THEORETICAL PHYSICS

Quantum Field Theory
SI2410
Fall 2014
Preparation Questions

The whole group

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1 SEMINAR 1: FUNCTIONAL INTEGRALS AND INTRODUCTION TO RENORMALIZATION

1.1 WHAT IS THE RELATIONSHIP BETWEEN THE FUNCTIONAL INTEGRAL FORMALISM AND THE N-POINT CORRELATION FUNCTION?

General generating functional is given by

$$Z[J] = \int D\phi \exp\left[i \int d^4x [L + J(x)\phi(x)]\right] \tag{1.1}$$

where L is the Lagrangian density and $J(x)\phi(x)$ is a source term.
The n-point correlation function is then given by

$$\langle 0|T\phi(x_1)...\phi(x_n)|0\rangle = \frac{1}{Z_0}\prod_{i=1}^{n}\left(-i\frac{\delta}{\delta J(x_i)}Z[J]\right) \tag{1.2}$$

where $Z_0 = Z[J=0]$. This yields

$$\langle 0|T\phi(x_1)...\phi(x_n)|0\rangle = \frac{\int D\phi\phi(x_1)...\phi(x_n)\exp\left[i\int d^4xL\right]}{\int D\phi\exp\left[i\int d^4xL\right]} \tag{1.3}$$

1.2 Given a Lagrangian density, how can the Feynman rules of a theory be computed using functional integral formalism?

The Feynman rules can be computed by Taylor expanding

$$\exp iS = \exp\left[i\int d^4xL_0 + L_{int}\right] \tag{1.4}$$

where S is the action, L_0 is the free Lagrangian density and L_{int} is the interaction Lagrangian density. In the case of $\lambda\phi^4$-theory we have the Lagrangian

$$L = L_0 - \frac{\lambda}{4!}\phi^4 \tag{1.5}$$

and by Taylor expansion

$$\exp iS = \exp\left[i\int d^4xL_0 - \frac{\lambda}{4!}\phi^4\right] = \exp\left[i\int d^4xL_0\right]\left(1 - i\int d^4x\frac{\lambda}{4!}phi^4 + ...\right) \tag{1.6}$$

where L_0 gives the propagators and all terms of order higher than two yields interaction. In this case we read of the vertex factor in momentum space as

$$-i\lambda(2\pi)^4\delta^4\left(\sum p\right) \tag{1.7}$$

1.3 What complications arise when using the functional integral formalism to quantize the electromagnetic field? How is it solved?

In QED the functional integral,

$$\int DAe^{iS[A]} \tag{1.8}$$

is badly divergent as a finite number of terms will have S[A] = 0 due to gauge invariance,

$$A_\mu \to A_\mu + \frac{1}{e}\partial_\mu\alpha(x) \tag{1.9}$$

and thus $A_\mu = \frac{1}{e}\partial_\mu\alpha(x)$ is equal to $A_\mu = 0$. Through the Faddeev-Popov procedure we introduce some gauge fixing function G(A) = 0, this would be G(A) = $\partial_\mu A^\mu$ in the Lorentz gauge. By inserting $\delta G(A)$ into our functional integral by using,

$$1 = \int D\alpha\, \delta(G(A^\alpha))det\left[\frac{\delta G}{\delta\alpha}\right] \tag{1.10}$$

and a general G given by, $G(A) = \partial^\mu A_\mu - \omega(x)$ where ω is some scalar function. This gives as the photon propgator as,

$$\tilde{D}_F^{\mu\nu}(k) = \frac{-i}{k^2+i\epsilon}\left(g^{\mu\nu} - (1-\xi)\frac{k^\mu k^\nu}{k^2}\right) \tag{1.11}$$

where the choice of ξ gives our gauge.

1.4 What are the properties of Grassmann numbers and how are they used to quantize spinor fields?

The Grassman numbers are anticommuting numbers with the following properties,

$$\begin{aligned} \theta\eta &= -\eta\theta \\ \theta^2 &= 0 \\ \int 1 d\theta &= 0 \\ \int \theta d\theta & \\ \int \frac{\partial f}{\partial \theta} d\theta &= 0 \end{aligned} \tag{1.12}$$

To quantize spinor fields a grassman valued source field is introduced into $Z[\eta, \bar{\eta}]$ such that,

$$Z[\eta, \bar{\eta}] = \int D\eta \int D\bar{\eta} e^{i \int d^4 x [L + \bar{\eta}\psi + \bar{\psi}\eta]}. \tag{1.13}$$

A Grassman field is given by,

$$\eta(x) = \sum_i \eta_i \phi_i \tag{1.14}$$

where η_i is a grassman number and ϕ_i is a field basis and in the equation above the base is 4-spinor Dirac fields.

1.5 What is the superficial degree of divergence and how can it be computed? Use QED as an example.

The superficial degree of divergence is for a Feynmann diagram's integral defined as,

$$D = (\text{power of momenta in numerator} - \text{power of momenta in denominator}) \tag{1.15}$$

and tells us something about the divergence of a given diagram. In general d-dimensional QED this can be written as,

$$D = d + \frac{d-4}{2} V - \frac{d-2}{2} N_\gamma - \frac{d-1}{2} N_e \tag{1.16}$$

where V is the number of vertices and N_i the number of i = photon, electron external lines. D does not tell us everything about the divergence, three diagrams (c,f and g in fig 10.2 page 318) always diverge, D can only tell us if a given diagram diverge or not if the diagram does not contain any of those three.

1.6 How does renormalized perturbation theory relate the bare and physical masses?