

$$(a) \quad y_1 \vec{a}_1 + y_2 \vec{a}_2 = \vec{x}, \quad (x \in \mathbb{R}^n)$$

$$(-1 \leq y_1 \leq 1) \quad (-1 \leq y_2 \leq 1)$$

$$\therefore \text{polyhedron}(0)$$



$$(b) \quad \left. \begin{array}{l} \vec{1}^T \vec{x} = 1 \\ \vec{a}_1^T \vec{x} = b_1 \\ \vec{a}_2^T \vec{x} = b_2 \\ (-I) \vec{x} \leq 0 \end{array} \right\} \therefore \text{polyhedron}(0)$$

$$(c) \quad (-I) \vec{x} \leq 0.$$

$$\underbrace{\vec{y}^T \vec{x} \leq 1 \quad \|\vec{y}\|_2 = 1}_{\text{not linear.}} \rightarrow \underbrace{\|\vec{x}\|_2 \leq 1}_{\text{L2 Norm Ball}}$$

$$\therefore \text{polyhedron}(x)$$

2.8 Which of the following sets S are polyhedra? If possible, express S in the form $S = \{x \mid Ax \preceq b, Fx = g\}$.

(a) $S = \{y_1 a_1 + y_2 a_2 \mid -1 \leq y_1 \leq 1, -1 \leq y_2 \leq 1\}$, where $a_1, a_2 \in \mathbb{R}^n$. (o)

(b) $S = \{x \in \mathbb{R}^n \mid x \succeq 0, \mathbf{1}^T x = 1, \sum_{i=1}^n x_i a_i = b_1, \sum_{i=1}^n x_i a_i^2 = b_2\}$, where $a_1, \dots, a_n \in \mathbb{R}$ and $b_1, b_2 \in \mathbb{R}$. (o)

(c) $S = \{x \in \mathbb{R}^n \mid x \succeq 0, x^T y \leq 1 \text{ for all } y \text{ with } \|y\|_2 = 1\}$. (x)

(d) $S = \{x \in \mathbb{R}^n \mid x \succeq 0, x^T y \leq 1 \text{ for all } y \text{ with } \sum_{i=1}^n |y_i| = 1\}$. (o)

L1 Norm Ball. (no curvature)