Mechanics of Flexible Structures & Soft Robots
MAE-263F
Fall 2025

Homework 3

Due Date: 11:59 PM, 11/05/2025

You should create a single GitHub repository for this class and share it with the instructor (`khalidjm@seas.ucla.edu`). All the homeworks, reports, presentations, and proposal should be uploaded to this repository. Do not create a separate repository for each assignment. Within your repository, create a separate folder for each assignment (e.g., `homework_1`, `homework_2`, `homework_3`, `homework_4`, `homework_5`, `proposal`, `midterm_report`, and `final_report`).

Submission Instructions: Your submission on BruinLearn should only contain the URL to your GitHub repository. Your GitHub repository should include the following items:

1. **Report in PDF format:** Submit a report in `.pdf` format (file name should be `Homework1_LASTNAME.pdf`, replacing `LASTNAME` with your last name) addressing the questions asked in the deliverables. Include all the plots and figures requested in the assignment and discuss them in the report. See the syllabus for formatting requirements. As stated in the syllabus, you must use one of the provided templates.

2. **Source code:** The submission should include one main file named exactly as `HomeworkX.[ext]` (where `X` is the homework number) along with any additional files (e.g., functions or text files) as necessary. The main file should require no more than a single command to run or one click for execution.

3. **README file:** Add a `README.md` file on GitHub to provide clear instructions on how to run your code and describe the purpose of each file included in your repository.

Robotic Control of an Elastic Beam

In this homework, we will control the shape of a slender elastic beam with a planar robotic end-effector. The beam is modeled as a 2D mass–spring chain. You will impose time-varying Dirichlet boundary conditions at the right end (position and orientation) to drive a *middle* node along a prescribed trajectory.

Geometry & Material. Beam length $L = 1$ m. Circular tube with outer radius $R = 0.013$ m, inner radius $r = 0.011$ m. Young's modulus $E = 70$ GPa (aluminum), density $\rho = 2700$ kg/m^3. Area $A = \pi\,(R^2 - r^2)$. Area moment $I = \frac{\pi}{4}(R^4 - r^4)$.

Discretization. Let N be the number of nodes, indexed $1, \ldots, N$, with uniform spacing $\Delta L = L/(N-1)$. Each node carries a lumped mass

$$m \;=\; \frac{\rho\, A\, L}{N-1}.$$

Use your standard elastic energy for the springs between adjacent nodes (bending and stretching) consistent with the discussion in class.

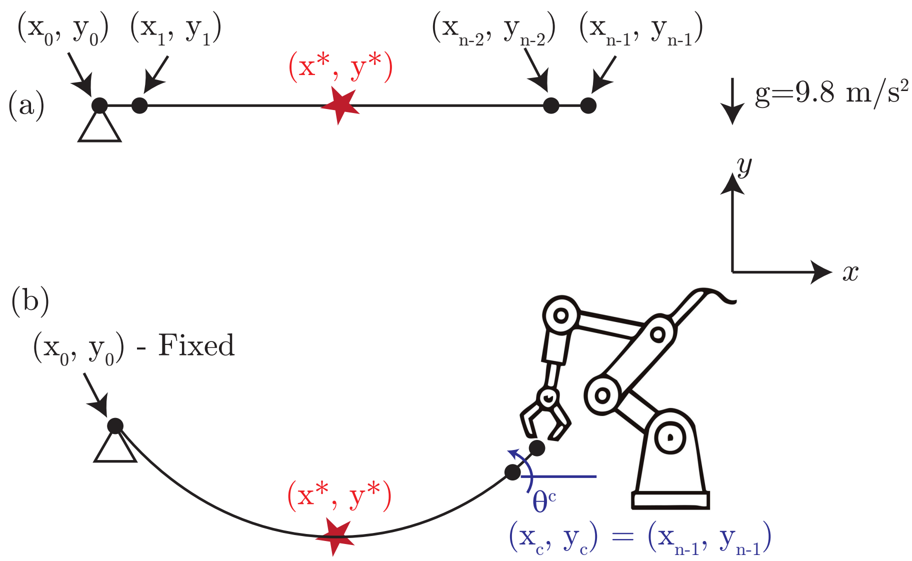

Figure 1: (a) Elastic beam and (b) its discrete mass–spring representation.

External Forces. Gravity acts on each node: $\mathbf{f}_g = [\,0,\,-mg\,]$ with $g = 9.81$ m/s^2.

Boundary Conditions. The left end is clamped at the origin:

$$x_1(t_{k+1}) = 0, \qquad y_1(t_{k+1}) = 0.$$

The right end is controlled by a planar robot via time-varying commands $x_c(t), y_c(t), \theta_c(t)$ with initial values $x_c(0) = L,\ y_c(0) = 0,\ \theta_c(0) = 0$. Enforce the following Dirichlet constraints on the *last two* nodes:

$$x_N(t_{k+1}) = x_c(t_{k+1}), \qquad y_N(t_{k+1}) = y_c(t_{k+1}),$$
$$x_{N-1}(t_{k+1}) = x_c(t_{k+1}) - \Delta L\,\cos\big(\theta_c(t_{k+1})\big), \qquad y_{N-1}(t_{k+1}) = y_c(t_{k+1}) - \Delta L\,\sin\big(\theta_c(t_{k+1})\big).$$

Target Node & Trajectory. Let the "middle" node be $* = \frac{N+1}{2}$ (e.g., $* = 10$ when $N = 19$; pick N odd for a unique middle). The desired trajectory of this middle node, denoted as (x^*, y^*), for $t \in [0, 1000]$ (in seconds) is

$$x^*(t) \;=\; \frac{L}{2}\cos\big(\tfrac{\pi}{2}\,\tfrac{t}{1000}\big), \qquad y^*(t) \;=\; -\,\frac{L}{2}\,\sin\big(\tfrac{\pi}{2}\,\tfrac{t}{1000}\big).$$

Task. Design a path planner that produces control inputs $\{x_c(t), y_c(t), \theta_c(t)\}$ so that the middle node tracks the prescribed $(x^*(t), y^*(t))$ under gravity.

Deliverables.

- A clear step-by-step description of your method, including pseudocode/algorithms for: (i) force/energy evaluation, (ii) time stepping, (iii) enforcement of Dirichlet constraints, and (iv) the path-planning or control law mapping the tracking error of node j to $\{x_c, y_c, \theta_c\}$.
- Plots of the control inputs $x_c(t), y_c(t), \theta_c(t)$ over time.
- At least five snapshots of the beam shape (node positions) during the motion.

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- A brief discussion of feasibility limits due to the robot's workspace and joint limits; explain how you handle infeasible commands (e.g., saturation or trajectory re-timing).

Notes. Multiple solutions exist. In practice, the robot's reachable workspace and rate limits restrict $\{x_c, y_c, \theta_c\}$; your implementation should respect these constraints.

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