

Question 1.

Algorithm: bonus(age, expe)
Requires: two positive integers
Returns: a positive integer, bonus amount

```
1. if age<=30 || expe<=5 then
2.     return 5000
3. elseif (age>30 && expe<=10) || (age<=40 && expe<=20) then
4.     return 7000
5. elseif (age>40 && expe<=20) || (age>=31 && age<=50 && expe<=30) then
6.     return 10000
7. else
8.     Return 15000
9. endif
```

TRACE:

bonus(45, 15)
line1: FALSE, GOTO line3
line3: FALSE, GOTO line5
line5: TRUE → **return 10000.**

Question 2.

a) Algorithm: length(L)
Requires: a list
Returns: a positive integer ≥ 0 , list length

```
1. if isEmpty(L) then
2.     return 0
3. else
4.     return 1+length(tail(L))
5. Endif
```

b) Algorithm: powerten(n)
Requires: a positive integer $n \geq 0$
Returns: a positive integer, 10^n

```
1. if n==0 then
2.     return 1
3. else
4.     return 10*powerten(n-1)
5. Endif
```

c) Algorithm: list2num(L) [main]
Requires: a list of positive integers
Returns: a positive n-digit integer

```
1. Let n=length(L)
2.   return listHelper(L,n)
```

Algorithm: listHelper(L,n) [helper]
Requires: a nonempty list and a positive integer $n \geq 1$
Returns: a positive n-digit integer

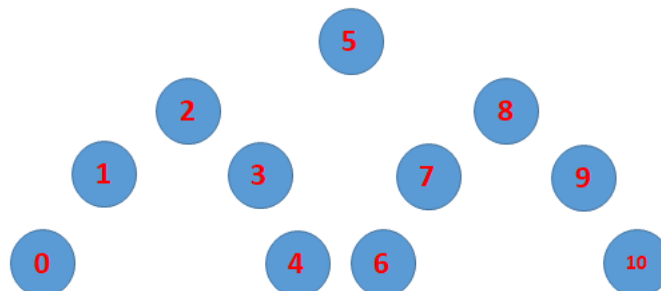
```
1. if n==1 then
2.   return value(L)
3. else
4.   return value(L)*powerten(n-1)+listHelper(tail(L),n-1)
5. Endif
```

Question 3.

- a)
- (i) Any sorted list with 5 elements, e.g. [1,2,3,4,5]
 - (ii) $n \log_2 n = 100 \times \lceil \log_2 100 \rceil = 700$
- b)
- (i) Because the list is sorted and binary search algorithm only works on sorted lists.
 - (ii) Find middle of the list: $18 > 11$; so keep the left half of the list: $L = [7, 11, 13, 14, 16]$
Find middle of the list: $13 > 11$; keep the left half of the list: $L = [7, 11]$
Find middle of the list: $7 < 11$; keep the right half of the list: $L = [11]$
Find middle of the list: $11 = 11$; return TRUE.
- c) $T = [30, 7, 11, 9, 20, 5, 10, 40]$
[7, 11, 9, 5, 20, 10] [30] [40]
[5] [7] [11, 9, 20, 10] [30] [40]
[5] [7] [9, 10] [11] [20] [30] [40]
[5] [7] [9] [10] [11] [20] [30] [40]
[5, 7, 9, 10, 11, 20, 30, 40]

Question 4.

- a)
- (i) $\lceil \log_2 n \rceil + 1$
 - (ii) Firstly sort the list: $Q = [0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10]$. There are 11 elements, find the middle to make the root.



(iii) Only 1 node, if the node value is less than 0 or greater than 10. Otherwise 4 nodes to fill the last row/generation.

b) A=31 or 32; B=49; C= no integer value; D=32 if A=31, no integer value if A=32.

c)

(i)

Algorithm: inOrder(bsT)
Requires: a binary search tree (BST)
Returns: a sorted list

```
1. if isLeaf(bsT) then
2.   return Nil
3. else
4.   return merge(inOrder(left(bsT)), merge(cons(root(BST), nil), inOrder(right(bsT))))
5. endif
```

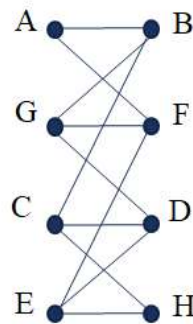
(ii) merge(merge([], merge([30], [35])), merge([40], merge([45], merge([50], []))))

Question 5.

a)

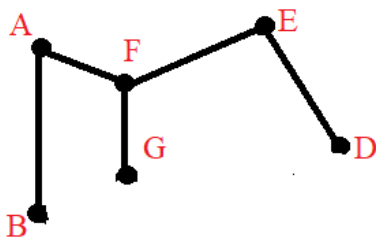
(i) It is not complete since not all vertices are connected. For example F is not connected to B.

(ii) The graph is bipartite according to the following diagram.



(iii) Neither ET nor EP. More than two vertices with odd degree.

b) 6 vertices so we need 5 edges. There are 7 edges, hence delete 2 without having a cycle.



$$\text{Number of spanning trees} = \binom{7}{5} - \binom{3}{2} - \binom{3}{2} = 21 - 3 - 3 = 15.$$

c) There are 8 vertices and we need 7 edges without cycles. Start at vertex A:

| VERTICES | EDGE SELECTED | # OF EDGES |
|------------------------------------|------------------|---------------------|
| A | AC=5 | 1 |
| A, C | CB=3 | 2 |
| A, C, B | CE=5 | 3 |
| A, C, B, E | ED=4 | 4 |
| A, C, B, E, D | DG=5 | 5 |
| A, C, B, E, D, G | GF=4 | 6 |
| A, C, B, E, D, G, F | GH=5 | 7 |
| <i>SHORTEST PATH LENGTH</i> | <i>31</i> | <i>STOP!</i> |

