멀티미디어수치해석 과제 05 : 11.2,6,13(a)
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11.2 다음 방정식의 역행렬을 구해라

$$\begin{cases} -8x_1 + x_2 - 2x_3 = -20 \\ 2x_1 - 6x_2 - x_3 = -38 \\ -3x_1 - x_2 + 7x_3 = -34 \end{cases}$$

$$\Leftrightarrow \begin{bmatrix} -8 & 1 & -2 \\ 2 & -6 & -1 \\ -3 & -1 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -20 \\ -38 \\ -34 \end{bmatrix}$$

$AX = I$, $X = [x_1 \; x_2 \; x_3]$

$A x_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $A x_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $A x_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$A = \begin{bmatrix} -8 & 1 & -2 \\ 2 & -6 & -1 \\ -3 & -1 & 7 \end{bmatrix}$

$R_2 = R_2 - (-\frac{1}{4})R_1$

$R_2 = [2 \; -6 \; -1]$

$-) -\frac{1}{4} R_1 = [2 \; -\frac{1}{4} \; \frac{1}{2}]$

$R_2 = 0 \; -\frac{23}{4} \; -\frac{3}{2}$

$R_3 = R_3 - (\frac{3}{8})R_1$

$R_3 = [-3 \; -1 \; 7]$

$-) \frac{3}{8} R_1 = [-3 \; \frac{3}{8} \; -\frac{3}{4}]$

$R_3 = 0 \; -\frac{11}{8} \; \frac{31}{4}$

$\begin{bmatrix} -8 & 1 & -2 \\ -\frac{1}{4} & -\frac{23}{4} & -\frac{3}{2} \\ \frac{3}{8} & -\frac{11}{8} & \frac{31}{4} \end{bmatrix} \leftarrow$

$R_3 = R_3 - \frac{11}{2 \cdot 23} R_2$

$= R_3 - \frac{11}{46} R_2$

$R_3 = 0 \; -\frac{11}{8} \; \frac{31}{4}$

$-) \frac{11}{46} R_2 = 0 \; -\frac{11}{8} \; -\frac{33}{92}$

$R_3 = 0 \; 0 \; \frac{373}{46}$

$\frac{31}{4} = \frac{713}{92}$, $\frac{713+33}{92} = \frac{746}{92} = \frac{373}{46}$

$\begin{bmatrix} -8 & 1 & -2 \\ -\frac{1}{4} & -\frac{23}{4} & -\frac{3}{2} \\ \frac{3}{8} & \frac{11}{46} & \frac{175}{23} \end{bmatrix}$

$A = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{4} & 1 & 0 \\ \frac{3}{8} & \frac{11}{46} & 1 \end{bmatrix} \begin{bmatrix} -8 & 1 & -2 \\ 0 & -\frac{23}{4} & -\frac{3}{2} \\ 0 & 0 & \frac{373}{46} \end{bmatrix}$

-3, $\frac{3}{8} - \frac{11}{8}$, $-\frac{3}{4} - \frac{33}{92} + \frac{373}{46}$, $\frac{-69-33+746}{92}$, $= \frac{644}{92}$

$LUX_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$LD_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$d_{11} = 1$

$-\frac{1}{4} d_{11} + d_{12} = 0$

$d_{12} = \frac{1}{4} d_1 = \frac{1}{4}$

$\frac{3}{8} d_{11} + \frac{11}{46} d_{12} + d_{13} = 0$

$d_{13} = -\frac{3}{8} d_{11} - \frac{11}{46} d_{12}$

$= -\frac{3}{8} \cdot 1 - \frac{11}{46} \cdot \frac{1}{4}$

$= -\frac{69+11}{8 \cdot 23}$

$= -\frac{80}{8 \cdot 23}$

$= -\frac{10}{23}$

$D_1 = \begin{bmatrix} 1 \\ \frac{1}{4} \\ -\frac{10}{23} \end{bmatrix}$

$UX_1 = D_1$

$\frac{373}{46} x_{13} = -\frac{10}{23}$ $\quad x_{13} = \frac{46}{373}(-\frac{10}{23}) = -\frac{20}{373}$

$-\frac{23}{4} x_{12} - \frac{3}{2} x_{13} = \frac{1}{4}$

$-\frac{23}{4} x_{12} = \frac{3}{2} x_{13} + \frac{1}{4} = \frac{3}{2}(-\frac{20}{373}) + \frac{1}{4} = \frac{-120+373}{4 \cdot 373}$

$= \frac{253}{4 \cdot 373}$

$x_{12} = -\frac{4}{23} \cdot \frac{253}{4 \cdot 373} = -\frac{11}{373}$

$-8x_{11} + x_{12} - 2x_{13} = 1$

$-8x_{11} = 1 - x_{12} + 2x_{13} = 1 + \frac{11}{373} - \frac{40}{373} = \frac{373+11-40}{373} = \frac{344}{373}$

$x_{11} = -\frac{1}{8} \cdot \frac{344}{373} = -\frac{1}{2} \cdot \frac{86}{373} = \frac{-43}{373}$

$X_1 = \begin{bmatrix} -\frac{43}{373} \\ -\frac{11}{373} \\ -\frac{20}{373} \end{bmatrix}$

11.2 계속

$LUX_2 = \begin{bmatrix}0\\1\\0\end{bmatrix}$

$LD_2 = \begin{bmatrix}0\\1\\0\end{bmatrix}$

$d_{21} = 0.$

$-\frac{1}{4}d_{21} + d_{22} = 1$

$d_{22} = 1 + \frac{1}{4}d_{21} = 1 + \frac{1}{4}\cdot 0 = 1$

$\frac{3}{8}d_{21} + \frac{11}{46}d_{22} + d_{23} = 0$

$d_{23} = -\frac{3}{8}d_{21} - \frac{11}{46}d_{22}$

$= -\frac{3}{8}0 - \frac{11}{46}\cdot 1 = -\frac{11}{46}$

$D_2 = \begin{bmatrix}0\\1\\-\frac{11}{46}\end{bmatrix}$

$UX_2 = D_2$

$\frac{373}{46}x_{23} = \frac{-11}{46}, \quad x_{23} = -\frac{11}{373}$

$-\frac{23}{4}x_{22} - \frac{3}{2}x_{23} = 1$

$-\frac{23}{4}x_{22} = 1 + \frac{3}{2}x_{23} = 1 + \frac{3}{2}\left(-\frac{11}{373}\right)$

$= \frac{746-33}{2\cdot 373} = \frac{713}{2\cdot 373}$

$x_{22} = -\frac{\cancel{4}^{2}}{23}\frac{713}{\cancel{2}\cdot 373} = -\frac{2\cdot 31}{373} = -\frac{62}{373}$ $\quad 23\overline{)713} = 31,\ 69,\ 23$

$-8x_{21} + x_{22} - 2x_{23} = 0$

$-8x_{21} = -x_{22} + 2x_{23}$

$= \frac{62}{373} - \frac{22}{373} = \frac{40}{373}$

$x_{21} = -\frac{1}{\cancel{8}}\frac{\cancel{40}^{5}}{373} = -\frac{5}{373}$

$X_2 = \begin{bmatrix}-\frac{5}{373}\\-\frac{62}{373}\\-\frac{11}{373}\end{bmatrix}$

$LUX_3 = \begin{bmatrix}0\\0\\1\end{bmatrix}$

$LD_3 = \begin{bmatrix}0\\0\\1\end{bmatrix}$

$d_{31} = 0$

$-\frac{1}{4}d_{31} + d_{32} = 0, \quad d_{32} = 0$

$\frac{3}{8}d_{31} + \frac{11}{46}d_{32} + d_{33} = 1 \quad d_{33} = 1$

$UX_3 = D_3 = \begin{bmatrix}0\\0\\1\end{bmatrix}$

$\frac{373}{46}x_{33} = 1 \quad x_{33} = \frac{46}{373}$

$-\frac{23}{4}x_{32} - \frac{3}{2}x_{33} = 0$

$-\frac{23}{4}x_{32} = +\frac{3}{2}x_{33} = +\frac{3}{\cancel{2}}\frac{\cancel{46}^{23}}{373}$

$x_{32} = -\frac{4}{\cancel{23}}\frac{3\cdot\cancel{23}}{373} = -\frac{12}{373}$

$-8x_{31} + x_{32} - 2x_{33} = 0$

$-8x_{31} = -x_{32} + 2x_{33}$

$= +\frac{12}{373} + \frac{92}{373} = \frac{104}{373}$

$x_{31} = -\frac{13}{373}$

$X_3 = \begin{bmatrix}-\frac{13}{373}\\-\frac{12}{373}\\\frac{46}{373}\end{bmatrix}$

$A^{-1} = X = \begin{bmatrix}X_1 & X_2 & X_3\end{bmatrix}$

$= \begin{bmatrix}-\frac{43}{373} & -\frac{5}{373} & -\frac{13}{373}\\ -\frac{11}{373} & -\frac{62}{373} & -\frac{12}{373}\\ -\frac{20}{373} & -\frac{11}{373} & \frac{46}{373}\end{bmatrix}$

$= -\frac{1}{373}\begin{bmatrix}43 & 5 & 13\\ 11 & 62 & 12\\ 20 & 11 & -46\end{bmatrix}$

11.6 $A = \begin{bmatrix} 8 & 2 & -10 \\ -9 & 1 & 3 \\ 15 & -1 & 6 \end{bmatrix}$

$\|A\|_f$, $\|A\|_1$, $\|A\|_\infty$ 를 구해라

놈을 구하기 전에 각 행의 최대원소를 1로 만들어라.

$$A = \begin{bmatrix} 1 & \frac{1}{4} & -\frac{5}{4} \\ -3 & \frac{1}{3} & 1 \\ 1 & -\frac{1}{15} & -\frac{2}{5} \end{bmatrix}$$

$$\|A\|_f = \sqrt{1 + \frac{1}{16} + \frac{25}{16} + 9 + \frac{1}{9} + 1 + 1 + \frac{1}{225} + \frac{4}{25}}$$

$$= 3.7283$$

$\|A\|_1 = |-3| = 3$

$\|A\|_\infty = \frac{13}{3}$

$|a_{11}| + |a_{12}| + |a_{13}| = 1 + \frac{1}{4} + \frac{5}{4} = 1 + \frac{6}{4} = \frac{10}{4} = \frac{5}{2} = \frac{75}{30}$

$|a_{21}| + |a_{22}| + |a_{23}| = 3 + \frac{1}{3} + 1 = \frac{13}{3} = \frac{130}{30}$

$|a_{31}| + |a_{32}| + |a_{33}| = 1 + \frac{1}{15} + \frac{2}{5} = \frac{15+1+6}{15} = \frac{22}{15} = \frac{44}{30}$

11.13 (a) 역행렬과 조건수를 구해라.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

(b) a_{33}을 9.1로 조금 바꾸어서 (a)를 반복해라.

(a)

$R_2 = R_2 - 4R_1$

$R_2 = [4\ 5\ 6]$

$-)\ 4R_1 = [4\ 8\ 12]$

$R_2 = 0\ -3\ -6$

$R_3 = R_3 - 7R_1$

$R_3 = [7\ 8\ 9]$

$-)\ 7R_1 = [7\ 14\ 21]$

$R_3 = [0\ -6\ -12]$

$$\begin{bmatrix} 1 & 2 & 3 \\ \boxed{4} & -3 & -6 \\ \boxed{7} & -6 & -12 \end{bmatrix}$$

$R_3 = R_3 - 2R_2$

$R_3 = 0\ -6\ -12$

$-)\ 2R_2 = 0\ -6\ -12$

$R_3 = 0\ 0\ 0$

$$\begin{bmatrix} 1 & 2 & 3 \\ \boxed{4} & -3 & -6 \\ \boxed{7} & \boxed{0} & \boxed{0} \end{bmatrix}$$

$$\det(A) = 1\begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix} - 2\begin{vmatrix} 4 & 6 \\ 7 & 9 \end{vmatrix} + 3\begin{vmatrix} 4 & 5 \\ 7 & 8 \end{vmatrix}$$

$= 45 - 48 - 2(36-42) + 3(32-35)$

$= -3 - 2(-6) + 3(-3)$

$= -3 + 12 - 9 = 0$

A^{-1}은 존재 안함

∴ 조건수도 없음

(b)

$$A' = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9.1 \end{bmatrix}$$

$R_2 = R_2 - 4R_1$

$R_2 = 4\ 5\ 6$

$-)\ 4R_1 = 4\ 8\ 12$

$R_2 = [0\ -3\ -6]$

$R_3 = R_3 - 7R_1$

$R_3 = 7\ 8\ 9.1$

$-)\ 7R_1 = 7\ 14\ 21$

$R_3 = [0\ -6\ -11.9]$

$$\begin{bmatrix} 1 & 2 & 3 \\ \boxed{4} & -3 & -6 \\ \boxed{7} & -6 & -11.9 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 \\ \boxed{4} & -3 & -6 \\ \boxed{7} & \boxed{2} & 0.1 \end{bmatrix}$$

$R_3 = R_3 - 2R_2$

$R_3 = 0\ -6\ -11.9$

$-)\ 2R_2 = 0\ -6\ -12$

$0\ 0\ 0.1$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 7 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & 0.1 \end{bmatrix}$$

$$LUX = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$LUX_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad LD_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$d_{11} = 1$

$4d_{11} + d_{12} = 0 \quad d_{12} = -4d_{11} = -4$

$7d_{11} + 2d_{12} + d_{13} = 0 \quad d_{13} = 0 - 7d_{11} - 2d_{12}$

$= 0 - 7 + 8$

$= 1$

$D_1 = \begin{bmatrix} 1 \\ -4 \\ 1 \end{bmatrix}$

$UX_{11} = D_1$

$0.1x_{13} = 1 \quad x_{13} = 10.$

$-3x_{12} - 6x_{13} = -4$

$-3x_{12} = -4 + 6x_{13} = -4 + 60 = 56$

$x_{12} = -\frac{56}{3}$

$x_{11} + 2x_{12} + 3x_{13} = 1$

$x_{11} = 1 - 2x_{12} - 3x_{13} = 1 + \frac{112}{3} - 30$

$= \frac{3 + 112 - 90}{3} = \frac{25}{3}$

$$X_1 = \begin{bmatrix} \frac{25}{3} \\ -\frac{56}{3} \\ 10 \end{bmatrix}$$

11.13 (b) 계속

$LUX_2 = LD_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$d_{21} = 0$

$4d_{21} + d_{22} = 1$

$d_{22} = 1 - 4d_{21} = 1$

$7d_{21} + 2d_{22} + d_{23} = 0$

$d_{23} = -7d_{21} - 2d_{22} = -2$

$D_2 = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$

$UX_2 = D_2$

$0.1x_{23} = -2 \quad x_{23} = -20$

$-3x_{22} - 6x_{23} = 1$

$-3x_{22} = 1 + 6x_{23} = 1 - 120 = -119$

$x_{22} = \frac{119}{3}$

$x_{21} + 2x_{22} + 3x_{23} = 0$

$x_{21} = -2x_{22} - 3x_{23}$

$= -2\frac{119}{3} + 60 = \frac{-238 + 180}{3} = -\frac{58}{3}$

$X_2 = \begin{bmatrix} -\frac{58}{3} \\ \frac{119}{3} \\ -20 \end{bmatrix}$

$LUX_3 = LD_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$d_{31} = 0$

$4d_{31} + d_{32} = 0 \quad d_{32} = 0$

$7d_{31} + 2d_{32} + d_{33} = 1 \quad d_{33} = 1$

$UX_3 = D_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$0.1x_{33} = 1 \quad x_{33} = 10$

$-3x_{32} - 6x_{33} = 0 \quad x_{32} = -2x_{33} = -20$

$x_{31} + 2x_{32} + 3x_{33} = 0$

$x_{31} = -2x_{32} - 3x_{33} = -2(-20) - 3 \cdot 10 = 40 - 30 = 10$

$X_3 = \begin{bmatrix} 10 \\ -20 \\ 10 \end{bmatrix}$

$A^{-1} = X = [X_1 \; X_2 \; X_3]$

$= \begin{bmatrix} \frac{25}{3} & -\frac{58}{3} & 10 \\ -\frac{56}{3} & \frac{119}{3} & -20 \\ 10 & -20 & 10 \end{bmatrix}$

$\|A\|_\infty = 24.1$

$|a_{11}| + |a_{12}| + |a_{13}| = 1+2+3 = 6$

$|a_{21}| + |a_{22}| + |a_{23}| = 4+5+6 = 15$

$|a_{31}| + |a_{32}| + |a_{33}| = 7+8+9.1 = 24.1$

$\|A^{-1}\|_\infty = \frac{195}{3} = 65$

$|a'_{11}| + |a'_{12}| + |a'_{13}| = \frac{25}{3} + \frac{58}{3} + 10 = \frac{25+58+30}{3} = \frac{113}{3}$

$|a'_{21}| + |a'_{22}| + |a'_{23}| = \frac{56}{3} + \frac{119}{3} + 20 = \frac{195}{3} = 65$

$|a'_{31}| + |a'_{32}| + |a'_{33}| = 10 + 20 + 10 = 40$

row-sum norm 조건수:

$cond(A) = \|A\|_\infty \|A^{-1}\|_\infty$

$= 24.1 \times 65 = 1566.5$

$\|A\|_f = \sqrt{1^2+2^2+3^2+4^2+5^2+6^2+7^2+8^2+9.1^2}$

$= \sqrt{1+4+9+16+25+36+49+64+82.81}$

≈ 16.9355

$\|A^{-1}\|_f = \sqrt{\frac{625}{9} + \frac{3364}{9} + 100 + \frac{3136}{9} + \frac{1461}{9} + 400 + 100 + 400 + 100}$

≈ 45.3211

프로베니우스 norm 조건수:

$cond(A) = \|A\|_f \|A^{-1}\|_f$

$= 16.9355 \times 45.3211 \approx 767.5355$