과제04_멀티미디어수치해석_2017113547_이정근 문제 10.3, 10.5, 10.13

10.3 나이브 가우스 소거법을 써서 다음 시스템을 분해하라
10.2 절의 기술대로 따라가라.
그리고 결과로나온 L과 U 행렬을 곱해서 A가 나오는지 판단해라.

$$\begin{cases} 10x_1 + 2x_2 - x_3 = 27 \\ -3x_1 - 6x_2 + 2x_3 = -61.5 \\ x_1 + x_2 + 5x_3 = -21.5 \end{cases}$$

$\Leftrightarrow$

$$\begin{bmatrix} 10 & 2 & -1 \\ -3 & -6 & 2 \\ 1 & 1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 27 \\ -61.5 \\ -21.5 \end{bmatrix}$$

$$A = \begin{bmatrix} 10 & 2 & -1 \\ -3 & -6 & 2 \\ 1 & 1 & 5 \end{bmatrix}$$

$\downarrow$

$$\begin{bmatrix} 10 & 2 & -1 \\ -\frac{3}{10} & -\frac{27}{5} & \frac{17}{10} \\ \frac{1}{10} & \frac{4}{5} & \frac{51}{10} \end{bmatrix}$$

$$\begin{cases} r_2 - \left(-\frac{3}{10}\right) r_1 \\ r_3 - \left(\frac{1}{10}\right) r_1 \end{cases}$$

$-\frac{3}{10} r_1 = \left[-3 \ -\frac{3}{5} \ +\frac{3}{10}\right]$

$r_2 = \left[-3 \ -\frac{30}{5} \ \frac{20}{10}\right]$

$r_2 - \left(-\frac{3}{10} r_1\right) = \left[0 \ -\frac{27}{5} \ \frac{17}{10}\right]$

$\frac{1}{10} r_1 = \left[1 \ \frac{1}{5} \ -\frac{1}{10}\right]$

$r_3 = \left[1 \ \frac{5}{5} \ \frac{50}{10}\right]$

$r_3 - \frac{1}{10} r_1 = \left[0 \ \frac{4}{5} \ \frac{51}{10}\right]$

$\downarrow$

$$\begin{bmatrix} 10 & 2 & -1 \\ -\frac{3}{10} & -\frac{27}{5} & \frac{17}{10} \\ \frac{1}{10} & -\frac{4}{27} & \frac{289}{54} \end{bmatrix}$$

$r_3 - \left(-\frac{4}{27}\right) r_2$

$-\frac{4}{27} r_2 = \left[0 \ \frac{4}{5} \ -\frac{34}{135}\right]$

$r_3 = \left[0 \ \frac{4}{5} \ \frac{1377}{270}\right]$

$-\frac{4}{27} r_2 = \left[0 \ \frac{4}{5} \ -\frac{68}{270}\right]$

$r_3 - \left(-\frac{4}{27}\right) r_2 = \left[0 \ 0 \ \frac{1445}{270}\right]$

$= \left[0 \ 0 \ \frac{289}{54}\right]$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{3}{10} & 1 & 0 \\ \frac{1}{10} & -\frac{4}{27} & 1 \end{bmatrix} \quad U = \begin{bmatrix} 10 & 2 & -1 \\ 0 & -\frac{27}{5} & \frac{17}{10} \\ 0 & 0 & \frac{289}{54} \end{bmatrix}$$

$$LU = \begin{bmatrix} 10 & 2 & -1 \\ -3 & -6 & 2 \\ 1 & 1 & 5 \end{bmatrix} = A$$

$-\frac{3}{5} - \frac{27}{5} = -\frac{30}{5} = -6$

$\frac{3}{10} + \frac{17}{10} = \frac{20}{10} = 2$

$\frac{1}{5} + \frac{4}{5} = 1$

$-\frac{1}{10} - \frac{34}{135} + \frac{289}{54} = \frac{-27 - 68 + 1445}{270}$

$= \frac{-95 + 1445}{270} = \frac{1350}{270} = 5$

10.5 LU분해와 partial pivoting으로 다음 방정식을 풀어라.

$$\begin{cases} 2x_1 - 6x_2 - x_3 = -38 \\ -3x_1 - x_2 + 7x_3 = -34 \\ -8x_1 + x_2 - 2x_3 = -40 \end{cases}$$

$$\Leftrightarrow \begin{bmatrix} 2 & -6 & -1 \\ -3 & -1 & 7 \\ -8 & 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -38 \\ -34 \\ -40 \end{bmatrix}$$

$$\Leftrightarrow \begin{bmatrix} -6 & 2 & -1 \\ -1 & -3 & 7 \\ 1 & -8 & -2 \end{bmatrix} \begin{bmatrix} x_2 \\ x_1 \\ x_3 \end{bmatrix} = \begin{bmatrix} -38 \\ -34 \\ -40 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$PA = \begin{bmatrix} 1 & -8 & -2 \\ -1 & -3 & 7 \\ -6 & 2 & -1 \end{bmatrix}$$

$R_2 = R_2 - (-R_1)$

$R_3 = R_3 - (-6R_1)$

$$\begin{array}{rl} R_2 = & [-1 \ \ -3 \ \ 7] \\ -)\ -R_1 = & [-1 \ \ 8 \ \ 2] \\ \hline R_2 = & 0 \ \ -11 \ \ 5 \end{array}$$

$$\begin{array}{rl} R_3 = & [-6 \ \ 2 \ \ -1] \\ -)\ -6R_1 = & [-6 \ \ 48 \ \ 12] \\ \hline & [0 \ \ -46 \ \ -13] \end{array}$$

$$\begin{bmatrix} 1 & -8 & -2 \\ -1 & -11 & 5 \\ -6 & -46 & -13 \end{bmatrix}$$

$R_3 = R_3 - \frac{46}{11} R_2$

$$\begin{array}{rl} R_3 = & [0 \ \ -46 \ \ -13] \\ -)\ \frac{46}{11}R_2 = & [0 \ \ -46 \ \ \frac{230}{11}] \\ \hline & [0 \ \ 0 \ \ \frac{-143-230}{11}] \\ = & [0 \ \ 0 \ \ -\frac{373}{11}] \end{array}$$

$$\begin{bmatrix} 1 & -8 & -2 \\ -1 & -11 & 5 \\ -6 & \frac{46}{11} & -\frac{373}{11} \end{bmatrix}$$

$$PA = LU = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -6 & \frac{46}{11} & 1 \end{bmatrix} \begin{bmatrix} 1 & -8 & -2 \\ 0 & -11 & 5 \\ 0 & 0 & -\frac{373}{11} \end{bmatrix}$$

$$PAx = Pb = \begin{bmatrix} -40 \\ -34 \\ -38 \end{bmatrix}$$

$$LUx = \begin{bmatrix} -40 \\ -34 \\ -38 \end{bmatrix} \qquad Ld = \begin{bmatrix} -40 \\ -34 \\ -38 \end{bmatrix}$$

$d_1 = -40$

$-d_1 + d_2 = -34 \qquad d_2 = -34 + d_1 = -34 - 40 = -74$

$-6d_1 + \frac{46}{11} d_2 + d_3 = -38$

$d_3 = -38 + 6d_1 - \frac{46}{11} d_2$

$= -38 - 240 - \frac{46}{11}(-74) = \frac{-418 - 2640 + 3404}{11}$

$= \frac{346}{11}$

$$Ux = d = \begin{bmatrix} -40 \\ -74 \\ \frac{346}{11} \end{bmatrix}$$

$-\frac{373}{11} x_3 = \frac{346}{11}, \quad x_3 = -\frac{346}{373}$

$-11x_1 + 5x_3 = -74 \quad \Big| \quad -11x_1 = -74 - 5x_3 = -74 + \frac{1730}{373} = \frac{-25872}{373} \quad \Big| \quad x_1 = \frac{+2352}{373}$

$x_2 - 8x_1 - 2x_3 = -40$

$x_2 = -40 + 8x_1 + 2x_3 = -40 + \frac{18816}{373} - \frac{692}{373}$

$= \frac{-14920 + 18816 - 692}{373} = \frac{3204}{373}$

답 $x_1 = \frac{2352}{373} \quad x_2 = \frac{3204}{373} \quad x_3 = -\frac{391}{373}$

10.13 Cholesky 분해로 다음을 만족하는 U를 결정하라

$$A = U^T U = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

$$U^T = \begin{bmatrix} u_{11} & 0 & 0 \\ u_{21} & u_{22} & 0 \\ u_{31} & u_{32} & u_{33} \end{bmatrix} \quad U = \begin{bmatrix} u_{11} & u_{21} & u_{31} \\ 0 & u_{22} & u_{32} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$U^T U = \begin{bmatrix} u_{11}^2 & * & * \\ u_{11}u_{21} & u_{21}^2 + u_{22}^2 & * \\ u_{11}u_{31} & u_{31}u_{21} + u_{32}u_{22} & u_{31}^2 + u_{32}^2 + u_{33}^2 \end{bmatrix}$$

$$u_{11} = \sqrt{a_{11}} = \sqrt{2}$$

$$u_{21} = \frac{a_{21}}{u_{11}} = \frac{-1}{\sqrt{2}}$$

$$u_{22} = \sqrt{a_{22} - u_{21}^2} = \sqrt{2 - \left(-\frac{1}{\sqrt{2}}\right)^2} = \sqrt{\frac{3}{2}}$$

$$u_{31} = \frac{a_{31}}{u_{11}} = 0$$

$$u_{32} = \frac{a_{32} - u_{31}u_{21}}{u_{22}} = \frac{-1-0}{\sqrt{\frac{3}{2}}} = \sqrt{\frac{2}{3}}$$

$$u_{33} = \sqrt{a_{33} - (u_{31}^2 + u_{32}^2)}$$

$$= \sqrt{2 - \left(0 + \frac{2}{3}\right)}$$

$$= \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}}$$

$$A = \begin{bmatrix} \sqrt{2} & 0 & 0 \\ -\frac{1}{\sqrt{2}} & \sqrt{\frac{3}{2}} & 0 \\ 0 & -\sqrt{\frac{2}{3}} & \frac{2}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} \sqrt{2} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & \sqrt{\frac{3}{2}} & -\sqrt{\frac{2}{3}} \\ 0 & 0 & \frac{2}{\sqrt{3}} \end{bmatrix}$$

$$U = \begin{bmatrix} \sqrt{2} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & \sqrt{\frac{3}{2}} & -\sqrt{\frac{2}{3}} \\ 0 & 0 & \frac{2}{\sqrt{3}} \end{bmatrix}$$