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Foundation Calculus and Mathematical Techniques CELEN037

Practice Problems SET-5 Sample Solution

Type 1: Simple Integration

1. Evaluate the following integrals: (i) $\int \left(\frac{3}{x} + \sqrt[3]{x} - \frac{4}{\sqrt{x}}\right)\, dx$

Solution:

$$\int \left(\frac{3}{x} + \sqrt[3]{x} - \frac{4}{\sqrt{x}}\right)\, dx$$

$$= \int \left(\frac{3}{x}\right)\, dx + \int \left(\sqrt[3]{x}\right)\, dx - \int \left(\frac{4}{\sqrt{x}}\right)\, dx$$

$$= 3\ln|x| + \frac{3}{4}x^{\frac{3}{4}} - 8\sqrt{x} + +C$$

Type 2: The method of substitution for integration

2. Evaluate the following integrals by using appropriate substitutions: (i) $\int (3x+2)^4\, dx$

Solution:

Let $t = 3x + 2$

$$\frac{dt}{dx} = 3$$

$$dx = \frac{1}{3}dt$$

$$I = \frac{1}{3}\int (t)^4\, dt$$

$$= \frac{1}{3}\cdot\frac{1}{5}t^5 + C$$

$$= \frac{1}{15}(3x+2)^5 + C$$

2. Evaluate the following integrals by using appropriate substitutions: (xi) $\int (x+3)^2(x-5)^5 \, dx$

Solution:

Let $t = x - 5$

$$\frac{dt}{dx} = 1$$

$$dx = dt$$

$$I = \int (t+8)^2 t^5 \, dt$$

$$= \int \left(t^7 + 16t^6 + 64t^5\right) \, dt$$

$$= \frac{t^8}{8} + \frac{16t^7}{7} + \frac{32t^6}{3} + C$$

$$= \frac{(x-5)^8}{8} + \frac{16(x-5)^7}{7} + \frac{32(x-5)^6}{3} + C$$

Type 3: Trigonometric integrals

3. Evaluate the following integrals by using appropriate substitutions: (i) $\int \frac{\sec^2 x}{\sqrt{\tan^2 x - 9}} \, dx$

Solution:

Let $t = \tan x$

$$\frac{dt}{dx} = \sec^2 x$$

$$\sec^2 x dx = dt$$

$$I = \int \frac{1}{\sqrt{t^2 - 3^2}} \, dt$$

$$= \ln \left| t + \sqrt{t^2 - 3^2} \right| + C$$

$$= \ln \left| \tan x + \sqrt{\tan^2 x - 9} \right| + C$$

Type 4: Integrals in the form $\int f(ax+b)\,dx$

4. Evaluate the following integrals: (i) $\int \sec(5x-1)\cdot\tan(5x-1)\ dx$

Solution:

As $\int \sec(x)\cdot\tan(x)\ dx = \sec(x)+C$

$$\int \sec(5x-1)\cdot\tan(5x-1)\ dx$$

$$= \frac{1}{5}\sec(5x-1)+C$$