University of Nottingham Ningbo China

CENTRE FOR ENGLISH LANGUAGE EDUCATION

PRELIMINARY YEAR, SEMESTER TWO, 2024-25

INTRODUCTION TO MATHEMATICAL SOFTWARE AND PROGRAMMING
END-OF-SEMESTER EXAM (Mock Exam)

Time allowed: TWO Hours

Candidates must write their ID number on the electronic file and fill-in their attendance card but must NOT work on anything else until the start of the exam is announced.

This paper contains FIVE questions. The total number of points is 100.

Only general bilingual dictionaries are allowed. Subject-specific dictionaries are not permitted.

No personal electronic devices can be used in this exam.

Do NOT open the examination paper until told to do so.

All work must be completed and submitted electronically.

ADDITIONAL MATERIAL:
Exam Instruction Sheet (electronic file).

INFORMATION FOR INVIGILATORS:
Please send the additional material electronically to students before the exam starts.
A 15-minute warning should be given before the end of the exam.
Please collect the question paper and collect all files electronically after the exam.

Submission Checklist

At the end of the exam, submit the following files along with the completed *Exam Instruction Sheet* (11 files in total).

Questions	**Saved Files**
Question 1	-
Question 2	**myPlot.m** **myPlot.png**
Question 3	**AnswerSheet.tex** **AnswerSheet.pdf**
Question 4	**isRepeated.m** (with any sub-functions) **idea.m** **test.m**
Question 5	**myFactorial.m** **myExp.m** **answer.m**

1. Use mathematical software GeoGebra/MATLAB to find the answers to the following questions. Keep a record of your computation results for later use in the $\LaTeX$ file in Question 3.

   (a) Find the area of the following ellipse. [4]

   $$\frac{(x-3)^2}{64} + \frac{(y-2)^2}{169} = 1$$

   (Round your answer to 2 decimal places.)

   (b) Find the partial fraction decomposition of the following rational expression. [4]

   $$\frac{2x^3}{(x-1)^2(x^2+1)}$$

   (c) Find the sum of the following series. [4]

   $$\sum_{n=1}^{20}(n^4 - 2n^3)$$

   (d) Let A be the matrix

   $$\begin{pmatrix} 1 & 0 & 2 \\ -2 & 4 & 0 \\ 0 & -1 & 0 \end{pmatrix}$$

   Find its inverse A^{-1} and the matrix product $A \cdot A$. [6]

   (e) What is the next prime number after 283? [2]

2. Write a MATLAB script **myPlot.m** that displays the overlaying plot of the following functions

   $$f(x) = 2\sin(2x)\cos(x), \quad g(x) = \sqrt{e^{-|x|}}$$

   on the interval $[-2\pi, 2\pi]$.
   Set your plot with the following configurations and export it as a **PNG** image file **myPlot.png**.

   - Use a red dashed line for f and a blue dotted line for g.
   - Set the line width for both curves as 2.
   - Add appropriate labels for the x-axis and y-axis.
   - Add an appropriate legend for f and g.
   - Show the gridline.
   - Set the title of the plot as "Two Curves". [20]

3. Create a $\LaTeX$ article based on your work in Question 1 and Question 2. It should contain contents as described below.

   Save your $\LaTeX$ source code as **AnswerSheet.tex** and the **PDF** file as **AnswerSheet.pdf**.

   (a) Set your article title as "My Answer Sheet".

   Display the article title, your Student ID, and today's date in the front page.

   Start a new page for each of the following sections in Question 3(b)(c), and display a table of contents in the front page. [4]

   (b) Create a section named "Question 1".

   In this section, typeset your answers to Question 1(a)(b)(c)(d)(e) using an ordered list.

   You should include all expressions/equations from the original questions. [12]

   (c) Create a section named "Question 2".

   In this section, insert the image file **myPlot.png** with an appropriate size and add the caption "MATLAB plot". Then include your MATLAB source code **myPlot.m** that is used in Question 2 for generating this image. [4]

4. (a) Write a MATLAB function **isRepeated.m** that checks if an input array has repeated values in it. For example,

   `isRepeated([1,3,7,3,8,1]) = 1`

   `isRepeated([5,2,9,1,7]) = 0`

   `isRepeated([ ]) = 0`

   - Do NOT use any of the built-in functions: `sort/max/min`.
   - For any sub-functions created based on your need, you must include them in the final submission of your electronical files. [12]

   (b) Write a MATLAB script **idea.m** to give a brief expanation about how your program works using MATLAB comment. [4]

   (c) Write a MATLAB script **test.m** to test your function in Question 4(a) with the following arrays.

   (i) The array `[3,7,3,8,1]`.
   (ii) An empty array.
   (iii) An array containing only one number.
   (iv) An array containing 5 integers that are randomly generated between 1 and 10. [4]

5. The Maclaurin expansion series for e^x is given by

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

(a) Write a MATLAB function **myFactorial.m** that computes $n!$ for a nonnegative integer n. For example,

```
myFactorial(0) = 1
myFactorial(1) = 1
myFactorial(3) = 6
```

- Do NOT use the built-in function: `factorial`. [6]

(b) Based on the expansion formula, we can approximate e^x by a polynomial with degree n:

$$e^x \approx 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!}$$

Write a MATLAB function **myExp.m** that takes a value x and a positive integer n as its input arguments. It should have two output arguments:

- the first one is the approximate value of e^x using the polynomial with degree n;
- the second one is the computing error obtained by taking the absolute difference between the approximate value and the actual value computed by `exp(x)`.

For example, calling `[approx,error] = myExp(0.5,3)` we should have

`approx = 1.6458    error = 0.0029`

because $e^{0.5} \approx 1 + 0.5 + \frac{0.5^2}{2!} + \frac{0.5^3}{3!}$, and $|1.6458 - \exp(0.5)| = 0.0029$.

- You must use your function in Question 5(a) for computing factorials. [10]

(c) Write a MATLAB script **answer.m** to answer the following questions with codes/comments.

(i) What is the approximate value of e^2 by using a polynomial of degree 4 in the expansion formula? What is the computing error?

(ii) We want to achieve a better approximation to e^2 so that the computing error is less than 10^{-5}, what will be the least degree of the expansion polynomial? (i.e. What is the smallest value n?) [4]

All work must be completed and submitted electronically.