THEORETICAL PHYSICS

Quantum Field Theory
SI2410
Fall 2014
Preparation Questions

The whole group

September 12, 2014

1 SEMINAR 1: FUNCTIONAL INTEGRALS AND INTRODUCTION TO RENORMALIZATION

1.1 WHAT IS THE RELATIONSHIP BETWEEN THE FUNCTIONAL INTEGRAL FORMALISM AND THE N-POINT CORRELATION FUNCTION?

General generating functional is given by

$$Z[J] = \int D\phi \exp\left[i \int d^4x [L + J(x)\phi(x)]\right] \tag{1.1}$$

where L is the Lagrangian density and $J(x)\phi(x)$ is a source term.
The n-point correlation function is then given by

$$\langle 0|T\phi(x_1)...\phi(x_n)|0\rangle = \frac{1}{Z_0} \prod_{i=1}^{n} \left(-i \frac{\delta}{\delta J(x_i)} Z[J]\right) \tag{1.2}$$

where $Z_0 = Z[J = 0]$. This yields

$$\langle 0|T\phi(x_1)...\phi(x_n)|0\rangle = \frac{\int D\phi\phi(x_1)...\phi(x_n) \exp\left[i \int d^4x L\right]}{\int D\phi \exp\left[i \int d^4x L\right]} \tag{1.3}$$

1.2 Given a Lagrangian density, how can the Feynman rules of a theory be computed using functional integral formalism?

The Feynman rules can be computed by Taylor expanding

$$\exp iS = \exp\left[i\int d^4x L_0 + L_{int}\right] \tag{1.4}$$

where S is the action, L_0 is the free Lagrangian density and L_{int} is the interaction Lagrangian density. In the case of $\lambda\phi^4$-theory we have the Lagrangian

$$L = L_0 - \frac{\lambda}{4!}\phi^4 \tag{1.5}$$

and by Taylor expansion

$$\exp iS = \exp\left[i\int d^4x L_0 - \frac{\lambda}{4!}\phi^4\right] = \exp\left[i\int d^4x L_0\right]\left(1 - i\int d^4x \frac{\lambda}{4!} phi^4 + ...\right) \tag{1.6}$$

where L_0 gives the propagators and all terms of order higher than two yields interaction. In this case we read of the vertex factor in momentum space as

$$-i\lambda(2\pi)^4\delta^4\left(\sum p\right) \tag{1.7}$$

1.3 What complications arise when using the functional integral formalism to quantize the electromagnetic field? How is it solved?

1.4 What are the properties of Grassmann numbers and how are they used to quantize spinor fields?

1.5 What is the superficial degree of divergence and how can it be computed? Use QED as an example.

1.6 How does renormalized perturbation theory relate the bare and physical masses?