1a. (i) $h(x) = (f \circ g)(x) = f(g(x))$

$$= f(x-2)$$
$$= 2(x-2)^2 + 3(x-2) - 2$$
$$= 2x^2 - 8x + 8 + 3x - 6 - 2$$
$$= 2x^2 - 5x$$

(ii) $h(x) = 2x^2 - 5x = 2(x^2 - \frac{5}{2}x)$

$$= 2(x^2 - \frac{5}{2}x + \frac{25}{16} - \frac{25}{16})$$
$$= 2(x - \frac{5}{4})^2 - \frac{25}{8}$$

(iii) As $(x - \frac{5}{4})^2 \geqslant 0$

$$2(x - \frac{5}{4})^2 \geqslant 0$$
$$2(x - \frac{5}{4})^2 - \frac{25}{8} \geqslant -\frac{25}{8}$$

$\therefore h(x) \geqslant -\frac{25}{8}$, $\quad R_h$ is $[-\frac{25}{8}, \infty)$

1b. (i) Let $y = f(x) = \sqrt{x+8} - 4$,

$$y + 4 = \sqrt{x+8}$$
$$(y+4)^2 = x + 8$$
$$x = (y+4)^2 - 8$$
$$\therefore f^{-1}(x) = (x+4)^2 - 8$$

(ii) $|x-2| \geqslant 5$

$\therefore x - 2 \geqslant 5$ or $x - 2 \leqslant -5$

$\therefore x \geqslant 7$ or $x \leqslant -3$ or $x \in R - (-3, 7)$

1c. (i) Let $e^x = t$, $\therefore \quad t^2 - 5t - 24 = 0$

$$t^2 - 8t + 3t - 24 = 0$$

$$(t-8)(t+3) = 0$$

$$t = 8 \text{ or } t = -3$$

as $t = e^x > 0$, $t = -3$ is not acceptable

$\therefore \quad t = 8$

$e^x = 8$, $x = \ln 8$

(ii) $\log \frac{6x}{4-x} = \log 3$

$\therefore \quad \frac{6x}{4-x} = 3$

$$6x = 3(4-x)$$

$$9x = 12$$

$x = \frac{4}{3}$ when $x = \frac{4}{3}$, $6x > 0$ and $4-x > 0$

$\therefore \quad x = \frac{4}{3}$.

2a. $\text{RHS} = \frac{2\sin\theta}{\sin^2\theta + \cos^2\theta - \sin^2\theta}$

$$= \frac{2\sin\theta}{\cos^2\theta}$$

$$= 2\,\frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\cos\theta} = 2\tan\theta\sec\theta$$

2b. Let $\cos\theta = t$, $2t^2 + t - 1 = 0$

$$2t^2 - t + 2t - 1 = 0$$

$$(2t-1)(t+1) = 0$$

$\therefore \quad t = \frac{1}{2}$ or $t = -1$

$\therefore \quad \cos\theta = \frac{1}{2}$ or $\cos\theta = -1$

as $\theta \in (0, \pi)$ $\therefore \cos\theta \in (-1, 1)$

$\therefore \quad \cos\theta = \frac{1}{2}$, $\therefore \theta = \frac{\pi}{3}$

2c. (i) $f(x) = R\cos x\cos\theta + R\sin x\sin\theta$

$\therefore \begin{cases} R\cos\theta = \sqrt{3} & \cdots ① \\ R\sin\theta = 1 & \cdots ② \end{cases}$

$①^2 + ②^2 = R^2(\sin^2\theta + \cos^2\theta) = 4$

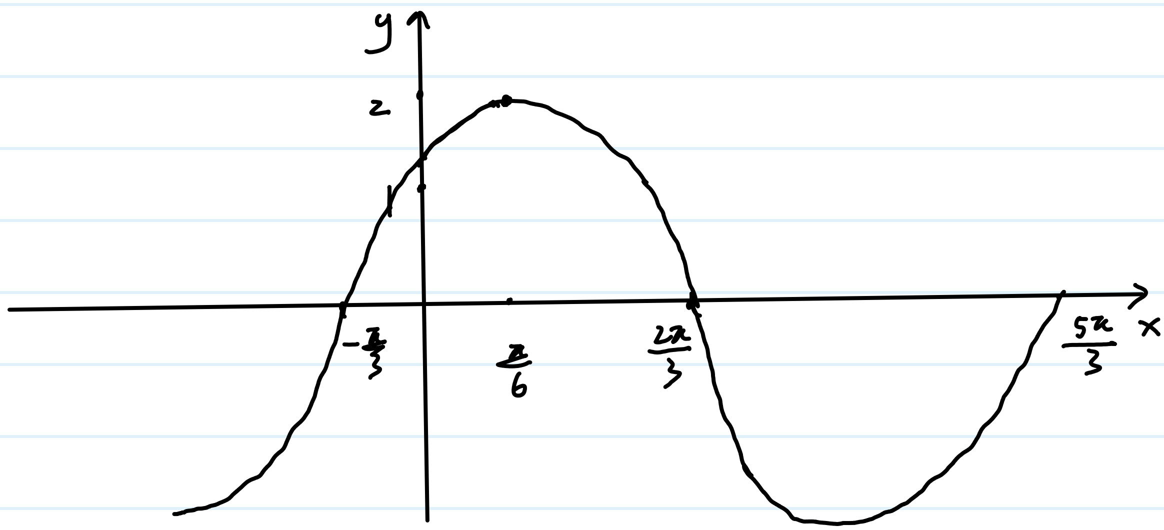
$\therefore R = 2$

$② \div ① = \dfrac{\sin\theta}{\cos\theta} = \tan\theta = \dfrac{\sqrt{3}}{3}$

$\therefore \theta = \tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$

$\therefore f(x) = 2\cos\left(x - \frac{\pi}{6}\right)$.

(ii)



2d. (i) $LHS = \sin 65° + \sin 25°$

$= 2\sin\left(\dfrac{65° + 25°}{2}\right)\cos\left(\dfrac{65° - 25°}{2}\right)$

$= 2\sin(45°)\cos(20°)$

$= \sqrt{2}\sin 70° = RHS$.

(ii) $\sin^{-1}\left(\sin\frac{2\pi}{3}\right) = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) =$

$= \sin^{-1}\left(\sin\left(\frac{\pi}{3}\right)\right)$

$= \frac{\pi}{3}$

3a (i) $x+2 = x-(-2) \quad \therefore c=-2$

$$\begin{array}{r|rrrr} & 6 & 5 & -12 & 4 \\ -2 & \downarrow & -12 & 14 & -4 \\ \hline & 6 & -7 & 2 & \boxed{0} \end{array}$$

$\therefore \quad 6x^3+5x^2-12x+4 = (x+2)(6x^2-7x+2)$

As the remaider is 0, therefore $x+2$ is a factor

(ii)

$p(x) = (x+2)(6x^2-7x+2)$

$= (x+2)(6x^2-3x-4x+2)$

$= (x+2)(3x-2)(2x-1)$

(iii) when $p(x)=0$, $x=-2, \frac{2}{3}$ or $\frac{1}{2}$.

3b $f(2) = 4+2a+b = -4 \quad \cdots ①$

$f(-4) = 16-4a+b = -4 \quad \cdots ②$

① − ②

$\therefore \; -12+6a=0 \quad \therefore a=2$

Let $a=2$ in ① $\quad \therefore \; 4+4+b=-4 \quad \therefore b=-12$

$\therefore a=2, \; b=-12$

3c $(x^2)^9 + \binom{9}{1}(x^2)^8(-1/2) + \binom{9}{2}(x^2)^7(-1/2)^2 + \binom{9}{3}(x^2)^6\left(-\frac{1}{2}\right)^3 + \boxed{\binom{9}{4}(x^2)^5\left(-\frac{1}{2}\right)^4}$

$+ \cdots\cdots \left(-\frac{1}{2}\right)^9$

$\therefore$ coefficient of $x^{10} = \binom{9}{4}\left(-\frac{1}{2}\right)^4$

$= 126\left(\frac{1}{16}\right)$

$= \frac{63}{8}$

4a.

$$(1-3x)^{-\frac{1}{4}} = 1+(-\tfrac{1}{4})\cdot(-3x)+\frac{(-\frac{1}{4})(-\frac{1}{4}-1)}{2!}\cdot(-3x)^2+\frac{(-\frac{1}{4})(-\frac{1}{4}-1)(-\frac{1}{4}-2)}{3!}\cdot(-3x)^3+\cdots$$

$$= 1+\frac{3}{4}x+\frac{45}{32}x^2+\frac{405}{128}x^3+\cdots$$

4b.

$$V+\delta V = \pi(r+\delta r)^2 h$$
$$= \pi(r+0.02r)^2 h$$
$$= \pi r^2 h(1+0.02)^2$$
$$= V\cdot\left(1+2\cdot 0.02+\frac{2(2-1)}{2!}\cdot(0.02)^2\right)$$
$$= V+0.0404V$$

$\therefore \delta V = 0.0404V$ $\quad\therefore$ error is 4.04% of volume.

4c. (i) $f(0) = 0-0+2 = 2$, $f(1) = 1-7+2 = -4$

$\therefore f(0)\cdot f(1) < 0$ $\quad\therefore$ there is at least a root in $(0,1)$

(ii) $x^3-7x+2=0$

$$7x = x^3+2$$
$$x = \frac{x^3+2}{7} \quad \therefore \quad x_{n+1} = \frac{x_n^3+2}{7}$$

(iii)

n	x_n
0	0.5
1	0.30357
2	0.28971
3	0.28919
4	0.28917
5	0.28917

the approximate value of x is 0.28917

5a. $A^T = \begin{pmatrix} 2 & 3 \\ 5 & -1 \end{pmatrix}$

$\therefore A^T B = \begin{pmatrix} 2 & 3 \\ 5 & -1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

$= \begin{pmatrix} 2\times1 + 3\times3 & 2\times2 + 3\times4 \\ 5\times1 + -1\times3 & 5\times2 + -1\times4 \end{pmatrix} = \begin{pmatrix} 11 & 16 \\ 2 & 6 \end{pmatrix}$

Let $C = \begin{pmatrix} 11 & 16 \\ k & 6 \end{pmatrix} = \begin{pmatrix} 11 & 16 \\ 2 & 6 \end{pmatrix}$ $\therefore k = 2$

5b. (i)

$\begin{pmatrix} 1 & -7 \\ 5 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -11 \\ -18 \end{pmatrix}$

(ii) $\det(A) = 1\times2 - (-7)\cdot5 = 37 \neq 0$

$\therefore A^{-1} = \frac{1}{\det(A)}\begin{pmatrix} 2 & 7 \\ -5 & 1 \end{pmatrix} = \begin{pmatrix} \frac{2}{37} & \frac{7}{37} \\ \frac{-5}{37} & \frac{1}{37} \end{pmatrix}$

(iii) $X = A^{-1}B = \frac{1}{37}\begin{pmatrix} 2 & 7 \\ -5 & 1 \end{pmatrix}\begin{pmatrix} -11 \\ -18 \end{pmatrix} = \frac{1}{37}\begin{pmatrix} -148 \\ 37 \end{pmatrix} = \begin{pmatrix} -4 \\ 1 \end{pmatrix}$

$\therefore x = -4, \quad y = 1$

5c. $\det(C) = 9\cdot(2k) - 2\cdot(1-k) = 0$

$\therefore k = \frac{1}{10}$

5d. $\frac{3x}{(2x+1)(x^2+2)} = \frac{A}{2x+1} + \frac{Bx+C}{x^2+2}$

$3x = A(x^2+2) + (Bx+C)(2x+1)$

Let $x = -\frac{1}{2}$, $\therefore A = -\frac{2}{3}$

Let $x = 0$ and $A = -\frac{2}{3}$, $\therefore C = \frac{4}{3}$

Let $x = 1$, $A = -\frac{2}{3}$ and $C = \frac{4}{3}$, $\therefore B = \frac{1}{3}$

$\therefore \frac{3x}{(2x+1)(x^2+2)} = -\frac{2}{3(2x+1)} + \frac{x+4}{3(x^2+2)}$

6a. $z = (i+3)^2 - 4i$

$= -1 + 6i + 9 - 4i = 8 + 2i$

6b (i) $2x - 4y - 2 + (3x - 2y)i = 0$

$\therefore \begin{cases} 2x - 4y - 2 = 0 & \cdots ① \\ 3x - 2y = 0 & \cdots ② \end{cases}$

$2 \times ② - ① \Rightarrow 4x + 2 = 0 \quad \therefore x = -\frac{1}{2}$

Let $x = -\frac{1}{2}$ in ② $\Rightarrow -\frac{3}{2} - 2y = 0 \quad \therefore y = -\frac{3}{4}$

(ii) $\left|-\frac{1}{2} - \frac{3}{4}i\right| = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(-\frac{3}{4}\right)^2} = \sqrt{\frac{1}{4} + \frac{9}{16}} = \sqrt{\frac{13}{16}} = \frac{\sqrt{13}}{4}$

6c. (i) $z = \frac{i}{1+i} = \frac{i(1-i)}{(1+i)(1-i)} = \frac{1+i}{2} = \frac{1}{2} + \frac{1}{2}i$

(ii) $r = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \frac{\sqrt{2}}{2}$, $\theta = \tan^{-1}\left|\frac{\frac{1}{2}}{\frac{1}{2}}\right| = \tan^{-1}(1) = \frac{\pi}{4}$

(iii) $\therefore z = \frac{\sqrt{2}}{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

6d.

$$|z_1| = \sqrt{3^2+4^2} = 5$$
$$|z_2| = \sqrt{6^2+3^2} = \sqrt{45} = 3\sqrt{5}$$
$$|z_3| = |\bar{z}_1 \cdot z_2| = |z_1| \cdot |z_2| = 15\sqrt{5}$$

$$\therefore \left|\frac{z_1^2 \cdot z_2}{\bar{z}_2 \cdot z_3}\right| = \frac{|z_1|^2 \cdot |z_2|}{|z_2| \cdot |z_3|} = \frac{5^2 \cdot 3\sqrt{5}}{3\sqrt{5} \cdot 15\sqrt{5}} = \frac{\sqrt{5}}{3}$$

7a. (i)

$$a_4 = a + (4-1)\cdot d = -20 \quad \cdots ①$$
$$a_8 = a + (8-1)\cdot d = -10 \quad \cdots ②$$
$$② - ① \Rightarrow 4d = 10$$
$$\therefore d = \frac{5}{2}$$
$$\therefore a = -\frac{55}{2}$$
$$\therefore a_{12} = -\frac{55}{2} + (12-1)\cdot\frac{5}{2} = 0$$

(ii)

$$S_{100} = \frac{100}{2}\left(2\cdot\left(-\frac{55}{2}\right) + (100-1)\cdot\frac{5}{2}\right) = 9625$$

7b (i)

$$a = 1 \qquad r = -\frac{1}{8} \div 1 = -\frac{1}{8}$$
$$\therefore a_6 = 1\cdot\left(-\frac{1}{8}\right)^{6-1} = -\frac{1}{32768}$$

(ii)

as $|r| = \frac{1}{8} < 1$

$$\therefore S = \frac{1}{1-\left(-\frac{1}{8}\right)} = \frac{8}{9}$$

7c (i)

$$\sum_{1}^{n} k(k-10) = \sum_{1}^{n} k^2 - 10\sum_{1}^{n} k$$
$$= \frac{n(n+1)(2n+1)}{6} - \frac{10n(n+1)}{2}$$
$$= \frac{n(n+1)(2n+1) - n(n+1)\cdot 30}{6} = \frac{n(n+1)(2n-29)}{6}$$

(ii) $$\sum_{n=15}^{30} n(n-10) = \sum_{n=1}^{30} n(n-10) - \sum_{n=1}^{14} n(n-10)$$

$$= \frac{30(30+1)(2\times 30-29)}{6} - \frac{14\cdot(14+1)\cdot(2\times 14-29)}{6}$$

$$= 4805 - (-35) = 4840$$

7d. $$LHS = \frac{1}{2}\left(f(1) - f(2) + f(2) - f(3) + \cdots + f(n) - f(n+1)\right)$$

$$= \frac{1}{2}\left(f(1) - f(n+1)\right)$$

$$= \frac{1}{2}\left(\frac{1}{1\cdot 2} - \frac{1}{(n+1)(n+2)}\right)$$

$$= \frac{1}{2}\left(\frac{(n+1)(n+2)-2}{2(n+1)(n+2)}\right) = \frac{n^2+3n}{4(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}.$$