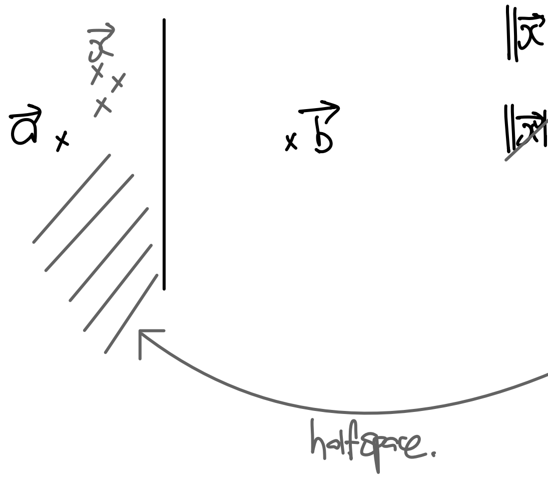
2.7 *Voronoi description of halfspace.* Let a and b be distinct points in $\mathbf{R}^n$. Show that the set of all points that are closer (in Euclidean norm) to a than b, *i.e.*, $\{x \mid \|x-a\|_2 \le \|x-b\|_2\}$, is a halfspace. Describe it explicitly as an inequality of the form $c^T x \le d$. Draw a picture.

$\vec{a}, \vec{b} \in \mathbb{R}^n$



$$\|\vec{x}-\vec{a}\|_2^2 \le \|\vec{x}-\vec{b}\|_2^2$$

$$\cancel{\|\vec{x}\|_2^2} - 2(\vec{a}^T \cdot \vec{x}) + \|\vec{a}\|_2^2 \le \cancel{\|\vec{x}\|_2^2} - 2(\vec{b}^T \cdot \vec{x}) + \|\vec{b}\|_2^2$$

$$2 \cdot (\vec{b}-\vec{a})^T \cdot \vec{x} \le \|\vec{b}\|_2^2 - \|\vec{a}\|_2^2$$

$$\therefore (\vec{b}-\vec{a})^T \cdot \vec{x} \le \frac{\|\vec{b}\|_2^2 - \|\vec{a}\|_2^2}{2}$$