University of Nottingham Ningbo China

CENTRE FOR ENGLISH LANGUAGE EDUCATION

PRELIMINARY YEAR, SEMESTER TWO, 2024-25

FOUNDATION CALCULUS AND MATHEMATICAL TECHNIQUES SAMPLE MID-SEMESTER EXAM

Time allowed: ONE HOUR

Candidates must write their ID number on this booklet and fill-in their attendance card but must NOT write anything else until the start of the exam is announced.

This paper contains TWENTY questions. The total number of points is 100. Answer all questions.

Only general bilingual dictionaries are allowed. Subject-specific dictionaries are not permitted. No electronic devices except for approved calculators (CASIO fx-82) can be used in this exam.

Do NOT open the examination paper until told to do so.

All answers must be written in this booklet.

ADDITIONAL MATERIAL: Formula Sheet

INFORMATION FOR INVIGILATORS:

- 1. A 15-minute warning should be given before the end of the exam.
- 2. Please collect this Booklet and Formula Sheet after the exam.
- 3. Please return this Booklets in ID order.

Student ID:	
Seminar Group (<i>e.g.</i> A35):	Marks (out of 100):

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Section A: Multiple Choice Questions. Choose the CORRECT option.

1.	Find	the limit $\lim_{\theta \to 0} \frac{\sin 2\theta}{\sin \theta}$.	[4]
	(A)	1	
	(B)	2	
	(C)	0	
	(D)	∞	
		Answer:	
2.	Find	the limit $\lim_{x \to \infty} \left(1 + \frac{2}{x}\right)^{3x}$	[4]
	(A)	∞	
	(B)	0	
	(C)	$-e^6$	
	(D)	e^6	
		Answer:	
3.	Giver	n that $y=(x+1)^2(2x-1)$, find $\frac{dy}{dx}$.	[4]
	(A)	$6x^2 + 6x$	
	(B)	$4x^2 + 3x$	
	(C)	$6x^2 + 3x$	
	(D)	$4x^2 + 6x$	
		Answer:	
4.	Giver	n that $y = \frac{\sqrt{x}}{1 - \sin x}$, find $\frac{dy}{dx}$.	[4]
	(A)	$\sqrt{x}(\frac{1}{2x}(1-\sin x)-\cos x)(1-\sin x)^{-2}$	
		$(\frac{1}{2x})\sqrt{x}((1-\sin x)+\cos x)(1-\sin x)^{-2}$	
		$\sqrt{x}(\frac{1}{2x}(1-\sin x)+\cos x)(1-\sin x)^{-2}$	
		$(\frac{1}{2x})\sqrt{x}((1-\sin x)-\cos x)(1-\sin x)^{-2}$	
		Answer:	

5.	Given $y=\sin{(\tan{(e^x)})}$, use the chain rule to find $\frac{dy}{dx}$.	[4]
	(A) $\cos(\tan(e^x)) \cdot \sec^2(e^x) \cdot e^x$	
	(B) $\cos\left(\sec^2\left(e^x\right)\right) \cdot \tan(e^x) \cdot e^x$	
	(C) $\sin(\tan(e^x)) \cdot \sec^2(e^x) \cdot e^x$	
	(D) $\sin\left(\sec^2\left(e^x\right)\right) \cdot \tan(e^x) \cdot e^x$	
	Answer:	
6.	Given $y=x^{99}$, find $\frac{d^{99}y}{dx^{99}}$.	[4]
	(A) $\frac{1}{99!}$	
	(B) 99!	
	(C) 1	
	(D) 0	
	Answer:	
7.	Given $f(1) = 2$, $f'(1) = 0$, $g(1) = 3$, and $g'(1) = 4$, find $h'(1)$ if $h(x) = \frac{f(x)}{g(x)}$.	[4]
	(A) $-\frac{8}{9}$	
	(B) $\frac{8}{9}$	
	(C) 1	

Answer:

8. Let $f(x) = (x - a)^2 \cdot \ln x$, and x = e is a local maximum, find the value of a. [4]

(A) -e

(D) 2

- (B) -3e
- (C) e
- (D) 3e

Answer: _____

9. Evaluate the indefinite integral
$$\int \left(3e^{3x} - \frac{6}{x} + \csc^2(2x)\right) dx$$
. [4]

(A)
$$-6 \ln |x| - \frac{\cot 2x}{2} + \frac{e^{3x}}{3} + C$$

(B)
$$-6 \ln |x| - \cot 2x + \frac{e^{3x}}{3} + C$$

(C)
$$-6 \ln |x| - \frac{\cot 2x}{2} + e^{3x} + C$$

(D)
$$-6 \ln |x| - \cot 2x + e^{3x} + C$$

Answer: _____

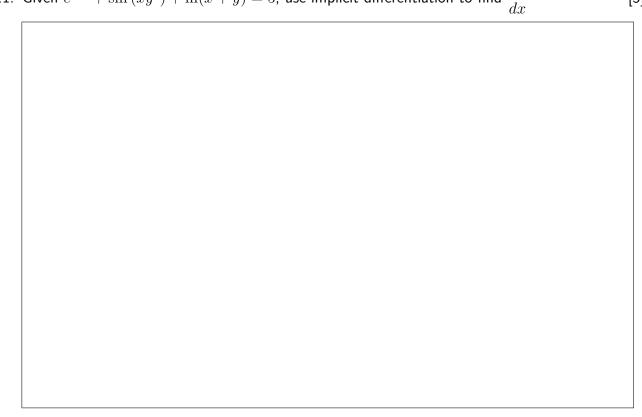
10. Evaluate
$$\int e^x \left(\frac{\cos x \sin x - \cos^2 x - \sin^2 x}{\sin^2 x} \right) dx$$
 by using the result $\int e^x (f(x) + f'(x)) dx = e^x (f(x)) + C$. [4]

- (A) $e^x(\sec x) + C$
- (B) $e^x(\csc x) + C$
- (C) $e^x(\cot x) + C$
- (D) $e^x(\tan x) + C$

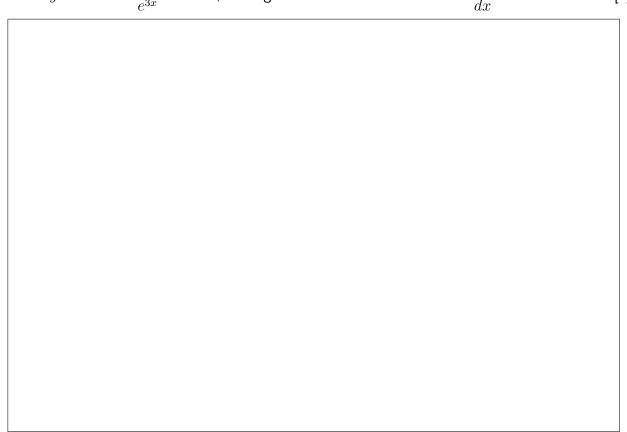
Answer: _____

Section B: Short Answer Questions. Answers must be written with necessary steps.

11.	Given $e^{x^2y} + \sin(xy^2) + \ln(x+y) = 5$, use implicit differentiation	to find $\frac{dy}{dx}$	[5]
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12. Given
$$y = \frac{\sqrt[3]{(x-1)^2} \cdot (x-3)^5}{e^{3x}}$$
, use logarithmic differentiation to find $\frac{dy}{dx}$. [5]



b) Hence, find $\left.\frac{dy}{dx}\right _{t=0}$. (2) Also, find the equation of the tangent line to the curve when $t=0$.	a) find $\frac{dy}{dx}$.				
) Hence, find	$\frac{dy}{dx}$.			
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	Also, find th	ne equation of the ta	angent line to the c	curve when $t = 0$.	

	r depth $h = 6m$? $(V = \frac{1}{3}\pi r^2 h)$	
Evaluate the integral	$\int \tan^5 x \cdot \sec^2 x dx \text{ by using the substitution } \tan x = t.$	
Evaluate the integral	$\int \sin 6x \cdot \sin 3x dx$	
Evaluate the integral		

Evaluate the integr	$\int \sin^5 x dx.$	
	$\int \ln(\ln x)$	
Use an appropriate	e substitution to evaluate the integral $\int \frac{\ln(\ln x)}{x \ln x} dx$.	

(a) Consid	er solving $f(x)$	$x) = x^3 + 3x $	-1=0, show th	$x_{n-1} = x_n - \frac{f(x_n)}{f'(x_n)},$ nat $x_{n+1} = \frac{2x_n^3 - 1}{3x_n^2 + 3}$	
h) Starting	r with $x = 1$	dotormino th	x = x = x = x = x = x = x = x = x = x =	— O that lies in the	intorval (1 2)
b) Starting	g with $x_0=1$ decimal plac	, determine the es. List all x_n	he root of $f(x)$ values until the	=0 that lies in the approximation is ac	interval $(-1,3)$, hieved.
b) Starting correct to 5	g with $x_0=1$ decimal plac	, determine the es. List all x_n	he root of $f(x)$ values until the	=0 that lies in the approximation is ac	interval $(-1,3)$, hieved.
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b) Starting correct to 5	g with $x_0=1$ of decimal place	, determine the es. List all x_n	te root of $f(x)$ values until the	= 0 that lies in the approximation is ac	interval $(-1,3)$, hieved.

•	laclaurin's expansio	f(x) up to the	terms with x	
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	itution $x=\dfrac{1}{16}$ in the correct to 4 decima		to approximate the val	ue of $\sqrt[4]{15}$.
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You may use this space for rough work.