# Machine Intelligence 1

Assignment 1

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## 1.7

### (a)

The difference between supervised and unsupervised learning boils down to having labels for the data or not. For every data-point, supervised learning provides a label that associates the point with a class. These classes may be binary but can also be many. As a result, supervised Algorithms do classification in most cases (like SVM's and such).

Unsupervised learning does not have labels for its data. Algorithms need to find some structure in the data to make assumptions/decisions. Many of these do dimensionality reduction to find latent variables.

Reinfocement learning however has ratings instead of labels. An algorithm of this kind chooses certain actions based on observations. If an action is applied, a rating is provided to let the algorithm lern if its action was good or bad. This type of learning is often seen in agent-environments.

### (b)

**To teach a dog to catch a ball**

*reinforcement learning*

A dog observes its environment (**data**) and performs certain tasks (**actions**). If the dog does good, it may be rewarded with a treat (**positive rating**) or if he does bad, yelling (**negative rating**)

**To read hand written addresses from letters**

*supervised learning*

Ths is a typical application of supervised algorithms. The adresses are usually split up to the character level. Here, the real character (**label**) that is encoded as an image of the handwritten character (**data**) is known. Therefor a classification can be learned.

**To identify groups of users with the same taste of music**

*unsupervised learning*

In this scenario we have to find the labels that are not given. just a collection of **data** is given that hopefully contains some not very noisy data about users, the music they listend to and maybe other preferences or user interactions.

MI 1 Blatt 1

1.4
$$A = \begin{pmatrix} 5 & 8 & 16 \\ 4 & 1 & 5 \\ -4 & -4 & -11 \end{pmatrix}$$

$$\det A = (5\cdot(-11)) + (4\cdot(-4)\cdot 16) + (-4\cdot 8\cdot 8) - [(-4\cdot 1\cdot 16) + (4\cdot 8\cdot(-11)) + (5\cdot(-4)\cdot 8)]$$

$$= -55 - 256 - 256 - [-64 - 352 - 160]$$

$$= -567 + 576$$

$$= \underline{9}$$

1.1
$$p(x) := \begin{cases} c\cdot\sin(x), & x\in[0,\pi] \\ 0, & \text{else} \end{cases}$$

a) $$1 \stackrel{!}{=} \int_{-\infty}^{\infty} p(x)\,dx = \int_0^\pi c\cdot\sin(x)\,dx = c\int_0^\pi \sin(x)\,dx$$

$$= c\,[\,-\cos(\pi) - (-\cos(0))]$$

$$= c\,[\,-(-1) - (-1)]$$

$$= 2c$$

$$\Rightarrow 2c \stackrel{!}{=} 1$$

$$\underline{\underline{c = \tfrac{1}{2}}}$$

b) $$\langle X\rangle_p := \int_{\mathbb{R}} x\,p(x)\,dx = \int_0^\pi \frac{x}{2}\sin(x)\,dx$$

$$= \frac{1}{2}\int_0^\pi \overset{g}{x}\,\overset{f'}{\sin(x)}\,dx \overset{\text{partiell}}{=} \frac{1}{2}\left([-\cos(x)\,x]_0^\pi - \int_0^\pi -\cos(x)\cdot 1\,dx\right)$$

$$= \frac{1}{2}\left(\pi + \int_0^\pi \cos(x)\,dx\right) = \frac{1}{2}\left(\pi + [\sin(x)]_0^\pi\right) = \frac{\pi}{2}$$

c) $\mathrm{Var}(X) \overset{\text{Verschiebungssatz}}{=} \langle X^2\rangle_p - \langle X\rangle_p^2$

$$\langle X^2\rangle_p = \int_{\mathbb{R}} x^2 \underbrace{\tfrac{1}{2}\sin(x)}_{p(x)}\,dx = \frac{1}{2}\int_0^{\pi} \underset{g}{x^2}\, \underset{f'}{\sin(x)}\,dx$$

$$= \frac{1}{2}\left([-x^2\cos(x)]_0^{\pi} - \int_0^{\pi} \underset{g}{2x}\,\underset{f'}{(-\cos(x))}\,dx\right) = \frac{1}{2}\left(\pi^2 + [-2x\sin(x)]_0^{\pi} - \int_0^{\pi} 2(-\sin(x))\,dx\right)$$

$$= \frac{1}{2}\left(\pi^2 + 0 + 2\int_0^{\pi}\sin(x)\,dx\right) = \frac{\pi^2}{2} + \int_0^{\pi}\sin(x)\,dx$$

$$= \frac{\pi^2}{2} + [-\cos(x)]_0^{\pi} = \frac{\pi^2}{2} + [-\cos(\pi) - (-\cos(0))]$$

$$= \frac{\pi^2}{2} + [-(-1) - (-(1))] = \frac{\pi^2}{2} + 2$$

$$\Rightarrow \mathrm{Var}(X) = \frac{\pi^2}{2} + 2 - \frac{\pi}{2}^2$$

1.2

$$p(z) = \begin{cases} 3/7\,(2-x)(x+y), & x\in[0,2],\ y\in[0,1] \\ 0 & ,\ \text{else} \end{cases}$$

a) $$p_x(x) = \int_{\mathbb{R}} p(\cancel{x}y)\,dy = \frac{3}{7}\int_0^1 (2-x)(x+y)\,dy$$

$$= \frac{3}{7}(2-x)\int_0^1 x+y\,dy = \underline{\frac{3}{7}(2-x)\left(x+\frac{1}{2}\right)}$$

$$p_y(y) = \frac{3}{7}\int_0^2 \underset{f'}{(2-x)}\underset{g}{(x+y)}\,dx = \left[\frac{3}{7}\left([(2x-x^2)(x+y)]_0^2 - \int_0^2 (2x-x^2)(1+0)\,dx\right)\right.$$

$$= \frac{3}{7}\left(\underbrace{(2\cdot 2-2^2)(2+y)-0}_{0}\right) - \int_0^2 (2x-x^2\,dx = \frac{3}{7}\left([x^2]_0^2 - \left[\frac{1}{3}x^3\right]_0^2\right)$$

$$\left.= \frac{3}{7}\left(4 - \frac{8}{3}\right)\right]$$

$$= \frac{3}{7}\int_0^2 -x^2 + (2-y)x + 2y\,dx = \frac{3}{7}\left(\left[-\frac{x^3}{3}\right]_0^2 + \left[(2-y)\frac{x^2}{2}\right]_0^2 + [2yx]_0^2\right)$$

$$= \frac{3}{7}\left(-\frac{8}{3} + (2-y)2 + 4y\right) = \underline{\underline{\frac{3}{7}\left(-\frac{8}{3} + 4 + 2y\right)}}$$

1.3 $f(x) = \sqrt{1+x}$ $\quad x_0 = 0$

$$T_f(x, x_0) = \sum_{i=0}^{n} \frac{f^{(i)}(x_0)}{i!}(x-x_0)^i$$

$$f' = \frac{1}{2\sqrt{x+1}}$$

$$f'' = -\frac{1}{4(x+1)^{3/2}}$$

$$f''' = \frac{3}{8(x+1)^{5/2}}$$

$$T_f(x, x_0) = \frac{\sqrt{1+0}}{0!}(x-0)^0 + \frac{\frac{1}{2\sqrt{0+1}}}{1!}(x-0)^1 + \frac{\frac{1}{4(0+1)^{3/2}}}{2!}(x-0)^2$$

$$+ \frac{\frac{3}{8(0+1)^{5/2}}}{3!}(x-0)^3$$

$$= \underline{\underline{1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3}}$$

1.5

$f(x,y) = c + x^2 + y^2$

$g(x,y) = c + x^2 - y^2$

$f_{xx} = 2$ , $f_{xy} = 0$

$f_{yx} = 0$ , $f_{yy} = 2$

$f_x = 2x$ , $f_y = 2y$ $g_{xx} = 2$ , $g_{xy} = 0$

$g_x = 2x$ , $g_y = -2y$ $g_{yx} = 0$ , $g_{yy} = -2$

a)

$a = (0,0)$ is a critical point iff $grad(f(a)) = 0$

$$grad(f(a)) = \begin{pmatrix} f_x(a) \\ f_y(a) \end{pmatrix} = \begin{pmatrix} 2x \\ 2y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$grad(g(a)) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$\Rightarrow$ a is a critical point for both functions

b)

$$H_f = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

The Eigenvalues of a Triangular Matrix are the elements of the main diagonal

$\Rightarrow ev_1 = 2 = ev_2$

$\Rightarrow H_f$ is positive definite $\Rightarrow$ f has a local Minimum at a

$$H_g = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix} \Rightarrow ev_1 = 2 , ev_2 = -2$$

$\Rightarrow H_g$ is not definite $\Rightarrow$ g has a saddle point at a

16

$P(D) = 0{,}01 \qquad P(\bar{D}) = 0{,}99$

$P(+|D) = 0{,}95 \qquad P(-|D) = 0{,}05$

$P(+|\bar{D}) = 0{,}001 \qquad P(-|\bar{D}) = 0{,}999$

$P(+) = P(+|D)\,P(D) + P(+|\bar{D})\,P(\bar{D}) = 0{,}95 \cdot 0{,}01 + 0{,}001 \cdot 0{,}99$
$= 0{,}01049$

- $P(D|+) = \dfrac{P(+|D)\,P(D)}{P(+)} = \dfrac{0{,}95 \cdot 0{,}01}{0{,}01049} = \underline{\underline{0{,}9056}}$

$\Rightarrow P(\bar{D}|+) = 1 - 0{,}9056 = \underline{\underline{0{,}0944}}$

~~• P(D̄|−) =~~

$P(-) = P(-|D)\,P(D) + P(-|\bar{D})\,P(\bar{D}) = 0{,}999 \cdot 0{,}99 + 0{,}05 \cdot 0{,}01$
$= 0{,}98951$

$P(\bar{D}|-) = \dfrac{P(-|\bar{D})\,P(\bar{D})}{P(-)} = \dfrac{0{,}999 \cdot 0{,}99}{0{,}98951} = \underline{\underline{0{,}9995}}$

$\Rightarrow P(D|-) = 1 - 0{,}9995 = \underline{\underline{0{,}0005}}$