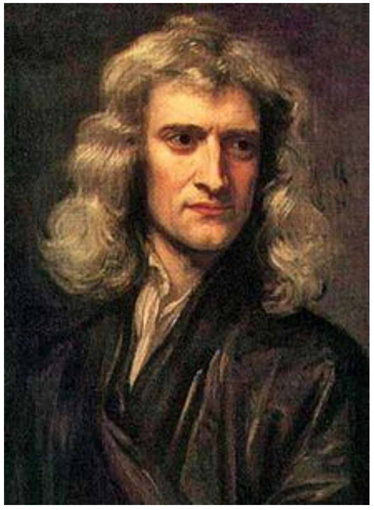
Science A Physics

Lecture 10: The Electrical Force

Aims of today's lecture

1. The Electric Field
2. Electric Fields and Conductors.
3. Motion of a Charged Particle in an Electric Field.

A Changing World-view

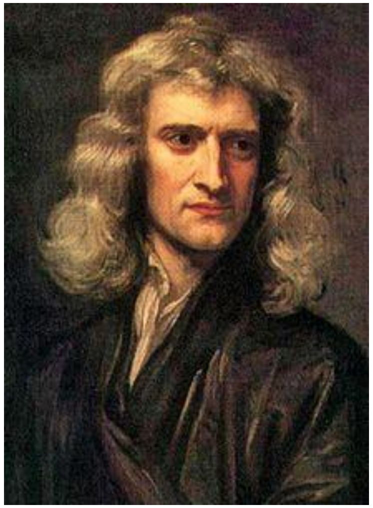


Newton, 1642-1727

Faraday, 1791-1867

- As we have seen, many common forces can be referred to as ‘contact’ forces, where one object ‘directly interacts’/’makes contact’ with another object. For example, a tennis racket hitting a tennis ball.
- In contrast, gravitational force and electrical force are ‘non-contact’ forces, acting over a distance: there is a force between two objects even when the objects are not touching.

A Changing World-view

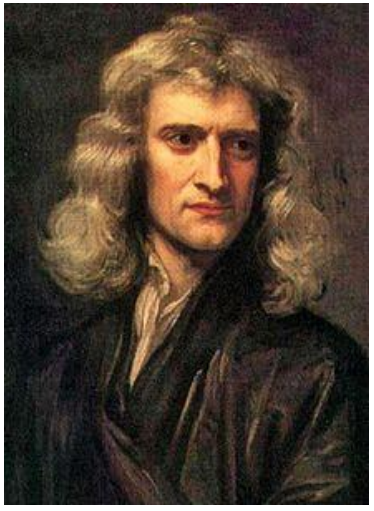


Newton, 1642-1727

Faraday, 1791-1867

- It is interesting to note that Newton felt uneasy about force acting over a distance despite laying the foundation for this world-view at the time when he published his law of universal gravitation. The obvious question to ask, which no-one had an answer for, was what mediating the gravitational force?

A Changing World-view



Newton, 1642-1727

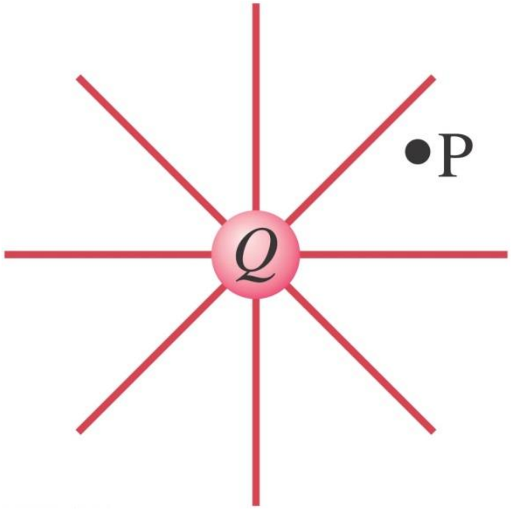
Faraday, 1791-1867

- It was Faraday who introduced the idea of a **field** to explain how a force can be mediated over a distance.
- According to Faraday, an electric field extends outward from every charge and permeates all of space; let's look at this idea in more detail.

1. The Electric Field

The Electric Field

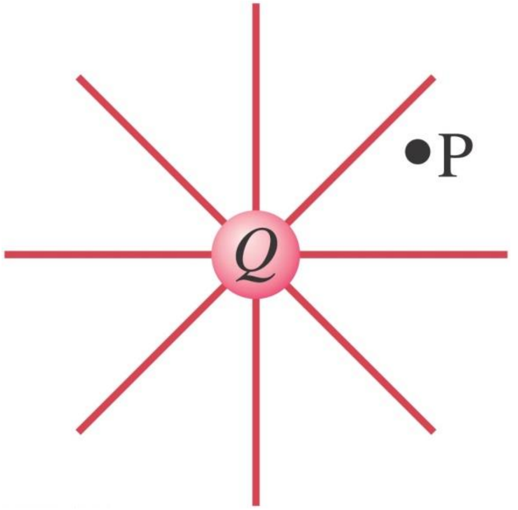
Faraday, 1791-1867



- In the figure on the right, if a second charge (call it P) is placed near the first charge, it feels a force exerted by the electric field that is there (say, at point P). The electric field at point P is considered to interact directly with charge Q to produce the force on P (and Q).

The Electric Field

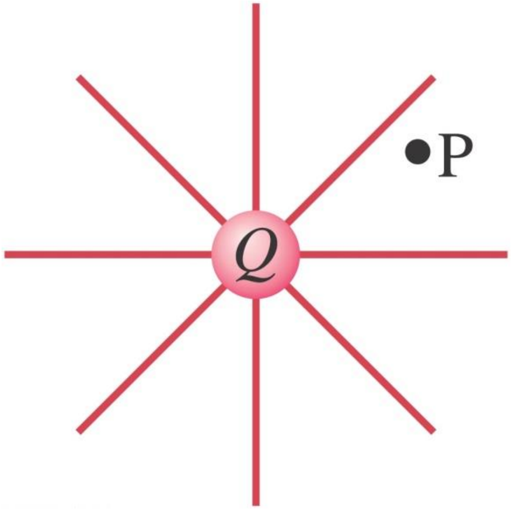
Faraday, 1791-1867



- We can, in principle, investigate the electric field surrounding a charge or group of charges by measuring the force on a small positive test charge at rest.
- By a test charge, we mean a charge so small that the force it exerts does not significantly affect the charges that create the field.

The Electric Field

Faraday, 1791-1867



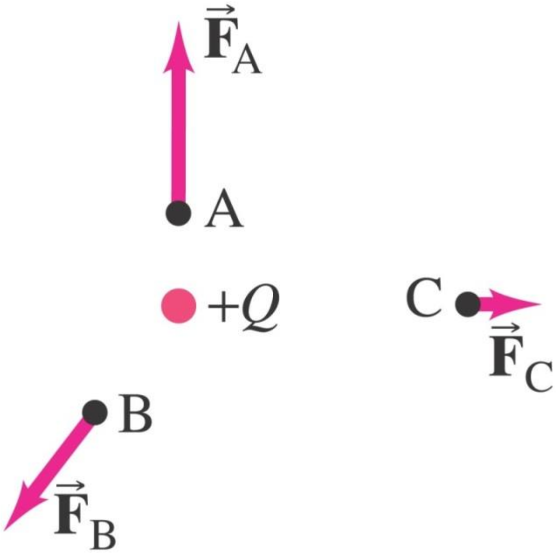
- The electric field is defined in terms of the force on such a positive test charge. In particular, the electric field, $\vec{E}$, at any point in space is defined as the force, $\vec{F}$, exerted on a tiny positive test charge placed at that point divided by the magnitude of the test charge q:

$$\vec{E} = \frac{\vec{F}}{q}$$

The Electric Field

Faraday, 1791-1867

$$\vec{E} = \frac{\vec{F}}{q}$$



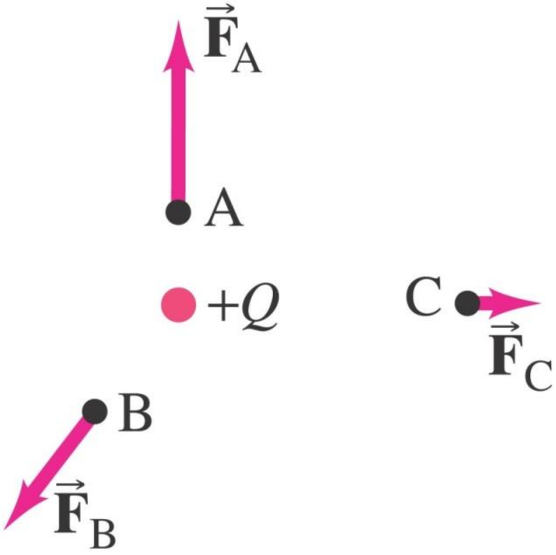
- The electric field at any point in space can be measured, based on the above equation. For simple situations involving one or several point charges, we can calculate $\vec{E}$. For example, the electric field at a distance, r , from a single point charge, Q, would have magnitude

$$E = \frac{F}{q} = \frac{kqQ/r^2}{q} = k\frac{Q}{r^2}$$

The Electric Field

Faraday, 1791-1867

$$\vec{E} = \frac{\vec{F}}{q}$$



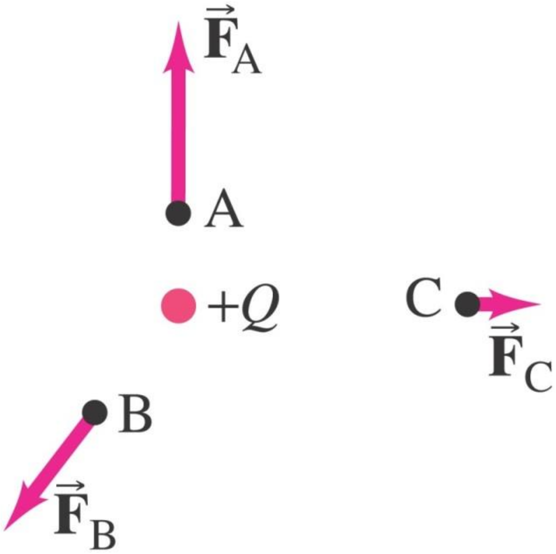
$$E = \frac{F}{q} = \frac{kqQ/r^2}{q} = k\frac{Q}{r^2}$$

- E depends only on the charge Q which produces the field, and not on the value of the test charge q. We can also refer to the above equation as the electric field form of Coulomb’s law.

The Electric Field

Faraday, 1791-1867

$$\vec{E} = \frac{\vec{F}}{q}$$

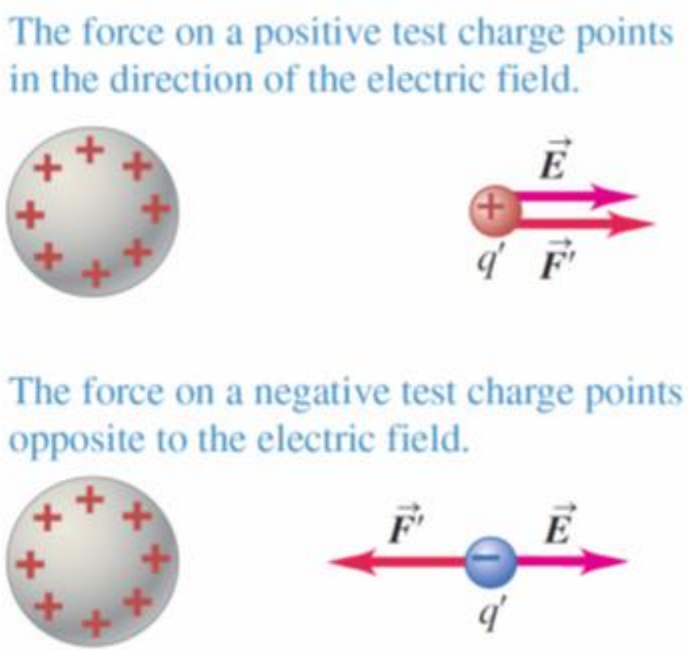


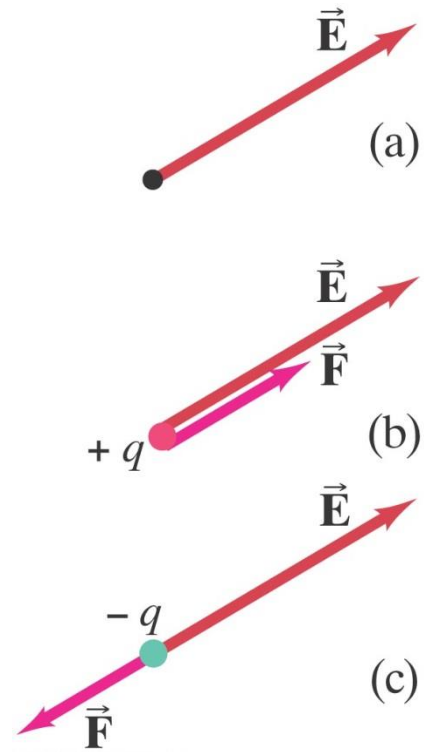
$$E = \frac{F}{q} = \frac{kqQ/r^2}{q} = k\frac{Q}{r^2}$$

- If we are given the electric field $\vec{E}$ at a given point in space, then we can calculate the force $\vec{F}$ on any charge q placed at that point by writing

$$\vec{F} = q\vec{E}$$

The Electric Field





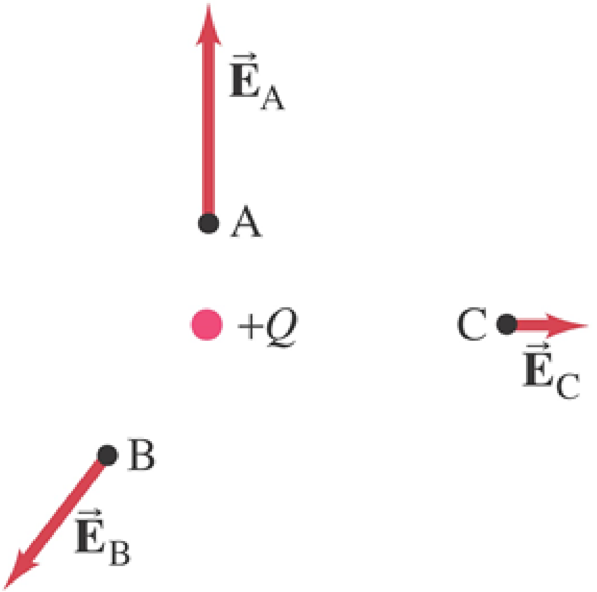
$$\vec{F} = q\vec{E}$$

- The above equation is valid even if q is not small as long as q does not cause the charges creating $\vec{E}$ to move. If q is positive, $\vec{F}$ and $\vec{E}$ point in the same direction. If q is negative, $\vec{F}$ and $\vec{E}$ point in opposite directions.

The Electric Field

Faraday, 1791-1867

$$\vec{E} = \frac{\vec{F}}{q}$$



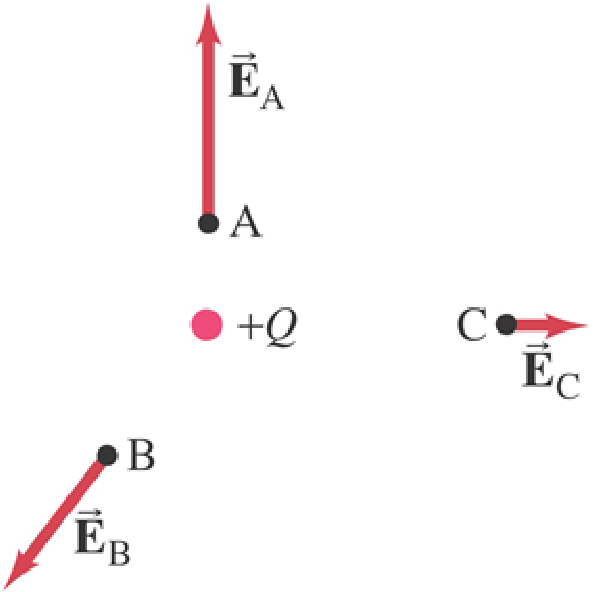
$$E = \frac{F}{q} = \frac{kqQ/r^2}{q} = k\frac{Q}{r^2}$$

- Since the electric field is a vector, it is sometimes referred to as a vector field. We could indicate the electric field with arrows at various points in a given situation, such as at A, B, and C above.

The Electric Field

Faraday, 1791-1867

$$\vec{E} = \frac{\vec{F}}{q}$$



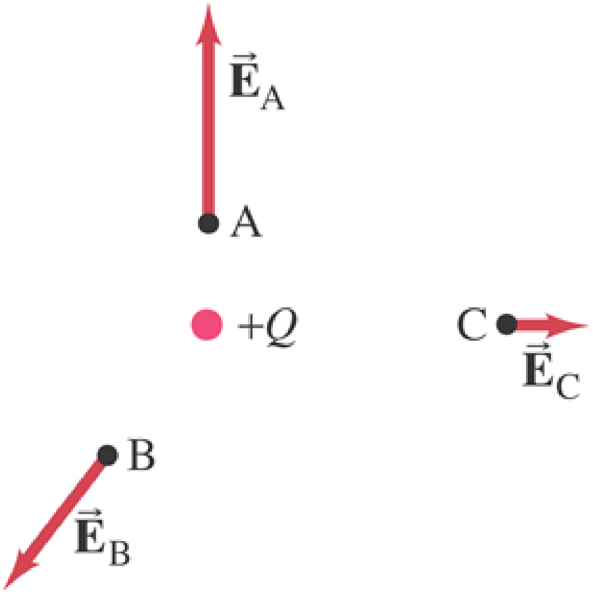
$$E = \frac{F}{q} = \frac{kqQ/r^2}{q} = k\frac{Q}{r^2}$$

- The directions of the electric field at points A, B and C are the same as that of the forces shown earlier, but the magnitudes (arrow lengths) are different, since we divide the forces at points A, B and C in the previous slide by q to get the electric field.

The Electric Field

Faraday, 1791-1867

$$\vec{E} = \frac{\vec{F}}{q}$$

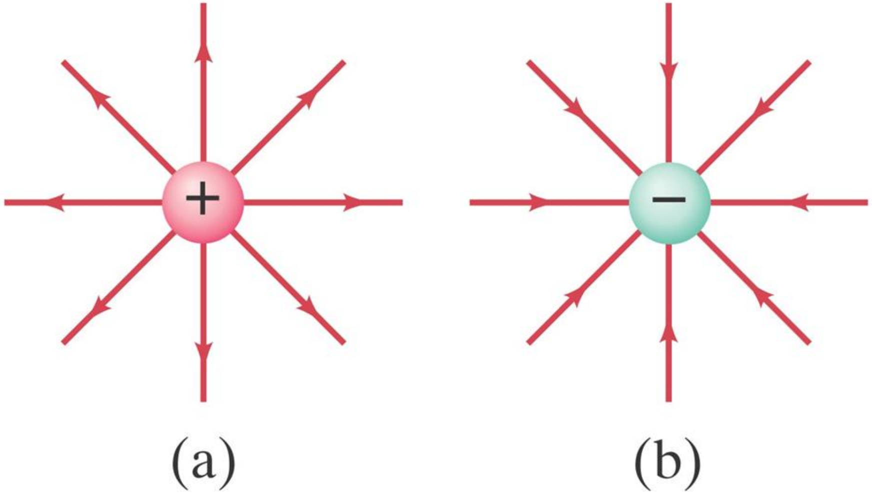


$$E = \frac{F}{q} = \frac{kqQ/r^2}{q} = k\frac{Q}{r^2}$$

- To indicate the electric field in such a way at many points, however, would result in many arrows, which might appear complicated or confusing. To avoid this, we use another technique, that of field lines.

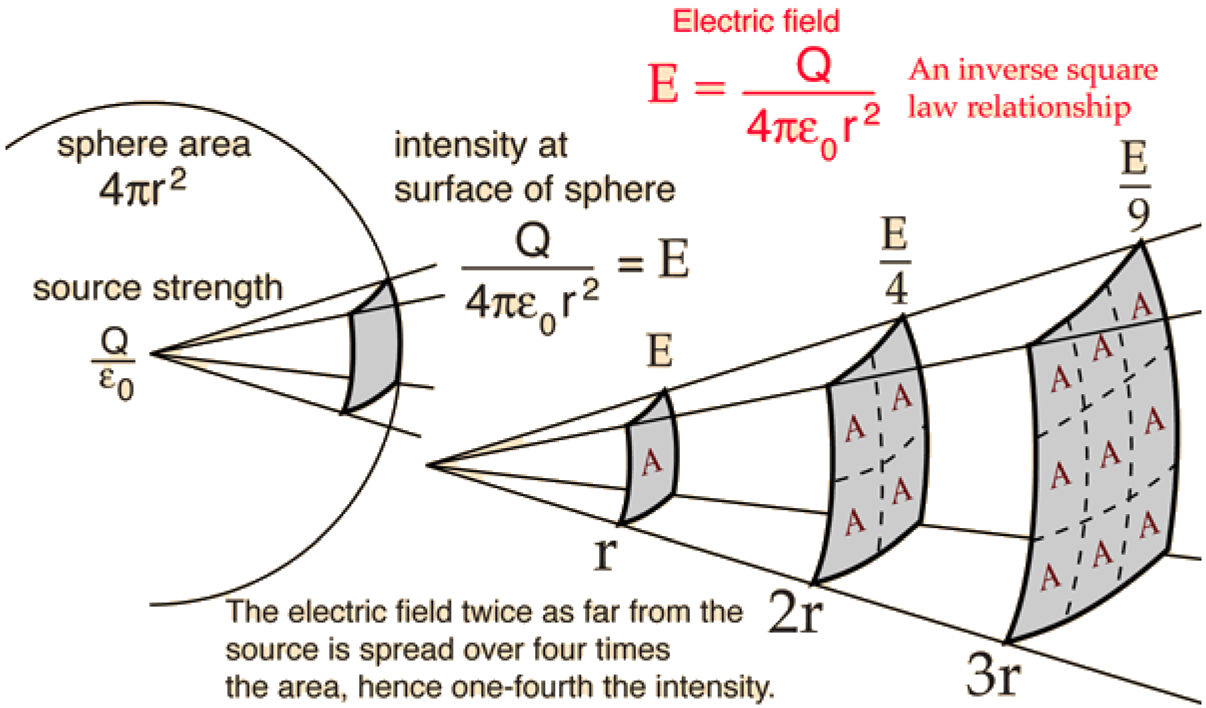
The Electric Field

Faraday, 1791-1867



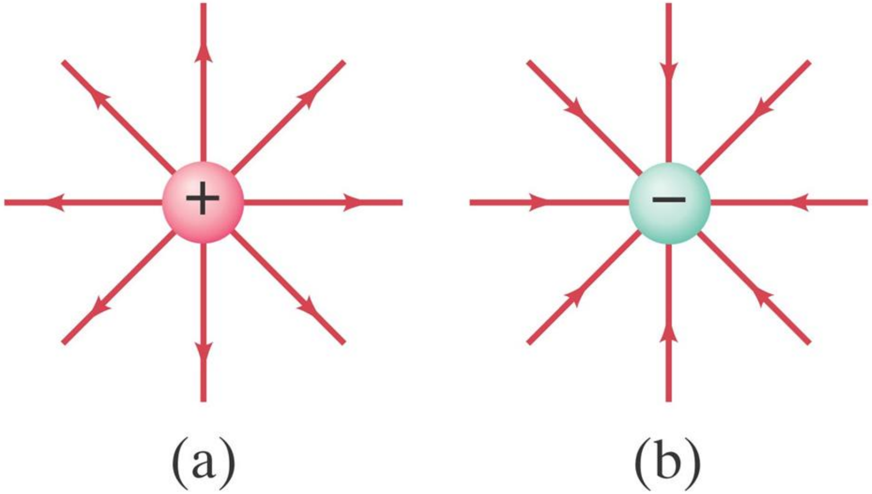
- To visualise the electric field, we draw a series of lines to indicate the direction of the electric field at various points in space. These **electric field lines** (sometimes called **lines of force**) are drawn so that they indicate the direction of the force due to the given field on a positive test charge.

Inverse Square Law



The Electric Field

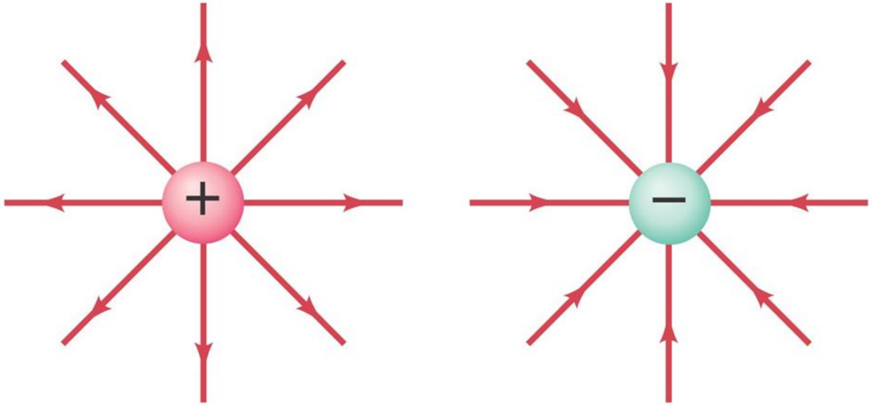
Faraday, 1791-1867



- In part (a), the lines point radially outward from the charge, and in part (b), they point radially inward toward the charge because that is the direction the force would be on a positive test charge in each case.
- We can draw the lines so that the number of lines starting on a positive charge, or ending on a negative charge, is proportional to the magnitude of the charge.

The Electric Field

Faraday, 1791-1867

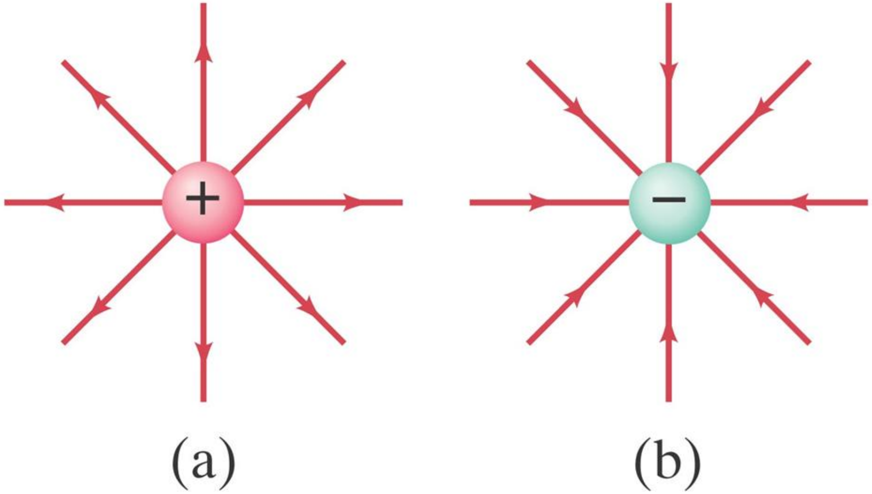


(a) (b)

- Nearer the charge, where the electric field is greater $(F \propto \frac{1}{r^2})$, the lines are closer together. This is a general property of electric field lines: the closer together the lines are, the stronger the electric field in that region.

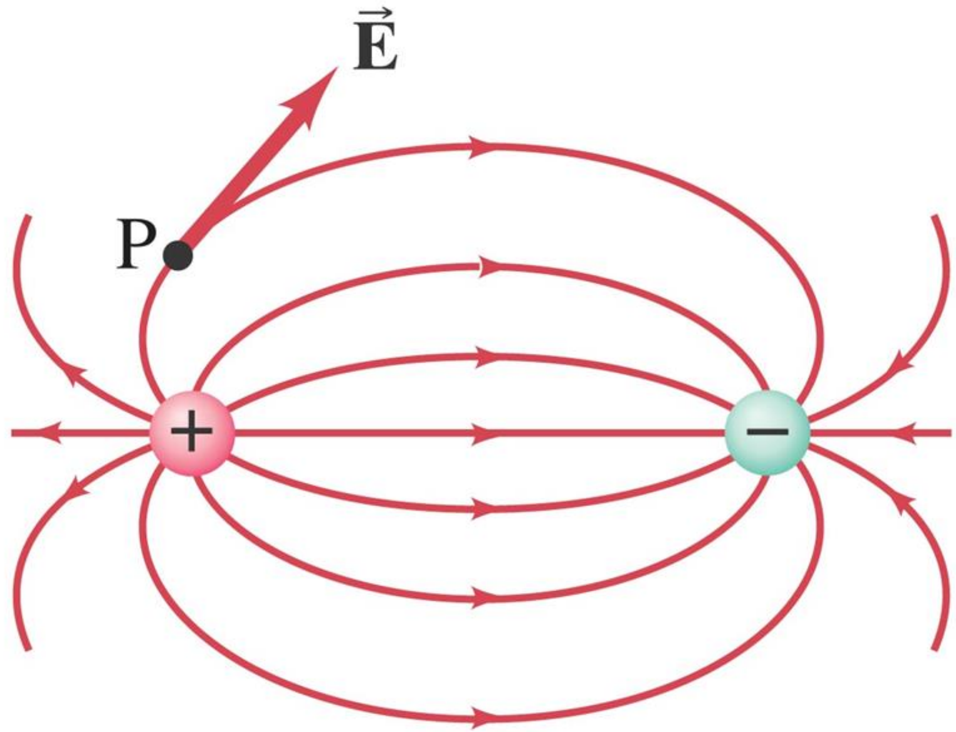
The Electric Field

Faraday, 1791-1867



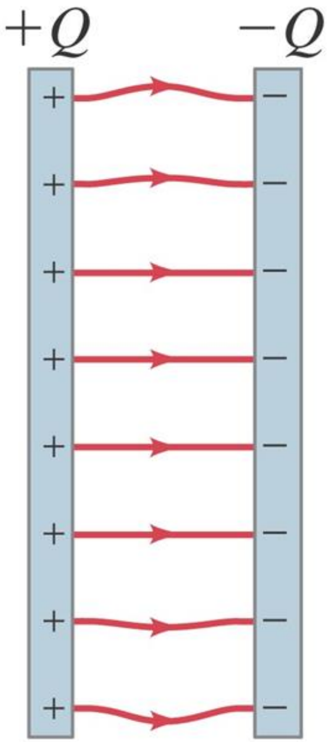
- In fact, electric field lines can be drawn so that the number of lines crossing any unit area perpendicular to $\vec{E}$ is proportional to the magnitude of the electric field.

The Electric Field



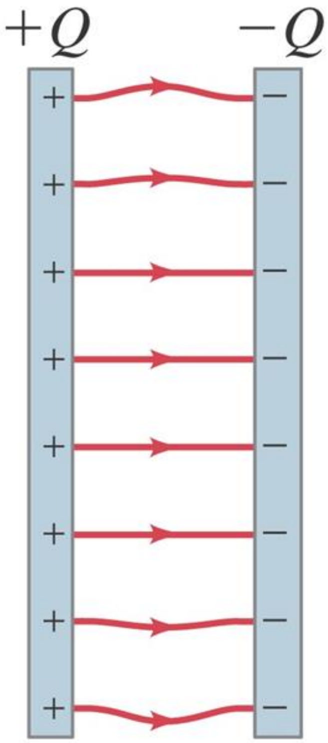
- The electric field lines are curved in this case, and are directed from the positive charge to the negative charge. The direction of the electric field at any point is tangent to the field line at that point, as shown by the vector arrow $\vec{E}$ at point P.

The Electric Field



- For the two plates, the electric field lines start out perpendicular to the surface of the metal plates, and go directly from one plate to the other, as we expect.
- The field lines between two close plates are parallel and equally spaced in the central region, but fringe outward near the edges.

The Electric Field



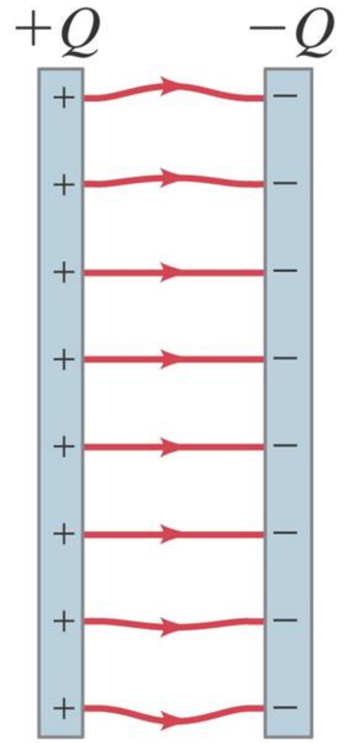
- Thus, in the central region, the electric field has the same magnitude at all points, and we can write

$$E = constant = \frac{\sigma}{\epsilon_0}$$ σ is the charge/ unit area

The Electric Field

$$E = constant = \frac{\sigma}{\epsilon_0}$$

σ is the charge/ unit area

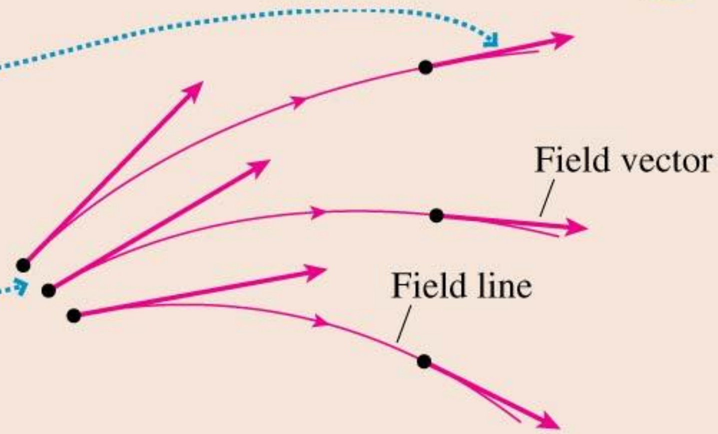


- The fringing of the field near the edges can often be ignored, particularly if the separation of the plates is small compared to their height and width.

Have-a-Read: The Electric Field

TACTICS BOX 26.1 **Drawing and using electric field lines**

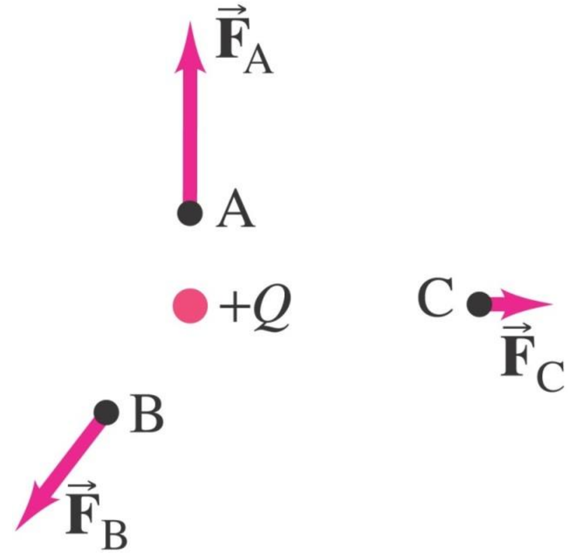
1. Electric field lines are continuous curves drawn tangent to the electric field vectors. Conversely, the electric field vector at any point is tangent to the field line at that point.
2. Closely spaced field lines represent a larger field strength, with longer field vectors. Widely spaced lines indicate a smaller field strength.
3. Electric field lines never cross.
4. Electric field lines start from positive charges and end on negative charges.



The Electric Field

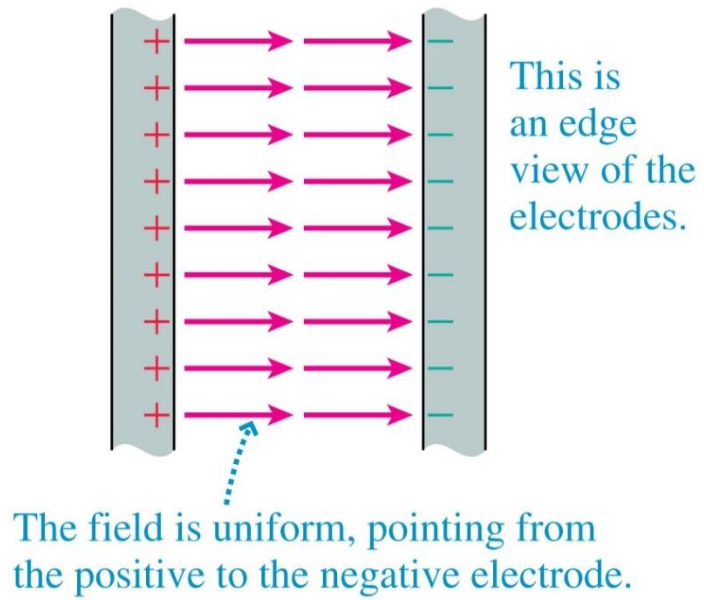
Faraday, 1791-1867

$$\vec{E} = \frac{\vec{F}}{q}$$



- If the electric field at a given point in space is due to more than one charge, the individual fields (call them $\vec{E}_1$, $\vec{E}_2$, etc.) due to each charge are added vectorially (**the superposition principle**) to get the total field at that point: $$\vec{E} = \vec{E}_1 + \vec{E}_2 + \cdots$$
- Let's now see how we calculate the electric field for a continuous charge distribution.

Electric Field Calculations



- In many cases, we can treat charge as being distributed continuously.
- We can divide a charge distribution into infinitesimal charges dQ, each of which we assume to be a tiny point charge.

Electric Field Calculations

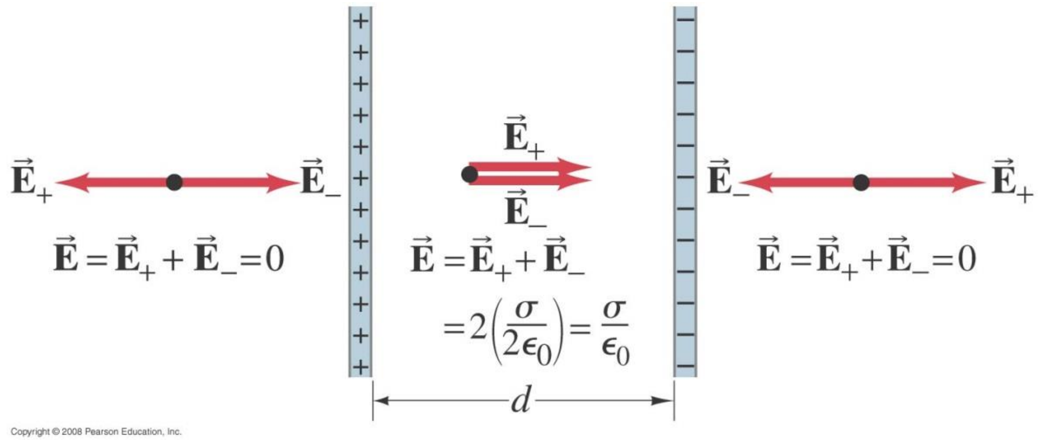
- The contribution to the electric field at a distance r from each dQ is

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dQ}{r^2}$$

- Then the electric field, $\vec{E}$, at any point is obtained by summing over all the infinitesimal contributions, which is the integral

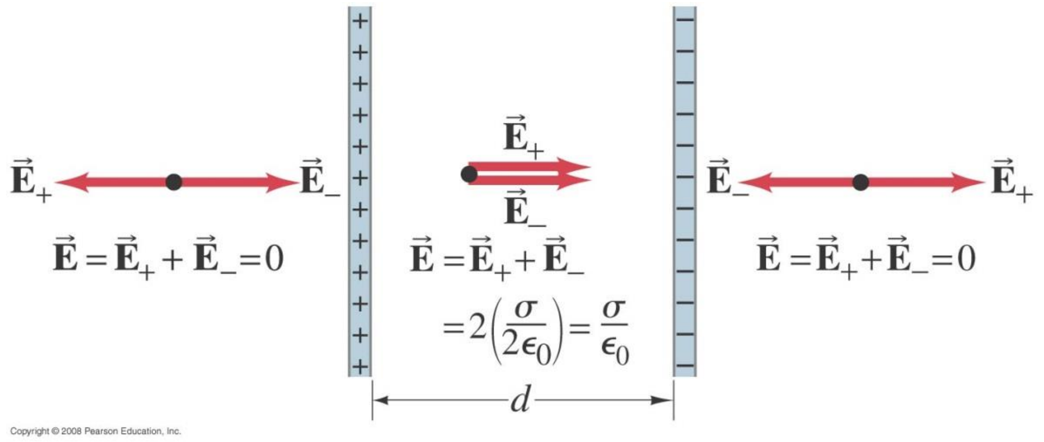
$$\vec{E} = \int d\vec{E}$$

Electric Field Calculations for Two Parallel Plates



- Each plate sets up an electric field of magnitude $\sigma/2\epsilon_o$.
- The field due to the positive plate points away from that plate whereas the field due to the negative plate points towards that plate.

Electric Field Calculations for Two Parallel Plates

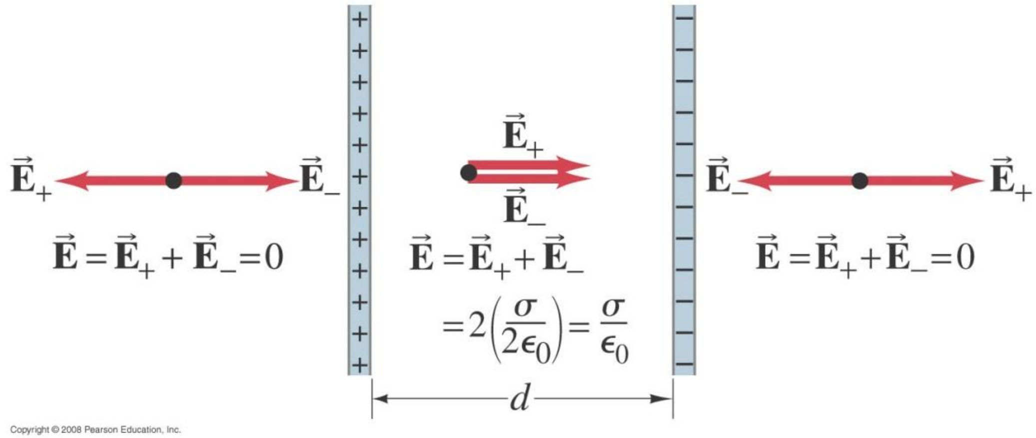


- In the region between the plates, the fields add together as shown.

$$E = E_+ + E_- = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$$

- The field is uniform, since the plates are very large compared to their separation, so this result is valid for any point, whether near one or the other of the plates, or midway between them as long as the point is far from the ends.

Electric Field Calculations for Two Parallel Plates



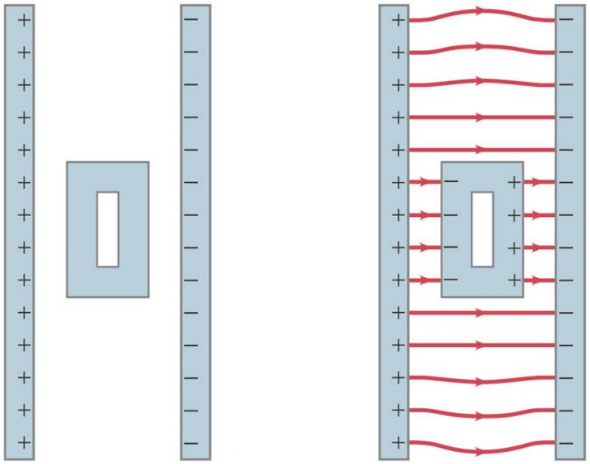
- Outside the plates, the fields cancel,

$$E = E_+ + E_- = \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0} = 0$$

- Both these results (as shown in the above figure) are valid ideally for infinitely large plates, but in the real world they are good approximations for finite plates if the separation is much less than the dimensions of the plate and for points not too close to the edge.

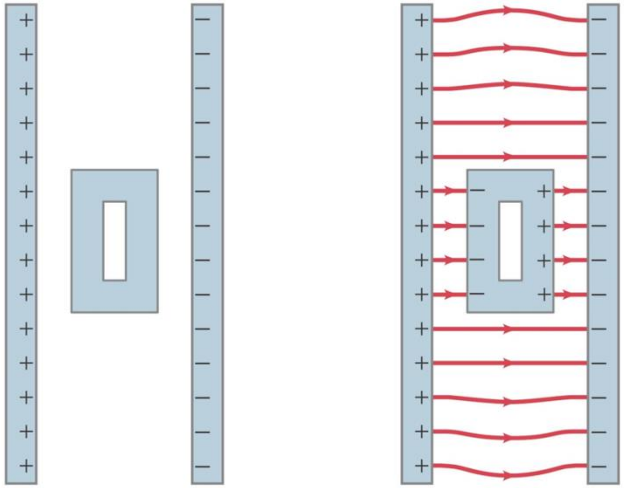
2. Electric Fields and Conductors

Electric Fields and Conductors



- The electric field inside a conductor is zero in the static situation– that is, when the charges are at rest.
- If there were an electric field within a conductor, there would be a force on the free electrons. The electrons would move until they reached positions where the electric field, and therefore the electric force on them, did become zero.
- This reasoning has some interesting consequences that are borne out by experiment.

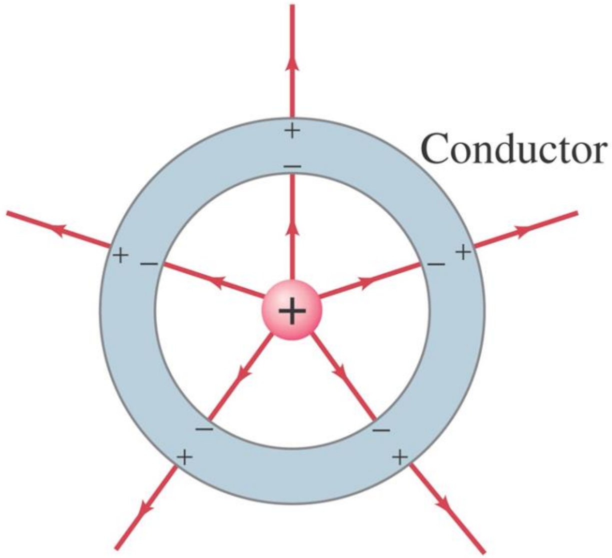
Electric Fields and Conductors



- Any net charge on a conductor distributes itself on the surface. If there were charges inside, there would be an electric field.

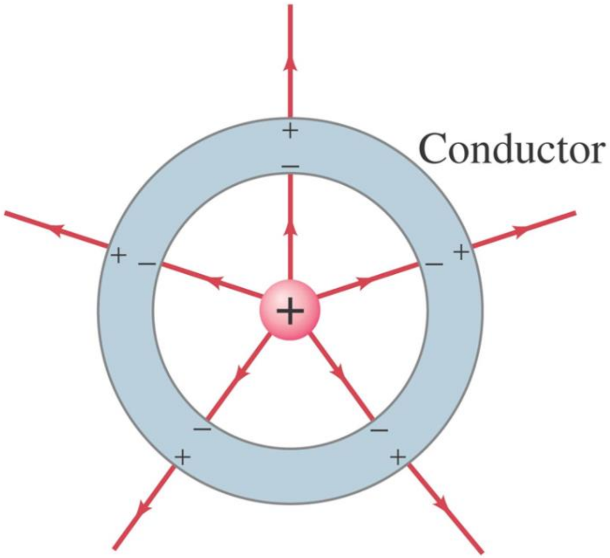
- **E.g.** For a negatively charged conductor, you can imagine that the negative charges repel one another and race to the surface to get as far from one another as possible.

Electric Fields and Conductors



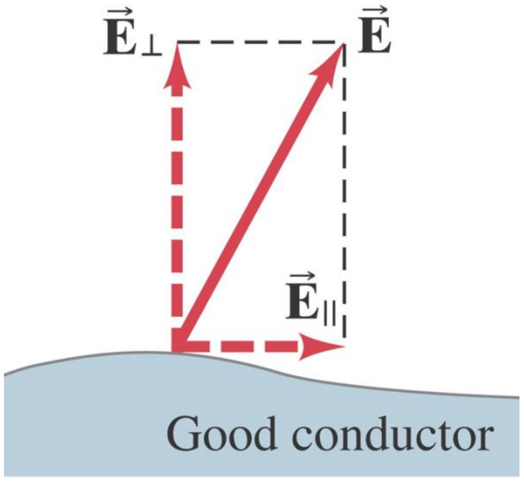
- Suppose that a positive charge Q is surrounded by an isolated uncharged metal conductor whose shape is a spherical shell.
- Because there can be no field within the metal, the lines leaving the central positive charge must end on negative charges on the inner surface of the metal. Thus, an equal amount of negative charge, $-Q$, is induced on the inner surface of the spherical shell.

Electric Fields and Conductors



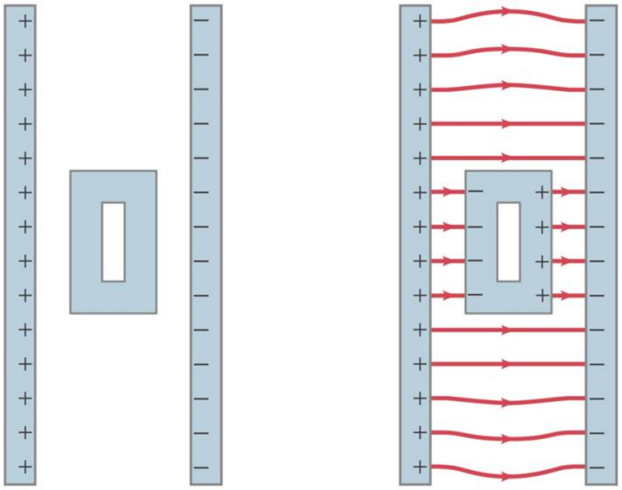
- Then, since the shell is neutral, a positive charge of the same magnitude, $+Q$, must exist on the outer surface of the shell.
- Thus, although no field exists in the metal itself, an electric field exists outside of it, as shown above, as if the metal were not even there.

Electric Fields and Conductors



- A related property of static electric fields and conductors is that the electric field is always perpendicular to the surface outside of a conductor.
- If there were a component of $\vec{E}$ parallel to the surface, it would exert a force on free electrons at the surface, causing the electrons to move along the surface until they reached positions where no net force was exerted on them parallel to the surface– that is until the electric field was perpendicular to the surface.

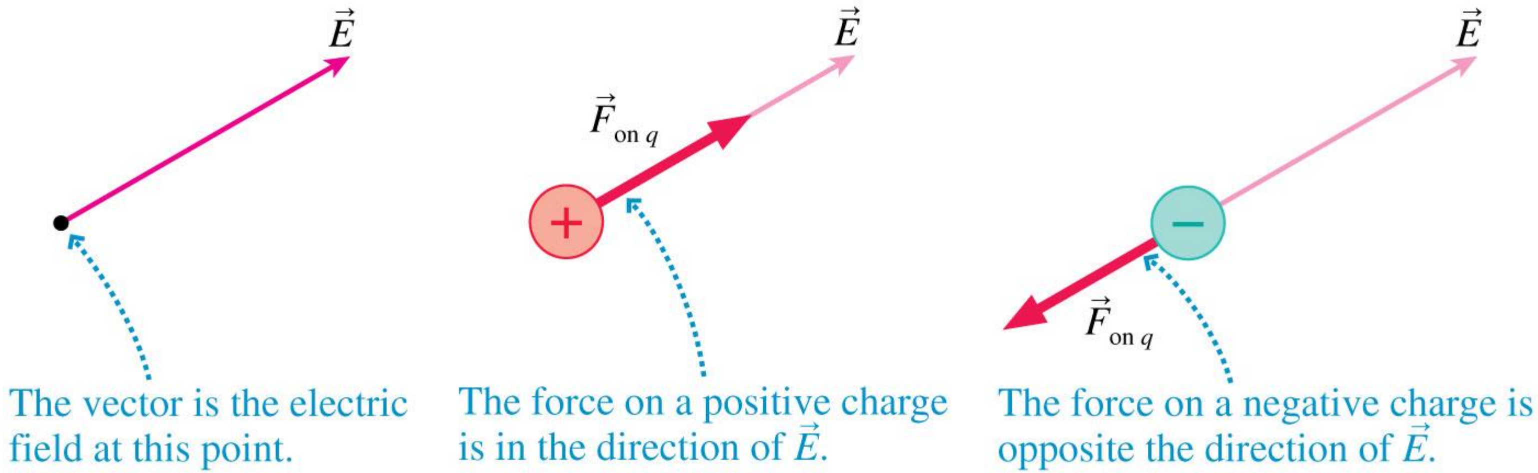
Electric Fields and Conductors



- As a consequence of what we’ve just talked about, a conducting box, as shown above, is an effective device for shielding delicate instruments and electronic circuits from unwanted external electric fields. Such a device is known as a ‘Faraday cage’.

3. Motion of a Charged Particle in an Electric Field

Motion of a Charged Particle in an Electric Field



- If an object having an electric charge, q, is at a point in space where the electric field is $\vec{E}$, the force on the object is given by

$$\vec{F}_{on\ q} = q\vec{E}$$

Motion of a Charged Particle in an Electric Field

- The electric field exerts a force $\vec{F}_{on\ q} = q\vec{E}$ on a charged particle.
- If this is the only force acting on $\boldsymbol{q}$, it causes the charged particle to accelerate with

$$\vec{a} = \frac{\vec{F}_{\text{on } q}}{m} = \frac{q}{m}\vec{E}$$

- In a uniform field, the acceleration is constant:

$$a = \frac{qE}{m} = \text{constant}$$

Summary of today's Lecture

1. The Electric Field
2. Electric Field Calculations for Continuous Charge Distributions.
3. Electric Fields and Conductors.
4. Motion of a Charged Particle in an Electric Field.

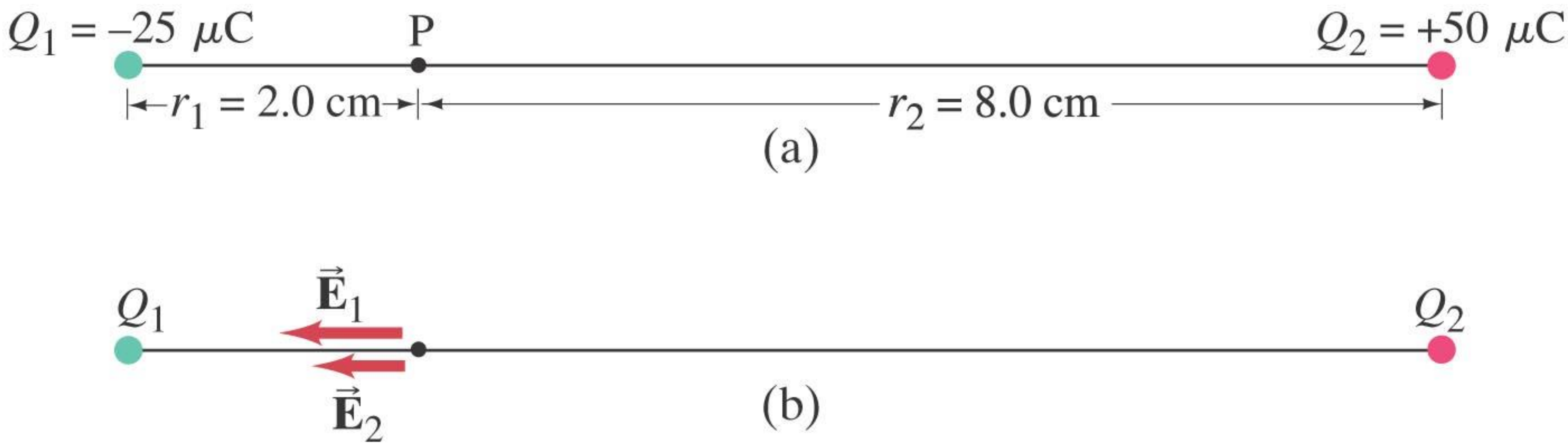
Lectures 14-15: Optional Reading

- **Ch. 21.7**, Electric Field Calculations for Continuous Charge Distributions; p.660-662
- **Ch. 21.9**, Electric Fields and Conductors; p.665
- **Ch. 21.10**, Motion of a Charged Particle in an Electric Field; p.666

Home Work

Do not forget to attempt the **Additional Problems** for this lecture before logging in to **Mastering Physics** to complete your assignments.

Possible Exam Question: Have a Read (p.658)

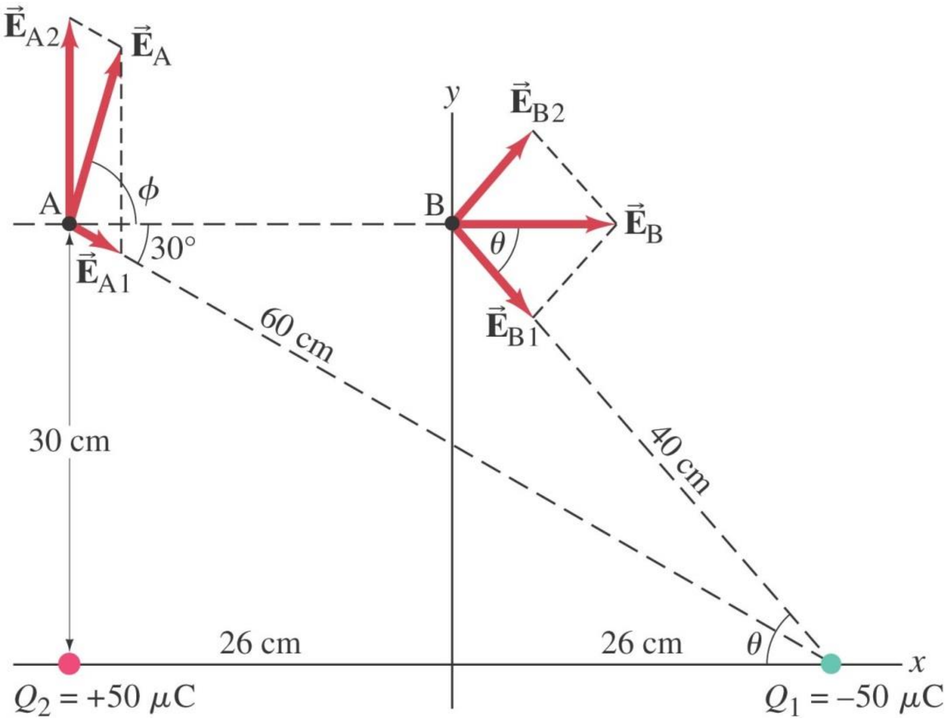


Q.1 Two point charges are separated by a distance of 10.0 cm. One has a charge of $-25\mu C$ and the other $+50\mu C$.

(a) Determine the direction and magnitude of the electric field at a point P between the two charges that is 2.0 cm from the negative charge (as shown in figure (a)).

(b) If an electron ($\text{mass} = 9.11 \times 10^{-31}\text{kg}$) is placed at rest at P and then released (as in figure (b)), what will be its initial acceleration (direction and magnitude)?

Possible Exam Question: Have a Read (p.658)

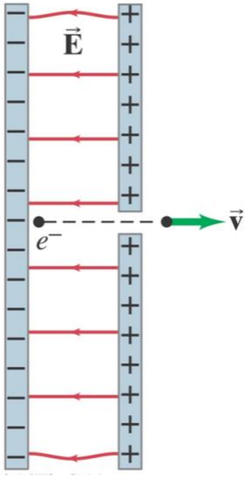


Q.2 Calculate the total electric field

(a) at point A and

(b) at point B due to both charges, Q_1 and Q_2.

Possible Exam Question: Have a Read (p.666)



Q.3 An electron (mass $\mathrm{m} = 9.1 \times 10^{-31}\mathrm{kg}$) is accelerated in the uniform field $\vec{E}$ ($\mathrm{E} = 2.0 \times 10^{4}\mathrm{N/C}$) between two parallel charged plates. The separation of the plates is 1.5 cm. The electron is accelerated from rest near the negative plate and passes through a tiny hole in the positive plate.

(a) With what speed does it leave the hole?

(b) Show that the gravitational force can be ignored. Assume the hole is so small that it does not affect the uniform field between the plates.