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Foundation Algebra for Physical Sciences & Engineering CELEN036

Practice Problems SET-4 Sample Solution

Type 1: Trigonometric Identities

1. Prove the following trigonometric identities: (i)

Proof:

$$\begin{aligned} LHS &= \frac{\cos x}{1-\sin x} + \frac{1-\sin x}{\cos x} \\ &= \frac{\cos^2 x + (1-\sin x)^2}{\cos^2 x(1-\sin x)^2} \\ &= \frac{cos^2 x + \sin^2 x + 1 - 2\sin x}{(1-\sin x)\cos x} \\ &= \frac{2-2\sin x}{(1-\sin x)\cos x} \\ &= \frac{2}{\cos x} \\ &= 2\sec x \\ &= RHS \end{aligned}$$

Type 2: Conversion formulae

10. Convert the following degrees to radians or vice-versa: $(i) \quad -160^\circ$

Solution:

Use the degree to radian formula:

$$\theta = \pi \times \left(\frac{-160^\circ}{180^\circ}\right)$$
$$= -\frac{8\pi}{9}$$

Type 3: Finding values of trigonometric function

11. Find the range and period of the following trigonometric functions: $(i) \quad 3\,\mathrm{cosec}(3-4x)$

Solution:

Range: for any $\theta \quad \mathrm{cosec}\theta \in \mathbb{R} - (-1,1)$

$\therefore \mathrm{cosec}\,(3-4x) \in \mathbb{R} - (-1,1)$

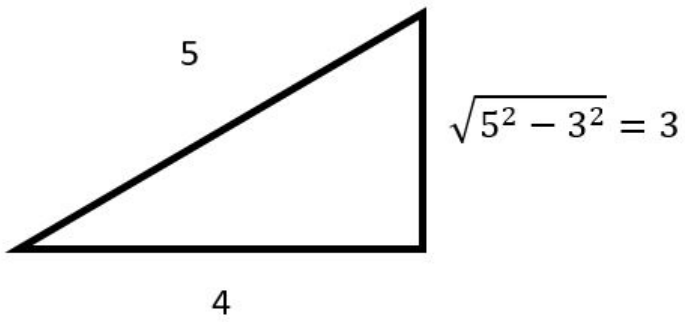
$\therefore 3\,\mathrm{cosec}\,(3-4x) \in \mathbb{R} - (-3,3)$

The principal period of $3\,\mathrm{cosec}\,(3-4x)$ is $\dfrac{2\pi}{|-4|} = \dfrac{\pi}{2}$

Type 4: Finding Values of Trigonometric Functions

12. Given $\cos\theta = -\dfrac{4}{5}$; $\dfrac{\pi}{2} < \theta < \pi$. Find the value of $\mathrm{cosec}\,\theta + \cot\theta$.

Solution:



$\because \dfrac{\pi}{2} < \theta < \pi$

$\theta \in$ Quadrant II

$\therefore \mathrm{cosec}\,\theta > 0,\ \cot\theta < 0$

$\because \cos\theta = -\dfrac{4}{5}$

From the triangle: Opposite side = 3, Adjacent side = 4 and Hypotenus = 5

$\therefore \mathrm{cosec}\,\theta + \cot\theta = (+\dfrac{5}{3}) + (-\dfrac{4}{3}) = \dfrac{1}{3}$

Type 5: Solving Trigonometric Equations

17. Solve the equation $\cos^2 x + \cos x = \sin^2 x$ for $0 \leq x \leq \pi$.

Solution:

$\cos^2 x + \cos x = \sin^2 x$

$\implies \cos^2 x + \cos x = 1 - \cos^2 x$

$\implies 2\cos^2 x + \cos x - 1 = 0$

Let $t = \cos x$: $\implies 2t^2 + t - 1 = 0$

$\implies (2t - 1)(t + 1) = 0$

$\implies t = \frac{1}{2}$ or $t = -1$

$\implies \cos x = \frac{1}{2}$ or $\cos x = -1$

$\because 0 \leq x \leq \pi$

$x = \frac{\pi}{3}$ or $x = \pi$