6. 1.21.

a) $4u_{xx} + 12u_{xy} + 5u_{yy} = \underbrace{6u_x - u_y}$

Para este exercício vou ignorar!

Vou me concentrar em esta parte!

$a(x,y) = 4$

$b(x,y) = 6$

$c(x,y) = 5$

$\Rightarrow 4u_{xx} + 12u_{xy} + 5u_{yy} = a(x,y)u_{xx} + 2b(x,y)u_{xy} + c(x,y)u_{yy}.$

$\Rightarrow b^2 - ac = 36 - 20 = 16 > 0$

$\Rightarrow$ $\boxed{\text{Hiperbólica.}}$ 2 famílias curvas!

$$4\mu^2 - 12\mu + 5 = 0$$

Formula discriminante:

$$\mu = \frac{-12 \pm 8}{2 \cdot 4} = \frac{-12 \pm 8}{8}$$

$\Rightarrow \mu_1 = -\frac{5}{2}, \quad \mu_2 = -\frac{1}{2}$

$\frac{dy}{dx} = \mu_1 \Rightarrow y = -\frac{5}{2}x + \text{const}.$

$= \mu_2 \Rightarrow y = -\frac{x}{2} + \text{const}.$

Escaneado con CamScanner

1.58 • $\langle \phi_k, \phi_m \rangle = \int_0^{\ell} \operatorname{Sen}\left(\frac{k\pi x}{\ell}\right) \operatorname{Sen}\left(\frac{m\pi x}{\ell}\right) dx$ (1)

$$= \begin{cases} 0, & \text{si } k \neq m \\ \ell/2, & \text{si } k = m \end{cases}$$

*Nota: Asumimos $k \neq m$, $m \neq 0$

Integração por partes

$$\int_0^{\ell} \operatorname{Sen}\left(\frac{k\pi x}{\ell}\right) \operatorname{Sen}\left(\frac{m\pi x}{\ell}\right) dx$$

$f(x) = \operatorname{Sen}\left(\frac{k\pi x}{\ell}\right)$

$g(x) = \frac{\ell}{m\pi} \cos\left(\frac{m\pi x}{\ell}\right)$

$g'(x) = \operatorname{Sen}\left(\frac{m\pi x}{\ell}\right)$

$$= \frac{\ell}{m\pi} \operatorname{Sen}\left(\frac{k\pi x}{\ell}\right) \cos\left(\frac{m\pi x}{\ell}\right)\Big|_0^{\ell} \xrightarrow{} 0$$

$$- \int_0^{\ell} - \cos\left(\frac{k\pi x}{\ell}\right)\left(\frac{k\pi}{\ell}\right)\left(\frac{\ell}{m\pi}\right) \cos\left(\frac{m\pi x}{\ell}\right) dx$$

$$= \frac{k}{m} \int_0^{\ell} \cos\left(\frac{k\pi x}{\ell}\right) \cos\left(\frac{m\pi x}{\ell}\right) dx \qquad f(x) =$$

$$= \frac{k}{m}\left(-\cos\left(\frac{k\pi x}{\ell}\right) \operatorname{Sen}\left(\frac{m\pi x}{\ell}\right)\left(\frac{\ell}{m\pi}\right)\Big|_0^{\ell} \xrightarrow{} 0\right.$$

$$\left. - \int_0^{\ell} \left(\frac{k\pi}{\ell}\right) \operatorname{Sen}\left(\frac{k\pi x}{\ell}\right) \operatorname{Sen}\left(\frac{m\pi x}{\ell}\right)\left(\frac{\ell}{m\pi}\right) dx\right)$$

$$= \frac{k^2}{m^2} \int_0^{\ell} \operatorname{Sen}\left(\frac{k\pi x}{\ell}\right) \operatorname{Sen}\left(\frac{m\pi x}{\ell}\right) dx$$

$$\Rightarrow \left(1 - \frac{k^2}{m^2}\right) \int_0^{\ell} \operatorname{Sen}\left(\frac{k\pi x}{\ell}\right) \operatorname{Sen}\left(\frac{m\pi x}{\ell}\right) dx = 0$$

$$\Rightarrow \int_0^{\ell} \operatorname{Sen}\left(\frac{k\pi x}{\ell}\right) \operatorname{Sen}\left(\frac{m\pi x}{\ell}\right) dx = 0. \quad \left(\text{pois } k \neq m \Rightarrow \frac{k^2}{m^2} \neq 1\right)$$

~~* $k \neq m$, $m = 0$,~~

$$\cancel{\int_0^{\ell} \operatorname{Sen}\left(\frac{k\pi x}{\ell}\right) \operatorname{Sen}(0)\, dx = 0.}$$

* $k = m$

$$\int_0^l \operatorname{sen}\left(\frac{k\pi x}{l}\right)^2 dx = \frac{l}{k\pi}\int_0^{k\pi} \operatorname{sen}^2(y)\, dy =$$

$$\frac{l}{k\pi}\left(\int_0^{k\pi} (\operatorname{sen}(y)\cos(y))' + \cos^2(y)\, dy\right) =$$

$$\frac{l}{k\pi}\left(\cancel{\operatorname{sen}(y)\cos(y)\Big|_0^{k\pi}}^{\,0} + \int_0^{k\pi} \cos^2(y)\, dy\right) =$$

$$= \frac{l}{k\pi}\left(\int_0^{k\pi} 1 - \operatorname{sen}^2(y)\, dy\right) = \frac{l}{k\pi}\left(k\pi - \int_0^{k\pi} \operatorname{sen}^2(y)\, dy\right.$$

$$= l - \int_0^l \operatorname{sen}\left(\frac{k\pi x}{l}\right)^2 dx$$

$$\Rightarrow 2\int_0^l \operatorname{sen}\left(\frac{k\pi x}{l}\right)^2 dx = l$$

$$\Rightarrow \int_0^l \operatorname{sen}\left(\frac{k\pi x}{l}\right)^2 dx = \frac{l}{2}.$$

Nota: Use o fato de $(\operatorname{sen}(x)\cos(x))' = \operatorname{sen}^2(x) - \cos^2(x)$

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* Ejercicio Neuman:

- Heat equation (Section 1.4, pág 36).
  ↳ Entender a idea!

$$\begin{cases} u \in C^2((0,+\infty)\times(0,l)) \cap C^1([0,+\infty)\times[0,l]) \\ \partial_t u(t,x) = \partial_x^2 u(t,x), \quad (t,x) \in (0,+\infty)\times(0,l) \\ \partial_x u(t,0) = \partial_x u(t,l) = 0, \quad t \geq 0 \\ u(0,x) = \phi(x) \end{cases}$$

$$\begin{cases} X \in C^2((0,\ell)) \cap C^1([0,\ell]) \\ -X''(x) = \lambda X(x) \\ X'(0) = X'(\ell) = 0 \end{cases}$$

ojo: o livro escreveu errado!

- $\boxed{1.59 \text{ e } 1.60}$ $\longrightarrow$ Section 1.4
Basearse

* 1.59

a) Primera parte do corolario 1.55!
Ojo: Agora $\lambda \in [0, +\infty)$ ¿Por que não podemos concluir $\lambda > 0$?

b) $\boxed{\text{Ver pagina 40.}}$

$$X(x) = C\cos(\sqrt{\lambda}x) + D\,\mathrm{sen}(\sqrt{\lambda}x)$$

$$X'(x) = C\cdot(\sqrt{\lambda})\,\mathrm{sen}(\sqrt{\lambda}x) - D\cdot(\sqrt{\lambda})\cos(\sqrt{\lambda}x)$$

$$\Rightarrow X'(0) = -D(\sqrt{\lambda})\cos(0) = -D\sqrt{\lambda}$$

$$\Rightarrow -D\sqrt{\lambda} = 0$$

- $\sqrt{\lambda} = 0 \Rightarrow X(x) \equiv \text{constante}.$
$\Rightarrow X$ é da forma $C\cos(\sqrt{\lambda}x)$ com $\lambda = 0$

- $\lambda > 0 \Rightarrow D = 0$.

Assim asumimos $C \neq 0$ para não obter a solução trivial.

4

$\therefore X(x) = C\cos(\sqrt{\lambda}x)$.

Agora $X'(\ell) = C(\sqrt{\lambda})\operatorname{Sen}(\sqrt{\lambda}\ell) = 0$

$\Rightarrow \operatorname{Sen}(\sqrt{\lambda}\ell) = 0$ (pois $c \neq 0$, e $\sqrt{\lambda} \geq 0$ je o caso $\lambda = 0$ já foi feito)

$\Rightarrow \sqrt{\lambda}\ell = \pi K \quad K \in \mathbb{N}$

$\Rightarrow \lambda = \left(\frac{\pi K}{\ell}\right)^2$.

$\therefore \lambda_K = \left(\frac{\pi . K}{\ell}\right)^2, \quad K \in \mathbb{N}$

$X_K(x) = C\cos\left(\frac{\pi K}{\ell} . x\right), \quad K \in \mathbb{N}$.

*1.60.

a) $\boxed{\text{Ver página 41}}$. Tome $C = 1$

Temos $\phi(x) = \frac{a_0}{2} + \sum_{K=1}^{\infty} a_K \cos\left(\frac{K\pi x}{\ell}\right)$

- $\langle X_0, X_m \rangle = \int_0^\ell X_0(x) . X_m(x)\, dx$

1.58

$= \int_0^\ell \cos\left(\frac{\pi m x}{\ell}\right) dx = \begin{cases} \ell, & m = 0 \\ 0, & m \neq 0 \end{cases}$

- $\langle X_m, X_K \rangle = \int_0^\ell X_m(x) X_K(x)\, dx$

$= \int_0^\ell \cos\left(\frac{m\pi x}{\ell}\right)\cos\left(\frac{K\pi x}{\ell}\right) dx = \begin{cases} \frac{\ell}{2} & K = m > 0 \\ 0 & K \neq m \end{cases}$

1.58

fixo $m \geq 0$

Observe que $\phi(x) = \frac{a_0}{2} X_0(x) + \sum_{k=1}^{\infty} a_k X_k(x)$

$$\Rightarrow \langle \phi, X_m \rangle = \left\langle \frac{a_0}{2} X_0 + \sum_{k=1}^{\infty} a_k X_k, X_m \right\rangle$$

$$= \frac{a_0}{2} \langle X_0, X_m \rangle + \left\langle \sum_{k=1}^{\infty} a_k X_k, X_m \right\rangle \quad \text{(linealidad)}$$

$$= \frac{a_0}{2} \langle X_0, X_m \rangle + \sum_{k=1}^{\infty} a_k \langle X_k, X_m \rangle \quad \text{(linealidad e continuidad)}$$

$$\cancel{= \frac{a_0}{2} \langle X_0, X_m \rangle + a_m}$$

$$\cancel{m = 0}$$

$$\cancel{\Rightarrow \langle \phi, X_0 \rangle = a_0}$$

- $m = 0$

$$\Rightarrow \langle \phi, X_0 \rangle = \frac{a_0}{2} \langle X_0, X_m \rangle = \frac{a_0}{2} \cdot \ell$$

$$\Rightarrow a_0 = \frac{2}{\ell} \langle \phi, X_0 \rangle = \frac{2}{\ell} \int_0^{\ell} \phi(x)\, dx$$

- $m \neq 0$

$$\Rightarrow \langle \phi, X_m \rangle = a_m \langle X_m, X_m \rangle$$

$$= a_m \cdot \frac{\ell}{2} \Rightarrow a_m = \frac{2}{\ell} \langle \phi, X_m \rangle$$

$$= \frac{2}{\ell} \int_0^{\ell} \phi(x) \cos\left(\frac{m\pi x}{\ell}\right) dx.$$

6 b) $\boxed{\text{Ver proposição 1.56}}$

Prova é análoga.

* Criterio de M-Weierstrass:

Seja $\{f_n\}$ uma secuencia de funcoês. Suponha q) existem $\{M_n\}$ uma secuencia de constantes tq:

$$|f_n(x)| \leq M_n \quad \forall x \text{ no dominio. } \forall n \geq 1$$

Se $\sum_{n=1}^{\infty} M_n$ converge então $\sum_{n=1}^{\infty} f_n(x)$ converge absoluta e uniformemente.

* Ojo! $\phi'(0) = \phi'(\ell) = 0$.

- $$u(t,x) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos\left(\frac{k\pi x}{\ell}\right) \exp\left(-\frac{k^2\pi^2}{\ell^2}t\right)$$
- $$\partial_t u(t,x) = \sum_{k=1}^{\infty} a_k \left(\frac{-k^2\pi^2}{\ell^2}\right) \cos\left(\frac{k\pi x}{\ell}\right) \exp\left(\frac{-k^2\pi^2 t}{\ell^2}\right)$$

c) Objetivo: obter o kernel associado!

Sustituir $a_k$ en $u(t,x)$ e intercambiar Suma com integral (posso fazer pela convergencia uniforme).