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Foundation Algebra for Physical Sciences & Engineering CELEN036

Practice Problems SET-8 Sample Solution

Type 1: Expansion using the Binomial theorem with $\binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

2. Expand the following expressions using the Binomial theorem: (v) $(4-3x)^4$

Solution:

$$\begin{aligned}(4-3x)^4 \quad &= 4^4 \cdot (-3x)^0 \cdot \binom{4}{4} + 4^3 \cdot (-3x)^1 \cdot \binom{4}{3} + 4^2 \cdot (-3x)^2 \cdot \binom{4}{2} \\ &+ 4^1 \cdot (-3x)^3 \cdot \binom{4}{1} + 4^0 \cdot (-3x)^4 \cdot \binom{4}{0} \\ &= 256 - 768x + 864x^2 - 432x^3 + 81x^4\end{aligned}$$

Type 2: To find the coefficient of certain term in the expansion ×

7. Find the coefficient of x^3 in the expansion of $\left(2x - \dfrac{1}{3x}\right)^9$.

Solution:

The formula for term k + 1 in binomial expansion $(a+b)^n$: $T_{k+1} = a^k \cdot b^{n-k} \cdot \binom{n}{k}$

$\therefore a = 2x,\ b = (-\dfrac{1}{3} \cdot x^{-1}),\ n = 9$

Substitute above in term k + 1 formula: $T_{k+1} = (2x)^k \cdot (-\dfrac{1}{3} \cdot x^{-1})^{9-k} \cdot \binom{9}{k} = 2^k \cdot (-\dfrac{1}{3})^{9-k} \cdot \binom{9}{k} \cdot x^{2k-9}$

As we are looking for term x^3 therefore $x^{2k-9} = x^3 \implies 2k - 9 = 3 \implies k = 6$

Therefore the coefficient of term x^3 is the seventh term T_7 with $k = 6$ is $2^6 \cdot (-\dfrac{1}{3})^{9-6} \cdot \binom{9}{6} = -\dfrac{1792}{9}$

Type 3: Application of the generalized Binomial theorems

20. The radius of a sphere is measured as r, with an error of $\delta r = 1.2\%$ of r. The volume of the sphere $V = \dfrac{4}{3}\pi r^3$ is then calculated using the measured r. Use the approximation $(1+x)^n \approx 1 + nx + \frac{n(n-1)}{2} \cdot x^2$ to find the resulting error δV in the calculated volume.

Solution:

$$V + \delta V = \frac{4}{3}\pi(r+\delta r)^3$$

As $\delta r = 0.012r$

$$V + \delta V = \frac{4}{3}\pi(r+0.012r)^3 = \frac{4}{3}\pi r^3(1+0.012)^3 = V \cdot (1+0.012)^3$$

Now use generalised binomial expansion to find the approximation of $(1+0.012)^3$

$$(1+0.012)^3 \approx 1 + 3 \times 0.012 + \frac{3 \times (3-1)}{2} \times 0.012^2$$

$$= 1.0364$$

$$V + \delta V = 1.0364V$$

$$\delta V = 0.0364V$$

Therefore the error δv is 3.64% of volume