Foundation Algebra for Physical Sciences & Engineering CELEN036

Practice Problems SET-10 **Topic:** Partial Fractions, Sequences

Type 1: Finding terms of a sequence

1. Find the first five terms (i.e., for $n = 1, 2, 3, 4, 5$) of the following sequence:

   (i) $f(n) = 3n + 2$ (ii) $f(n) = \dfrac{n+2}{n(n+1)}$

   (iii) $f(n) = 1 + (-1)^n$ (iv) $f(n) = (-1)^n(n+1)$

2. Find the nth term's formula of the following sequence:

   (i) $3,\ 7,\ 11,\ 15,\ 19, \cdots$ (ii) $5,\ 2,\ -1,\ -4,\ -7, \cdots$

   (iii) $1,\ \frac{1}{2},\ \frac{1}{4},\ \frac{1}{8}, \cdots$ (iv) $4,\ -8,\ 16,\ -32,\ 64, \cdots$

Type 2: Arithmetic progression

3. Find the 10^{th} term of the sequence: $3,\ 15,\ 27,\ 39, \cdots$

4. Find the 14^{th} term of the sequence: $-19,\ -16,\ -13,\ -10, \cdots$

5. Find the 12^{th} term of the sequence: $95,\ 80,\ 65,\ 50, \cdots$

6. Obtain the twelfth, the twenty-seventh, and the fortieth terms of the arithmetic sequence:

   $3,\ 7,\ 11,\ 15, \cdots$

7. The first term of an arithmetic sequence is 7 and its common difference is 5. If the n^{th} term $a_n = 462$, find n.

8. For an arithmetic sequence $152,\ 145,\ 138,\ 131, \cdots$, if the n^{th} term $a_n = 12$, find n.

9. Find the common difference, the fifth term, the nth term and the 100th term of the arithmetic progression (AP).

   $$\frac{x}{x^2+1},\quad \frac{2x^2+x+1}{x^3+x^2+x+1},\quad \frac{3x^2+x+2}{x^3+x^2+x+1},\quad \frac{4x^2+x+3}{x^3+x^2+x+1}, \ldots$$

Type 3: Geometric progression

10. The first term of a geometric sequence is 2 and the common ratio is 3. Find the fourth term.

11. The first term of a geometric sequence is -3 and the third term is -12. Find the second term a_2.

12. The sixth term of a geometric sequence is 16 and the third term is 2. Find the first term a and the common ratio r.

13. For a geometric sequence $\frac{1}{3}, \frac{1}{6}, \frac{1}{12}, \frac{1}{24}, \cdots$, if the n^{th} term $a_n = \frac{1}{3072}$, find n.

14. Find the negative common ratio of a geometric progression whose first term is 8 and the fifth term is $\frac{1}{2}$.

15. The second term of a geometric progression with only positive terms is $\frac{1}{4}$ and the sum of the first four terms is $\frac{1}{16}$ of the sum of the next four terms. Find the G.P.

16. The sum and product of three consecutive terms in a geometric progression are 52 and 1728 respectively. Find these three terms.

17. Find the nth term expression $a_n = ar^{(n-1)}$ of the geometric progression (GP) using the given values: $a_5 = -4, \quad a_9 = 16$.

Type 4: Find the n^{th} term

18. Write down the first three terms of the following series:

   (i) $\displaystyle\sum_{r=1}^{\infty} \frac{x^{2r}}{(2r-1)(2r+1)}$

   (ii) $\displaystyle\sum_{r=0}^{\infty} \frac{(-1)^{r-1}\, 2^{2r}\, x^{2r-1}}{(2r)!}$

   (iii) $\displaystyle\sum_{n=1}^{\infty} 2^{n/2} \cdot \sin\left(\frac{n\pi}{4}\right) \cdot \frac{x^n}{n!}$

19. From the following formula for the series S_n, obtain the formula a_n for the sequence:

   (i) $S_n = n^3 - 2n$ $\qquad$ (ii) $S_n = \frac{1-2^n}{3}$

20. Find an Arithmetic Progression (A.P.) the sum of whose first n terms is $2n^2 + n$.

Type 5: Arithmetic Series

21. If the sixth and the tenth term of an A.P. are 23 and 39 respectively, find a_{16} and S_{19}.

22. The eighth term of an A.P. is 5 and the sum of the first 14 terms is 49. Find the first term.

23. Obtain an A.P. whose fourth term is 4 and the sum of the first eight terms is $\frac{2}{5}$ times the sum of the first four terms.

24. The sequence obtained by taking successive differences of $4,\ 6,\ 11,\ 19,\ 30, \cdots$ (for example, $6 - 4 = 2$, $11 - 6 = 5$) is an A.P. Find the sequence and the sum of the first n terms.

25. Find the sum of all the integers between 100 and 600 that are multiples of 11.

Type 6: Geometric Series

26. If $a = 25$, $r = \frac{1}{5}$, and $a_n = \frac{1}{625}$, find n and S_4.

27. If $r = \frac{1}{3}$ and $S_4 = 150$, find the first term a.

28. If $a = 16$ and $a_5 = 81$, find $r > 0$ and S_3.

29. If for a geometric sequence, $a_2 = 6$ and $a_5 = 48$, find S_5.

30. Find the sum of the following infinite geometric series:

    $(i)\quad \frac{1}{4} + \frac{1}{20} + \frac{1}{100} + \cdots \qquad (ii)\quad \frac{1}{3} + \frac{1}{6} + \frac{1}{12} + \cdots$

Type 7: Power Series

31. Find the sum of the integers from 1 to 1000.

32. Find the sum: $1 + 3 + 5 + 7 + 9 + 11 + \cdots$ (up to n terms).

33. Find the sum: $1 + (1 + 3) + (1 + 3 + 5) + \cdots$ (up to n terms).

34. Find the sum: $1 + (3 + 5) + (7 + 9 + 11) + \cdots$ (up to n terms).

35. Find the sum: $1 \cdot 3 \cdot 7 + 2 \cdot 5 \cdot 11 + 3 \cdot 7 \cdot 15 + \cdots$ (up to n terms).

Type 8: Method of Difference

36. Express $\dfrac{2}{4r^2 - 1}$ in partial fractions, then show that: $\displaystyle\sum_{r=1}^{n} \frac{2}{4r^2 - 1} = \frac{2n}{2n + 1}$.

37. Show that $4r^3 = r^2(r + 1)^2 - (r - 1)^2 r^2$. Hence, show that $\displaystyle\sum_{r=1}^{n} r^3 = \frac{n^2(n + 1)^2}{4}$.

Answers

1 (i) 5, 8, 11, 14, 17 (ii) $\frac{3}{2}, \frac{2}{3}, \frac{5}{12}, \frac{3}{10}, \frac{7}{30}$ (iii) 0, 2, 0, 2, 0 (iv) $-2, 3, -4, 5, -6$

2 (i) $4n-1$ (ii) $8-3n$ (iii) $\left(\frac{1}{2}\right)^{n-1}$ (iv) $(-2)^{n+1}$

3 111

4 20

5 -70

6 47, 107, 159

7 92

8 21

9 $d = \frac{1}{x+1}$, $a_5 = \frac{5x^2+x+4}{x^3+x^2+x+1}$, $a_n = \frac{x}{x^2+1} + (n-1)\frac{1}{x+1}$, $a_{100} = \frac{100x^2+x+99}{x^3+x^2+x+1}$.

10 54

11 ± 6

12 $a = \frac{1}{2}$, $r = 2$

13 11

14 $-\frac{1}{2}$

15 $a_n = \frac{1}{8} \cdot 2^{n-1}$

16 4, 12, 36 and 36, 12, 4

17 $a_n = (\pm 1 \pm i)^{(n-1)}$

18 (i) $\frac{x^2}{3}, \frac{x^4}{15}, \frac{x^6}{35}$ (ii) $-\frac{1}{x}, 2x, -\frac{2x^3}{3}$ (iii) $x, x^2, \frac{x^3}{3}$

19 (i) $a_n = 3n^2 - 3n - 1$ (ii) $-\frac{2^{n-1}}{3}$

20 $a_n = 4n - 1$

21 $a_{16} = 63$, $S_{19} = 741$

22 $a = -16$

23 $a_n = -4n + 20$

24 $a_n = 3n - 1$, $S_n = \frac{n(3n+1)}{2}$

25 15840

26 $n = 7$, $S_4 = \frac{156}{5}$

27 $\frac{405}{4}$

28 $r = \frac{3}{2}$, $S_3 = 76$

29 93

30 (i) $\frac{5}{16}$ (ii) $\frac{2}{3}$

31 500500

32 n^2

33 $\dfrac{n(n+1)(2n+1)}{6}$

34 $\dfrac{n^2(n+1)^2}{4}$

35 $\dfrac{1}{6}\,n\,(n+1)(12n^2+32n+19)$