Topic 1: Definition of Derivatives

Key Formula: $\boxed{\dfrac{dy}{dx} = f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}}$

Illustration 1: Given $y = (x+1)^2$, find $\dfrac{dy}{dx}$ using definition of derivatives.

$$
\begin{aligned}
y &= f(x) = (x+1)^2 \\
\frac{dy}{dx} &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\
&= \lim_{h\to 0} \frac{(x+h+1)^2-(x+1)^2}{h} \\
&= \lim_{h\to 0} \frac{x^2+h^2+1+2xh+2x+2h-(x^2+2x+1)}{h} \\
&= \lim_{h\to 0} \frac{h^2+2xh+2h}{h} \\
&= \lim_{h\to 0} \frac{h(h+2x+2)}{h} \\
&= \lim_{h\to 0} (h+2x+2) = 2x+2
\end{aligned}
$$

1. $y = \cos x$	2. $y = \tan x$
Answer:	**Answer:**

Find dy/dx of the following functions using definition of derivatives.

1. $y = \dfrac{1}{x^2}$ **Answer:**	2. $y = \sqrt{x}$ **Answer:**
3. $y = \sqrt{x+1}$ **Answer:**	4. $y = \dfrac{1}{\sqrt{x}}$ **Answer:**

Topic 1: Definition of Derivatives

Equivalent form: $$\boxed{\frac{dy}{dx} = f'(x) = \lim_{t \to x} \frac{f(t) - f(x)}{t - x}}$$

Illustration 2: Given $y = \sin x^2$, find $\frac{dy}{dx}$ using definition of derivatives.

Suppose $y = f(x) = \sin x^2 \Rightarrow f(t) = \sin t^2$

$$\begin{aligned}
\frac{dy}{dx} &= \lim_{t \to x} \frac{f(t) - f(x)}{t - x} \\
&= \lim_{t \to x} \frac{\sin(t^2) - \sin(x^2)}{t - x} \\
&= \lim_{t \to x} \frac{2\cos\left(\frac{t^2+x^2}{2}\right)\sin\left(\frac{t^2-x^2}{2}\right)}{(t-x)} \cdot \frac{(t+x)}{(t+x)} \\
&= \lim_{t \to x}(t+x) \lim_{t \to x} \cos\left(\frac{t^2+x^2}{2}\right) \lim_{t \to x} \frac{\sin\left(\frac{t^2-x^2}{2}\right)}{\left(\frac{t^2-x^2}{2}\right)} \\
&= 2x\cos(x^2) \cdot (1) \\
\therefore \frac{dy}{dx} &= 2x\cos(x^2)
\end{aligned}$$

1. $y = \sin^2 x$ **Answer:**	2. $y = \cos(x^2)$ **Answer:**

Find dy/dx of the following functions using definition of derivatives.

1. $y = \cos(x+1)$ **Answer:**	2. $y = \sin(x^2)$ **Answer:**
3. $y = e^{5x}$ **Answer:**	4. $y = \ln 2x$ **Answer:**

Find dy/dx of the following functions using definition of derivatives.

1. $y = x^n$, hint: binomial expansion

Answer:

2. $y = 5^{ex}$

Answer:

Topic 2: The Sum and Difference Rules

Key Formula: $\boxed{\dfrac{d}{dx}(u \pm v) = \dfrac{du}{dx} \pm \dfrac{dv}{dx}}$

Illustration: Given $y = (x+5)\cdot(3x-1)$, find $\dfrac{dy}{dx}$

$$y = (x+5)\cdot(3x-1) = 3x^2 + 14x - 5$$

$$\text{Hence} \quad \frac{dy}{dx} = \frac{d}{dx}(3x^2 + 14x - 5)$$

$$= 3\frac{d}{dx}(x^2) + 14\frac{d}{dx}(x) - 5\frac{d}{dx}(1)$$

$$= 6x + 14$$

1. $y = \sqrt[4]{x} - 2\sec x$ **Answer:**	2. $y = 6\ln x + 6^x + 6^6$ **Answer:**

Find dy/dx of the following functions.

1. $y = x^4 - \dfrac{1}{x}$ **Answer:**	2. $y = \dfrac{2x^3 + x - 3}{x^2}$ **Answer:**
3. $y = x^2 - \dfrac{4}{\sqrt[3]{x^2}}$ **Answer:**	4. $y = \dfrac{(x-2) \cdot (x-3)}{x}$ **Answer:**

Find dy/dx of the following functions.

1. $y = 5x^{-\frac{1}{3}} - 3\cos x$ **Answer:**	2. $y = \sqrt[4]{x^3} + 2\tan x$ **Answer:**
3. $y = \ln\left(\dfrac{x-5}{x+1}\right)$ **Answer:**	4. $y = \sqrt{x} + 2^x - \cot x - \dfrac{1}{x}$ **Answer:**

Topic 3: The Product Rule

Key Formula: $$\boxed{\frac{d}{dx}(u \cdot v) = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}}$$

Illustration: Given $y = \ln x \cdot \sec x$, find $\dfrac{dy}{dx}$

Let $u = \ln x$ and $v = \sec x$

$$\begin{aligned}\frac{dy}{dx} &= \ln x \cdot \frac{d}{dx}(\sec x) + \sec x \cdot \frac{d}{dx}(\ln x)\\ &= \ln x \cdot \sec x \cdot \tan x + \sec x \cdot \frac{1}{x}\\ &= \sec x \left(\ln x \cdot \tan x + \frac{1}{x} \right)\end{aligned}$$

1. $y = \dfrac{1}{x} \cdot \sec x$, **hint:** $\dfrac{d}{dx}\left(\dfrac{1}{x}\right) = -\dfrac{1}{x^2}$

Answer:

2. $y = \dfrac{1}{x} \cdot \sin x$, **hint:** $\dfrac{d}{dx}\left(\dfrac{1}{x}\right) = -\dfrac{1}{x^2}$

Answer:

Find dy/dx of the following functions.

1. $y = x^3 \cdot \tan x$

Answer:

2. $y = \dfrac{1}{x} \cdot \ln x$, **hint:** $\dfrac{d}{dx}\left(\dfrac{1}{x}\right) = -\dfrac{1}{x^2}$

Answer:

3. $y = \sin 2x$, **hint:** use $\sin(2\theta)$ formula

Answer:

Find dy/dx of the following functions.

1. $3^x \cdot \cos x$

Answer:

2. $y = (x^8 + 2x - 3)e^x$

Answer:

3. $y = 2^x \cos x$

Answer:

1. $y = e^x \cdot \sin x \cdot \ln x$

Hint: $$\frac{d}{dx}(u \cdot v \cdot w) = u \cdot v \cdot \frac{dw}{dx} + v \cdot w \cdot \frac{du}{dx} + w \cdot u \cdot \frac{dv}{dx}$$

Answer:

2. $y = x \cdot \sin x \cdot \tan x$

Answer:

Topic 4: Quotient Rule

Key Formula: $$\boxed{\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \cdot \dfrac{du}{dx} - u \cdot \dfrac{dv}{dx}}{v^2}}$$

Illustration: Given $y = \dfrac{\sec x}{e^x}$, find $\dfrac{dy}{dx}$

Let $u = \sec x$ and $v = e^x$

$$\frac{dy}{dx} = \frac{e^x \cdot \frac{d}{dx}(\sec x) - \sec x \cdot \frac{d}{dx}(e^x)}{(e^x)^2}$$

$$= \frac{e^x \cdot \sec x \cdot \tan x - \sec x \cdot e^x}{(e^x)^2}$$

$$= \frac{\sec x \cdot \tan x - \sec x}{e^x}$$

1. $y = e^{-x} = \dfrac{1}{e^x}$ **Answer:**	2. $y = x \cdot e^{-x}$ **Answer:**

Find dy/dx of the following functions.

1. $y = \dfrac{\sqrt{x}}{e^x}$ **Answer:**	2. $y = \dfrac{1+e^x}{1-e^x}$ **Answer:**
3. $y = \dfrac{1+x}{1-x}$ **Answer:**	4. $y = \dfrac{1-x^2}{1+x^2}$ **Answer:**

Find dy/dx of the following functions.

1. $y = \dfrac{x^3 - 5x + 4\sqrt{x}}{x}$

Answer:

2. $y = \dfrac{\tan x + \sin x}{\sec x - \cos x}$

Answer: