminar you will study:

- Composition of Functions

- Inverse Functions

- Sketching Graphs of Functions

- Modulus Inequalities

Composition of functions

Example:

Given $f(x) = 2x + 3$ and $g(x) = x^2 + 1$, find $(f \circ g)(x)$ and $(g \circ f)(x)$.

Solution:

$$(f \circ g)(x) = f(g(x))$$

$$= f(x^2 + 1)$$

$$= 2(x^2 + 1) + 3$$

$$= 2x^2 + 5$$

$$(g \circ f)(x) = g(f(x))$$

$$= g(2x + 3)$$

$$= (2x + 3)^2 + 1$$

$$= 4x^2 + 12x + 10$$

Inverse functions

Example: Given $f(x) = \dfrac{2x-3}{x+4}, \ x \neq -4$, find the formula for $f^{-1}(x)$.

Solution:

Step 1: Let $y = f(x) = \dfrac{2x-3}{x+4}$

Step 2: Express x as a function of y : $\quad y = \dfrac{2x-3}{x+4} \ \Rightarrow xy + 4y = 2x - 3$

$$\Rightarrow x(y-2) = -3 - 4y$$
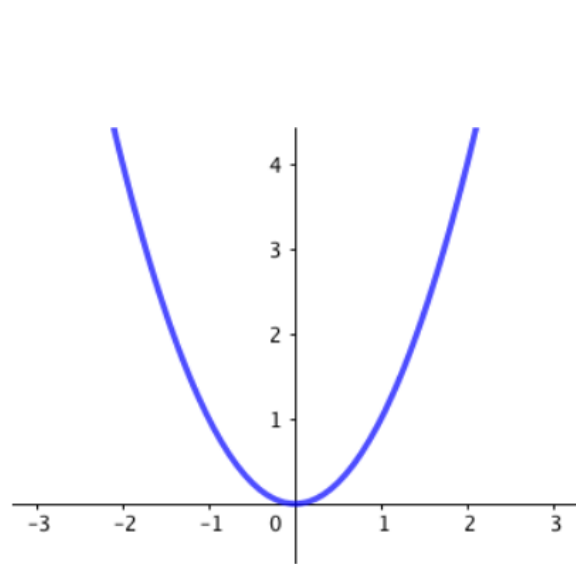
$$\Rightarrow x = \dfrac{4y+3}{2-y}$$

Step 3: $f^{-1}(x)$ is obtained by replacing y with x on the RHS:

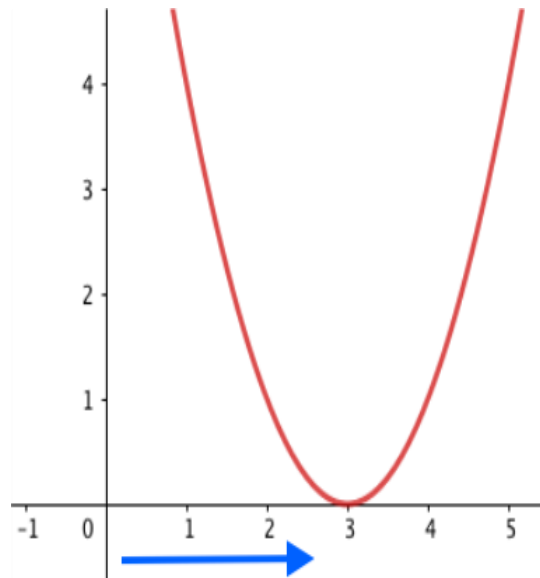$$f^{-1}(x) = \dfrac{4x+3}{2-x}, \ x \neq 2$$

Sketching graphs of functions

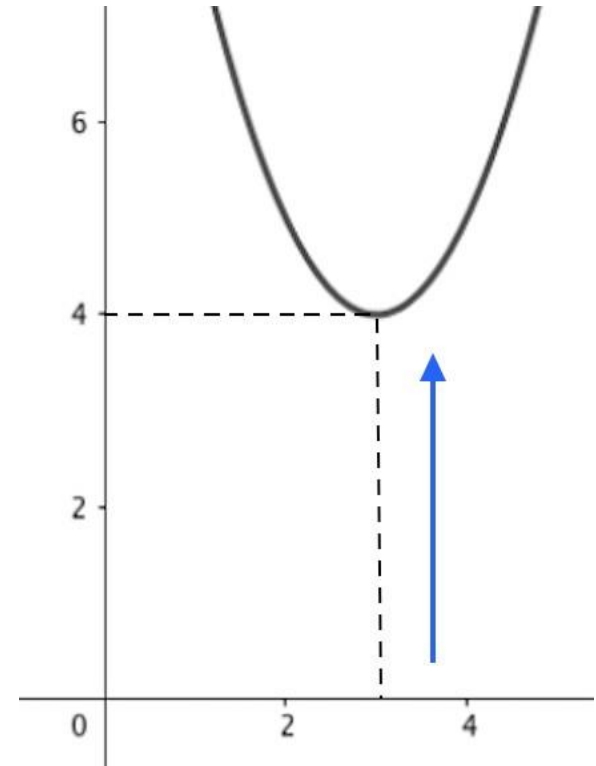**Example:** Sketch the graph of $y = (x - 3)^2 + 4$

Solution:

(a) To start with, draw the graph of the function $y = x^2$

(b) Shift the graph of $y = x^2$ to the right by 3 units to give $y = (x - 3)^2$

(c) Shift the resultant graph upwards by 4 units: $y = (x - 3)^2 + 4$

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Modulus Inequalities

Example: Solve $|4x - 1| \geq 7$.

Solution:

$$|4x - 1| \geq 7 \quad \Rightarrow \quad \pm(4x - 1) \geq 7$$

$$\Rightarrow (4x - 1) \geq 7 \qquad \text{or} \qquad -(4x - 1) \geq 7$$

$$\Rightarrow 4x \geq 8 \qquad\qquad\qquad \Rightarrow -4x + 1 \geq 7$$

$$\Rightarrow x \geq 2 \qquad\qquad\qquad \Rightarrow -4x \geq 6$$

$$\Rightarrow x \leq -\frac{3}{2}$$

Note the change in the inequality

Thus $x \leq -\dfrac{3}{2}$ or $x \geq 2$ i.e. $x \in \mathbb{R} - \left(-\dfrac{3}{2}, 2 \right)$