Introduction to Algorithms

CELEN086

Seminar 10
(w/c 16/12/2024)

Outline

In this seminar, we will study and review on following topics:

- Euler path and Euler circuit
- Spanning tree
- Kruskal's algorithm
- Prim's algorithm

You will also learn useful Math/CS concepts and vocabularies.

Euler path and Euler circuit

Euler path and Euler circuit cover all edges exactly once.

For a connected graph, we can check the degrees of all vertices to determine weather it has Euler path and Euler circuit.

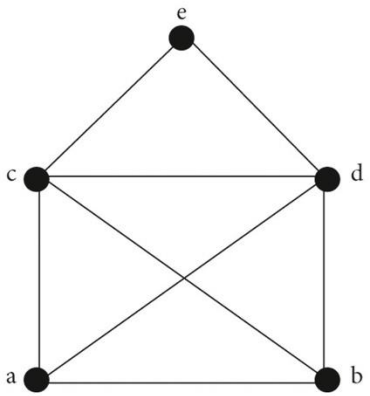
	Euler path	Euler circuit
• all vertices have even degrees	Yes	Yes
• two vertices have odd degrees	Yes	No
• more than two vertices have odd degrees	No	No

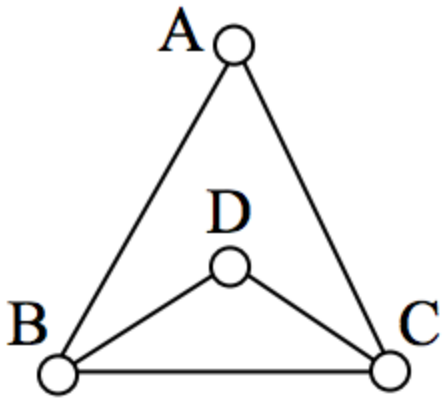
Note: Euler circuit is also Euler path.

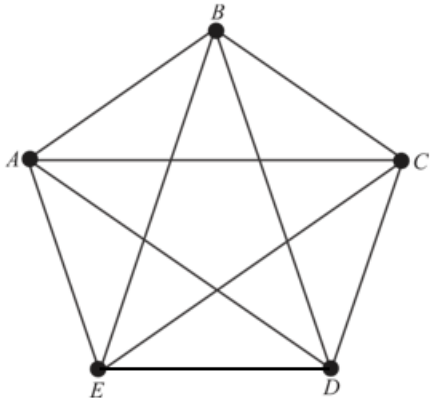
Practice

Determine if the following graphs have Euler path or Euler circuit.

Show one possible path/circuit on the graph if it exists.

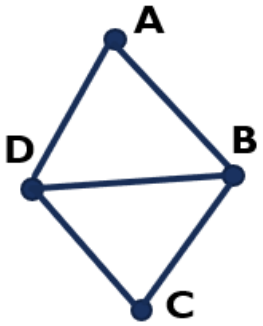






Counting spanning trees

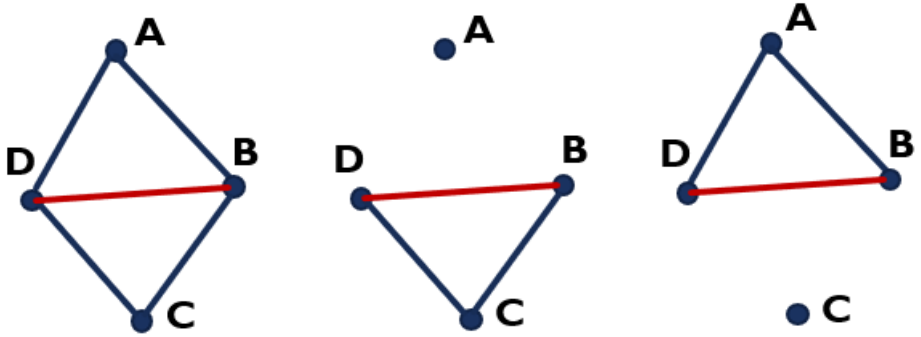
Find the total number of spanning trees of the following graph.



$$|V| = 4, |E| = 5, |E_s| = 3$$

Delete any two edges
(keep any three edges)

$$\binom{5}{2} = \binom{5}{3} = 10$$

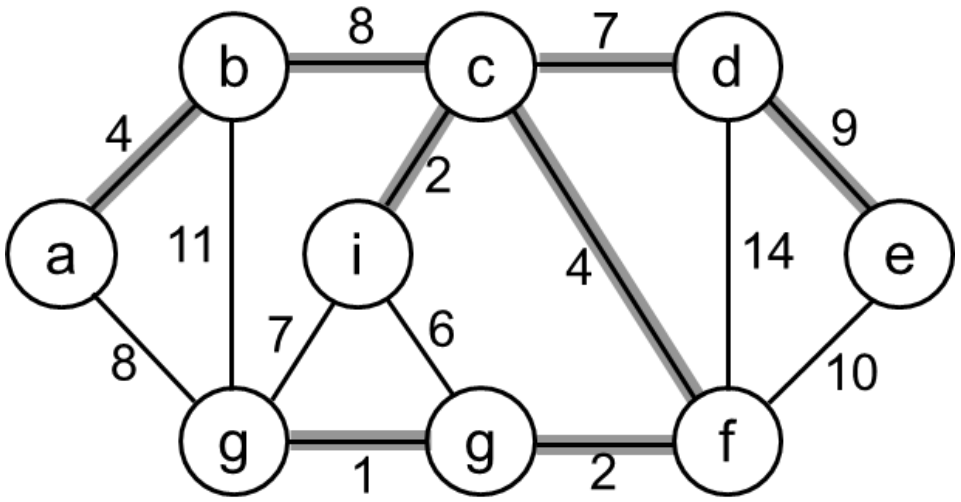


Can not delete both AD and AB, or both CD and CB.

$$10 - \binom{2}{2} - \binom{2}{2}$$
$$= 10 - 1 - 1 = 8$$

Minimum Spanning Trees

- Spanning Tree
  - A tree (i.e., connected, acyclic graph) which contains all the vertices of the graph.
- Minimum Spanning Tree
  - Spanning tree with the **minimum sum of weights**



Properties of Minimum Spanning Trees

Minimum spanning tree is not unique

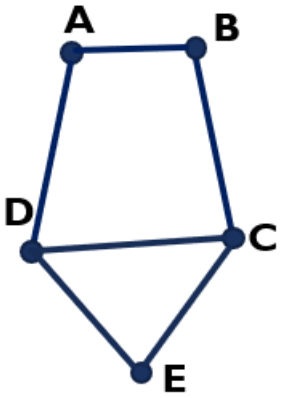
MST has no cycles – see why:

We can take out an edge of a cycle, and still have the vertices connected while reducing the cost.

# of edges in a MST: |V| - 1

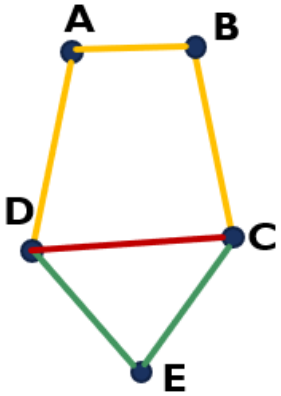
Practice

Consider the following graph, draw any 5 different spanning trees of it.



Compute total number of spanning trees.

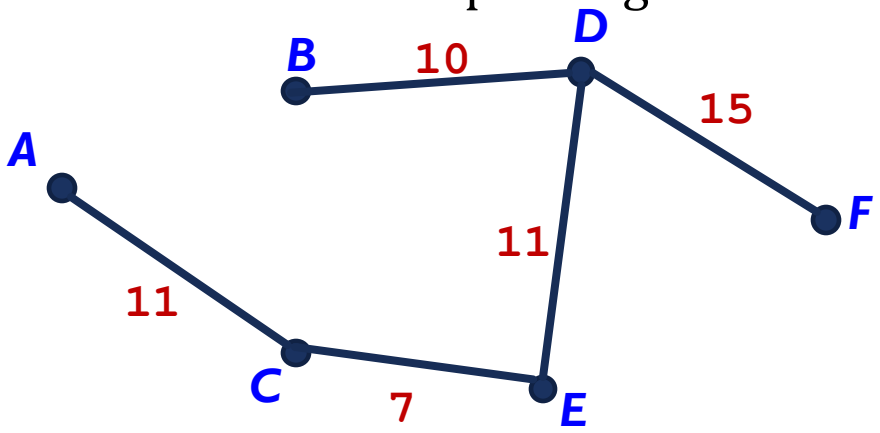
$$|V| = 5, |E| = 6, |E_s| = 4$$

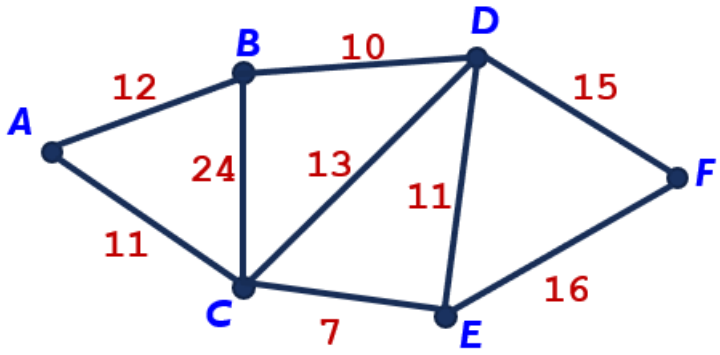


$$\binom{6}{2} - \binom{3}{2} - \binom{2}{2} = 15 - 3 - 1 = 11$$

Kruskal's algorithm

Minimum spanning tree:





$|E_s| = |V| - 1 = 6 - 1 = 5$

Edge selected: CE

BD

AC

DE

DF

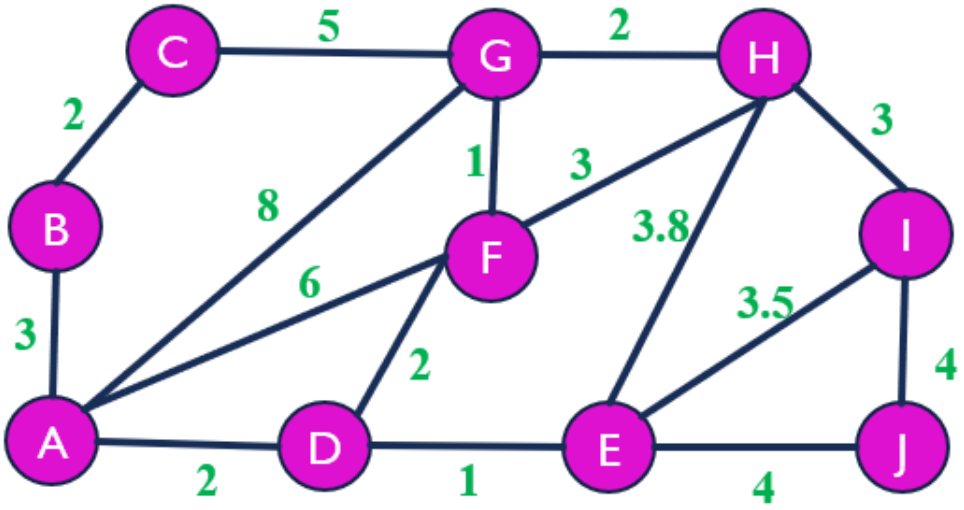
Minimum cost:

$7 + 10 + 11 + 11 + 15 = 54$

Note: in exam, you can present the process and final answer in a similar way.

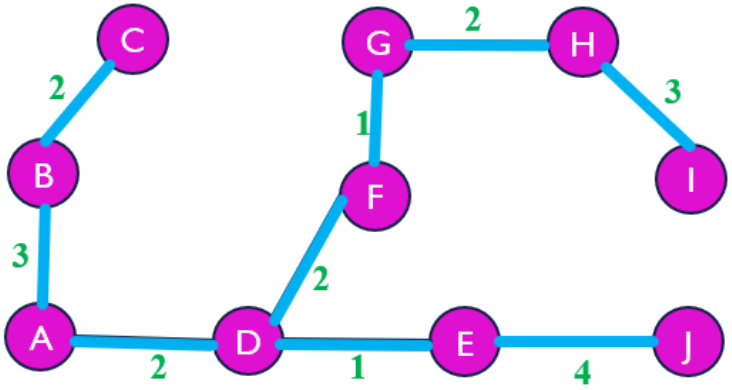
Practice

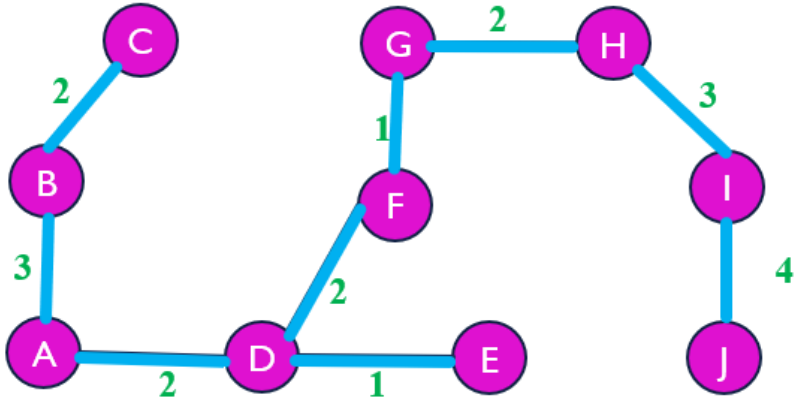
Apply Kruskal's algorithm to find the minimum spanning tree and compute the minimum cost.



Solution

(final answer)

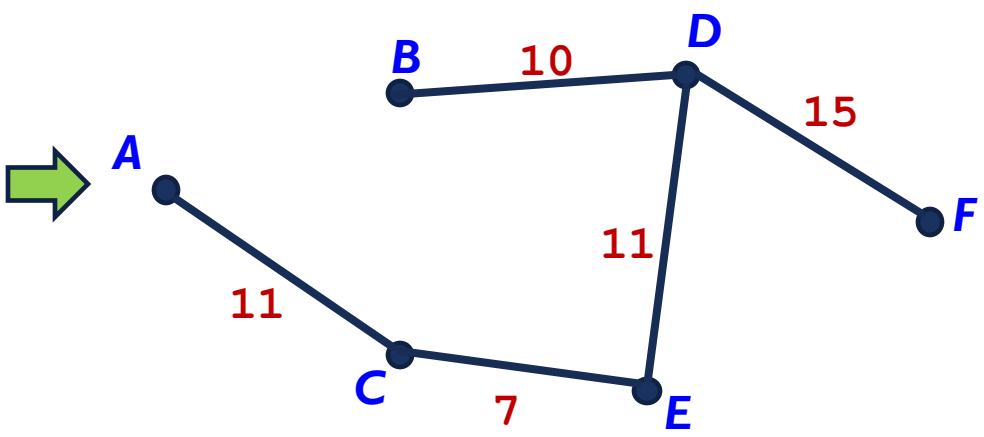


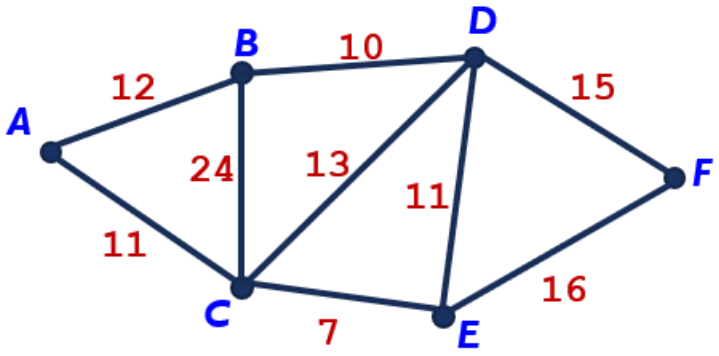


Minimum cost: 20

Note:
Either is fine. There might be more than one possible MST. You only need to show one of them.

Prim's algorithm





$|E_s| = |V| - 1 = 6 - 1 = 5$

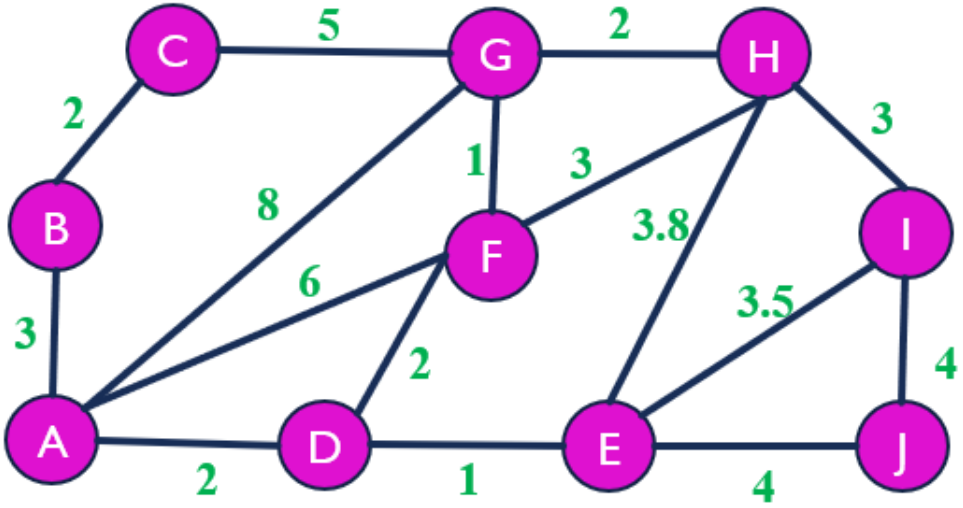
Edge selected: AC
CE
ED
DB
DF

Minimum cost:

$11 + 7 + 11 + 10 + 15 = 54$

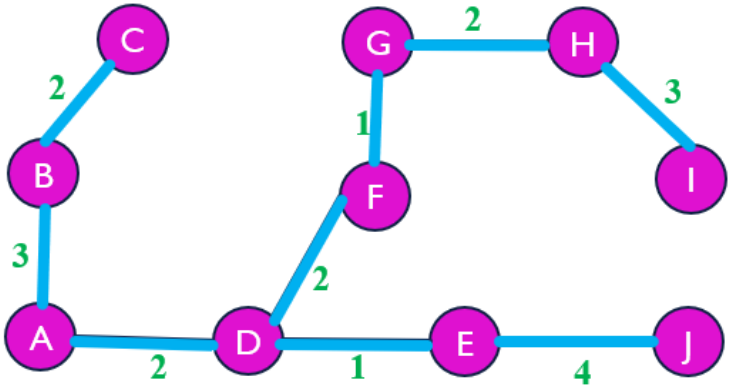
Practice

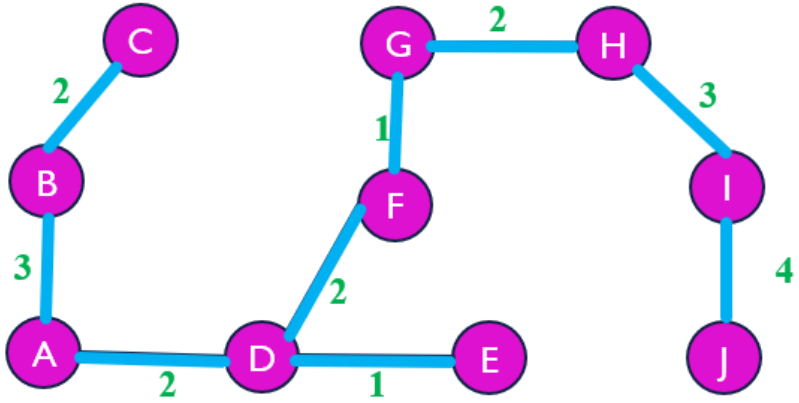
Apply Prim's algorithm (starting at Vertex B) to find the minimum spanning tree and compute the minimum cost.



Solution

(final answer)





Minimum cost: 20

Note:
The minimum cost of MST computed following Kruskal's, or Prim's algorithms will be the same.