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Foundation Calculus and Mathematical Techniques CELEN037

Practice Problems SET-8 Sample Solution

Type 1: Evaluating definite integrals

1. Evaluate the following definite integrals: $(i) \quad \displaystyle\int_2^3 \frac{1}{7-x^2}\, dx$

Solution:

$$\int_2^3 \frac{1}{7-x^2}\, dx = \left[\frac{1}{2\sqrt{7}} \ln\left|\frac{x+\sqrt{7}}{x-\sqrt{7}}\right|\right]_2^3$$

$$= \frac{1}{2\sqrt{7}} \ln\left|\frac{3+\sqrt{7}}{3-\sqrt{7}}\right| - \frac{1}{2\sqrt{7}} \ln\left|\frac{2+\sqrt{7}}{2-\sqrt{7}}\right|$$

$$= \frac{1}{2\sqrt{7}} \ln\left|\frac{-1-\sqrt{7}}{-1+\sqrt{7}}\right|$$

Type 2: Definite Integrals with Substitution

2. Evaluate the following definite integrals using the method of substitution:

$(i) \quad \displaystyle\int_0^{\frac{\sqrt{\pi}}{2}} x \cdot \sin(x^2)\, dx$

Solution:

$$t = x^2 \implies \frac{dt}{dx} = 2x,\ x \cdot dx = \frac{1}{2} dt$$

$$x = 0 \implies t = 0,\ x = \frac{\sqrt{\pi}}{2} \implies t = \frac{\pi}{4}$$

$$\int_0^{\frac{\sqrt{\pi}}{2}} x \cdot \sin(x^2)\, dx = \frac{1}{2} \int_0^{\frac{\pi}{4}} \sin(t)\, dt$$

$$= \frac{1}{2}\left[-\cos t\right]_0^{\frac{\pi}{4}} = \frac{1}{2}\cos 0 - \frac{1}{2}\cos\frac{\pi}{4} = \frac{1}{2} - \frac{\sqrt{2}}{4}$$

Type 3: Integration by Parts for Definite Integrals

3. Evaluate the following integrals using the method of integration by parts: $(i) \int_0^{\frac{1}{2}} \sin^{-1} x \ dx$

Solution:

$$u = \sin^{-1} x, \ \frac{dv}{dx} = 1 \implies \frac{du}{dx} = \frac{1}{\sqrt{1-x^2}} v = x$$

$$\int_0^{\frac{1}{2}} \sin^{-1} x \ dx = \left[x \cdot \sin^{-1} x\right]_0^{\frac{1}{2}} - \int_0^{\frac{1}{2}} \frac{x}{\sqrt{1-x^2}} \ dx$$

$$= \left[x \cdot \sin^{-1} x\right]_0^{\frac{1}{2}} - \left[-\sqrt{1-x^2}\right]_0^{\frac{1}{2}}$$

$$= \frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1$$

Type 4: Properties of Definite Integration

4. Evaluate the following integrals: $(ii) \quad \int\limits_2^4 \frac{\sqrt{x}}{\sqrt{6-x}+\sqrt{x}} \ dx$

Solution:

As $\int\limits_a^b f(x) \ dx = \int\limits_a^b f(a+b-x) \ dx$

$$\int\limits_2^4 \frac{\sqrt{x}}{\sqrt{6-x}+\sqrt{x}} \ dx = \int\limits_2^4 \frac{\sqrt{6-x}}{\sqrt{x}+\sqrt{6-x}} \ dx$$

Also $\int\limits_2^4 \frac{\sqrt{x}}{\sqrt{6-x}+\sqrt{x}} \ dx + \int\limits_2^4 \frac{\sqrt{6-x}}{\sqrt{x}+\sqrt{6-x}} \ dx = \int\limits_2^4 \frac{\sqrt{6-x}+\sqrt{x}}{\sqrt{6-x}+\sqrt{x}} \ dx = \int\limits_2^4 1 \ dx$

Therefore $2 \int\limits_2^4 \frac{\sqrt{x}}{\sqrt{6-x}+\sqrt{x}} \ dx = \int\limits_2^4 1 \ dx = [x]_2^4 = 2$

$$\int\limits_2^4 \frac{\sqrt{x}}{\sqrt{6-x}+\sqrt{x}} \ dx = 1$$