

15.1 포물선으로 14.1의 자료에 맞춰라. r^2 을 구하고 결과의 유효성에 대해 말하라.

표 14.1

$V, m/s$	10	20	30	40	50	60	70	80	Σ
F, N	25	70	380	550	610	1220	830	1450	5135
V^2	100	400	900	1600	2500	3600	4900	6400	20400
V^3	1000	8000	27000	64000	125000	216000	343000	512000	1296000
V^4	10000	160000	810000	2560000	6250000	12960000	24010000	40960000	87720000
VF	250	1400	11400	22000	30500	73200	58100	116000	312850
V^2F	2500	28000	342000	880000	1525000	4392000	4069000	9280000	20516500

$$f(V) = a_0 + a_1 V + a_2 V^2 \text{ 이자면}$$

$$\begin{bmatrix} 8 & 360 & 20400 \\ 360 & 20400 & 1296000 \\ 20400 & 1296000 & 87720000 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 5135 \\ 312850 \\ 20516500 \end{bmatrix} \Leftrightarrow A x = y$$

$$r_2 = r_2 - 45x r_1$$

$$\begin{array}{rrr} 360 & 20400 & 1296000 \\ -) 360 & 16200 & 918000 \\ \hline 0 & 4200 & 378000 \end{array}$$

$$r_3 = r_3 - 2550 r_1$$

$$\begin{array}{rrr} 20400 & 1296000 & 87720000 \\ -) 20400 & 918000 & 52020000 \\ \hline 0 & 378000 & 35700000 \end{array}$$

$$\begin{bmatrix} 8 & 360 & 20400 \\ 45 & 4200 & 378000 \\ 2550 & 378000 & 35700000 \end{bmatrix}$$

$$r_3 = r_3 - 90 r_2$$

$$\begin{array}{rrr} 0 & 378000 & 35700000 \\ -) 0 & 378000 & 34020000 \\ \hline 0 & 0 & 1680000 \end{array}$$

$$\begin{bmatrix} 8 & 360 & 20400 \\ 45 & 4200 & 378000 \\ 2550 & 90 & 1680000 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 45 & 1 & 0 \\ 2550 & 90 & 1 \end{bmatrix} \begin{bmatrix} 8 & 360 & 20400 \\ 0 & 4200 & 378000 \\ 0 & 0 & 1680000 \end{bmatrix}$$

$$L = L U$$

$$L d = y$$

$$d_1 = 5135$$

$$45d_1 + d_2 = 312850 \quad d_2 = 312850 - 45d_1 = 81775$$

$$2550d_1 + 90d_2 + d_3 = 20516500$$

$$d_3 = 20516500 - 2550d_1 - 90d_2 = 62500$$

$$d_1 = 5135 \quad d_2 = 81775 \quad d_3 = 62500$$

15.1 개월

$$UX = d$$

$$1680000a_2 = 62500$$

$$a_2 = 0.037$$

$$4200a_1 + 378000a_2 = 81775$$

$$a_1 = \frac{81775 - 378000 \times 0.037}{4200} = 16.14$$

$$8a_0 + 360a_1 + 20400a_2 = 5135$$

$$a_0 = \frac{5135 - 360a_1 - 20400a_2}{8} = -178.775$$

$$\therefore f(V) = -178.775 + 16.14V + 0.037V^2$$

$$\bar{F} = 641.875$$

$F - \bar{F}$	-616.875	-571.875	-261.875	-91.875	31.875	578.125	188.125	808.125
$(F - \bar{F})^2$	380534	327041	68578	8441	1016	334228	35391	653066

$$S_t = \sum (F - \bar{F})^2 = 1808294$$

V	10	20	30	40	50	60	70	80
$f(V)$	-13.675	158.825	338.725	526.025	720.725	922.825	1132.325	1349.225
$F - f(V)$	38.675	-88.825	41.275	23.975	-110.725	297.175	-302.325	100.775
$(F - f(V))^2$	1496	7890	1704	575	12260	88313	91400	10156

$$S_r = \sum (F - f(V))^2 = 213794$$

$$r^2 = \frac{S_t - S_r}{S_t} = \frac{1808294 - 213794}{1808294} = \frac{1594500}{1808294} = 0.8817$$

원래의 불확실성의 88.17%가 이 모델로 설명된다.

15.2

$$S_r = \sum (y_i - a_1 x_i - a_2 x_i^2)^2$$

$$\begin{aligned} \frac{\partial S_r}{\partial a_1} &= -2 \sum (y_i - a_1 x_i - a_2 x_i^2) x_i \\ &= -2 (\sum x_i y_i - a_1 \sum x_i^2 - a_2 \sum x_i^3) = 0 \\ (\sum x_i^2) a_1 + (\sum x_i^3) a_2 &= \sum x_i y_i \quad \text{--- ①} \end{aligned}$$

$$\begin{aligned} \frac{\partial S_r}{\partial a_2} &= -2 \sum (y_i - a_1 x_i - a_2 x_i^2) x_i^2 \\ &= -2 (\sum x_i^2 y_i - a_1 \sum x_i^3 - a_2 \sum x_i^4) = 0 \\ (\sum x_i^3) a_1 + (\sum x_i^4) a_2 &= \sum x_i^2 y_i \quad \text{--- ②} \end{aligned}$$

$$\sum x_i^3 \times \text{①} - \sum x_i^2 \times \text{②} :$$

$$\begin{aligned} \{ (\sum x_i^3)^2 - \sum x_i^2 \sum x_i^4 \} a_2 &= \sum x_i^3 \sum x_i y_i - \sum x_i^2 \sum x_i^2 y_i \\ a_2 &= \frac{\sum x_i^3 \sum x_i y_i - \sum x_i^2 \sum x_i^2 y_i}{(\sum x_i^3)^2 - \sum x_i^2 \sum x_i^4} \end{aligned}$$

$$(\sum x_i^3) a_2 = \frac{(\sum x_i^3)^2 \sum x_i y_i - \sum x_i^2 \sum x_i^3 \sum x_i^2 y_i}{(\sum x_i^3)^2 - \sum x_i^2 \sum x_i^4}$$

$$\sum x_i^2 a_1 = \sum x_i y_i - \sum x_i^3 a_2$$

$$\begin{aligned} &= \frac{(\cancel{\sum x_i^3})^2 \sum x_i y_i - \sum x_i^2 \sum x_i^4 \sum x_i y_i - \{ (\cancel{\sum x_i^3})^2 \sum x_i \sum y_i - \sum x_i^2 \sum x_i^3 \sum x_i^2 y_i \}}{(\sum x_i^3)^2 - \sum x_i^2 \sum x_i^4} \\ &= \frac{\sum x_i^2 \sum x_i^3 \sum x_i^2 y_i - \sum x_i^2 \sum x_i^4 \sum x_i y_i}{(\sum x_i^3)^2 - \sum x_i^2 \sum x_i^4} \end{aligned}$$

$$a_1 = \frac{\sum x_i^3 \sum x_i^2 y_i - \sum x_i^4 \sum x_i y_i}{(\sum x_i^3)^2 - \sum x_i^2 \sum x_i^4}$$

15.2 2015

$$a_1 = \frac{1296000 \times 20516500 - 87720000 \times 312850}{(1296000)^2 - 20400 \times 87720000} = \frac{-8.53818 \times 10^{10}}{-1.09872 \times 10^{10}}$$
$$= 7.771$$

$$a_2 = \frac{1296000 \times 312850 - 20400 \times 20516500}{(1296000)^2 - 20400 \times 87720000} = \frac{-1.3083 \times 10^{10}}{-1.09872 \times 10^{10}}$$
$$= 0.119$$