The University of Nottingham
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Science A Physics

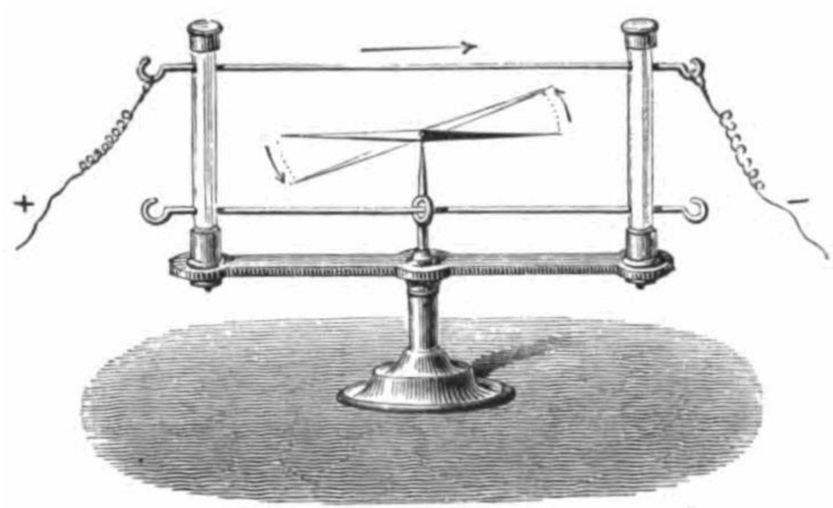
Lecture 18:

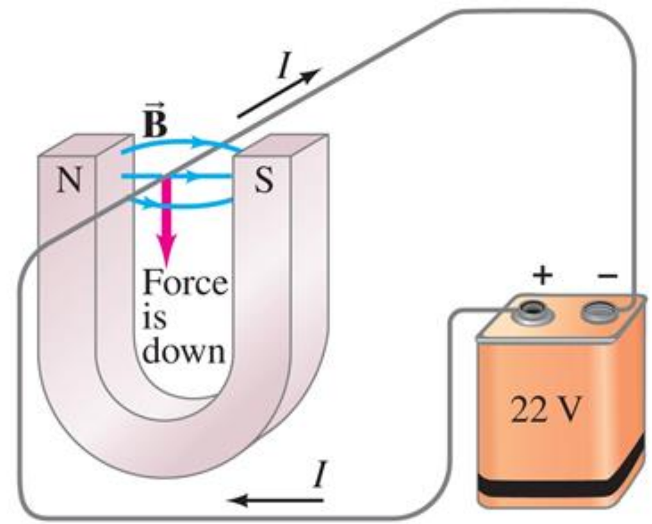
Magnetic Induction

Aims of today's lecture

1. Induced EMF
2. Faraday's Law of Induction, and Lenz's Law
3. EMF Induced in a Moving Conductor
4. Electric Generators
5. Back EMF and Counter Torque; Eddy Currents
6. Transformers and Transmission of Power

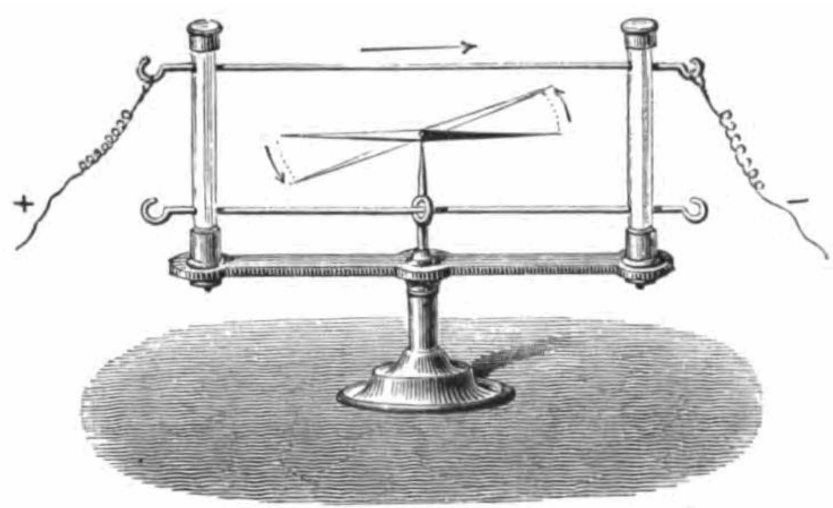
Electromagnetic Induction and Faraday's Law

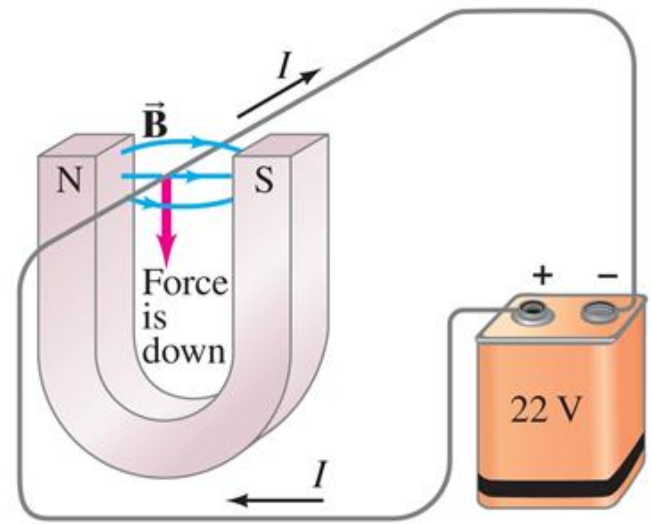




- As we have seen, it was discovered that there are two ways in which electricity and magnetism are related: (1) an electric current produces a magnetic field; and (2) a magnetic field exerts a force on an electric current or moving electric charge. These discoveries were made in the 1820s.

Electromagnetic Induction and Faraday's Law





- Scientists then began to wonder: if electric currents produce a magnetic field, is it possible that a magnetic field can produce an electric current?
- 10 years later, an American, Joseph Henry, and an Englishman, Michael Faraday, independently found that it was possible.

Electromagnetic Induction and Faraday's Law

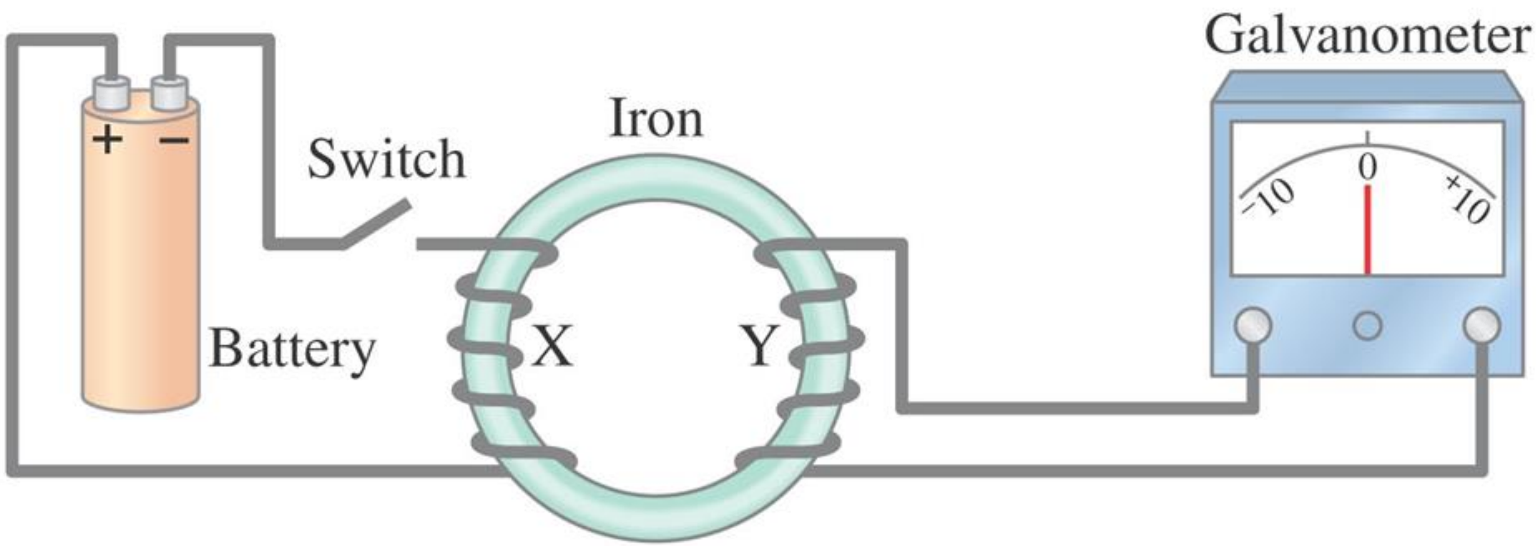
Henry (1797-1878)

Faraday (1791-1867)

- Interestingly, Henry actually made the discovery first, but Faraday published his results earlier and investigated the subject in more detail.
- The discovery of this phenomenon has produced some world-changing applications such as the electric generator.

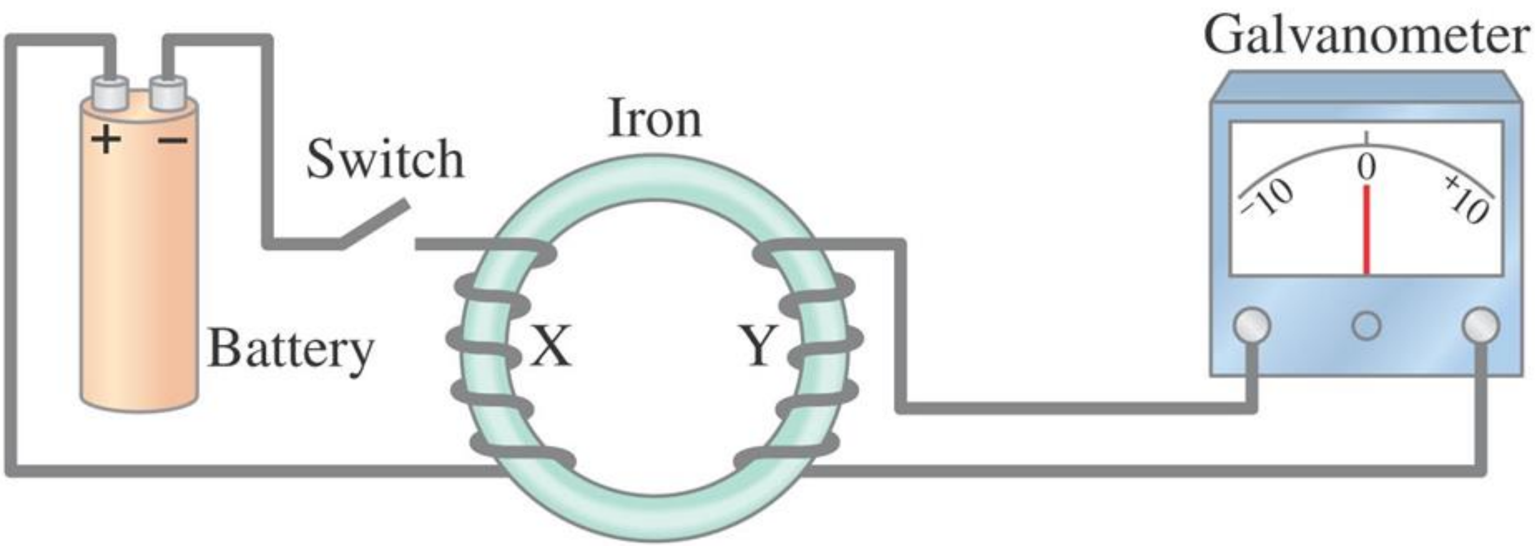
1. Induced EMF

Induced EMF



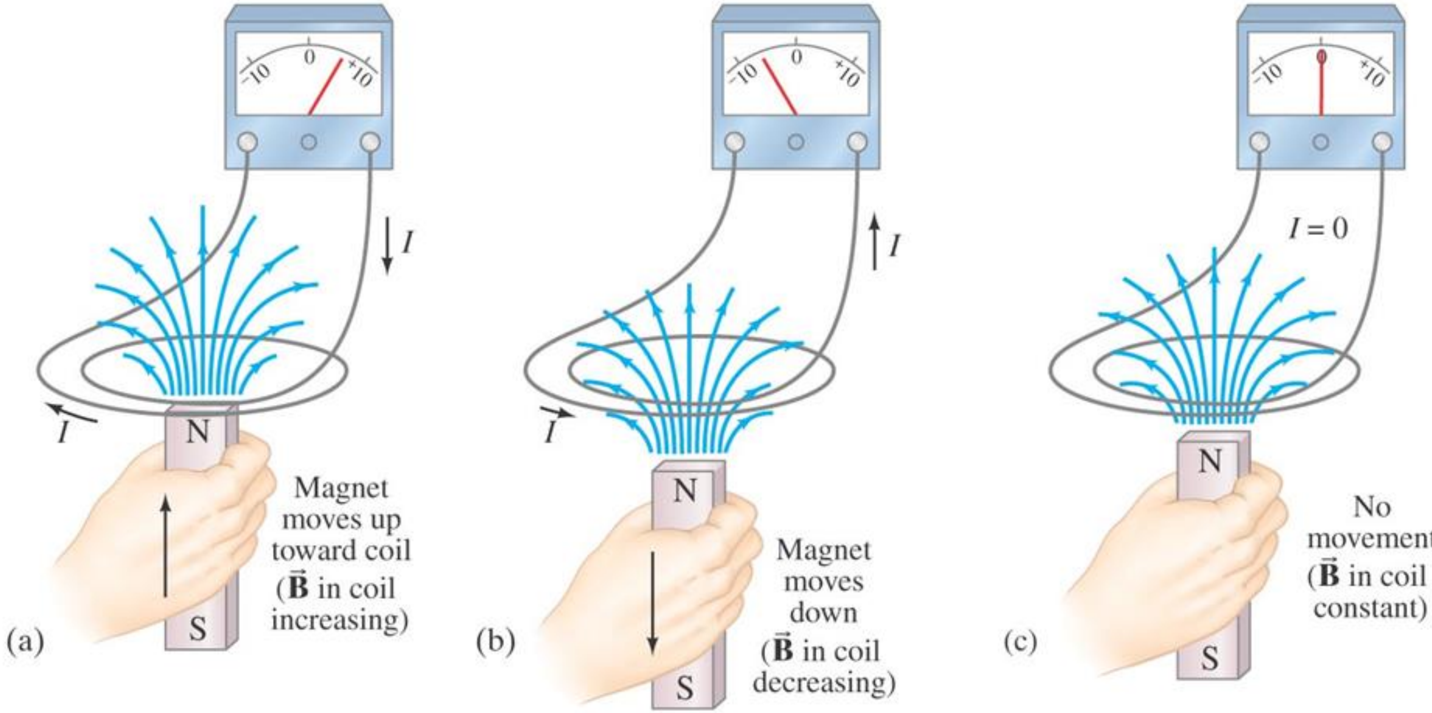
- In an attempt to produce an electric current from a magnetic field, Faraday used an apparatus like the one shown above.
- Faraday hoped that a strong steady current in X would produce a great enough magnetic field to produce a current in a second coil Y wrapped on the same iron ring.

Induced EMF



- Faraday concluded that although a constant magnetic field produces no current in a conductor, a changing magnetic field can produce an electric current. Such a current is called an **induced current**.
- When the magnetic field through coil Y changes, a current occurs in Y as if there were a source of emf in circuit Y. We therefore say that **a changing magnetic field induces an emf.**

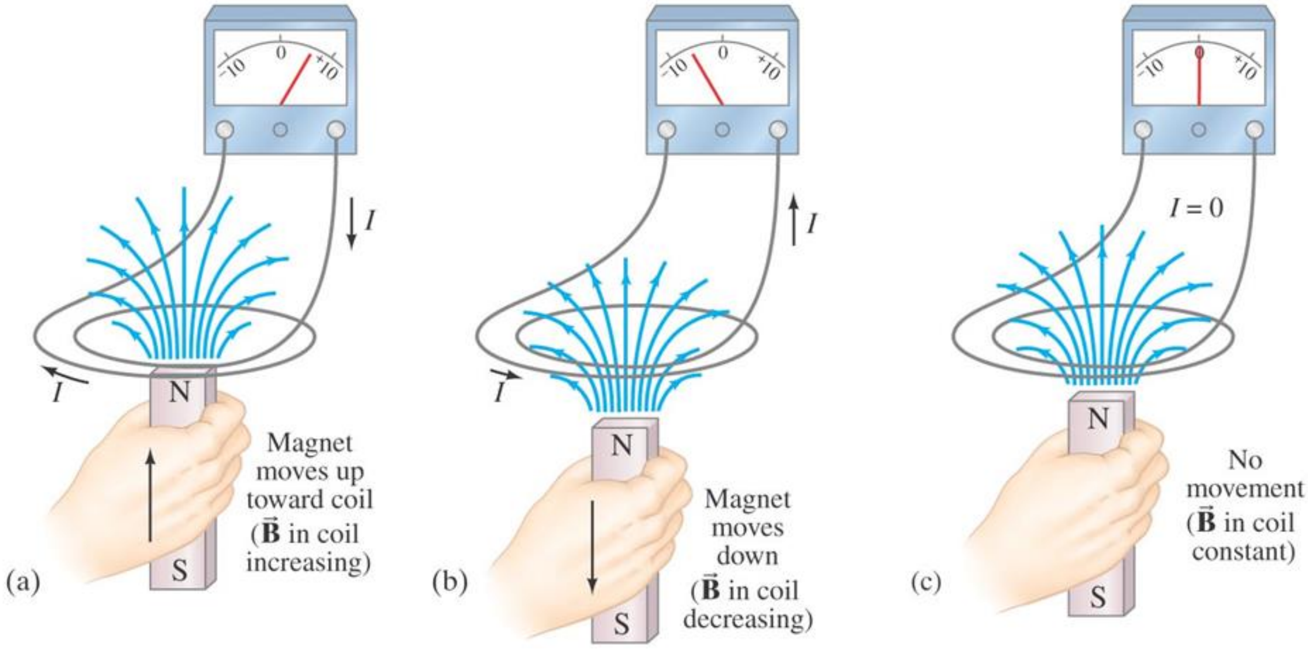
Induced EMF



- Faraday did further experiments on **electromagnetic induction**, as this phenomenon was called.
- Motion or change is required to induce an emf. It does not matter whether the magnet or the coil moves. It is their **relative motion** that counts.

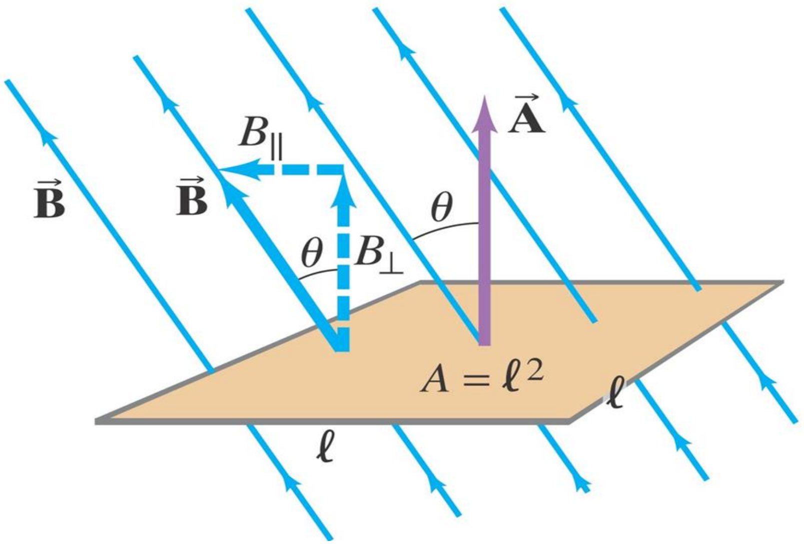
2. Faraday's Law of Induction, and Lenz's Law

Induced EMF



- Faraday investigated quantitatively what factors influence the magnitude of the emf induced.
- He found first of all that the more rapidly the magnetic field changes, the greater the induced emf.
- He also found that the induced emf depends on the area of the circuit loop.

Induced EMF

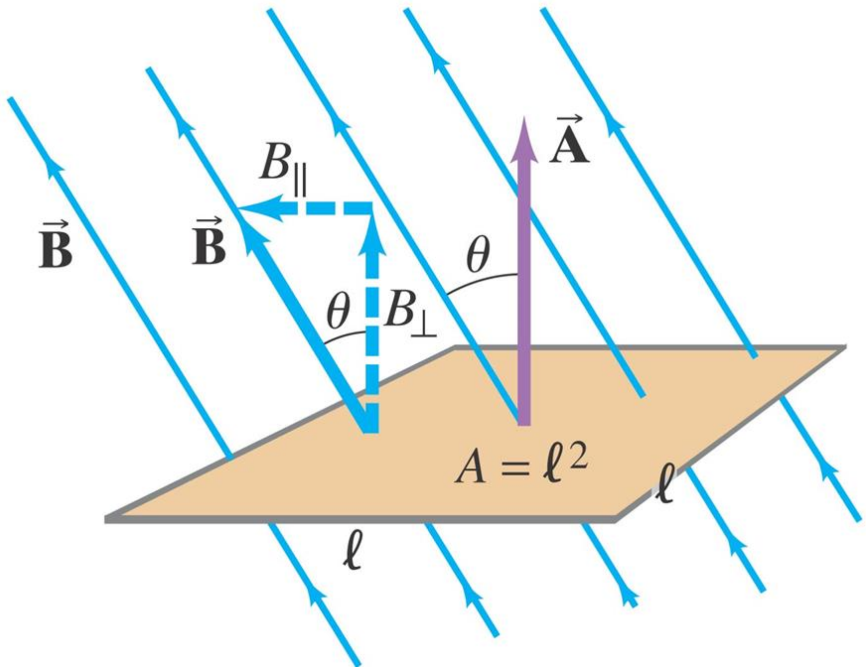


- Thus, we say that the emf is proportional to the rate of change of the magnetic flux, Φ_B, passing through the circuit or loop of area A.
- Magnetic flux for a uniform magnetic field is defined in the same way as for electric flux, namely as:

$$\Phi_B = B_{\perp}A = BAcos\theta = \vec{B} \cdot \vec{A}$$

- Here, $B_{\perp}$ is the component of the magnetic field $\vec{B}$ perpendicular to the face of the loop, and θ is the angle between $\vec{B}$ and the vector $\vec{A}$ (representing the area) whose direction is perpendicular to the face of the loop.

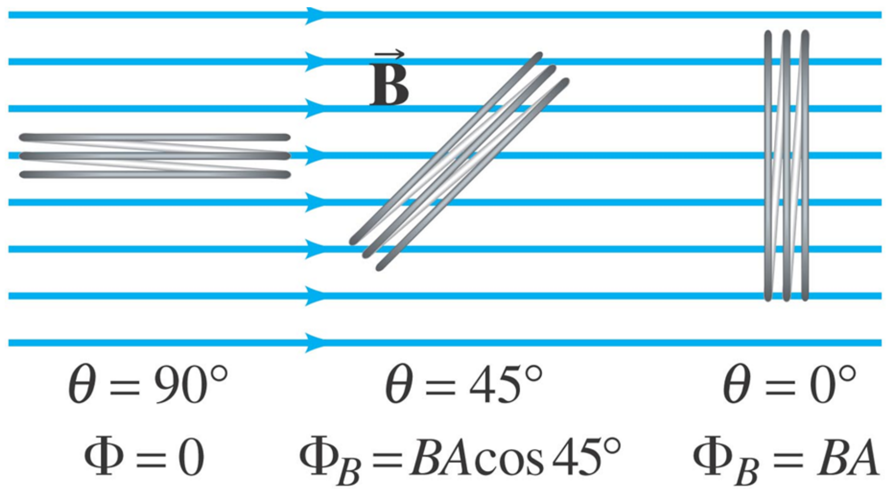
Induced EMF



- If the area is of some other shape, or $\vec{B}$ is not uniform, the magnetic flux can be written as

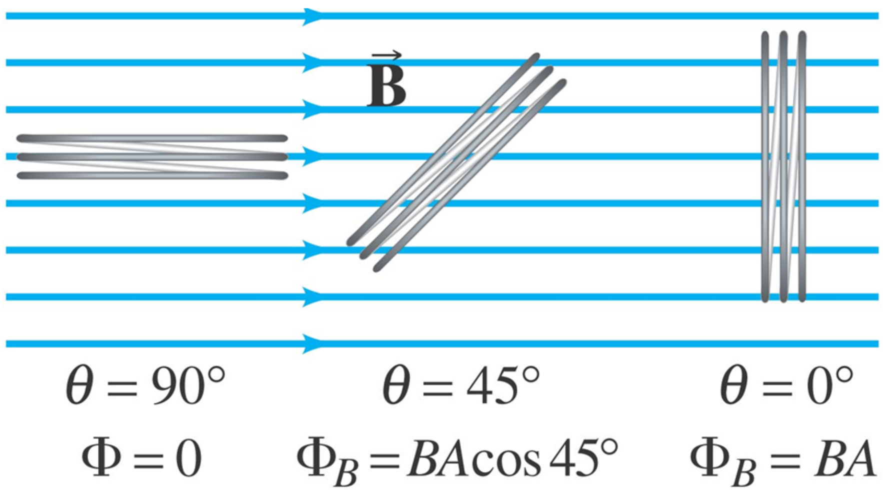
$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

Induced EMF



- The lines of $\vec{B}$ (like lines of $\vec{E}$) can be drawn such that the number of lines per unit area is proportional to the field strength.
- Then the flux Φ_B can be thought of as being proportional to the total number of lines passing through the area enclosed by the loop.
- The unit of magnetic flux is the tesla-metre^2; this is called a Weber: $1\ Wb = 1\ T \cdot m^2$.

Faraday's Law of Induction



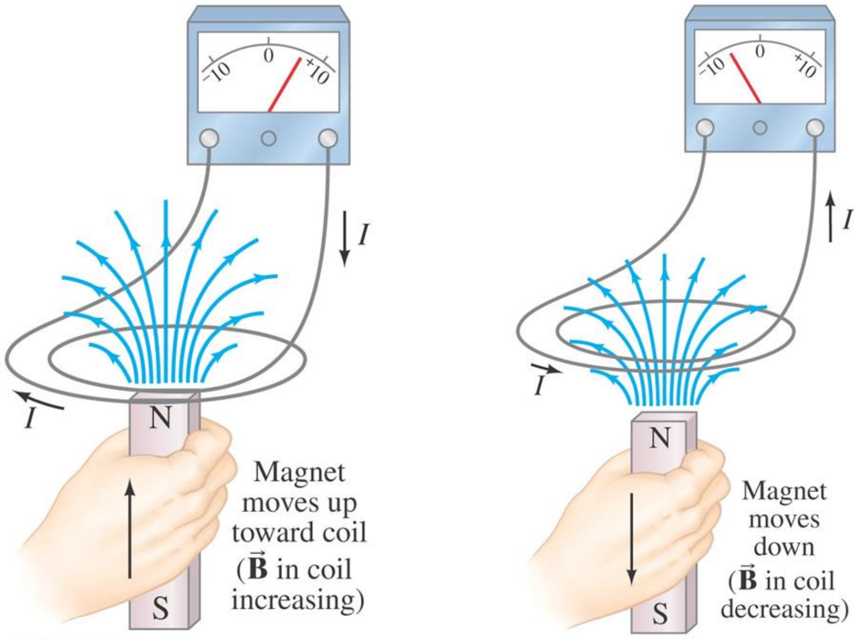
- The emf induced in a circuit is equal to the rate of change of magnetic flux through the circuit:

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

- If the circuit contains N loops that are closely wrapped so that the same flux passes through each, the $emf's$ induced in each loop add together, so

$$\varepsilon = -N\frac{d\Phi_B}{dt}$$

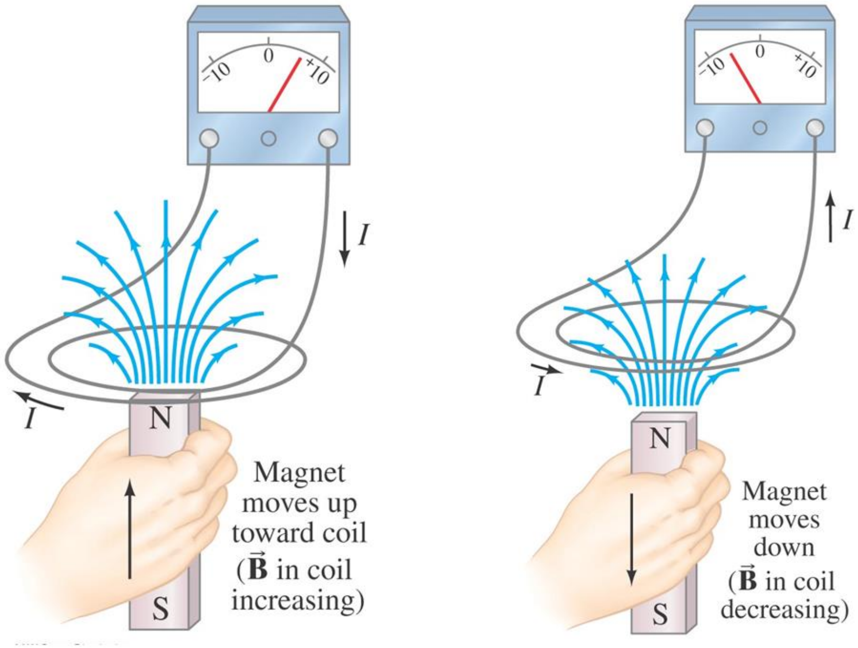
Faraday's Law of Induction



$$\varepsilon = -N\frac{d\Phi_B}{dt}$$

- The minus sign in the above equation reminds us in which direction the induced emf acts.
- Experiments show that a current produced by an induced emf moves in a direction so that the magnetic field created by that current opposes the original change in flux. This is known as **Lenz's law**.

Lenz's Law



Heinrich Lenz, 1804-1860

$$\varepsilon = -N\frac{d\Phi_B}{dt}$$

N.B.

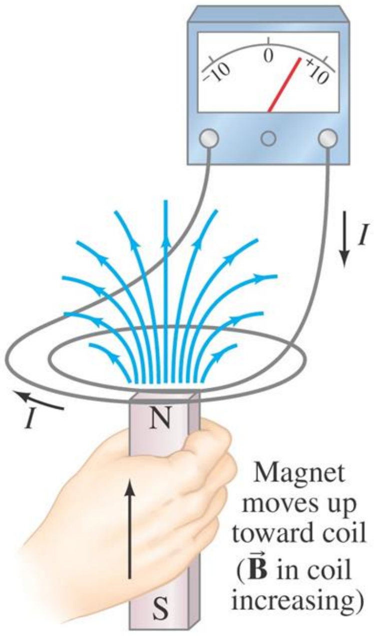
We are now discussing two distinct magnetic fields:

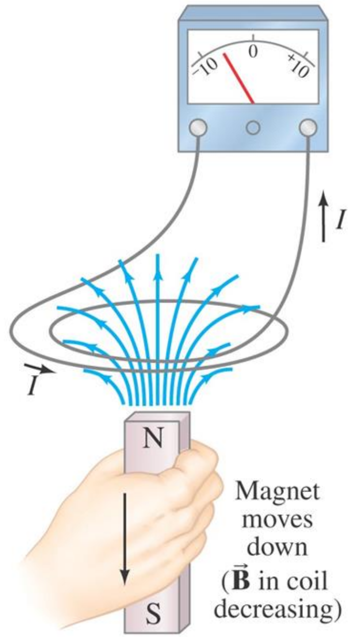
(1) the changing magnetic field or flux that induces the current, and

(2) the magnetic field produced by the induced current (all currents produce a field).

The second field opposes the change in the first.

Lenz's Law





Heinrich Lenz, 1804-1860

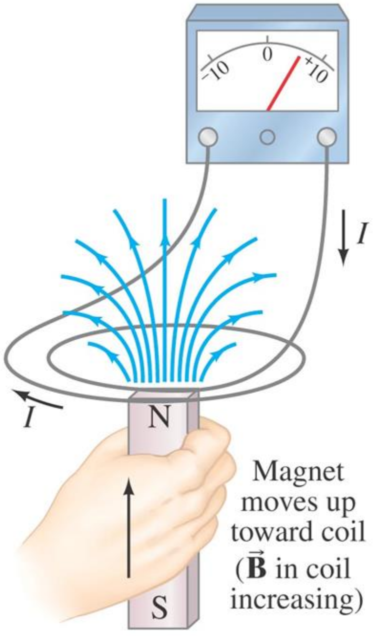
$$\varepsilon = -N\frac{d\Phi_B}{dt}$$

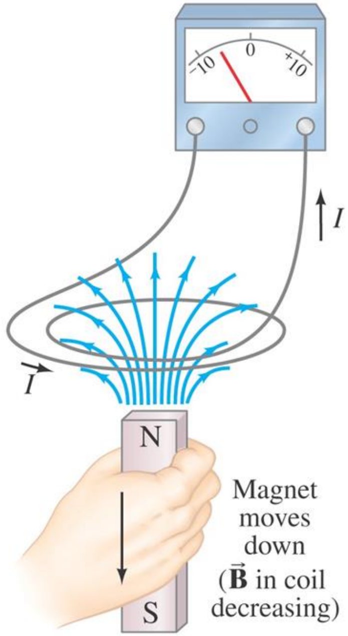
- Lenz's law:

  An induced emf is always in a direction that opposes the original change in flux that caused it.

- We can use Lenz's law and the right-hand rule to explain the above two observations in the above figure.

Lenz's Law



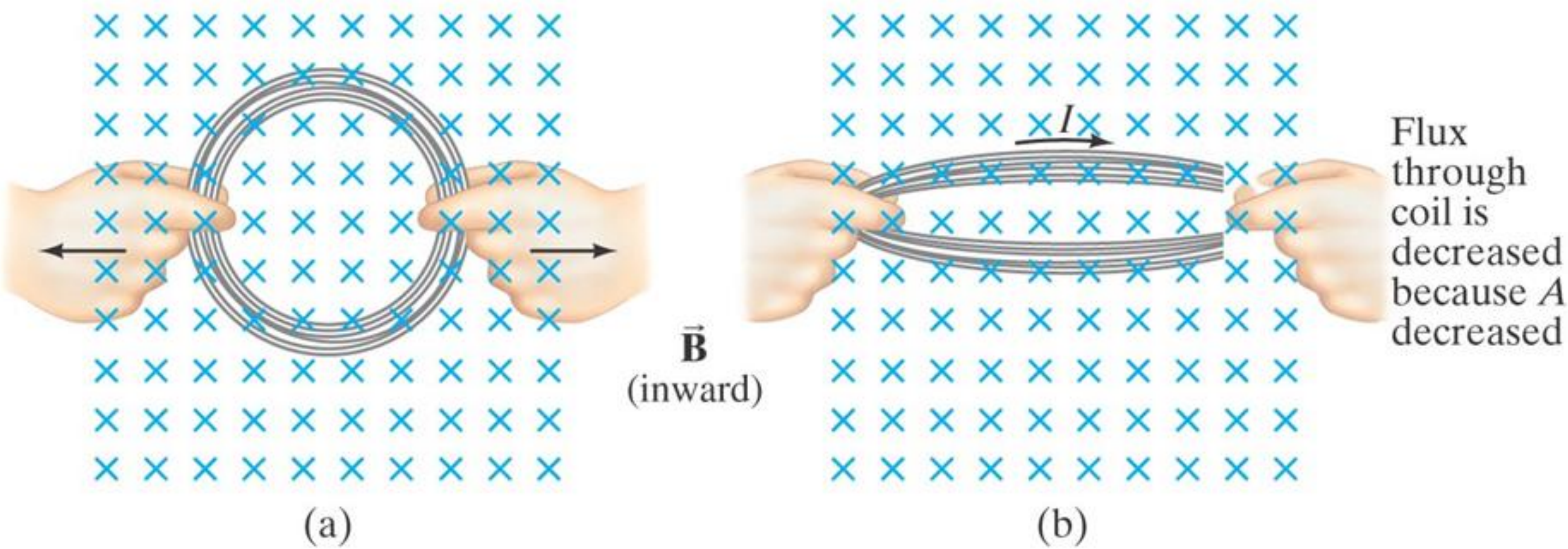


Heinrich Lenz, 1804-1860

$$\varepsilon = -N\frac{d\Phi_B}{dt}$$

- Since magnetic flux $\Phi_B = \int \vec{B} \cdot d\vec{A} = \int B cos\theta \, d\mathrm{A}$, we see that an emf can be induced in three ways:
- (1) by a changing magnetic field B;

Lenz's Law



Since magnetic flux $\Phi_B = \int \vec{B} \cdot d\vec{A} = \int B cos\theta \, d\mathrm{A}$, we see that an emf can be induced in three ways:

(1) by a changing magnetic field B;

(2) by changing the area A of the loop in the field;

(3) by changing the loop's orientation θ with respect to the field.

Lenz's Law: Problem Solving (P.881)

Lenz's law is used to determine the direction of the (conventional) electric current induced in a loop due to a change in magnetic flux inside the loop. To produce an induced current, you need

(a) A closed conducting loop, and

(b) An external magnetic flux through the loop that is changing in time.

1. Determine whether the magnetic flux ($\Phi_B = BAcos\theta$) inside the loop is decreasing, increasing, or unchanged.
2. The magnetic field due to the induced current:
   (a) points in the same direction as the external field if the flux is decreasing; (b) points in the opposite direction from the external field if the flux is decreasing; or (c) is zero if the flux is not changing.

Lenz's Law: Problem Solving (P.881)

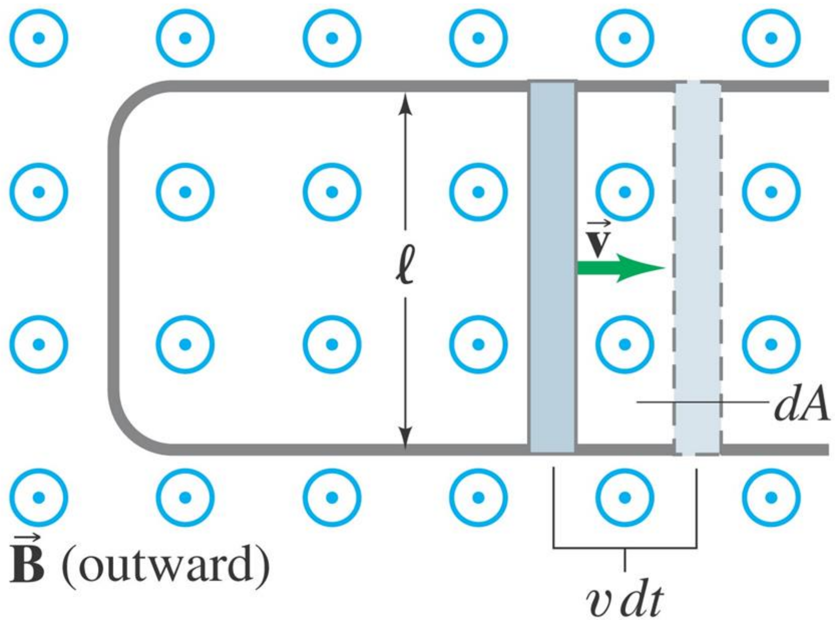
Lenz's law is used to determine the direction of the (conventional) electric current induced in a loop due to a change in magnetic flux inside the loop. To produce an induced current, you need

(a) A closed conducting loop, and
(b) An external magnetic flux through the loop that is changing in time.

3. Once you know the direction of the induced magnetic field, use the right-hand rule to find the direction of the induced current.
4. Always keep in mind that there are two magnetic fields: (1) an external field whose flux must be changing if it is to induce and electric current, and (2) a magnetic field produced by the induced current.

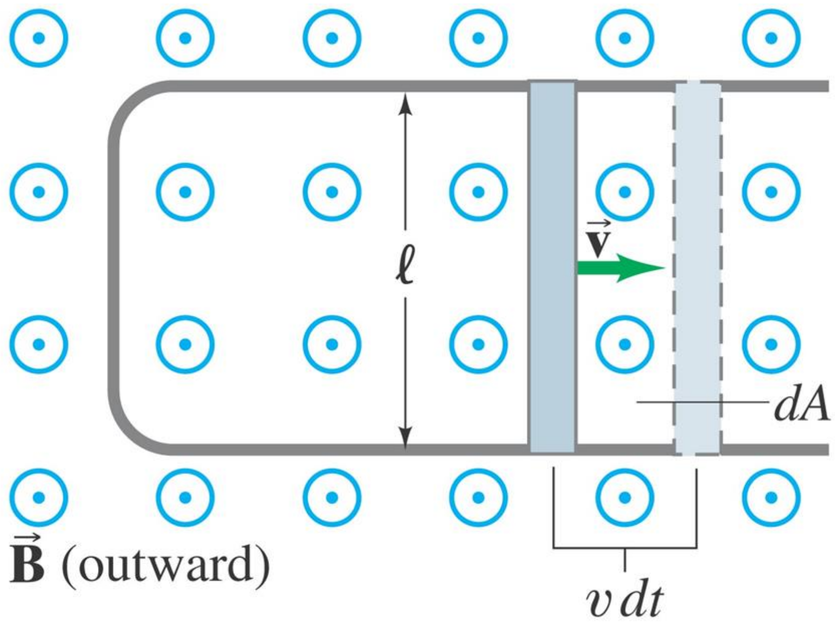
3. EMF Induced in a Moving Conductor

EMF Induced in a Moving Conductor



- Another way to induce an *emf* is shown above.
- We assume that a uniform magnetic field $\vec{B}$ is perpendicular to the area bounded by the U-shaped conductor and the movable rod resting on it.
- If the rod is made to move at a speed, v, it travels a distance $dx = vdt$ in a time dt.

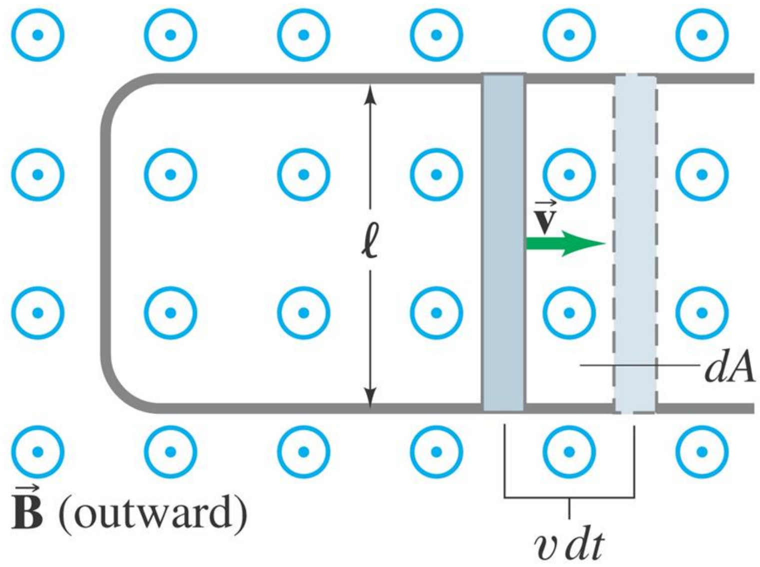
EMF Induced in a Moving Conductor



- Therefore, the area of the loop increases by an amount $dA = ldx = lvdt$ in a time dt.
- By Faraday's law, there is an induced *emf* whose magnitude is given by

$$\epsilon = \frac{d\Phi_B}{dt} = \frac{BdA}{dt} = \frac{Blvdt}{dt} = Blv$$

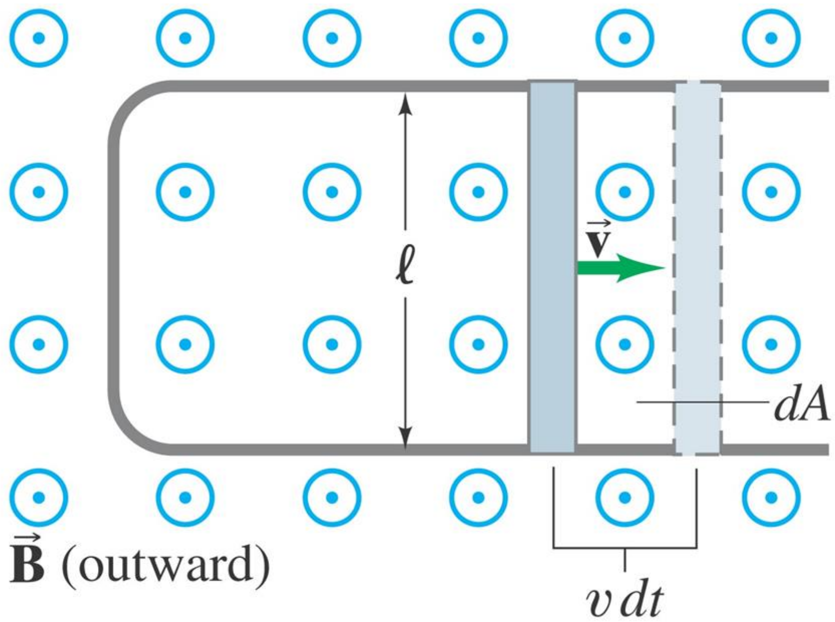
EMF Induced in a Moving Conductor



$$\epsilon = \frac{d\Phi_B}{dt} = \frac{BdA}{dt} = \frac{Blvdt}{dt} = Blv$$

- The above equation is valid as long as B, l, and v are mutually perpendicular (if they are not, we use only the components of each that are mutually perpendicular).
- An *emf* induced on a conductor moving in a magnetic field is sometimes called **motional emf.**

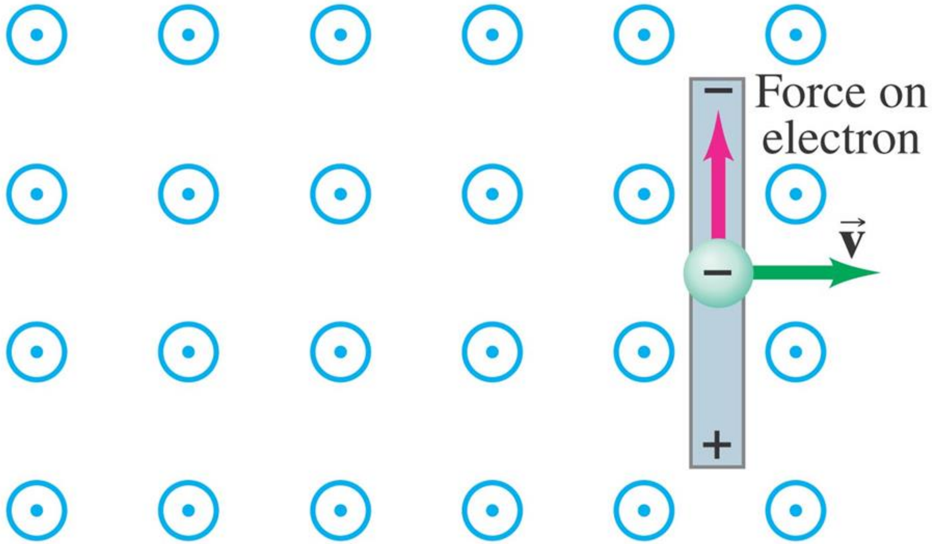
EMF Induced in a Moving Conductor



$$\epsilon = \frac{d\Phi_B}{dt} = \frac{BdA}{dt} = \frac{Blvdt}{dt} = Blv$$

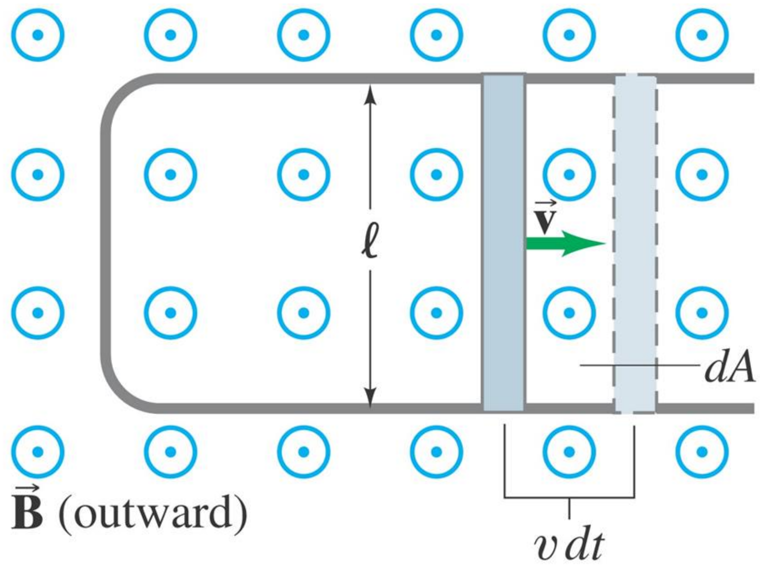
- We can also obtain the above equation without using Faraday's law.
- A charged particle moving perpendicular to a magnetic field B with speed v experiences a force $\vec{F} = q\vec{v} \times \vec{B}$.

EMF Induced in a Moving Conductor



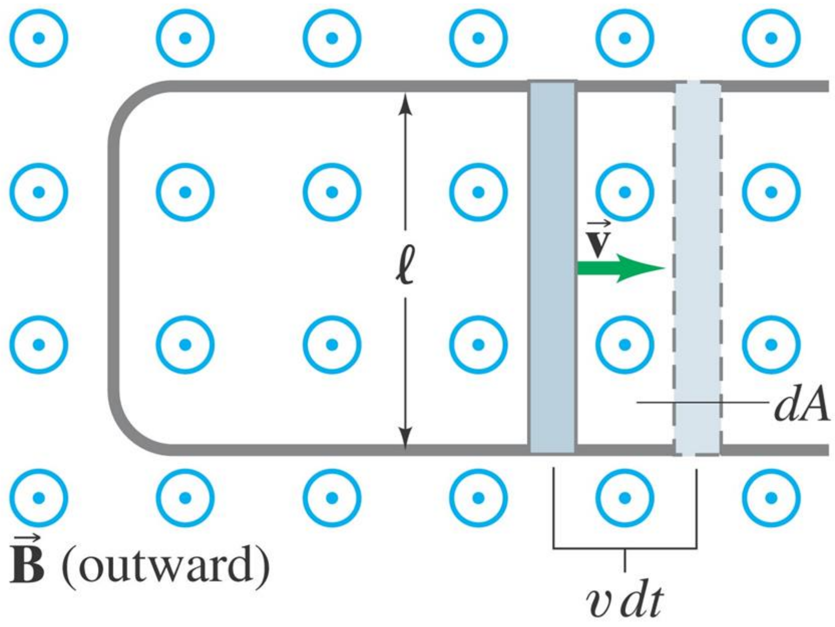
- When the rod moves to the right with speed v, the electrons in the rod also move with this speed. Therefore, since $\vec{v} \perp \vec{B}$, each electron feels a force $F = qvB$, which acts up the page, as shown above.
- If the rod was not in contact with the U-shaped conductor, electrons would collect at the upper end of the rod, leaving the lower end positive, as shown above.

EMF Induced in a Moving Conductor



- There must thus be an induced emf.
- If the rod is in contact with the u-shaped conductor, the electrons will flow into the U. There will then be a clockwise (conventional) current in the loop.
- To calculate the emf, we determine the work W needed to move a charge q from one end of the rod to the other against this potential difference: $W = force \times distance = (qvB)(l)$

EMF Induced in a Moving Conductor



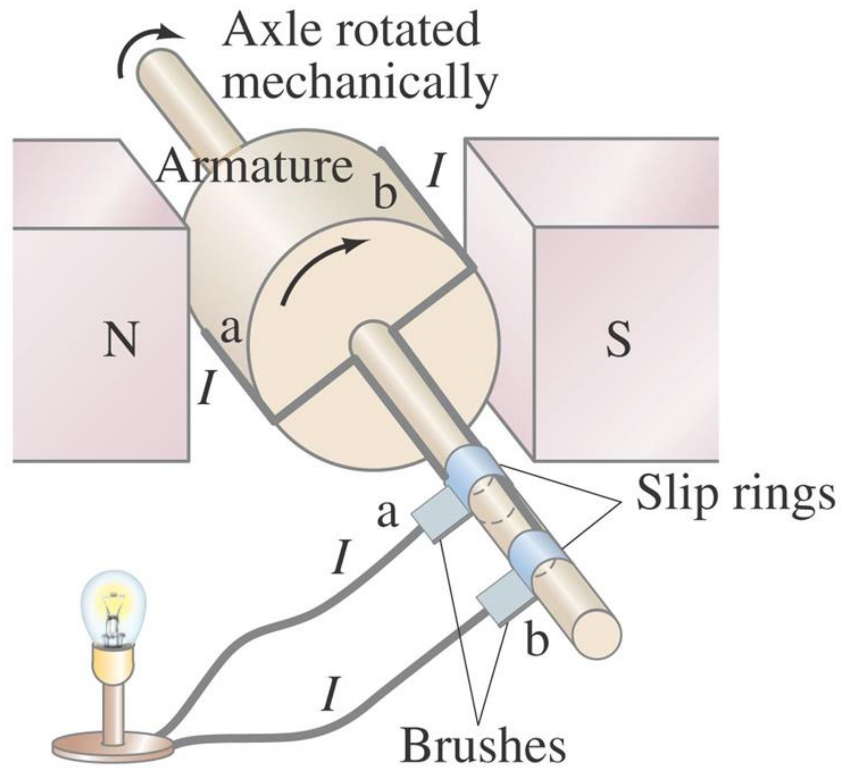
- The emf equals the work done per unit charge, so

$$emf = \frac{W}{q} = \frac{qvBl}{q} = Blv,$$

the same result as from Faraday's law.

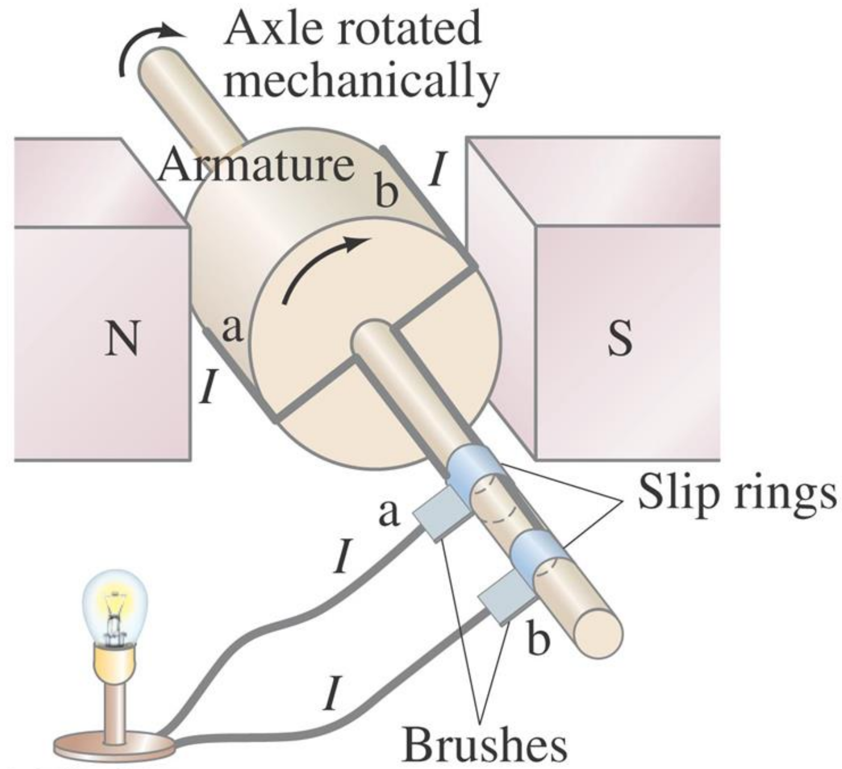
4. Electric Generators

Electric Generators



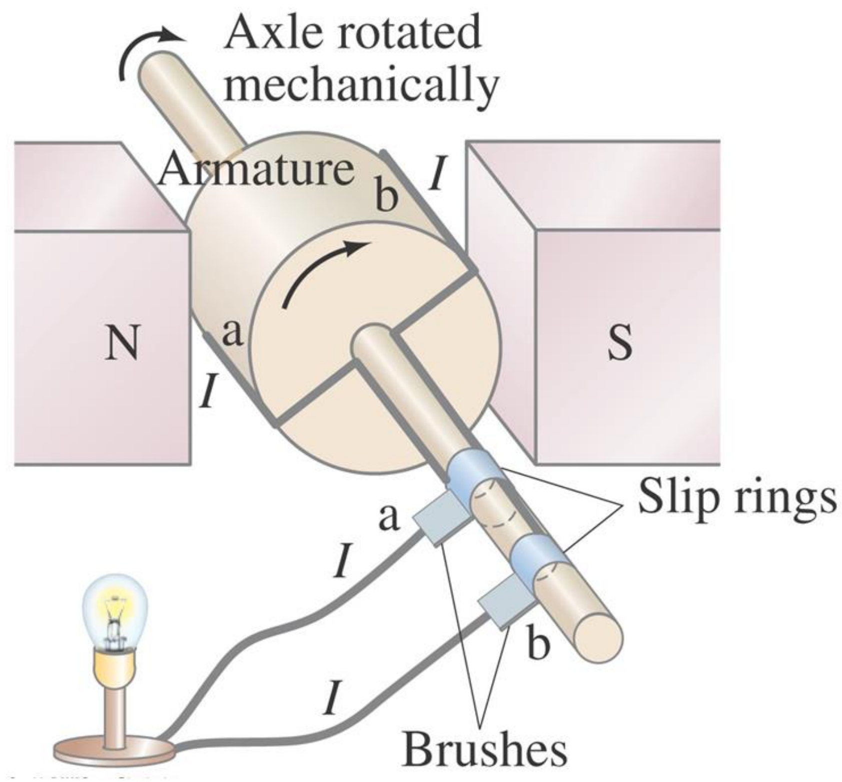
- Let’s now look at how **alternating currents (ac)** are generated: by an electric generator or dynamo, one of the most important practical results of Faraday’s great discovery.
- A generator transforms mechanical energy into electric energy, the opposite of what a motor does.

Electric Generators



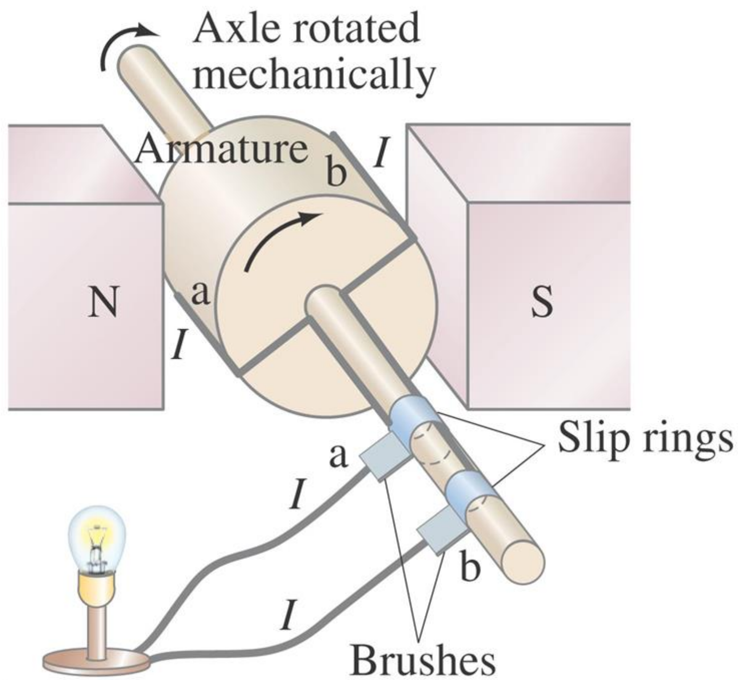
- A generator consists of many loops of wire (only one is shown above) wound on an armature that can rotate in a magnetic field.
- The axle is turned by some mechanical means (such as falling water, or high-pressure steam), and an emf is induced in the rotating coil.
- An electric current is thus the output of a generator.

Electric Generators



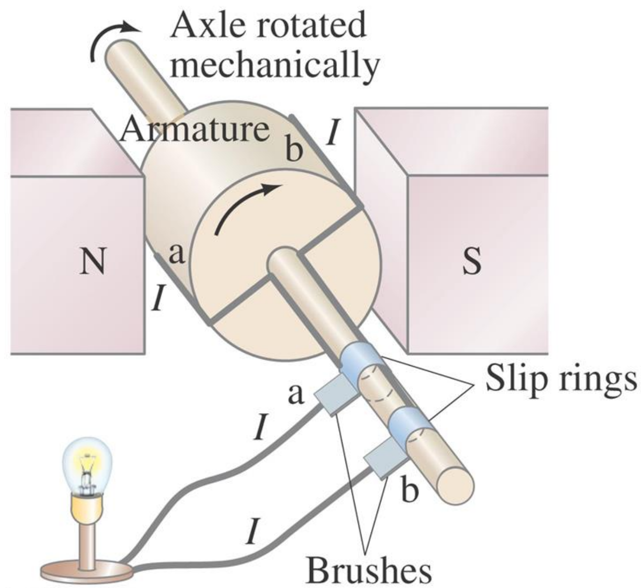
- Suppose in the above figure that the armature is rotating clockwise; then $\vec{F} = q\vec{v} \times \vec{B}$ applied to charged particles in the wire (or Lenz's law) tells us that the (conventional) current in the wire labelled b on the armature is outward, toward us; therefore, the current is outward from brush b.

Electric Generators



- Each brush is fixed and presses against a continuous slip ring that rotates with the armature.
- After one-half revolution, wire b will be where wire a is now in the drawing, and the current then at brush b will be inward.
- Thus, the current produced is alternating.

Electric Generators

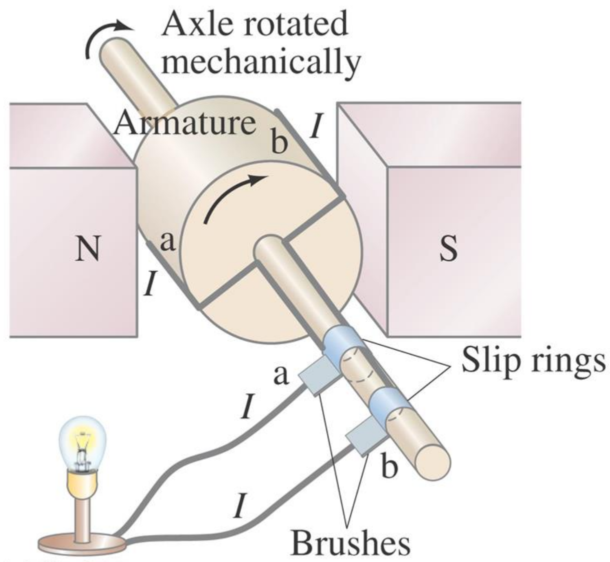


- Let us assume the loop is being made to rotate with constant angular velocity ω in a uniform magnetic field $\vec{B}$.
- From Faraday's law, the induced emf is

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{A} = -\frac{d}{dt}[BAcos\theta]$$

where A is the area of the loop and θ is the angle between $\vec{B}$ and $\vec{A}$

Electric Generators

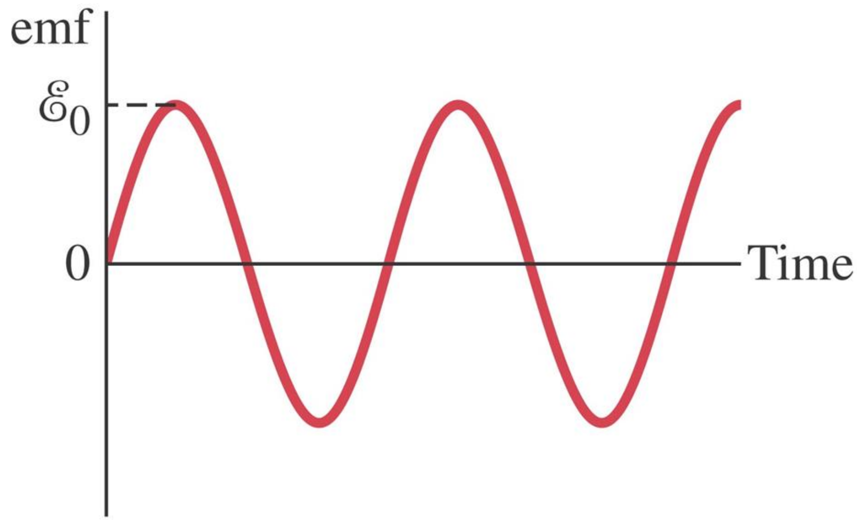


$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{A} = -\frac{d}{dt}[BAcos\theta]$$

Since $\omega = \frac{d\theta}{dt}$, then $\theta = \theta_0 + \omega t$, we can arbitrarily take $\theta_0 = 0$, so

$$\varepsilon = -BA\frac{d}{dt}(cos\omega t) = BA\omega sin\omega t$$

Electric Generators



- If the rotating coil contains N loops,

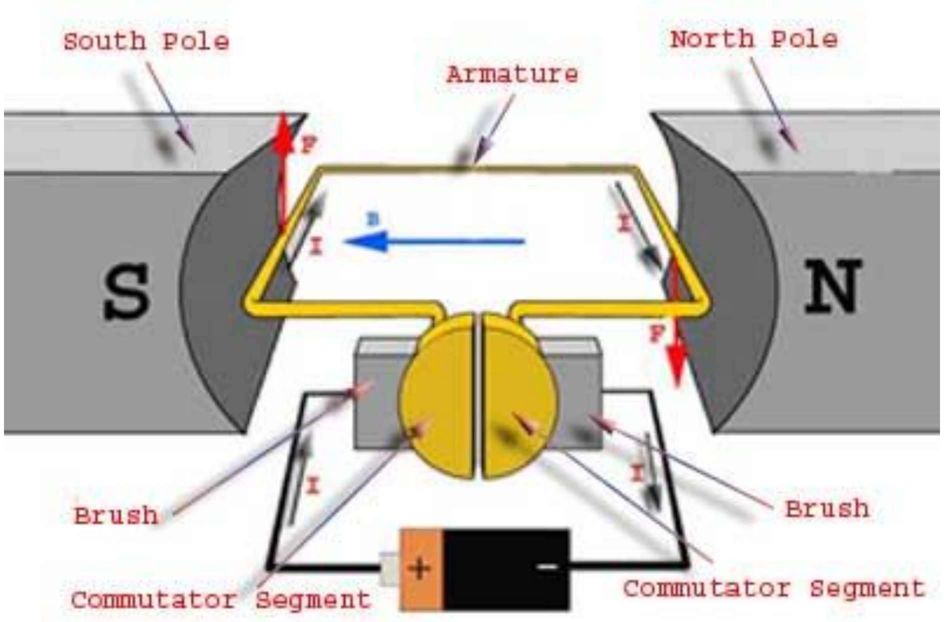
$$\varepsilon = NBA\omega sin\omega t = \varepsilon_0 sin\omega t$$

Thus, the output emf is sinusoidal with amplitude $\varepsilon_0 = NBA\omega$

- The frequency $f\left(=\frac{\omega}{2\pi}\right)$ is $60\ \mathrm{Hz}$ for general use in the United States and Canada, whereas $50\ \mathrm{Hz}$ is used in many countries.

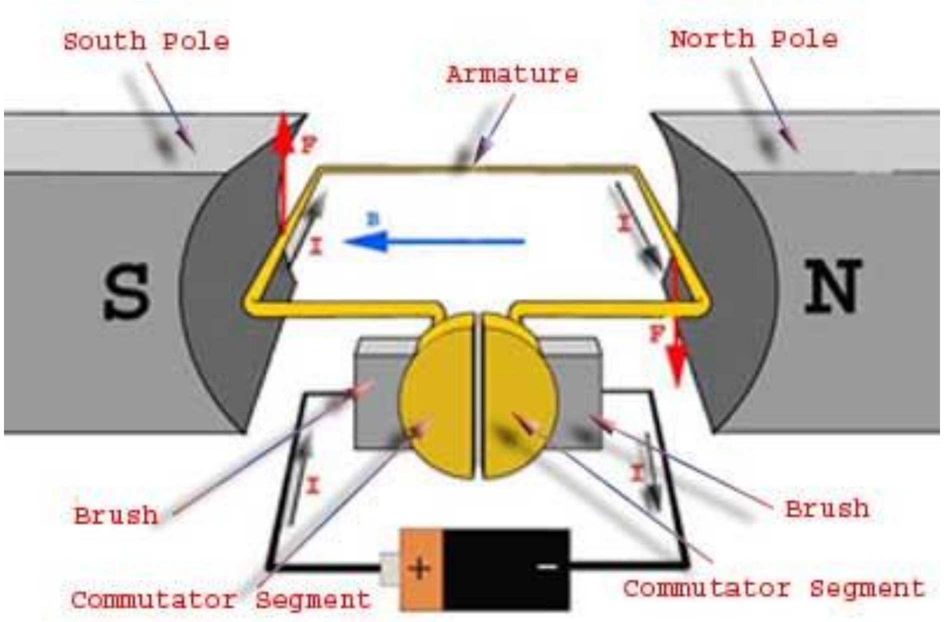
5. Back EMF and Counter Torque

Back EMF in a Motor



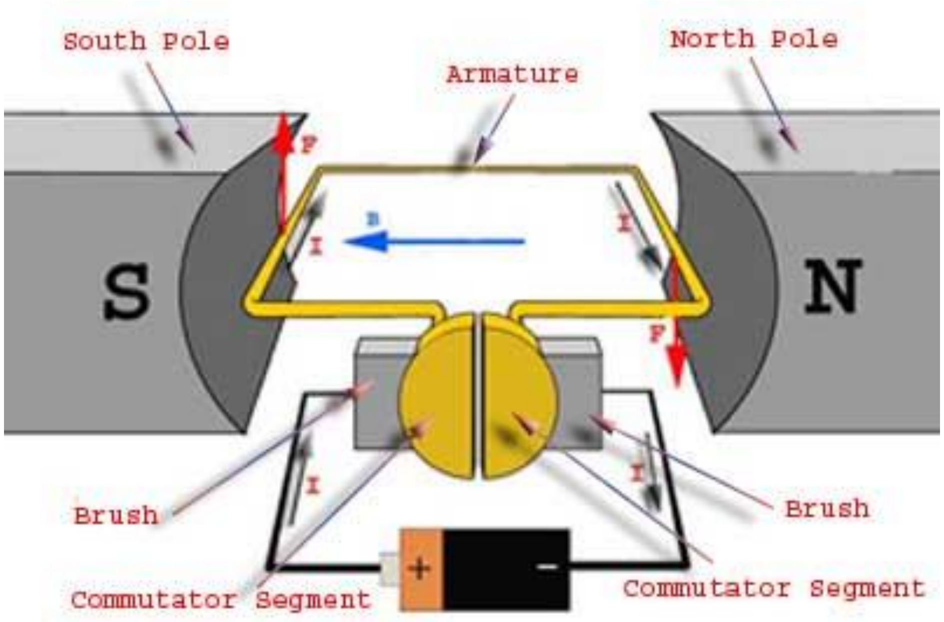
- As we have seen, a motor turns and produces mechanical energy when a current is made to flow in it.
- From a simple description of a dc motor, you might expect that the armature would accelerate indefinitely due to the torque on it.
- However, as the armature of the motor turns, the magnetic flux through the coil changes and an *emf* is generated.

Back EMF in a Motor



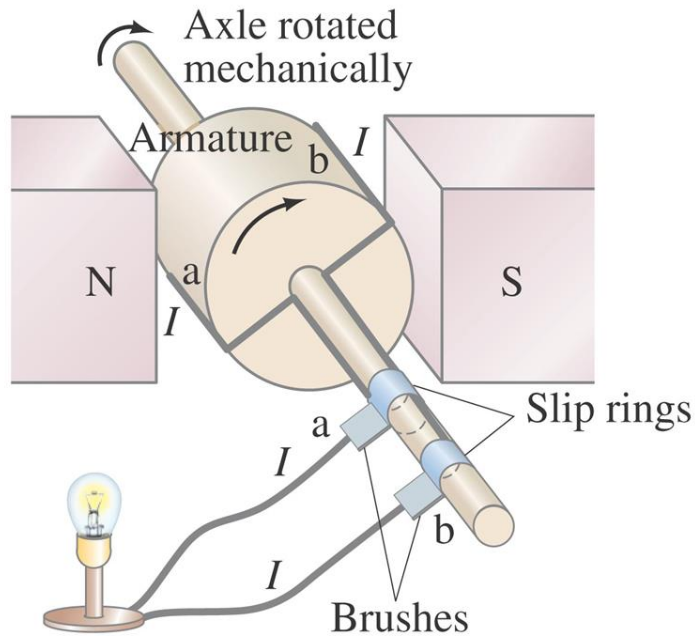
- This induced emf acts to oppose the motion (Lenz's law) and is called the **back *emf*** or **counter *emf***.
- The greater the speed of the motor, the greater the back *emf.*
- A motor normally turns and does work on something, but if there were no load, the motor's speed would increase until the back *emf* equalled the input voltage.

Back EMF in a Motor



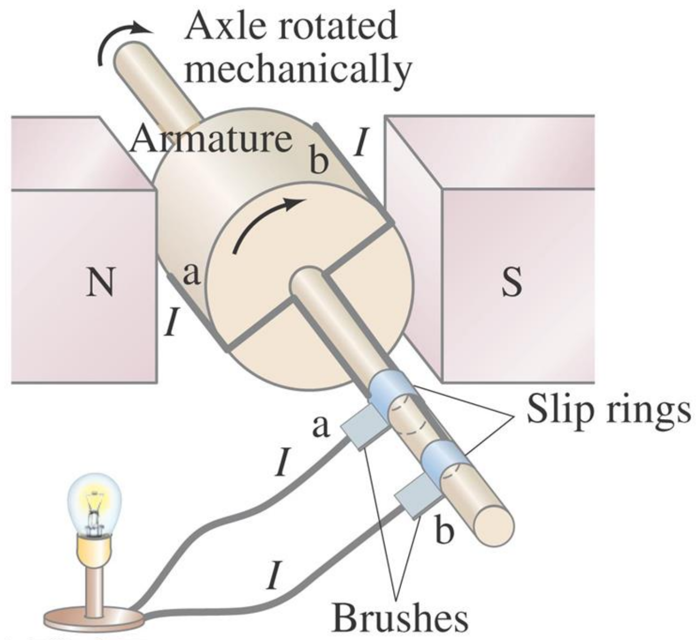
- When there is a mechanical load, the speed of the motor may be limited also by the load.
- The back *emf* will then be less than the external applied voltage.
- The greater the mechanical load, the slower the motor rotates and the lower is the back *emf* ($\varepsilon \propto \omega$).

Counter Torque



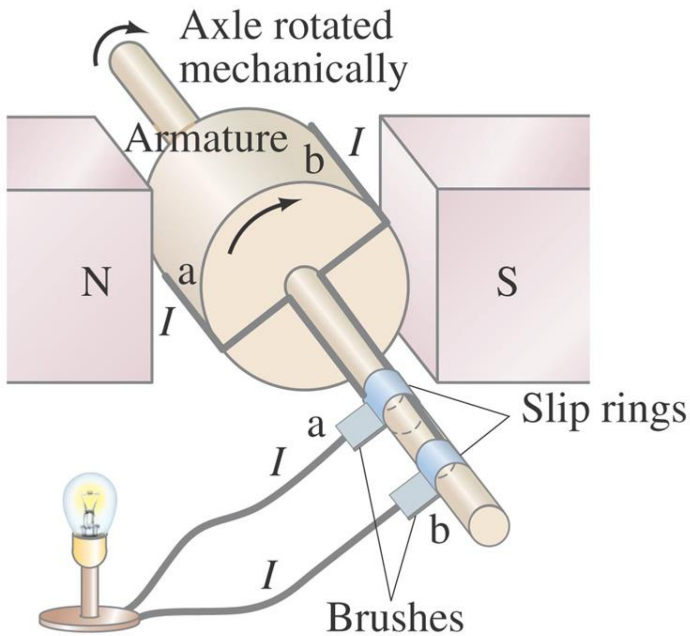
- In a generator, the situation is the reverse for that for a motor.
- As we have seen, the mechanical turning of the armature induces an *emf* in the loops, which is the output.
- If the generator is not connected to an external circuit, the *emf* exists at the terminals, but there is no current.
- In this case, it takes little effort to turn the armature.

Counter Torque



- But if the generator *is* connected to a device that draws current, then a current flows in the coils of the armature.
- Because this current-carrying coil is in an external magnetic field, there will be a torque exerted on it (as in a motor), and this torque opposes the motion (use the right-hand rule for the force on a wire). This is called a **counter torque**.

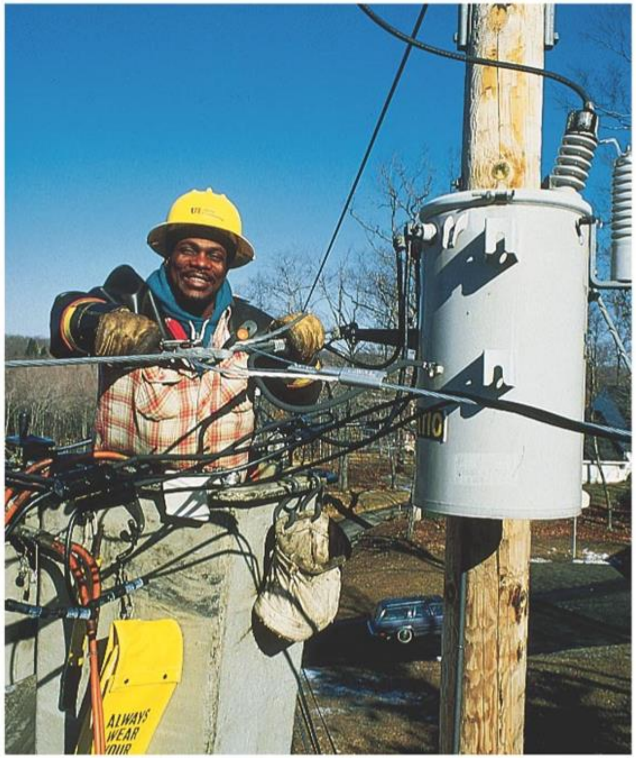
Counter Torque



- The greater the electrical load, that is, the more current that is drawn, the greater will be the counter torque. Hence, the external applied torque will have to be greater to keep the generator turning.
- This makes sense from the conservation of energy principle. More mechanical-energy input is needed to produce more electrical-energy output.

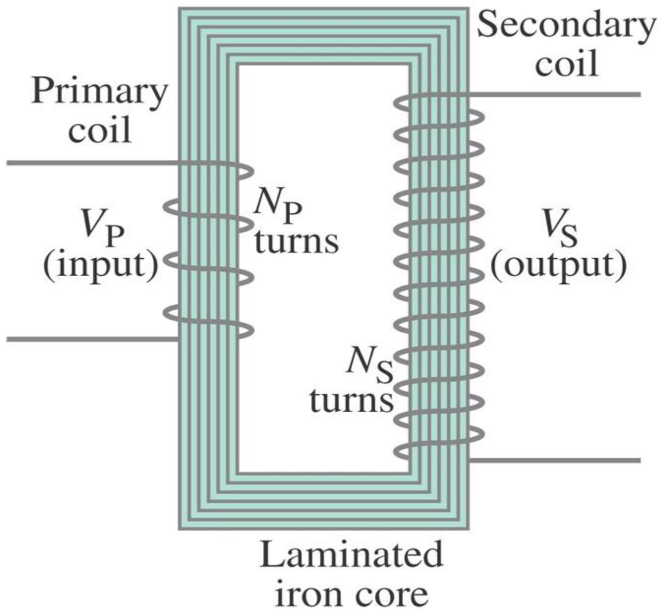
6. Transformers and Transmission of Power

Transformers and Transmission of Power



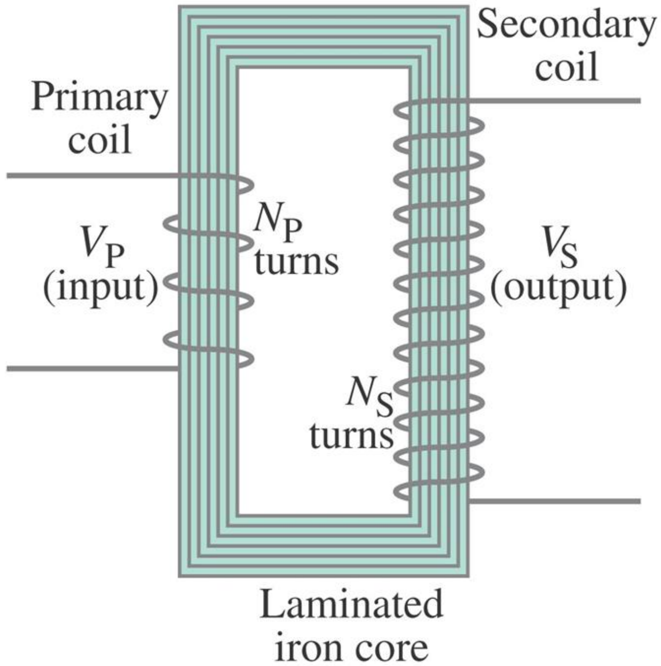
- A transformer is a device for increasing or decreasing an ac voltage.
- They have a wide variety of applications: for example, in chargers for cell phones; in laptops; and in your car to give the needed high voltage for the spark plugs.

Transformers and Transmission of Power



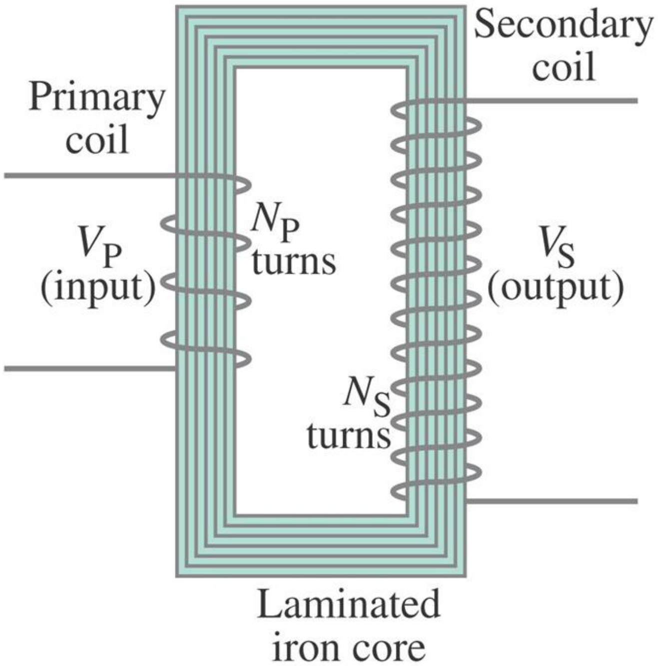
- A transformer consists of two coils of wire known as the **primary** and **secondary** coils.
- The two coils can be interwoven (with insulated wire); or they can be linked by an iron ore which is laminated to minimise eddy-current losses.

Transformers and Transmission of Power



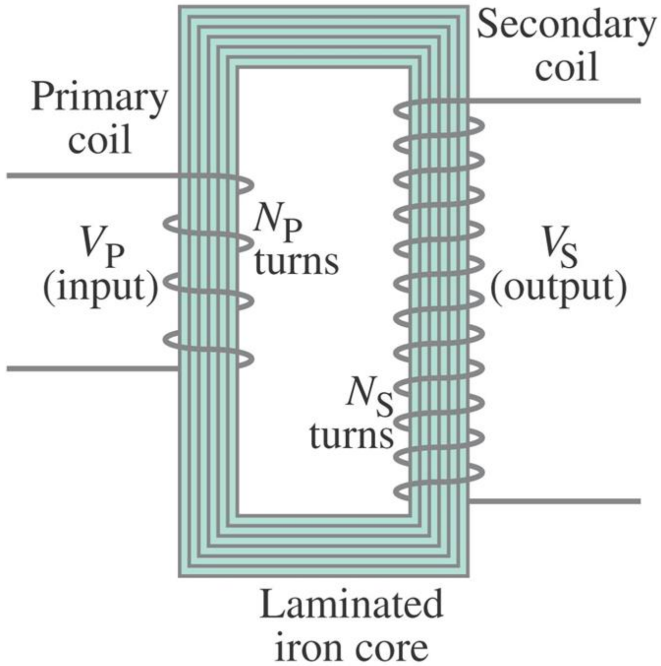
- Transformers are designed so that (nearly) all the magnetic flux produced by the current in the primary coil also passes through the secondary coil, and we can assume that this is true.
- We also assume that energy losses can be ignored.

Transformers and Transmission of Power



- When an ac voltage is applied to the primary coil, the changing magnetic field it produces will induce an ac voltage of the same frequency in the secondary coil.
- However, the magnitude of the voltage will be different according to the number of loops in each coil.

Transformers and Transmission of Power

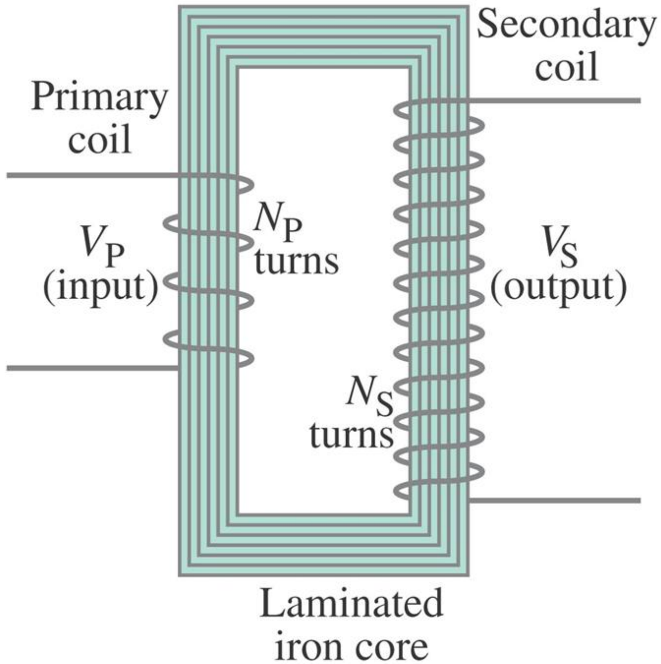


- From Faraday's law, the **magnitude** of the voltage or emf induced in the secondary coil is

$$V_s = N_s \frac{d\Phi_B}{dt},$$

where N_s is the number of turns in the secondary coil, and $d\Phi_B/dt$ is the rate at which the magnetic flux changes.

Transformers and Transmission of Power

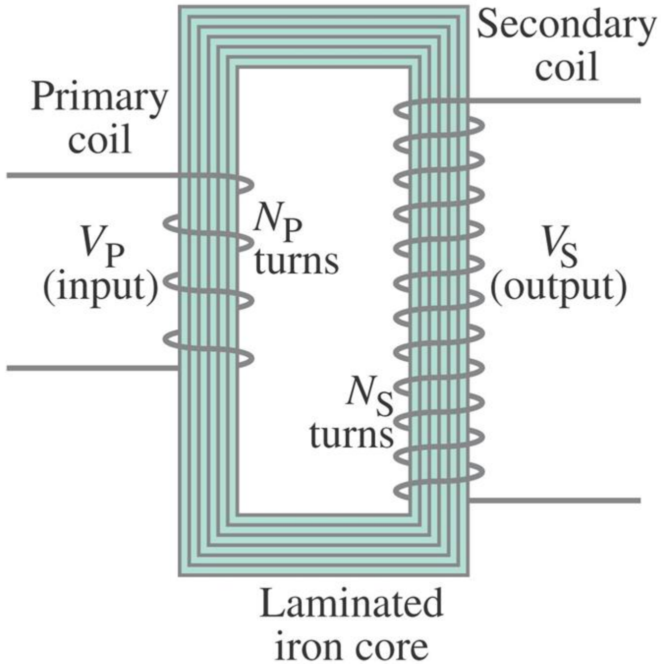


- The **magnitude of the input** primary voltage, V_p, is related to the rate at which the flux changes through it,

$$V_p = N_P \frac{d\Phi_B}{dt},$$

where N_P is the number of turns in the primary coil.

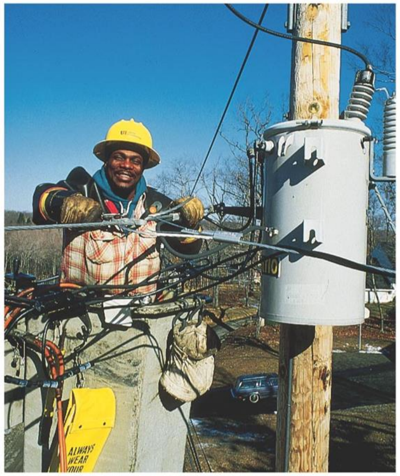
Transformers and Transmission of Power

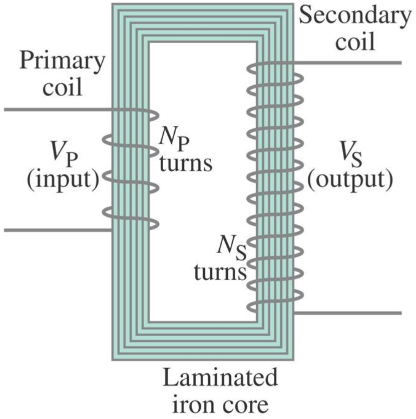


$$V_p = N_P \frac{d\Phi_B}{dt},$$

- This follows because the changing flux produces a ***back emf***, $N_p d\Phi_B/dt$, in the primary that exactly balances the applied voltage V_p if the resistance of the primary can be ignored (Kirchoff's rules).

Self-Inductance

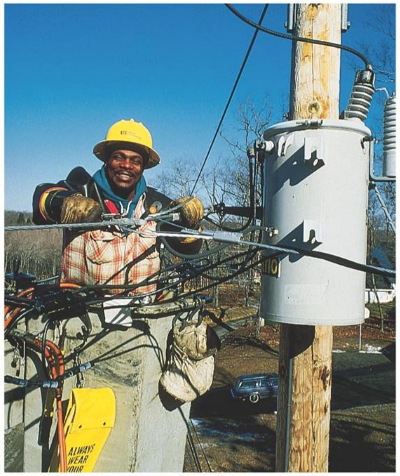


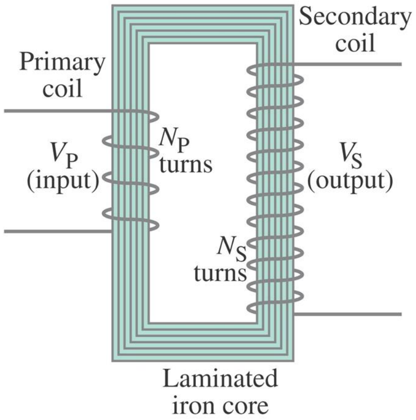


$$\varepsilon = -N\frac{d\Phi_B}{dt} = -L\frac{dI}{dt}$$

- The resistance of the primary in a transformer is usually quite small, perhaps less than 1Ω.
- If resistance alone limited the current in a transformer, tremendous currents would flow when a high voltage was applied.
- Indeed, a dc voltage applied to a transformer can burn it out.

Self-Inductance

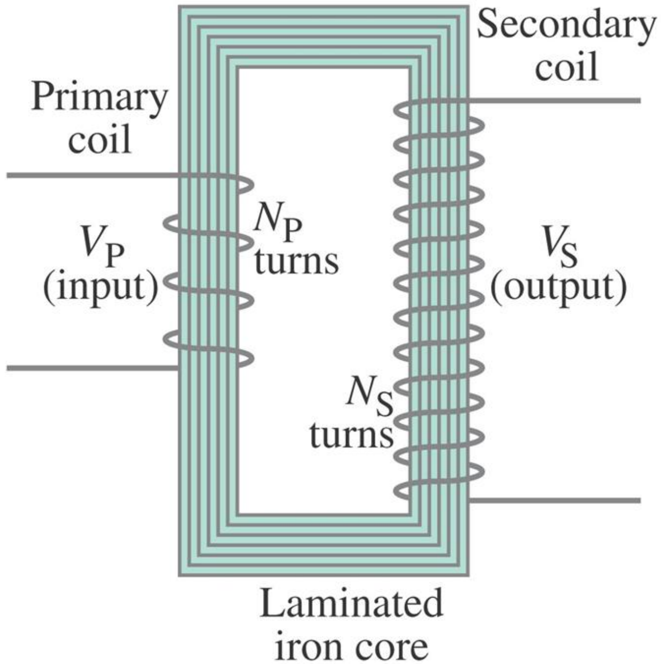




$$\varepsilon = -N\frac{d\Phi_B}{dt} = -L\frac{dI}{dt}$$

- It is the induced emf (or reactance) of the coil that limits the current to a reasonable value.
- Common inductors have inductances in the range from $1\mu\mathrm{H}$ to about $1\mathrm{H}$ (where $1\mathrm{H} = 1\mathrm{henry} = 1\Omega \cdot \mathrm{s}$)

Transformers and Transmission of Power



- Dividing the previous two equations, assuming little or no flux is lost, we can get the following

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}$$

- The transformer equation tells us how the secondary (output) voltage is related to the primary (input) voltage.

Summary of today’s Lecture

1. Induced EMF
2. Faraday’s Law of Induction, and Lenz’s Law
3. EMF Induced in a Moving Conductor
4. Electric Generators
5. Back EMF and Counter Torque; Eddy Currents
6. Transformers and Transmission of Power

Lecture 27: Optional Reading

- **Ch. 29.3**, EMF Induced in a Moving Conductor; p.883-884.
- **Ch. 29.4**, Electric Generators; p.884-886.
- **Ch. 29.5**, Back EMF and Counter Torque; Eddy Currents;p.886-888.
- **Ch. 29.6**, Transformers and Transmission of Power;p.888-891.
- **Ch. 29.1**, Induced EMF; p.877.
- **Ch. 29.2**, Faraday’s Law of Induction, and Lenz’s Law; p.878-882.

Home Work

Do not forget to attempt the **Additional Problems** for this lecture before logging in to **Mastering Physics** to complete your assignments.