Accommodating and extending various models for special effects within the Generalized Partially Confirmatory Factor Analysis (GPCFA) Framework

Yifan Zhang, Jinsong Chen

Introduction

- Special effects including **Method effects** and **testlet effects** are common issues in educational and psychological measurement.
- Existing models have lot limitations for special effects: Bifactor model, MTMM (multiple traits multiple methods) models, testlet models

Objectives

- Accommodating the GPCFA framework to special effects with added benefits:
  - Partially confirmatory knowledge
  - Local dependence
  - Mixed-type formats
  - Missingness
- Link to various Bifactor, MTMM and testlet effect models:
  - Standard Bifactor
  - CTCU (correlated trait correlated uniqueness)
  - CTUM (correlated trait uncorrelated method)
  - CTCM (correlated trait correlated method), CTC(M-1) (correlated trait correlated method model with one method less)
  - The general testlet model
  - 2PNO testlet model
  - Rasch testlet model
- Provide a subroutine to compute the equivalent effect size.

Theoretical Framework

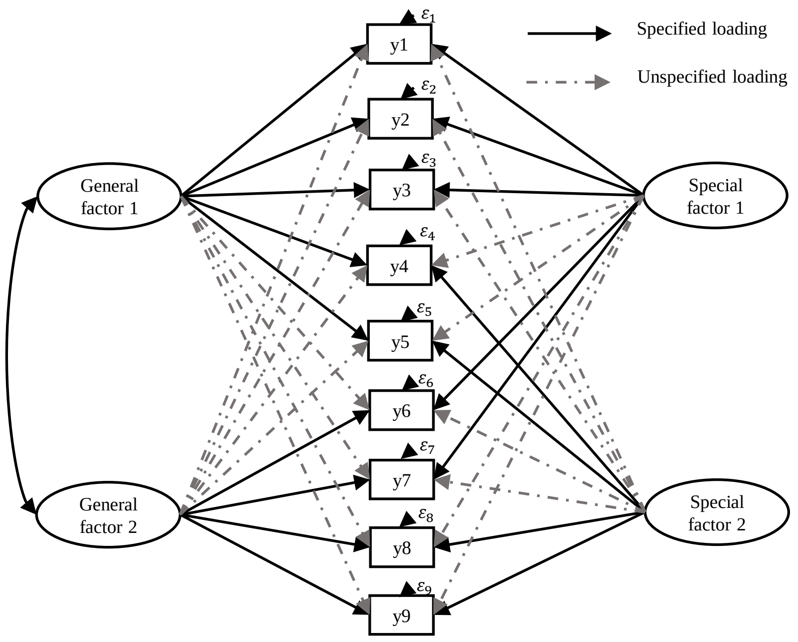
GPCFA for special effects:

$$\mathbf{Y} = \boldsymbol{\mu} + \boldsymbol{\Lambda}_g\mathbf{F}_\mathrm{g} + \boldsymbol{\Lambda}_\mathrm{s}\mathbf{F}_\mathrm{s} + \mathbf{E}$$

- $\mathbf{Y}$ is the observed variables
- $\boldsymbol{\mu}$ represents the $J \times 1$ intercept vector
- matrix $\boldsymbol{\Lambda}_\mathbf{g}$ $(\boldsymbol{\Lambda}_\mathbf{s})$ represents $J \times K_G$ $(J \times K_S)$ general (special) loading matrix
- $\mathbf{F}_\mathbf{g}$ $(\mathbf{F}_\mathbf{s})$ represents K_G (K_S) factors with the $K_G \times K_G$ $(K_S \times K_S)$ factorial covariance matrix $\boldsymbol{\Phi}$
- $\mathbf{E}$ represents the $J \times 1$ residuals with the $J \times J$ residual covariance matrix $\boldsymbol{\Psi}$

Effect Size:

$$\frac{\text{eigenvalue of special factors}}{\text{the number of indicators}}$$



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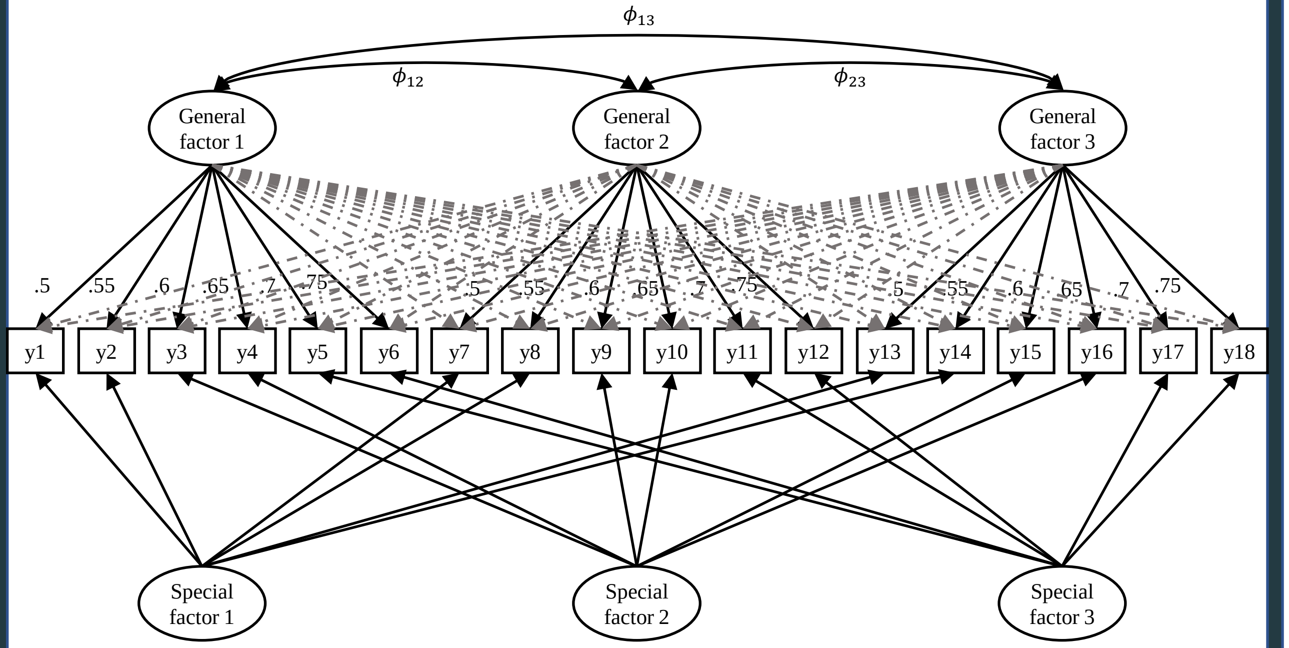
Simulation Study 1 Continuous variables

Data generation

- No. Replication = 200
- Sample size = 1000
- Interference $\psi_{13} = \psi_{31} = \psi_{23} = \psi_{32} = \psi_{46} = \psi_{64} = \psi_{56} = \psi_{65}$ =0.2



Simulation conditions

- Effect Size = 0.1 / 0.2
- Factorial correlation = .3 / .6

Results

		Effect Size =0.1							
		PHI=0.3				PHI=0.6			
Par	True	BIAS	RMSE	SE	SIG%	BIAS	RMSE	SE	SIG%
λ_{11}	0.5	0.018	0.045	0.095	0.995	0.006	0.044	0.146	1.000
λ_{12}	0.55	0.021	0.039	0.098	1.000	0.000	0.043	0.156	1.000
λ_{13}	0.6	0.039	0.053	0.101	1.000	0.033	0.054	0.152	1.000
λ_{14}	0.65	0.065	0.072	0.113	1.000	0.042	0.058	0.158	1.000
λ_{15}	0.7	-0.056	0.066	0.086	1.000	-0.104	0.112	0.122	1.000
λ_{16}	0.75	0.010	0.031	0.099	1.000	-0.031	0.048	0.140	1.000
λ_S	0.316	-0.022	0.101	0.122	0.650	-0.064	0.186	0.118	0.704
EF1	0.1	-0.075	0.075	0.052	1.000	-0.075	0.075	0.052	1.000
EF2	0.1	-0.076	0.076	0.065	1.000	-0.076	0.076	0.064	1.000
EF3	0.1	-0.073	0.073	0.073	1.000	-0.073	0.073	0.069	1.000
λ_0	0	0.010	0.031	0.097	0.000	0.025	0.042	0.131	0.000
ψ_{ij}	0.2	-0.066	0.073	0.082	0.274	-0.061	0.068	0.081	0.310
ψ_{ii}	0.650	-0.013	0.041	0.116	1.000	-0.013	0.039	0.115	1.000
$\phi_{oo'}$	0.3/0.6	-0.040	0.054	0.168	0.082	-0.088	0.094	0.196	0.968

		Effect Size =0.2							
		PHI=0.3				PHI=0.6			
Par	True	BIAS	RMSE	SE	SIG%	BIAS	RMSE	SE	SIG%
λ_{11}	0.5	0.022	0.054	0.104	0.990	0.000	0.069	0.179	0.920
λ_{12}	0.55	0.026	0.050	0.110	0.995	-0.002	0.064	0.191	0.925
λ_{13}	0.6	0.045	0.063	0.104	1.000	0.048	0.075	0.174	0.990
λ_{14}	0.65	0.220	0.224	0.095	1.000	0.197	0.203	0.138	0.995
λ_{15}	0.7	-0.100	0.107	0.073	1.000	-0.124	0.137	0.120	1.000
λ_{16}	0.75	0.054	0.064	0.078	1.000	0.033	0.062	0.122	1.000
λ_S	0.447	-0.025	0.102	0.122	0.632	-0.081	0.194	0.126	0.596
EF1	0.2	-0.158	0.158	0.083	1.000	-0.157	0.157	0.080	1.000
EF2	0.2	-0.167	0.168	0.192	1.000	-0.166	0.166	0.196	1.000
EF3	0.2	-0.168	0.168	0.128	1.000	-0.165	0.165	0.097	1.000
λ_0	0	0.007	0.060	0.131	0.000	0.042	0.089	0.194	0.000
ψ_{ij}	0.2	-0.102	0.110	0.050	0.398	-0.090	0.103	0.058	0.391
ψ_{ii}	0.550	-0.006	0.134	0.158	1.000	0.011	0.133	0.164	1.000
$\phi_{oo'}$	0.3/0.6	-0.025	0.079	0.210	0.158	-0.166	0.203	0.298	0.348

Note. λ_S = average of special factor loadings; EF: effect size; ψ_{ij} averaged across elements for $\psi_{13}, \psi_{23}, \psi_{46}, \psi_{56}$; ψ_{ii} averaged across elements for i = 1,7,13; for $\phi_{oo'}$, o and o'=1 to 3 and $o \neq o'$; RMSE: root mean square error; SE: standard error; SIG%: percent of estimates differed from zero significantly ($\alpha = .05$); the highlights are >0.1 for BIAS, RMSE or SE, and <0.9 for SIG%.

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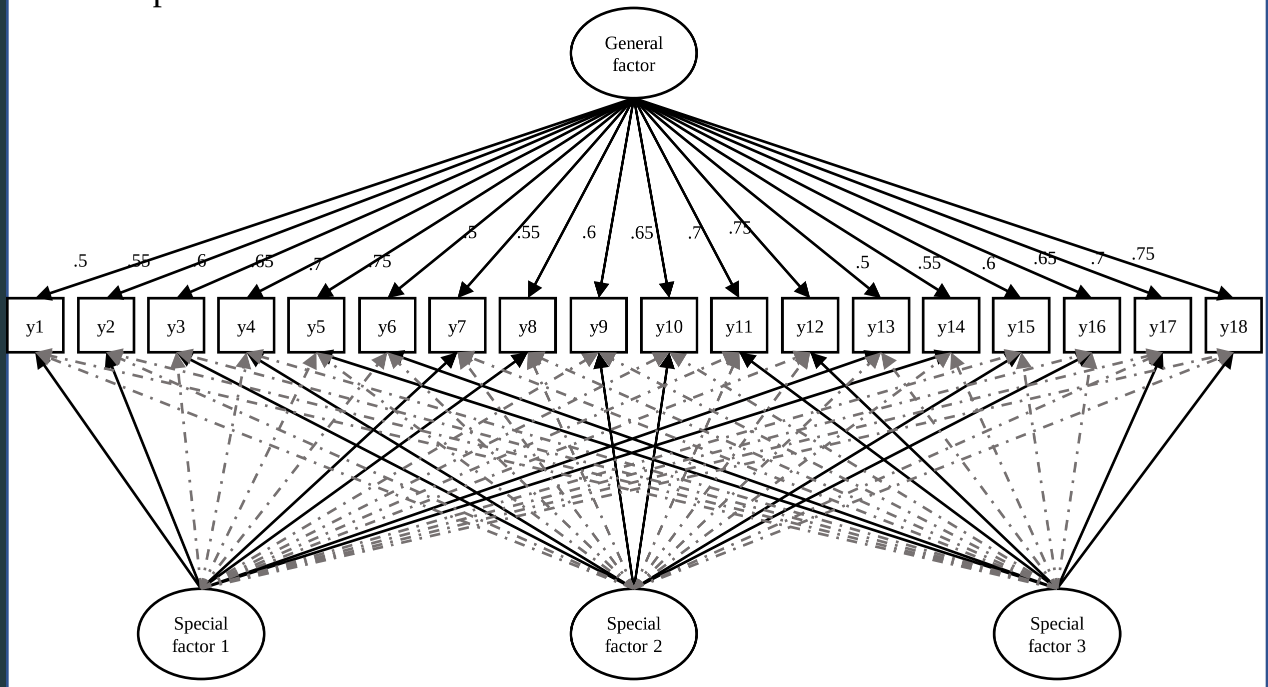
Simulation Study 2 Categorical variables

Data generation

- No. Replication = 200
- Sample size = 1000



Simulation conditions

- Effect Size = 0.1 / 0.2
- Number of categories = 2 / 4

Results

Par	True	Effect Size = 0.1 BIAS	RMSE	SE	SIG%	Effect Size = 0.2 BIAS	RMSE	SE	SIG%
		Number of categories = 2							
$\lambda_{\cdot 1}$	0.5	0.004	0.037	0.064	1.000	0.003	0.040	0.082	1.000
$\lambda_{\cdot 2}$	0.55	-0.002	0.035	0.063	1.000	-0.005	0.039	0.080	1.000
$\lambda_{\cdot 3}$	0.6	0.000	0.032	0.064	1.000	-0.007	0.037	0.083	1.000
$\lambda_{\cdot 4}$	0.65	-0.005	0.030	0.064	1.000	-0.014	0.038	0.083	1.000
$\lambda_{\cdot 5}$	0.7	0.004	0.029	0.064	1.000	0.021	0.041	0.088	1.000
$\lambda_{\cdot 6}$	0.75	0.001	0.027	0.064	1.000	0.014	0.037	0.088	1.000
EF1	0.1/0.2	-0.070	0.070	0.024	1.000	-0.146	0.146	0.049	1.000
EF2	0.1/0.2	-0.066	0.066	0.021	1.000	-0.139	0.139	0.041	1.000
EF3	0.1/0.2	-0.058	0.058	0.016	1.000	-0.122	0.122	0.016	1.000
λ_0	0	0.019	0.037	0.113	0.000	0.010	0.034	0.106	0.001
		Number of categories = 4							
$\lambda_{\cdot 1}$	0.5	0.001	0.030	0.060	1.000	-0.003	0.034	0.078	1.000
$\lambda_{\cdot 2}$	0.55	-0.005	0.029	0.059	1.000	-0.012	0.035	0.077	1.000
$\lambda_{\cdot 3}$	0.6	0.002	0.026	0.061	1.000	-0.006	0.031	0.080	1.000
$\lambda_{\cdot 4}$	0.65	-0.004	0.024	0.061	1.000	-0.013	0.031	0.081	1.000
$\lambda_{\cdot 5}$	0.7	0.002	0.023	0.063	1.000	0.012	0.032	0.086	1.000
$\lambda_{\cdot 6}$	0.75	-0.001	0.021	0.062	1.000	0.004	0.027	0.086	1.000
EF1	0.1/0.2	-0.070	0.070	0.023	1.000	-0.146	0.146	0.059	1.000
EF2	0.1/0.2	-0.063	0.063	0.017	1.000	-0.135	0.135	0.039	1.000
EF3	0.1/0.2	-0.056	0.056	0.013	1.000	-0.121	0.121	0.016	1.000
λ_0	0	0.018	0.032	0.099	0.000	0.013	0.029	0.094	0.000

Note. $\lambda_{\cdot i}$ = average of three parts of general factor loadings (e.g. $\lambda_{\cdot 1}$ present the average of $\lambda_{11}, \lambda_{17}, \lambda_{1,13}$); EF: effect size, value before '/' for effect size = 0.1, values after '/' for effect size = 0.2; RMSE: root mean square error; SE: standard error; SIG%: percent of estimates differed from zero significantly ($\alpha = .05$); the highlights are >0.1 for BIAS, RMSE or SE, and <0.9 for SIG%.

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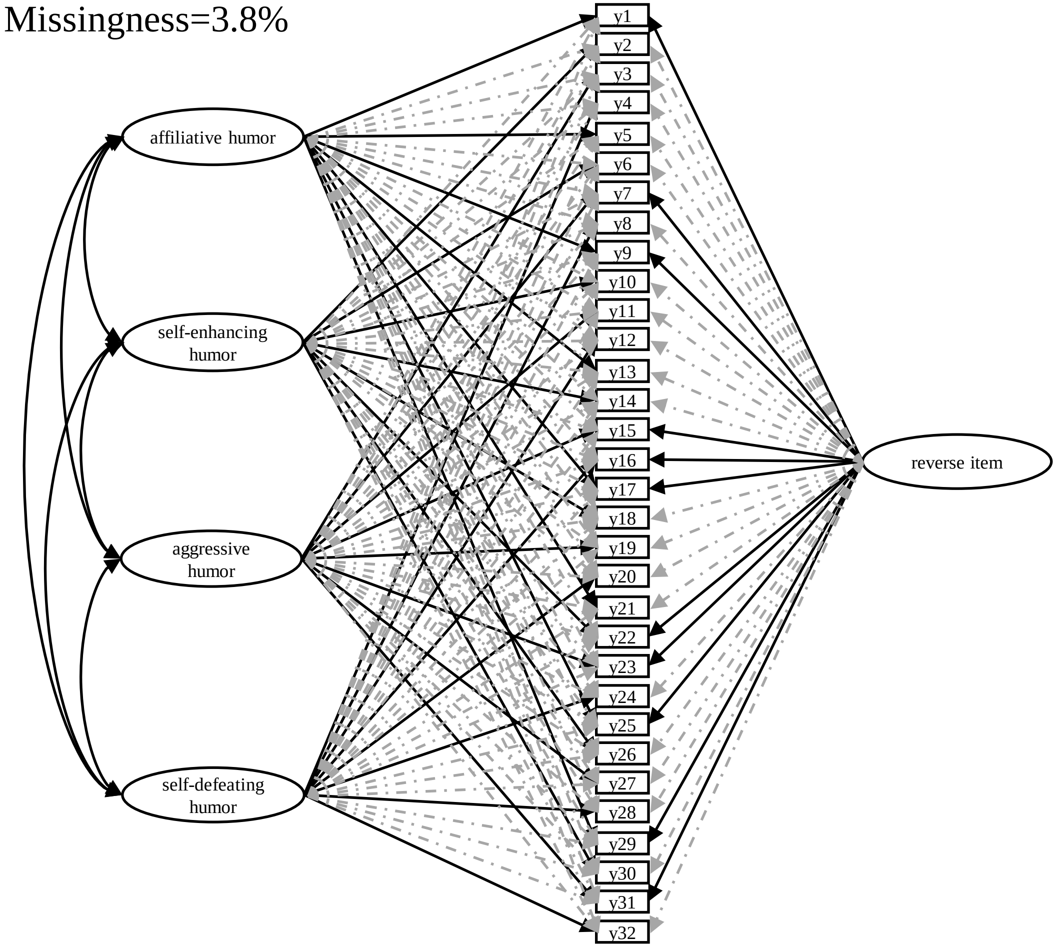
Empirical Study 3 Humor Styles Questionnaire

- N=1070
- Missingness=3.8%



Results

	Local Independent					Local Dependent						
Item	F1	F2	F3	F4	R	F1	F2	F3	F4	R	LD	effect
1	0.637				0.110	0.650				0.137	$\Psi_{24,4}$	0.098
2		0.636					0.620				$\Psi_{29,5}$	0.136
3			0.534					0.492		0.141	$\Psi_{30,6}$	0.247
4				0.634					0.626		$\Psi_{20,8}$	0.145
5	0.631					0.600					$\Psi_{18,10}$	0.113
6	0.219	0.383				0.236	0.350				$\Psi_{25,13}$	0.198
7			0.593					0.553		0.135	$\Psi_{21,17}$	0.078
8				0.792					0.729		$\Psi_{29,23}$	-0.082
9	0.456					0.438				0.155	$\Psi_{28,27}$	0.079
10		0.806					0.736					
11			0.551					0.551				
12				0.648					0.642			
13	0.584				0.221	0.600						
14		0.655					0.648					
15			0.676					0.571		0.275		
16				0.563					0.576	0.188		
17	0.738				0.119	0.742				0.163		
18		0.821					0.754					
19			0.449					0.456				
20				0.789					0.723			
21	0.658					0.689						
22		0.369					0.408			0.157		
23			0.490		0.121			0.412		0.243		
24	-0.171			0.482					0.478	-0.119		
25	0.662				0.265	0.652				0.167		
26		0.678					0.653					
27			0.548					0.519				
28		0.183	0.168	0.247				0.173	0.240			
29	0.580		0.185	-0.194		0.528			-0.190	0.173		
30		0.458					0.442					
31			0.686		0.109			0.595		0.256		
32				0.668					0.661			
ES					0.0482					0.0589		

Note. F1 = affiliative humor; F2 = self-enhancing humor; F3 = aggressive humor; F4 = self-defeating humor; R = reverse item; LD = local dependence (only significant terms were presented); ES: effect size; underscored in general factors (F1-F4) are cross-loadings; underscored in special factor (R) are specified items; only significant loadings at general factors and loading absolute value above 0.1 at special factors are presented.

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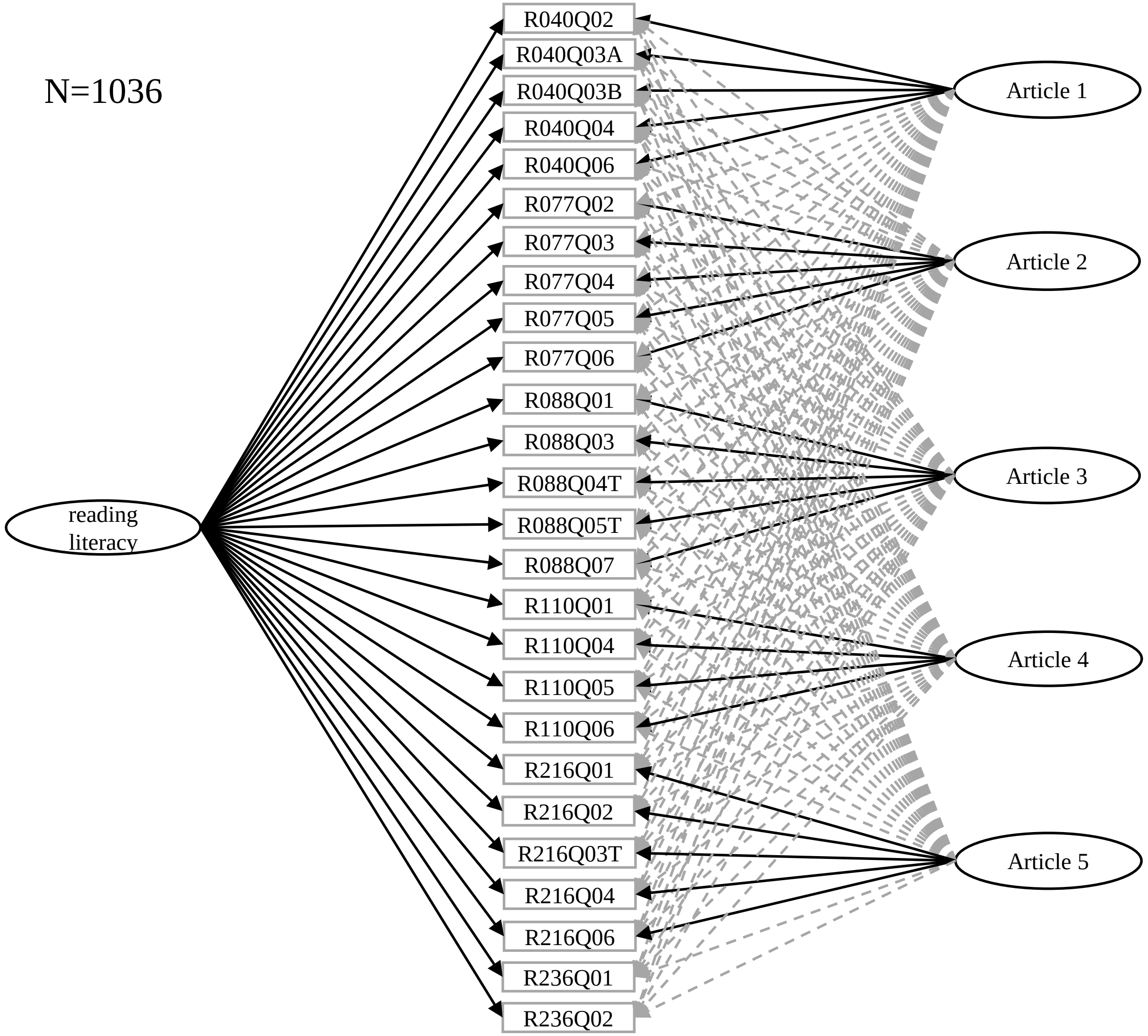
Empirical Study 4 PISA Reading Assessment (UK 2000)

- N=1036



Results

Item	Item code	G1	A1	A2	A3	A4	A5
1	R040Q02	0.541	0.343				
2	R040Q03A	0.625	0.407		0.103	-0.104	
3	R040Q03B	0.749	0.450				
4	R040Q04	0.604	0.336				
5	R040Q06	0.554	0.291				
6	R077Q02	0.658		0.130			
7	R077Q03	0.712		0.216			
8	R077Q04	0.558					
9	R077Q05	0.542		0.380			
10	R077Q06	0.519		0.260			
11	R088Q01	0.683			0.122		
12	R088Q03	0.681	0.117		0.121		
13	R088Q04T	0.648	0.111		0.191		
14	R088Q05T	0.639			0.280		
15	R088Q07	0.646			0.212		
16	R110Q01	0.648				0.168	
17	R110Q04	0.741				0.251	
18	R110Q05	0.767				0.249	
19	R110Q06	0.569				0.275	
20	R216Q01	0.649					0.280
21	R216Q02	0.804					
22	R216Q03T	0.829					0.105
23	R216Q04	0.751					0.306
24	R216Q06	0.600					0.479
25	R236Q01	0.695				0.125	
26	R236Q02	0.639					
ES	R040Q02		0.102	0.056	0.049	0.058	0.076

Note. G1= Reading literacy; A1-5 = 5 different articles; D1=Reverse item; D2=5-point; ES: effect size; underscored are specified loadings; only significant loadings at general factors and loading absolute value above 0.1 at special factors are presented.

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Summary

- Multiple general factors and special factors with different constraints on factorial correlation and residual
- Loading matrix, local dependence, mixed types of variables, missingness
- Regularization of loading structure
- Partially confirmatory structure

Recommendations for Practitioners

- effect size < 0.05 can be negligible
- small effect size (~.1) is ok, GPCFA will achieve good model estimation
- large effect size (~.2) might lead to overestimating for some parameters

Limitation

- Time-consuming
- Raw data are required

Further Plan

- Compare the performance of GPCFA with other generalized models
- Explore more large-scale empirical evidence