

$$\text{logit}(\pi) = \beta_0 + \beta_1 X + \beta_2 X^2$$

$$= \log\left(\frac{\pi}{1-\pi}\right)$$

$$\log(\text{OR}_c) = \log\left[\frac{\pi_c/(1-\pi_c)}{\pi/(1-\pi)}\right]$$

$$= \text{logit}(\pi_c) - \text{logit}(\pi)$$

$$= \beta_0 + \beta_1(X+c) + \beta_2(X+c)^2 - [\beta_0 + \beta_1 X + \beta_2 X^2]$$

$$= \beta_0 + \beta_1 X + \beta_1 c + \beta_2 X^2 + \beta_2 \cdot 2cX + \beta_2 c^2 - [\beta_0 + \beta_1 X + \beta_2 X^2]$$

$$= \beta_1 c + \beta_2 2cX + \beta_2 c^2$$

$$= \beta_1 c + \beta_2 (2cX + c^2)$$

change the result in terms of OR : $\exp[\beta_1 c + \beta_2 (2cX + c^2)]$