Note 4. MIMO Systems – Multiplexing

Diversity vs. Multiplexing

- Multiple input multiple output (MIMO) systems
  1. Increase ***data rates*** through ***multiplexing***

     $\Rightarrow$ Independent signaling paths that can be used to send independent data
  2. Improve ***performance*** through ***diversity*** $\Rightarrow$ Diversity gain

- Up to now, diversity schemes for MIMO systems were introduced.
  - Receiver Diversity – MRC, EGC, SC
    - Channel information at the receiver
      - Open-loop systems
  - Transmit Diversity – MRT, Antenna selection
    - Channel information at the receiver & the transmitter
      - Close-loop systems
  - Transmit Diversity – STBC
    - Channel information at the receiver
      - Open-loop systems

Spatial Multiplexing

- Spatially multiplexed MIMO systems (2x2 example)

$$\mathbf{y}_1 = \mathbf{H}_1\mathbf{x}_1 + \mathbf{n}_1 \qquad \mathbf{y}_2 = \mathbf{H}_2\mathbf{x}_2 + \mathbf{n}_2$$

- MIMO System with *N* TX & *M* RX antennas
  - 매 전송시간마다 *N*개의 독립적인 송신 심볼이 전송될 수 있음.
  - 하지만, Diversity(MISO/SIMO) 기법들과는 다르게, 이제 각 심볼들은 서로에게 영향을 주는 형태로 들어오며, Alamouti 기법처럼 간단하게 분리할 수도 없음
  - ML (Maximum-likelihood), MMSE (Minimum mean-square-error), ZF (Zero-Forcing) 등의 검출 기술이 필요

Linear Receiver

- MIMO System Model with N TX & M RX antennas (frequency flat fading is assumed)

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

  - where $\mathbf{y}$: $M \times 1$ received signal vector
    $\mathbf{H}$: $M \times N$ MIMO channel matrix
    $\mathbf{x}$: $N \times 1$ transmitted signal vector
    $\mathbf{n}$: $M \times 1$ AWGN (additive white Gaussian noise) vector

- Linear receiver for spatial multiplexing
  - Using the linear filter with low computational complexity
    - Separate the transmitted data streams, and then independently decode each stream
  - Zero-Forcing(ZF)
  - Minimum Mean Square Error (MMSE)

ZF Detection (1/3)

- ZF (Zero-Forcing) Detection
  - ZF Filter

$$\mathbf{G}_{ZF} = \begin{cases} \mathbf{H}^{-1} & M = N \\ \mathbf{H}^{\dagger} = \left(\mathbf{H}^H\mathbf{H}\right)^{-1}\mathbf{H}^H & M > N \end{cases}$$

  - Output of the ZF receiver

$$\mathbf{z} = \mathbf{G}_{ZF}\mathbf{y} = \begin{cases} \mathbf{H}^{-1}\left(\mathbf{Hx}+\mathbf{n}\right) = \mathbf{x}+\mathbf{H}^{-1}\mathbf{n} & M = N \\ \left(\mathbf{H}^H\mathbf{H}\right)^{-1}\mathbf{H}^H\left(\mathbf{Hx}+\mathbf{n}\right) = \mathbf{x}+\left(\mathbf{H}^H\mathbf{H}\right)^{-1}\mathbf{H}^H\mathbf{n} & M > N \end{cases}$$

  - Cannot be applied when $M < N$
  - Low computational complexity
    - Main complexity – Matrix inversion & multiplication
  - Completely eliminate the inter-stream interference.
  - Noise Enhancement

$$\mathbf{n}_{after\ \mathrm{ZF}} = \begin{cases} \mathbf{H}^{-1}\mathbf{n} & M = N \\ \left(\mathbf{H}^H\mathbf{H}\right)^{-1}\mathbf{H}^H\mathbf{n} & M > N \end{cases}$$

ZF Detection (2/3)

- 2x2 BPSK (Binary Shift Phase Keying) example

$$\mathbf{y} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \ \mathbf{H} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$$

  - ZF filter – Inverse of $\mathbf{H}$

$$\mathbf{G}_{ZF} = \mathbf{H}^{-1} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

  - Applying ZF filter to the receive signal vector

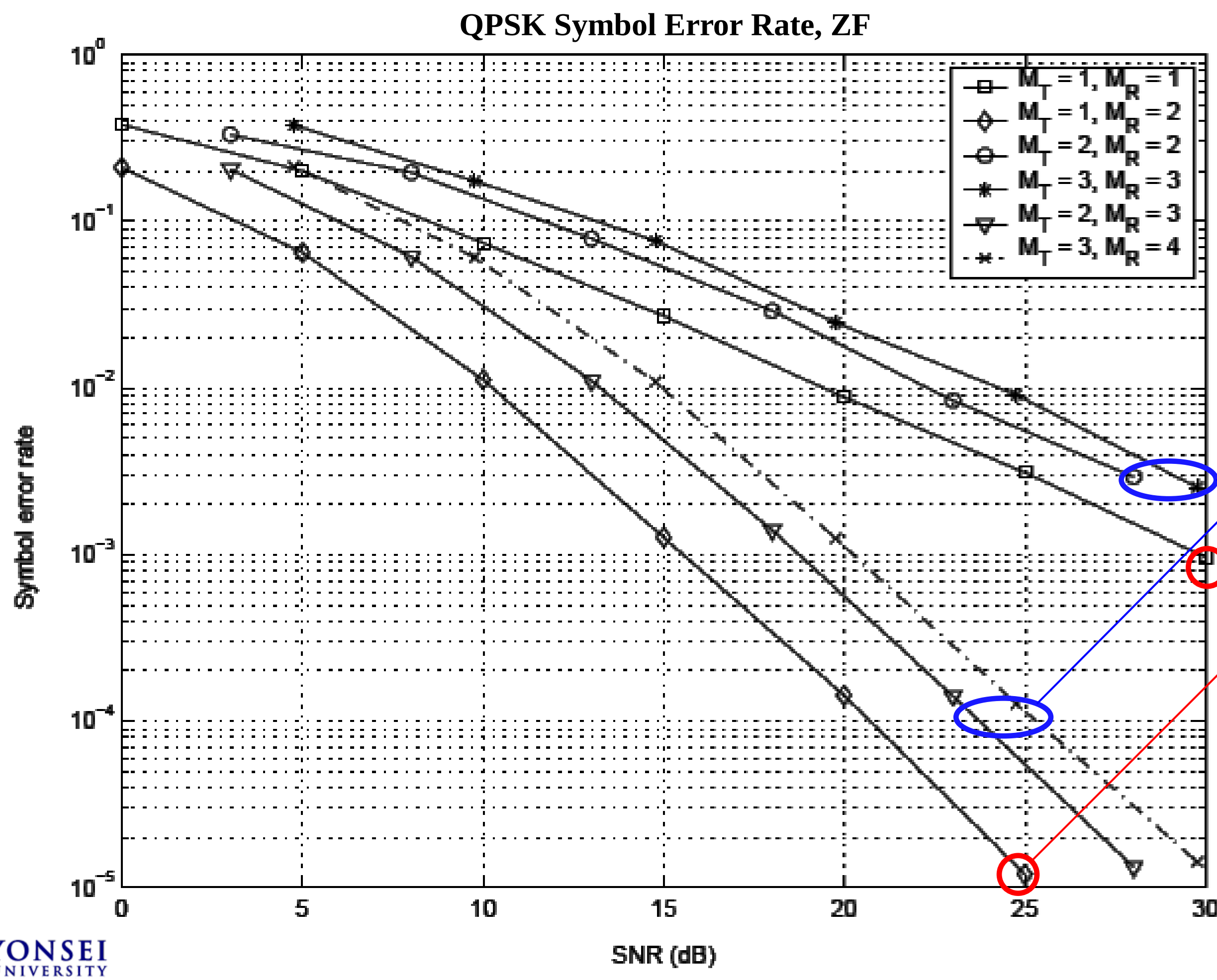
$$\mathbf{G}_{ZF}\mathbf{y} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 6 \\ 9 \end{bmatrix}$$

  - Find the closest possible TX symbol

$$\mathbf{G}_{ZF}\mathbf{y} = \begin{bmatrix} \boxed{6} \\ \boxed{9} \end{bmatrix} \rightarrow \begin{bmatrix} \{\boxed{1}, -1\} \\ \{\boxed{1}, -1\} \end{bmatrix} \rightarrow \begin{bmatrix} \boxed{1} \\ \boxed{1} \end{bmatrix}$$

ZF Detection (3/3)

- Diversity order of ZF detection – $M - N + 1$
  - SIMO – ZF is equivalent to MRC



The slope of the curve (diversity order) with $M \times N$ MIMO with ZF

$||$

The slope of the curve (diversity order) with $M - N + 1$ SIMO using MRC

MMSE Detection (1/3)

- MMSE (Minimum Mean-Square-Error) Detection
  - MMSE Filter (In fact, Linear MMSE filter)

$$\mathbf{G}_{MMSE} = \arg\min_{\mathbf{G}} E\left\{\left\|\mathbf{G}\mathbf{y}-\mathbf{x}\right\|_F^2\right\}$$

$$\mathrm{E}\left\{(\mathbf{G}\mathbf{y}-\mathbf{x})\mathbf{y}^H\right\} = \mathbf{0}_{N,M} \longleftarrow$$

The orthogonality principle: Sufficient & Necessary condition to achieves minimum mean-square error → between error & observations

$$\mathrm{E}\left\{\mathbf{G}\mathbf{y}\mathbf{y}^H\right\} - \mathrm{E}\left\{\mathbf{x}\mathbf{y}^H\right\} = \mathbf{0}_{N,M}$$

$$\begin{aligned}
\mathrm{E}\left\{\mathbf{G}\mathbf{y}\mathbf{y}^H\right\} &= \mathbf{G}\mathrm{E}\left\{\mathbf{y}\mathbf{y}^H\right\} \\
&= \mathbf{G}\mathrm{E}\left\{(\mathbf{H}\mathbf{x}+\mathbf{n})(\mathbf{H}\mathbf{x}+\mathbf{n})^H\right\} \\
&= \mathbf{G}\mathrm{E}\left\{\mathbf{H}\mathbf{x}\mathbf{x}^H\mathbf{H}^H + \mathbf{n}\mathbf{n}^H + \mathbf{n}\mathbf{x}^H\mathbf{H}^H + \mathbf{H}\mathbf{x}\mathbf{n}^H\right\} \\
&= \mathbf{G}\left[\mathrm{E}\left\{\mathbf{H}\mathbf{x}\mathbf{x}^H\mathbf{H}^H\right\} + \mathrm{E}\left\{\mathbf{n}\mathbf{n}^H\right\} + \mathrm{E}\left\{\mathbf{n}\mathbf{x}^H\mathbf{H}^H\right\} + \mathrm{E}\left\{\mathbf{H}\mathbf{x}\mathbf{n}^H\right\}\right] \\
&= \mathbf{G}(\mathbf{H}\mathbf{H}^H + \sigma_n^2\mathbf{I})
\end{aligned}$$

$$\begin{aligned}
\mathrm{E}\left\{\mathbf{x}\mathbf{y}^H\right\} &= \mathrm{E}\left\{\mathbf{x}(\mathbf{H}\mathbf{x}+\mathbf{n})^H\right\} \\
&= \mathrm{E}\left\{\mathbf{x}\mathbf{x}^H\mathbf{H}^H + \mathbf{x}\mathbf{n}^H\right\} \\
&= \mathrm{E}\left\{\mathbf{x}\mathbf{x}^H\right\}\mathbf{H}^H + \mathrm{E}\left\{\mathbf{x}\mathbf{n}^H\right\} = \mathbf{H}^H
\end{aligned}$$

$$\mathbf{G}(\mathbf{H}\mathbf{H}^H + \sigma_n^2\mathbf{I}) - \mathbf{H}^H = \mathbf{0}_{N,M}$$

$$\mathbf{G}(\mathbf{H}\mathbf{H}^H + \sigma_n^2\mathbf{I}) = \mathbf{H}^H$$

$$\boxed{\begin{aligned}\mathbf{G}_{MMSE} &= \mathbf{H}^H(\mathbf{H}\mathbf{H}^H + \sigma_n^2\mathbf{I})^{-1} \\ &= (\mathbf{H}^H\mathbf{H} + \sigma_n^2\mathbf{I})^{-1}\mathbf{H}^H\end{aligned}}$$

$$\begin{bmatrix} \mathbf{A}_{r\times r} & \mathbf{B}_{r\times s} \\ \mathbf{C}_{s\times r} & \mathbf{D}_{s\times s} \end{bmatrix}$$

The matrix inversion lemma :

$$(\mathbf{A}-\mathbf{B}\mathbf{D}^{-1}\mathbf{C})^{-1}\mathbf{B}\mathbf{D}^{-1} = \mathbf{A}^{-1}\mathbf{B}(\mathbf{D}-\mathbf{C}\mathbf{A}^{-1}\mathbf{B})^{-1}$$

MMSE Detection (2/3)

- 2x2 BPSK (Binary Shift Phase Keying) example

$$\mathbf{y} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \ \mathbf{H} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$$

  - Linear MMSE filter (when $\sigma_n^2 = 1$)

$$\mathbf{G}_{MMSE} = (\mathbf{H}^H\mathbf{H} + \mathbf{I})^{-1}\mathbf{H}^H = \begin{bmatrix} 1/3 & 0 \\ 0 & 1/3 \end{bmatrix}$$

  - Applying MMSE filter to the receive signal vector

$$\mathbf{G}_{MMSE}\mathbf{y} = \begin{bmatrix} 1/3 & 0 \\ 0 & 1/3 \end{bmatrix}\begin{bmatrix} 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

  - Find the closest possible TX symbol

$$\mathbf{G}_{MMSE}\mathbf{y} = \begin{bmatrix} \boxed{1} \\ \boxed{1} \end{bmatrix} \rightarrow \begin{bmatrix} \{\boxed{1,} -1\} \\ \{\boxed{1,} -1\} \end{bmatrix} \rightarrow \begin{bmatrix} \boxed{1} \\ \boxed{1} \end{bmatrix}$$

MMSE Detection (3/3)

- MMSE (Minimum Mean-Square-Error) Detection
  - Can be applied when $M < N$
  - Low computational complexity – Almost same to ZF detection
    - Main complexity – Matrix inversion & multiplication
  - Balance the interference mitigation and the noise enhancement
    - Achieving better Post-Processing SINR than ZF → Better error performance
    - Diversity order – $M - N + 1$ → Equal to ZF detection
  - At low SNR, MMSE becomes MF (matched filter) detection

$$\left(\mathbf{H}^H\mathbf{H} + \frac{1}{SNR}\mathbf{I}\right)^{-1}\mathbf{H}^H \xrightarrow[SNR\to 0]{} \frac{1}{SNR}\mathbf{H}^H$$

  - At high SNR, MMSE becomes ZF detection

$$\left(\mathbf{H}^H\mathbf{H} + \frac{1}{SNR}\mathbf{I}\right)^{-1}\mathbf{H}^H \xrightarrow[SNR\to \infty]{} \left(\mathbf{H}^H\mathbf{H}\right)^{-1}\mathbf{H}^H$$

  - Disadvantage (Comparing with ZF) → Estimation of noise variance

ML Detection (1/4)

- MIMO System Model with N TX & M RX antennas (frequency flat fading is assumed)

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

  - where $\mathbf{y}$: $M \times 1$ received signal vector

    $\mathbf{H}$: $M \times N$ MIMO channel matrix

    $\mathbf{x}$: $N \times 1$ transmitted signal vector

    $\mathbf{n}$: $M \times 1$ AWGN (additive white Gaussian noise) vector

- ML (Maximum-likelihood) detection → Non-linear receivers
  - Optimal detection for MIMO systems
  - Algorithm to find the possible transmitted signal vector that maximizes the likelihood function.

  → Find $\arg\min_{\mathbf{x}' \in \mathbf{X}} \|\mathbf{y} - \mathbf{H}\mathbf{x}'\|_F^2$, where $\mathbf{X}$ is the set of possible TX vectors

ML Detection (2/4)

- 2x2 BPSK (Binary Shift Phase Keying) example

$$\mathbf{y} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \ \mathbf{H} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$$

  - The possible elements of **X** are (Symbol Energy = 1 for simple notations)

$$\mathbf{X} = \left\{ \mathbf{x}_1\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right), \mathbf{x}_2\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right), \mathbf{x}_3\left(\begin{bmatrix} -1 \\ 1 \end{bmatrix}\right), \mathbf{x}_4\left(\begin{bmatrix} -1 \\ -1 \end{bmatrix}\right) \right\}$$

  - ML Detection results: $\mathbf{x}_1$

$$\boxed{\mathbf{y} - \mathbf{H}\mathbf{x}_1 = \begin{bmatrix} 3 \\ 3 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \rightarrow 4}$$

$$\mathbf{y} - \mathbf{H}\mathbf{x}_2 = \begin{bmatrix} 3 \\ 3 \end{bmatrix} - \begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \end{bmatrix} \rightarrow 25$$

$$\mathbf{y} - \mathbf{H}\mathbf{x}_3 = \begin{bmatrix} 3 \\ 3 \end{bmatrix} - \begin{bmatrix} -3 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \end{bmatrix} \rightarrow 37$$

$$\mathbf{y} - \mathbf{H}\mathbf{x}_4 = \begin{bmatrix} 3 \\ 3 \end{bmatrix} - \begin{bmatrix} -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix} \rightarrow 16$$

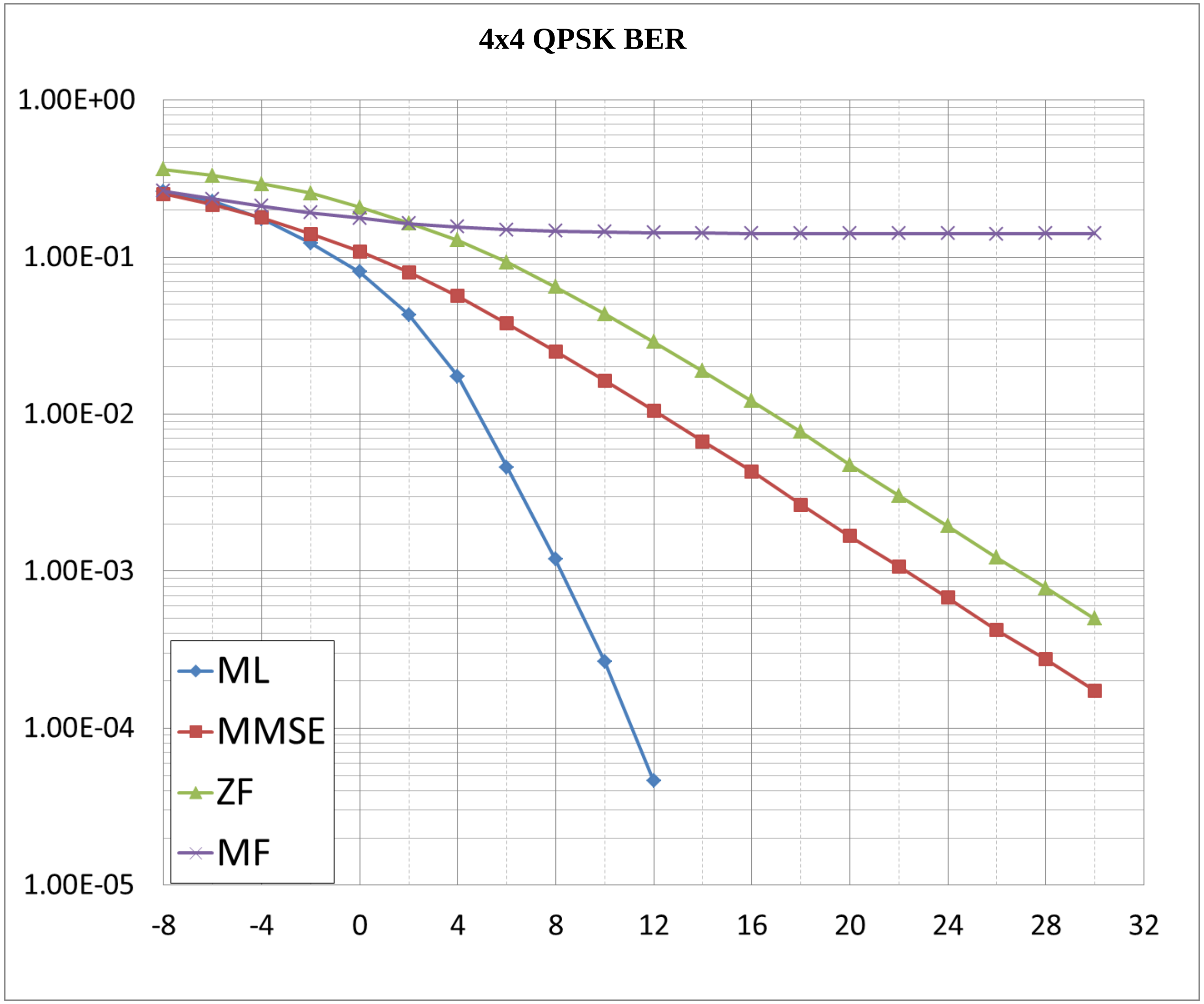
ML Detection (3/4)

- Problems of ML detection – Computational Complexity
  - In the previous example, 4 computations were performed, since the number of possible TX vectors was 4.
  - If the modulation utilizes a constellation with 2b points to transmit b bits, the number of possibilities for **x** is 2^{bN}
    - *N*: # of TX antennas
    - BPSK: 2 points to transmit 1 bits
    - QPSK: 4 points to transmit 2 bits
    - 16-QAM: 16 points to transmit 4 bits
    - ….
  - Ex) For 4 transmit antennas and 16-QAM,
    - The number of possible TX vectors – 2^{16}=65536

    → Impractical as *N* or *b* increases

ML Detection (4/4)

- Diversity order of ML detection – M



SIC Receiver (1/5)

- ML detection
  - Optimal performance, but too complicated!
- MMSE / ZF detections
  - Low computational complexity, but poor performance!
- Successive Interference Cancellation (Serial IC)
  - Based on the Linear Receiver (MMSE, ZF)
  - To achieve a find performance with a moderate computational complexity
  - Sequentially decode and subtract the TX symbol
- Sequential Detection Process
  - 1) Estimate one of TX symbols using Linear Filter
  - 2) Cancel it from the RX vectors, as if the TX symbol was not actually transmitted! → Go to 1)

SIC Receiver (2/5)

- 2x2 BPSK (Binary Shift Phase Keying) example (1/3)
  - ZF Filter Assumption

$$\mathbf{y} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \quad \mathbf{H} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$$

  - ZF filter – Inverse of $\mathbf{H}$

$$\mathbf{G}_{ZF} = \mathbf{H}^{-1} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

  - Applying ZF filter to the receive signal vector

$$\mathbf{G}_{ZF}\mathbf{y} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 6 \\ 9 \end{bmatrix}$$

  - Find the closest possible TX symbol

$$\mathbf{G}_{ZF}\mathbf{y} = \begin{bmatrix} \boxed{6} \\ \boxed{9} \end{bmatrix} \rightarrow \begin{bmatrix} \{\boxed{1,}-1\} \\ \{\boxed{1,}-1\} \end{bmatrix} \rightarrow \begin{bmatrix} \boxed{1} \\ \boxed{1} \end{bmatrix}$$

SIC Receiver (3/5)

- 2x2 BPSK (Binary Shift Phase Keying) example (2/3)
  - Select any TX symbol. (1st TX symbol in this example)

$$x_1 = 1$$

  - Cancel it from RX signals as if it was not actually transmitted.
    - Interference cancellation step

$$\mathbf{y} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \ \mathbf{H} = \begin{bmatrix} \boxed{\begin{matrix} 2 \\ -1 \end{matrix}} & \boxed{\begin{matrix} -1 \\ 1 \end{matrix}} \end{bmatrix} \rightarrow \mathbf{y} - \mathbf{h}_1 x_1 = \begin{bmatrix} 3 \\ 3 \end{bmatrix} - \begin{bmatrix} \boxed{\begin{matrix} 2 \\ -1 \end{matrix}} \end{bmatrix} \cdot 1 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

  - Also, reconfigure the channel matrix

$$\mathbf{H} = \begin{bmatrix} \boxed{\begin{matrix} 2 \\ -1 \end{matrix}} & \boxed{\begin{matrix} -1 \\ 1 \end{matrix}} \end{bmatrix} \rightarrow \mathbf{H}' = \begin{bmatrix} \boxed{\begin{matrix} -1 \\ 1 \end{matrix}} \end{bmatrix}$$

  - Then, the system model after the IC step becomes

$$\mathbf{y}' = \begin{bmatrix} 1 \\ 4 \end{bmatrix}, \ \mathbf{H}' = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

SIC Receiver (4/5)

- 2x2 BPSK (Binary Shift Phase Keying) example (3/3)

$$\mathbf{y}' = \begin{bmatrix} 1 \\ 4 \end{bmatrix}, \ \mathbf{H}' = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

  - ZF filter – Pseudo Inverse of $\mathbf{H}'$

$$\mathbf{G}_{ZF}' = ((\mathbf{H}')^H (\mathbf{H}'))^{-1} (\mathbf{H}')^H = \begin{bmatrix} -0.5 & 0.5 \end{bmatrix}$$

  - Applying ZF filter to the residual RX signal vector

$$\mathbf{G}_{ZF}'\mathbf{y}' = \begin{bmatrix} -0.5 & 0.5 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \end{bmatrix} = 1.5$$

  - Find the closest possible TX symbol

$$x_2 = 1$$

SIC Receiver (5/5)

- SIC (Successive Interference Cancellation)
  - Performance – Outperforms simple ZF and MMSE
  - Complexity – Increased from ZF and MMSE, but still requires a significantly lower computational complexity than ML.

- Main drawback of SIC
  - Error propagation
    - If there was an error in symbol detection in the early stage, it propagates to the future stages.
    - Example) Suppose $x_1 = -1$ in the previous example.
    - Then, $$\mathbf{y} - \mathbf{h}_1 x_1 = \begin{bmatrix} 3 \\ 3 \end{bmatrix} - \begin{bmatrix} 2 \\ -1 \end{bmatrix} \cdot -1 = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$
    - ZF filtering → $\mathbf{G}_{ZF}\,'\mathbf{y}' = \begin{bmatrix} -0.5 & 0.5 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \end{bmatrix} = -1.5 \rightarrow \boxed{x_2 = -1}$

Error!

OSIC Receiver (1/4)

- Ordered SIC (Successive Interference Cancellation)
  - To minimize the occurrence of error propagations in SIC
  - How? → Select the most reliable TX symbol among the residual TX symbols

- At every stage, the symbol(stream) with the highest SINR is decoded.
  - Compute SINR on all streams.
  - Choose a stream with the highest SINR.
  - Perform IC step.
  - Repeat with updated channel matrix until all streams are decoded.

OSIC Receiver (2/4)

- OSIC receiver
  - Step 1 : Initialization

$i=1,\quad \mathbf{r}_1=\mathbf{r},\quad \mathbf{H}_1=\mathbf{H}$

Calculate the initial weight matrix : $G_1=\begin{cases}\mathbf{H}^{\dagger} & ZF\\ \mathbf{H}^H\left(\mathbf{H}\mathbf{H}^H+\sigma^2\mathbf{I}\right)^{-1} & MMSE\end{cases}$

$(\mathbf{A})_k$: k-th column of matrix **A**

$(\mathbf{A})^j$: j-th row of matrix **A**

$\mathbf{A}_{\bar{k}}$: deleting the k-th column of **A**

  - Step 2 : OSIC

For $i=1:M$

Determine which substream to detect first

Ordering : $k_i=\begin{cases}\underset{j\notin\{k_1,\ldots,k_{i-1}\}}{\arg\max}\left(SNR_j\right) & ZF \longrightarrow SNR_j=\dfrac{1}{\sigma^2\left\|\left(\mathbf{G}_i\right)^j\right\|^2}\\ \underset{j\notin\{k_1,\ldots,k_{i-1}\}}{\arg\max}\left(SINR_j\right) & MMSE \longrightarrow SINR_j=\dfrac{\left\|\left(\mathbf{G}_i\mathbf{H}_i\right)_{j,j}\right\|^2}{\sum_{m=1,m\neq j}^{M_t}\left\|\left(\mathbf{G}_i\mathbf{H}_i\right)_{j,m}\right\|^2+\sigma^2\left\|\left(\mathbf{G}_i\right)^j\right\|^2}\end{cases}$

$\begin{pmatrix} i\ :\ \text{detection order}\\ j:\ \text{stream order}\end{pmatrix}$

Nulling vector : $\mathbf{w}_{k_i}=\left(\mathbf{G}_i\right)^{k_i}$ — **k_ith nulling vector is k_ith row of G**

Nulling : $y_{k_i}=\mathbf{w}_{k_i}\mathbf{r}_i$ — **Compute k_ith decision statistic**

Decision : $\hat{x}_{k_i}=Q\left(y_{k_i}\right)$ — **Estimate k_ith component of x** $\left(Q(\bullet)\ \text{: quantization function}\right)$

SIC : $\mathbf{r}_{i+1}=\mathbf{r}_i-\left(\mathbf{H}\right)_{k_i}\hat{x}_{k_i}$ — **Cancel detected component**

Upcate the channel matrix : $\mathbf{H}_{i+1}=\mathbf{H}_{\bar{k}_i}$ — **Candidate nulling vectors for deflated system**

Calculate the weight matrix : $\mathbf{G}_{i+1}=\begin{cases}\mathbf{H}_{i+1}^{\dagger} & ZF\\ \mathbf{H}_{i+1}^H(\mathbf{H}_{i+1}\mathbf{H}_{i+1}^H+\sigma^2\mathbf{I})^{-1} & MMSE\end{cases}$

end

OSIC Receiver (3/4)

- 2x2 BPSK (Binary Shift Phase Keying) example (1/3)
  - ZF Filter Assumption

$$\mathbf{y} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \ \mathbf{H} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$$

  - ZF filter – Inverse of $\mathbf{H}$

$$\mathbf{G}_{ZF} = \mathbf{H}^{-1} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

  - Calculate the SINR (in fact, SNR for ZF) after the ZF filter

$$SNR_1 = 1/(\sigma^2(1^2 + 1^2))$$

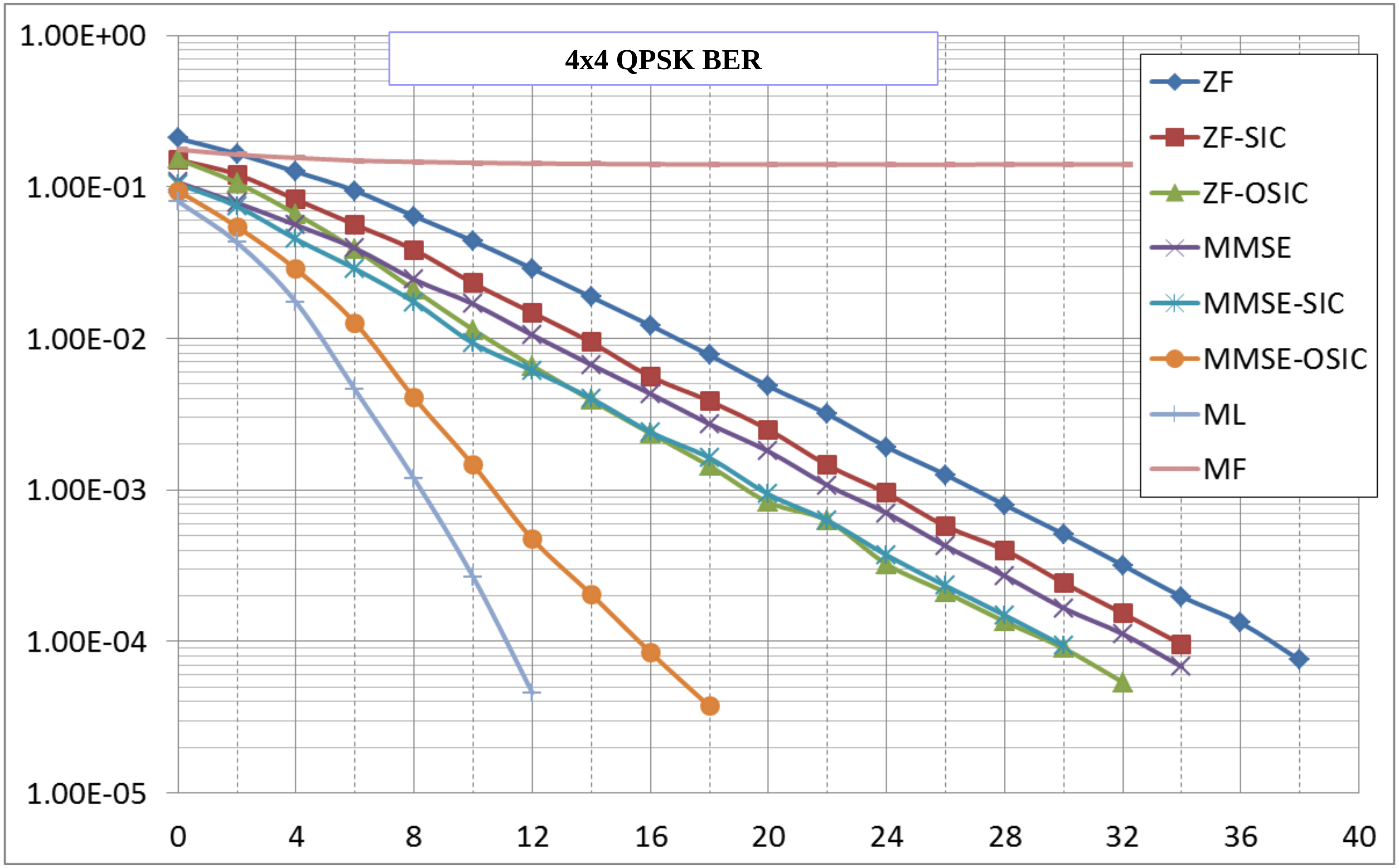
$$SNR_2 = 1/(\sigma^2(1^2 + 2^2))$$

  - 1^{st} TX symbol has a higher SINR than 2^{nd} TX symbol

  → Select the 1^{st} TX symbol & decoding / IC stages
    - The remainder is equivalent to SIC

OSIC Receiver (4/4)

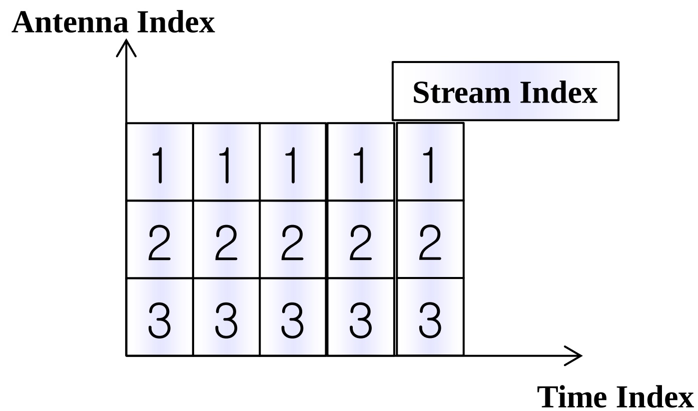
- Diversity order of SIC – Same as linear filter: $M-N+1$
- Diversity order of OSIC – Between ML and SIC



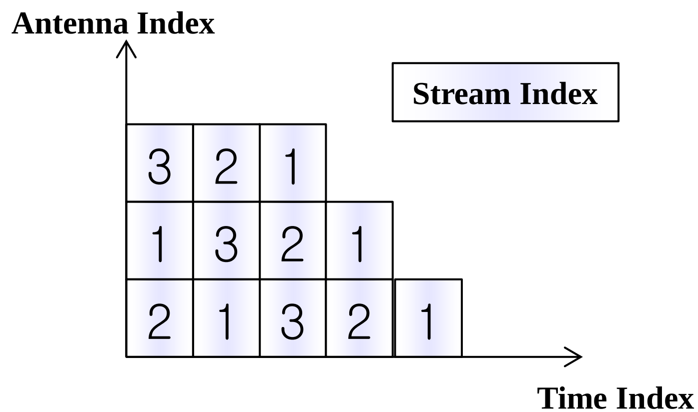
YONSEI UNIVERSITY

V-BLAST/D-BLAST Architecture
- Way to perform spatial multiplexing

- Vertical Bell Labs Layered Space-Time Architecture
  - Horizontal Layering
    - Smaller transmit diversity
    - No space-time waste
  - OSIC: originally for V-BLAST
    - OSIC is often referred as V-BLAST



- Diagonal Bell Labs Layered Space-Time Architecture
  - Diagonal Layering
    - Full Transmit Diversity → Reliability ↑
    - Space-time waste → Rate ↓
  - Detect sub-streams as its order
    - 1^{st} stream is detected first, 2^{nd}, 3^{rd}, …



To do list

- 2x2 ML, MMSE, ZF, MMSE-OSIC, MMSE-SIC, ZF-SIC, ZF-OSIC BER simulations
  - *N*x*M*: TX Antennas – *N*, RX Antennas – *M*
- SNR = 1 / σ^2 (E_s/N_o)
- 5 dB step (From 0dB ~20dB)

Thank You!