THEORETICAL PHYSICS

Quantum Field Theory
SI2410
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Preparation Questions

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1 SEMINAR 1: FUNCTIONAL INTEGRALS AND INTRODUCTION TO RENORMALIZATION

1.1 WHAT IS THE RELATIONSHIP BETWEEN THE FUNCTIONAL INTEGRAL FORMALISM AND THE N-POINT CORRELATION FUNCTION?

General generating functional is given by

$$Z[J] = \int D\phi \exp\left[i \int d^4x [L + J(x)\phi(x)]\right] \tag{1.1}$$

where L is the Lagrangian density and $J(x)\phi(x)$ is a source term.
The n-point correlation function is then given by

$$\langle 0|T\phi(x_1)...\phi(x_n)|0\rangle = \frac{1}{Z_0}\prod_{i=1}^{n}\left(-i\frac{\delta}{\delta J(x_i)}Z[J]\right) \tag{1.2}$$

where $Z_0 = Z[J=0]$. This yields

$$\langle 0|T\phi(x_1)...\phi(x_n)|0\rangle = \frac{\int D\phi\phi(x_1)...\phi(x_n)\exp\left[i\int d^4xL\right]}{\int D\phi\exp\left[i\int d^4xL\right]} \tag{1.3}$$

1.2 Given a Lagrangian density, how can the Feynman rules of a theory be computed using functional integral formalism?

The Feynman rules can be computed by Taylor expanding

$$\exp[iS] = \exp\left[i\int d^4xL_0 + L_{int}\right] \tag{1.4}$$

where S is the action, L_0 is the free Lagrangian density and L_{int} is the interaction Lagrangian density. In the case of $\lambda\phi^4$-theory we have the Lagrangian

$$L = L_0 - \frac{\lambda}{4!}\phi^4 \tag{1.5}$$

and by Taylor expansion

$$\exp iS = \exp\left[i\int d^4xL_0 - \frac{\lambda}{4!}\phi^4\right] = \exp\left[i\int d^4xL_0\right]\left(1 - i\int d^4x\frac{\lambda}{4!}phi^4 + ...\right) \tag{1.6}$$

where L_0 gives the propagators and all terms of order higher than two yields interaction. In this case we read of the vertex factor in momentum space as

$$-i\lambda(2\pi)^4\delta^4\left(\sum p\right) \tag{1.7}$$

1.3 What complications arise when using the functional integral formalism to quantize the electromagnetic field? How is it solved?

In QED the functional integral,

$$\int DAe^{iS[A]} \tag{1.8}$$

is badly divergent as a finite number of terms will have S[A] = 0 due to gauge invariance,

$$A_\mu \to A_\mu + \frac{1}{e}\partial_\mu\alpha(x) \tag{1.9}$$

and thus $A_\mu = \frac{1}{e}\partial_\mu\alpha(x)$ is equal to $A_\mu = 0$. Through the Faddeev-Popov procedure we introduce some gauge fixing function G(A) = 0, this would be G(A) = $\partial_\mu A^\mu$ in the Lorentz gauge. We can insert $\delta G(A)$ into our functional integral by using,

$$1 = \int D\alpha\,\delta(G(A^\alpha))det\left[\frac{\delta G}{\delta\alpha}\right] \tag{1.10}$$

and a general G given by, $G(A) = \partial^\mu A_\mu - \omega(x)$ where ω is some scalar function. This gives as the photon propgator as,

$$\tilde{D}_F^{\mu\nu}(k) = \frac{-i}{k^2+i\epsilon}\left(g^{\mu\nu} - (1-\xi)\frac{k^\mu k^\nu}{k^2}\right) \tag{1.11}$$

where the choice of ξ gives our gauge.

1.4 What are the properties of Grassmann numbers and how are they used to quantize spinor fields?

The Grassman numbers are anticommuting numbers with the following properties,

$$\begin{aligned} \theta\eta &= -\eta\theta \\ \theta^2 &= 0 \\ \int 1 d\theta &= 0 \\ \int \theta d\theta & \\ \int \frac{\partial f}{\partial \theta} d\theta &= 0 \end{aligned} \tag{1.12}$$

To quantize spinor fields a grassman valued source field is introduced into $Z[\eta, \bar{\eta}]$ such that,

$$Z[\eta, \bar{\eta}] = \int D\eta \int D\bar{\eta} e^{i\int d^4x[L + \bar{\eta}\psi + \bar{\psi}\eta]}. \tag{1.13}$$

A Grassman field is given by,

$$\eta(x) = \sum_i \eta_i \phi_i \tag{1.14}$$

where η_i is a grassman number and ϕ_i is a field basis and in the equation above the base is 4-spinor Dirac fields.

1.5 What is the superficial degree of divergence and how can it be computed? Use QED as an example.

The superficial degree of divergence is for a Feynmann diagram's integral defined as,

$$D = (\text{power of momenta in numerator} - \text{power of momenta in denominator}) \tag{1.15}$$

and tells us something about the divergence of a given diagram. In general d-dimensional QED this can be written as,

$$D = d + \frac{d-4}{2} V - \frac{d-2}{2} N_\gamma - \frac{d-1}{2} N_e \tag{1.16}$$

where V is the number of vertices and N_i the number of i = photon, electron external lines. D does not tell us everything about the divergence, three diagrams (c,f and g in Fig. 1.1) always diverge, D can only tell us if a given diagram diverge or not if the diagram does not contain any of those three.

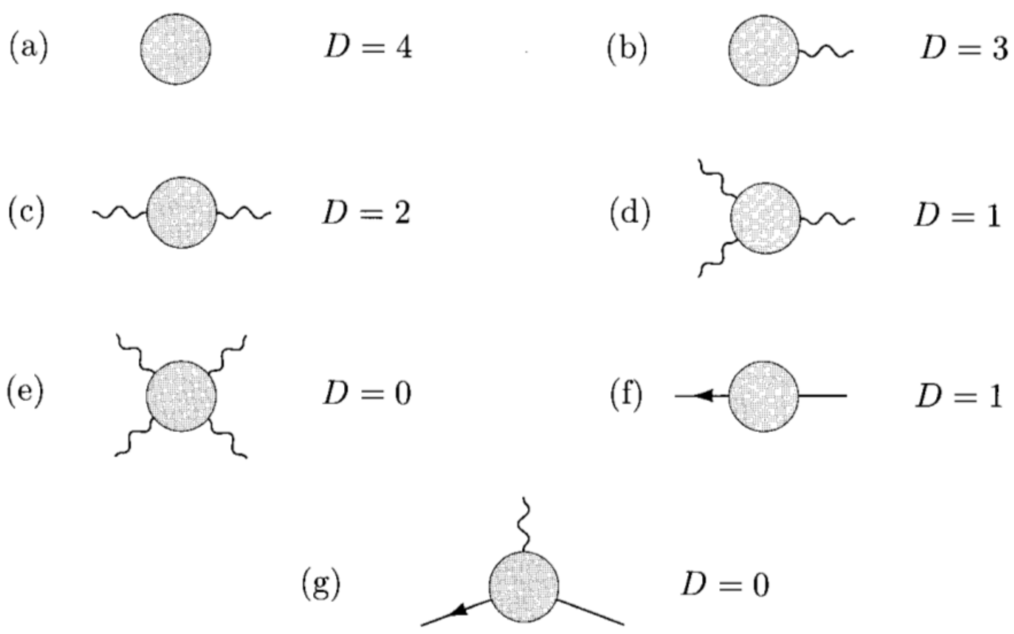

Figure 1.1: The seven QED amplitudes whose superficial degree of divergence (D) is ≥ 0.

1.6 How does renormalized perturbation theory relate the bare and physical masses?

In renormalized perturbation theory we begin by introducing new fields to rescale the original Lagrangian which only contains bare parameters. The next step is to introduce counterterms containing the physical parameters which splits the Lagrangian into two parts where one part now contains all infinities. The final step is to specify renormalization conditions which then fixes the physical parameters such as mass, charge or field strength.

The common conditions to specify the physical mass is where the pole of the propagator is, in $\lambda\phi^4$-theory this simply implies that $m^2 = p^2$ since the propagator is given by $\frac{i}{p^2 - m^2 + i\epsilon}$. This will define the physical mass m as a function of the bare mass m_0 and the mass-counterterm δ_m.