



THEORETICAL PHYSICS

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# Quantum Field Theory

## SI2410

## Fall 2014

## Preparation Questions

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### 1 SEMINAR 1: FUNCTIONAL INTEGRALS AND INTRODUCTION TO RENORMALIZATION

#### 1.1 WHAT IS THE RELATIONSHIP BETWEEN THE FUNCTIONAL INTEGRAL FORMALISM AND THE N-POINT CORRELATION FUNCTION?

General generating functional is given by

$$Z[J] = \int D\phi \exp \left[ i \int d^4x [L + J(x)\phi(x)] \right] \quad (1.1)$$

where  $L$  is the Lagrangian density and  $J(x)\phi(x)$  is a source term.

The n-point correlation function is then given by

$$\langle 0 | T \phi(x_1) \dots \phi(x_n) | 0 \rangle = \frac{1}{Z_0} \prod_{i=1}^n \left( -i \frac{\delta}{\delta J(x_i)} Z[J] \right) \quad (1.2)$$

where  $Z_0 = Z[J=0]$ . This yields

$$\langle 0|T\phi(x_1)\dots\phi(x_n)|0\rangle = \frac{\int D\phi \phi(x_1)\dots\phi(x_n) \exp\left[i \int d^4x L\right]}{\int D\phi \exp\left[i \int d^4x L\right]} \quad (1.3)$$

## 1.2 GIVEN A LAGRANGIAN DENSITY, HOW CAN THE FEYNMAN RULES OF A THEORY BE COMPUTED USING FUNCTIONAL INTEGRAL FORMALISM?

The Feynman rules can be computed by Taylor expanding

$$\exp[iS] = \exp\left[i \int d^4x L_0 + L_{int}\right] \quad (1.4)$$

where  $S$  is the action,  $L_0$  is the free Lagrangian density and  $L_{int}$  is the interaction Lagrangian density. In the case of  $\lambda\phi^4$ -theory we have the Lagrangian

$$L = L_0 - \frac{\lambda}{4!}\phi^4 \quad (1.5)$$

and by Taylor expansion

$$\exp iS = \exp\left[i \int d^4x L_0 - \frac{\lambda}{4!}\phi^4\right] = \exp\left[i \int d^4x L_0\right] \left(1 - i \int d^4x \frac{\lambda}{4!}\phi^4 + \dots\right) \quad (1.6)$$

where  $L_0$  gives the propagators and all terms of order higher than two yields interaction. In this case we read off the vertex factor in momentum space as

$$-i\lambda(2\pi)^4\delta^4(\sum p) \quad (1.7)$$

## 1.3 WHAT COMPLICATIONS ARISE WHEN USING THE FUNCTIONAL INTEGRAL FORMALISM TO QUANTIZE THE ELECTROMAGNETIC FIELD? HOW IS IT SOLVED?

In QED the functional integral,

$$\int DA e^{iS[A]} \quad (1.8)$$

is badly divergent as a finite number of terms will have  $S[A] = 0$  due to gauge invariance,

$$A_\mu \rightarrow A_\mu + \frac{1}{e}\partial_\mu\alpha(x) \quad (1.9)$$

and thus  $A_\mu = \frac{1}{e}\partial_\mu\alpha(x)$  is equal to  $A_\mu = 0$ . Through the Faddeev-Popov procedure we introduce some gauge fixing function  $G(A) = 0$ , this would be  $G(A) = \partial_\mu A^\mu$  in the Lorentz gauge. We can insert  $\delta G(A)$  into our functional integral by using,

$$1 = \int D\alpha \delta(G(A^\alpha)) \det\left[\frac{\delta G}{\delta\alpha}\right] \quad (1.10)$$

and a general  $G$  given by,  $G(A) = \partial^\mu A_\mu - \omega(x)$  where  $\omega$  is some scalar function. This gives as the photon propagator as,

$$\tilde{D}_F^{\mu\nu}(k) = \frac{-i}{k^2 + i\epsilon} \left(g^{\mu\nu} - (1 - \xi) \frac{k^\mu k^\nu}{k^2}\right) \quad (1.11)$$

where the choice of  $\xi$  gives our gauge.

#### 1.4 WHAT ARE THE PROPERTIES OF GRASSMANN NUMBERS AND HOW ARE THEY USED TO QUANTIZE SPINOR FIELDS?

The Grassman numbers are anticommuting numbers with the following properties,

$$\begin{aligned}
 \theta\eta &= -\eta\theta \\
 \theta^2 &= 0 \\
 \int 1 d\theta &= 0 \\
 \int \theta d\theta &= 0 \\
 \int \frac{\partial f}{\partial \theta} d\theta &= 0
 \end{aligned}
 \tag{1.12}$$

To quantize spinor fields a grassman valued source field is introduced into  $Z[\eta, \bar{\eta}]$  such that,

$$Z[\eta, \bar{\eta}] = \int D\eta \int D\bar{\eta} e^{i \int d^4x [L + \bar{\eta}\psi + \bar{\psi}\eta]}. \tag{1.13}$$

A Grassman field is given by,

$$\eta(x) = \sum_i \eta_i \phi_i \tag{1.14}$$

where  $\eta_i$  is a grassman number and  $\phi_i$  is a field basis and in the equation above the base is 4-spinor Dirac fields.

#### 1.5 WHAT IS THE SUPERFICIAL DEGREE OF DIVERGENCE AND HOW CAN IT BE COMPUTED? USE QED AS AN EXAMPLE.

The superficial degree of divergence is for a Feynmann diagram's integral defined as,

$$D = (\text{power of momenta in numerator} - \text{power of momenta in denominator}) \tag{1.15}$$

and tells us something about the divergence of a given diagram. In general d-dimensional QED this can be written as,

$$D = d + \frac{d-4}{2}V - \frac{d-2}{2}N_\gamma - \frac{d-1}{2}N_e \tag{1.16}$$

where V is the number of vertices and  $N_i$  the number of  $i$  = photon, electron external lines. D does not tell us everything about the divergence, three diagrams (c,f and g in Fig. ??) always diverge, D can only tell us if a given diagram diverge or not if the diagram does not contain any of those three.

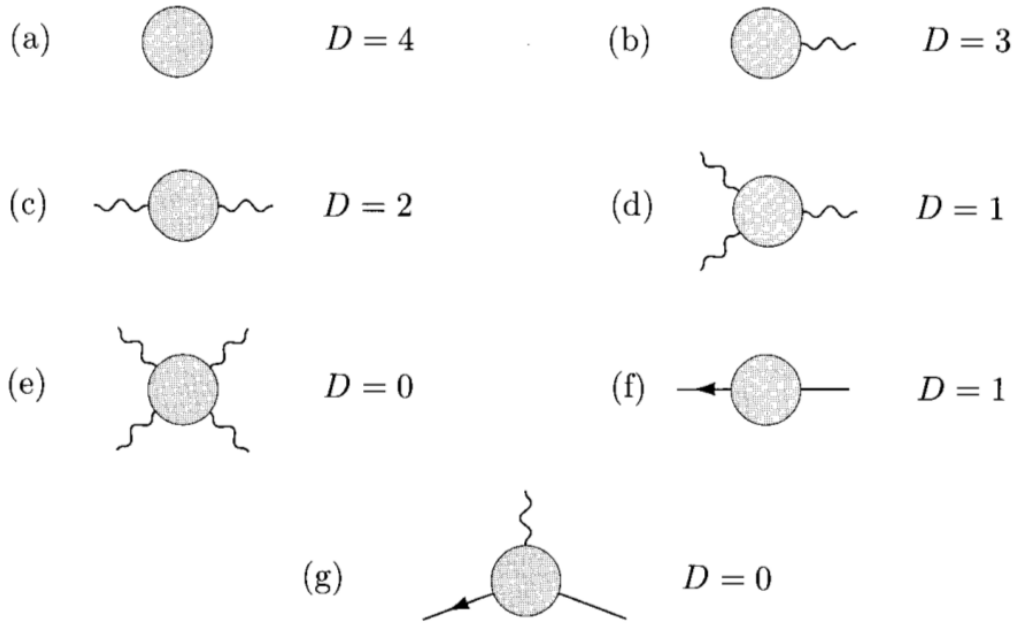


Figure 1.1: The seven QED amplitudes whose superficial degree of divergence ( $D$ ) is  $\geq 0$ .

### 1.6 HOW DOES RENORMALIZED PERTURBATION THEORY RELATE THE BARE AND PHYSICAL MASSES?

In renormalized perturbation theory we begin by introducing new fields to rescale the original Lagrangian which only contains bare parameters. The next step is to introduce counterterms containing the physical parameters which splits the Lagrangian into two parts where one part now contains all infinities. The final step is to specify renormalization conditions which then fixes the physical parameters such as mass, charge or field strength.

The common conditions to specify the physical mass is where the pole of the propagator is, in  $\lambda\phi^4$ -theory this simply implies that  $m^2 = p^2$  since the propagator is given by  $\frac{i}{p^2 - m^2 + i\epsilon}$ . This will define the physical mass  $m$  as a function of the bare mass  $m_0$  and the mass-counterterm  $\delta_m$ .

## 2 RENORMALIZATION AND SPONTANEOUS SYMMETRY BREAKING

### 2.1 WHAT IS SPONTANEOUS SYMMETRY BREAKING? GIVE A CONCRETE EXAMPLE OF A SPONTANEOUSLY BROKEN THEORY.

Subcutaneous symmetry breaking occurs when the potential term in the Lagrangian has a minima which is asymmetric with respect to the Lagrangian. Since the ground state is not at zero one needs to re-parametrize the Lagrangian around the minima in order to achieve this but this brakes the symmetry of the Lagrangian.

Consider the linear sigma model which has  $N$  symmetries with Lagrangian

$$L = \frac{1}{2} (\partial_\mu \phi^i)^2 + \mu^2 (\phi^i)^2 - \frac{\lambda}{4} [(\phi^i)^2]^2 \quad (2.1)$$

where the potential term is

$$V(\phi^i) = -\mu^2 (\phi^i)^2 + \frac{\lambda}{4} [(\phi^i)^2]^2 \quad (2.2)$$

which has it's minimas at  $(\phi_0^i)^2 = \frac{\mu^2}{\lambda}$ . Re-parametrizing the Lagrangian around one of this minimas breaks the  $N$  symmetries and thus the symmettries are spontaneously broken.

### 2.2 WHAT IS GOLDSTONE'S THEOREM AND ITS IMPLICATIONS?

Goldstone's theorem states that every spontaneously broken symmetry with  $N$  symmetries give rise to  $N - 1$  massless particles and 1 massive.

This implies that spontaneously broken theories can only have one massive particle, e.g. LSW-theory where the only massive particle is the Higgs boson. The  $Z$ - and  $W^\pm$ -bosons are massless but are given mass by the Higgs mechanism.

### 2.3 WHAT ARE THE REASONS THAT SPONTANEOUSLY BROKEN THEORIES ARE RENORMALIZABLE? (ASSUMING THAT THE UN-BROKEN THEORY IS.)

Assuming that the unbroken theory is renormalizable the broken theory will also remain renormalizable since the structure of the divergent parts of the Feynman diagrams are unaffected [p.360]. The reparametrization of the Lagrangian will yield several more new divergent diagrams but they all have the same structure and thus all infinities can be absorbed into the same counterterm which renders the broken theory renormalizable as well.

### 2.4 WHAT ARE THE BASICS OF WILSON'S APPROACH TO RENORMALIZATION?

The method of Wilson's approach to renormalization follows the following steps:

- Introduce high momenta cut-off  $\Lambda$
- Separate fields into  $\hat{\phi}(k)$  where  $b\Lambda < |k| < \Lambda$ , 0 otherwise and  $\phi(k)$  where  $|k| < b\Lambda$ , 0 otherwise and  $0 < b < 1$ .

- Integrate out  $\hat{\phi}(k)$  to obtain an effective Lagrangian  $L_{eff}$
- Rescale  $L_{eff}$  to obtain recursion relations for the coefficients.

## 2.5 WHAT IS THE CALLAN-SYMANZIK EQUATION AND HOW DOES IT RELATE TO THE RUNNING OF PHYSICAL QUANTITIES?

HELLO WORLD!!

## 2.6 DESCRIBE HOW YOU WOULD CHOSE SUITABLE RENORMALIZATION CONDITIONS.