Science A Physics

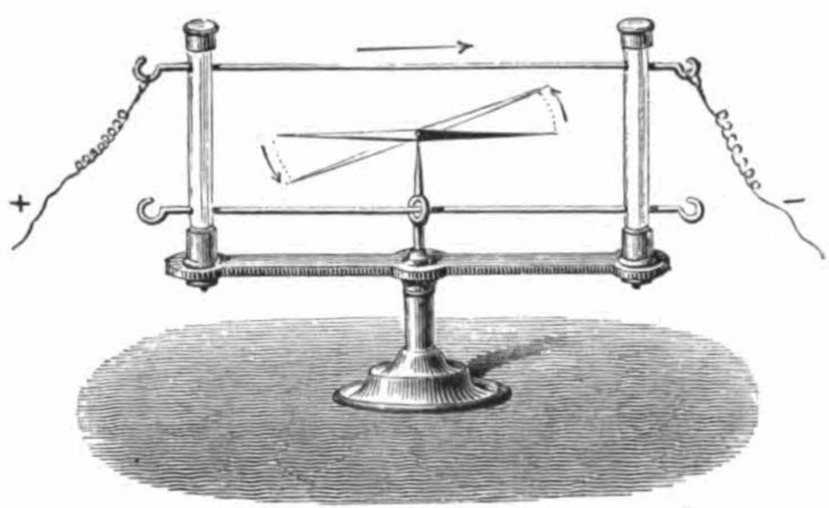
Lecture 17:

Magnetic Fields; Part 1

Aims of today's lecture

1. Magnetic Field due to a Straight Wire
2. Force between Two Parallel Wires
3. Definition of the Ampere and the Coulomb
4. Ampère's Law
5. Magnetic Field of a Solenoid
6. Electromagnets and Solenoids—Applications

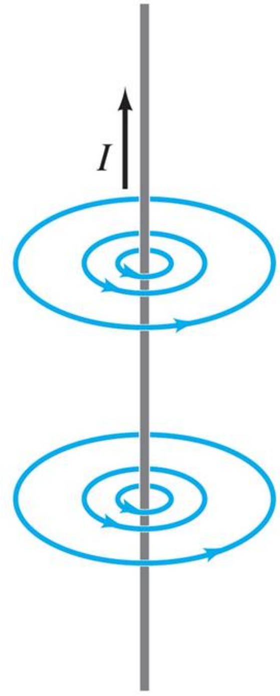
Sources of Magnetic Field



- As we have seen, there are effects (forces and torques) that a magnetic field has on electric currents and on moving charges.
- We've also seen that magnetic fields are produced not only by magnets, but also by electric currents (Ørsted's discovery).
- It is this aspect of magnetism, the production of magnetic fields, that we now discuss.

1. Magnetic Field Due to a Straight Wire

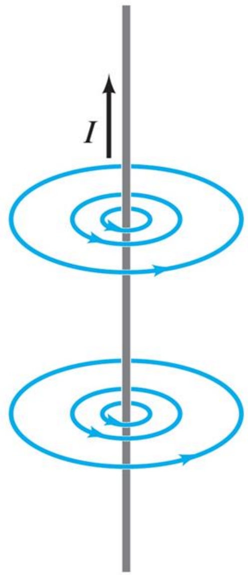
Magnetic Field Due to a Straight Wire



- Careful experiments show that the magnetic field $\boldsymbol{B}$ due to a long straight wire at a point near it is directly proportional to the current $\boldsymbol{I}$ in the wire and inversely proportional to the distance $\boldsymbol{r}$ from the wire:

$$B \propto \frac{I}{r}$$

Magnetic Field Due to a Straight Wire



$$B \propto \frac{I}{r}$$

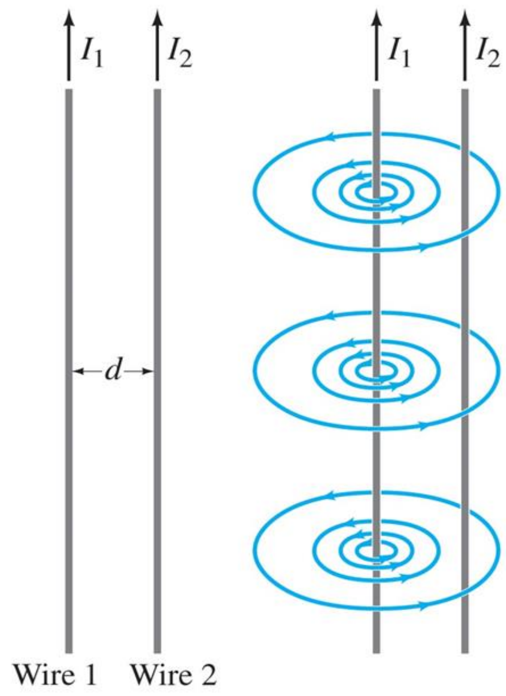
- The relation $B \propto l/r$ is valid as long as r, the perpendicular distance to the wire, is much less than the distance to the ends of the wire (i.e., the wire is long)
- The proportionality constant is written as $\mu_0/2\pi$; thus,

$$B = \frac{\mu_0}{2\pi}\frac{I}{r}$$

- The value of the constant μ_0, which is called the **permeability of free space**, is $\mu_0 = 4\pi \times 10^{-7}\mathrm{T} \cdot \mathrm{m/A}$.

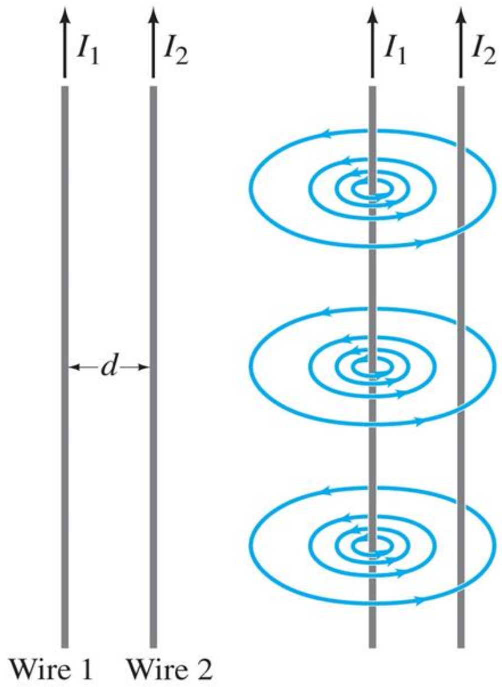
2. Force between Two Parallel Wires

Force Between Two Parallel Wires



- A wire carrying a current produces a magnetic field, and a current-carrying wire feels a force when placed in a magnetic field.
- Thus, two current-carrying wires will exert a force on each other.
- In the above figure, each current produces a magnetic field that is 'felt' by the other, so each must exert a force on the other.

Force Between Two Parallel Wires



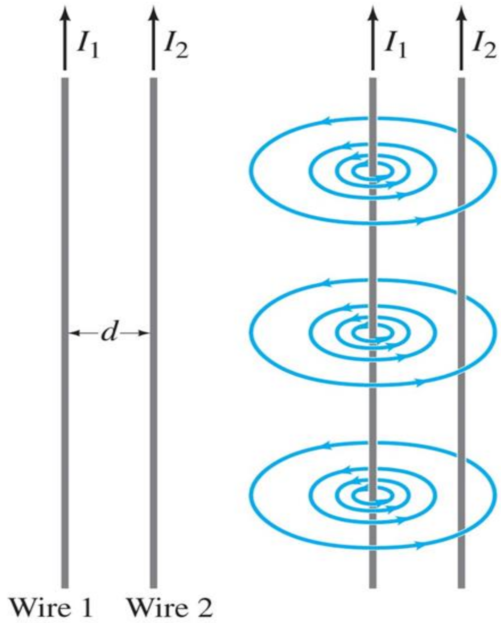
- For example, the magnetic field B_1 produced by I_1 at the location of wire 2 is

$$B = \frac{\mu_0}{2\pi}\frac{I_1}{d}$$

- According to $F_{max} = IlB$, the force F_2 exerted by B_1 on a length l_2 of wire 2, carrying current I_2, is

$$F_2 = I_2 B_1 l_2$$

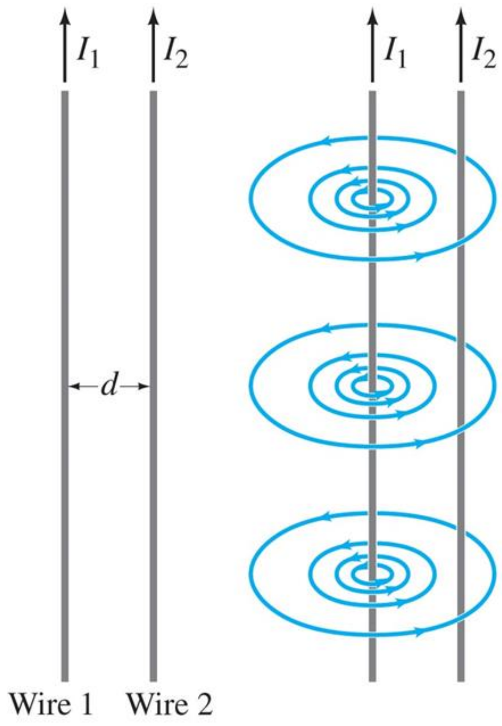
Force Between Two Parallel Wires



- Note that the force on I_2 is due only to the field produced by I_1. Of course, I_2 also produces a field, but it does not exert a force on itself.
  - We can substitute B_1 into the formula for F_2 and find that the force on a length l_2 of wire 2 is

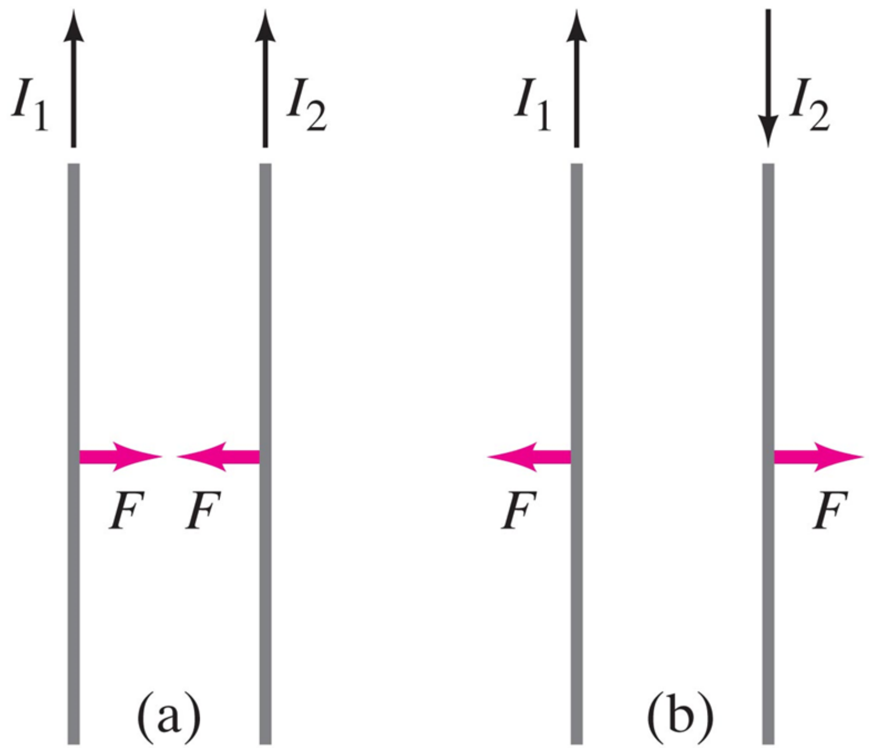
$$F_2 = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d} l_2$$

Force Between Two Parallel Wires



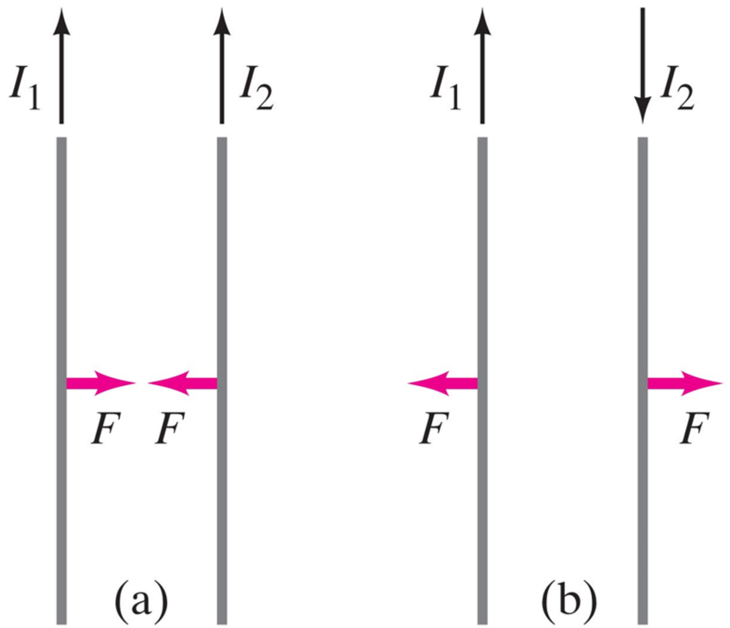
- If we use right-hand-rule-1, we see that the lines of B_1 are as shown above. Then using right-hand-rule-2, we see that the force exerted on I_2 will be to the left. That is, I_1 exerts an attractive force on I_2, which is true as long as the currents are in the same direction.

Force Between Two Parallel Wires



- If I_2 is in the opposite direction, the right-hand rule indicates that the force is in the opposite direction. That is, I_1 exerts a repulsive force on I_2, as shown in (b).

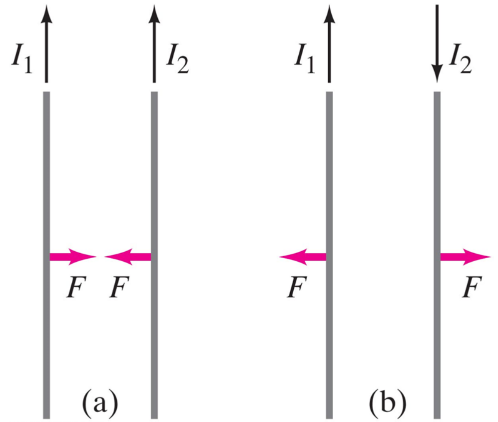
Force Between Two Parallel Wires



- Reasoning similar to the previous slide shows that the magnetic field produced by I_2 exerts an equal but opposite force on I_1.
- We also expect this to be true from Newton's 3rd law.
- Thus, parallel currents in the same direction attract each other, whereas parallel currents in the opposite direction repel.

3. Definitions of the Ampere and the Coulomb

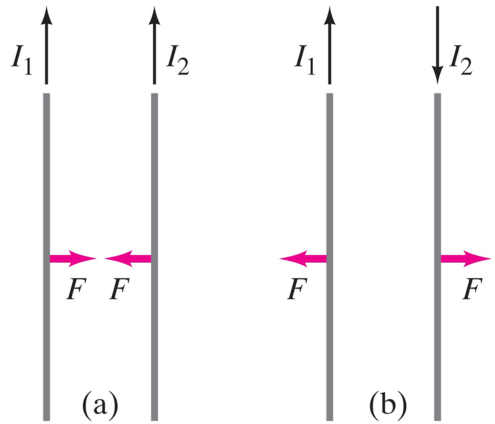
Definitions of the Ampere and the Coulomb



- We use the force between two parallel current-carrying wires, to define the ampere precisely. If $I_1 = I_2 = 1\mathrm{A}$ exactly, and the two wires are exactly 1 m apart, then

$$\frac{F}{l} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d} = \frac{(4\pi \times 10^{-7}\mathrm{T}\cdot\frac{\mathrm{m}}{\mathrm{A}})}{2\pi}\frac{(1\mathrm{A})(1\mathrm{A})}{1\mathrm{m}} = 2 \times 10^{-7}\mathrm{N/m}$$

Definitions of the Ampere and the Coulomb

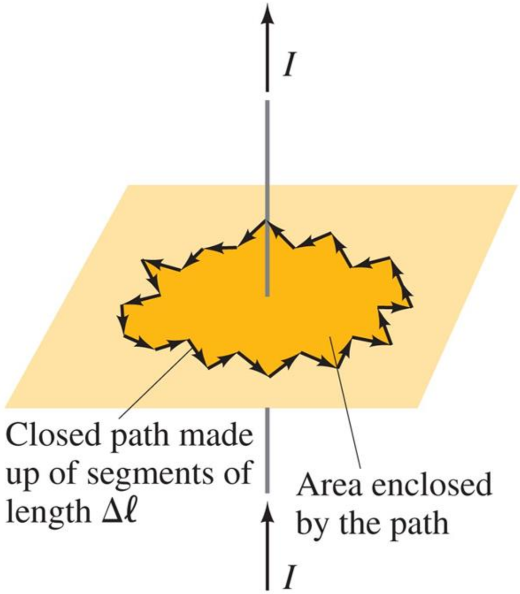


$$\frac{F}{l} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d} = \frac{(4\pi \times 10^{-7}\mathrm{T}\cdot\frac{\mathrm{m}}{\mathrm{A}})}{2\pi}\frac{(1\mathrm{A})(1\mathrm{A})}{1\mathrm{m}} = 2 \times 10^{-7}\mathrm{N/m}$$

- Thus, one **ampere** is defined as that current flowing in each of two long parallel wires 1 m apart, which results in a force of exactly $2 \times 10^{-7}\mathrm{N}$ per metre of length of wire.
- The **coulomb** is then defined as being exactly one ampere-second: $1\mathrm{C} = 1\mathrm{A}\cdot\mathrm{s}$.

4. Ampère’s Law

Ampère’s Law

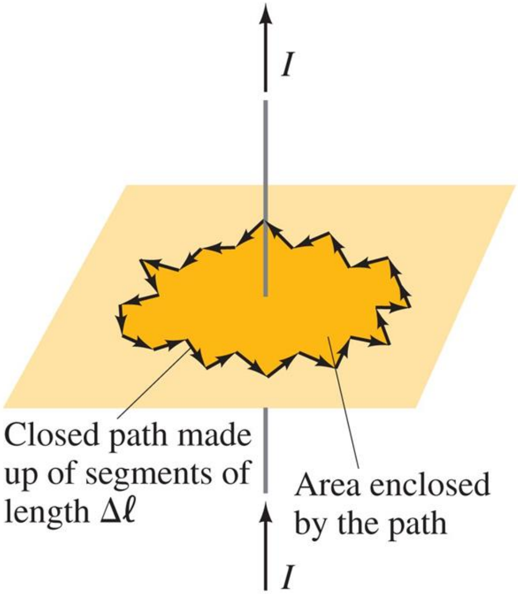


Andrè Marie Ampere (1775-1836)

- In section 1, we looked at the equation that gives the relation between the current in a long straight wire and the magnetic field it produces. This equation is valid only for a long straight wire.

Q. Is there a general relation between a current in a wire of any shape and the magnetic field around it?

Ampère's Law

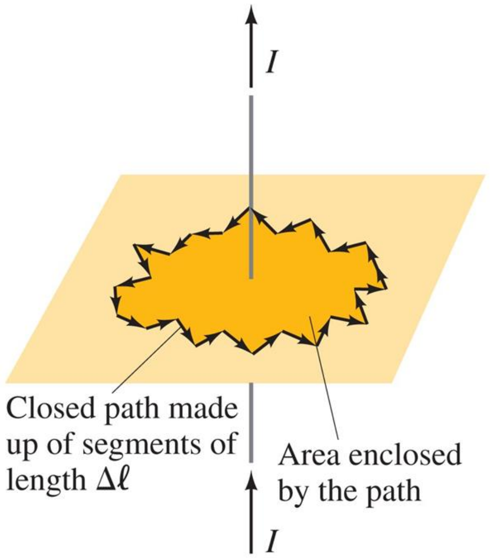


André Marie Ampere (1775-1836)

Q. Is there a general relation between a current in a wire of any shape and the magnetic field around it?

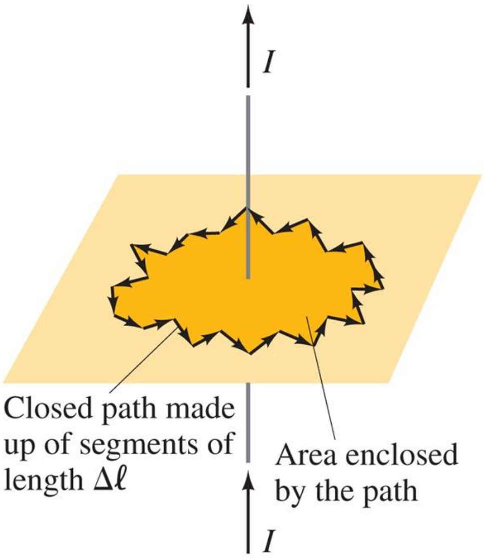
- Ampere proposed such a relation shortly after Øersted's discovery.

Ampère's Law



- Consider an arbitrary closed path around a current as shown in the above figure, and imagine this path as being made up of short segments each of length Δl.
- First, we take the product of the length of each segment times the component of $\vec{B}$ parallel to that segment (call this component $B_{\parallel}$).

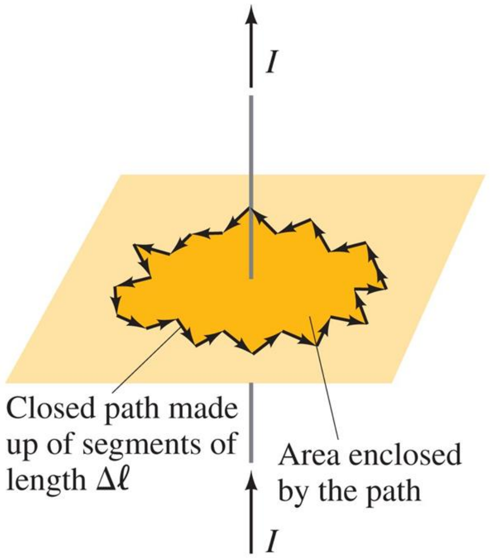
Ampère's Law



- If we now sum all these terms, according to Ampère, the result will be equal to μ_0 times the net current I_{encl} that passes through the surface enclosed by the path:

$$\sum B_{\parallel}\Delta l = \mu_0 I_{encl}$$

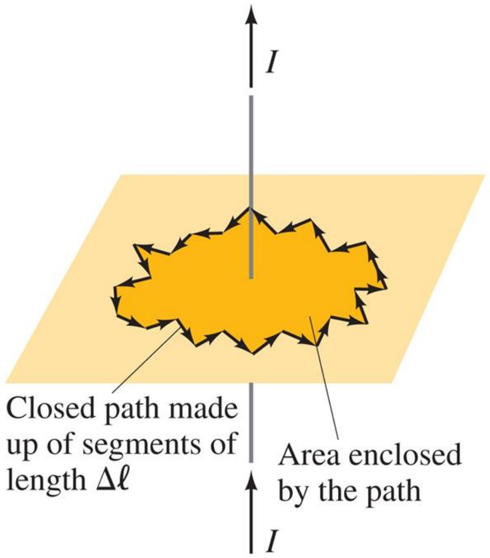
Ampère's Law



$$\sum B_{\parallel}\Delta l = \mu_0 I_{encl}$$

- The lengths Δl are chosen so that $B_{\parallel}$ is essentially constant along each length. The sum must be made over a closed path; and I_{encl} is the net current passing through the surface bounded by this closed path (orange in the above figure).

Ampère's Law

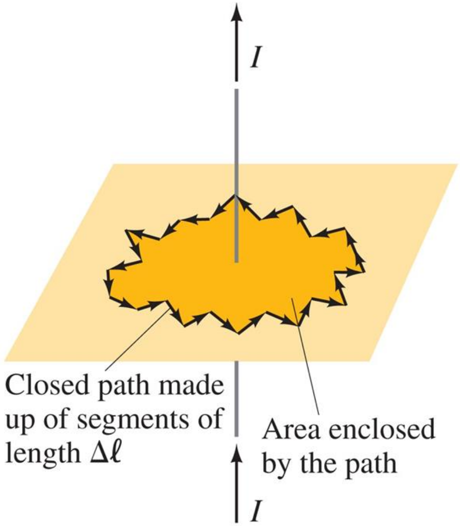


$$\sum B_{\parallel}\Delta l = \mu_0 I_{encl}$$

- In the limit $\Delta l \to 0$, this relation becomes

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

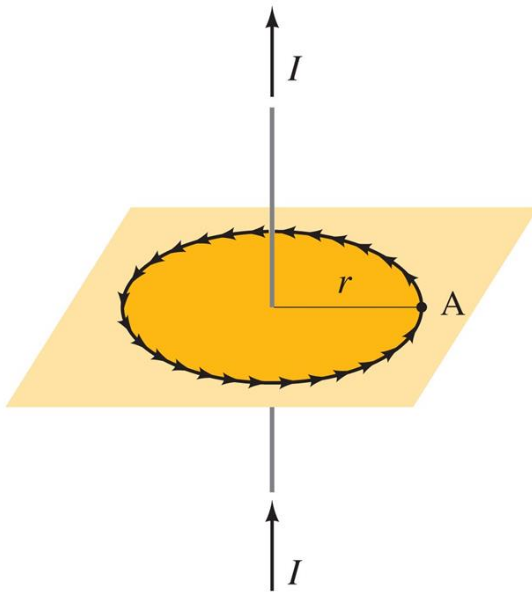
Ampère's Law



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

- The above equation is known as Ampère's law.
- The integrand is taken around a closed path, and I_{encl} is the current passing through the space enclosed by the chosen path or loop.

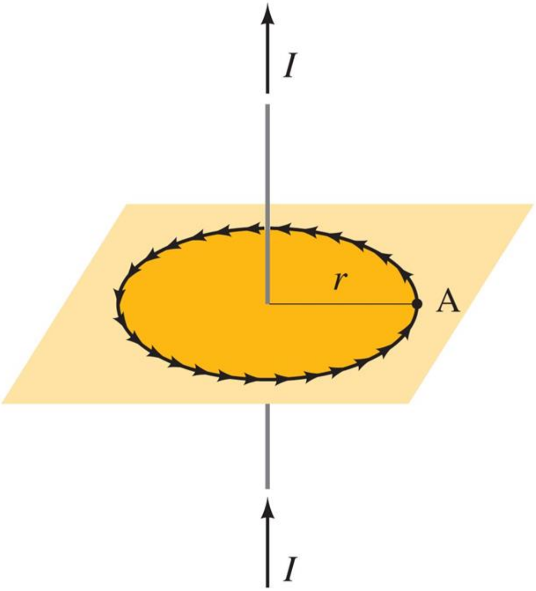
Ampère's Law



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

- To understand Ampère's law better, let us apply it to the simple case of a single long straight wire carrying a current I which we've already examined.
- Suppose we want to find the magnitude of $\vec{B}$ at some point A which is a distance r from the wire, as shown above.
- We know the magnetic field lines are circles with the wire at their centre.

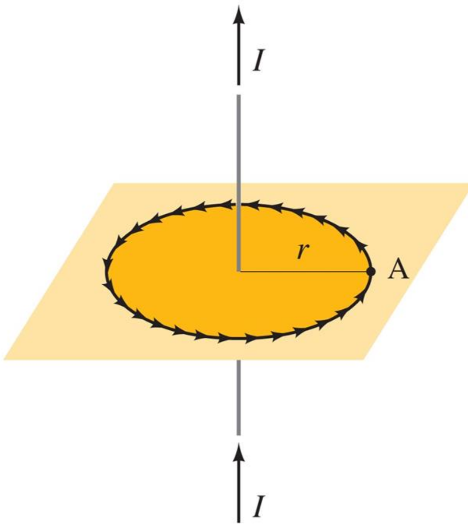
Ampère's Law



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

- To apply the above equation, we choose as our path of integration a circle of radius r.
- The choice of path is ours, so we choose one that will be convenient: at any point on this circular path, $\vec{B}$ will be tangent to the circle.
- Furthermore, since all points on the path are the same distance from the wire, by symmetry, we expect B to have the same magnitude at each point.

Ampère's Law



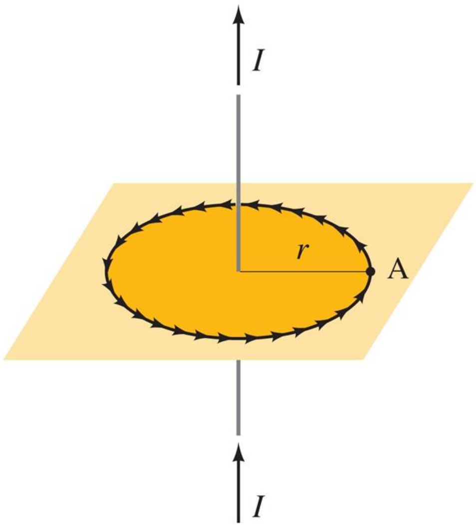
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

- Thus, for any short segment of the circle, $\vec{B}$ will be parallel to that segment, and (setting $I_{encl} = I$)

$$\mu_0 I = \oint \vec{B} \cdot d\vec{l}$$

$$= \oint B dl = B \oint dl = B(2\pi r)$$

Ampère's Law



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

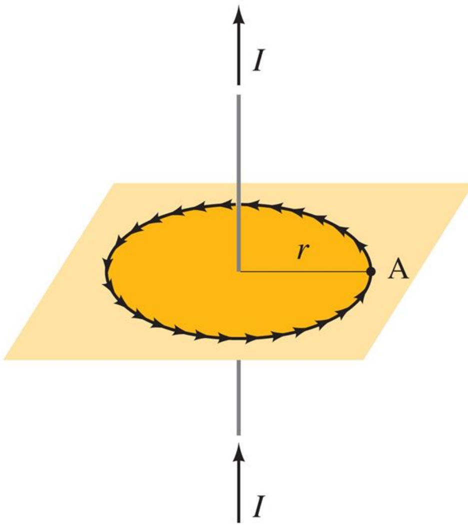
$$\mu_0 I = \oint \vec{B} \cdot d\vec{l}$$

$$= \oint B dl = B \oint dl = B(2\pi r)$$

where $\oint dl = 2\pi r$, the circumference of the circle. We solve for B, and obtain

$$B = \frac{\mu_0 I}{2\pi r}$$

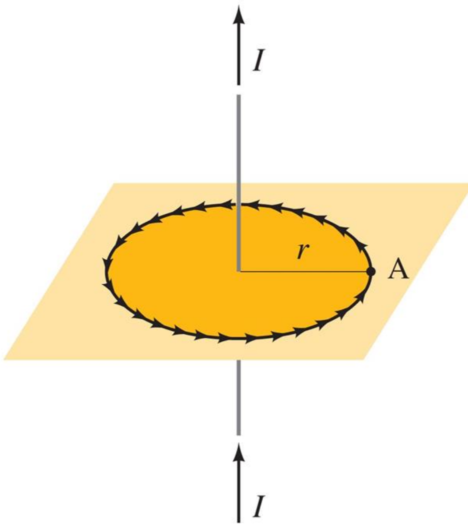
Ampère’s Law



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

- A great many experiments indicate that Ampère’s law is valid in general.
- However, as with Gauss’s law for the electric field, its practical value as a means to calculate the magnetic field is limited mainly to simple or symmetric situations.

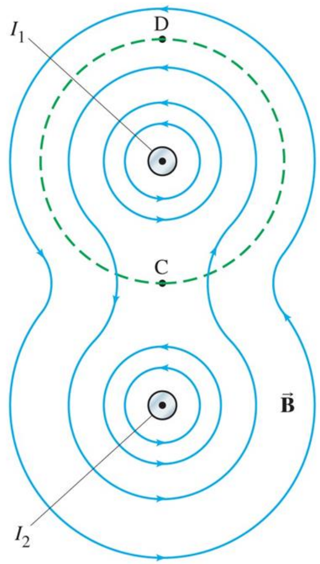
Ampère's Law



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

- Its importance is that it relates the magnetic field to the current in a direct and mathematically elegant way.
- Ampère's law is thus considered one of the basic laws of electricity and magnetism.
- It is valid for any situation where the currents and fields are steady and not changing in time, and no magnetic materials are present.

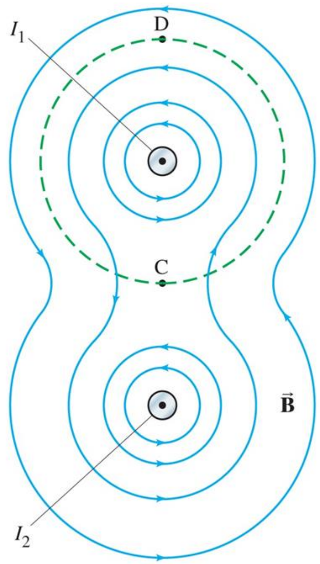
Ampère's Law



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

- It should be noted that the $\vec{B}$ in Ampère's law is not necessarily due to the current I_{encl}.
- $\vec{B}$ is the field at each point in space along the chosen path due to all sources—including the current I enclosed by the path, but also due to any other sources.
- For example, the field surrounding two parallel current-carrying wires is the vector sum of the fields produced by each, the field lines of which are shown in the above figure.

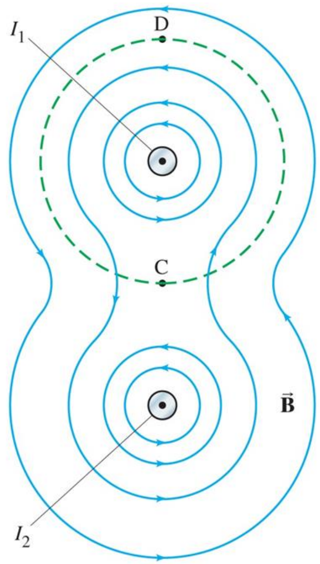
Ampère's Law



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

- If the path chosen for the integral is a circle centred on one of the wires, with radius less than the distance between the wires (the dashed line in the above figure), only the current (I_1) in the encircled wire is included on the right side of the equation.
- $\vec{B}$ on the left side of the equation must be the total $\vec{B}$ at each point due to both wires.

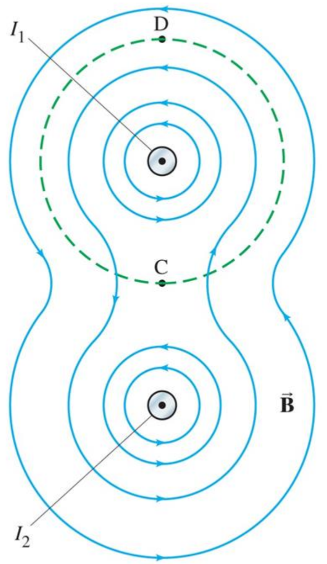
Ampère's Law



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

- Note also that $\oint \vec{B} \cdot d\vec{l}$ for the path shown in the above is the same whether the second wire is present or not (in both cases, it equals $\mu_0 I_1$ according to Ampère's law). How can this be?

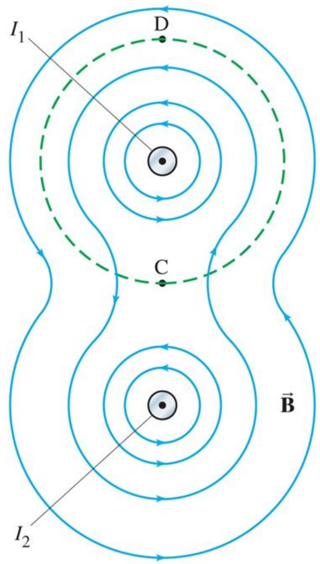
Ampère's Law



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

- It can be so because the fields due to the two wires partially cancel one another at some points between them, such as point C in the diagram ($\vec{B} = 0$ at a point midway between the wires if $I_1 = I_2$).
- At other points, such as D, the fields add together to produce a larger field.

Ampère's Law



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

- In the sum, $\oint \vec{B} \cdot d\vec{l}$, these effects just balance so that $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_1$, whether the second wire is there or not.
- The integral $\oint \vec{B} \cdot d\vec{l}$ will be the same in each case, even though $\vec{B}$ will not be the same at every point for each of the two cases.

Ampère’s Law: Problem Solving
(For reference only, as calculus will NOT be asses sed)

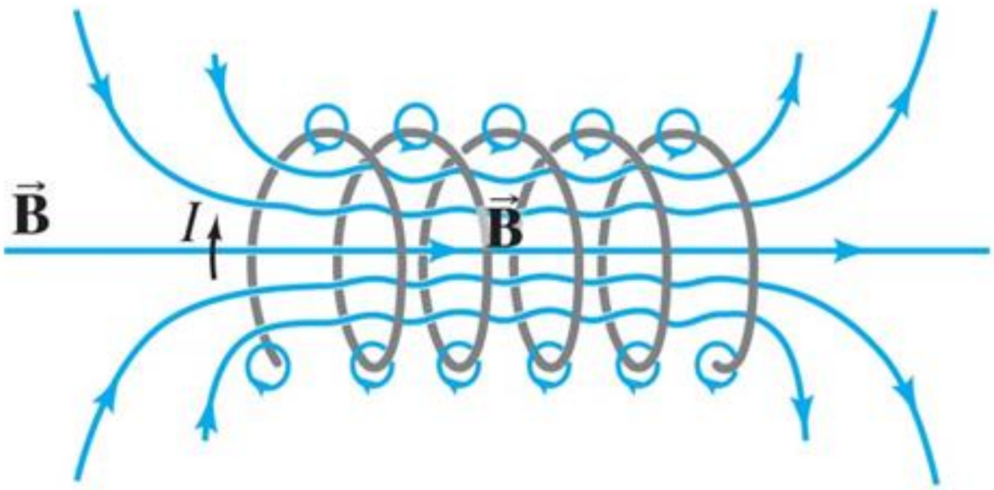
1) Ampère’s law, like Gauss’s law, is always a valid statement. But as a calculation tool, it is limited mainly to systems with a high degree of symmetry. The first step in applying Ampère’s law is to identify useful symmetry.
2) Chose an integration path that reflects the symmetry. Search for paths where B has constant magnitude along the entire path or along segments of the path. Make sure your integration path passes through the point where you wish to evaluate the magnetic field.

Ampère’s Law: Problem Solving

3) Use symmetry to determine the direction of $\vec{B}$ along the integration path. With a smart choice of path, $\vec{B}$ will be either parallel or perpendicular to the path.
4) Determine the enclosed current, I_{encl}. Be careful with signs. Let the fingers of your right hand curl along the direction of $\vec{B}$ so that your thumb shows the direction of positive current. If you have a solid conductor and your integration path does not enclose the full current, you can use the current density (current per unit area) multiplied by the enclosed area, as in question 5.

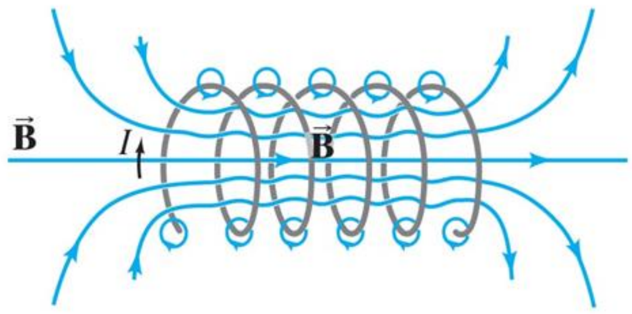
5. Magnetic Field of a Solenoid

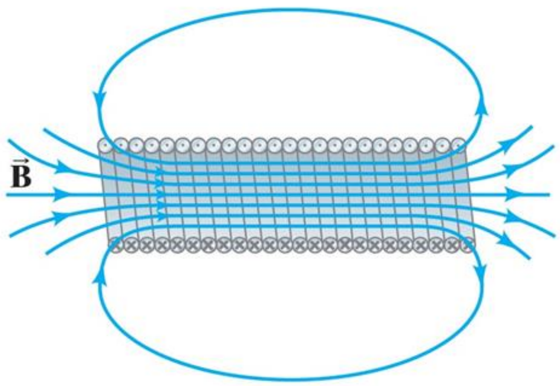
Magnetic Field of a Solenoid



- A long coil of wire consisting of many loops is called a **solenoid**.
- Each loop produces a magnetic field, as shown.
- The above figure shows the field due to a solenoid when the coils are far apart.
- Near each wire, the field lines are very nearly circles as for a straight wire (that is, at distances that are small compared to the curvature of the wire).

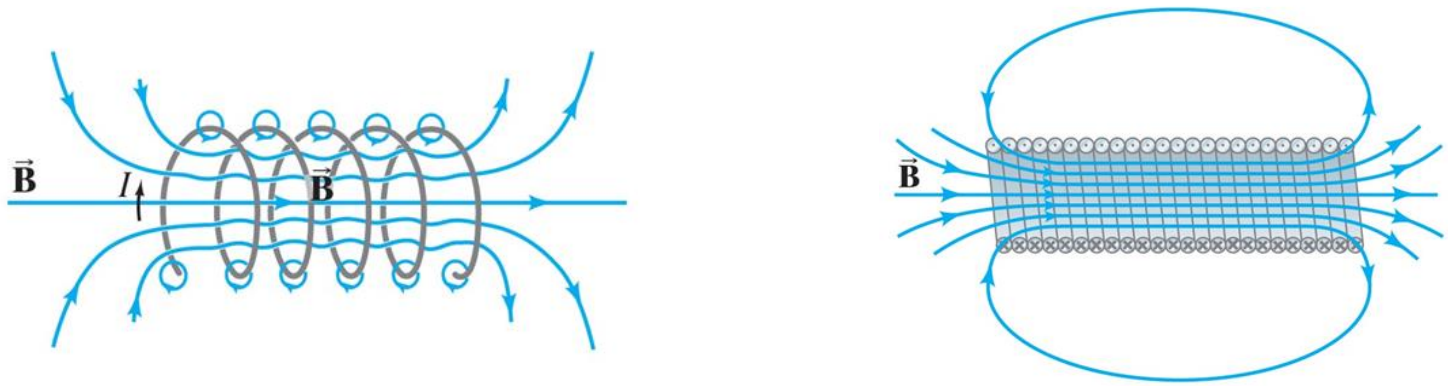
Magnetic Field of a Solenoid





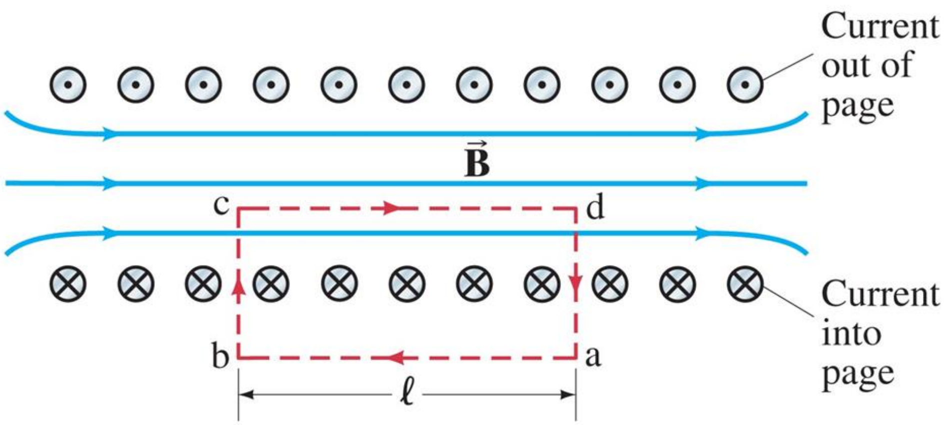
- Toward the centre of the solenoid, the fields add up to give a field that can be fairly large and fairly uniform.
- For a long solenoid with closely packed coils, as shown in the figure to the right, the field is nearly uniform and parallel to the solenoid axis within the entire cross section.
- The field outside the solenoid is very small compared to the field inside, except near the ends.

Magnetic Field of a Solenoid



- The same number of field lines that are concentrated inside the solenoid spread out into the vast open space outside.
- We can use Ampère's law to determine the magnetic field inside a very (ideally, infinitely long) closely packed solenoid.

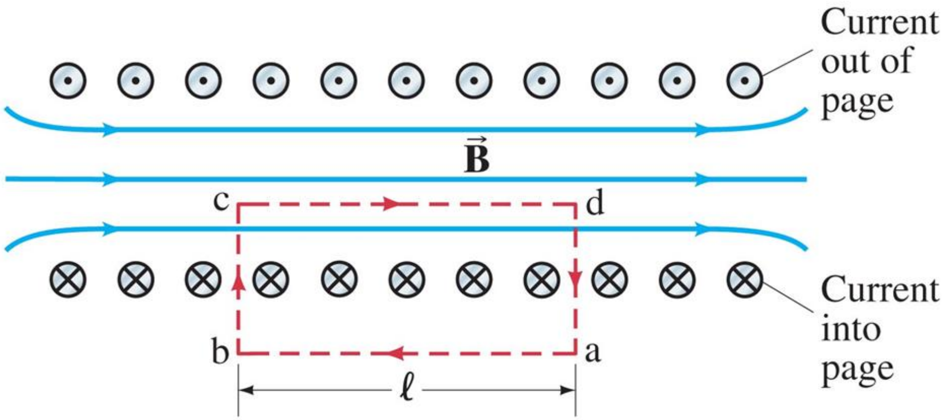
Magnetic Field of a Solenoid



- We choose the path $abcd$, shown above, far from either end, for applying Ampère's law.
- We will consider this path as made up of four segments, the sides of the rectangle: ab, bc, cd, da.
- Then the left side of the equation for Ampère's law becomes

$$\oint \vec{B} \cdot d\vec{l} = \int_a^b \vec{B} \cdot d\vec{l} + \int_b^c \vec{B} \cdot d\vec{l} + \int_c^d \vec{B} \cdot d\vec{l} + \int_d^a \vec{B} \cdot d\vec{l}$$

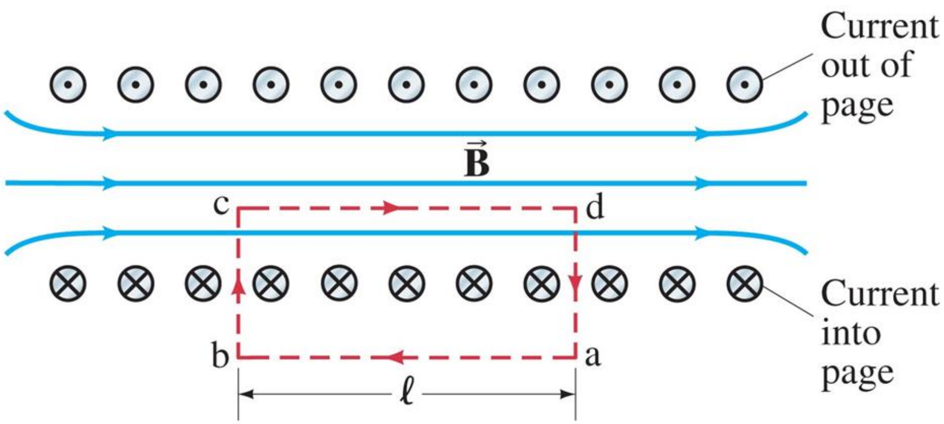
Magnetic Field of a Solenoid



$$\oint \vec{B}\cdot d\vec{l} = \int_a^b \vec{B}\cdot d\vec{l} + \int_b^c \vec{B}\cdot d\vec{l} + \int_c^d \vec{B}\cdot d\vec{l} + \int_d^a \vec{B}\cdot d\vec{l}$$

- The field outside the solenoid is so small as to be negligible compared to the field inside. Thus, the first term in this sum will be zero.
- Furthermore, $\vec{B}$ is perpendicular to the segments bc and da inside the solenoid, and is nearly zero between and outside the coils, so these terms too are zero.

Magnetic Field of a Solenoid



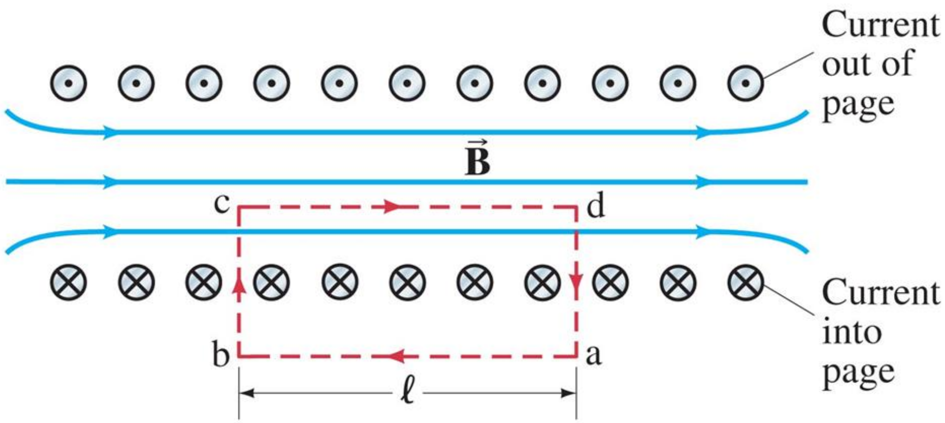
$$\oint \vec{B} \cdot d\vec{l} = \int_a^b \vec{B} \cdot d\vec{l} + \int_b^c \vec{B} \cdot d\vec{l} + \int_c^d \vec{B} \cdot d\vec{l} + \int_d^a \vec{B} \cdot d\vec{l}$$

- Therefore, we have reduced the integral to the segment cd where $\vec{B}$ is the nearly uniform field inside the solenoid, and is parallel to $d\vec{l}$, so

$$\oint \vec{B} \cdot d\vec{l} = \int_c^d \vec{B} \cdot d\vec{l} = Bl$$

where l is the length cd.

Magnetic Field of a Solenoid

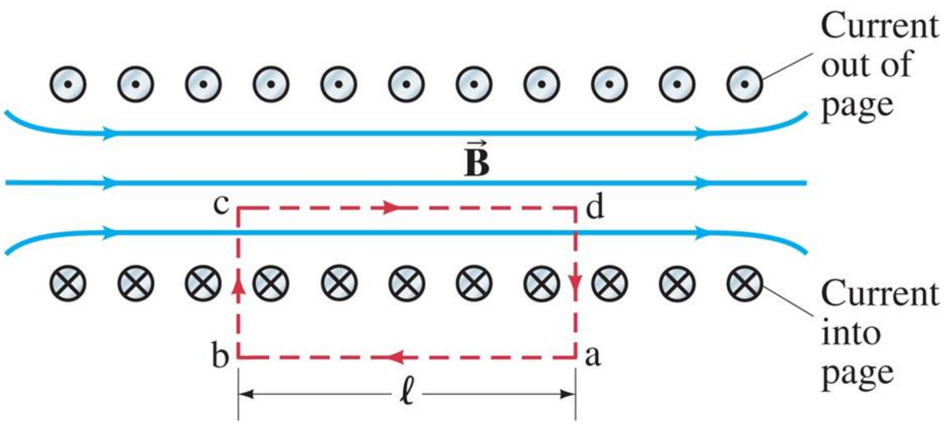


$$\oint \vec{B} \cdot d\vec{l} = \int_c^d \vec{B} \cdot d\vec{l} = Bl$$

- Now we determine the current enclosed by this loop for the right side of Ampère's law.
- If a current I flows in the wire of the solenoid, the total current enclosed by our path $abcd$ is NI where N is the number of loops our path encircles (five in the above figure). Thus, Ampère's law gives us

$$Bl = \mu_0 NI$$

Magnetic Field of a Solenoid

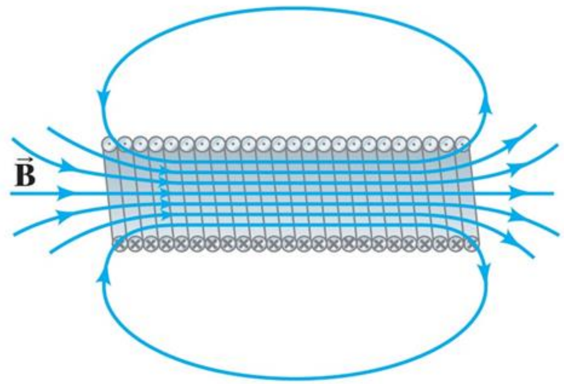


- If we let $n = N/l$ be the number of loops per unit length, then

$$B = \mu_0 n I$$

- This is the magnitude of the magnetic field within a solenoid. Note that B depends only on the number of loops per unit length, n, and the current I.
- The field does not depend on position within the solenoid, so B is uniform. This is strictly true only for an infinite solenoid, but it is a good approximation for real ones for points not close to the ends.

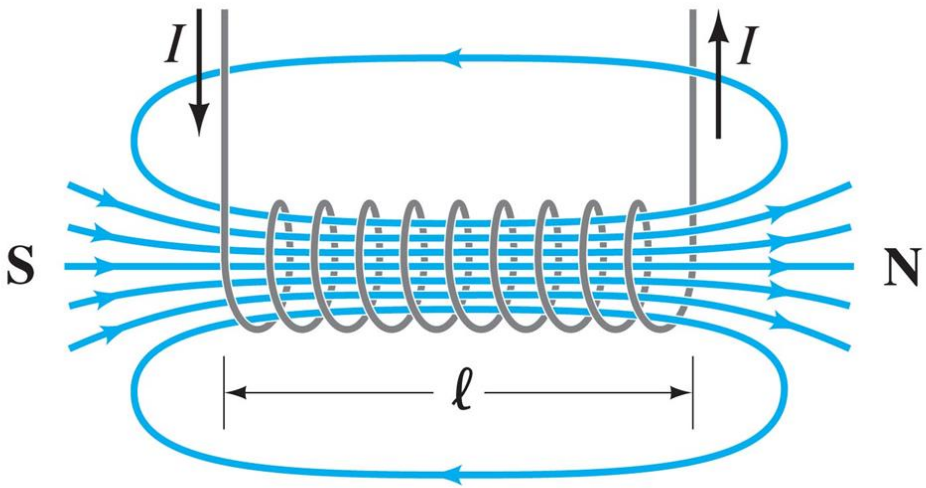
Magnetic Field of a Solenoid



- Looking at the above figure, the field outside of a solenoid is much like that of a bar magnet.
- Indeed, a solenoid acts like a magnet, with one end acting as a north pole and the other as south pole, depending on the direction of the current in the loop.
- Since magnetic field lines leave the north pole of a magnet, the north pole of the solenoid in the above figure is on the right.

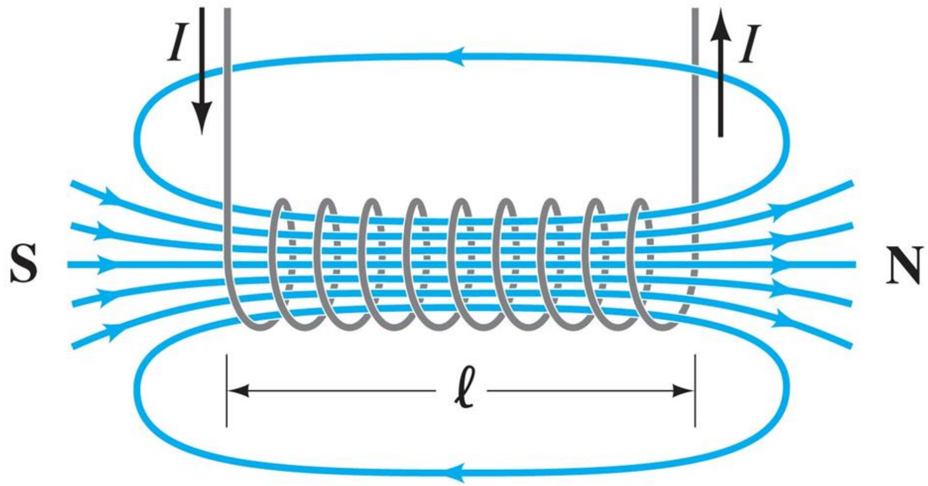
6. Electromagnets and Solenoids—Applications

Electromagnets and Solenoids—Applications



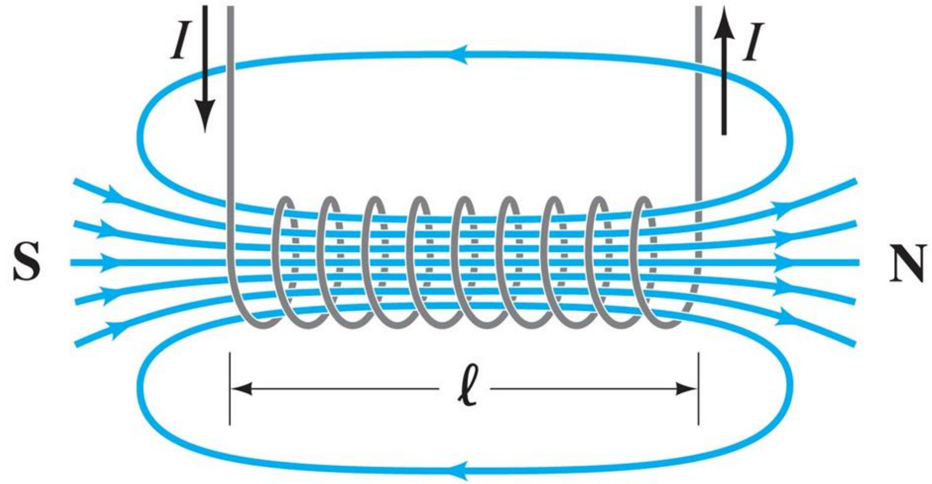
- If a piece of iron is placed inside a solenoid, the magnetic field is increased greatly because the domains of the iron are aligned by the magnetic field produced by the current.
- The resulting magnetic field is the sum of that due to the current and that due to the iron, and can be hundreds or thousands of times larger than that due to the current alone. This arrangement is called an **electromagnet**.

Electromagnets and Solenoids—Applications



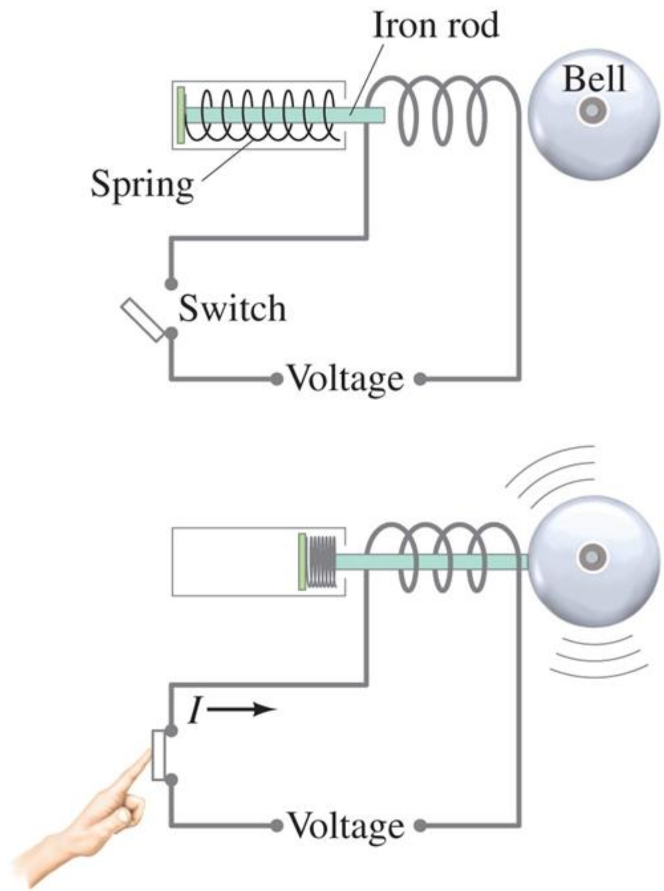
- The alloys of iron used in electromagnets acquire and lose their magnetism quite readily when the current is turned on or off, and so are referred to as ‘soft iron’.
- Iron that holds its magnetism even when there is no externally applied field is called ‘hard iron’. Hard iron is used in permanent magnets. Soft iron is usually used in electromagnets so that the field can be turned on and off readily.

Electromagnets and Solenoids—Applications



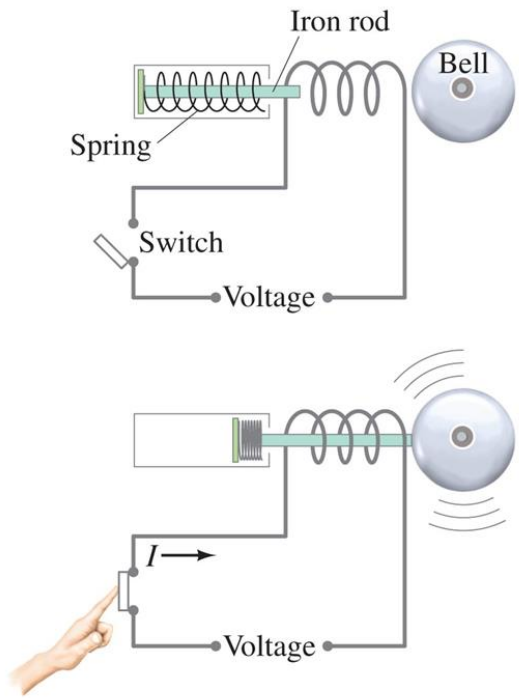
- Electromagnets have many practical applications, from use in motors and generators to producing large magnetic fields for research.

Electromagnets and Solenoids—Applications



- Another useful device consists of a solenoid into which a rod of iron is partially inserted. This combination is also referred to as a solenoid. One simple use is as a doorbell.

Electromagnets and Solenoids—Applications



- When the circuit is closed by pushing the button, the coil effectively becomes a magnet and exerts a force on the iron rod. The rod is pulled into the coil and strikes the bell.
- Solenoids are used as switches in many devices, as they have the advantage of moving mechanical parts quickly and accurately.

Summary of today's Lecture

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1. Magnetic Field Due to a Straight Wire
2. Force between Two Parallel Wires
3. Definition of the Ampere and the Coulomb
4. Ampère's Law
5. Magnetic Field of a Solenoid
6. Electromagnets and Solenoids—Applications

Lecture 17: Optional Reading

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- **Ch. 28.1**, Magnetic Field Due to a Straight Wire; p.846-847.
- **Ch. 28.2**, Force between Two Parallel Wires; p.847-848.
- **Ch. 28.3**, Definition of the Ampere and the Coulomb; p.848-849.
- **Ch. 28.4**, Ampère’s Law; p.849-852.
- **Ch. 28.5**, Magnetic Field of a Solenoid and a Toroid; p.853-854.
- **Ch. 28.8**, Electromagnets and Solenoids – Applications; p.859.

Home Work

Do not forget to attempt the **Additional Problems** for this lecture before logging in to **Mastering Physics** to complete your assignments.