

Exercises

$$f: \text{Convex} \Leftrightarrow \text{dom } f \text{ is convex, } f(\theta \vec{x}_1 + (1-\theta)\vec{x}_2) \leq \theta f(\vec{x}_1) + (1-\theta)f(\vec{x}_2) \quad (0 \leq \theta \leq 1)$$

\updownarrow

$$f: \text{quasi-convex} \Leftrightarrow \text{dom } f \text{ is convex, } \underline{\text{sublevel sets}} \text{ are convex for } \forall \alpha.$$

$$S_\alpha = \{\vec{x} \in \text{dom } f \mid f(\vec{x}) \leq \alpha\}$$

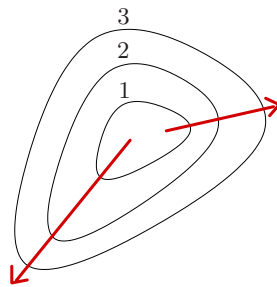
$$f: \text{concave} \Leftrightarrow \text{dom } f \text{ is convex, } f(\theta \vec{x}_1 + (1-\theta)\vec{x}_2) \geq \theta f(\vec{x}_1) + (1-\theta)f(\vec{x}_2)$$

\updownarrow

$$f: \text{quasi-concave} \Leftrightarrow \text{dom } f \text{ is convex, } \underline{\text{superlevel sets}} \text{ are convex for } \forall \alpha.$$

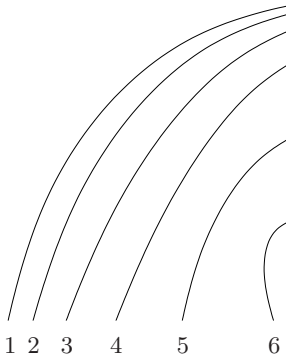
$$S_\alpha = \{\vec{x} \in \text{dom } f \mid f(\vec{x}) \geq \alpha\}$$

3.2 Level sets of convex, concave, quasiconvex, and quasiconcave functions. Some level sets of a function f are shown below. The curve labeled 1 shows $\{x \mid f(x) = 1\}$, etc.



Convex (~~possible~~) (X)
 quasi-convex (possible)
 concave (X)
 quasi-concave (X)

Could f be convex (concave, quasiconvex, quasiconcave)? Explain your answer. Repeat for the level curves shown below.



convex (X)
 quasi-convex (X)
 concave (possible)
 quasi-concave (possible)