On the Relation Between PDA and MMSE-ISDIC

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***Abstract*—The probabilistic data association (PDA) and the minimum-mean-square-error filtering based iterative soft decision interference cancellation (MMSE-ISDIC) are two popular detectors for systems with co-channel interference. In this letter, the relation between the two detectors is analytically derived for the V-BLAST systems. The two detectors perform in a similar manner, whereas differ in the metric calculation. It is proved that for both original signal model and decorrelated signal model, the PDA and the MMSE-ISDIC detectors are equivalent in terms of performance for both uncoded and coded systems.**

***Index Terms*—Iterative soft decision interference cancellation, MIMO, probabilistic data association, V-BLAST.**

I. Introduction

THE minimum-mean-square-error (MMSE) filtering based iterative soft decision interference cancellation (MMSE-ISDIC) is a well-known suboptimum detector for code-division multiplex access (CDMA) systems [1]. Recently, the probabilistic data association (PDA), which is a popular algorithm for target tracking, has attracted great interests for suboptimum detection of CDMA [2] and the Vertical Bell Labs Layered Space-Time (V-BLAST) systems [3], [4]. In this letter, a detailed comparison is drawn between the PDA and the MMSE-ISDIC detectors in respect of the V-BLAST systems. The two detectors perform in a similar manner, whereas differ in the metric calculation. In the PDA, vector Gaussian approximation is adopted for metric calculation instead of scalar Gaussian approximation as in the MMSE-ISDIC. With a detailed analysis, we prove that for both original signal model and decorrelated signal model, the two detectors are equivalent in terms of performance for both uncoded and coded systems (to the best of our knowledge, this proposition has never been proved in any literature). In the following, we will use $(\cdot)^T$, $(\cdot)^H$, and $(\cdot)^{-1}$ to denote transpose, conjugate transpose, and matrix inversion operation, respectively.

II. System Model

We consider a V-BLAST system with N_T transmit antennas and N_R receive antennas (for generality, $N_T \leq N_R$ is not assumed here). At the transmitter, information bit streams are first coded, interleaved, and mapped to symbols; the symbols are then fed to N_T transmit antennas by serial/parallel transform. The received vector $\boldsymbol{r} = [r_1, r_2, \ldots, r_{N_R}]^T$ is

$$\boldsymbol{r} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{w} \tag{1}$$

where $\boldsymbol{H}$ is an $N_R \times N_T$ channel matrix and its (i,j)th entry $h_{i,j}$ represents the channel gain from transmit antenna j to receive antenna i, assuming rich scattering, the entries of $\boldsymbol{H}$ are modeled as i.i.d. circular symmetric complex Gaussian variables with zero mean and unit variance. We assume that $\boldsymbol{H}$ remains constant during a transmission burst, and varies randomly from one burst to another. $\boldsymbol{x} = [x_1, x_2, \ldots, x_{N_T}]^T$ is the transmitted symbol vector with the elements taken from a unit average energy constellation set $\Omega = \{a_1, a_2, \ldots, a_Q\}$, $x_i (i = 1, 2, \ldots, N_T)$ is mapped by $C = \log_2 Q$ bits, i.e., $b_{i,c} (c = 1, 2, \ldots, C)$. $\boldsymbol{w} = [w_1, w_2, \ldots, w_{N_R}]^T$ is an i.i.d. circular symmetric complex Gaussian noise vector with zero-mean and covariance matrix $N_0 \boldsymbol{I}_{N_R}$.

III. Relation Between PDA and MMSE-ISDIC

Before our analysis, let us draw a brief review of the MMSE-ISDIC and the PDA detectors. The MMSE-ISDIC proceeds as follows [1].

1) Initialize the soft decision vector $\widetilde{\boldsymbol{x}} = [\widetilde{x}_1, \widetilde{x}_2, \ldots, \widetilde{x}_{N_T}]^T$ to all zeros and the residual interference variance vector $\boldsymbol{v} = [v_1, v_2, \ldots, v_{N_T}]^T$ to all ones.
2) For $i = 1, 2, \ldots, N_T$ (for simplicity of analysis, suppose here that the detection order is $1, 2, \ldots, N_T$), calculate the MMSE filtering vector on soft interference-cancelled signal vector $\boldsymbol{r}_{\mathrm{IC},i} = \boldsymbol{r} - \sum_{j=1, j\neq i}^{N_T} \boldsymbol{h}_j \widetilde{x}_j$ for detecting x_i as follows:

$$\boldsymbol{f}_i^M = \boldsymbol{h}_i^H \left(\boldsymbol{H}\boldsymbol{\mathcal{D}}_i \boldsymbol{H}^H + N_0 \boldsymbol{I}_{N_R} \right)^{-1} = \boldsymbol{h}_i^H \boldsymbol{R}_i^{-1} \tag{2}$$

and then update the soft decision $\widetilde{x}_i$ and the residual interference variance v_i, respectively, as follows:

$$\begin{aligned} \varphi_{i,q}^M &= -\left| \boldsymbol{f}_i^M \boldsymbol{r}_{\mathrm{IC},i} - \mu_i^M a_q \right|^2 / \left[\mu_i^M \left(1 - \mu_i^M \right) \right] \\ &\qquad q = 1, 2, \ldots, Q \\ \widetilde{x}_i &= \left[\sum_{q=1}^{Q} a_q \exp\left(\varphi_{i,q}^M \right) \right] / \left[\sum_{q=1}^{Q} \exp\left(\varphi_{i,q}^M \right) \right] \\ v_i &= \left[\sum_{q=1}^{Q} |a_q - \widetilde{x}_i|^2 \exp\left(\varphi_{i,q}^M \right) \right] / \left[\sum_{q=1}^{Q} \exp\left(\varphi_{i,q}^M \right) \right]. \end{aligned} \tag{3}$$

In (2), $\boldsymbol{h}_i$ is the ith column of $\boldsymbol{H}$, $\boldsymbol{R}_i = (\boldsymbol{H}\mathcal{D}_i \boldsymbol{H}^H + N_0 \boldsymbol{I}_{N_R})$, $\mathcal{D}_i$ is a diagonal matrix with $\boldsymbol{v}$ on its diagonal except that the ith diagonal element is set to 1; in (3), $\mu_i^M = \boldsymbol{f}_i^M \boldsymbol{h}_i$ is the bias of the MMSE filtering for detecting x_i,

Manuscript received December 6, 2006; revised January 18, 2007. This work was supported by The work was supported by NSFC (60496316), NSFC (60572146), RFDP (20050701007), TRAPOYT, MOE (107103). The associate editor coordinating the review of this paper and approving it for publication was Dr. Philip Schiter.

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Digital Object Identifier 10.1109/LSP.2007.896157

and $\varphi_{i,q}^{M}$ is called the metric of the MMSE-ISDIC in this letter.

3) Repeat the second step for a certain number of iterations.
4) Calculate decision output.

Note that in the MMSE-ISDIC, $n_i = \sum_{j=1,j\neq i}^{N_T} \boldsymbol{f}_i^M \boldsymbol{h}_j(x_j - \widetilde{x}_j) + \boldsymbol{f}_i^M \boldsymbol{w}$, is the residual interference plus noise at the MMSE filtering output for detecting x_i, which is approximated as a Gaussian variable, i.e., $n_i \sim \mathrm{N}(0, \mu_i^M(1-\mu_i^M))$. The PDA proceeds in the similar manner as the MMSE-ISDIC and differs from the MMSE-ISDIC only in metric calculation. The metric of the PDA $\varphi_{i,q}^{P}$ is calculated directly based on the soft interference-cancelled signal vector $\boldsymbol{r}_{\mathrm{IC},i}$ and no MMSE filtering is performed [4]

$$\varphi_{i,q}^{P} = -(\boldsymbol{r}_{\mathrm{IC},i} - \boldsymbol{h}_i a_q)^H \boldsymbol{R}_{\boldsymbol{n}_i}^{-1} (\boldsymbol{r}_{\mathrm{IC},i} - \boldsymbol{h}_i a_q), \quad q = 1,2,\ldots,Q \tag{4}$$

where $\boldsymbol{R}_{\boldsymbol{n}_i} = \sum_{j=1,j\neq i}^{N_T} v_j \boldsymbol{h}_j \boldsymbol{h}_j^H + N_0 \boldsymbol{I}_{N_R}$ (The PDA detectors in [2] and [3] are proposed for BPSK modulation and square/rectangular QAM modulation, respectively. In this letter, the conventional PDA detector as in [4] is adopted, i.e., no real-valued transformation as in [2] and [3] is performed for generalization and easier comprehensiveness). Note that in the PDA, $\boldsymbol{n}_i = \sum_{j=1,j\neq i}^{N_T} \boldsymbol{h}_j(x_j - \widetilde{x}_j) + \boldsymbol{w}$ is the residual interference plus noise vector after soft interference cancellation for detecting x_i, which is approximated as a Gaussian vector, i.e., $\boldsymbol{n}_i \sim \mathrm{N}(\boldsymbol{0}, \boldsymbol{R}_{\boldsymbol{n}_i})$. The decision output calculations for the MMSE-ISDIC and the PDA are the same. For uncoded systems, the $a_q (q = 1,2,\ldots,Q)$ with the maximum metric in (3) or (4) is selected as decision output. For coded systems, the logarithm likelihood ratio (LLR) of the coded bits $b_{i,c} (i = 1,2,\ldots,N_T; c = 1,2,\ldots,C)$ for channel decoding is calculated as

$$L_{i,c}^{P} = \log \frac{\sum_{a_q \in \Omega_c^+} \exp\left(\varphi_{i,q}^{P}\right)}{\sum_{a_q \in \Omega_c^-} \exp\left(\varphi_{i,q}^{P}\right)} \text{ (for PDA)}$$
$$L_{i,c}^{M} = \log \frac{\sum_{a_q \in \Omega_c^+} \exp\left(\varphi_{i,q}^{M}\right)}{\sum_{a_q \in \Omega_c^-} \exp\left(\varphi_{i,q}^{M}\right)} \text{ (for MMSE} - \text{ISDIC)} \tag{5}$$

where $\Omega_c^+ \in \{\Omega | b_{i,c} = +1\}$ and $\Omega_c^- \in \{\Omega | b_{i,c} = -1\}$. Note that Gaussian approximation is adopted by both the MMSE-ISDIC and the PDA: the MMSE-ISDIC adopts Gaussian approximation to a scalar (i.e., n_i) while the PDA to a vector (i.e., $\boldsymbol{n}_i$). For the MMSE-ISDIC, once the MMSE filtering is performed, $\varphi_{i,q}^{M} (q = 1,2,\ldots,Q)$ can be easily calculated with only scalar multiplications; whereas for the PDA, the calculations of $\varphi_{i,q}^{P}$ need matrix-vector multiplications for each $q (q = 1,2,\ldots,Q)$.

We now analyze the relation between the PDA and the MMSE-ISDIC. Without loss of generality, suppose that we are to detect $x_i (1 \le i \le N_T)$ at the kth $(k = 1,2,3\ldots)$ iteration (k is omitted in the following for ease of notation). For the MMSE-ISDIC, the metric $\varphi_{i,q}^{M}$ in (3) can be derived as

$$\begin{aligned}
\varphi_{i,q}^{M} &= -\left|\boldsymbol{f}_i^M \boldsymbol{r}_{\mathrm{IC},i} - \mu_i^M a_q\right|^2 / \left[\mu_i^M\left(1-\mu_i^M\right)\right] \\
&= \Big[-\boldsymbol{r}_{\mathrm{IC},i}^H \left(\boldsymbol{f}_i^M\right)^H \boldsymbol{f}_i^M \boldsymbol{r}_{\mathrm{IC},i} + \boldsymbol{r}_{\mathrm{IC},i}^H \left(\boldsymbol{f}_i^M\right)^H \mu_i^M a_q \\
&\quad + a_q^H \mu_i^M \boldsymbol{f}_i^M \boldsymbol{r}_{\mathrm{IC},i} - a_q^H \mu_i^M \mu_i^M a_q\Big] / \left[\mu_i^M\left(1-\mu_i^M\right)\right] \\
&= \Big[-\boldsymbol{r}_{\mathrm{IC},i}^H \left(\boldsymbol{f}_i^M\right)^H \boldsymbol{f}_i^M \boldsymbol{r}_{\mathrm{IC},i} / \mu_i^M + \boldsymbol{r}_{\mathrm{IC},i}^H \left(\boldsymbol{f}_i^M\right)^H a_q \\
&\quad + a_q^H \boldsymbol{f}_i^M \boldsymbol{r}_{\mathrm{IC},i} - a_q^H \mu_i^M a_q\Big] / \left(1-\mu_i^M\right), \\
&\quad q = 1,2,\ldots,Q
\end{aligned} \tag{6}$$

where we have used the fact that the MMSE filtering bias μ_i^M is a real number. For the PDA, the metric $\varphi_{i,q}^{P}$ in (4) can be derived as

$$\begin{aligned}
\varphi_{i,q}^{P} &= -(\boldsymbol{r}_{\mathrm{IC},i} - \boldsymbol{h}_i a_q)^H \boldsymbol{R}_{\boldsymbol{n}_i}^{-1} (\boldsymbol{r}_{\mathrm{IC},i} - \boldsymbol{h}_i a_q) \\
&= \Big(-\boldsymbol{r}_{\mathrm{IC},i}^H \boldsymbol{R}_{\boldsymbol{n}_i}^{-1} \boldsymbol{r}_{\mathrm{IC},i} + \boldsymbol{r}_{\mathrm{IC},i}^H \boldsymbol{R}_{\boldsymbol{n}_i}^{-1} \boldsymbol{h}_i a_q \\
&\quad + a_q^H \boldsymbol{h}_i^H \boldsymbol{R}_{\boldsymbol{n}_i}^{-1} \boldsymbol{r}_{\mathrm{IC},i} - a_q^H \boldsymbol{h}_i^H \boldsymbol{R}_{\boldsymbol{n}_i}^{-1} \boldsymbol{h}_i a_q\Big) \\
&= \Big[-\boldsymbol{r}_{\mathrm{IC},i}^H \boldsymbol{R}_{\boldsymbol{n}_i}^{-1} \boldsymbol{r}_{\mathrm{IC},i} \left(1-\mu_i^M\right) + \boldsymbol{r}_{\mathrm{IC},i}^H \left(\boldsymbol{f}_i^M\right)^H a_q \\
&\quad + a_q^H \boldsymbol{f}_i^M \boldsymbol{r}_{\mathrm{IC},i} - a_q^H \mu_i^M a_q\Big] / \left(1-\mu_i^M\right), \\
&\quad q = 1,2,\ldots,Q
\end{aligned} \tag{7}$$

where we have used the fact that $\boldsymbol{h}_i^H \boldsymbol{R}_{\boldsymbol{n}_i}^{-1} = \boldsymbol{f}_i^M / (1-\mu_i^M)$, which is proven in Appendix A. To continue, we define the "vital items" as the items that include a_q, and define the "trivial items" as the items that do not include a_q. With a careful examination of the last lines of (6) and (7), we see that the "vital items" of $\varphi_{i,q}^{M}$ and $\varphi_{i,q}^{P}$ are just the same. It is clear that only the "vital items" in $\varphi_{i,q}^{M}$ and $\varphi_{i,q}^{P}$ affect the update of $\widetilde{x}_i$ and v_i in (3), and the hard decision and LLR calculation in (5) are also affected by only the "vital items" in $\varphi_{i,q}^{M}$ and $\varphi_{i,q}^{P}$. (In detail, when we update $\widetilde{x}_i$ and v_i with (3), the "trivial items" can be extracted from both numerator and denominator and eventually cancelled, the same instance also happens when we calculate LLR with (5); it is evident that the "trivial items" have no effect on hard decision since the "trivial items" are unchanged for all Q tries). We therefore arrive at the proposition that for any modulation constellation, the MMSE-ISDIC will become equivalent to the PDA in terms of performance for both uncoded and coded systems if the same detection order is adopted by both detectors. We also note that $\varphi_{i,q}^{M}$ does not equal to $\varphi_{i,q}^{P}$ since the "trivial items" in the last lines of (6) and (7) do not equal (i.e., $-\boldsymbol{r}_{\mathrm{IC},i}^H (\boldsymbol{f}_i^M)^H \boldsymbol{f}_i^M \boldsymbol{r}_{\mathrm{IC},i} / [\mu_i^M(1-\mu_i^M)] \neq -\boldsymbol{r}_{\mathrm{IC},i}^H \boldsymbol{R}_{\boldsymbol{n}_i}^{-1} \boldsymbol{r}_{\mathrm{IC},i}$), these "trivial items", however, have no effect on iterative update of $\widetilde{x}_i$ and v_i in (3) and the final decision output (both hard decision and LLR calculation), therefore our performance equivalence proposition between the two detectors holds regardless of the "trivial items" in the metrics.

Till now, we have analyzed the relation between the MMSE-ISDIC and the PDA based on the original signal model (1), when $N_T \le N_R$, the complexities of both the MMSE-ISDIC and the PDA can be reduced by adopting the decorrelated signal model

$$\widehat{\boldsymbol{r}} = (\boldsymbol{H}^H \boldsymbol{H})^{-1} \boldsymbol{H}^H \boldsymbol{r} = \boldsymbol{x} + (\boldsymbol{H}^H \boldsymbol{H})^{-1} \boldsymbol{H}^H \boldsymbol{w}. \tag{8}$$

In the following, we will analyze the relation between the MMSE-ISDIC and the PDA based on the decorrelated signal model (8). Now the soft interference-cancelled signal for detecting x_i $(i = 1,2,\ldots,N_T)$ is $\widehat{\boldsymbol{r}}_{\mathrm{IC},i} = \widehat{\boldsymbol{r}} - \sum_{j=1,j\neq i}^{N_T} \boldsymbol{e}_j \widetilde{x}_j = (\boldsymbol{H}^{\mathrm{H}} \boldsymbol{H})^{-1} \boldsymbol{H}^{\mathrm{H}} \boldsymbol{r}_{\mathrm{IC},i}$, where $\boldsymbol{e}_j$ is an $N_T \times 1$ vector whose the jth element is 1 and all other elements are zeros. For the MMSE-ISDIC, the MMSE filtering vector on $\widehat{\boldsymbol{r}}_{\mathrm{IC},i}$ for detecting x_i is

$$\widehat{\boldsymbol{f}}_i^M = \left[\left(\boldsymbol{\mathcal{D}}_i + N_0(\boldsymbol{H}^H\boldsymbol{H})^{-1}\right)^{-1}\right]_i \tag{9}$$

where $[\cdot]_i$ denotes the ith line of a matrix. $\widehat{\boldsymbol{f}}_i^M$ and $\boldsymbol{f}_i^M$ are related by

$$\begin{aligned}\widehat{\boldsymbol{f}}_i^M(\boldsymbol{H}^H\boldsymbol{H})^{-1}\boldsymbol{H}^H &= \left[\left(\boldsymbol{\mathcal{D}}_i + N_0(\boldsymbol{H}^H\boldsymbol{H})^{-1}\right)^{-1}\right]_i \\ &\quad\times(\boldsymbol{H}^H\boldsymbol{H})^{-1}\boldsymbol{H}^H \\ &= \left[\left(\boldsymbol{H}^H\boldsymbol{H}\boldsymbol{\mathcal{D}}_i + N_0\boldsymbol{I}_{N_T}\right)^{-1}\boldsymbol{H}^H\boldsymbol{H}\right]_i \\ &\quad\times(\boldsymbol{H}^H\boldsymbol{H})^{-1}\boldsymbol{H}^H \\ &= \left[\left(\boldsymbol{H}^H\boldsymbol{H}\boldsymbol{\mathcal{D}}_i + N_0\boldsymbol{I}_{N_T}\right)^{-1}\right]_i\boldsymbol{H}^H \\ &= \boldsymbol{h}_i^H\left(\boldsymbol{H}\mathcal{D}_i\boldsymbol{H}^H + N_0\boldsymbol{I}_{N_R}\right)^{-1} \\ &= \boldsymbol{f}_i^M\end{aligned} \tag{10}$$

where we have used the fact that $(\boldsymbol{H}^H\boldsymbol{H}\mathcal{D}_i + N_0\boldsymbol{I}_{N_T})^{-1}\boldsymbol{H}^H = \boldsymbol{H}^H(\boldsymbol{H}\mathcal{D}_i\boldsymbol{H}^H + N_0\boldsymbol{I}_{N_R})^{-1}$, which is proved in Appendix B. Based on the decorrelated signal model, the metric of the MMSE-ISDIC $\widehat{\varphi}_{i,q}^M$ is

$$\widehat{\varphi}_{i,q}^M = -\left|\widehat{\boldsymbol{f}}_i^M\widehat{\boldsymbol{r}}_{\mathrm{IC},i} - \left(\widehat{\boldsymbol{f}}_i^M\right)_i a_q\right|^2 \Big/ \left[\left(\widehat{\boldsymbol{f}}_i^M\right)_i\left(1-\left(\widehat{\boldsymbol{f}}_i^M\right)_i\right)\right], \quad q = 1,2,\ldots,Q \tag{11}$$

where $(\cdot)_i$ denotes the ith element of a vector. It is not difficult to find that $\widehat{\boldsymbol{f}}_i^M\widehat{\boldsymbol{r}}_{\mathrm{IC},i} = \boldsymbol{f}_i^M\boldsymbol{r}_{\mathrm{IC},i}$ and $(\widehat{\boldsymbol{f}}_i^M)_i = \boldsymbol{f}_i^M\boldsymbol{h}_i = \mu_i^M$ from (10), we then have $\widehat{\varphi}_{i,q}^M = \varphi_{i,q}^M$, therefore the MMSE-ISDIC for the original signal model (1) and the decorrelated signal model (8) are equivalent in terms of performance. For the PDA, the metric based on the decorrelated signal model (8) $\widehat{\varphi}_{i,q}^P$ is

$$\begin{aligned}\widehat{\varphi}_{i,q}^P = -(\widehat{\boldsymbol{r}}_{\mathrm{IC},i} - \boldsymbol{e}_i a_q)^H&\left[\sum_{j=1,j\neq i}^{N_T} v_j\boldsymbol{e}_j\boldsymbol{e}_j^H + N_0(\boldsymbol{H}^H\boldsymbol{H})^{-1}\right]^{-1} \\ &\times(\widehat{\boldsymbol{r}}_{\mathrm{IC},i} - \boldsymbol{e}_i a_q), \quad q = 1,2,\ldots,Q.\end{aligned} \tag{12}$$

By defining $\boldsymbol{A} = (\boldsymbol{H}^H\boldsymbol{H})^{-1}\boldsymbol{H}^H$ and $\boldsymbol{Q}_{\boldsymbol{n}_i} = \boldsymbol{H}(\boldsymbol{H}^H\boldsymbol{R}_{\boldsymbol{n}_i}\boldsymbol{H})^{-1}\boldsymbol{H}^H$, (12) can be rewritten as

$$\begin{aligned}\widehat{\varphi}_{i,q}^P &= -\left[\boldsymbol{A}(\boldsymbol{r}_{\mathrm{IC},i} - \boldsymbol{h}_i a_q)\right]^H\left(\boldsymbol{A}\boldsymbol{R}_{\boldsymbol{n}_i}\boldsymbol{A}^H\right)^{-1}\left[\boldsymbol{A}(\boldsymbol{r}_{\mathrm{IC},i} - \boldsymbol{h}_i a_q)\right] \\ &= -(\boldsymbol{r}_{\mathrm{IC},i} - \boldsymbol{h}_i a_q)^H\boldsymbol{H}\left(\boldsymbol{H}^H\boldsymbol{R}_{\boldsymbol{n}_i}\boldsymbol{H}\right)^{-1}\boldsymbol{H}^H(\boldsymbol{r}_{\mathrm{IC},i} - \boldsymbol{h}_i a_q) \\ &= \Big(-\boldsymbol{r}_{\mathrm{IC},i}^H\boldsymbol{Q}_{\boldsymbol{n}_i}\boldsymbol{r}_{\mathrm{IC},i} + \boldsymbol{r}_{\mathrm{IC},i}^H\boldsymbol{Q}_{\boldsymbol{n}_i}\boldsymbol{h}_i a_q \\ &\qquad + a_q^H\boldsymbol{h}_i^H\boldsymbol{Q}_{\boldsymbol{n}_i}\boldsymbol{r}_{\mathrm{IC},i} - a_q^H\boldsymbol{h}_i^H\boldsymbol{Q}_{\boldsymbol{n}_i}\boldsymbol{h}_i a_q\Big) \\ &= \Big(\boldsymbol{r}_{\mathrm{IC},i}^H\boldsymbol{Q}_{\boldsymbol{n}_i}\boldsymbol{r}_{\mathrm{IC},i} + \boldsymbol{r}_{\mathrm{IC},i}^H\boldsymbol{R}_{\boldsymbol{n}_i}^{-1}\boldsymbol{h}_i a_q \\ &\qquad + a_q^H\boldsymbol{h}_i^H\boldsymbol{R}_{\boldsymbol{n}_i}^{-1}\boldsymbol{r}_{\mathrm{IC},i} - a_q^H\boldsymbol{h}_i^H\boldsymbol{R}_{\boldsymbol{n}_i}^{-1}\boldsymbol{h}_i a_q\Big) \\ &= \Big[-\boldsymbol{r}_{\mathrm{IC},i}^H\boldsymbol{Q}_{\boldsymbol{n}_i}\boldsymbol{r}_{\mathrm{IC},i}\left(1-\mu_i^M\right) + \boldsymbol{r}_{\mathrm{IC},i}^H\left(\boldsymbol{f}_i^M\right)^H a_q \\ &\qquad + a_q^H\boldsymbol{f}_i^M\boldsymbol{r}_{\mathrm{IC},i} - a_q^H\mu_i^M a_q\Big] \Big/ \left(1-\mu_i^M\right), \\ &\qquad q = 1,2,\ldots,Q\end{aligned} \tag{13}$$

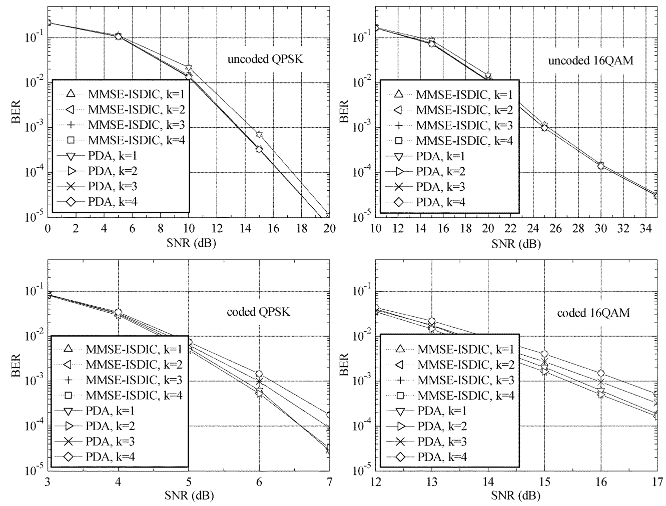

Fig. 1. BER versus SNR performance comparisons between the MMSE-ISDIC and the PDA, k =1, 2, 3, 4.

where we have used the fact that $\boldsymbol{h}_i^H\boldsymbol{R}_{\boldsymbol{n}_i}^{-1} = \boldsymbol{h}_i^H\boldsymbol{Q}_{\boldsymbol{n}_i}$, which is proven in Appendix C. By comparing (13) with (7), we see that $\widehat{\varphi}_{i,q}^P$ and $\varphi_{i,q}^P$ have the same "vital items", therefore the PDA for the original signal model (1) and the decorrelated signal model (8) are equivalent in terms of performance. Based on the above analysis, it is straightforward that the performance equivalence proposition between the MMSE-ISDIC and the PDA holds for both uncoded and coded systems when the decorrelated signal model is adopted.

IV. Simulation Results

We consider a V-BLAST system with $N_R = N_T = 8$. The channel coding adopts a rate-1/2 turbo code with generators 7 and 5 in octal notation. Random interleave are used for both turbo code interleave and channel interleave. Each channel interleave block occupies one burst (with a length of $L = 50$ symbol durations) and each channel interleave block contains one turbo coding block. A BCJR turbo decoding with eight decoding iterations is adopted. QPSK and 16-QAM modulation schemes are tested, and results with k =1, 2, 3, 4 iterations are shown. The optimal detection order as in [2], [3] is adopted for both the MMSE-ISDIC and the PDA. We assume that $\boldsymbol{H}$ and N_0 are known at the receiver. In Fig. 1, bit error rate (BER) versus average received signal-to-noise ratio (SNR) (defined as N_T/N_0 in this letter) performance comparisons between the two detectors are given for both uncoded and coded systems (at least 3×10^5 randomly generated channel realizations are tested for each SNR). We see that the BER curves of the two detectors overlap each other at each k (i.e., the two detectors have the same trajectory of convergence), which coincides with our analytical results. Note that $k = 2$ is enough to acquire the best performance, and further iterations do not necessarily improve performance, which can be explained by the unreliable soft decisions produced by the previous iteration [4, Sec. 4].

V. Conclusions

In this letter, we have drawn a detailed comparison between the PDA and the MMSE-ISDIC detectors in respect of the V-BLAST systems. The two detectors perform in a similar manner and differ only in the metric calculation. We have proved that for both the original signal model and the decorrelated signal model, the two detectors are equivalent in terms of performance if the same detection order is adopted by both detectors. The performance equivalence proposition between the two detectors holds for both uncoded and coded systems and has no restriction on modulation constellation.[1]

APPENDIX

A) By using the matrix inversion lemma, it is straightforward that

$$\begin{aligned}\boldsymbol{h}_i^H\boldsymbol{R}_{\boldsymbol{n}_i}^{-1} &= \boldsymbol{h}_i^H\left(\sum_{j=1,j\neq i}^{N_T} v_j\boldsymbol{h}_j\boldsymbol{h}_j^H + N_0\boldsymbol{I}_{N_R}\right)^{-1}\\ &= \boldsymbol{h}_i^H\left(\boldsymbol{R}_i - \boldsymbol{h}_j\boldsymbol{h}_j^H\right)^{-1}\\ &= \boldsymbol{h}_i^H\left[\boldsymbol{R}_i^{-1} + \boldsymbol{R}_i^{-1}\boldsymbol{h}_i\boldsymbol{h}_i^H\boldsymbol{R}_i^{-1}/\left(1-\boldsymbol{h}_i^H\boldsymbol{R}_i^{-1}\boldsymbol{h}_i\right)\right]\\ &= \boldsymbol{f}_i^M + \mu_i^M\boldsymbol{f}_i^M/\left(1-\mu_i^M\right)\\ &= \boldsymbol{f}_i^M/\left(1-\mu_i^M\right)\end{aligned} \tag{A1}$$

where $\boldsymbol{f}_i^O = \boldsymbol{h}_i^H\boldsymbol{R}_{\boldsymbol{n}_i}^{-1}$ is the optimum combination (OC) filtering [5] on $\boldsymbol{r}_{\mathrm{IC},i}$ for detecting x_i, we have

$$\boldsymbol{f}_i^O = \boldsymbol{f}_i^M/\left(1-\mu_i^M\right). \tag{A2}$$

B) By definition, the residual interference variance $v_i(i = 1,2,\ldots,N_T)$ is nonzero, hence it is valid to assume that $\mathcal{D}_i$ is invertible, we then have

$$\begin{aligned}&\boldsymbol{\mathcal{D}}_i\boldsymbol{H}^H\left(\boldsymbol{H}\boldsymbol{\mathcal{D}}_i\boldsymbol{H}^H + N_0\boldsymbol{I}_{N_R}\right)^{-1}\\ &= \boldsymbol{\mathcal{D}}_i^{1/2}\left(\boldsymbol{H}\boldsymbol{\mathcal{D}}_i^{1/2}\right)^H\left[\boldsymbol{H}\mathcal{D}_i^{1/2}\left(\boldsymbol{H}\boldsymbol{\mathcal{D}}_i^{1/2}\right)^H + N_0\boldsymbol{I}_{N_R}\right]^{-1}\\ &= \boldsymbol{\mathcal{D}}_i^{1/2}\left[\left(\boldsymbol{H}\boldsymbol{\mathcal{D}}_i^{1/2}\right)^H\boldsymbol{H}\boldsymbol{\mathcal{D}}_i^{1/2} + N_0\boldsymbol{I}_{N_\mathrm{T}}\right]^{-1}\left(\boldsymbol{H}\boldsymbol{\mathcal{D}}_i^{1/2}\right)^H\\ &= \boldsymbol{\mathcal{D}}_i^{1/2}\left(\boldsymbol{\mathcal{D}}_i^{1/2}\boldsymbol{H}^H\boldsymbol{H}\boldsymbol{\mathcal{D}}_i^{1/2} + N_0\boldsymbol{I}_{N_\mathrm{T}}\right)^{-1}\boldsymbol{\mathcal{D}}_i^{1/2}\boldsymbol{H}^H\\ &= \boldsymbol{\mathcal{D}}_i^{1/2}\left(\boldsymbol{H}^H\boldsymbol{H}\boldsymbol{\mathcal{D}}_i^{1/2} + N_0\boldsymbol{\mathcal{D}}_i^{-1/2}\right)^{-1}\boldsymbol{H}^H\\ &= \boldsymbol{\mathcal{D}}_i\left(\boldsymbol{H}^H\boldsymbol{H}\boldsymbol{\mathcal{D}}_i + N_0\boldsymbol{I}_{N_\mathrm{T}}\right)^{-1}\boldsymbol{H}^H.\end{aligned} \tag{A3}$$

Therefore, we arrive at $\boldsymbol{H}^H(\boldsymbol{H}\mathcal{D}_i\boldsymbol{H}^H + N_0\boldsymbol{I}_{N_R})^{-1} = (\boldsymbol{H}^H\boldsymbol{H}\mathcal{D}_i + N_0\boldsymbol{I}_{N_T})^{-1}\boldsymbol{H}^H$.

C) Let us introduce the match-filtered signal model $\check{\boldsymbol{r}} = \boldsymbol{H}^H\boldsymbol{r} = \boldsymbol{H}^H\boldsymbol{H}x + \boldsymbol{H}^H\boldsymbol{w}$, the MMSE filtering on soft interference-cancelled signal vector $\check{\boldsymbol{r}}_{\mathrm{IC},i} = \check{\boldsymbol{r}} - \sum_{j=1,j\neq i}^{N_T}\boldsymbol{H}^H\boldsymbol{h}_j\tilde{x}_j$ for detecting x_i is

$$\begin{aligned}\check{\boldsymbol{f}}_i^M &= \boldsymbol{h}_i^H\boldsymbol{H}(\boldsymbol{H}^H\boldsymbol{H}\boldsymbol{\mathcal{D}}_i\boldsymbol{H}^H\boldsymbol{H} + N_0\boldsymbol{H}^H\boldsymbol{H})^{-1}\\ &= \boldsymbol{h}_i^H\boldsymbol{H}(\boldsymbol{H}^H\boldsymbol{H})^{-1}\left(\boldsymbol{H}^H\boldsymbol{H}\boldsymbol{\mathcal{D}}_i + N_0\boldsymbol{I}_{N_\mathrm{T}}\right)^{-1}\\ &= \left[\left(\boldsymbol{H}^H\boldsymbol{H}\boldsymbol{\mathcal{D}}_i + N_0\boldsymbol{I}_{N_\mathrm{T}}\right)^{-1}\right]_i\end{aligned} \tag{A4}$$

where we use the assumption that $\boldsymbol{H}^H\boldsymbol{H}$ is invertible. $\check{\boldsymbol{f}}_i^M$ and $\boldsymbol{f}_i^M$ are related by

$$\begin{aligned}\check{\boldsymbol{f}}_i^M\boldsymbol{H}^H &= \left[\left(\boldsymbol{H}^H\boldsymbol{H}\boldsymbol{\mathcal{D}}_i + N_0\boldsymbol{I}_{N_\mathrm{T}}\right)^{-1}\boldsymbol{H}^H\right]_i\\ &= \boldsymbol{h}_i^H\left(\boldsymbol{H}\boldsymbol{\mathcal{D}}_i\boldsymbol{H}^H + N_0\boldsymbol{I}_{N_R}\right)\\ &= \boldsymbol{f}_i^M.\end{aligned} \tag{A5}$$

Let us denote $\check{\boldsymbol{f}}_i^O$ as the OC filtering on $\check{\boldsymbol{r}}_{\mathrm{IC},i}$ for detecting x_i, we have

$$\begin{aligned}\check{\boldsymbol{f}}_i^O &= \boldsymbol{h}_i^H\boldsymbol{H}\left(\boldsymbol{H}^H\boldsymbol{H}\boldsymbol{\mathcal{D}}_i\boldsymbol{H}^H\boldsymbol{H} - \boldsymbol{H}^H\boldsymbol{h}_i\boldsymbol{h}_i^H\boldsymbol{H} + N_0\boldsymbol{H}^H\boldsymbol{H}\right)^{-1}\\ &= \boldsymbol{h}_i^H\boldsymbol{H}\left(\boldsymbol{H}^H\boldsymbol{R}_{\boldsymbol{n}_i}\boldsymbol{H}\right)^{-1}.\end{aligned} \tag{A6}$$

By using the matrix inversion lemma as in Appendix A, after some calculations, we arrive at

$$\check{\boldsymbol{f}}_i^O = \check{\boldsymbol{f}}_i^M/\left(1-\check{\mu}_i^M\right) \tag{A7}$$

where $\check{\mu}_i^M = \check{\boldsymbol{f}}_i^M\boldsymbol{H}^H\boldsymbol{h}_i = \boldsymbol{f}_i^M\boldsymbol{h}_i = \mu_i^M$. From (A2), (A5), (A6), and (A7), we arrive at

$$\begin{aligned}\boldsymbol{f}_i^O &= \boldsymbol{h}_i^H\boldsymbol{R}_{\boldsymbol{n}_i}^{-1} = \boldsymbol{f}_i^M/(1-\mu_i^M)\\ &= \check{\boldsymbol{f}}_i^M\boldsymbol{H}^H/\left(1-\mu_i^M\right) = \check{\boldsymbol{f}}_i^O\boldsymbol{H}^H\\ &= \boldsymbol{h}_i^H\boldsymbol{H}\left(\boldsymbol{H}^H\boldsymbol{R}_{\boldsymbol{n}_i}\boldsymbol{H}\right)^{-1}\boldsymbol{H}^H = \boldsymbol{h}_i^H\boldsymbol{Q}_{\boldsymbol{n}_i}.\end{aligned} \tag{A8}$$

[1]In addition, we would like to note that for square/rectangular QAM modulation, the performance equivalence between the two detectors also holds if real-valued transformation as in [3] is adopted (the MMSE-ISDIC for real-valued signal model is just a straightforward extension of [1]). That is, for real-valued signal model, the performance equivalence between the two detectors holds for both original and decorrelated models, and both uncoded and coded systems, which is validated by our numerous simulation results (not shown for page limitation) and can be proved straightforwardly based on the methods in this letter.

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