A Math Forum Web Unit
Allan Adler's

What is a Magic Square?

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Definition, Special Properties

A magic square is an arrangement of the numbers from **1** to **n^2** (n-squared) in an **n**x**n** matrix, with each number occurring exactly once, and such that the sum of the entries of any row, any column, or any main diagonal is the same. It is not hard to show that this sum must be **n(n^2+1)/2**.

The simplest magic square is the **1x1** magic square whose only entry is the number **1**.

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The next simplest is the **3x3** magic square

8	1	6
3	5	7
4	9	2

and those derived from it by symmetries of the square. This **3x3** square is definitely magic and satisfies the definition given above.

The **4x4 Dürer magic square** (or, what is essentially the same, the 4x4 magic square I use) has many interesting special properties that are not shared by magic squares in general. They are so interesting that they are often pointed out when this square is presented. That is good, but can sometimes lead to misunderstandings as to which is the meat and which is the gravy. The meat is the definition I gave above. The gravy (or some of it), suggested by Jerome S. Meyer in his book, *Fun with Mathematics* , follows.

In the case of this **4x4** magic square:

1	15	14	4
12	6	7	9
8	10	11	5
13	3	2	16

in addition to having the sum 34 (= 4(4^2+1)/2) in each row, column and main diagonal,

1. The four corners add to 34.
2. The four numbers in the center add to 34.
3. The 15 and 14 in the top row and the 3 and 2 facing them in the bottom row add to 34.
4. The 12 and 8 in the first column and the 9 and 5 facing them in the last column add to 34.
5. The four squares in the corners add to 34.
6. If you go clockwise around the square and choose the first squares away from the corners (15,9,2,8), they add to 34. The same holds if you go counterclockwise.
7. If you replace each entry by its square, you get the following square:

1	225	196	16
144	36	49	81
64	100	121	25
169	9	4	256

This square is not magic, but it has some noteworthy properties:

1. The first and last column have the same sum; likewise the 2nd and 3rd columns have the same sum;
2. The same holds with rows instead of columns (although the sums one gets are different from the sums for the columns);
3. From (1) and (2), it follows that the left half of the square has the same sum as the right half and that this is one-half the sum of all of the numbers in the square. The same holds for the top half and the bottom half; hence the left half equals the right half equals the top half equals the bottom half. Furthermore, this also equals the sum of the 8 numbers on the diagonals and the sum of the 8 numbers off the diagonals.

8. If instead you replace each entry by its cube, you get the following square:

1	3375	2744	64
1728	216	343	729
512	1000	1331	125
2197	27	8	4096

This square has the property that the sum of the 8 numbers on the diagonals equals the sum of the 8 numbers off the diagonals.

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