



Student Evaluation of Module (SEM)

Foundation Algebra for Physical Sciences & Engineering



- This is an evaluation of the module for **CELEN036**
- There are 5 questions and opportunity for some comments
- Scan the QR code below, login using your university account, and complete the survey.



Seminar 9

In this seminar you will study:

- Algebra of Matrices
- Inverse of 2×2 matrices
- Solving 2×2 systems of linear equations

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Algebra of Matrices: Equality of Matrices

Example: Given matrices $A = \begin{pmatrix} x & 2 \\ 5 & y \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 3 \\ z & -1 \end{pmatrix}$,

find the constants x , y , and z , if $3A = 2B$.

Solution:

$$3A = \begin{pmatrix} 3x & 6 \\ 15 & 3y \end{pmatrix}$$

$$2B = \begin{pmatrix} 2 & 6 \\ 2z & -2 \end{pmatrix}$$

$$3A = 2B \Rightarrow \begin{pmatrix} \boxed{3x} & 6 \\ \boxed{15} & \boxed{3y} \end{pmatrix} = \begin{pmatrix} \boxed{2} & 6 \\ \boxed{2z} & \boxed{-2} \end{pmatrix}$$

$$\Rightarrow \begin{cases} 3x = 2 \\ 3y = -2 \\ 2z = 15 \end{cases}$$

$$\Rightarrow x = \frac{2}{3}, \quad y = -\frac{2}{3}, \quad z = \frac{15}{2}$$

Algebra of Matrices: The Transpose of a Matrix

Example: Given matrices $A = \begin{pmatrix} 3 & 2 \\ -4 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 6 \\ 2 & -1 \end{pmatrix}$, find A^T , B^T , and $(A + B)^T$.

Hence show that $(A + B)^T = A^T + B^T$.

Solution:

$$A^T = \begin{pmatrix} 3 & 2 \\ -4 & 0 \end{pmatrix}^T = \begin{pmatrix} 3 & -4 \\ 2 & 0 \end{pmatrix}$$

$$B^T = \begin{pmatrix} 1 & 6 \\ 2 & -1 \end{pmatrix}^T = \begin{pmatrix} 1 & 2 \\ 6 & -1 \end{pmatrix}$$

$$A + B = \begin{pmatrix} 4 & 8 \\ -2 & -1 \end{pmatrix}$$

$$\Rightarrow (A + B)^T = \begin{pmatrix} 4 & -2 \\ 8 & -1 \end{pmatrix}$$

$$A^T + B^T = \begin{pmatrix} 3 & -4 \\ 2 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 6 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & -2 \\ 8 & -1 \end{pmatrix}$$

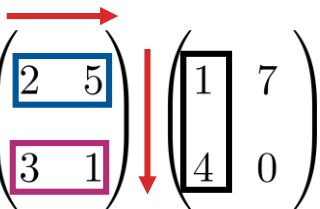
$$\Rightarrow (A + B)^T = A^T + B^T$$

Algebra of Matrices: Matrix multiplication

Example: Given matrices $A = \begin{pmatrix} 2 & 5 \\ 3 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 7 \\ 4 & 0 \end{pmatrix}$,

find AB and BA .

Solution:

$$AB = \begin{pmatrix} \boxed{2} & \boxed{5} \\ \boxed{3} & \boxed{1} \end{pmatrix} \begin{pmatrix} \boxed{1} & 7 \\ \boxed{4} & 0 \end{pmatrix}$$


Similarly
↓

$$= \begin{pmatrix} \boxed{2 \times 1 + 5 \times 4} & 2 \times 7 + 5 \times 0 \\ \boxed{3 \times 1 + 1 \times 4} & 3 \times 7 + 1 \times 0 \end{pmatrix}$$
$$= \begin{pmatrix} 22 & 14 \\ 7 & 21 \end{pmatrix}$$

$$BA = \begin{pmatrix} 1 & 7 \\ 4 & 0 \end{pmatrix} \begin{pmatrix} 2 & 5 \\ 3 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \times 2 + 7 \times 3 & 1 \times 5 + 7 \times 1 \\ 4 \times 2 + 0 \times 3 & 4 \times 5 + 0 \times 1 \end{pmatrix}$$

$$= \begin{pmatrix} 23 & 12 \\ 8 & 20 \end{pmatrix}$$

In general, $AB \neq BA$



Inverse of 2×2 Matrices

Example: Given $A = \begin{pmatrix} 7 & 4 \\ 5 & 3 \end{pmatrix}$ find the inverse matrix A^{-1} , if it exists.

Solution:

Step 1 Find the determinant of A: $\det(A) = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$ given that $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

Step 2 Use the formula for the inverse: If $\det(A) \neq 0$, then $A^{-1} = \frac{1}{\det(A)} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

Note: If $\det(A) = 0$, A is singular, and its inverse does not exist.

Here, $\det(A) = 7 \times 3 - 4 \times 5 = 1 \neq 0$

Thus, A^{-1} exists,

$$\Rightarrow A^{-1} = \frac{1}{1} \begin{pmatrix} 3 & -4 \\ -5 & 7 \end{pmatrix} = \begin{pmatrix} 3 & -4 \\ -5 & 7 \end{pmatrix}$$



Solving 2×2 Systems of Linear Equations

Example: Express the system of linear equations $\begin{cases} 3x + y = 4 \\ 5x - 4y = 1 \end{cases}$ into the matrix form

$AX = B$, and use the matrix method ($AX = B \Rightarrow X = A^{-1}B$) to solve it.

Solution:

Step 1 Matrix form $AX = B$:

$$\begin{pmatrix} 3 & 1 \\ 5 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

$\downarrow \quad \downarrow \quad \downarrow$
 $A \quad X \quad B$

Step 2 Find the determinant of A : $\det(A) = -12 - 5 = -17 \neq 0 \quad \therefore A^{-1}$ exists

Step 3 Find A^{-1} :

Since $\det(A) \neq 0$, then $A^{-1} = -\frac{1}{17} \begin{pmatrix} -4 & -1 \\ -5 & 3 \end{pmatrix}$



Solving 2×2 Systems of Linear Equations

Step 4 Find X :

$$X = A^{-1}B \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{-17} \begin{pmatrix} -4 & -1 \\ -5 & 3 \end{pmatrix} \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

$$= \frac{1}{-17} \begin{pmatrix} -4 \times 4 + -1 \times 1 \\ -5 \times 4 + 3 \times 1 \end{pmatrix}$$

$$= \frac{1}{-17} \begin{pmatrix} -17 \\ -17 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Always show the matrix multiplication process in the exams

$$\therefore x = 1, y = 1$$



THANKS FOR YOUR ATTENTION

More about linear transformation: [Video of linear transformation](#)
(beyond the requirement of this module)