$$
{}^{0}T_4 = \begin{bmatrix} \cos\theta_1 \cos(\theta_2+\theta_3+\theta_4) & -\cos\theta_1 \sin(\theta_2+\theta_3+\theta_4) & \sin\theta_1 & 200(8\cos\theta_2 + 9\cos(\theta_2+\theta_3) + 5\cos(\theta_2+\theta_3+\theta_4))\cos\theta_1 \\ \sin\theta_1 \cos(\theta_2+\theta_3+\theta_4) & -\sin\theta_1 \sin(\theta_2+\theta_3+\theta_4) & \cancel{-\cos\theta} \; -\cos\theta_1 & 200(8\cos\theta_2 + 9\cos(\theta_2+\theta_3) + 5\cos(\theta_2+\theta_3+\theta_4))\sin\theta_1 \\ \sin(\theta_2+\theta_3+\theta_4) & \cos(\theta_2+\theta_3+\theta_4) & 0 & 1600\sin\theta_2 + 1800\sin(\theta_2+\theta_3) + 1000\sin(\theta_2+\theta_3+\theta_4) + 800 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{0}T_4 = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \left[\begin{matrix} \cancel{-\cos\theta_1} & \cancel{\cos\theta_1}\ 0 & \cancel{\sin\theta_1} \end{matrix} \right.
$$

$$\left(1600\cos\theta_2 + 1800\cos(\theta_2+\theta_3) + 1000\cos(\theta_2+\theta_3+\theta_4)\right)\cdot\cos\theta_1$$

$\cancel{l_2\cos\theta_2}$

$$\begin{cases} \left[l_2 c_2 + l_3\cos(\theta_2+\theta_3)\,\cancel{\theta}\right] c_1 = P_x & ① \\ \left[l_2 c_2 + l_3\cos(\theta_2+\theta_3)\right] s_1 = P_y & ② \\ l_2 s_2 + l_3\sin(\theta_2+\theta_3)\,\cancel{\theta} - l_4 + d_1 = \cancel{d_1}\, P_z & ③ \end{cases}$$

$$\Rightarrow \theta_1 = \begin{cases} \arctan\dfrac{P_y}{P_x}, & P_x \neq 0 \\ \pm 90^\circ, & P_x = 0 \end{cases}$$

①, ③ 整理 $\Longrightarrow$

$$l_2 c_2 = \frac{P_x}{c_1} - l_3\cos(\theta_2+\theta_3) = \frac{P_x}{c_1} - l_3 s_4 \quad ④$$

$$l_2 s_2 = P_z \,\cancel{+d}\, - d_1 + l_4 - l_3\sin(\theta_2+\theta_3) = P_z - d_1 + l_4 + l_3 c_4 \quad ⑤$$

$④^2 + ⑤^2$

$$\Rightarrow l_2^2 = \left(\frac{P_x}{c_1} - l_3 s_4\right)^2 + \left(P_z - d_1 + l_4 + l_3 c_4\right)^2$$

$$= \frac{P_x^2}{c_1^2} + l_3^2 \cancel{s} s_4^2 - 2\frac{P_x l_3}{c_1} s_4 + \left(P_z - d_1 + l_4\right)^2 + l_3^2 c_4^2 + 2l_3(P_z - d_1 + l_4) c_4$$

$$= \frac{P_x^2}{c_1^2} + l_3^2 \underbrace{- 2\frac{P_x l_3}{c_1}}_{a} s_4 + \underbrace{2l_3(P_z - d_1 + l_4)}_{b} c_4$$

$$\Rightarrow a s_4 + b c_4 = l_2^2 - l_3^2 - \frac{P_x^2}{c_1^2}$$

$\cancel{a \sin 4 + b}$ cos

$\cancel{a s_4}$

$$a s_4 + b c_4 = \begin{cases} \sqrt{a^2+b^2} \sin\left(x + \arctan\frac{b}{a}\right), & a > 0 \\ \sqrt{a^2+b^2} \sin\left(\arctan\left(-\frac{b}{a}\right) - x\right), & a < 0 \\ b \cos x, & a = 0 \end{cases}$$

$$\Longrightarrow l_2^2 - l_3^2 - \frac{P_x^2}{c_1^2} = \begin{cases} \sqrt{\frac{4P_x^2 l_3^2}{c_1^2} + 4l_3^2 (P_z - d_1 + l_4)^2} \cdot \sin\left(\theta_4 + \arctan\frac{-c_1(P_z - d_1 + l_4)}{P_x}\right) & (P_x < 0) \\ \sqrt{\frac{4P_x^2 l_3^2}{c_1^2} + 4l_3^2 (P_z - d_1 + l_4)^2} \cdot \sin\left(\arctan\frac{c_1(P_z - d_1 + l_4)}{P_x} - \theta_4\right) & (P_x > 0) \\ 2l_3 (P_z - d_1 + l_4) \cos\theta_4 & (P_x = 0) \end{cases}$$

$$\theta_4 = \begin{cases} \arcsin\left(\frac{l_2^2 - l_3^2 - \frac{P_x^2}{c_1^2}}{\sqrt{\frac{4P_x^2 l_3^2}{c_1^2} + 4l_3^2 (P_z - d_1 + l_4)^2}}\right) - \arctan\frac{-c_1(P_z - d_1 + l_4)}{P_x} & (P_x < 0) \\ \arctan\frac{c_1(P_z - d_1 + l_4)}{P_x} - \arcsin\frac{l_2^2 - l_3^2 - \frac{P_x^2}{c_1^2}}{\sqrt{\frac{4P_x^2 l_3^2}{c_1^2} + 4l_3^2 (P_z - d_1 + l_4)^2}} & (P_x > 0) \\ \pm \arccos\frac{l_2^2 - l_3^2 - \frac{P_x^2}{c_1^2}}{2l_3(P_z - d_1 + l_4)} & (P_x = 0) \end{cases}$$

$$\left.\begin{aligned} l_2 c_2 &= \frac{r_x}{c_1} - l_3 s_4 \\ l_2 s_2 &= r_z - d_1 + l_4 + l_3 c_4 \end{aligned}\right\} \Rightarrow \theta_4 \text{ 已算出，} \cancel{\theta_2 \text{ 可}}$$

$$\theta_2 = \arccos\left(\frac{r_x}{c_1 l_2} - \frac{l_3 s_4}{l_2}\right)$$

$$\cancel{\Rightarrow l_3 c_4 = l_2 s_2 - r_z + d_1 - l_4}$$

$$\theta_3 = 270^\circ - \theta_2 - \theta_4$$