Tabletop-game-Finans

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1 Introduction

The tabletop-game *Finans* is a Scandinavian tabletop-game that resembles the widely known game *Monopoly* a lot. The goal of the game is to have the highest total value of assets at the end of the game. This is done by buying and selling grounds, building houses and buying bonds. In this article, I will briefly explain the process of calculating the best strategies to win.

2 The game Finans

The game *Finans* resembles Monopoly a lot. The main difference between the to games, is the amount of choices a player is faced. In Finans, players are faced with more choices, thus, making the game more complex.

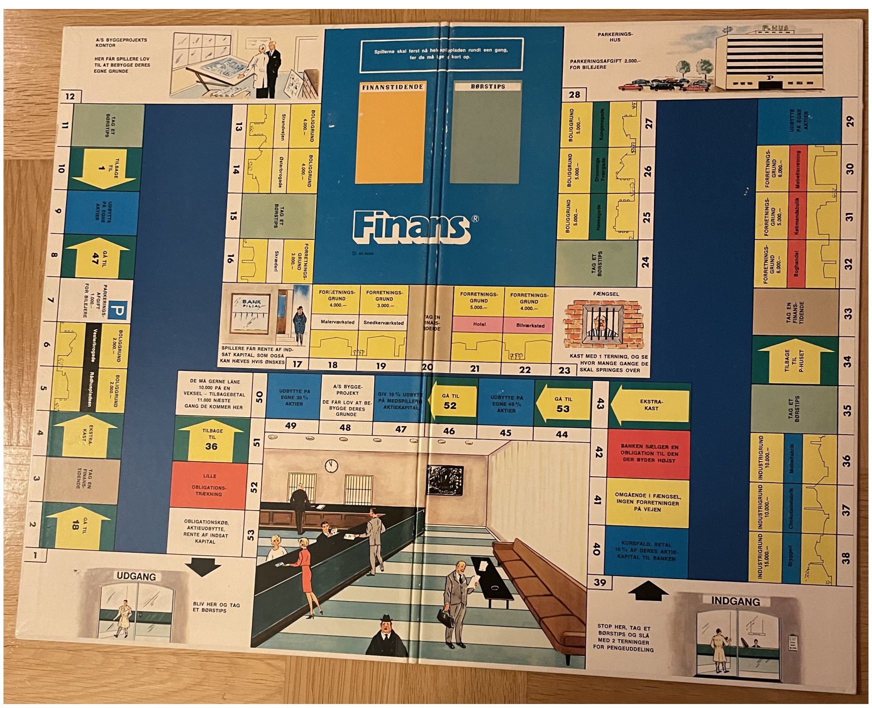

Figure 1: Picture of the game map.

2.1 Rules

For more detailed explanation of rules I will refer to the file: *rules.pdf*. If the rules does not interest you in the slightest and you simply want to beat your friends and family in Finans, simply jump to Results and discussion. Two main things will be covered in this article:

- Which bonds to buy
- Which grounds to buy

Normally all player travel around the board using only one die, but players have the option to purchase a car, if they do so, they will throw two dies instead of one each round. For the sake of simplifying calculations, we will assume no players have cars. In addition to that, it is also beneficial to not have a car while driving through the bank, as each round when you are inside the bank, you can purchase grounds. Walking or driving slowly through the bank gives you more opportunities to purchase ground.

2.1.1 Rules: Bond-raffle

Bonds can be bought and sold, although they do not resemble real-life bonds. There are bonds ranging from 1 to 12. There is a small and a big *bond-raffle*. They both have three prizepools. The bond-raffle consists of throwing to dies. The biggest prizepool is given to the player with the bond of the same number of the sum of the two dies, the second prizepool is given to the player with the bond of the same number as the biggest die, and the smallest prizepool is given to the player with the bond of the same number as the smallest die.

2.1.2 Rules: Grounds and houses

Players can buy grounds which will yield a cash flow whenever other players land on that ground. After having bought the ground, one can buy a house on the ground to increase the amount players have to pay when landing on it.

3 Theory

The game of Finans can be modelled as a Markov Chain. We are interested in the long-term distribution of where players will be, ie. the limiting distribution. We analyze the Markov Chain of the game where players move from one state to another, and we see that the Markov Chain satisfies:

- Irreducible
- Aperiodic
- Finite state space

It's trivial that we have irreducibility, since any state can be reached from any other state after a random walk. We will not rigorously prove Aperiodicity, but intuitively the Markov Chain is aperiodic due to the multitude of ways one could return to the same state. Lastly it is trivial that the state space is finite, since each square on the map represents a state. This implies that the Markov Chain is ergodic and we can use the Fundamental Limit Theorem for Markov Chains:

Let $X_0, X_1, \ldots$ be an ergodic Markov chain. There exists a unique, positive, stationary distribution π, which is the limiting distribution of the chain. That is,

$$\pi_j = \lim_{n\to\infty} P_{ij}^n \quad \text{for all } i, j.$$

The unique stationary distribution can be computed by solving the following equation:

$$\pi P = \pi$$

Since the state space consists of 53 states, we use R to numerically compute the unique stationary distribution.

After having calculated the limiting distribution, we choose to only look at some elements of the limiting distribution, since some of the squares in the game simply map to another square. For example, square 8: *"Go to square 48"*, thus, this element in the limiting distribution would not be relevant for us.

4 Results and Discussion

4.1 Bond-raffle

By using conditioned expected value we get the following barplot of which bonds are most worth buying, and how much they are worth:

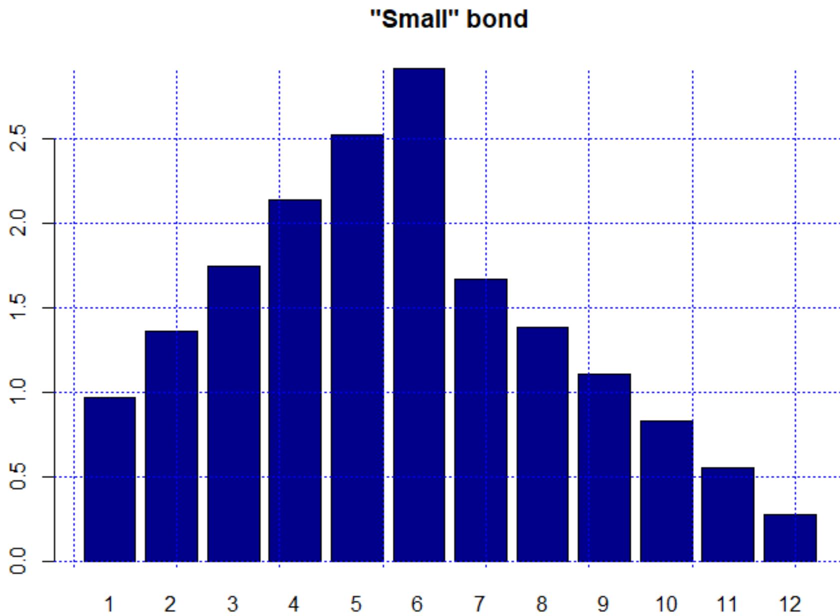

Figure 2: Expected return of small bonds.

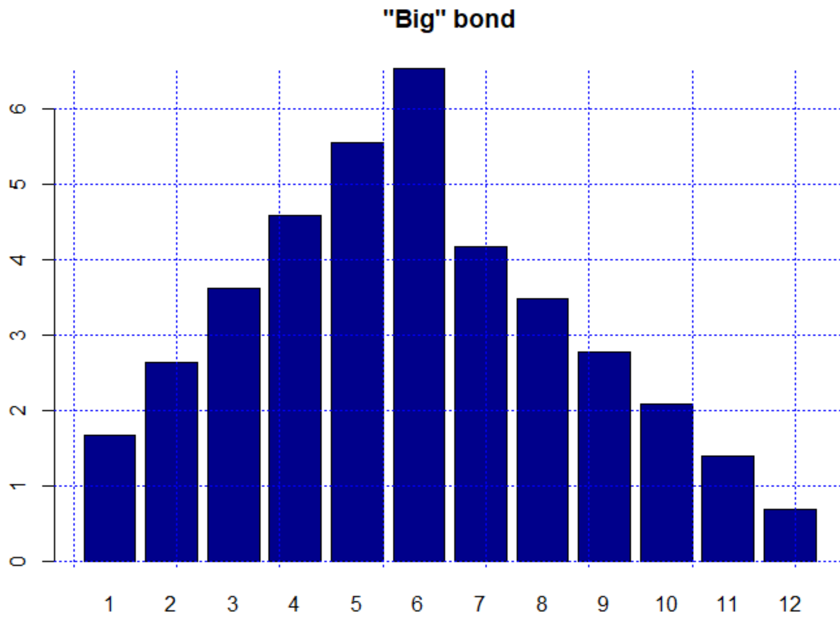

Figure 3: Expected return of big bonds.

As we can see, bond number 6 is optimal in both the small and big bond raffle.

4.2 Grounds and houses

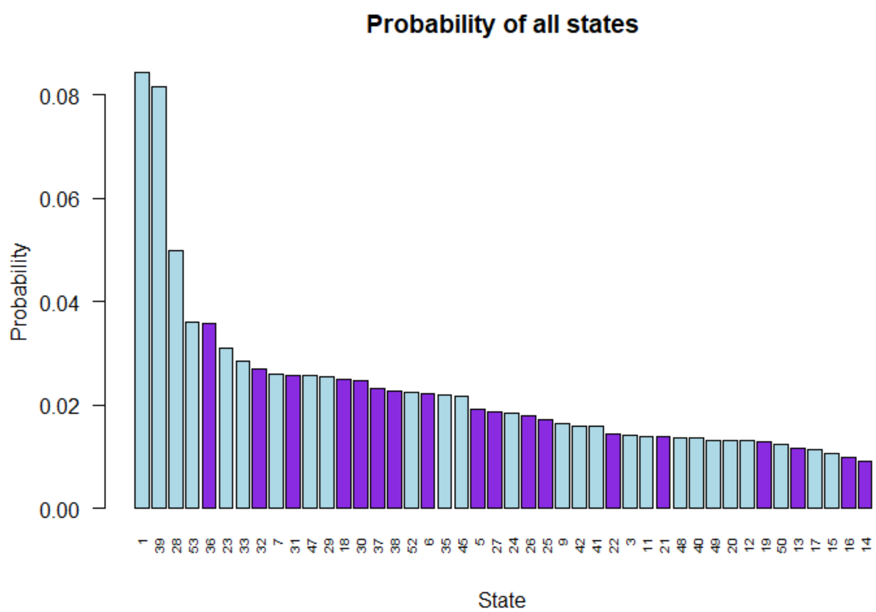

Figure 4: Probability of being in a given state over time. Purple: States with properties. Blue: States with no properties.

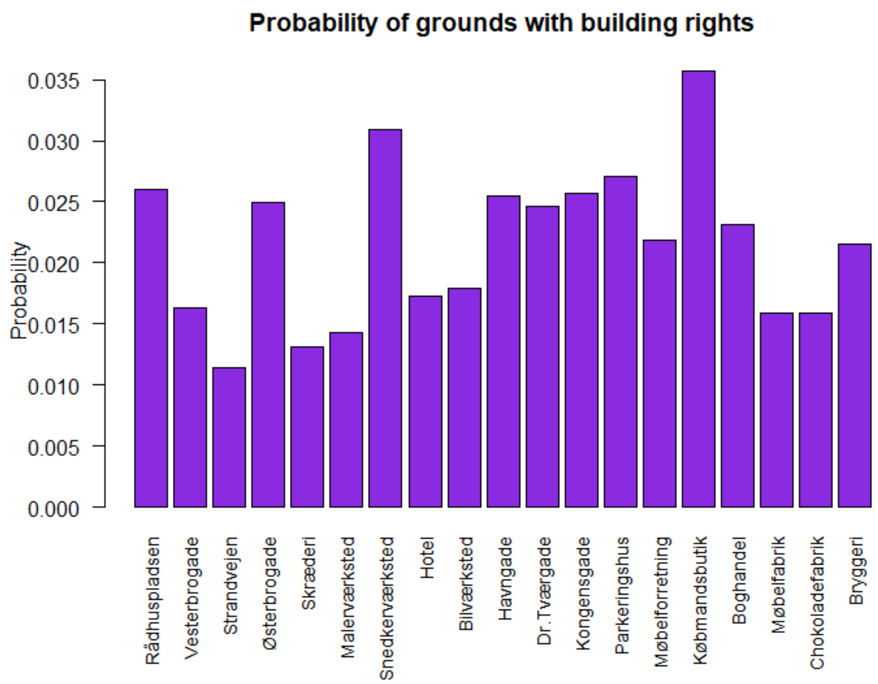

Figure 5: Long-term probability of purchasable states.

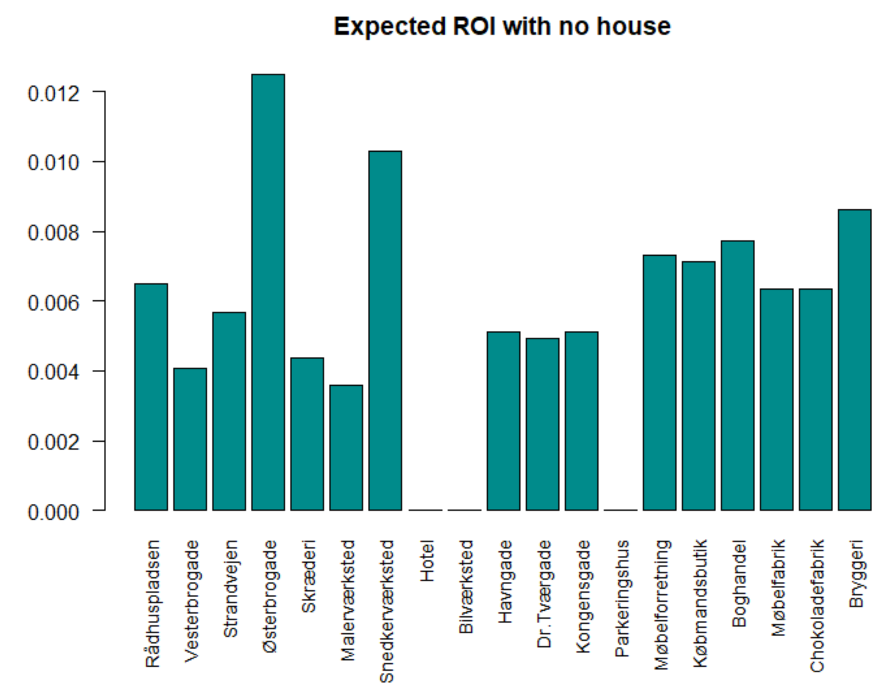

Figure 6: Expected ROI assuming no house is built on the ground.

Figure 4.2 shows expected ROI assuming no house is built on the ground. Expected ROI for state (i) is simply calculated by the following formula:

$$E_i[ROI\ no\ house] = p_i * \frac{Return_i}{Cost\ of\ ground_i} \tag{1}$$

The value of the Expected ROI doesn't have any usefull interpretation alone in this example, but it becomes useful when comparing different properties against each other.

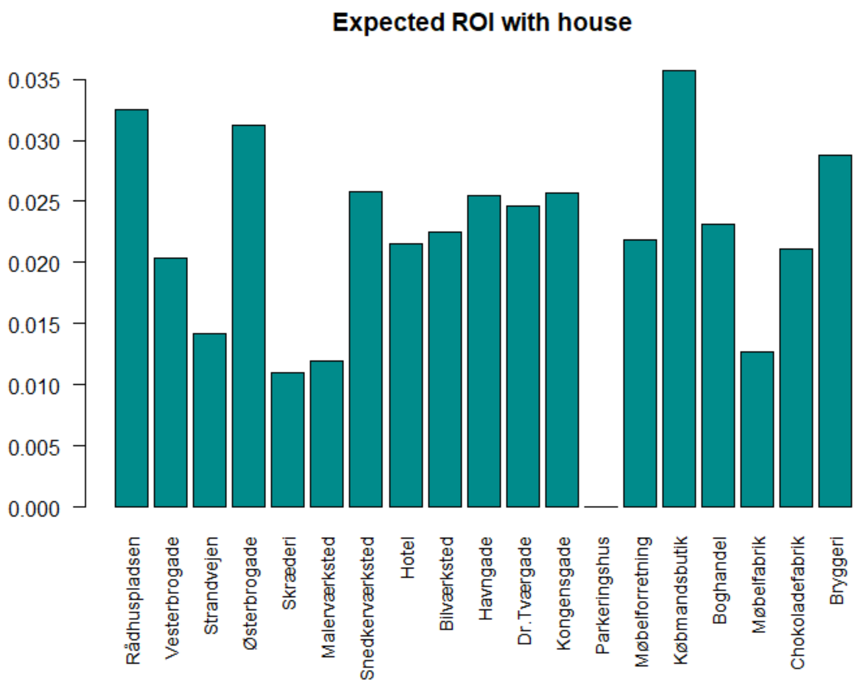

Figure 7: Expected ROI assuming having built house.

$$E_i[ROI\ with\ house] = p_i * \frac{Return\ with\ house_i}{Cost\ of\ ground_i\ +\ Cost\ of\ house_i} \tag{2}$$

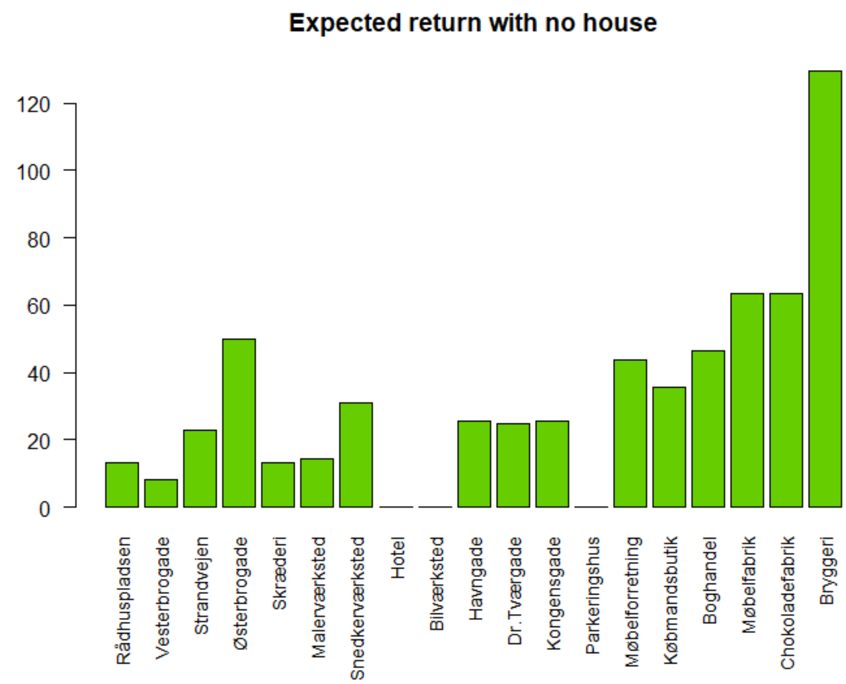

Figure 8: Expected Return assuming no house is built.

The above plot does not take into consideration how much the ground costs. Although ROI might seem to be a better estimate to whether or not a player should buy a property, only looking at the return might actually be more useful. At the end of the game, all assets are evaluated, grounds and houses are evaluated at the same price at which they are bought. Thus, buying a property does not affect your net asset value. Likewise for Expected return, the lone value of expected return is not useful alone, but becomes useful when compared with other states.

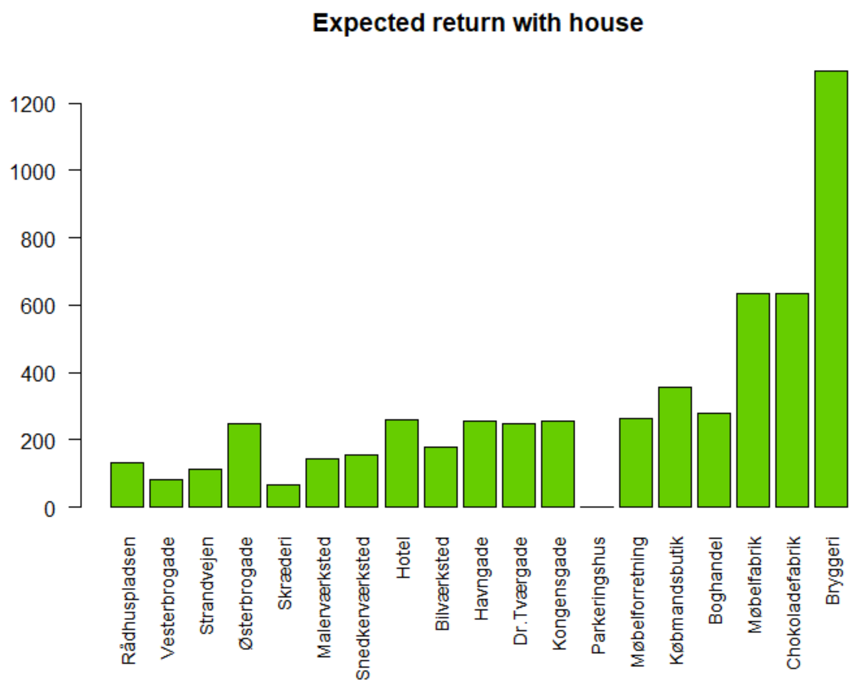

Figure 9: Expected Return assuming house is built.

The above plot shows the expected return after building a house on the ground.

The best strategy for a player would probably be to try to buy grounds and then prioritizing building houses above buying new grounds. By comparing Figure 4.2 and Figure 4.2, we see that a much better ROI is achieved by purchasing a house.