Foundation Algebra for Physical Sciences & Engineering CELEN036

Practice Problems SET-5 **Topic:** Trigonometry II

Type 1: Addition and Factor formulae

1. Prove the following trigonometric identities:

$$(i) \quad \cos(270^\circ - \theta) = -\sin\theta$$

$$(ii) \quad \tan\left(\frac{\pi}{4} + \alpha\right) = \frac{\cos\alpha + \sin\alpha}{\cos\alpha - \sin\alpha}$$

$$(iii) \quad \tan 63^\circ = \frac{\cos 18^\circ + \sin 18^\circ}{\cos 18^\circ - \sin 18^\circ}$$

$$(iv) \quad \cot 5^\circ = \frac{\sqrt{3}\cos 25^\circ + \sin 25^\circ}{\cos 25^\circ - \sqrt{3}\sin 25^\circ}$$

2. Given $3\sin(x-y) - \sin(x+y) = 0$. Show that $\tan x = 2\tan y$.

3. Prove the following results:

$$(i) \quad \frac{\sin 6\theta - \sin 4\theta}{\sin\theta} = 2\cos 5\theta$$

$$(ii) \quad \frac{\sin 80^\circ + \sin 20^\circ}{\cos 20^\circ - \cos 80^\circ} = \sqrt{3}$$

$$(iii) \quad \cos\left(\frac{\pi}{8}\right) + \cos\left(\frac{3\pi}{8}\right) + \cos\left(\frac{5\pi}{8}\right) + \cos\left(\frac{7\pi}{8}\right) = 0$$

$$(iv) \quad \sin\left(\frac{\pi}{6} + \theta\right) + \cos\left(\frac{\pi}{3} + \theta\right) = \cos\theta$$

4. Given that A and B are angles, such that $\sin A = \dfrac{4}{5}, \quad \dfrac{\pi}{2} < A < \pi$, and

$\cos B = -\dfrac{5}{13}, \quad \pi < B < \dfrac{3\pi}{2}$. Find each of the following.

(i) $\sin(A+B)$ (ii) $\tan(A+B)$ (iii) the quadrant of the angle $A+B$

Type 2: Multi-angle formulae

5. Prove the following results:

(i) $\dfrac{1-\cos 2\theta+\sin 2\theta}{1+\cos 2\theta+\sin 2\theta}=\tan\theta$

(ii) $\dfrac{\tan\theta}{1+\tan^2\theta}=\dfrac{1}{2}\sin 2\theta$

(iii) $1+\dfrac{4\tan^2\theta}{(1-\tan^2\theta)^2}=\dfrac{1}{1-4\sin^2\theta\cos^2\theta}$

(iv) $\dfrac{\sin\theta+\sin 3\theta+\sin 5\theta+\sin 7\theta}{\cos\theta+\cos 3\theta+\cos 5\theta+\cos 7\theta}=\tan 4\theta$

(v) $\cos 80^\circ+\sin 50^\circ=\cos 20^\circ$

(vi) $\cos\theta=\dfrac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$

6. Prove that: $\left(\sin\dfrac{x}{2}+\cos\dfrac{x}{2}\right)^2=1+\sin x$.

7. Simplify: $\sqrt{2+\sqrt{2+2\cos 4\theta}}$. (Take $\sqrt{X^2}=X$)

8. Prove that $\cos\theta=\sqrt{\dfrac{1}{2}+\sqrt{\dfrac{1}{8}+\dfrac{1}{8}\cos 4\theta}}$. (Take $\sqrt{X^2}=X$)

9. Find all solutions of $\cos 2x=\cos x$ over the interval of $[0,2\pi)$.

10. Solve $4\sin\theta\cos\theta=\sqrt{3}$ in the interval of $\left[0,\dfrac{\pi}{2}\right]$.

11. Give exact solutions of $3\tan 3x=\sqrt{3}$ over the interval of $\left[0,\dfrac{\pi}{2}\right]$.

Type 3: Inverse Trigonometric functions

12. Without using a calculator, find the values of:

(i) $\cos\left[\sin^{-1}\left(-\dfrac{1}{2}\right)\right]$ (ii) $\tan\left[\cos^{-1}\left(-\dfrac{1}{2}\right)\right]$

(iii) $\sin^{-1}\left[\sin\left(\dfrac{2\pi}{3}\right)\right]$ (iv) $\sin\left[2\sin^{-1}\left(\dfrac{1}{\sqrt{2}}\right)\right]$

13. Evaluate the following expression using addition or factor formulae: $\cos\left(\tan^{-1}\sqrt{3}+\sin^{-1}\dfrac{1}{3}\right)$.

14. Evaluate the following expression using multi-angle formulae: $\tan\left(2\sin^{-1}\dfrac{2}{5}\right)$.

15. Solve $\cos^{-1}x=\sin^{-1}\dfrac{1}{2}$.

16. Find the solution of $\sin^{-1}x-\cos^{-1}x=\dfrac{\pi}{6}$.

Type 4: Expressing $a\cos x + b\sin x$ in the form $r\cos(x \pm \theta)$ or $r\sin(x \pm \theta)$

17. Express $2\cos\theta + 5\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$, and $0 < \alpha < 90°$. Hence solve the equation $2\cos\theta + 5\sin\theta = 3 \quad (0 < \theta < 360°)$.

18. Show that $\cos\theta - \sqrt{3}\sin\theta$ can be written in the form $R\cos(\theta + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Hence sketch the graph of $f(\theta) = \cos\theta - \sqrt{3}\sin\theta \quad (0 < \theta < 2\pi)$.

19. Given that $5\sin\theta + 12\cos\theta \equiv R\cos(\theta - \alpha)$, find $R > 0$ and $\alpha \in \left[0, \frac{\pi}{2}\right]$.

20. Express $5\sin x + 12\cos x$ in the form $R\sin(x + \theta)$, where $R > 0$ and $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ are to be determined.

Answers

4 (i) $\sin(A+B) = \frac{16}{65}$ (ii) $\tan(A+B) = \frac{16}{63}$ (iii) quadrant I

7 $2\cos\theta$

9 $0, \frac{2\pi}{3}, \frac{4\pi}{3}$

10 $\frac{\pi}{6}, \frac{\pi}{3}$

11 $\frac{\pi}{18}, \frac{7\pi}{18}$

12 (i) $\frac{\sqrt{3}}{2}$ (ii) $-\sqrt{3}$ (iii) $\frac{\pi}{3}$ (iv) 1

13 $\frac{2\sqrt{2} - \sqrt{3}}{6}$

14 $\frac{4\sqrt{21}}{17}$

15 $\frac{\sqrt{3}}{2}$

16 $\frac{\sqrt{3}}{2}$

17 $R = 2,\ \alpha = 86.2°$ Roots $\theta = 124.35°$ or $12.05°$

18 $R = 2,\ \alpha = \frac{\pi}{3}$

19 $R = 13,\ \alpha = 0.3948$

20 $R = 13,\ \theta = 1.18$