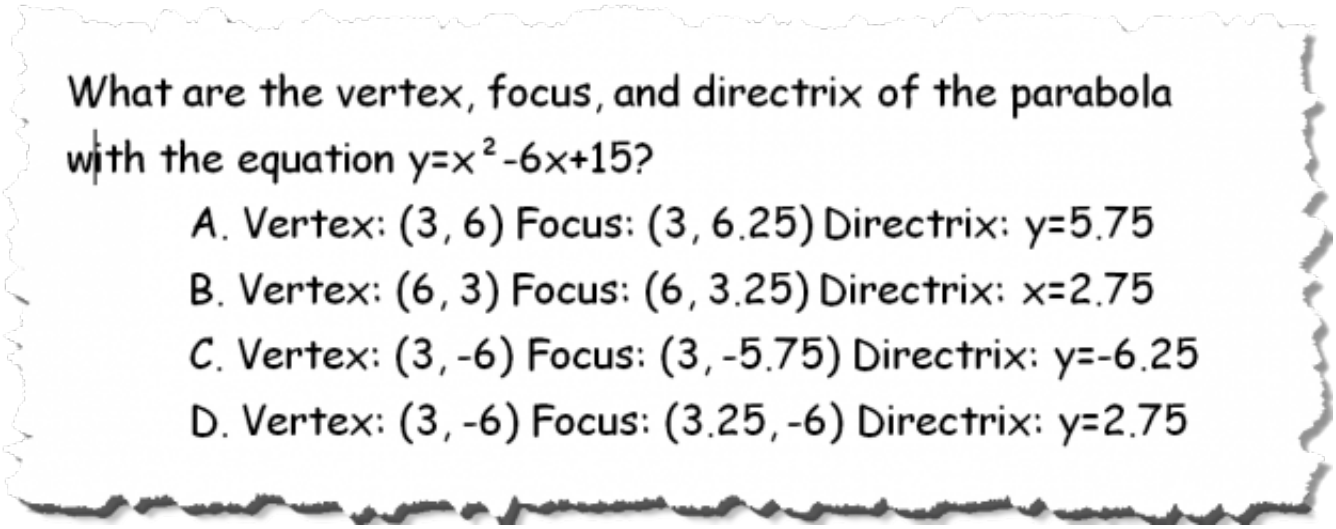
# Finding the Vertex, Focus, and Directrix of a Parabola Given in Standard Form

by Keith Roberts in Algebra, Homework Questions




Watch two things in these types of problems. First, the geometry of the situation — i.e. the orientation of the parabola and its various parts; and second, make sure your formulas are properly applied, as there are many k's, p's and a's floating around and are a lot to contend with.

**SEE ALSO: How to find the directrix of a parabola, given its vertex and focus.**

The geometry in this case is straight forward — we have a parabola that is opening upward in the plane. This is because we are quadratic in x (as opposed to y — see what to do here when it's not), and the leading coefficient a is positive. **See Figure A.**

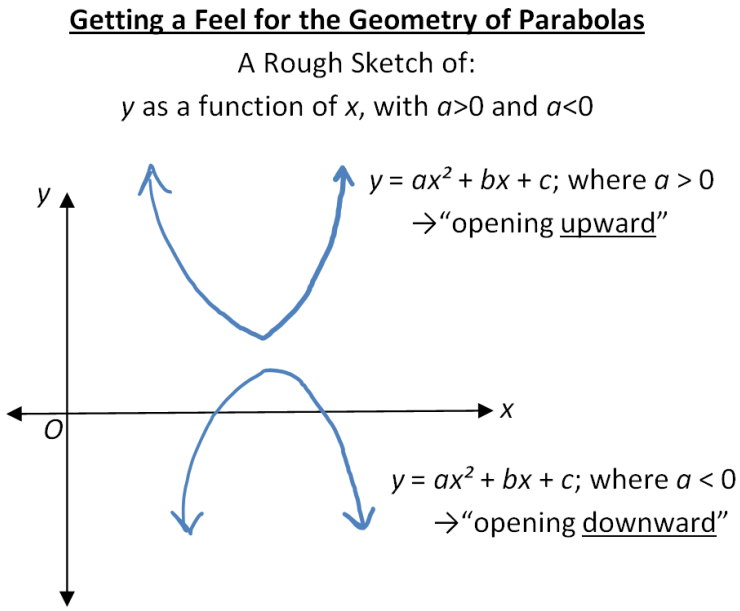



Figure A: Determining orientation of parabola based on value of $a$.

Because the parabola is opening upward, our focus will have the highest y-coordinate, the vertex will be south of the focus, and finally the directrix will be furthest south of them all.

With that in mind, to actually find the vertex, focus, and directrix, you can either transform your $y=x^2-6x+15$ into vertex form, or you can leave it, and use formulas for these from standard form. The consideration that I make when deciding which to use is whether the standard form is "easily" made into vertex form. When I look at $y=x^2-6x+15$, I do not see a quick way to turn it into vertex form through factoring (meaning we would have to complete the square). Thus, I would keep $y=x^2-6x+15$ in standard form and use formulas from there. To help us with our calculations, we've put together a quick reference "cheat sheet" in **Figure B.**

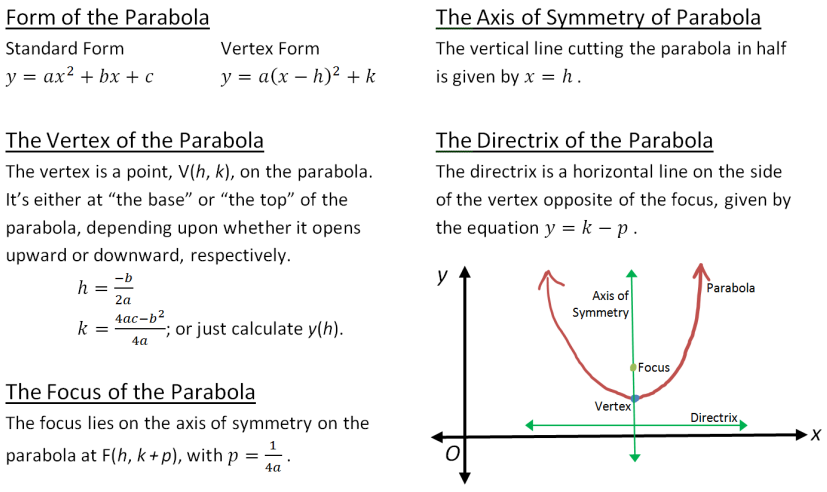



Figure B: Parabola cheat sheet for vertically oriented parabolas:

focus, vertex, axis of symmetry, and directrix.

We're also solving this problem in the context of multiple choice, which is great because the answer is somewhere already on the paper! It's even better because sometimes you won't have to derive the entire answer to rule out certain ones. (See the **Test Taking Tips** below, for some tips about how to use multiple choice answers to your advantage.)

**How to Find the Vertex**

First, we'll work on the vertex. The vertex x-coordinate is given by x=h=-b/(2a). Recall that when in standard form, the convention is to use coefficients a, b, and c as in $ax^2+bx+c$. Thus, a=1, b=-6, and c=15 in our example, and the x-coordinate of the focus is:

$x = -b / (2a) = – (-6) / (2*1) = 6/2 = 3.$

To find the y-coordinate of the vertex, k, we can either use the formula $(4ac-b^2)/(4a)$ in **Figure B** or plug the x-coordinate we found into $y=ax^2+bx+c$. We'll do the latter, using x=3 we just found:

$y = ax^2 + bx + c = 1*(3)^2 – 6\ (3) + 15 = 9 – 18 + 15 = 6.$

Thus, the vertex is (3,6).

> **TEST TAKING TIP #1:** Notice here that we have only one answer that has vertex (3,6), and that answer is A. Thus, at this point, I would double check my calculations very carefully (as always, but especially in this case, as we would be betting the house on it), and pick that answer. Then move on to the next question from there. I'll do this especially when the test is timed. We'll continue the rest of the calculations, however, to be sure you know how to do the rest.

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**How to Find the Focus**

The focus lies on the axis of symmetry of the parabola, and has the y-coordinate k+1/(4a). Because we just found the vertex to be (3,6), we know the axis of symmetry to be x=3, and the focus lies on that line. See **Figure B.** Its x-coordinate is therefore x=3. Because in our problem a = 1, the y-coordinate of the focus is 1/(4a) = 1/(4*1) = 1/4 north of the vertex. Thus, y = 6 + 1/4 = 6.25, and our focus is (3,6.25), which is what we were expecting from our selection of multiple choice answer A.

> **TEST TAKING TIP #2:** Note that we know that the vertex x-coordinate and focus x-coordinate are always the same. So even before we made any calculations in this problem, once we knew the parabola opened upward, we could rule out multiple choice answer D, as that option presents a vertex and focus with different x-coordinates.

## How to Find the Directrix

Finally, we can find the directrix of a parabola by noting that it will be a horizontal line and south of the vertex of the upward opening parabola, as we said above. Once again, see **Figure B.** Once you know the y=coordinate of the vertex, k, it is given by y = k – p, where p = 1/(4a). Thus, as we calculated for the focus, above:

p = 1/(4a) = 1/(4*1) = 1/4

and the directrix is y = 6 – 1/4 = 5.75, once again confirming our answer of multiple choice option A.

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Finding the Directrix of a Parabola, Given its Vertex and Focus

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**Brian Anderson** · 2 months ago

Find an equation in standard form of the parabola described.
Vertex at (–5, 8); directrix at y = 3
how do i get this standard form please?

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**The ProTutor Company** Mod Brian Anderson · 2 months ago

Hi Brian -- We start by visualizing the vertex at (-5, 8). We are given a directrix, and note that because it is y=something, the directrix is horizontal. Because the directrix is horizontal, we know the parabola opens either upward or downward. Since we are given that the vertex y-coordinate is 8, the directrix at y=3 is below the vertex. Parabolas always open the opposite direction of the directrix. Thus, we know the parabola opens upward.

Note that visualizing this helps inform our intuition about what is taking place geometrically. Unless you are also working with horizontal parabolas, you don't have to invest in this step. But it does give us a nice frame of reference as we make our calculations. It also gives us a picture of the end result, and facilitates a reality check, once we've calculated a solution. Calculations really begin with the next step.

Since we are given a vertex for a parabola we now know to be vertically oriented, we can use it to find the vertex form of the parabola:

y = a(x+5)^2 +8

Note that we are missing only the value "a" -- which we will obtain from the given

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**Kala** • 6 months ago

What about this standard form?

(x-h)^2=4p(y-k) Has vertical axis

(y-k)^2=4p(x-h) Has horizontal axis

I can't figure out how to find my directrix or focus. I found the vertex though.
(y+2)^2=(x-5) Vertex is (5,-2)

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**The ProTutor Company** Mod ↗ Kala • 6 months ago

Hi Kala -- When you are working with the forms

$(x-h)^2 = 4p(y-k)$ and
$(y-k)^2 = 4p(x-h)$

the same relationship, p = 1/(4a) applies.

This is because these relationships are the translated versions of

$X^2 = (4p)Y$ and
$Y^2 = (4p)X$

and if we rearrange each of these by dividing each side by 4p, we have

$[1/(4p)]X^2 = Y$ and
$[1/(4p)]Y^2 = X$.

The 1/(4p) is just the "a" we are used to seeing when the equations are written in the form

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**Tracy Smith** · 7 months ago

Vertex at origin, Directrix y equals -1/16 how do I write this in a standard form equation for a parabola

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**The ProTutor Company** Mod ➚ Tracy Smith · 7 months ago

Hi Tracy -- One way is to start by finding the vertex form of the parabola, and then convert it to standard form.

First, note that the directrix is y="something", and is a horizontal line, meaning that the parabola opens either upward or downward. Because the vertex is (0,0) and above the directrix y=-1/16, we know the parabola opens upward.

Because the parabola opens vertically, the vertex form for this parabola is

$y = a(x-h)^2 + k$,

where (h,k) is the vertex of the parabola. In this case, with vertex at the origin, (h,k) = (0,0). This gives us

$y=a(x-0)^2+0 = ax^2$.

This means the last piece of information we need to find is a.

For a vertically oriented parabola, the general directrix equation is y=k-p, where p = 1/(4a). In our case, the vertex is (h,k) = (0,0), so k=0. Then y=0-p=-p=-1/16, so -p=-1/16 or p=1/16. Then because p=1/(4a), 1/(4a)=1/16. When we solve this for a, we get a=4.

Thus our equation in vertex form, as well as in standard form (in this example) is $y=4x^2$. Keith

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**Tracy Smith** ➚ The ProTutor Company · 7 months ago

Where did the 4a come from. I guess I do not completely understand where you have p= 1/(4a)=1/16

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**The ProTutor Company** Mod → Tracy Smith • 7 months ago

It comes from the definition of p. p=1/(4a) is for every parabola. It gets its own letter because geometrically its absolute value gives the distance between the vertex and the focus, and the vertex and the directrix. Then the general directrix equation for a vertically oriented parabola is y=k-p, where p=1/(4a). I edited the previous comment to clarify this.

Basically the problem gives us the y=-1/16, which gives us p=1/16. Then the definition of p gives us the link between p and a that we need to get a, the last info we need to get our equation.

If the text you are using hasn't used p before, then you can always write the general equation of the directrix as just y=k-1/(4a). Remember k is the y-coordinate of the vertex (h,k), and is k=0 in our problem.

See Figure B from the main article, for a cheat sheet of parabola formulas to see an illustration of this.

see more

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**Seth may** • 7 months ago

How do you find the vertex given only the focus (-2,1) and the directrix x=-3 for a parabola?

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**The ProTutor Company** Mod → Seth may • 7 months ago

Hi Seth -- Since our directrix is vertical, we know the parabola opens either to the left or right. Because the directrix is x=-3, and the focus is at (-2,1), we know the focus of the parabola is to the right of the directrix. Thus, the parabola opens to the right.

Next, find the point on the directrix that has same y-coordinate as the focus. Notice the points on the directrix all have the form (-3,y). Thus the point on the directrix that has same y-coordinate as the focus is (-3,1).

Now, to find the vertex of the parabola, simply find the midpoint of this point and the focus. This should be (-5/2,1), if my arithmetic is correct. Keith

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**Seth may** ➚ The ProTutor Company • 7 months ago
Awesome
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**Vanessa Racquel** • a year ago
I'm trying to solve this word problem. I'm not given an equation, I have to figure it out myself. But it doesn't give me any information about a directrix or focus, so I don't know how to find a. The only information I received from the problem is the maximum height is 8 yds and it lands 50 yds away. The equation I have right now is y-8=-1/4p(x-25)^2. Please help.
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**The ProTutor Company** Mod ➚ Vanessa Racquel • a year ago
Hi Vanessa -- You're already so close! You've got the vertex form of the equation up to a, like you said. Now use the fact that our projectile lands 50 feet away. That means that x = 50 and y = 0. Plug these values into the equation you have, and solve for a (or p), and you will have it!
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**Vanessa Racquel** ➚ The ProTutor Company • a year ago
Thank you so much!
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**Dawn McCullough** • 2 years ago
If given a problem like (y-2)^=8(x+1), I know the vertex is (-1,2) but how do I go about finding the focus and the directrix? I know part of the focus contains the 2, like (a,2). How do I find a? and the directrix from here?
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**The ProTutor Company** Mod ➚ Dawn McCullough • a year ago
Hi Dawn -- If you already have the vertex, the key to finding the focus and directrix is to find the value of p. Once you have p, then you just add and subtract it from the vertex to find the focus and directrix. In your example, we are given the equation of the parabola in the form, $(y-2)^2 = 8(x+1)$. Since this is of the form $y^2$ = something, we know $8 = 4p$, or $p = 2$. This is because the general version of this form of the horizontal parabola is $y^2 = 4px$ (for the parent function), and $(y-k)^2 = 4p(x-h)$ (in the translated case).

Because $p > 0$, we know that the parabola opens to the right. So we add $p = 2$, to the x-coordinate of the vertex to get the focus. So the focus will have the x-coordinate $-1 + 2 = 1$. The focus coordinates, therefore, will be (1,2).

The directrix will then be a vertical line, to the left of the vertex: p units to the left, to be exact. So in this case, the directrix will be the line x = -1 - 2, or x = -3.

To check, we can use the focus and directrix we just found, and generate a couple of points that must lie on the parabola determined by these. Then we can test that those points we generated are indeed solutions to $(y-2)^2 = 8(x+1)$. Alternatively (and more rigorously), we can just use the directrix and focus we have found to derive the equation of the parabola from the definition. In the case of a directrix of x = -3 and a focus of (1,2),

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**The ProTutor Company** Mod • 2 years ago

Another really good question. We received this question on January 27, 2014, and again, had trouble getting the email out to a Yahoo! email address. Here is is:

**How do you find the equation of a parabola given only the focus, direction it opens, and a point on the parabola?**

Thanks for reaching out. Great question. These can be toughies. Basically what we want to do is start with the vertex form of the parabola equation, and use the information we are given to find the h, k, and a that are required to uniquely determine the equation. It's easiest to see with an example, so I'll come up with one and walk you through it.

OK, here we go. Suppose you have the focus F(2,3), and are given a point on the parabola is (3,5), and we are told the parabola opens upward. Here's how we'd find the equation of the parabola:

First, note that since we have the focus F(h,k+p) = F(2,3), this means we have h=2, and k+p=3. (See my parabola formula cheat sheet: http://www.protutorcompany.com... Note that the y-coordinate is k+p (as opposed to k-p) because the parabola is opening upward.

see more

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**The ProTutor Company** Mod • 2 years ago

On August 14, 2013, we received an interesting related question on graphing parabolas. Unfortunately, the Yahoo! email address we received wasn't working so we couldn't get this out to the actual asker of this question. Hopefully he'll see it here, and everyone can also benefit from seeing the discussion. As always, chime in if you have questions yourself, or have comments, or find that I've made a calculation error. I've been known to make them on occasion! Here it is:

**HOW DO I FIND THE CO-ORDINATES OF ITS FOCUS AND DIRECTION OF PARABOLA GIVEN THAT THE VERTEX IS AT (4,-8)?**

In this case, assuming you know that the parabola is vertically oriented, you're going to need another piece of information in order to uniquely determine the parabola, determining its opening direction, or determine its focus. To see this: If we think for a moment and imagine the coordinate plane, and then imagine the point (4,-8) in quadrant IV, we can also visualize at least two parabolas that have this point as a vertex. The parabola with this vertex could open upward; or, the parabola with this vertex could open downward. Consequently, the parabola could have a focus that is above the point (4,-8) or below the point (4,-8). And, with only the single requirement that this parabola have the vertex (4,-8), there is no limit as to how wide or narrow the parabola will open when it either goes upward or downward. So we're going to need some additional info, if the point (4,-8) is the vertex of the parabola. About the only thing we do know about this parabola is that if it's horizontally oriented, the axis of symmetry is given by x=4.

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**susmith v.v** • a month ago

How to find the equation of parabola whose vertex is (-2,-1) and focus is (-1,-1) ?

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**The ProTutor Company** Mod ↗ susmith v.v • a month ago

The first thing to note is that when we visualize these two points in the plane, they are located horizontally from one another. We observe that both points, V(-2,-1) and F(-1,-1), are in quadrant III, and the focus F(-1,-1) is to the right of the vertex V(-2,-1). Because the focus is to the right of the the vertex, we know the parabola opens to the right. Thus, we know the equation of our parabola, in vertex form, will be:

$x = a(y-k)^2 + h.$

From V(-2,-1), we have h = -2, k = -1, so:

$x = a(y+1)^2 - 2.$

Note, we need only to find "a", and we will have our equation for the parabola. To find it, we note that the focus is always a distance of 1/(4a) away from the vertex. In this case, because the parabola opens to the right, the focus is 1/(4a) units to the right of the vertex. Since the vertex x-coordinate, h, is -2, the focus x-coordinate must be -2 + 1/(4a), which we use in conjunction with the fact the focus x-coordinate is given to be -1. So we can find "a" by solving the equation:

$-2 + 1/(4a) = -1.$

When we do, we find a = 1/4.

Thus, our equation is x = (1/4)(y+1)^2 - 2.

We can then expand this to find standard form, if we need it:

x = (1/4)y^2 +(1/2)y - (7/4).

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