

Supplementary Material

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1 Ablation Study on dAoA Integration

To validate the contributions of the *Temporal-dAoA* and *Spatial-dAoA* modules, we conducted an ablation study comparing variants of the proposed framework.

1.1 Experimental Setup

The evaluated variants are:

- **Temporal-Only:** The framework with only the local KF tracking enabled for velocity constraints, disabling the WLS-based map correction.
- **Spatial-Only:** The framework with only the periodic WLS map correction enabled, disabling the temporal velocity constraints.
- **Proposed (Full):** The complete dAoA-SLAM system integrating both modules.

Results are averaged over Monte Carlo trials.

1.2 Quantitative Results

Table 1 presents the comparative results. In addition to average errors, we report the **Standard Deviation (Std)** of the RMSE to assess algorithm stability, and the **Final** errors to evaluate convergence.

Table 1: Quantitative Comparison of Ablation Variants

Method	Agent RMSE [m]		Map OSPA [m]		Time/Run /Epoch [ms]
	Avg \pm Std	Final	Avg	Final	
Temporal-Only	0.247 ± 0.233	0.429	0.909	0.571	36.98
Spatial-Only	0.257 ± 0.195	0.436	0.939	0.603	35.40
Proposed (Full)	0.225 ± 0.069	0.367	0.895	0.540	40.06

1.3 Analysis

The results demonstrate the necessity of integrating both modules:

Stability and Robustness: The most significant improvement lies in the stability of the system. While the *Temporal-Only* and *Spatial-Only* methods achieve reasonable average RMSE (0.247 m and 0.257 m), they exhibit high standard deviations (± 0.233 and ± 0.195), indicating susceptibility to measurement noise and occasional divergence. In contrast, the **Proposed (Full)** method dramatically reduces the standard deviation to ± 0.069 . This confirms that the coupling of temporal dynamics and spatial geometric constraints effectively suppresses outliers, ensuring robust performance across varying runs.

Accuracy and Convergence: The full framework outperforms the single-module variants in all accuracy metrics. It achieves the lowest Final RMSE (0.367 m) and Final OSPA (0.540 m). Although the computational cost increases slightly, the substantial gains in precision and reliability justify the integration.

2 Computational Complexity Analysis

This section compares the computational complexity of the baseline *Angle-SLAM* and the proposed *dAoA-SLAM*. Both methods utilize the **Rao-Blackwellized Particle Filter (RBPF)**.

2.1 Variable Definitions

To facilitate the analysis, we define the following variables representing the scale of the problem:

- N_p : Number of particles. This is the dominant factor in RBPF-based methods (e.g., $N_p = 15,000$).
- K_n : Number of map features (anchors) existing at time step n .
- M_n : Number of AoA measurements received at time step n .
- H : Size of the sliding history window of keyframes used for WLS optimization.
- D_x : Dimension of the agent state ($D_x = 4$ for 2D position and velocity).
- D_a : Dimension of the feature state ($D_a = 2$ for 2D position).

2.2 Complexity of Angle-SLAM (Baseline)

The baseline involves three steps per iteration. The complexity is dominated by data association:

1. **Particle Prediction:** Linear w.r.t particle count: $\mathcal{O}(N_p \cdot D_x^2) \approx \mathcal{O}(N_p)$.
2. **Data Association:** Evaluating likelihoods for all measurements vs. features: $\mathcal{O}(N_p \cdot M_n \cdot K_n)$.

3. **Map Update (EKF)**: Updating Gaussian beliefs: $\mathcal{O}(N_p \cdot M_n \cdot D_a^2) \approx \mathcal{O}(N_p \cdot M_n)$.

Total Complexity:

$$C_{Baseline} \approx \mathcal{O}(N_p \cdot M_n \cdot K_n) \quad (1)$$

2.3 Complexity of dAoA-SLAM (Proposed)

The proposed method adds two modules to the base RBPF ($C_{Base} \approx C_{Baseline}$):

Module 1: Temporal-dAoA (Local KF)

Maintains a low-dimensional Kalman Filter (2×2 state: angle and angular velocity) for each feature to extract the temporal dAoA.

- **Cost:** Constant time $\mathcal{O}(1)$ per feature per step.
- **Total Cost:** $\mathcal{O}(K_n)$ (pre-filtered). This adds a linear term relative to the map size.

Module 2: Spatial-dAoA (WLS Correction)

Performs WLS optimization using H historical frames. Solving the linear system $\hat{a} = (A^T W A)^{-1} A^T W b$. Constructing the 2×2 matrix $A^T W A$ involves traversing H history frames ($\mathcal{O}(H)$), and inversion is $\mathcal{O}(1)$.

- **Cost:** $\mathcal{O}(N_p \cdot K_n \cdot H)$ for constructing and solving the linear system.
- **Frequency:** Triggered **periodically** (e.g., every 10 steps). The amortized cost is negligible compared to the per-step data association.

Total Complexity: The total complexity sums the base RBPF and additional modules. Since the Spatial module is periodic, the asymptotic complexity remains:

$$\begin{aligned} C_{Proposed} &\approx C_{Base} + C_{Temporal} + C_{Spatial} \\ &\approx \mathcal{O}(N_p \cdot M_n \cdot K_n) + \mathcal{O}(N_p \cdot K_n) + \mathcal{O}(N_p \cdot K_n \cdot H)_{periodic} \\ &\approx \mathcal{O}(N_p \cdot M_n \cdot K_n) \end{aligned} \quad (2)$$

2.4 Conclusion

The analysis demonstrates that **dAoA-SLAM maintains the same asymptotic time complexity** as the baseline. The added modules introduce only linear, lightweight computations.

Table 2: Computational Complexity Comparison

Algorithm Component	Angle-SLAM (Baseline)	dAoA-SLAM (Proposed)	Remark
Main RBPF Loop	$\mathcal{O}(N_p \cdot M_n \cdot K_n)$	$\mathcal{O}(N_p \cdot M_n \cdot K_n)$	Dominated by particle count N_p
Temporal Module	N/A	$\mathcal{O}(N_p \cdot K_n)$	Lightweight 1D-KF
Spatial Module	N/A	$\mathcal{O}(N_p \cdot K_n \cdot H)$	Periodic, low amortized cost
Overall Complexity	$\mathcal{O}(N_p \cdot M_n \cdot K_n)$	$\mathcal{O}(N_p \cdot M_n \cdot K_n)$	Same Asymptotic Order

3 List of Symbols

Table 3: Summary of Notations and Definitions

Symbol	Definition / Description	First Context
1. Indices and Sets		
n	Discrete time step index ($n = 1, \dots, N$)	Sec. 2.1
j	Index of physical anchor ($j = 1, \dots, J$)	Sec. 2.1
k	Index of a legacy map feature (Virtual or Physical Anchor)	Sec. 2.1
m	Index of a measurement in the current scan ($m = 1, \dots, M_n^{(j)}$)	Sec. 2.1
i	Index of a keyframe in the sliding history window	Sec. 3.2.4
\mathcal{H}_k	Set of historical keyframes for feature k used in WLS optimization	Sec. 3.2.4
\mathcal{N}	Set of frames with reliable dAoA estimates	Sec. 3.2.3
2. System State and Map Variables		
\mathbf{x}_n	Agent state at time n , defined as $\mathbf{x}_n \triangleq [\mathbf{p}_n^T, \mathbf{v}_n^T]^T$	Eq. (1)
\mathbf{p}_n	Agent position vector $[p_{x,n}, p_{y,n}]^T$	Sec. 2.1
\mathbf{v}_n	Agent velocity vector $[v_{x,n}, v_{y,n}]^T$	Sec. 2.1
$\mathbf{a}_k^{(j)}$	Position of the k -th legacy feature associated with anchor j	Sec. 2.1
$\tilde{\mathbf{a}}_{m,n}^{(j)}$	Position candidate of a potential new feature derived from meas. m	Sec. 2.1
$K_n^{(j)}$	Number of legacy features for anchor j at time n	Sec. 2.1
$P_{k,n}^{(j)}$	Confidence score of feature k	Sec. 2.1
P_s	Probability of feature survival (persistence probability)	Sec. 2.1
3. Measurements and Data Association		
$M_n^{(j)}$	Total number of AoA measurements received from anchor j at time n	Sec. 2.1
$\theta_{m,n}^{(j)}$	The m -th Angle-of-Arrival (AoA) measurement	Sec. 2.1
$\omega_{k,n}^{(j)}$	dAoA (Differential AoA) : The angular velocity of feature k	Sec. 2.1
$c_{k,n}^{(j)}$	Feature-oriented association variable ($c_k = m$: feature k generates meas. m)	Sec. 2.2
$b_{m,n}^{(j)}$	Measurement-oriented association variable ($b_m = k$: meas. m from feature k)	Sec. 2.2
$\Psi_n^{(j)}$	Global decomposable exclusion constraints for data association	Eq. (2)
$\psi(\cdot)$	Indicator function ensuring DA consistency ($c_k = m \iff b_m = k$)	Eq. (2)
4. Algorithm and Belief Propagation (BP)		
Θ_n	Set of all latent variables at time n : $\{\mathbf{x}_n, \mathbf{a}_n, \tilde{\mathbf{a}}_n, \mathbf{c}_n, \mathbf{b}_n, \mathbf{z}_n\}$	Sec. 3.1
$g(\cdot)$	Factor potential for legacy features (incorporating detection prob. P_d)	Eq. (5)
$h(\cdot)$	Factor potential for new features/clutter (incorporating spatial prior)	Eq. (6)
P_d	Probability of detection	Eq. (5)
f_{FA}	Probability density of false alarms (clutter intensity)	Eq. (6)
α_n	Predicted belief of the agent state	Sec. 3.2.1

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Table 3 – continued from previous page

Symbol	Definition / Description	First Context
$\alpha_{k,n}^{(j)}$	Predicted belief of the feature state	Sec. 3.2.1
$\beta_{k,n}^{(j)}$	Message integrated from measurement m to feature k	Eq. (9)
$\xi_{m,n}^{(j)}$	Message integrated for new feature generation	Eq. (9)
$\nu_{m,k}^{(p)}$	BP Message from measurement node m to feature node k at iteration p	Eq. (10)
$\zeta_{k,m}^{(p)}$	BP Message from feature node k to measurement node m at iteration p	Eq. (10)
$\eta_{k,n}^{(j)}$	Final DA marginal probability for feature k (product of incoming messages)	Sec. 3.2.2
$\varsigma_{m,n}^{(j)}$	Final DA marginal probability for measurement m (product of incoming messages)	Sec. 3.2.2
5. Specific Modules (KF Tracking & WLS Optimization)		
$z_{k,n}^{(j)}$	Local Tracker State: $z \triangleq [\theta, \omega]^T$ (Pre-filtered AoA and dAoA)	Eq. (11)
H_{KF}	Measurement matrix for the local Kalman Filter [1, 0]	Sec. 2.1
\mathcal{L}_{dAoA}	Likelihood function derived from temporal dAoA constraints	Eq. (12)
ω_{pred}	Predicted angular velocity based on agent state and map	Eq. (13)
A_i	Linearized Jacobian-like matrix for WLS at keyframe i : $[\sin \theta_i, -\cos \theta_i]$	Eq. (15)
b_i	Target vector for WLS: $b_i = A_i p_i$	Eq. (15)
W_i	Weight matrix for WLS (inverse of variance σ_i^2)	Eq. (15)
q_{prop}	Proposal distribution generated by WLS for particle rejuvenation	Sec. 3.2.4