Solutions or Roots of Quadratic Equations

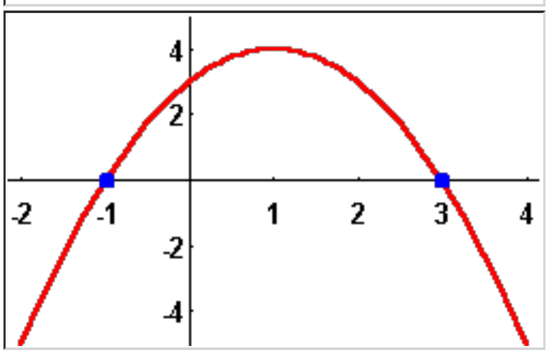
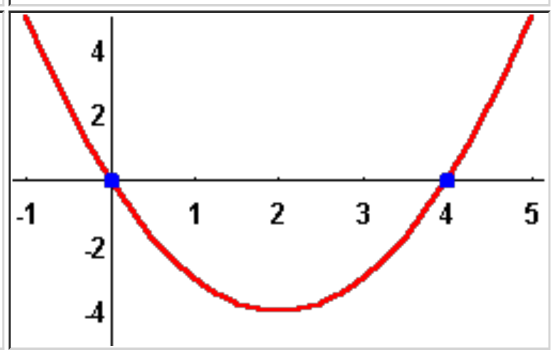
Consider the quadratic equation

$$ax^2 + bx + c = 0 .$$

A real number x will be called a solution or a root if it satisfies the equation, meaning $ax^2 + bx + c = 0$.

It is easy to see that the roots are exactly the x-intercepts of the quadratic function $f(x) = ax^2 + bx + c$,

that is the intersection between the graph of the quadratic function with the x-axis.

$a<0$	$a>0$
	

Example 1: Find the roots of the equation

$$x^2 - 1 = 0 .$$

Solution. This equation is equivalent to

$$x^2 = 1 .$$

Since 1 has two square-roots $\{1, -1\}$, the solutions for this equation are

$$x = 1 \text{ or } x = -1 .$$

Example 2: Find the roots of the equation

$$x^2 - 2x - 2 = 0 .$$

Solution. This example is somehow trickier than the previous one but we will see how to work it out in the general case. First note that we have

$$x^2 - 2x - 2 = x^2 - 2x + 1 - 3 = (x-1)^2 - 3 .$$

Therefore the equation is equivalent to

$$(x-1)^2 - 3 = 0$$

which is the same as

$$(x-1)^2 = 3 .$$

Since 3 has two square-roots $\{\sqrt{3}, -\sqrt{3}\}$, we get

$$x - 1 = \sqrt{3} \quad \text{or} \quad x - 1 = -\sqrt{3} ,$$

which give the solutions to the equation

$$x = 1 + \sqrt{3} \quad \text{or} \quad x = 1 - \sqrt{3} .$$

We may then wonder whether any quadratic equation may be reduced to the simplest ones described in the previous examples. The answer is somehow more complicated but it was known for a very longtime (to the Babylonians about 2000 B.C.). Their idea was based mainly on **completing the square** which we did in solving the second example.

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