Seminar 5

In this seminar you will study:

- Addition and factor formulae
- Multi-angle and half-angle formulae
- Inverse trigonometric functions
- Expressing $f(x) = a\cos x + b\sin x$ in the forms of $r\cos(x-\theta)$ or similar forms

Mid-Semester Examination

Wednesday 13th — Nov. from 15:00 to 16:00.

Confirm your Exam room from the email sent by CPSO

Questions in the mid-semester examinations will cover all the topics

treated in **Lecture** and **Seminar 1 to 5**

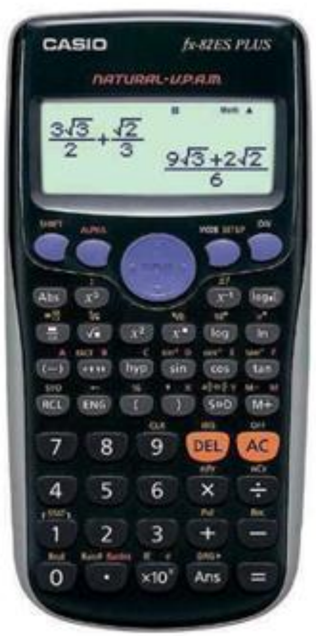
Format: 20 short answer questions

Mid-Semester Examination

Only permissible calculators are allowed in the examination

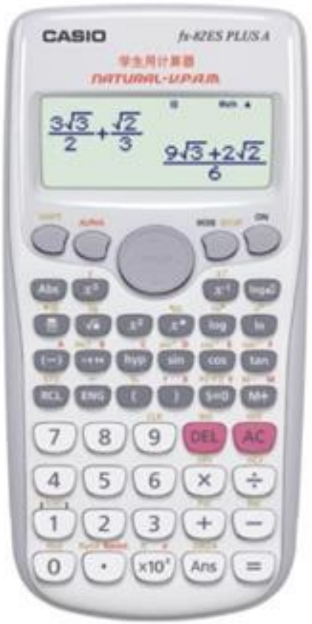
Permissible calculator are the $fx - 82$ series

$fx - 82ES\ PLUS$



$fx - 82ES\ PLUS\ 2nd\ edition$

$fx - 82ES\ PLUS\ A$



Addition formulae

$$
\begin{aligned}
\sin(A+B) &= \sin A\cos B + \cos A\sin B\\
\sin(A-B) &= \sin A\cos B - \cos A\sin B\\
\cos(A+B) &= \cos A\cos B - \sin A\sin B\\
\cos(A-B) &= \cos A\cos B + \sin A\sin B\\
\tan(A+B) &= \frac{\tan A + \tan B}{1-\tan A\tan B}\\
\tan(A-B) &= \frac{\tan A - \tan B}{1+\tan A\tan B}
\end{aligned}
$$

Addition formulae

Example: Prove that $\tan 29^\circ = \dfrac{\cos 16^\circ - \sin 16^\circ}{\cos 16^\circ + \sin 16^\circ}$.

Solution:

$$\begin{aligned}
\text{LHS} &= \tan 29^\circ \\
&= \tan(45^\circ - 16^\circ) \\
&= \frac{\tan 45^\circ - \tan 16^\circ}{1 + \tan 45^\circ \cdot \tan 16^\circ} \\
&= \frac{1 - \dfrac{\sin 16^\circ}{\cos 16^\circ}}{1 + \dfrac{\sin 16^\circ}{\cos 16^\circ}} \\
&= \frac{\dfrac{\cos 16^\circ - \sin 16^\circ}{\cos 16^\circ}}{\dfrac{\cos 16^\circ + \sin 16^\circ}{\cos 16^\circ}} = \frac{\cos 16^\circ - \sin 16^\circ}{\cos 16^\circ + \sin 16^\circ} = \text{RHS}
\end{aligned}$$

$$\left[\text{using } \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}\right]$$

Factor formulae

$$\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$$

$$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

$$\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$$

$$\cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

Allied angle formulae:
(Not in Fomula Sheet)

$$\sin(90^\circ - \theta) = \cos\theta$$

$$\cos(90^\circ - \theta) = \sin\theta$$

Factor formulae

Example: Prove that $\sin 20^\circ + \cos 50^\circ = \sin 80^\circ$

Solution:

$$\begin{aligned}
\text{LHS} &= \sin 20^\circ + \cos 50^\circ \\
&= \cos(90^\circ - 20^\circ) + \cos 50^\circ \\
&= \cos 70^\circ + \cos 50^\circ \\
&= 2\cos\left(\frac{70^\circ + 50^\circ}{2}\right)\cos\left(\frac{70^\circ - 50^\circ}{2}\right) \\
&= 2\cos 60^\circ \cos 10^\circ \\
&= 2 \cdot \frac{1}{2}\sin(90^\circ - 10^\circ) \\
&= \sin 80^\circ = \text{RHS}
\end{aligned}$$

$$\left[\text{using } \cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)\right]$$

Multi-angle and half-angle formulae

Multi(double)-angle formulae

$$1 + \cos 2\theta = 2\cos^2\theta$$

$$1 - \cos 2\theta = 2\sin^2\theta$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

Half-angle formulae

$$1 + \cos\theta = 2\cos^2\left(\frac{\theta}{2}\right)$$

$$1 - \cos\theta = 2\sin^2\left(\frac{\theta}{2}\right)$$

$$\sin\theta = 2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)$$

Use $\sqrt{x^2} = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

Multi-angle and half-angle formulae

Example: Prove that $\sqrt{\dfrac{1-\cos 2\theta}{1+\cos 2\theta}} = -\tan\theta, \quad \theta \in \left(\dfrac{\pi}{2}, \pi\right)$

Solution:

$$\text{LHS} = \sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}$$

$$= \sqrt{\frac{2\sin^2\theta}{2\cos^2\theta}} \qquad \text{using} \quad \boxed{\begin{array}{c} 1-\cos 2\theta = 2\sin^2\theta \\ 1+\cos 2\theta = 2\cos^2\theta \end{array}}$$

$$= \sqrt{\tan^2\theta}$$

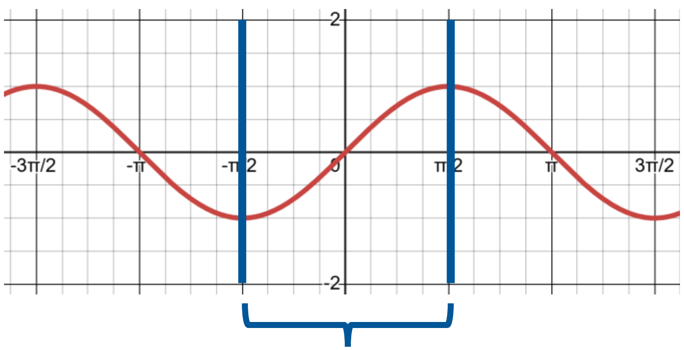
$$= |\tan\theta|$$

$$= -\tan\theta \qquad \left[\text{since } \tan\theta < 0 \text{ for } \theta \in \left(\frac{\pi}{2}, \pi\right)\right]$$

$$= \text{RHS}$$

Inverse Trigonometric functions

Trigonometric function are periodic and many to one functions



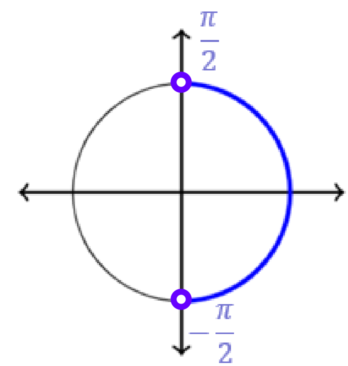
Restricting the domain makes it a one to one function

- The restricted domain of sin function is $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$.

- The restricted domain of tan function is $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$.

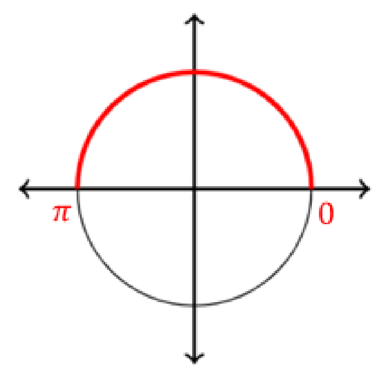
- The restricted domain of cos function is $[0, \pi]$.

$[-\frac{\pi}{2}, \frac{\pi}{2}]$, used for $\sin^{-1}$



$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, used for $\tan^{-1}$

$[0, \pi]$, used for $\cos^{-1}$



Inverse Trigonometric functions

Example 1: Find $\sin^{-1}\left(\sin\frac{3\pi}{4}\right)$

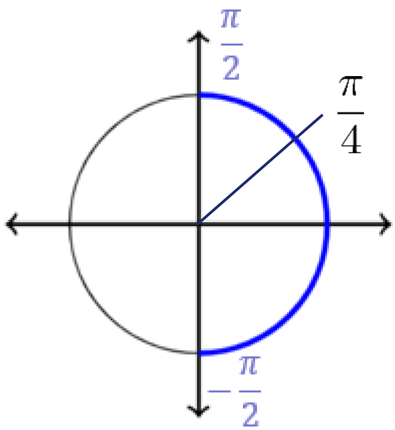
Solution:

$$\sin^{-1}\left(\sin\frac{3\pi}{4}\right) \neq \frac{3\pi}{4} \quad \text{since} \quad \boxed{\frac{3\pi}{4} \notin \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]}$$

But
$$\sin^{-1}\left(\sin\frac{3\pi}{4}\right) = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$$

$$= \sin^{-1}\left(\sin\frac{\pi}{4}\right) \quad \boxed{\because \quad f^{-1}(f(x)) = x}$$

$$= \frac{\pi}{4} \quad \boxed{\because \quad \frac{\pi}{4} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]}$$



Inverse Trigonometric functions

Example 2: Find $\tan\left(\tan^{-1}2+\tan^{-1}3\right)$.

Solution:

Let $\tan^{-1}2=\alpha$ and $\tan^{-1}3=\beta$

$$\tan\left(\tan^{-1}2+\tan^{-1}3\right)=\tan\left(\alpha+\beta\right)$$

$$=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$$\boxed{\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}}$$

As $\tan\alpha=\tan(\tan^{-1}2)=2,\ \tan\beta=\tan(\tan^{-1}3)=3$ $\boxed{\because\ f\left(f^{-1}(x)\right)=x}$

$$\therefore\qquad=\frac{2+3}{1-2\cdot3}=-1$$

Expressing $f(x) = a\cos x + b\sin x$ in the form $r\cos(x-\theta)$ or similar forms

Example: Express $f(x) = \sqrt{3}\sin x + \cos x$ in the form $r\cos(x-\theta)$, where $r > 0$, $\theta \in \left(0, \frac{\pi}{2}\right)$.

Also, find the period and range of $f(x)$, and sketch the curve of $f(x)$.

Solution:

$$f(x) = \boxed{\sqrt{3}\sin x} + \boxed{\cos x} \equiv r\cos(x-\theta) = \boxed{r\cos x\cos\theta} + \boxed{r\sin x\sin\theta}$$

$$\Rightarrow \begin{cases} r\cos\theta = 1 \\ r\sin\theta = \sqrt{3} \end{cases} \Rightarrow (r\cos\theta)^2 + (r\sin\theta)^2 = r^2(\cos^2\theta + \sin^2\theta) = 1^2 + (\sqrt{3})^2$$

$$\therefore \quad r = 2$$

$$\Rightarrow \begin{cases} \cos\theta = \dfrac{1}{2} \\ \sin\theta = \dfrac{\sqrt{3}}{2} \end{cases} \Rightarrow \tan\theta = \sqrt{3} \quad \therefore \quad \theta = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$$

$$\because \theta \in \left(0, \frac{\pi}{2}\right)$$

Note: Set calculator to RADIAN mode: **Shift Mode 4**

Thus, $f(x) = \sqrt{3}\sin x + \cos x = 2\cos\left(x - \frac{\pi}{3}\right)$

Expressing $f(x) = a\cos x + b\sin x$ in the form $r\cos(x-\theta)$ or similar forms

Example: Express $f(x) = \sqrt{3}\sin x + \cos x$ in the form $r\cos(x-\theta)$, where $r > 0,\ \theta \in \left(0, \dfrac{\pi}{2}\right)$.

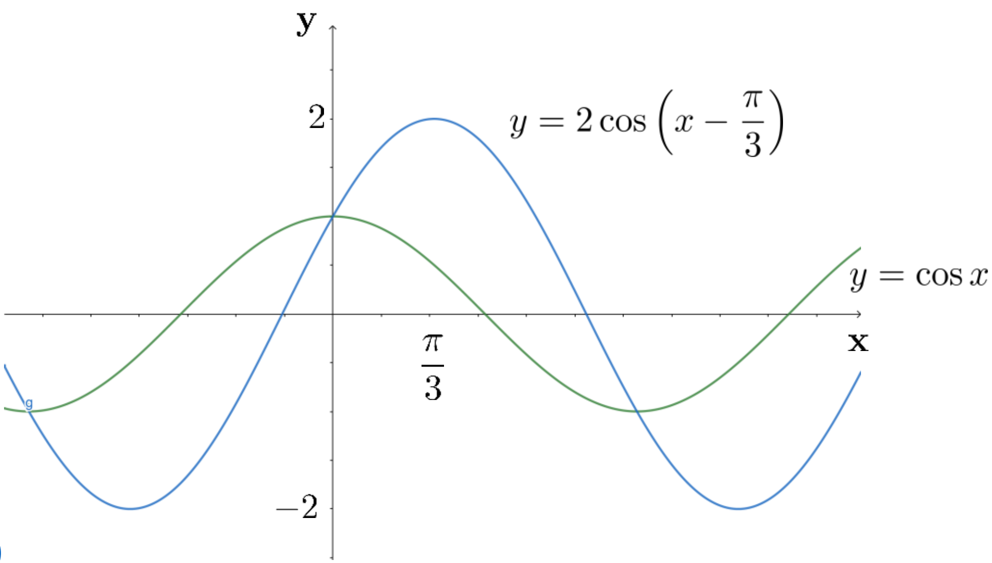
Also, find the period and range of $f(x)$, and sketch the curve of $f(x)$.

Solution:

$$f(x) = \sqrt{3}\sin x + \cos x = 2\cos\left(x - \frac{\pi}{3}\right)$$

Period of f is $\dfrac{2\pi}{|\,1\,|} = 2\pi$

Range of f is $[-2, 2]$



Click the Moodle Link for:
[Video on calculator use (Trigonometry)]()

Independent Learning Week (ILW)

Week 9 w/c 11 - Nov. 2024

CELE has named the week commencing on Monday 11th November as your Independent Learning Week.

You will have **no lectures and seminars** throughout the Independent Learning Week, but you will be given some learning activities to engage in.

The goal is for you to use this week to reflect on what learning independently outside the classroom means to you, sharpen your study skills, and reinforce your preparation moving toward the end of the semester. To help you:

1) Instructions will be posted on Moodle on Monday 11th November.

2) Office hours will remain at the usual times in case you need supports from tutors.

Office Hour

Monday	1 pm to 2 pm	YFB 412
Monday	3 pm to 4 pm	PB 112
Tuesday	9 am to 10 am	YFB 412
Tuesday	11 am to 12 noon	Trent 437
Tuesday	1 pm to 2 pm	YFB 412
Tuesday	3 pm to 4 pm	PB 205
Wednesday	10 am to 11 am	PMB 449
Wednesday	11 am to 12 noon	YFB 412
Wednesday	12 noon to 1 pm	YFB 412
Wednesday	1 pm to 2 pm	YFB 104
Friday	4 pm to 5 pm	YFB 219

THANKS FOR YOUR ATTENTION