CELEN037
Foundation Calculus and Mathematical Techniques

Lecture 7

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Main content

- Integrals of the form $\int \frac{1}{\alpha \cos(x) + \beta \sin(x) + \gamma} \, dx$
- Integrals of the form $\int \frac{1}{\alpha \cos^2(x) + \beta \sin^2(x) + \gamma} \, dx$
- Integration by partial fractions
- Integration by parts

Integrals of the form $\int \frac{1}{\alpha \cos(x) + \beta \sin(x) + \gamma} \, dx$

Suppose that we want to evaluate an integral of the form

$$\int \frac{1}{\alpha \cos(x) + \beta \sin(x) + \gamma} \, dx$$

where α, β and γ are real numbers, at least two of which are nonzero.

We can do so by using the substitution $t = \tan\left(\dfrac{x}{2}\right)$, the trigonometric identity $\sec^2\left(\dfrac{x}{2}\right) = 1 + \tan^2\left(\dfrac{x}{2}\right)$ and one or both of:

- $\dfrac{2t}{1+t^2} = \sin(x)$
  $$\left(\text{since } \frac{2\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)} = \frac{2\tan\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)\cos^2\left(\frac{x}{2}\right)} = \frac{2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}{\cos^2\left(\frac{x}{2}\right)+\sin^2\left(\frac{x}{2}\right)}\right)$$
- $\dfrac{1-t^2}{1+t^2} = \cos(x)$
  $$\left(\text{since } \frac{1-\tan^2\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)} = \frac{\left(1-\tan^2\left(\frac{x}{2}\right)\right)\cos^2\left(\frac{x}{2}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)\cos^2\left(\frac{x}{2}\right)} = \frac{\cos^2\left(\frac{x}{2}\right)-\sin^2\left(\frac{x}{2}\right)}{\cos^2\left(\frac{x}{2}\right)+\sin^2\left(\frac{x}{2}\right)}\right)$$

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Problem 1

Evaluate $\displaystyle\int \frac{1}{3\cos(x) - 4\sin(x) + 5}\, dx$.

Let $t = \tan\left(\frac{x}{2}\right)$.

Then $\dfrac{dt}{dx} = \dfrac{1}{2}\sec^2\left(\dfrac{x}{2}\right) = \dfrac{1}{2}\left(1 + \tan^2\left(\dfrac{x}{2}\right)\right)$ and $\dfrac{2}{1+t^2}dt = dx$.

$$
\begin{aligned}
\therefore \int \frac{1}{3\cos(x) - 4\sin(x) + 5}\, dx &= \int \frac{2}{\left(3\frac{1-t^2}{1+t^2} - 4\frac{2t}{1+t^2} + 5\right)(1+t^2)}\, dt \\
&= \int \frac{2}{3 - 3t^2 - 8t + 5 + 5t^2}\, dt \\
&= \int \frac{2}{8 - 8t + 2t^2}\, dt \\
&= \int \frac{1}{t^2 - 4t + 4}\, dt \\
&= \int \frac{1}{(t-2)^2}\, dt \\
&= -\frac{1}{t-2} + C \\
&= \frac{1}{2 - \tan\left(\frac{x}{2}\right)} + C
\end{aligned}
$$

Problem 2

Evaluate $\displaystyle\int \frac{1}{\cos(x)+1}\,dx$.

Let $t = \tan\left(\frac{x}{2}\right)$.

Then $\dfrac{dt}{dx} = \dfrac{1}{2}\sec^2\left(\dfrac{x}{2}\right) = \dfrac{1}{2}\left(1+\tan^2\left(\dfrac{x}{2}\right)\right)$ and $\dfrac{2}{1+t^2}dt = dx$.

$$
\begin{aligned}
\therefore \int \frac{1}{\cos(x)+1}\,dx &= \int \frac{2}{\left(\frac{1-t^2}{1+t^2}+1\right)(1+t^2)}\,dt \\
&= \int \frac{2}{1-t^2+1+t^2}\,dt \\
&= \int \frac{2}{2}\,dt \\
&= \int 1\,dt \\
&= t+C \\
&= \tan\left(\frac{x}{2}\right)+C
\end{aligned}
$$

Integrals of the form $\int \frac{1}{\alpha \cos^2(x) + \beta \sin^2(x) + \gamma}\, dx$

Suppose that we want to evaluate an integral of the form

$$\int \frac{1}{\alpha \cos^2(x) + \beta \sin^2(x) + \gamma}\, dx$$

where α, β and γ are real numbers, at least two of which are nonzero. We can do this by dividing the numerator and the denominator of the integrand by $\cos^2(x)$, using the trigonometric identity $\sec^2(x) = 1 + \tan^2(x)$ and using the substitution $t = \tan(x)$.

Problem 3

Evaluate $\displaystyle\int \frac{1}{1+3\sin^2(x)}\,dx$.

Let $t = \tan(x)$.
Then $\dfrac{dt}{dx} = \sec^2(x)$ and $dt = \sec^2(x)\,dx$.

$$
\begin{aligned}
\therefore \int \frac{1}{1+3\sin^2(x)}\,dx &= \int \frac{\frac{1}{\cos^2(x)}}{\frac{1+3\sin^2(x)}{\cos^2(x)}}\,dx \\
&= \int \frac{\sec^2(x)}{\sec^2(x)+3\tan^2(x)}\,dx \\
&= \int \frac{\sec^2(x)}{1+4\tan^2(x)}\,dx \\
&= \int \frac{1}{1+4t^2}\,dt \\
&= \int \frac{1}{(2t)^2+1^2}\,dt \\
&= \frac{1}{2}\tan^{-1}(2t)+C \\
&= \frac{1}{2}\tan^{-1}(2\tan(x))+C
\end{aligned}
$$

Problem 4

Evaluate $\displaystyle\int \frac{\sin(x)}{\sin(3x)}\,dx$.

We first note that

$$\frac{\sin(x)}{\sin(3x)} = \frac{\sin(x)}{3\sin(x) - 4\sin^3(x)} = \frac{1}{3 - 4\sin^2(x)}.$$

Let $t = \tan(x)$.
Then $\displaystyle\frac{dt}{dx} = \sec^2(x)$ and $dt = \sec^2(x)\,dx$.

$$
\begin{aligned}
\therefore \int \frac{\sin(x)}{\sin^3(x)}\,dx &= \int \frac{\frac{1}{\cos^2(x)}}{\frac{3-4\sin^2(x)}{\cos^2(x)}}\,dx \\
&= \int \frac{\sec^2(x)}{3\sec^2(x) - 4\tan^2(x)}\,dx \\
&= \int \frac{\sec^2(x)}{3 - \tan^2(x)}\,dx \\
&= \int \frac{1}{3 - t^2}\,dt \\
&= \int \frac{1}{\left(\sqrt{3}\right)^2 - t^2}\,dt \\
&= \frac{1}{2\sqrt{3}} \ln\left|\frac{t+\sqrt{3}}{t-\sqrt{3}}\right| + C \\
&= \frac{1}{2\sqrt{3}} \ln\left|\frac{\tan(x)+\sqrt{3}}{\tan(x)-\sqrt{3}}\right| + C
\end{aligned}
$$

Integration by partial fractions

Suppose that we want to evaluate an integral of the form

$$\int \frac{r(x)}{q(x)}\,dx$$

where r is a polynomial of degree m and q is a polynomial of degree n where $m < n$.

We can do this by finding the partial fraction decomposition of $\dfrac{r(x)}{q(x)}$ from which we can proceed to evaluate the integral.

The results that can be used to do this include the results on the next three pages.

Integration by partial fractions: non-repeated linear factors

- If α and β are distinct real numbers and

$$q(x) = (x+\alpha)(x+\beta)$$

then there exist real numbers A and B for which

$$\frac{r(x)}{q(x)} = \frac{A}{x+\alpha} + \frac{B}{x+\beta}.$$

- If α, β and γ are distinct real numbers and

$$q(x) = (x+\alpha)(x+\beta)(x+\gamma)$$

then there exist real numbers A, B and D for which

$$\frac{r(x)}{q(x)} = \frac{A}{x+\alpha} + \frac{B}{x+\beta} + \frac{D}{x+\gamma}.$$

- If α and β are real numbers, γ and δ are nonzero real numbers, $\alpha\delta \neq \beta\gamma$ and

$$q(x) = (\gamma x+\alpha)(\delta x+\beta)$$

then there exist real numbers A and B for which

$$\frac{r(x)}{q(x)} = \frac{A}{\gamma x+\alpha} + \frac{B}{\delta x+\beta}.$$

Integration by partial fractions: repeated linear factor

- If α and β are distinct real numbers and

$$q(x) = (x+\alpha)^2(x+\beta)$$

then there exist real numbers A, B and D for which

$$\frac{r(x)}{q(x)} = \frac{A}{x+\alpha} + \frac{B}{(x+\alpha)^2} + \frac{D}{x+\beta}.$$

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Integration by partial fractions: non-repeated quadratic factor

- If α is a nonzero real number, β, γ and δ are real numbers, $\gamma^2 - 4\delta < 0$ and

$$q(x) = (\alpha x + \beta)(x^2 + \gamma x + \delta)$$

then there exist real numbers A, B and D for which

$$\frac{r(x)}{q(x)} = \frac{A}{\alpha x + \beta} + \frac{Bx + D}{x^2 + \gamma x + \delta}.$$

Problem 5

Use partial fractions to evaluate $\displaystyle\int \frac{1}{x^2+x-2}\,dx$.

We first note that

$$\frac{1}{x^2+x-2} = \frac{1}{(x+2)(x-1)} = \frac{A}{x+2} + \frac{B}{x-1}$$

$$\Rightarrow A(x-1) + B(x+2) = 1.$$

Now,

$$x = -2 \Rightarrow -3A = 1 \Rightarrow A = -\frac{1}{3}$$

and

$$x = 1 \Rightarrow 3B = 1 \Rightarrow B = \frac{1}{3}.$$

$$
\begin{aligned}
\therefore \int \frac{1}{x^2+x-2}\,dx &= \frac{1}{3}\int \left(\frac{1}{x-1}-\frac{1}{x+2}\right) dx \\
&= \frac{1}{3}(\ln|x-1|-\ln|x+2|)+C \\
&= \frac{1}{3}\ln\left|\frac{x-1}{x+2}\right|+C
\end{aligned}
$$

Problem 6

Evaluate $\int \frac{x^2+2}{x(x+2)(x-1)}\,dx$.

We first note that

$$\frac{x^2+2}{x(x+2)(x-1)} = \frac{A}{x} + \frac{B}{x+2} + \frac{D}{x-1}$$

$$\Rightarrow A(x+2)(x-1) + Bx(x-1) + Dx(x+2) = x^2+2.$$

Now,

$$x = 0 \Rightarrow -2A = 2 \Rightarrow A = -1,$$

$$x = -2 \Rightarrow 6B = 6 \Rightarrow B = 1$$

and

$$x = 1 \Rightarrow 3D = 3 \Rightarrow D = 1.$$

$$
\begin{aligned}
\therefore \int \frac{x^2+2}{x(x+2)(x-1)}\,dx &= \int\left(\frac{1}{x+2}-\frac{1}{x}+\frac{1}{x-1}\right)dx \\
&= \ln|x+2|-\ln|x|+\ln|x-1|+C \\
&= \ln\left|\frac{(x+2)(x-1)}{x}\right|+C
\end{aligned}
$$

Problem 7

Evaluate $\displaystyle\int \frac{x+2}{x^3-2x^2}\,dx$.

We first note that

$$\frac{x+2}{x^3-2x^2} = \frac{x+2}{x^2(x-2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{D}{x-2}$$

$$\Rightarrow Ax(x-2) + B(x-2) + Dx^2 = x+2.$$

Now,

$$x = 2 \Rightarrow 4D = 4 \Rightarrow D = 1,$$

$$x = 0 \Rightarrow -2B = 2 \Rightarrow B = -1$$

and

$$x = 1 \Rightarrow -A + 1 + 1 = 3 \Rightarrow A = -1.$$

$$
\begin{aligned}
\therefore \int \frac{x+2}{x^3-2x^2}\,dx &= \int \left(\frac{1}{x-2} - \frac{1}{x} - \frac{1}{x^2} \right) dx \\
&= \ln|x-2| - \ln|x| + \frac{1}{x} + C \\
&= \ln\left|\frac{x-2}{x}\right| + \frac{1}{x} + C
\end{aligned}
$$

Problem 8

Evaluate $\displaystyle\int \frac{13}{(2x+3)(x^2+1)}\,dx$.

We first note that

$$\frac{13}{(2x+3)(x^2+1)} = \frac{A}{2x+3} + \frac{Bx+D}{x^2+1}$$

$$\Rightarrow A\big(x^2+1\big) + (Bx+D)(2x+3) = 13.$$

Now,

$$x = -\frac{3}{2} \Rightarrow \frac{13}{4}A = 13 \Rightarrow A = 4,$$

$$x = 0 \Rightarrow 4 + 3D = 13 \Rightarrow D = 3$$

and

$$x = 1 \Rightarrow 8 + 5B + 15 = 13 \Rightarrow B = -2.$$

$$
\begin{aligned}
\therefore \quad & \int \frac{13}{(2x+3)(x^2+1)}\,dx \\
= \; & \int \left(\frac{4}{2x+3} - \frac{2x}{x^2+1} + \frac{3}{x^2+1} \right) dx \\
= \; & 4\int \frac{1}{2x+3}\,dx - \int \frac{2x}{x^2+1}\,dx + 3\int \frac{1}{x^2+1}\,dx \\
= \; & 2\ln|2x+3| - \ln\left(x^2+1\right) + 3\tan^{-1}(x) + C
\end{aligned}
$$

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Problem 9

Evaluate $\displaystyle\int \frac{1}{(x-1)(x^2+1)}\,dx$.

We first note that

$$\frac{1}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+D}{x^2+1}$$

$$\Rightarrow A\big(x^2+1\big) + (Bx+D)(x-1) = 1.$$

Now,

$$x = 1 \Rightarrow 2A = 1 \Rightarrow A = \frac{1}{2},$$

$$x = 0 \Rightarrow \frac{1}{2} - D = 1 \Rightarrow D = -\frac{1}{2}$$

and

$$x = -1 \Rightarrow 1 + 2B + 1 = 1 \Rightarrow B = -\frac{1}{2}.$$

$$
\begin{aligned}
\therefore \qquad & \int \frac{1}{(x-1)(x^2+1)}\,dx \\
= \quad & \frac{1}{2}\int\left(\frac{1}{x-1}-\frac{x}{x^2+1}-\frac{1}{x^2+1}\right)dx \\
= \quad & \frac{1}{2}\int\frac{1}{x-1}\,dx-\frac{1}{4}\int\frac{2x}{x^2+1}\,dx-\frac{1}{2}\int\frac{1}{x^2+1}\,dx \\
= \quad & \frac{1}{2}\ln|x-1|-\frac{1}{4}\ln\left(x^2+1\right)-\frac{1}{2}\tan^{-1}(x)+C
\end{aligned}
$$

Integration by parts: Justification

Suppose that $u = f(x)$ and $v = g(x)$. Then, for x in the domain of f and the domain of g for which $f'(x)$ and $g'(x)$ exist,

$$\frac{d}{dx}(uv) = \frac{du}{dx}v + u\frac{dv}{dx}$$

by the product rule, and hence,

$$u\frac{dv}{dx} = \frac{d}{dx}(uv) - \frac{du}{dx}v.$$

Therefore,

$$\int u\frac{dv}{dx}\,dx = uv - \int \frac{du}{dx}v\,dx.$$

Integration by parts: Result

We can (attempt to) evaluate an integral of the form

$$\int y(x)z(x)\,dx$$

by letting u be (the appropriate) one of $y(x)$ and $z(x)$, letting $\dfrac{dv}{dx}$ be the other of $y(x)$ and $z(x)$, and using the integration by parts formula

$$\int u\frac{dv}{dx}\,dx = uv - \int \frac{du}{dx}v\,dx.$$

Integration by parts: LIATE

While not always the correct choice, it is often the correct choice to choose u to be which of $y(x)$ and $z(x)$ comes first in the list:

- Logarithmic functions (for example, $\ln(x)$ and $\log_a(x)$)
- Inverse trigonometric functions (for example, $\sin^{-1}(x)$ and $\cos^{-1}(x)$)
- Algebraic functions (for example, polynomials and rational functions)
- Trigonometric functions (for example, $\sin(x)$, $\cos(x)$ and $\tan(x)$)
- Exponential functions (for example, e^x and a^x)

The acronym LIATE may help with remembering the order of this list.

Integration by parts: detail

The acronym detail is an alternative way to help with remembering a method for arriving at the same choice as the method on the previous page.

- $\dfrac{dv}{dx}$ is taken to be which of $y(x)$ and $z(x)$ comes first in the list:
- exponential functions (for example, e^x and a^x)
- trigonometric functions (for example, $\sin(x)$, $\cos(x)$ and $\tan(x)$)
- algebraic functions (for example, polynomials and rational functions)
- inverse trigonometric functions (for example, $\sin^{-1}(x)$ and $\cos^{-1}(x)$)
- logarithmic functions (for example, $\ln(x)$ and $\log_a(x)$)

Problem 10

Evaluate $\displaystyle\int x\cos(x)\,dx$.

Let $u = x$ and $\dfrac{dv}{dx} = \cos(x)$.

Then $\dfrac{du}{dx} = 1$ and we can let $v = \sin(x)$.

$$\begin{aligned} \therefore \int x\cos(x)\,dx &= x\sin(x) - \int \sin(x)\,dx \\ &= x\sin(x) + \cos(x) + C \end{aligned}$$

Problem 11

Evaluate $\displaystyle\int x\ln(x)\,dx$.

Let $u = \ln(x)$ and $\dfrac{dv}{dx} = x$.

Then $\dfrac{du}{dx} = \dfrac{1}{x}$ and we can let $v = \dfrac{1}{2}x^2$.

$$
\begin{aligned}
\therefore \int x\ln(x)\,dx &= \frac{1}{2}x^2\ln(x) - \int \frac{x^2}{2x}\,dx \\
&= \frac{1}{2}x^2\ln(x) - \frac{1}{2}\int x\,dx \\
&= \frac{1}{2}x^2\ln(x) - \frac{1}{4}x^2 + C \\
&= x^2\left(\frac{\ln(x)}{2} - \frac{1}{4}\right) + C
\end{aligned}
$$

Problem 12

Evaluate $\int xe^x\,dx$.

Let $u = x$ and $\dfrac{dv}{dx} = e^x$.

Then $\dfrac{du}{dx} = 1$ and we can let $v = e^x$.

$$
\begin{aligned}
\therefore \int xe^x\,dx &= xe^x - \int e^x\,dx \\
&= xe^x - e^x + C \\
&= e^x(x-1) + C
\end{aligned}
$$

Problem 13

Evaluate $\displaystyle\int x^2 \tan^{-1}(x)\,dx$.

Let $u = \tan^{-1}(x)$ and $\dfrac{dv}{dx} = x^2$.

Then $\dfrac{du}{dx} = \dfrac{1}{x^2+1}$ and we can let $v = \dfrac{1}{3}x^3$.

$$
\begin{aligned}
\therefore \int x^2 \tan^{-1}(x)\,dx &= \frac{1}{3}x^3 \tan^{-1}(x) - \frac{1}{3}\int \frac{x^3}{x^2+1}\,dx \\
&= \frac{1}{3}x^3 \tan^{-1}(x) - \frac{1}{3}\int \frac{(x^2+1)x - x}{x^2+1}\,dx \\
&= \frac{1}{3}x^3 \tan^{-1}(x) - \frac{1}{3}\int x\,dx + \frac{1}{3}\int \frac{x}{x^2+1}\,dx \\
&= \frac{1}{3}x^3 \tan^{-1}(x) - \frac{1}{3}\int x\,dx + \frac{1}{6}\int \frac{2x}{x^2+1}\,dx \\
&= \frac{x^3}{3}\tan^{-1}(x) - \frac{x^2}{6} + \frac{\ln\left(x^2+1\right)}{6} + C
\end{aligned}
$$

Problem 14

Evaluate $\int \ln(x)\,dx$.

Let $u = \ln(x)$ and $\frac{dv}{dx} = 1$.
Then $\frac{du}{dx} = \frac{1}{x}$ and we can let $v = x$.

$$\begin{aligned}
\therefore \int \ln(x)\,dx &= x\ln(x) - \int \frac{x}{x}\,dx \\
&= x\ln(x) - \int 1\,dx \\
&= x\ln(x) - x + C \\
&= x(\ln(x) - 1) + C
\end{aligned}$$

Problem 15

Evaluate $\int x^2 e^x \, dx$.

Let $u = x^2$ and $\dfrac{dv}{dx} = e^x$.

Then $\dfrac{du}{dx} = 2x$ and we can let $v = e^x$.

$$\therefore \int x^2 e^x \, dx = x^2 e^x - 2 \int x e^x \, dx.$$

Now let $u = x$ and $\dfrac{dv}{dx} = e^x$.

Then $\dfrac{du}{dx} = 1$ and we can let $v = e^x$.

$$\begin{aligned} \therefore \int x^2 e^x \, dx &= x^2 e^x - 2\left(x e^x - \int e^x \, dx \right) \\ &= x^2 e^x - 2x e^x + 2e^x + C \\ &= e^x \left(x^2 - 2x + 2 \right) + C \end{aligned}$$

Problem 16

Evaluate $\displaystyle\int e^x \cos(x)\,dx$.

Let $u = \cos(x)$ and $\dfrac{dv}{dx} = e^x$.

Then $\dfrac{du}{dx} = -\sin(x)$ and we can let $v = e^x$.

$$\therefore \int e^x \cos(x)\,dx \quad = \quad e^x \cos(x) + \int e^x \sin(x)\,dx.$$

Now let $u = \sin(x)$ and $\dfrac{dv}{dx} = e^x$.

Then $\dfrac{du}{dx} = \cos(x)$ and we can let $v = e^x$.

$$\therefore \int e^x \cos(x)\,dx \quad = \quad e^x \cos(x) + e^x \sin(x) - \int e^x \cos(x)\,dx$$

So,

$$I \quad = \quad e^x(\cos(x) + \sin(x)) - I$$

where

$$I = \int e^x \cos(x)\, dx.$$

Hence,

$$\int e^x \cos(x)\, dx \quad = \quad \frac{e^x}{2}(\cos(x) + \sin(x)) + C$$

Problem 17

Use an appropriate substitution and integration by parts to evaluate $\int x^3 e^{x^2}\,dx$.

Let $t = x^2$. Then $\dfrac{dt}{dx} = 2x$ and $\dfrac{1}{2}dt = x\,dx$.

$$\therefore \int x^3 e^{x^2}\,dx \;=\; \frac{1}{2}\int te^t\,dt$$

Let $u = t$ and $\dfrac{dv}{dt} = e^t$.
Then $\dfrac{du}{dt} = 1$ and we can let $v = e^t$.

$$\begin{aligned}
\therefore \int x^3 e^{x^2}\, dx &= \frac{1}{2}\left(te^t - \int e^t\, dt\right) \\
&= \frac{1}{2}\left(te^t - e^t\right) + C \\
&= \frac{e^{x^2}}{2}(x^2 - 1) + C
\end{aligned}$$

A solution to the class activity of Lecture 5

Let $t = x^2 + 1$.
Then $\dfrac{dt}{dx} = 2x$ and $\dfrac{1}{2}dt = x\,dx$.

$$\begin{aligned}\therefore \int \frac{x}{x^2+1}\,dx &= \frac{1}{2}\int \frac{1}{t}\,dt \\ &= \frac{1}{2}\ln|t| \\ &= \frac{1}{2}\ln\left(x^2+1\right) + C.\end{aligned}$$

So, (1) is correct.

Now,

$$\begin{aligned}\frac{1}{2}\ln\left(2x^2+2\right) &= \frac{1}{2}\ln\left(2\left(x^2+1\right)\right)\\ &= \frac{1}{2}\left(\ln(2)+\ln\left(x^2+1\right)\right)\\ &= \frac{1}{2}\ln(2)+\frac{1}{2}\ln\left(x^2+1\right).\end{aligned}$$

Hence,

$$\frac{d}{dx}\left(\frac{1}{2}\ln\left(2x^2+2\right)\right) = \frac{d}{dx}\left(\frac{1}{2}\ln\left(x^2+1\right)\right).$$

So, (2) is also correct.

Moreover,

$$
\begin{aligned}
\frac{1}{2}\ln\left(\frac{x^2+1}{2}\right) &= \frac{1}{2}\big(\ln\big(x^2+1\big)-\ln(2)\big) \\
&= \frac{1}{2}\ln\big(x^2+1\big)-\frac{1}{2}\ln(2).
\end{aligned}
$$

Hence,

$$
\frac{d}{dx}\left(\frac{1}{2}\ln\left(\frac{x^2+1}{2}\right)\right) = \frac{d}{dx}\left(\frac{1}{2}\ln\big(x^2+1\big)\right).
$$

So, (3) is also correct.

A solution to the class activity of Lecture 6

$$\begin{aligned}\int \frac{3}{4x^2-25}\,dx &= 3\int \frac{1}{(2x)^2-5^2}\,dx \\ &= \frac{3}{20}\ln\left|\frac{2x-5}{2x+5}\right| + C.\end{aligned}$$

So, (2) is correct.

Now,

$$\begin{aligned}
\frac{3}{20}\ln\left|\frac{6x-15}{2x+5}\right| &= \frac{3}{20}\ln\left|\frac{3(2x-5)}{2x+5}\right| \\
&= \frac{3}{20}\ln\left(3\left|\frac{2x-5}{2x+5}\right|\right) \\
&= \frac{3}{20}\left(\ln(3)+\ln\left|\frac{2x-5}{2x+5}\right|\right) \\
&= \frac{3}{20}\ln(3)+\frac{3}{20}\ln\left|\frac{2x-5}{2x+5}\right|.
\end{aligned}$$

Hence,

$$\frac{d}{dx}\left(\frac{3}{20}\ln\left|\frac{6x-15}{2x+5}\right|\right) = \frac{d}{dx}\left(\frac{3}{20}\ln\left|\frac{2x-5}{2x+5}\right|\right).$$

So, (1) is also correct.

Moreover,

$$\frac{1}{20}\ln\left|\frac{6x-15}{2x+5}\right| = \frac{1}{20}\ln|6x-15| - \frac{1}{20}\ln|2x+5|.$$

Therefore,

$$\frac{d}{dx}\left(\frac{1}{20}\ln\left|\frac{6x-15}{2x+5}\right|\right) = \frac{3}{10(6x-15)} - \frac{1}{10(2x+5)}.$$

When $x = 0$,

$$\begin{aligned}\frac{3}{10(6x-15)} - \frac{1}{10(2x+5)} &= -\frac{3}{150} - \frac{1}{50} \\ &= -\frac{1}{25}\end{aligned}$$

but

$$\frac{3}{4x^2-25} = -\frac{3}{25}.$$

So, (3) is incorrect.

Independent Learning Week

This semester's Independent Learning Week (ILW) is week 8 of this semester (the week commencing 7 April 2025).

There will be no CELEN037 lectures, seminars or problem solving classes during the Independent Learning Week but you will be given some learning activities to engage in.

Tasks will be posted on Moodle on 7 April 2025.

CELEN037 mid-semester exam

- When: Wednesday **9th April 2025** from 15:00 to 16:00.
- Where: See the email from CELE Professional Services Office (CPSO).
- Bring: Student ID, pens and a Casio fx-82 series calculator (CELEN037 Calculator).
- Format: 10 multiple choice questions and 10 short-answer questions assessing topics from weeks 1 to 6 of CELEN037, excluding extended topics.
- Weight: 30% of your CELEN037 mark.

Ensure that you have read and have understood the Policy on academic misconduct and the Exam Instructions for Students (with the amendments that "approved documents" should be "a Casio fx-82 series calculator" and "All electronic devices" should be "All electronic devices except Casio fx-82 series calculators".

Note that above there are 3 links (in blue) that can be clicked on.