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Foundation Algebra for Physical Sciences & Engineering — CELEN036

Practice Problems SET-7 Sample Solution

Type 1: Intermediate value theorem

2. Given that the function $f(x) = x + \sin x - 1$ is continuous at $(-\infty, +\infty)$, prove that there exist at least one real root of the equation $x + \sin x - 1 = 0$.

Solution:

$$f\left(\frac{\pi}{2}\right) = \frac{\pi}{2} + \sin\left(\frac{\pi}{2}\right) - 1 = \frac{\pi}{2} + 1 - 1 = \frac{\pi}{2}$$

$$f(0) = 0 + \sin(0) - 1 = 0 + 0 - 1 = -1$$

$$\therefore\quad f\left(\frac{\pi}{2}\right) \cdot f(0) = -\frac{\pi}{2} < 0$$

Therefore, there is at least one root in interval $\left(\frac{\pi}{2}, 0\right)$

As $\left(\frac{\pi}{2}, 0\right) \in (-\infty, +\infty)$

Therefore, there is at least one root.

Type 2: Bisection method

4. Find the root of $f(x) = e^{-x}(3.2 \sin x - 0.5 \cos x)$ on the interval $[3, 4]$ in 3 decimal places. (write 3 rows)

Solution:

$$f(3) \cdot f(4) \approx 0.0471 \times -0.0384 = -0.0018 < 0$$

Therefore, there is at least a root in $(3, 4)$

n	a	b	$c = \frac{a+b}{2}$	$f(a)$	$f(b)$	$f(c)$	Decision:
0	3	4	3.500	> 0	< 0	< 0	Replace b by c
1	3	3.500	3.250	> 0	< 0	> 0	Replace a by c
2	3.250	3.500	3.375	> 0	< 0	< 0	Replace b by c

Type 3: Iteration method

6. Given the equation of $2x^3 - 2x - 5 = 0$, find the estimated root within 4 d.p. using the iterative formula $x_{n+1} = \left(\dfrac{2x_n + 5}{2}\right)^{\frac{1}{3}}$ and $x_0 = 1.5$.

Solution:

$$x_1 = \left(\frac{2x_0 + 5}{2}\right)^{\frac{1}{3}} = 1.5874$$

$$x_2 = \left(\frac{2x_1 + 5}{2}\right)^{\frac{1}{3}} = 1.5989$$

$$x_3 = \left(\frac{2x_2 + 5}{2}\right)^{\frac{1}{3}} = 1.6004$$

$$x_4 = \left(\frac{2x_3 + 5}{2}\right)^{\frac{1}{3}} = 1.6006$$

$$x_5 = \left(\frac{2x_4 + 5}{2}\right)^{\frac{1}{3}} = 1.6006$$

As $x_4 = x_5 = 1.6006,$ therefore the approximate root is $x^* = 1.6006$