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Foundation Algebra for Physical Sciences & Engineering CELEN036

Practice Problems SET-10 Sample Solution

Type 1: Finding terms of a sequence

1. Find the first five terms (i.e., for $n = 1, 2, 3, 4, 5$) of the following sequence: $(i)\ f(n) = 3n + 2$

Solution:

$f(1) = 5,\ f(2) = 8,\ f(3) = 11,\ f(4) = 14,\ f(5) = 17,$

The first five terms are: $5, 8, 11, 14, 17, \cdots$

Type 2: Arithmetic progression

3. Find the 10^{th} term of the sequence: $3,\ 15,\ 27,\ 39, \cdots$

Solution:

First term $a = 3$,

Common difference $d = 15 - 3 = 12$,

Therefore, the n^{th} term fomula is:$a_n = 3 + (n - 1) \cdot 12$.

The 10^{th} term is:$a_{10} = 3 + (10 - 1) \cdot 12 = 111$.

9. Find the common difference, the fifth term, the nth term and the 100th term of the arithmetic progression (AP).

$$\frac{x}{x^2+1},\quad \frac{2x^2+x+1}{x^3+x^2+x+1},\quad \frac{3x^2+x+2}{x^3+x^2+x+1},\quad \frac{4x^2+x+3}{x^3+x^2+x+1}, \ldots$$

Solution:

First term $a = \dfrac{x}{x^2+1}$,

Common difference $d = \dfrac{2x^2+x+1}{x^3+x^2+x+1} - \dfrac{x}{x^2+1} = \dfrac{1}{x+1}$,

Therefore, the n^{th} term fomula is:$a_n = \dfrac{x}{x^2+1} + (n-1) \cdot \dfrac{1}{x+1} = \dfrac{nx^2+x+n-1}{x^3+x^2+x+1}$.

The 15^{th} term is:$a_{15} = \dfrac{15x^2+x+14}{x^3+x^2+x+1}$.

The 100^{th} term is:$a_{15} = \dfrac{100x^2+x+99}{x^3+x^2+x+1}$.

Type 3: Geometric progression

16. The sum and product of three consecutive terms in a geometric progression are 52 and 1728 respectively. Find these three terms.

Solution:

Let the three consecutive terms in a G.P. to be $a,\ ar,\ ar^2$

$\therefore\ a + ar + ar^2 = 52,\ \ a \cdot ar \cdot ar^2 = 1728$

$a \cdot ar \cdot ar^2 = 1728 \implies a^3r^3 = 12^3 \implies ar = 12 \implies a = \dfrac{12}{r}$

$\therefore\ \dfrac{12}{r} \cdot (1 + r + r^2) = 52.$

As $r \neq 0$, therefore $12(1 + r + r^2) = 52r \implies 3r^2 - 10r + 3 = 0.$

$\therefore r = 3, r = \dfrac{1}{3}.$

$\therefore a = \dfrac{12}{r} = 36,$ or 4.

These three terms are: $36, 12, 4$ or $4, 12, 36$.

Type 4: Find the n^{th} term

20. Find an Arithmetic Progression (A.P.) the sum of whose first n terms is $2n^2 + n$.

Solution:

Let $n = 1,\ \therefore S_1 = a_1 = a = 3$

Substitute into the formula of sum of A.P.: $S_n = \dfrac{n}{2} \cdot [\,2 \times 3 + (n-1) \cdot d\,] \equiv 2n^2 + n$

$\therefore d = 4$

$\therefore a_n = a + (n-1)d = 3 + 4(n-1) = 4n - 1$

Type 5: Arithmetic Series

25. Find the sum of all the integers between 100 and 600 that are multiples of 11.

Solution:

The first integer larger than 100 that are multiples of 11 are 110, therefore $a = 110,\ d = 11$.

$\therefore a_n = 11n + 99$

Let $a_n = 600, \implies n \approx 45.55$

$\therefore a_{45} = 594$

$\therefore S_{45} = \dfrac{45}{2} \cdot [\,2 \times 110 + (45-1) \cdot 11\,] = 15840$

Type 6: Geometric Series

30. Find the sum of the following infinite geometric series: $(i)\ \frac{1}{4} + \frac{1}{20} + \frac{1}{100} + \cdots$.

Solution:

$a = \frac{1}{4}, \quad d = \frac{1}{20} \div \frac{1}{4} = \frac{1}{5}$

As $|r| < 1, \therefore S = \frac{a}{1-r} = \frac{5}{16}$

Type 7: Power Series

35. Find the sum: $1 \cdot 3 \cdot 7 + 2 \cdot 5 \cdot 11 + 3 \cdot 7 \cdot 15 + \cdots$ (up to n terms).

Solution:

The n^{th} term of the series is: $n \cdot (2n+1) \cdot (4n+3) = 8n^3 + 10n^2 + 3n$

$$\sum_{k=1}^{n} \big(8k^3 + 10k^2 + 3k\big) = 8 \cdot \sum_{k=1}^{n} k^3 + 10 \cdot \sum_{k=1}^{n} k^2 + 3 \cdot \sum_{k=1}^{n} k$$

$$= 8 \cdot \frac{n^2(n+1)^2}{4} + 10 \cdot \frac{n(n+1)(2n+1)}{6} + 3 \cdot \frac{n(n+1)}{2}$$

$$= \frac{12n^2(n+1)^2 + 10n(n+1)(2n+1) + 9n(n+1)}{6} = \frac{n(n+1)[\,12n(n+1) + 10(2n+1) + 9\,]}{6}$$

$$= \frac{n(n+1)(12n^2 + 32n + 19)}{6}$$

Type 8: Method of Difference

36. Express $\frac{2}{4r^2-1}$ in partial fractions, then show that: $\sum_{r=1}^{n} \frac{2}{4r^2-1} = \frac{2n}{2n+1}$.

Solution:

$$\frac{2}{4r^2-1} = \frac{A}{2r+1} + \frac{B}{2r-1} \implies 2 = A(2r-1) + B(2r+1)$$

Let $r = \frac{1}{2} \implies B = 1$

Let $r = -\frac{1}{2} \implies A = -1$

$$\therefore \sum_{r=1}^{n} \frac{2}{4r^2-1} = \sum_{r=1}^{n} \left(\frac{1}{2r-1} - \frac{1}{2r+1}\right)$$

$$= \frac{1}{1} - \frac{1}{3} + \frac{1}{3} - \frac{1}{5} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{1}{2n-1} - \frac{1}{2n+1}$$

$$= \frac{1}{1} - \frac{1}{2n+1} = \frac{(2n+1)-1}{2n+1} = \frac{2n}{2n+1}$$