Exercises

Definition of convexity

2.2 Show that a set is convex if and only if its intersection with any line is convex. Show that a set is affine if and only if its intersection with any line is affine.

($\rightarrow$) Set is convex $\longrightarrow$ intersection with any line is convex.

$x_1, x_2 \in C \cap L$ (C: convex set, L: any line)

$\left.\begin{array}{l} \alpha x_1 + (1-\alpha) x_2 \in C \ (0 \le \alpha \le 1) \\ \alpha x_1 + (1-\alpha) x_2 \in L \ (0 \le \alpha \le 1) \end{array}\right\} \therefore \alpha x_1 + (1-\alpha) x_2 \in C \cap L \ (0 \le \alpha \le 1)$

($\leftarrow$) Set is convex $\longleftarrow$ intersection with any line is convex.

$\left.\begin{array}{l} x_1, x_2 \in S \cap L \ (S: \text{arbitrary set}) \\ \alpha x_1 + (1-\alpha) x_2 \in S \cap L \ (0 \le \alpha \le 1) \end{array}\right\} S \cap L \subseteq C \quad \therefore S$ is convex set.

($\rightarrow$) Set is affine $\longrightarrow$ intersection with any line is affine.

$x_1, x_2 \in A \cap L$ (A: affine set)

$\left.\begin{array}{l} \theta x_1 + (1-\theta) x_2 \in A \ (\theta \in \mathbb{R}) \\ \theta x_1 + (1-\theta) x_2 \in L \ (\theta \in \mathbb{R}) \end{array}\right\} \theta x_1 + (1-\theta) x_2 \in A \cap L \ (\theta \in \mathbb{R})$

($\leftarrow$) Set is affine $\longleftarrow$ intersection with any line is affine.

$\left.\begin{array}{l} x_1, x_2 \in S \cap L \\ \theta x_1 + (1-\theta) x_2 \in S \cap L \ (\theta \in \mathbb{R}) \end{array}\right\} S \cap L \subseteq A \quad \therefore S$ is affine set.