Benchmark Orthogonality Properties

All benchmarks are not alike.

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W e define a manager's benchmark as a passive representation of the manager's investment process. The benchmark possesses the prominent investment characteristics that a manager's portfolio would display in the absence of active management. Hence, the benchmark is a reflection of the manager's investment style.[1]

The evaluation of an investment manager's performance proceeds from the implicit assumption that the manager's returns are to be compared against a valid benchmark. That is, (knowledgeable) performance evaluators insist that an unambiguous, investable, relevant benchmark, specified in advance of the evaluation period, be available for analysis.

Many plan sponsors, their consultants, and their managers who are under review, though, spend little time ascertaining the appropriateness of the benchmarks used in the evaluation of the managers' investment results. We find this lack of attentiveness to be a serious mistake. In this article, we discuss certain fundamental properties of valid benchmarks and their implications for manager performance evaluation.

DISAGGREGATING A MANAGER'S PORTFOLIO

Let us start with the simple identity of an investment manager's portfolio:[2]

$$P = P \tag{1}$$

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Now, consider an appropriately selected benchmark B. Adding to and subtracting B from the right-hand side of Equation (1) gives:

$$P = B + (P - B) \qquad (2)$$

If we define the manager's active investment judgments A as being the difference between the manager's portfolio P and the benchmark B, so that A = (P - B), then Equation (2) becomes:

$$P = B + A \qquad (3)$$

Equation (3) states that the manager's portfolio can be partitioned into two elements: the manager's benchmark (or investment style) and the manager's active management decisions (or investment skill).[3]

The manager's benchmark is based on publicly available information concerning the manager's investment process. For example, the manager may advertise that it is a small-capitalization, value manager. Further, the manager may disclose in its presentations to current and prospective clients that it typically buys stocks with market capitalizations below $100 million and price-to-book ratios below 2.5. The manager may also make known that it does not follow companies in the biotechnology or health care industries, and that securities held in its portfolios are usually equal-weighted.

These (and other) investment characteristics are incorporated into the manager's benchmark. Conversely, proprietary aspects of the manager's investment process, which relate to how the manager discerns mispriced securities, are excluded from the benchmark.

By definition then, the manager's benchmark must not contain valuable information that would lead either the manager or objective third parties to expect that the benchmark will perform, in a predictable fashion, differently from any other publicly accessible portfolio, after accounting for risk and investment management costs. In particular, the manager's choice of an investment style should not give the manager any advantage (or disadvantage) over the rest of the market.

This no-value condition of the benchmark is vital to disaggregating the manager's portfolio into the two components shown in Equation (3): investment style and investment skill. If this condition did not hold, then the manager would naturally incorporate the valuable information into its active portfolio. A revised benchmark would have to be constructed that did not contain this information.

In the final analysis, active managers charge their clients fees for delivering investment skill, not the investment styles contained in their benchmarks.[4]

BENCHMARK ORTHOGONALITY PROPERTIES

The disaggregation of the manager's portfolio leads directly to two properties of a valid benchmark. We will refer to these properties as *benchmark orthogonality properties*. First, the performance of the value-added component should be uncorrelated with the performance of the benchmark. Second, the performance of the value-added component should be uncorrelated with the performance of a market proxy M (for example, the S&P 500), which we can think of as simply another benchmark distributed freely by some third-party vendor.

Upon reflection, both of these benchmark orthogonality properties arise out of common sense and are symmetrical. If the benchmark has adequately captured the manager's investment style, then the manager's performance relative to that benchmark should be unrelated to how either its investment style or the broad market performs. Despite their simplicity, a closer examination of these properties leads to several interesting conclusions concerning benchmark quality and performance evaluation.

Property 1: COV[B, A] = 0

The first property implies that the long-term beta of the portfolio with respect to the benchmark should be 1. That is:

$$\beta[P, B] = 1$$

To show this, from Equation (3), we have:

$$P = B + A \qquad (3)$$

Consequently:

$$COV[P, B] = COV[(B + A), B]$$

$$= VAR[B] + COV[A, B] \qquad (4)$$

Calling upon the definition of one portfolio's beta rel-

ative to another gives us the beta of the manager's portfolio relative to the benchmark. That is:

$$\beta[P, B] = COV[P, B]/VAR[B] \qquad (5)$$

If we substitute Equation (4) for the numerator of Equation (5), and apply the zero covariance of Property 1, then we have:

$$\beta[P, B] = \{VAR[B] + COV[A, B]\}/VAR[B]$$

$$= VAR[B]/VAR[B]$$

$$= 1$$

Therefore, in determining whether the long-run time series of a benchmark's returns is appropriate for a particular investment discipline, we need only regress the actual portfolio's returns in excess of the risk-free rate against the benchmark's excess returns, and test the statistical significance of the estimated regression coefficient against a theoretical value of 1. This is a simple benchmark quality test that performance evaluators should apply to any prospective benchmarks.

Property 2: COV[A, M] = 0

The second property implies that the portfolio's beta with respect to the market proxy is the same as the benchmark's beta with respect to the market proxy. That is:

$$\beta[P, M] = \beta[B, M]$$

To show this, again we use Equation (3) to produce:

$$COV[P, M] = COV[(B + A), M]$$

$$= COV[B, M] + COV[A, M]$$

Dividing both sides by VAR[M], and applying the zero covariance of Property 2, we have:

$$COV[P, M]/VAR[M] = COV[B, M]/VAR[M] \qquad (6)$$

Noting that the left and right sides of Equation (6) are simply the betas of P and B with respect to the market proxy, we have:

$$\beta[P, M] = \beta[B, M]$$

This equality implies another benchmark quality test: The systematic risk of the portfolio over the long run should not deviate in a statistically significant sense from that of the benchmark.

Equation (6) has another interesting implication. It allows the benchmark builder to determine the appropriate cash position for the benchmark. Given a desired list of benchmark securities and associated weights (excluding cash), then the necessary cash position is simply the position that will produce the beta equality described in Equation (6).

Many plan sponsors and consultants, even those otherwise quite knowledgeable about benchmarks, are reluctant to permit cash to be included in manager benchmarks. The second benchmark orthogonality property demonstrates that the benchmark will be mis-specified without including the manager's appropriate long-run cash position. For managers who hold some cash on a regular basis (and most do, even those who describe themselves as fully invested), the exclusion of cash from their benchmarks will cause the benchmark to tend to overstate the systematic risk of the manager's investment style.

SOME ADDITIONAL CONCLUSIONS

Taken together, the benchmark orthogonality properties greatly simplify analysis of whether a particular benchmark is appropriate for a specific investment discipline.[5] They also have several other implications critical to the evaluation of manager performance.

Referring again to Equation (3), we can disaggregate the portfolio into an additional component. That is, by introducing the market proxy on the right-hand side, we have:

$$P = M + (B - M) + A \qquad (7)$$

Thus we can view the portfolio as being composed of a *systematic* element M, a *style bias (or misfit)* element (B − M), and an *active management* element A.

Given the no–value condition of the benchmark, if we take the return expectations of the components of Equation (7), then:

$$E[r(P)] = E[r(M)] + E[r(A)] \qquad (8)$$

Consequently, the only way the manager can expect to outperform the market proxy is if the manager expects

to outperform its benchmark, that is, $E[r(A)] > 0$.

Many members of the institutional investment community are interested merely in evaluating a manager's results relative to a market proxy and in testing the significance of that relative performance. Defining X as the difference between the manager's portfolio and the market proxy, we have:

$$X = P - M$$

Therefore:

$$r(X) = r(P) - r(M)$$

Thus the expectation of X is:

$$E[r(X)] = E[r(P)] - E[r(M)] \qquad (9)$$

Substituting Equation (8) on the right-hand side of Equation (9), and rearranging terms, gives:

$$E[r(X)] = E[r(A)] \qquad (10)$$

The volatility of X is given by:

$$VAR[r(P) - r(M)] = VAR[r(B - M) + r(A)]$$

$$= VAR[r(B - M)] + VAR[r(A)]$$

$$+ 2 \times COV[r(B - M), r(A)]$$

From the benchmark orthogonality properties, we have $COV[r(B - M), r(A)] = 0$. Therefore:

$$VAR[r(X)] = VAR[r(P) - r(M)]$$

$$= VAR[r(B - M)] + VAR[r(A)] \qquad (11)$$

As $VAR[r(B - M)] \geq 0$, Equation (11) demonstrates that the portfolio's returns relative to the market proxy can be no less variable than the portfolio's returns relative to the benchmark, despite the fact that they both have the same expectation.

The benefits of evaluating a manager's performance relative to a valid benchmark instead of a market proxy are emphasized if we introduce the concept of the *information ratio*. The information ratio measures a manager's excess return versus a benchmark compared to the variability of that excess return.[6] It is the appro-

priate measure of a manager's value-added. On an ex ante basis for a portfolio Z:

$$IR[Z] = E[Z] \Big/ \Big\{ \sqrt{VAR[Z]} \Big\} \qquad (12)$$

From Equations (10) and (11), it must be true that:

$$IR[r(A)] \geq IR[r(X)] \qquad (13)$$

In essence, if we are evaluating the investment skill of a manager, our test can never be less exact, and likely will be more exact, using returns relative to a properly constructed benchmark than returns relative to a market proxy.

As a final point, our experience has been that, on average:

$$VAR[r(B - M)] \cong VAR[r(A)]$$

In other words, the variability of a manager's benchmark returns relative to a market proxy (that is, the manager's style bias or misfit) is typically at least as volatile as the active management returns generated by the manager. If the two return series' volatilities were of equal size, then in terms of the information ratio:

$$IR[r(A)] \cong \sqrt{2 \times IR[r(X)]}$$

We can interpret this result to mean that it will take only 70% of the time $(1/\sqrt{2})$ to establish the statistical significance of a manager's value-added process if one compares the manager's portfolio returns to an appropriate benchmark instead of comparing them to a market proxy. We consider such reductions to be extremely valuable in the uncertain world of investment management, and we believe they are well worth the effort it takes to construct valid manager benchmarks.

ENDNOTES

[1]For a discussion of basic benchmark issues, see Bailey, Richards, and Tierney [1988] and Divecha and Grinold [1989].

[2]Equation (1) and the equations that follow can be thought of as referring to returns or security holdings, as the context of the discussion dictates.

[3]The investment management ramifications of disaggregating a plan sponsor's total portfolio into investment style and investment skill are explored in Tierney and Winston [1990].

[4]One can conceive of interesting applications of an "augmented" benchmark that contains some of the manager's active management information (for example, see Bailey and Tierney [1993]). For purposes of this discussion, however, and for performance evaluation in general, we will assume we are dealing with "baseline" benchmarks that satisfy the no-value condition.

[5]The evaluation of benchmark quality is described in Bailey [1992].

[6]The information ratio is discussed at length in Grinold [1989] and Sharpe [1994].

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