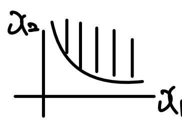

2.11 *Hyperbolic sets.* Show that the *hyperbolic set* $\{x \in \mathbf{R}^2_+ \mid x_1 x_2 \geq 1\}$ is convex. As a generalization, show that $\{x \in \mathbf{R}^n_+ \mid \prod_{i=1}^n x_i \geq 1\}$ is convex. *Hint.* If $a, b \geq 0$ and $0 \leq \theta \leq 1$, then $a^\theta b^{1-\theta} \leq \theta a + (1-\theta) b$; see §3.1.9.

hyperbolic set (H) : $\{\vec{x} \in \mathbb{R}^2_+ \mid x_1 x_2 \geq 1\}$

let $\vec{a}, \vec{b} \in H$. $(a_1 a_2 \geq 1,\ b_1 b_2 \geq 1)$

$$\alpha \cdot \vec{a} + (1-\alpha)\vec{b} = \alpha \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + (1-\alpha)\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} \alpha \cdot a_1 + (1-\alpha) b_1 \\ \alpha \cdot a_2 + (1-\alpha) b_2 \end{bmatrix} \geq \begin{bmatrix} a_1^\alpha \cdot b_1^{(1-\alpha)} \\ a_2^\alpha \cdot b_2^{(1-\alpha)} \end{bmatrix}$$

$(0 \leq \alpha \leq 1)$

$$(a_1 a_2)^\alpha \cdot (b_1 b_2)^{(1-\alpha)} \geq 1$$

$\therefore \alpha \cdot \vec{a} + (1-\alpha)\vec{b} \in H$ $(0 \leq \alpha \leq 1)$ $\Rightarrow$ H is convex set.

(Same proof for $\vec{x} \in \mathbb{R}^n_+$)