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Foundation Algebra for Physical Sciences & Engineering CELEN036

Practice Problems SET-9 Sample Solution

Type 1: Algebra of matrices

9. Given matrices $A = \begin{pmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$,

show that $AB = I$, where I is the identity matrix of the same order as A and B.

Solution:

$$AB = \begin{pmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{pmatrix} \begin{pmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} =$$

$$\begin{pmatrix} 1\times 7+3\times(-1)+3\times(-1) & 1\times(-3)+3\times 1+3\times 0 & 1\times(-3)+3\times 0+3\times 1 \\ 1\times 7+4\times(-1)+3\times(-1) & 1\times(-3)+4\times 1+3\times 0 & 1\times(-3)+4\times 0+3\times 1 \\ 1\times 7+3\times(-1)+4\times(-1) & 1\times(-3)+3\times 1+4\times 0 & 1\times(-3)+3\times 0+4\times 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I$$

Type 2: Inverse matrices

13. Show that the inverse matrix of $A = \begin{pmatrix} -1 & 1 \\ -2 & 0 \end{pmatrix}$ is $B = \begin{pmatrix} 0 & -0.5 \\ 1 & -0.5 \end{pmatrix}$.

Solution:

Determinant of A is $\det(A) = (-1)\times 0 - 1\times(-2) = 2$

Therefore inverse matrix of A is $A^{-1} = \frac{1}{2}\begin{pmatrix} 0 & -1 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 0 & -0.5 \\ 1 & -0.5 \end{pmatrix} = B.$

Type 3: Solving 2×2 systems of equations using matrix method

14. Solve the following systems of linear equations using matrix method: (i) $\begin{cases} x + 2y &= 13 \\ 2x - 5y &= 8 \end{cases}$

Solution:

Convert the linear system into matrix form $AX = B$

$A = \begin{pmatrix} 1 & 2 \\ 2 & -5 \end{pmatrix}$, $X = \begin{pmatrix} x \\ y \end{pmatrix}$ and $B = \begin{pmatrix} 13 \\ 8 \end{pmatrix}$

Determinant of A is det$(A) = 1 \times (-5) - 2 \times 2 = -9 \neq 0$

Therefore the inverse matrix $A^{-1} = \dfrac{1}{-9} \begin{pmatrix} -5 & -2 \\ -2 & 1 \end{pmatrix}$

$$X = A^{-1}B = \frac{1}{-9} \begin{pmatrix} -5 & -2 \\ -2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 13 \\ 8 \end{pmatrix} = -\frac{1}{9} \begin{pmatrix} (-5) \times 13 + (-2) \times 8 \\ (-2) \times 13 + 1 \times 8 \end{pmatrix} = \begin{pmatrix} 9 \\ 2 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

Therefore $x = 9,\ y = 2$.