Identification of β- Decay in 8- Isomer 136 Cs and Possibilities of Unknown States in 136 Ba

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I. Introduction

The Only State in 136 Cs Known to Undergo β- Decay Prior to This Report is 5+, the Ground State.

By Further Analyzing the Collected Data Such As the 136 Cs Gamma Energy Spectrum From the Experiment Conducted at the ISOLDE Facility at CERN in 2011, Which Attempted to Better Understand the Properties of 8- Isomer 136 Cs Through Measurements of the Excitation Energy of the 8- State As Well As the Internal Decay Time for 8- to Reach the 5+ Ground State, We Determine Whether the 8- Isomeric State in 136 Cs, In Addition to the 5+ State, Also Undergo β- Decay to 136 Ba.

We Also Analyze the Same Data to Determine If There are Any Unknown States in 136 Ba That are Visited From the β- Decays in the 8- Isomeric State of 136 Cs.

II. Analysis

A. Identification of β- Decay in 136 Cs

For a Possible Indication of β- Decay of 8- Isomer 136 Cs, the Initial Goal Was to Identify the Known Gamma Decay Radiations With the Highest Intensities in the β- Decay Scheme of 5+ 136 Cs Using the Detected Energy Plot From the Experiment.

Table 1. Branching Ratios From the β- Decay in 5+ 136 Cs [1]

Energy [KeV]	Intensities [%]
818.51	99.04
1048.07	80.00

340.55	42.20
1235.36	20.00
176.60	10.00

As It Can Be Seen From Table 1, the Decay Radiations With the Highest Intensities Were 818 KeV (~ 100%) and 1048 KeV (~ 80%).

For This Analysis, It Was Important to Check First If There are Any Peaks at 818 KeV and 1048 KeV in the Detected Energy Plots of the Background Runs Before Checking the Plots From the Experiment In Order to Confirm That Peaks at 818 KeV and 1048 KeV in the Plots From the Experiment are From the β- Decay of 136 Cs and Not From Different Background Sources.

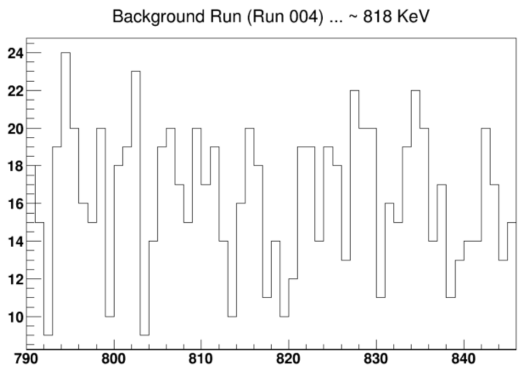

FIG 1. Background Radiation at 818 KeV. As Shown, There is No Peak at 818 KeV.

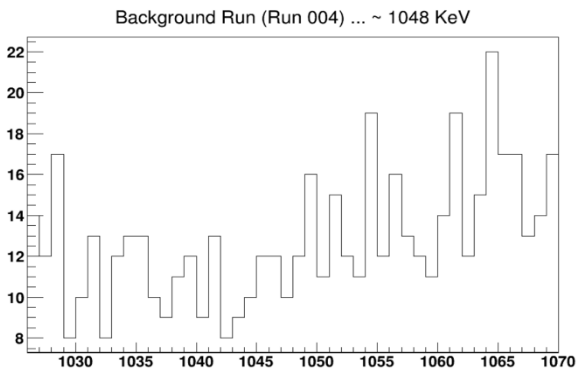

FIG 2. Background Radiation at 1048 KeV. As Shown, There is No Peak at 1048 KeV.

As Shown in Figures 1 and 2, There Were No Peaks at Energies 818 KeV and 1048 KeV. These Results Confirm That the Background Sources Did Not Largely Affect the Counts of 818 KeV and 1048 KeV From the Experiment.

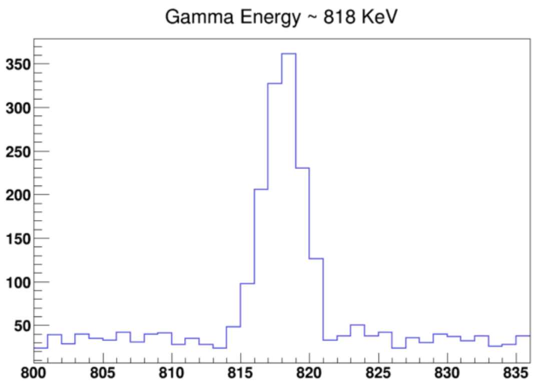

FIG 3. Peak at 818 KeV From the Experiment

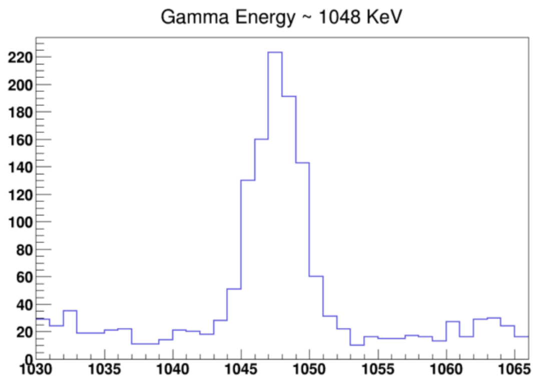

FIG 4. Peak at 1048 KeV From the Experiment

As Seen in Figures 3 and 4, the Peaks at 818 KeV and 1048 KeV From the Experiment are Clearly Differentiable From Other Energies; This is an Indication That There Were 136 Cs That Underwent β- Decay During the Experiment.

B. Evidence of β- Decay in 8- Isomer 136 Cs

It Has Been Confirmed That There Were 136 Cs That Underwent β- Decay, But This Does Not Necessarily Imply That There Were β- Decay Specifically in the 8- Isomeric State of 136 Cs.

There is In Fact a Possibility That All of These β- Decays Occurred in the 5+ 136 Cs Instead of Any in the 8- 136 Cs. Therefore, Identification of These Known Decay Radiations in the Detected Energy Plots is Not Sufficient Amount of Information to Confirm That β- Decays Occurred in the 8- Isomeric State.

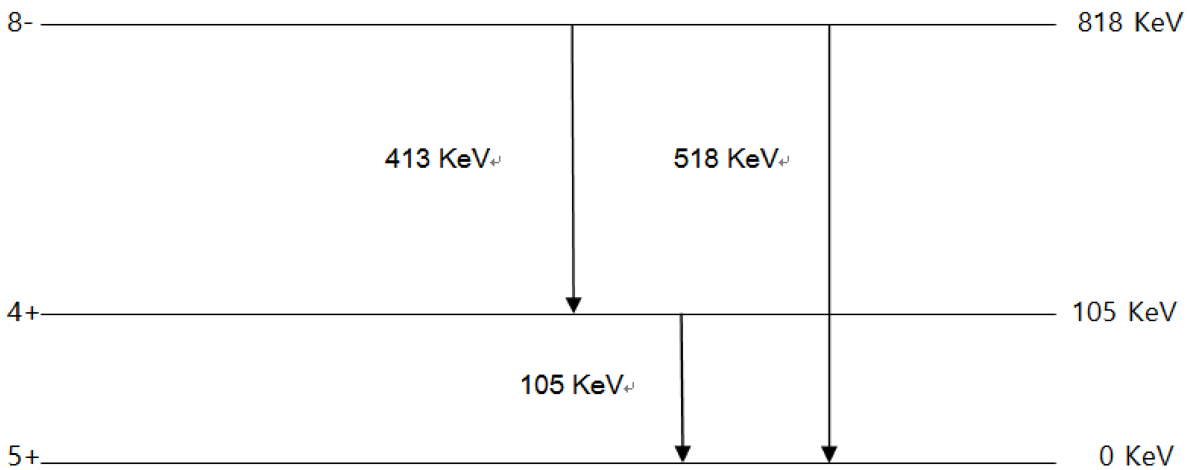

FIG 5. Energy Levels in 136 Cs. According to [2], Most of the Internal Branching From the 8- State is Directly to the 5+ State, and There are Only Very Few Branching From the 8- State to the 4+ State. This Implies That 4+ State is Very-Lowly Populated Compared to the 8+ State As Well As the 5+ State.

One Way to Confirm the β- Decay in 8- 136 Cs is Through Proof By Contradiction, By Calculating the β- Decay Branching Ratios Using the Experimental Data With an Assumption That All β- Decays are From 5+ State, and Then Comparing Its Results With the β- Branching Ratios in 5+ 136 Cs As Given in Table 1. If β- Decay Only Occurred in 5+ 136 Cs, Then the Computed β- Branching Ratios Must Correlate With the Theoretical β- Branching Ratios in 5+ 136 Cs As Shown in Table 1; If There is No Correlation However, It is an Indication That β- Decay is Also Occurring in Additional State in 136 Cs, Most Likely in 8-, As 8- is a More Popular Isomeric State Than 4+ in 136 Cs (See Figure 5) [2].

Therefore, the Next Step Was to Calculate the Branching Ratios of the β- Decay Radiations With Relatively High Intensities, Such As 1048 KeV (~80%), 340 KeV (~42%), 1235 KeV (~20%) and 176 KeV (~10%) As Shown in Table 1 Using the Detected Energy Plot From the Experiment; For Simplicity, These Energies Will Be Denoted As X.

In Summary, the Branching Ratios of Each Decay Radiation Can Be Calculated Using the Following Equation:

$$\mathrm{BR_X} = \frac{\#\text{ of }\beta-\text{ Decays That Emitted X (X=176 KeV,340 KeV,1048 KeV,or 1235 KeV)}}{\text{Total }\#\text{ of }\beta-\text{ Decays}}$$

$$= \frac{\text{Counts of Decay Radiation X (X=176 KeV,340 KeV,1048 KeV,or 1235 KeV)}}{\text{Counts of Decay Radiation 818 KeV}} \quad .... \text{ Eq1}$$

Note That The Total Counts of 818 KeV is Assumed to Be Equivalent to the Total Number of β- Decays As the β- Decay Scheme of 5+ 136 Cs in Table 1 Shows That Approximately All (99%) of the β- Decays Emit 818 KeV.

The 1st Step in Calculating the Branching Ratios Was Obtaining the Detected Counts of Each of the Considering Energy. Because the Detected Energy Plots are in the Units of Counts/KeV vs KeV, the Peak of the Corresponding Energy Had to Be Integrated In Order to Determine the Counts.

Note That Like Previously Done With 818 KeV and 1048 KeV to Account for Any Large Noise, It Was Checked Whether There Were Peaks at 176 KeV, 340 KeV and 1235 KeV in the Background Runs;

It Have Been Confirmed That There Were No Peaks At Any of These Energies.

Furthermore, It Has Been Verified That There Were Peaks at 176 KeV, 340 KeV and 1235 KeV From the Experiment.

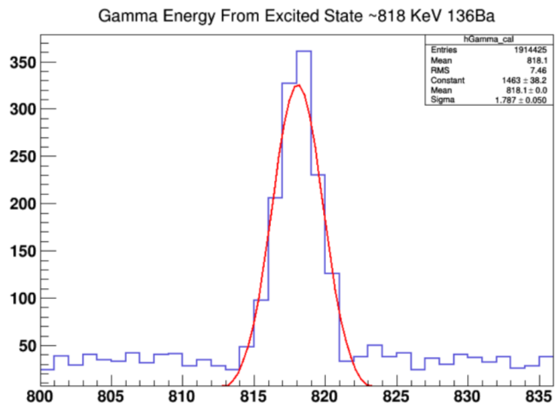

FIG 6. The Peak at 818 KeV is Being Fitted Using Gaussian-Normal Function. The Focus Was Making the Area of the Fit As Close As Possible to the Area of the Peak.

FIG 7.

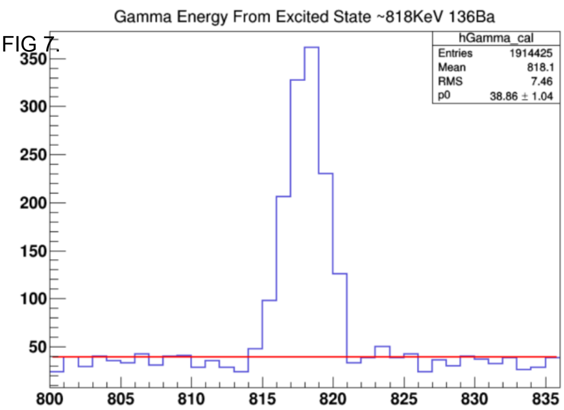

FIG 7. The Background Radiation Surrounding the Peak 818 KeV is Being is Fitted Using a Constant Function.

As Shown in Figure 6, the Energy Peaks Were Fitted Using Gaussian-Normal Function, and the Fit Was Integrated to Approximate the Counts of 818 KeV and X; However, As Shown in Figure 7, There Were Also Background Radiations Present During the Experiment. This Means That the Counts of 818 KeV and X Obtained From Integrating the Gaussian-Normal Fit Also Includes the Counts of the Background Radiation. To Filter These Unwanted Counts, the Background Radiations Surrounding 818 KeV and X Were Fitted Using a Constant Function As Shown in Figure 7, Integrated, and Removed From the Integrated Value of the Gaussian-Normal Fit to Achieve the Accurate Detected Counts of 818 KeV and X.

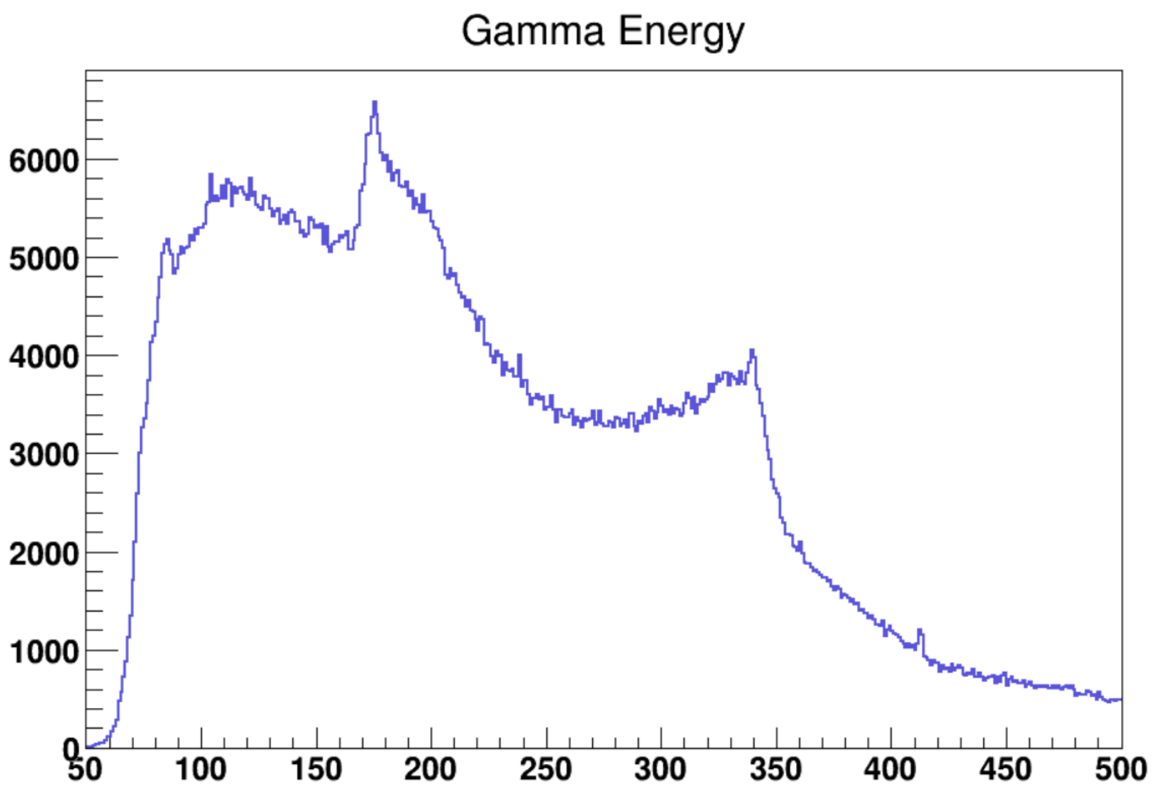

FIG 8. Energy Plot From the Experiment Between 50 KeV and 500 KeV.
It is Very Evident That There are Peaks Around 176 KeV, 340 KeV and 413 KeV (Decay From 8- to 4+ in 136 Cs), But Backscattering and Compton Edge Around 176 KeV and 340 KeV Makes Them Especially Difficult for Modeling and Computing the Counts.

Unfortunately, As It Can Be Seen in Figure 8, the Counts/KeV at 176 KeV and 340 KeV Were Not Only Accounting for the Background Radiation, But Also Additionally Backscattering and Compton Edge, Respectively.

Because It Was Especially Extremely Difficult to Predict and Model the Counts Due to Backscattering, 176 KeV Was Neglected From Further Analysis; 340 KeV Was Carefully Evaluated, But There Still Exists a Low Possibility of a Relatively Large Error in the Measurement.

The Detected Count Measurements of 818 KeV and X are Shown in Table 2.

Table 2. Detected Counts of the Peak Energies From the Experiment

Energy [KeV]	Detected Counts
818	1227.87
340	370.49
1048	808.59
1235	150.56

However, It is Not Possible to Determine the Branching Ratios With Only the Detected Counts Since All of These Energies Have Different Absolute Detection Efficiencies; Note That In Order to Calculate the Branching Ratios, the Absolute Detection Efficiencies of 818 KeV and X Must Be Equivalent As the Ratio

Between the Counts With Same Efficiencies is Equivalent to the Ratio Between the Overall Counts. Therefore, to Modify the Detected Counts of X Accordingly Such That These Values are of the Same Efficiency As 818 KeV, the Ratios Between the Absolute Detection Efficiencies of 818 KeV and X Were Determined.
As Derived in Appendix B, the Ratio Between the Relative Efficiencies is Equivalent to the Ratio Between the Absolute Detection Efficiencies.
Therefore, to Find the Ratios Between the Absolute Detection Efficiencies of 818 KeV and X, the Next Step Was to Determine the Relative Efficiencies of 818 KeV and X.

Before Running the Experiment With 136 Cs, the HPGe Gamma Ray Detector Was Calibrated Using 152 Eu, a Standard Calibration Source in Gamma Ray Spectroscopy.

Table 3. Branching Ratios [3] and the Detected Counts of the 152 Eu Decay Radiations

Energy [KeV]	Intensities [%]	Detected Counts
1408.01	31.25	28999.10
121.78	28.41	259547.00
344.27	26.59	119342.00
1112.08	16.10	22781.20
964.07	14.50	27541.50
778.91	12.97	28856.50
1085.84	8.82	19095.60
244.69	7.55	47835.20

Using the Counts of the Peak Energies From the Calibration Runs (Which Were Determined By Applying the Same Method As With the 136 Cs Experiment) in Table 3, the Intensities in Table 3, and Relative Efficiency Formula:

$$\varepsilon_{\mathrm{Rel}} = \frac{\text{Counts of E= M KeV}}{\text{Counts of Reference E=N KeV}} \; \frac{\text{BR of Reference E= N KeV}}{\text{BR of E=M KeV}} \text{ .... Eq 2 (Derivation in Appendix A)}$$

With Reference Energy at 121 KeV, the Relative Efficiency of Each Peak Energy Was Found;
the Values Can Be Seen in Table 4.

Table 4. Relative Efficiencies of the 152 Eu Decay Radiations

Energy [KeV]	Relative Efficiency [%]
1408.01	15.22
121.78	100.00
344.27	49.13
1112.08	18.60
964.07	20.79

778.91	24.35
1085.84	20.63
244.69	69.35

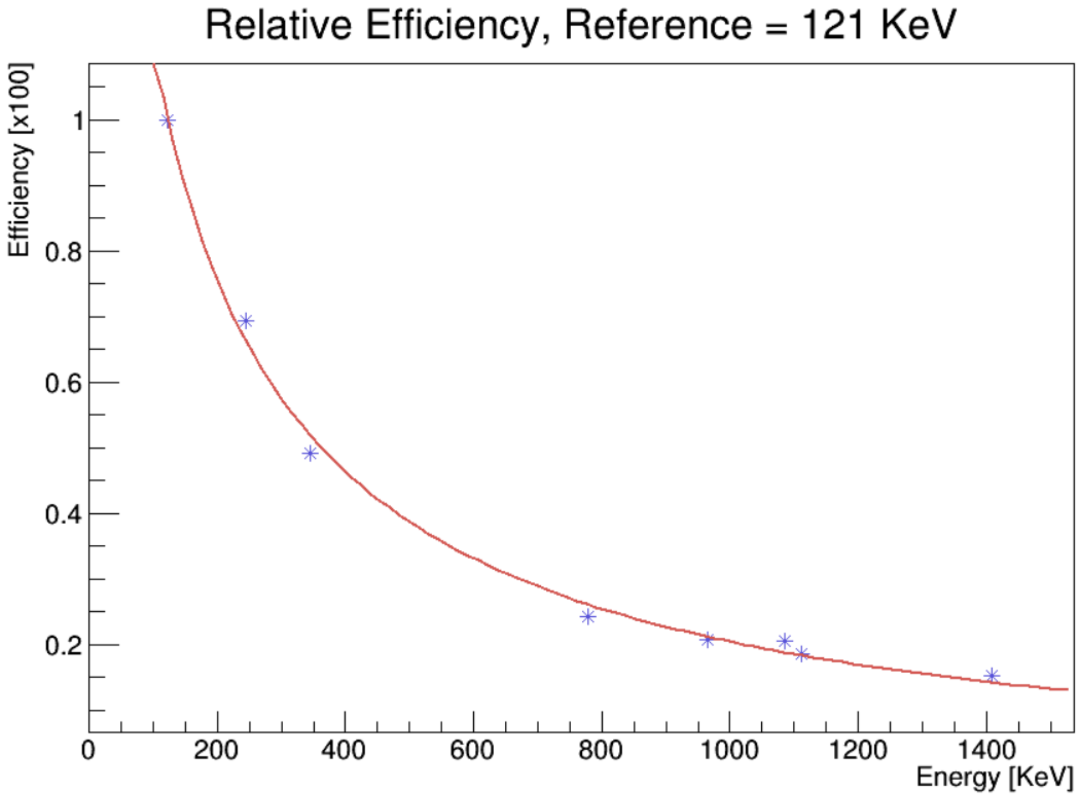

FIG 9. The Relative Efficiencies of the Peak Energies From 152 Eu Decay are Plotted (Blue Points). These Relative Efficiency Points Were Best-Fitted Using Eq 3 (Red Line), and Was Used to Approximate the Relative Efficiencies of the Peak Energies From 136 Cs Decay.

As Shown in Figure 9, the Plot of the Values From Table 4 Was Best-Fitted With the Equation:

$$\varepsilon_{\mathrm{Rel}} = \exp(1.049 - 0.047\, log(E) - 0.006\, (log(E))^2 - 0.006\, (log(E))^3) \quad \text{.............................. Eq 3}$$

and the Relative Efficiencies of 818 KeV and X Were Approximated Using This Fit.

The Calculated Ratios Between the Absolute Detection Efficiencies (Relative Efficiencies) of 818 KeV and X are Shown in Table 5.

Table 5: Ratios Between the Absolute Detection Efficiencies of 818 KeV and X

Absolute Detected Efficiency of 818 KeV /	Ratio
Absolute Detected Efficiency of 340 KeV	0.477
Absolute Detected Efficiency of 1048 KeV	1.276
Absolute Detected Efficiency of 1235 KeV	1.514

Substituting the Ratios Between the Absolute Detection Efficiencies From Table 5 and the Detected Counts From Table 2 Into the Following Equation That Aims to Determine the Ratio Between the Counts With Same Detection Efficiencies:

$$\mathrm{BR_X} = \frac{\text{Counts of Decay Radiation X}}{\text{Counts of Decay Radiation 818 KeV}}$$

$$= \frac{\text{Detected Counts of Decay Radiation X}}{\text{Detected Counts of Decay Radiation 818 KeV}} \frac{\varepsilon_{\mathrm{Abs}}\text{ of Decay Radiation 818 KeV}}{\varepsilon_{\mathrm{Abs}}\text{ of Decay Radiation X}} \quad \text{.......................... Eq 4}$$

the β- Decay Branching Ratios of X Were Determined;

Table 6 Shows the Comparison Between the Calculated Branching Ratios and the Theoretical Branching Ratios.

Table 6: Comparison Between the Calculated Branching Ratios and the Theoretical Branching Ratios of the β- Decay in 5+ 136 Cs

Energy [KeV]	Computed Branching Ratio	Theoretical Branching Ratio
340	0.144	0.422
1048	0.840	0.800
1235	0.186	0.200

As Shown in Table 6, the Computed Branching Ratios are Not Equivalent to the Theoretical β- Decay Branching Ratios of 5+ 136 Cs.

With This Result, It Can Be Concluded That Not All of the β- Decays Occurred in the 5+ State of 136 Cs, But Also in the 8- Isomeric State of 136 Cs.

Note That the Measured Branching Ratios of 1048 KeV and 1235 KeV Does Not Add Up to 1 (ie. Slightly Above, 1.026), Which Should Not Be the Case According to the Current 136 Cs Decay Scheme That Says That All 2+ State Move Directly to the 0+ State in 136 Ba. However, It Has Been Confirmed After Accounting the Fit Uncertainties and Considering the Range of Possible Counts of All 818 KeV, 1048 KeV and 1235 KeV That It is Possible for the Measured Branching Ratios to Add Up to 1.

C. Ratio Between the Internal Conversion and β- Decay in 8- Isomer 136 Cs

Now That It Has Been Confirmed That 8- Isomer 136 Cs Undergo β- Decay, Additional Question Arises: What is the Ratio Between the Internal Conversion (De-Excitation From 8- State to the 5+ State of 136 Cs) and the β- Decay in 8- Isomer 136 Cs? This is a Useful Problem to Consider Because With This Outcome, It is Possible to Predict How Much Radioactive 136 Ba There is to Properly Dispose at the End of Similar Experiments With Short Run Times Relative to the Half-Life of 136 Cs.

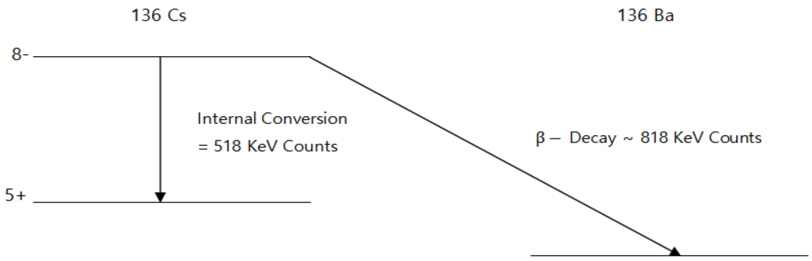

FIG 10. Scheme Showing the Internal Conversion From 8- to 5+ in 136 Cs and β- Decay in 8- 136 Cs

So to Find This Ratio, the Detected Counts of 518 KeV From the Experiment Was First Determined (380900) Using the Same Method As Did With the Other Peak Energies, and the Relative Efficiency of 518 KeV Was Determined Using Figure 9 (0.379). Using These Values, the Ratio Between the Absolute Detection Efficiencies of 518 KeV and 818 KeV Was Found (1.505), and the Detected Counts of 818 KeV Was Modified Accordingly Such That the Counts is of the Same Detection Efficiency As the 518 KeV. Using the Counts of 518 KeV and 818 KeV With the Same Detection Efficiencies, the Ratio Between the Internal Conversion and the β- Decay in 8- Isomer 136 Cs Resulted to Be Approximately 380900:1846, ~ 0.485%, Which is an Extremely Small Value.

However, Before Drawing to a Conclusion That This is the Ratio Between the Internal Conversion and the β- Decay in 8- Isomer 136 Cs, There is One More Problem to Consider: As Mentioned Earlier, There is a Possibility That the Counts of 818 KeV is Not Only Including the Decay Radiations Due to the β- Decay of 8- Isomer 136 Cs, But Also the Decay Radiations Due to the β- Decay of 5+ 136 Cs.

As Mentioned in [2], the Half-Life of 136 Cs is 13.16 Days. Using This Information and the Radioactive Half-Life Decay Formula:

Final Amount [/1] = (Initial Amount = 1) $(0.5)^{\frac{\text{Run Time of the Experiment}}{\text{Half Life of the Isotope}}}$ .... Eq 5

, It Shows That By the End of the Experiment, Which Lasted Approximately 25 Minutes, Only 0.1% of the Initial Amount of 5+ 136 Cs Would Have Underwent β- Decay.

Using This Calculation and Estimating the Number of 5+ 136 Cs β- Decay Per 1% Detection Efficiency to Be the Quotient Between the Overall 5+ 136 Cs β- Decay Counts and 100%, It Can Be Approximated That Out of the 380900 5+ 136 Cs Formed From the Internal Conversion, Approximately 381 β- Decayed to Emit 818 KeV. Therefore, Removing This Value From the Modified Counts of 818 KeV, It Can Be Concluded That the Ratio Between the Internal Conversion and the β- Decay in 8- Isomer 136 Cs is Approximately 380900:1465, ~ 0.385% (Realistically Slightly Less As It is Likely That There Were Also 136 Cs 5+ That Were Not From the Internal Conversion, Which is a Factor Difficult to Consider).

D. Possibilities of Other Unknown States in 136 Ba

To Check If There are Any Indications of Other Unknown States in 136 Ba, All Peak Energies Were Compared With the Known Decay Radiations From the Current 136 Cs Decay Scheme Model [1].

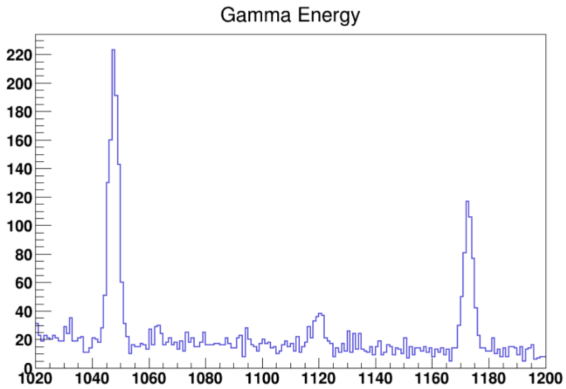

FIG 11. Peak at 1172 KeV (Right) From the Experiment

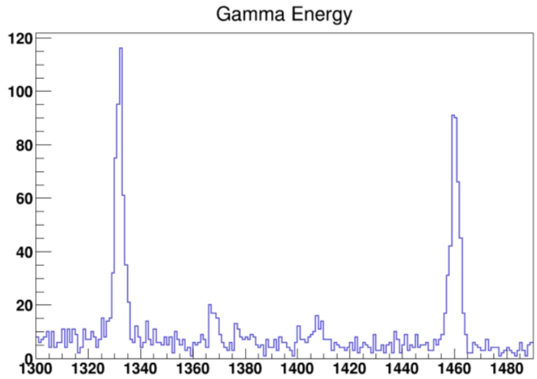

FIG 12. Peaks at 1332 KeV (Left) and 1460 KeV (Right) From the Experiment

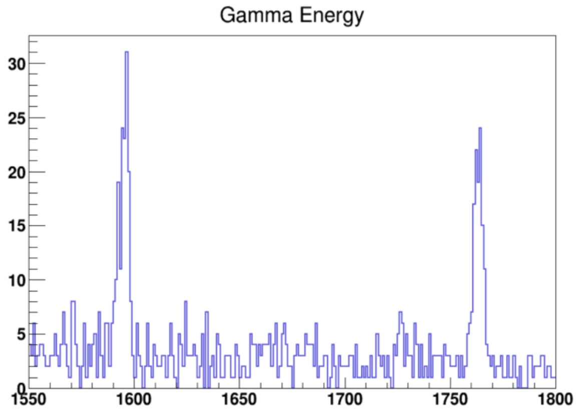

FIG 13. Peaks at 1594 KeV (Left) and 1762 KeV (Right) From the Experiment

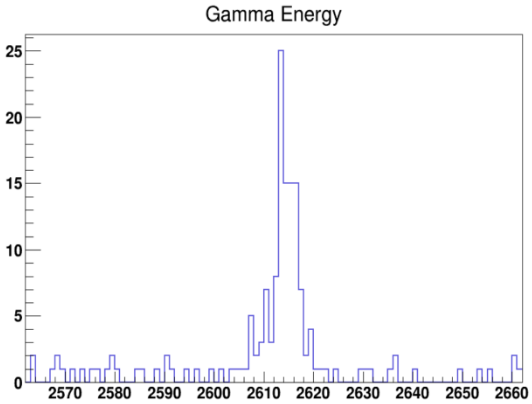

FIG 14. Peak at 2204 KeV From the Experiment

There Were Multiple Peak Energies That Were Not Outlined in the Current Decay Scheme, Such As ~ 1172 KeV, 1332 KeV, 1460 KeV, 1594 KeV, 1762 KeV and 2204 KeV, As It Can Be Seen in Figures 11,12,13 and 14 Respectively.

However, Before Drawing to a Conclusion That These Energies are From Unknown Decays, It is Important to Consider Cases Like Natural Background Sources and Decays of Neutron-Rich Isotopes Other Than 136 Cs.

From the Natural Background Radiation Measurements Made From the HPGe Detectors in the Past, It is Very Likely That the Peak Energies 1172 KeV, 1332 KeV and 1460 KeV are From the Decays of 60 Co, 60 Co and 40 K, Respectively [4].

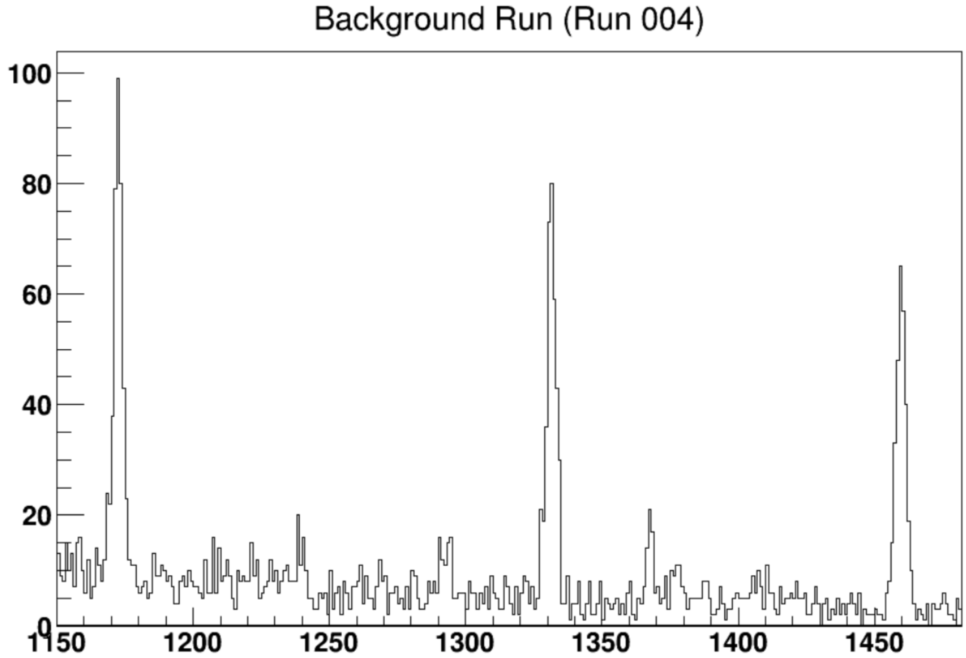

FIG 15. Peaks at 1172 KeV (Left), 1332 KeV (Center), 1460 KeV (Right) From the Longest Background Run. It is Very Noticeable That There Were Peaks at All of These Energies.

Furthermore, the Counts of These Energies From the Background Runs Were High Compared to Its Surroundings As It Can Be Seen in Figure 15, Which is an Additional Evidence That These Were Not Decay Radiations From 136 Cs.

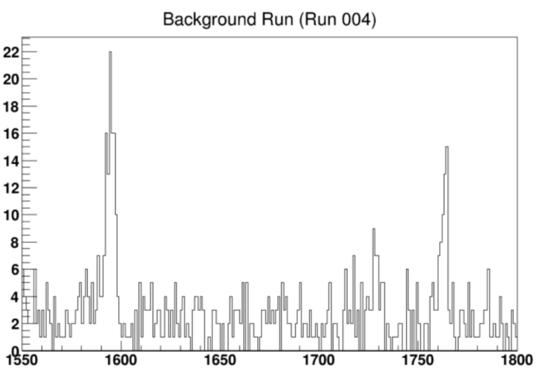

FIG 16. Peaks at 1594 KeV (Left) and 1762 KeV (Right) From the Longest Background Run.

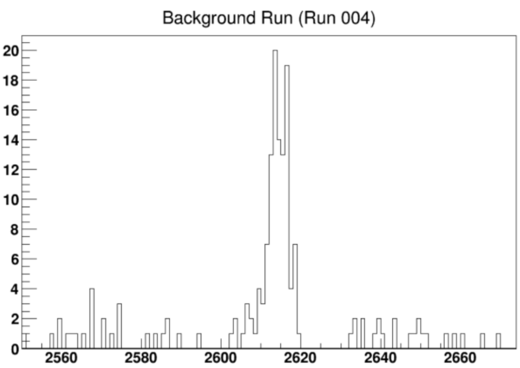

FIG 17. Peak at 2612 KeV From the Longest Background Run.

In Addition to This, Even Though It is Difficult to Identify From Which Isotopes the Decay Radiations 1594 KeV, 1762 KeV and 2612 KeV are From As They are Not Outlined in the Natural Background Radiation List [4], It Can Be Seen in Figures 16 and 17 That the Counts of These Energies From the Background Runs are High Compared to Its Surroundings. From This Observation, It Can Be Concluded That These Energies are Not From the Decay of 136 Cs, But Instead From the Unknown Background Sources.

There is One Energy That Has Not Been Detected in the Background Runs: 2204 KeV.

There are 2 Possibilities for the Detection:

1. Neutron-Rich Isotope With a Mass Close to 136 Cs Emitted 2205 KeV During Its Decay.

2. Background Runs Were Not Run Long Enough to Detect All of the Background Sources.

1st Possibility to Consider is the Decay of a Different Neutron-Rich Isotope That Has a Similar Mass As 136 Cs.

To Collect Only the 136 Cs From All the Other Isotopes That Were Produced From the Collision of Uranium Atoms in the Experiment, the Incoming Isotopes Were Filtered Based on Their Momentum (Mass) With the Use of Electromagnetic Fields.

Therefore, It is Highly Probable That Isotopes With Nearly the Same Mass As 136 Cs Passed Through the Filter and Emitted the Decay Radiation of 2204 KeV.

The Only Isotope Satisfying Both of These Conditions is 137 Nd, Which Emits 2205 KeV With 0.05% Branching Ratio [5].

If 2204 KeV Which Has an Extremely Small Branching Ratio Was Detected, Then the Detector Must Have Also Made Many Measurements of Decay Radiations With Slightly Higher Decay Branching Ratios of 137 Nd, Such As 505 KeV (Intensity: 9.00%) and 781 KeV (Intensity: 9.30%) [5].

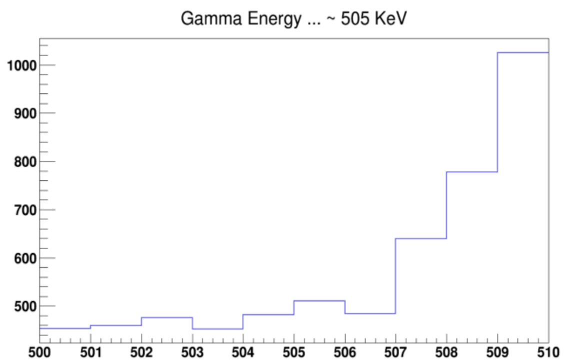

FIG 18. Energy Plot From the Experiment Near 505 KeV. As Shown, There is No Peak at 505 KeV. The Increase in Counts Between 507 KeV is Also Due to the Background Radiation.

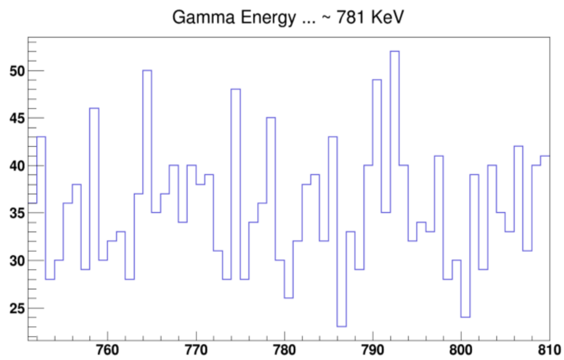

FIG 19. Energy Plot From the Experiment Near 781 KeV. As Shown, the Count at 781 KeV is Not Differentiable From the Counts at Surrounding Energies; This Implies That Count at 781 KeV is From a Background Source.

However, As It Can Be Seen in Figures 18 and 19, There Were No Peaks at These Energies; Only Background Radiations Were Present.

With This Result, It Can Be Concluded That 137 Nd Was Not the Reason for the Detection of 2204 KeV.

Another Possibility for the Detection of 2204 KeV is That the Background Runs Did Not Collect Sufficient Amount of Data to Detect All of the Background Sources Due to Its Relatively Short Runtime.

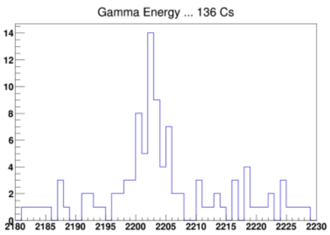

FIG 20. Peak at 2205 KeV From the Experiment

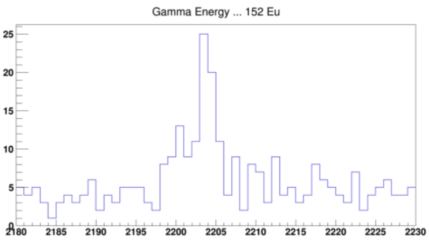

FIG 21. Peak at 2205 KeV From the Calibration Run

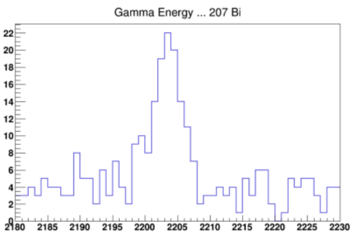

FIG 22. Peak at 2205 KeV From a Different Experiment

As Shown in Figures 20, 21 and 22, It is Evident That a Significant Number of 2204 KeV Detections Were Made in Each of the Energy Plots From 136 Cs Experiment (1500 Seconds), 152 Eu Calibration Run (3605 Seconds) and 207 Bi (Different Experiment) Run (3600 Seconds), Which All Had Runtimes Approximately 1.5 to 15 Times Greater Than the Runtimes of the Background Runs (885 Seconds, 235 Seconds).

This Result, Which Shows That 2204 KeV Was Detected Not Only in the 136 Cs Experiment But Also in Other Runs, Proves That 2204 KeV is Not From the Decay of 136 Cs But Instead From a Background Source.

Furthermore, the Large Differences Between the Runtimes Hint That the Short Duration of the Background Runs is the Reason for Not Being Able to Detect the Background Source Emitting 2204 KeV.

The Emission of 2204 KeV is Predicted to Be From the Decay of 214 Bi, Which is an Isotope That Has Been Previously Identified As a Background Source Known For Emitting Natural Background Radiation of 1538 KeV [4].

III. Conclusion

In Summary, We Have Determined That 8- Isomer 136 Cs, In Addition to 5+ 136 Cs, Also Undergo β- Decay to 136 Ba, and Through Further Analysis Approximated the Ratio Between the Internal Conversion and the β- Decay in 8- Isomer 136 Cs to Be 380900:1465, ~ 0.385%.

We Have Also Further Studied the Energy Peaks From the Experiment That are Not Outlined in the Current 136 Cs Decay Scheme Model to Check If Any of Them are Potentially From an Unknown Decay, But Have Verified That They Were All Instead From the Background Sources.

IV. Appendix

A. Derivation: Finalized Form of the Relative Efficiency Formula

Rel. Efficiency is the Ratio Between the Abs. Efficiencies:

$$\varepsilon_{\mathrm{Rel}} = \frac{\varepsilon_{\mathrm{Abs}} \text{ of E= } \alpha \text{ KeV}}{\varepsilon_{\mathrm{Abs}} \text{ of E= } \beta \text{ KeV}} \quad \text{.......... Eq A}$$

Since $\varepsilon_{\mathrm{Abs}}$ Can Be Expressed: $\frac{\text{Counts of E= } \alpha \text{ KeV}}{\text{t x A x } \mathrm{BR}_{\alpha}}$, t = Total Decay Time of the Material [s]

A = Activity of the Material [Bq]

BR_α= Branching Ratio of Decay Energy α [KeV]

The Rel. Efficiency Formula Can Be Re-Expressed:

$$\varepsilon_{Rel} = \frac{\dfrac{\text{Counts of E}= \alpha \text{ KeV}}{\text{t x A x BR}_\alpha}}{\dfrac{\text{Counts of E}= \beta \text{ KeV}}{\text{t x A x BR}_\beta}}$$

Because t x A Cancel, the Rel. Efficiency Formula Can Be Re-Expressed in the Final Form:

$$\varepsilon_{Rel} = \frac{\text{Counts of E}= \alpha \text{ KeV}}{\text{Counts of E}= \beta \text{ KeV}} \frac{BR_\beta}{BR_\alpha} \quad \text{.......... Eq 2}$$

B. Proof: Ratio Between Relative Efficiencies = Ratio Between Absolute Efficiencies

Given the Ratio Between Rel. Efficiencies of Energies α and β:

$$\frac{\varepsilon_{Rel} \text{ of E}= \alpha \text{ KeV}}{\varepsilon_{Rel} \text{ of E}= \beta \text{ KeV}} = \frac{M}{N}$$

Using Eq A From Section A, the Expression Becomes:

$$\frac{\dfrac{\varepsilon_{Abs} \text{ of E}= \alpha \text{ KeV}}{\varepsilon_{Abs} \text{ of E}= \gamma \text{ KeV}}}{\dfrac{\varepsilon_{Abs} \text{ of E}= \beta \text{ KeV}}{\varepsilon_{Abs} \text{ of E}= \gamma \text{ KeV}}} = \frac{M}{N}$$

Because ε_{Abs} of E = γ KeV Cancel, the Expression Becomes the Ratio Between Abs. Efficiencies:

$$\frac{\varepsilon_{Abs} \text{ of E}= \alpha \text{ KeV}}{\varepsilon_{Abs} \text{ of E}= \beta \text{ KeV}} = \frac{M}{N} \quad \text{QED}$$

V. References

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