Lec19-2-6(b)

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$$\mu = \exp(\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2 + \beta_4 X_3)$$

Take natural log on both sides, we get:

$$\log \mu = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2 + \beta_4 X_3$$

So, a c-unit change in X_2 has effect:

$$\log \mu_{X_2+c} - \log \mu_{X_2} \quad \Big| \; *$$

$$= \beta_0 + \beta_1 X_1 + \beta_2 (X_2 + c) + \beta_3 X_1 (X_2 + c) + \beta_4 X_3$$
$$- \beta_0 - \beta_1 X_1 - \beta_2 X_2 - \beta_3 X_1 X_2 - \beta_4 X_3$$
$$= c \cdot \beta_2 + c \cdot \beta_3 X_1$$
$$= c \cdot (\beta_2 + \beta_3 X_1)$$

Now convert $*$ back to a function of μ.

$$\exp[\log \mu_{X_2+c} - \log \mu_{X_2}] = \exp[c \cdot (\beta_2 + \beta_3 X_1)]$$

$$\exp\left[\frac{\log \mu_{X_2+c}}{\log \mu_{X_2}}\right] = \exp[c \cdot (\beta_2 + \beta_3 X_1)]$$

$$\frac{\mu_{X_2+c}}{\mu_{X_2}} = \exp[c \cdot (\beta_2 + \beta_3 X_1)]$$

<u>Therefore</u>:

The multiplicative change in mean response for a c-unit change in X_2 is: $\dfrac{\mu_{X_2+c}}{\mu_{X_2}} = \exp[c \cdot (\beta_2 + \beta_3 X_1)]$