$(n=1)$ $A = [a_1]$ $\quad a_1 y^2 \geq 0, \quad a_1 \geq 0.$

$(n=2)$ $A = \begin{bmatrix} a_1 & a_2 \\ a_2 & a_3 \end{bmatrix}$ $\quad a_1 \geq 0, a_3 \geq 0, \det A \geq 0.$

$a_1 a_3 - a_2^2 \geq 0.$

$(n=3)$ $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_2 & a_4 & a_5 \\ a_3 & a_5 & a_6 \end{bmatrix}$ $\quad a_1 \geq 0. \ a_4 \geq 0. \ a_6 \geq 0.$

$a_1 a_4 - a_2^2 \geq 0, \quad a_1 a_6 - a_3^2 = 0, \quad a_4 a_6 - a_5^2 = 0.$

$\det A \geq 0. \ a_1(a_4 a_6 - a_5^2) - a_2(a_2 a_6 - a_3 a_5) + a_3(a_2 a_5 - a_3 a_4) \geq 0.$

Sylvester's Criterion. (for symmetric matrix)

Convex cones and generalized inequalities

2.28 *Positive semidefinite cone for* $n = 1, 2, 3$. Give an explicit description of the positive semidefinite cone $\mathbf{S}_+^n$, in terms of the matrix coefficients and ordinary inequalities, for $n = 1, 2, 3$. To describe a general element of $\mathbf{S}^n$, for $n = 1, 2, 3$, use the notation

$$x_1, \qquad \begin{bmatrix} x_1 & x_2 \\ x_2 & x_3 \end{bmatrix}, \qquad \begin{bmatrix} x_1 & x_2 & x_3 \\ x_2 & x_4 & x_5 \\ x_3 & x_5 & x_6 \end{bmatrix}.$$

$A \in S_+^n \ (A \succeq 0) \rightarrow \vec{x}^T A \cdot \vec{x} \geq 0$ for $\forall \vec{x} \in \mathbb{R}^n$, A is convex cone.