3.5 [RV73, page 22] *Running average of a convex function.* Suppose $f : \mathbf{R} \to \mathbf{R}$ is convex, with $\mathbf{R}_+ \subseteq \mathbf{dom}\, f$. Show that its *running average* F, defined as

$$F(x) = \frac{1}{x}\int_0^x f(t)\, dt, \qquad \mathbf{dom}\, F = \mathbf{R}_{++},$$

is convex. *Hint.* For each s, $f(sx)$ is convex in x, so $\int_0^1 f(sx)\, ds$ is convex.

Sum of all convex functions are convex.

$t = sx$

$dt = x \cdot ds$

If $f(x)$ is convex, then $f(ax+b)$ is convex

$F(x) = \int_0^1 f(sx) \cdot ds \approx f(0.1x) + f(0.2x) + \dots \quad \forall s \in [0,1]$

$\therefore F(x)$ is convex function.