Introduction to Algorithms (CELEN086)

Problem Sheet 1

Topics: Computing basics; Boolean statement; IF-statement; Simple algorithm design

1. You have learned elementary basics of algorithms. Pick up one that you are familiar with (could be a simple one, or one you are interested in) and note it down **before attending Seminar 1**. You need to demonstrate it to your classmates. For example,

   - What task is your algorithm solving?
   - What kinds of data are you dealing with in the algorithm?
   - What are the inputs/outputs?
   - (Optional) Any other approaches for solving the same problem?

2. Let x=3, y=4, z=5. Evaluate the following Boolean/logical statements.
   i. x>y || y!=z
   ii. (x*x+y*y == z*z) && (x+y!=z)
   iii. !(x<z)
   iv. y>z || !(y>x)

3. Let P and Q be two Boolean variables. Suppose P is True and Q is False.
   For the following compounded statements, True or False?
   i. !P || Q && P
   ii. P && !P || Q
   iii. P && (!P || Q) || !Q
   iv. Can you design a compounded Boolean statement that involves both P and Q, and make it always True regardless of whether P, Q are true or false?

4. Write an algorithm called **modulus()** that computes modulus (absolute value) of number **x**.

   Note: the modulus function is given by

$$f(x) = \begin{cases} x; & if\ x \geq 0 \\ -x; & if\ x < 0 \end{cases}$$

5. Write an algorithm called **reverse** () takes a 3-digit integer **n** as its input, and returns a new integer with reversed digits. For example if your input is 973 then your algorithm should return 379. Your algorithm should first check that the last digit of your input is 0 or not; if the last digit is 0, your algorithm should return -1.

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6. Write an algorithm **NOT(P)**, which returns the negation of an input Boolean variable P.

7. Write an algorithm for **AND(P,Q).** It should take two Boolean variable P and Q, and return the compounded result True or False.

8. Write an algorithm for **OR(P,Q)**. It should take two Boolean variable P and Q, and return the compounded result True or False.

9. A linear model processes numbers from data Set X and produces some odd numbers in data Set Y, shown as follows:

Set X	Linear Model	Set Y
3		5
5	>>>>>>>>>>>>>	9
?? (missing)	data processing…	19
25	>>>>>>>>>>>>>	?
?? (missing)		47

i. Can you formulate an equation or function for this model?
ii. Use this model to predict what should be the result (?) of processing number 25.
iii. Suppose we have known a few computed results (e.g., 19 and 47) in Set Y.
We are interested in finding a way to figure out those missing data (??) in the original Set X. Can you propose a method for handling this task?
iv. What is the mathematical concept that you are applying here?

*Note: linear model can be represented by an equation y=a*x+b or function f(x)=a*x+b.*

10. Write an algorithm **isTriangle()** that takes three positive numbers a, b, c as its inputs, and returns True if we can form a triangle with lengths a, b, c; False otherwise.

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11. In digital electronics, a NAND gate (NOT-AND) is a logic gate which produces an output which is False only if all its inputs are True. Draw up the truth table for **NAND(P,Q)** and write an algorithm for it.

12. The exclusive OR gate (XOR) is a special case of OR such that it is True if only one of the arguments is True. Write an algorithm to implement **XOR(P,Q)**.

13. The annual income tax rate in some state is shown in the table below:

Annual Income	$1 - $10000	$10001 – $25000	$25001 – $45000	$45001 – $65000	$65001 – $90000	Above $90000
Tax Rate	**5%**	**7.5%**	**11%**	**15%**	**17.5%**	**Not Defined**

The tax rate on each bracket (income interval) is applied to the difference from previous bracket. For Example if your annual income is $68'000.00 then your tax will be calculated as following:

$$\$68'000.00 = \$10'000 + \$15'000 + \$20'000 + \$20'000 + \$3'000,$$
$$TAX = 10000 \times 0.05 + 15000 \times 0.075 + 20000 \times 0.11 + 20000 \times 0.15 + 3000 \times 0.175$$
$$= \$500 + \$1125 + \$2200 + \$3000 + \$525 = \$7350.$$

Write an algorithm called **taxCalc()** that takes a positive integer and returns the annual tax value. If the income is greater than $90000, your algorithm should return -1 meaning 'Please visit your tax office.'

14. Current month starts on Thursday, 1 September 2022. Write an algorithm called **whatDay()** that takes an integer n ranging from 1 to 30, and returns an integer ranging from 1 to 7 representing Monday to Sunday respectively.

For example, calling whatDay(30) should return 5, because the last day of this month is Friday.