Latent Dirichlet Allocation
-Topic model

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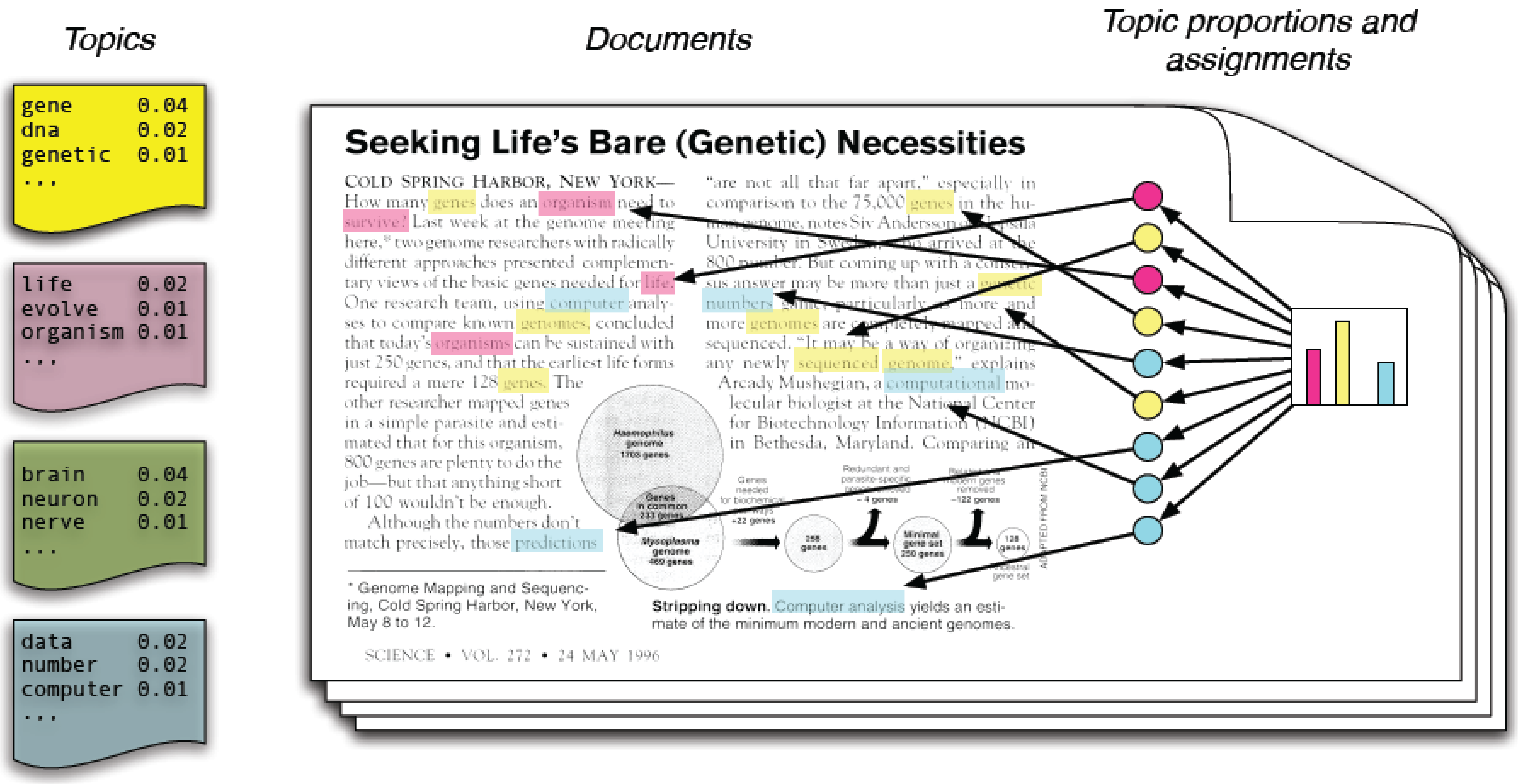
01
방법론 소개

History



- David Blei, Andrew Ng, and Michael I. Jordan in 2003
- Natural Language Process(NLP) 모델

Characteristics

- Document, Topic
  - Document : 다양한 Topic들의 혼합
  - Topic : Word들의 분포

- Generative
  - Observed로 모수를 학습하고 다시 Observed를 만든다.
  - 잘 알고 있는 GAN ~ image generative

- Statistical
  - word에 topic 배정시 Gibbs sampler 등

Objective

"Arts"	"Budgets"	"Children"	"Education"
NEW	MILLION	CHILDREN	SCHOOL
FILM	TAX	WOMEN	STUDENTS
SHOW	PROGRAM	PEOPLE	SCHOOLS
MUSIC	BUDGET	CHILD	EDUCATION
MOVIE	BILLION	YEARS	TEACHERS
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LOVE	CONGRESS	LIFE	HAITI

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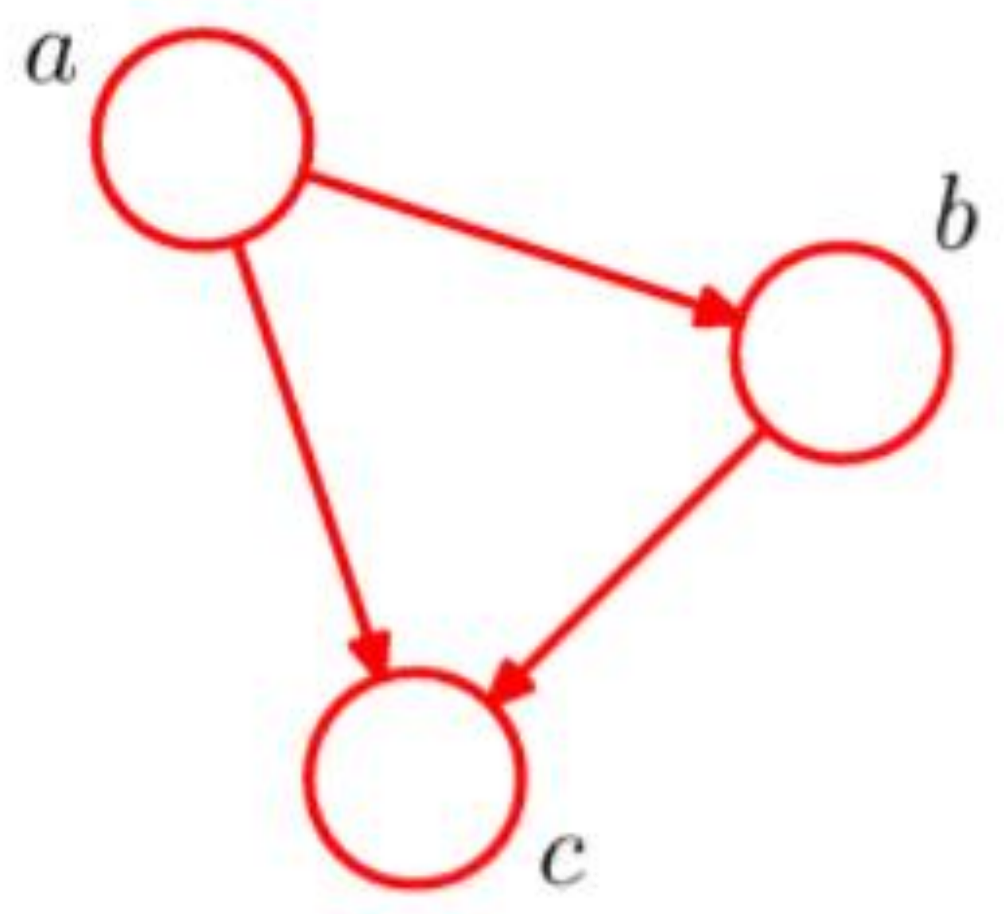
- How to assign a topic to a word ?

Blei, et al., Jordan, Michael I (January 2003). Lafferty, John, ed. "Latent Dirichlet Allocation". Journal of Machine Learning Research. 3 (4–5): pp. 993–1022

02
Bayesian Network

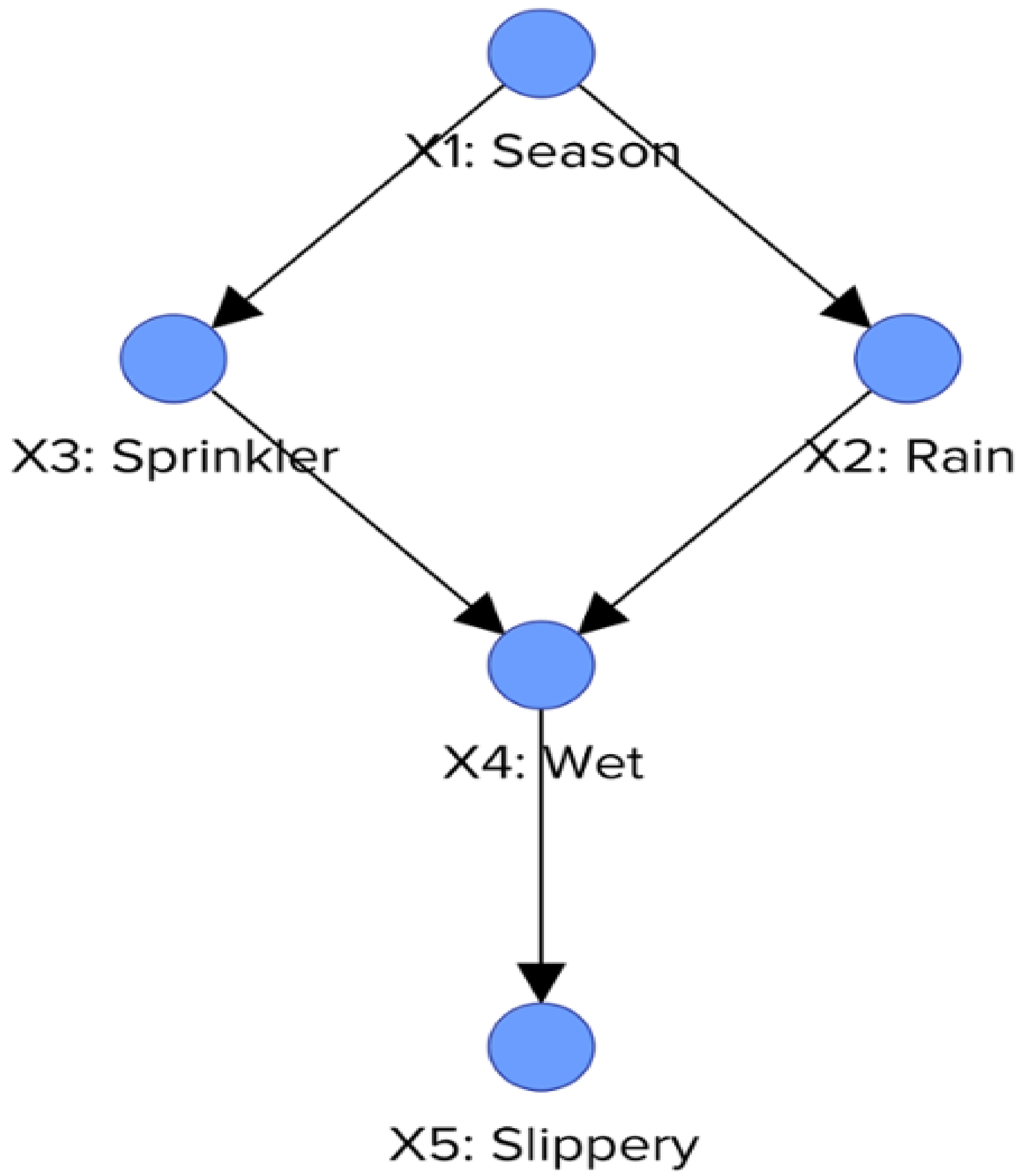
Graphical Representation

$$
\begin{aligned}
p(a,b,c) &= p(c|a,b)p(a,b) \\
&= p(c|a,b)p(b|a)p(a)
\end{aligned}
\quad \rightleftarrows
$$



- 확률분포를 그래픽적 모형으로 나타낼 수 있다.
- 시각적 직관적으로 모델을 파악할 수 있다.
- node는 확률변수, edge는 변수들 사이의 조건부 관계를 의미한다.

Bayesian Network



$$p(x_1, x_2, x_3, x_4, x_5)$$
$$= p(x_3|x_1)p(x_2|x_1)p(x_4|x_2, x_3)p(x_5|x_4)$$

- Design our Belif to graphical structure
- 결합확률분포를 얻을 수있다.

http://www.bayesia.com/bayesian-networks-examples

03

Gibbs Sampling

Intuition

$$\theta_1^{(j)} \sim p\left(\theta_1 | \theta_2^{(j-1)}, \ldots, \theta_K^{(j-1)}\right)$$

$$\theta_2^{(j)} \sim p\left(\theta_2 | \theta_1^{(j)}, \theta_3^{(j-1)}, \ldots, \theta_K^{(j-1)}\right)$$

$$\vdots$$

$$\theta_k^{(j)} \sim p\left(\theta_k | \theta_1^{(j)} \ldots, \theta_{k-1}^{(j)}, \theta_{k+1}^{(j-1)}, \ldots, \theta_K^{(j-1)}\right)$$

$$\vdots$$

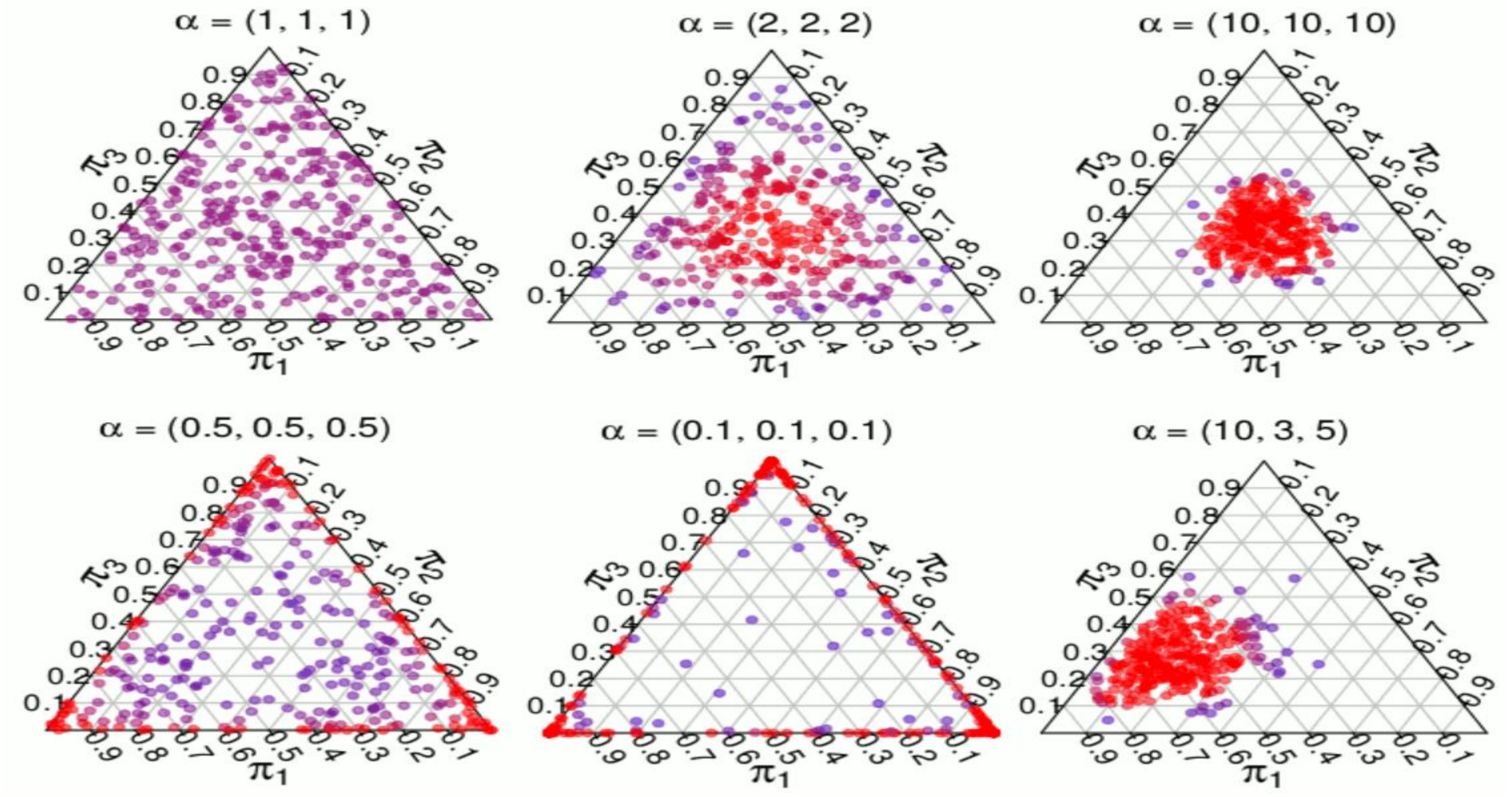
$$\theta_K^{(j)} \sim p\left(\theta_K | \theta_1^{(j)}, \ldots, \theta_{K-1}^{(j)}\right)$$

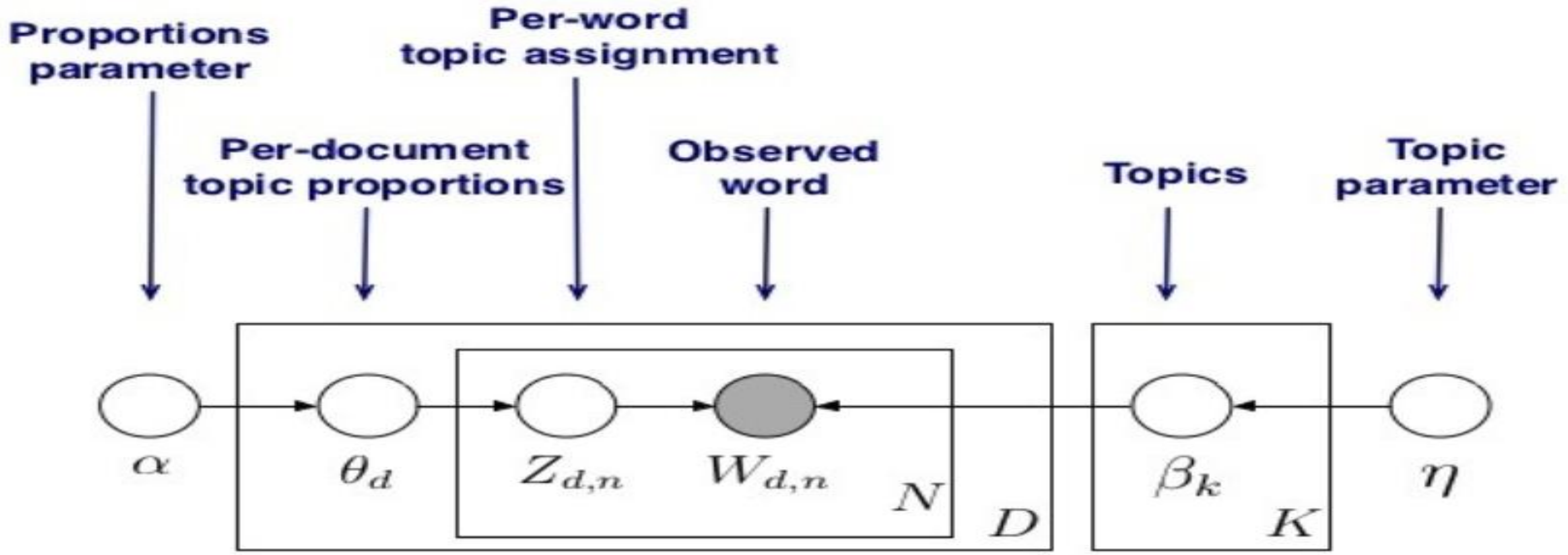
- 고차원 결합확률 분포는 계산하기 어렵다.
- 거기서 sample들을 뽑아보자
- 그런데 한 번에 뽑기 어려우니 다른 것들이 주어 졌다고 치고 하나씩 뽑자
- 다 뽑으면 그것이 하나의 sample이다.
- 여기 까지만 알자.

04
Model Structure

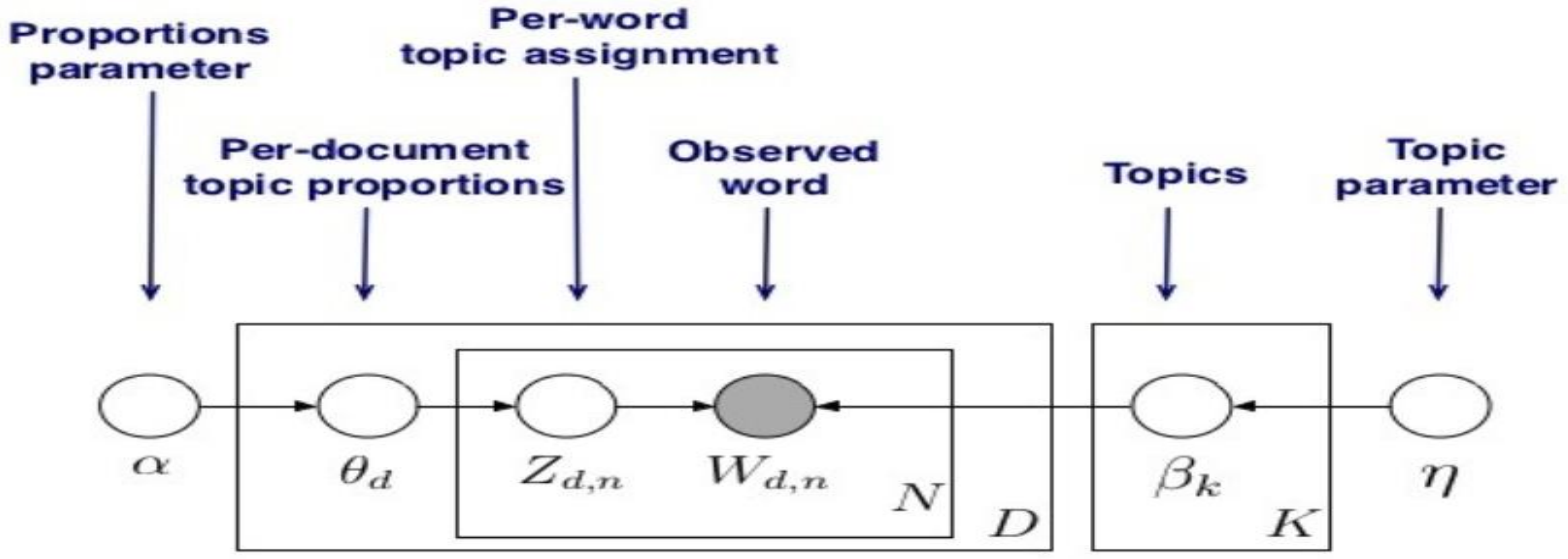
Dirichlet Distribution

$$f(x_1,...,x_k;\alpha_1,...,\alpha_k) = \frac{\prod_{i=1}^{k}\Gamma(\alpha_i)}{\Gamma(\sum_{i=1}^{k}\alpha_i)}\prod_{i=1}^{k}x_i^{\alpha_i-1}, \sum_{i=1}^{k}x_i = 1, x_i \geq 0$$





- $\alpha \in \mathbb{R}^K$: 문서별 주제에 대한 Dirichlet 사전 분포 모수
- $\eta \in \mathbb{R}^V$: 주제별 단어에 대한 Dirichlet 사전 분포
- $\theta_d \in \mathbb{R}^K$: 문서 d에 대한 주제 분포
- $\beta_k \in \mathbb{R}^V$: 주제 k에 대한 단어 분포
- $Z_{d,n}$: 문서 d의 n번째 단어의 주제
- $W_{d,n}$: 문서 d의 n번째 단어



$$p(\beta, \theta, z, w|\alpha, \eta) = \prod_{i=1}^{K} p(\beta_i|\eta) \prod_{d=1}^{D} p(\theta_d|\alpha)(\prod_{n=1}^{N} p(z_{d,n}|\theta_d)p(w_{d,n}|\beta_{1:K}, z_{d,n}))$$

05

Tedious Computation

1. Marginalization

Start with initializing latent topic to all words

$$P(z,w|\alpha,\eta) = \int_\theta \int_\beta p(\beta,\theta,z,w|\alpha,\eta)\,d\beta\, d\theta$$

$$= \underline{\int_\beta \prod_{i=1}^{K} p(\beta_i|\eta) \prod_{d=1}^{D} \prod_{n=1}^{N} p(w_{d,n}|\beta_{1:K}, z_{d,n})\,d\beta} \times \underline{\int_\theta \prod_{d=1}^{D} p(\theta_d|\alpha) \prod_{n=1}^{N} p(z_{d,n}|\theta_d)\,d\theta}$$

$$\int_\beta \prod_{i=1}^{K} p(\beta_i|\eta) \prod_{i=1}^{K} \prod_{n=1}^{N} p(w_{d,n}|\beta_{1:K}, z_{d,n})\,d\beta = \prod_{i=1}^{K} \int_{\beta_i} p(\beta_i|\eta) \prod_{d=1}^{D} \prod_{n=1}^{N} p(w_{d,n}|\beta_{1:K}, z_{d,n})\,d\beta_i$$

$$\int_\theta \prod_{d=1}^{D} p(\theta_d|\alpha) \prod_{n=1}^{N} p(z_{d,n}|\theta_d)\,d\theta = \prod_{d=1}^{D} \int_{\theta_d} p(\theta_d|\alpha) \prod_{n=1}^{N} p(z_{d,n}|\theta_d)\,d\theta_d$$

2. Build Gibbs Sampler

$$P(Z,W|\alpha,\eta)=\prod_{d=1}^{D}\frac{\Gamma(\sum_{i=1}^{K}\alpha_i)}{\prod_{i=1}^{K}\Gamma(\alpha_i)}\frac{\prod_{i=1}^{K}\Gamma(n_d^i+\alpha_i)}{\Gamma(\sum_{i=1}^{K}(n_d^i+\alpha_i))}\times\prod_{i=1}^{K}\frac{\Gamma(\sum_{v=1}^{V}\eta_v)}{\prod_{v=1}^{V}\Gamma(\eta_v)}\frac{\prod_{v=1}^{V}\Gamma(n_v^i+\eta_v)}{\Gamma(\sum_{v=1}^{V}n_v^i+\eta_v)}$$

This enable you calculate below form

$$P(Z_{(d,n)}=k|Z_{-(d,n)},W,\alpha,\eta)\propto P(Z_{(d,n)}=k,Z_{-(d,n)},W,\alpha,\eta)$$

$$\propto(n_{d,-(d,n)}^{k}+\alpha_k)\times\frac{n_{v,-(d,n)}^{k}+\eta_v}{\sum_{v'=1}^{V}(n_{v',-(d,n)}^{k}+\eta_{v'})}$$

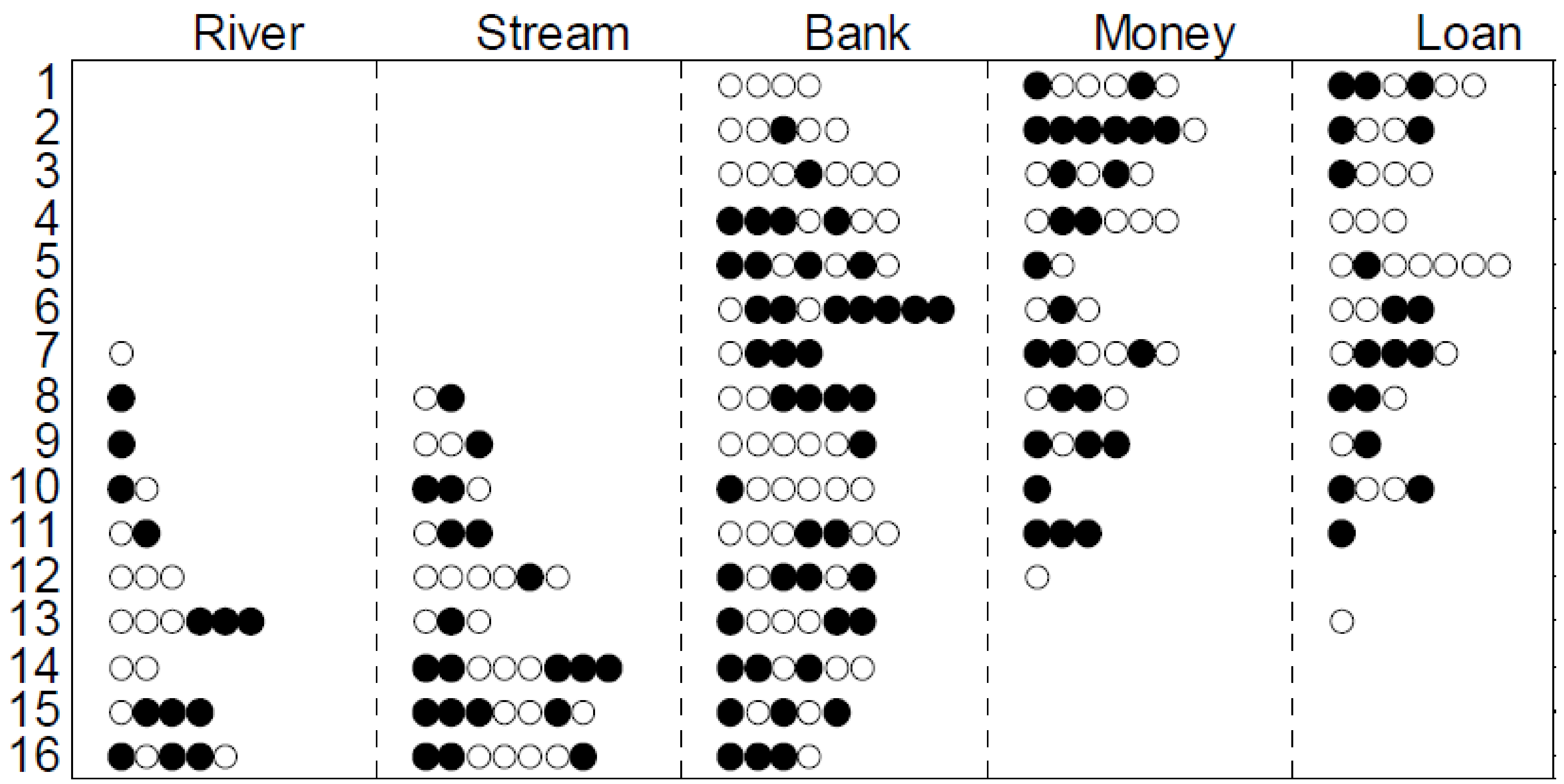
- $n_{d,-(d,n)}^{k}+\alpha_k$: $W_{d,n}$를 제외하고 문서 d에서 topic k인 단어의 개수 + 문서 d가 topic k인 경향에 비례한다.
- $n_{v,-(d,n)}^{k}+\eta_v$: $W_{d,n}$를 제외하고 단어 뭉치 중 하나인 v가 topic k인 개수 + 단어 v가 나오는 경향
- $\sum_{v'=1}^{V}(n_{v',-(d,n)}^{k}+\eta_{v'})$: 확률화

Compute Other Distribution

<u>After all, we have latent topic for each word.</u>
For computing topic-word distribution and document-topic distribution, just counting is enough.

- topic k의 단어 분포 $\in \mathbb{R}^V$: $\beta_k = \frac{n_v^k + \eta}{\sum_{v=1}^{V} n_v^k + V\eta}$
  - 문서 전체에서 topic k로 할당된 단어 뭉치 속의 단어 수
- 문서 d의 주제 분포 $\in \mathbb{R}^K$: $\theta_d = \frac{n_d^k + \alpha}{\sum_{i=1}^{K} n_d^i + K\alpha}$
  - 문서 내에서 topic k로 할당된 단어의 수

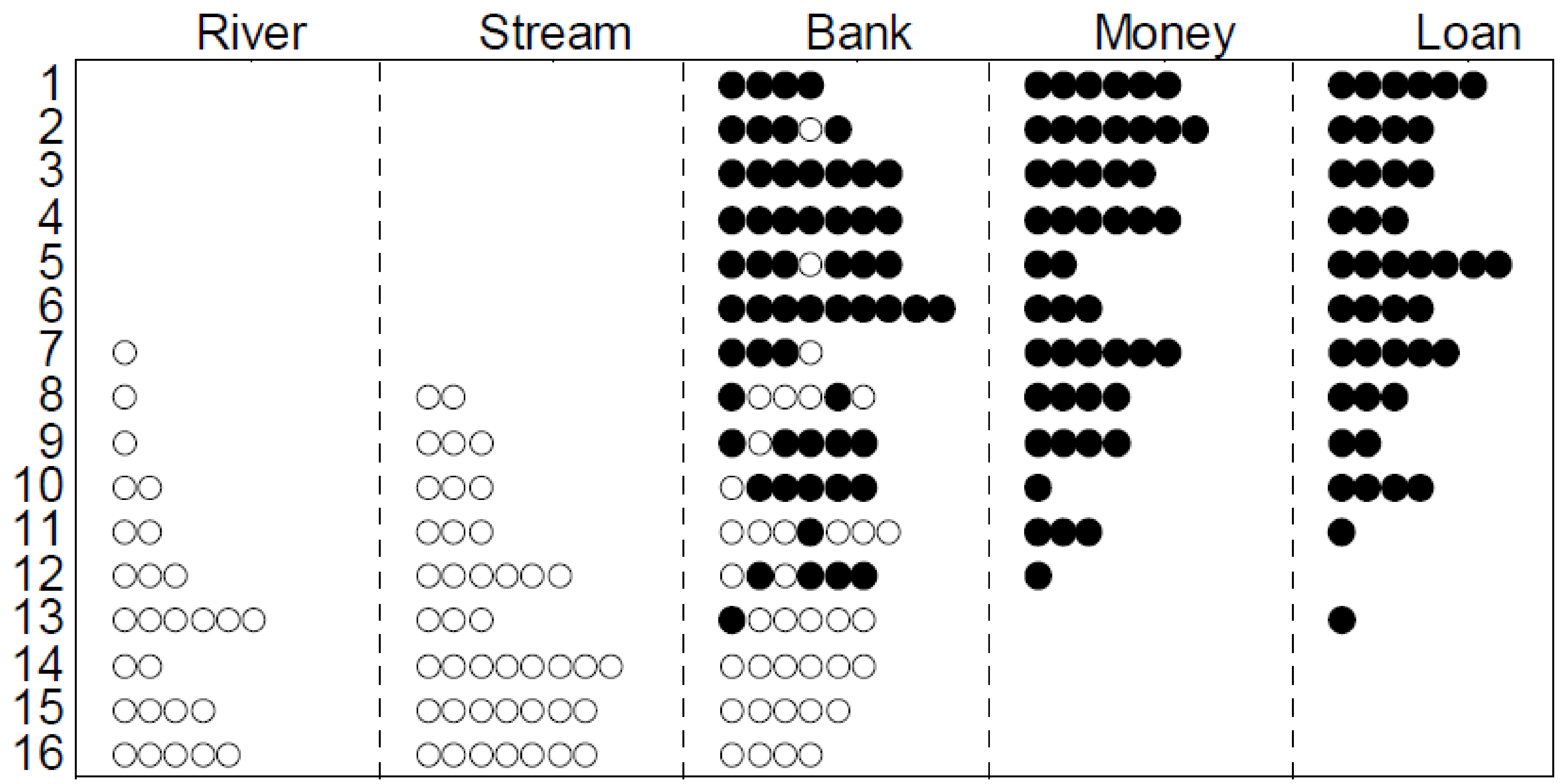
EXAMPLE



$$\theta, \beta$$

estimation

Iteration



Variational Inference

- approximation to distribution

$$P(Z|X) \approx Q(Z)$$

- minimize distance between two distributions

$$D_{KL}(Q||P) = \sum_Z Q(Z) \log \frac{Q(Z)}{P(Z|X)}$$

$$D_{KL}(Q||P) = \sum_Z Q(Z)[\log \frac{Q(Z)}{P(Z,X)} + \log P(X)]$$

Variational Inference

- X is already observed i.e. **constant**

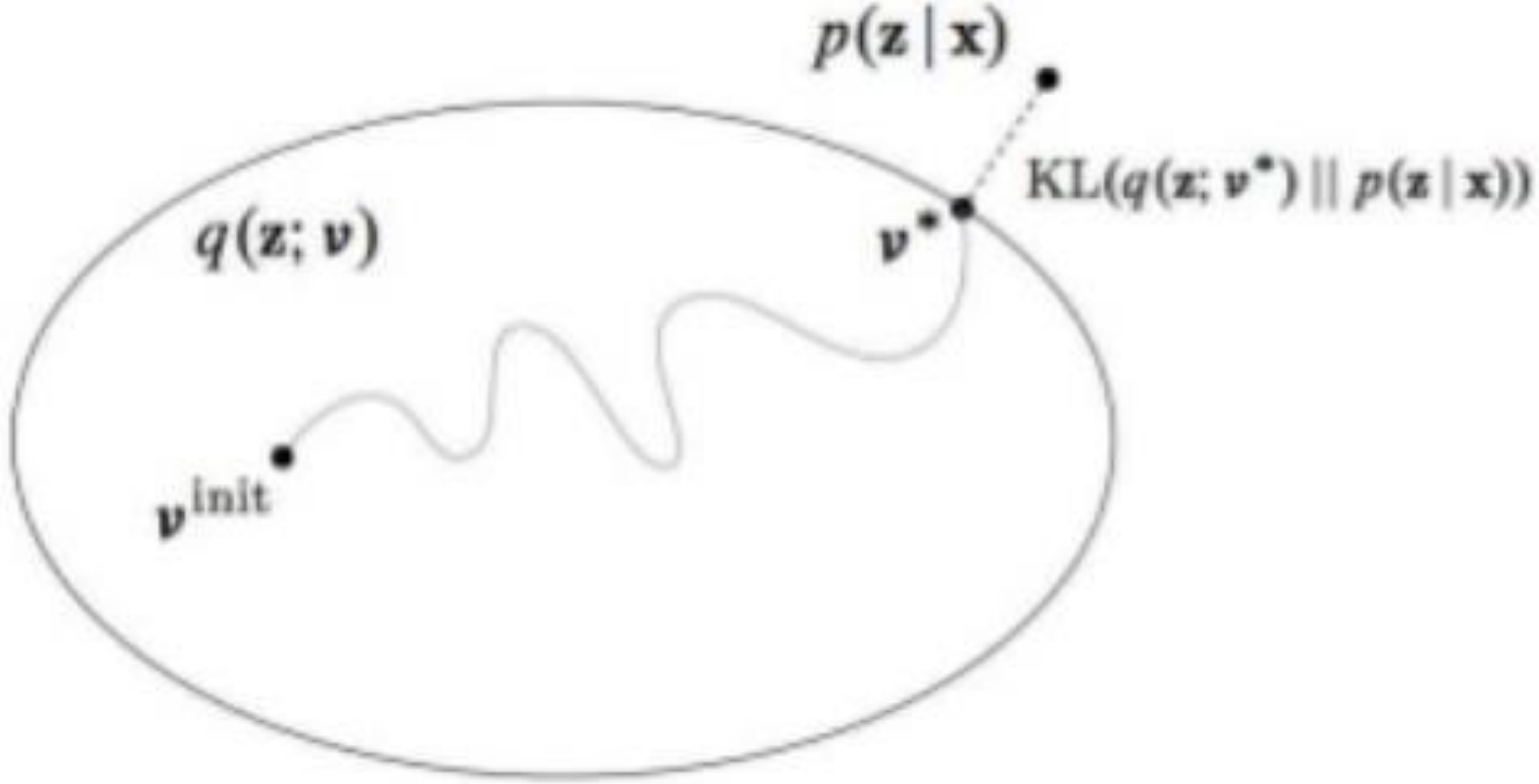
$$P(X)$$

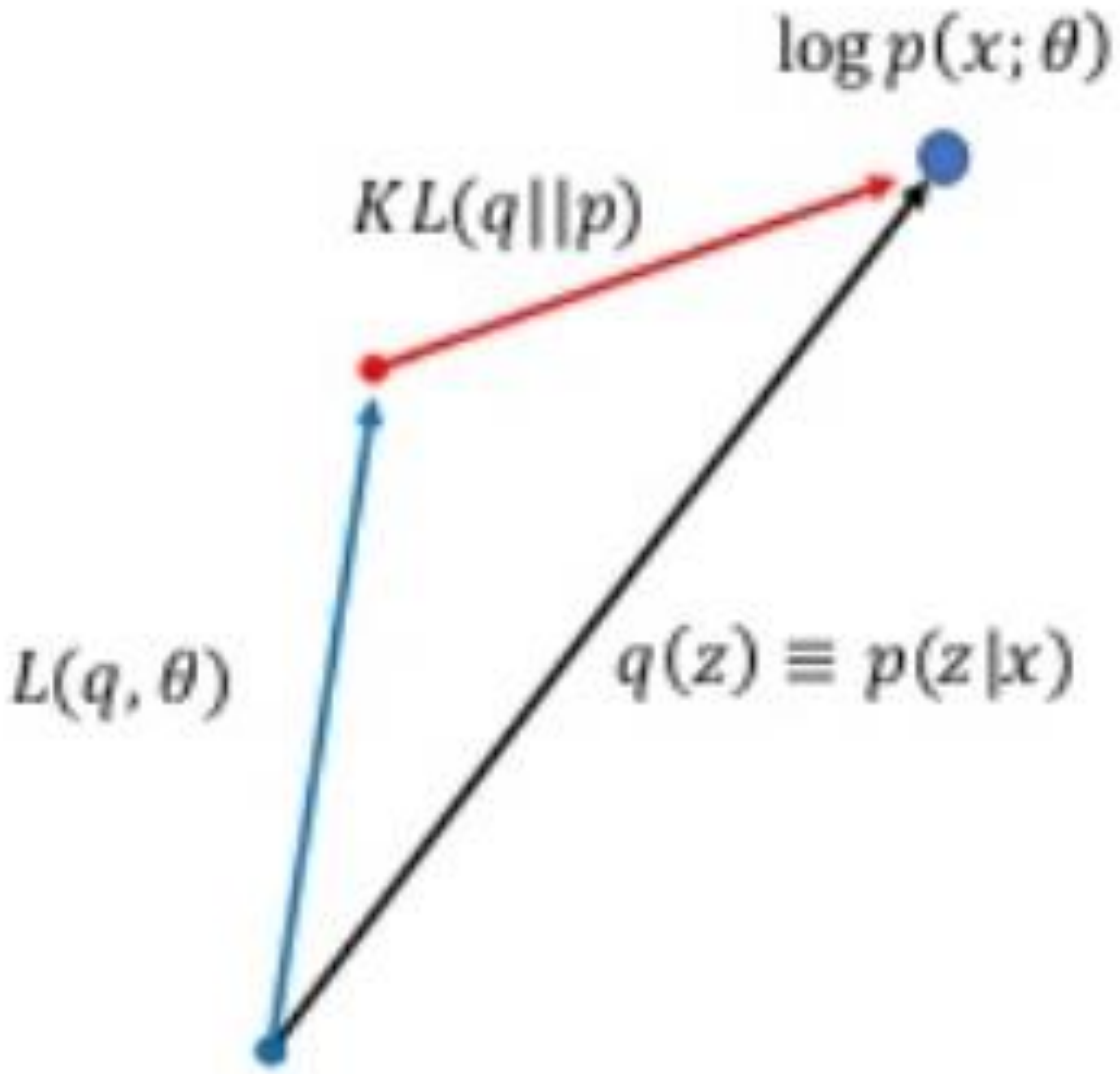
- To minimizing distance between two distributions, maximize right term

$$D_{KL}(Q||P) = \sum_Z Q(Z)[\log \frac{Q(Z)}{P(Z,X)} + \log P(X)]$$

$$\log P(X) = D_{KL}(Q||P) - \sum_Z Q(Z)[\log \frac{Q(Z)}{P(Z,X)}]$$

Variational Inference





06
Conclusion

Again Objective

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- We now have assigned topics to words

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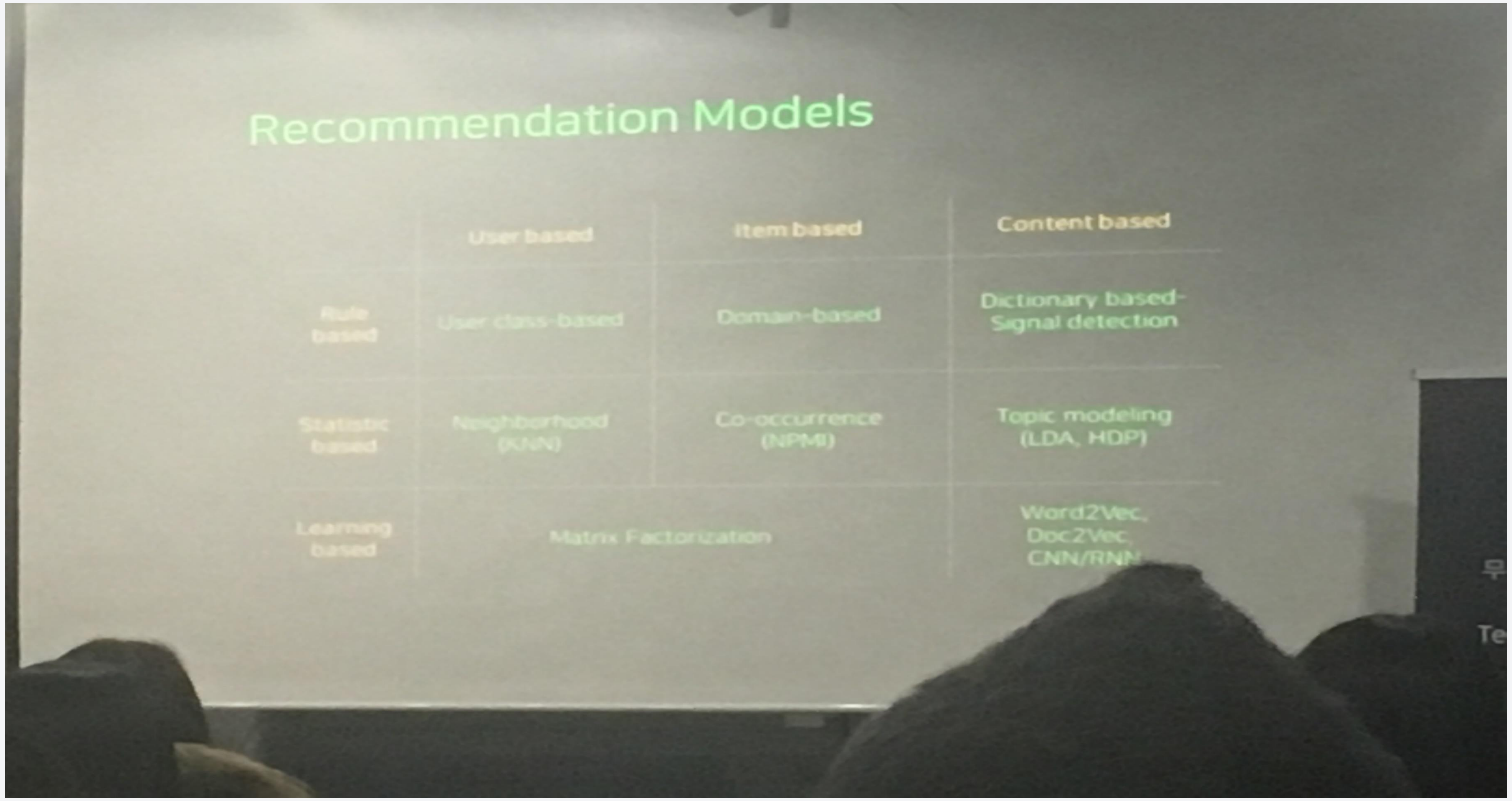
Application



- various field including NLP, experiment

Thank You