Lec22_1(b)

April 3, 2019 4:43 PM

1) Represent descriptions as ratios of mean ratios:

$$GI\ VC:M \mid F:M = \frac{\mu_{VC,F} / \mu_{M,F}}{\mu_{VC,M} / \mu_{M,M}}$$

$$GI\ M:VL \mid F:M = \frac{\mu_{M,F} / \mu_{VL,F}}{\mu_{M,M} / \mu_{VL,M}}$$

2) Proof of these ratios are equivalent to linear combinations of parameters.

- The model of interest is:

$$\log(\mu_{ij}) = \beta_0 + \beta_i^G + \beta_j^I + \beta_{ij}^{GI}$$

where $i = 1, \dots, I$ in G

$j = 1, \dots, J$ in I

- the ratio of 2 means is:

$$\frac{\mu_{ij} / \mu_{i'j}}{\mu_{ij'} / \mu_{i'j'}} = \frac{\mu_{ij} \cdot \mu_{i'j'}}{\mu_{i'j} \cdot \mu_{ij'}} \qquad (*)$$

- Work this on the log scale:

$$\log(*) = \log(\mu_{ij}) + \log(\mu_{i'j'}) - \log(\mu_{i'j}) - \log(\mu_{ij'})$$

$$= \cancel{\beta^0} + \cancel{\beta_i^G} + \cancel{\beta_j^I} + \beta_{ij}^{GI} + \cancel{\beta^0} + \cancel{\beta_{i'}^G} + \cancel{\beta_{j'}^I} + \beta_{i'j'}^{GI}$$

$$- \cancel{\beta^0} - \cancel{\beta_{i'}^G} - \cancel{\beta_j^I} - \beta_{i'j}^{GI} - \cancel{\beta^0} - \cancel{\beta_i^G} - \cancel{\beta_{j'}^I} - \beta_{ij'}^{GI}$$

$$= \beta_{ij}^{GI} + \beta_{i'j'}^{GI} - \beta_{i'j}^{GI} - \beta_{ij'}^{GI}$$

$$= \beta_{ij}^{GI} + \beta_{i'j'}^{GI} - \beta_{i'j}^{GI} - \beta_{ij'}^{GI}$$

- Coerce it back to the mean ratio:

$$\frac{\mu_{ij} / \mu_{i'j}}{\mu_{ij'} / \mu_{i'j'}} = \exp\left[\beta_{ij}^{GI} + \beta_{i'j'}^{GI} - \beta_{i'j}^{GI} - \beta_{ij'}^{GI}\right]$$

$\Longrightarrow$ Inside $\exp(\cdot)$ is a linear combination of interaction parameters!

- Therefore, the ratio of mean ratios can be represented as exponentiated linear combinations of certain parameters in our model.