

A Unified Empirical Framework to Study Segregation

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Abstract

We study the determinants of race and income segregation in the San Francisco Bay area from 1990 to 2004. Our framework incorporates the endogenous feedback loop at the core of the seminal Schelling (1969) model of segregation into a dynamic model of neighborhood choice, thus allowing for data to be observed in transition toward a steady state. We assess the relative importance of a variety of mechanisms that generate segregation – endogenous sorting on the basis of the socioeconomic composition of neighbors, sorting on the basis of other neighborhood amenities, differential responses to prices – and the frictions that mediate these mechanisms – moving costs and incomplete information. Identification of households’ endogenous responses to the socioeconomic compositions of neighbors is facilitated by novel instrumental variables that exploit the logic of a dynamic choice model with frictions. Sorting based on unobserved neighborhood amenities is the most important factor generating segregation followed distantly by endogenous sorting on the basis of the socioeconomic composition of neighbors. Frictions, primarily moving costs, play a central role in keeping segregation in check as they disproportionately mitigate endogenous sorting.

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1 Introduction

Neighborhood demographics are often in a state of flux. In Figure 1, we present the evolution of the socioeconomic compositions of several San Francisco Bay Area neighborhoods over a fifteen year period. As suggested by these selected neighborhoods, there is rich heterogeneity in the trends of the race and income compositions across neighborhoods. What explains these trends? In his seminal work, Schelling (1969) proposed a concise answer to this question: the composition of neighborhoods may change due to the presence of discrimination. That is, households sort on the basis of the race or income of their neighbors. If, for instance, Hispanic households prefer Hispanic neighbors relative to non-Hispanic ones, then an increase in the Hispanic share of a neighborhood might induce additional relative inflows of Hispanic households, which would in turn trigger further inflows of Hispanics in the future, and so on. This endogenous positive feedback loop could generate the observed serial correlation in socioeconomic composition we see in West Richmond (top left panel) by itself.

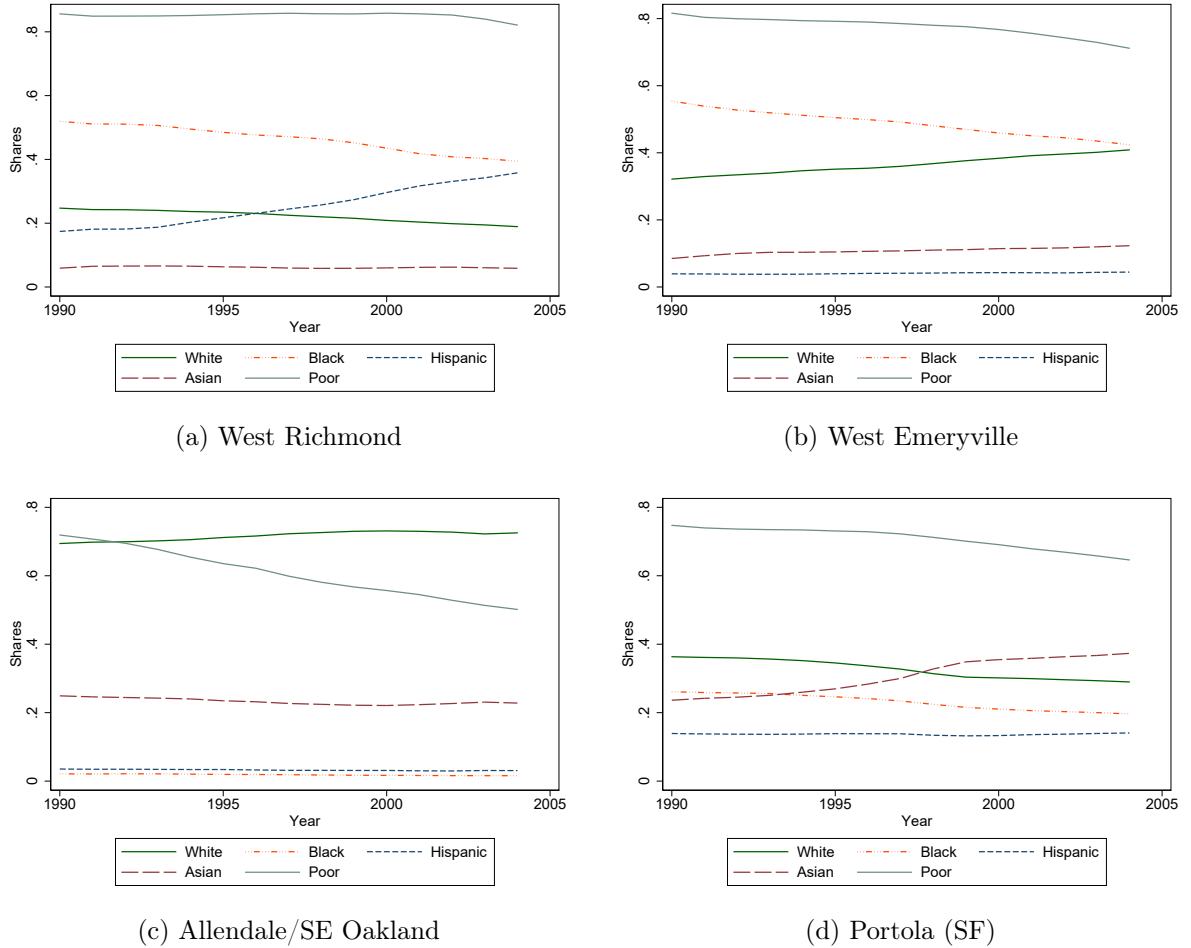
Meanwhile, a rich parallel literature on residential choice has developed to study sorting on the basis of local amenities (including the socioeconomic compositions of neighborhoods) and prices.¹ A common assumption in this literature is that neighborhoods are observed in *steady state*, i.e., in the absence of future amenity shocks, the demographic compositions of the neighborhoods will not change. This leaves no room for the endogenous mechanism discussed above, so the trends shown in Figure 1 would be attributed to serially correlated, exogenous changes in the amenities of these neighborhoods. For instance, we would conclude that some West Richmond amenity that Hispanics disproportionately like has gradually increased over the sample period in some manner outside of the model of residential choice.

This paper studies the determinants of segregation within a new framework that bridges these two literatures. In particular, we incorporate into empirical models of residential choice the endogenous feedback loop that fuels the dynamics suggested by Schelling (1969). This allows us to study the dynamic trajectory toward steady state, which may have important implications for segregation. This should be of particular interest to policymakers since any policy that influences the socioeconomic compositions of neighborhoods today may also impact the trajectories of their future socioeconomic

¹See, for example, McGuire (1974); Epple, Filimon and Romer (1984); Kiel and Zabel (1996); Epple and Sieg (1999); Epple, Romer and Sieg (2001, 2003); Bayer, McMillan and Rueben (2004*a, b*); Bayer and Timmins (2005, 2007); Bayer, Ferreira and McMillan (2007); Bayer et al. (2016); Caetano (2019).

compositions even if no other actions are taken later on. This implies that the short-run effect of such a policy may be completely different from its long-run effect.

Figure 1: Socioeconomic Composition of Selected San Francisco Bay Area Neighborhoods Over Time, 1990-2004



Source: See Section 2.

We modify certain assumptions in the residential choice literature to accommodate this endogenous mechanism. Specifically, we weaken assumptions about households' expectations over the socioeconomic compositions of neighborhoods at the time they make their residential choices. Because these expectations impact the choices that households make, which in turn translate into future expectations of the socioeconomic compositions of neighborhoods, restrictions on how expectations are formed may influence the

strength of the feedback loop. For instance, information frictions may attenuate the ability of discrimination to translate into segregation because of *ex-ante* uncertainty: each household may not individually respond as strongly to the socioeconomic compositions of neighborhoods because they may not be able to perfectly forecast the sorting decisions of others (leading to a coordination problem) or the extent to which amenities will change in the future due to changes in the socioeconomic compositions. This friction is reinforced by moving costs since it is costly to undo a choice that was made with an expectation that did not get realized.

Identifying this endogenous feedback loop requires us to isolate households' responses to the socioeconomic compositions of neighborhoods versus their responses to other neighborhood amenities. To this end, we propose a new instrumental variable (IV) that follows the logic of a dynamic choice model with frictions. Our identification strategy relies on the assumption that information from the more distant past (e.g., two years ago) does not directly affect valuations of neighborhoods today *conditional on valuations from the more recent past* (e.g., in the past year). This translates into isolating the component of a neighborhood's socioeconomic composition that is due to *mismatched households*, i.e., those who currently reside in their neighborhood for reasons that are no longer relevant to new inflows; although they made optimal choices in the past given their expectations at that time, these households are now stranded in their homes because of moving costs in spite of that neighborhood having become less attractive to them in the meantime. We perform many robustness checks that support the validity of the identification strategy in our application.

We analyze a monthly data set of residential sales (Bayer et al. (2016)) across 224 neighborhoods in the San Francisco Bay Area from 1990-2004 that allows us to observe the heterogeneous sorting of eight socioeconomic groups over time: rich and poor Whites, Blacks, Hispanics and Asians. Our empirical framework combines a dynamic model of neighborhood choice with a simulation procedure that allows us to isolate specific determinants of segregation (and their interactions) by analyzing simulated counterfactuals. We summarize the framework here. First, households form expectations about the endogenous characteristics of neighborhoods (their race and income compositions, and their expected effects on prices and other amenities) as well as unobserved neighborhood characteristics. Based on these expectations, they decide if they should move and, if so, which neighborhood is best for them. They may sort heterogeneously on the basis of both the expected race and income compositions of their neighbors and unobserved amenities. All of this sorting is mediated by two frictions: moving costs

and incomplete information. We estimate two key sets of parameters for each of the eight socioeconomic groups of households: their moving costs and their responses to different types of neighbors. Following Bayer et al. (2016), we identify moving costs from the decisions of households who chose to move instead of staying in their current houses. We identify the responses of households to their neighbors with the IV strategy discussed above.

Given these estimated parameters and our model, we simulate what would happen endogenously to the socioeconomic compositions of neighborhoods under various counterfactuals that include: different initial allocations of households across neighborhoods, different responses to neighbors (e.g., no race and/or income discrimination), different levels of moving costs, and different price sensitivities. For a simulation starting today, we explicitly model the fact that sorting today affects the choices of households next month, which in turn may affect the choices of households in two months, repeating this endogenous feedback loop indefinitely until reaching a new steady state. We simulate this entire dynamic re-sorting process to uncover the resulting trajectories of neighborhoods under these different counterfactuals. Comparing these trajectories across counterfactuals allow us to identify the relative roles of each factor in explaining segregation.

We find robust evidence of discriminatory sorting: households tend to respond positively to neighbors of the same race and income, though to different degrees depending on their race and their income. There is also substantial heterogeneity in the responses to neighbors of other types, and some of these responses are not reciprocated, which leads to complex dynamics. We also find some evidence of heterogeneous sorting due to prices. However, we find much greater heterogeneity in the responses to unobserved amenities. All of these drivers of segregation, especially discrimination, are mitigated by frictions. Absent moving costs, there would be much more sorting across neighborhoods, which would dramatically reshape their socioeconomic compositions. This in turn would trigger further discriminatory sorting, which would grow in prominence in the long-run. By attenuating this feedback loop, moving costs carve out a greater role for the other neighborhood amenities as determinants of segregation.

Relevant Literature

Our paper bridges two distinct but related literatures on residential choice and segregation. We briefly review some of the most relevant studies.

Empirical Models of Residential Choice and Neighborhood Sorting

Because segregation is an outcome of neighborhood sorting, we build upon the prolific literature on the determinants of residential choice.² This literature is largely interested in estimating the marginal willingness to pay for neighborhood amenities. Three papers in this literature are particularly related to our study. Bayer, McMillan and Rueben (2004a) develop a framework to estimate horizontal models of neighborhood choice by building on insights from the empirical industrial organization literature (Berry (1994); Berry, Levinsohn and Pakes (1995)). This framework has been widely applied and extended in this literature (e.g., Bayer, Ferreira and McMillan (2007); Bayer, Keohane and Timmins (2009); Ferreira (2010); Bayer and McMillan (2012); Ringo (2013); Bayer et al. (2016); Caetano (2019)). Bayer and Timmins (2005) study the existence and uniqueness of equilibrium in sorting models with endogenous amenities such as the demographic composition of a neighborhood; Bayer and Timmins (2007) discuss estimation in empirical models like these and suggest an IV approach for identification based on the logic of a static model of neighborhood choice.

Following this literature, we employ a discrete choice framework that enables us to study the relative importance of racial and income compositions, prices and unobserved amenities in explaining the sorting patterns that lead to segregation. This also allows us to embed moving costs as an additional friction that prevents sorting. A key departure lies in our weakening of assumptions on households' expectations when residential decisions are made. This is crucial for our purpose, as it renders our approach compatible with residential choices being observed in the process of convergence toward steady state via the endogenous mechanism described above. Another related departure is that we take a different strategy to estimate a dynamic model of residential choice with moving costs. Although this is not the first paper to do so in the context of neighborhood choice (e.g., Bayer et al. (2016) and Caetano (2019)), we show that many standard assumptions in dynamic demand estimation can be avoided when the goal is to study segregation (as opposed to uncovering the value of amenities as is typical in these studies). Finally, the IV approach that we develop is novel, and it follows from the logic of a dynamic model of neighborhood choice with frictions. These IVs can be created with no additional data requirements and they can also be used to identify price responses.

²Kuminoff, Smith and Timmins (2013) provide a comprehensive review of the growing literature on neighborhood sorting.

Dynamic Models of Segregation

A largely theoretical literature based on the seminal Schelling model (Schelling (1969, 1971)) has sought to explore how segregation can arise and evolve when households care about the characteristics of their neighbors. In the Schelling model, heterogeneous agents select where to live by simple rules of thumb. Although this purely heuristic model is not explicitly based on the optimization of an objective, it generates a valuable insight into a fundamental social force that may drive segregation: agents of different types react systematically differently to the composition of their neighbors. Schelling also explicitly models a friction, myopia, to ensure that neighborhoods gradually evolve toward a steady state.

Subsequent theoretical papers have embedded this intuition into a more standard economic framework (e.g., Becker and Murphy (2000); Bayer and Timmins (2005)), and there have been some recent attempts to estimate these models of segregation in reduced-form and structural contexts (e.g., Card, Mas and Rothstein (2008a); Banzhaf and Walsh (2013); Caetano and Maheshri (2017, 2020)). Banzhaf and Walsh (2013) discuss the role of other amenities in generating segregation under no moving costs. Caetano and Maheshri (2017) and Caetano and Maheshri (2020) study school segregation in a framework that embeds the key insight of Schelling (1969) and discuss how policies may have completely different effects on segregation in the short- and the long-run because of the endogenous feedback loop. In this paper, we generalize and extend that framework in four directions. First, we analyze segregation along multiple dimensions simultaneously. Second, we make fewer assumptions on households' expectations, thus imposing fewer restrictions on the way that race and income compositions of neighborhoods may evolve. Third, we explicitly model realistic frictions such as moving costs, which motivates novel IVs. Finally, we extend the framework to account for heterogeneous and endogenous responses to prices.

The rest of the paper proceeds as follows. In Section 2, we describe our data. In Section 3 we present an empirical model of neighborhood segregation, articulate the specific assumptions required for identification, and discuss the estimation and simulation of this model. We present our baseline results in Section 4 and consider different counterfactuals in order to assess the importance of various determinants of segregation in Section 5. In Section 6, we extend our framework to explicitly incorporate prices and present additional results before concluding in Section 7.

2 Data

We use a monthly sample of all San Francisco Bay Area neighborhoods from January 1990 to November 2004. We define the San Francisco Bay Area as the six core counties (Alameda, Contra Costa, Marin, Santa Clara, San Francisco and San Mateo counties) that comprise the major cities of San Francisco, Oakland and San Jose and their surroundings, which are divided into neighborhoods by merging contiguous Census tracts until each resulting neighborhood contains approximately 10,000 households. Those neighborhoods with fewer than six annual home sales in our sample period are dropped leaving a total of 224 neighborhoods.

For each neighborhood in each month, we compute estimates of their race and income composition following the approach described in Bayer et al. (2016). Because high frequency data on the socioeconomic composition of neighborhoods is unavailable from standard sources (e.g., the Census) we must merge information from two main sources in order to construct these variables. The first source is Dataquick Information Services, a national real estate data service. Dataquick provides a detailed listing of all real estate transactions in the Bay Area including buyers' and sellers' names, buyer's mortgage information and property locations. The second source is a dataset on mortgage applications published in accordance with the Home Mortgage Disclosure Act (HMDA) of 1975. Notably, HMDA data contains demographic information on mortgage applicants and the locations of properties that the applicants are buying. By linking these datasets on buyer's mortgage information and property locations, we can estimate how the demographics of neighborhoods change with each real estate transaction. With neighborhood-level estimates of the flows of households of different groups, we estimate the actual socioeconomic composition of each neighborhood by anchoring our flow estimates to the actual socioeconomic composition of each neighborhood per the 1990 US Census.³

We classify households into eight groups on the basis of four races (Whites, Blacks, Hispanics and Asians) and two income designations (rich or poor, depending on whether household income is greater than \$50,000 in 1990 dollars).⁴ For expositional simplicity,

³Bayer et al. (2016) report the results of several diagnostic tests that ensure the validity of this estimation procedure.

⁴We obtain the race and income of the original stock of households as of 1990 from the 1990 Census. From 1990 onward, all changes in the income of the neighborhood are measured based on income data from HMDA deflated to 1990 levels. We chose an income threshold of \$50,000 because it resulted in the most balance of rich and poor among all available thresholds in the 1990 Census.

we refer to Hispanics as a race rather than an ethnicity, and the other three racial groups include only non-Hispanic households.⁵ For each race-income group g , neighborhood j and month t , we observe the total number of homeowners, the total numbers of homeowners who moved into a new house, and the total number of homeowners who stayed in the same home since last month.⁶ We also observe the total number of households of each group who chose to exit the Bay Area homeownership market in each month.⁷

We summarize our data in Table 1. The majority of homeowners in the Bay Area are White, although there are sizable Asian and Hispanic populations as well. Roughly 47% of homeowners in the Bay Area are classified as rich, though this share is much smaller for Blacks and Hispanics. The socioeconomic compositions of neighborhoods also change over time in our sample as reflected in monthly inflow rates ranging from 0.1% for poor Whites to 0.7% for rich Asians.

The high variance in the average number of homeowners of each group reflects substantial cross-sectional heterogeneity in the socioeconomic composition of neighborhoods, i.e., segregation. We calculate the dissimilarity index for each of the eight socioeconomic groups defined by race and income and summarize it in Table 1.⁸ We choose this widely used measure of segregation because it is easy to interpret. For instance, a rich White dissimilarity index of 0.29 indicates that 29% of rich Whites would have to be relocated (holding all other households' locations fixed) in order to distribute them evenly across all Bay Area neighborhoods (i.e., to ensure that the share of rich Whites was the same in all neighborhoods). The index ranges from zero to one,

⁵We are unable to observe populations at the race-ethnicity-income group-tract level in the 1990 Census. Instead, we are able to observe populations at the race-income group-tract level, at the ethnicity-income group-tract level, and at the race-ethnicity-tract level. As such, our raw counts of rich and poor Whites, Blacks and Asians in each neighborhood include Hispanics. To address this, we reweight each group uniformly across neighborhoods to ensure that the number of rich Whites plus the number of poor Whites is equal to the number of non-Hispanic Whites (and do the same for Blacks and Asians), and we uniformly reweight each group to ensure that the number of rich Hispanics plus the number of poor Hispanics is equal to the total number of Hispanics. Our results are effectively unchanged if we assume all Hispanics to be White and adjust the population numbers accordingly.

⁶Households who move between houses within the same neighborhood counted as inflows (but not stayers).

⁷They are the households who are observed to move out of some neighborhood in $t - 1$ but not observed to move into any neighborhood in t .

⁸If N_{gj} is the total number of group g households residing in neighborhood j , then the dissimilarity index for group g households is defined as $\frac{1}{2} \sum_j \left| \frac{N_{gj}}{\sum_k N_{gk}} - \frac{N_j - N_{gj}}{\sum_k (N_k - N_{gk})} \right|$ where $N_j = \sum_g N_{gj}$. Note that a group may corresponds to a race-income combination (e.g., rich Whites), a race (e.g., rich Whites plus poor Whites) or an income level (e.g., poor households of all races).

and a higher value means that households of a given socioeconomic group are more concentrated in certain neighborhoods. Blacks are the most concentrated racial group, followed distantly by Asians, Hispanics and Whites. While rich Whites and Asians tend to be more concentrated than their poor counterparts, the opposite is true for rich Blacks and Hispanics.

Table 1: Summary Statistics

	White		Black		Hispanic		Asian	
	Rich	Poor	Rich	Poor	Rich	Poor	Rich	Poor
Share of Homeowners	0.38 (0.24)	0.33 (0.22)	0.02 (0.02)	0.04 (0.04)	0.04 (0.04)	0.06 (0.06)	0.08 (0.07)	0.05 (0.05)
Average Num. of Homeowners	2,196 (2,089)	1,879 (1,161)	108 (145)	244 (370)	217 (280)	340 (330)	471 (342)	315 (342)
Average Monthly Inflows	8.71 (10.57)	2.36 (3.36)	0.38 (0.92)	0.30 (0.78)	1.26 (2.71)	0.90 (1.73)	3.58 (6.60)	1.13 (2.02)
Average Monthly Stayers	2,187 (2,081)	1,877 (1,160)	108 (145)	243 (370)	216 (278)	340 (329)	467 (702)	314 (341)
Dissimilarity Index	0.29	0.19	0.41	0.57	0.27	0.34	0.36	0.33
Num. of Observations	39,872							

Note: Each observation is a neighborhood-month from January 1990 to November 2004. Poor households have an income of less than \$50,000 in 1990 dollars. Standard deviations are presented in parentheses.

3 Empirical Framework

We start with an overview of our framework before delving into the details of identification and estimation in the remainder of this section. A city is divided into J neighborhoods populated by households of G different demographic groups. Let N_{gjt}

represent the number of group g households who reside in neighborhood j in period t . Each neighborhood possesses a multidimensional endogenous amenity: the socioeconomic (race and income) composition of residents denoted as the vector of shares \mathbf{s}_{jt} , where s_{gjt} is an element of this vector representing the share of group g :

$$s_{gjt} = \frac{N_{gjt}}{\sum_{g'} N_{g'jt}} \quad (1)$$

(Hereafter, we refer to all vectors and matrices in bold type.) The compositions of all neighborhoods in the city can be represented by the matrix \mathbf{s}_t whose j th column is \mathbf{s}_{jt} . At the beginning of each period, households form expectations of their value of residing in each neighborhood and then choose where to reside.

We start from a generic function for the aggregated group g demand for neighborhood j in period t :

$$N_{gjt} = f_{gjt}(\mathbf{s}_t^e; \boldsymbol{\beta}_g, \phi_g) \quad (2)$$

where f_{gjt} is a function unique to each group-neighborhood-period. We denote $\mathbf{s}_t^e = \mathbb{E}_t[\mathbf{s}_t]$ as the expectation of \mathbf{s}_t that is formed by households *just before they make their decision in t* . This stands in contrast to \mathbf{s}_t , which is the *actual* composition *after* everyone made their decision in t .⁹ The parameter vector $\boldsymbol{\beta}_g$ corresponds to the marginal effect of \mathbf{s}_t^e in the absence of moving costs, and the parameter vector ϕ_g represents the moving cost that group g households would incur if they moved out of the house they occupied in $t - 1$. For ease of notation, we do not explicitly include other amenities in equation (2); below we allow for heterogeneous sorting by groups on the basis of these amenities, including unobserved ones.

Although N_{gjt} and \mathbf{s}_t are observed, \mathbf{s}_t^e is not, so it is infeasible to estimate $\boldsymbol{\beta}_g$ and ϕ_g directly from equation (2). To circumvent this issue, we use the actual, observed vector \mathbf{s}_t as a proxy for \mathbf{s}_t^e , yielding

$$N_{gjt} = f_{gjt}(\mathbf{s}_t; \boldsymbol{\beta}_g, \phi_g) + \underbrace{f_{gjt}(\mathbf{s}_t^e; \boldsymbol{\beta}_g, \phi_g) - f_{gjt}(\mathbf{s}_t; \boldsymbol{\beta}_g, \phi_g)}_{\text{error}_{gjt}} \quad (3)$$

With appropriate restrictions on f_{gjt} , the parameters $\boldsymbol{\beta}_g$ and ϕ_g can be estimated in a dynamic discrete choice model that we develop in Section 3.1.

⁹To simplify notation, we do not index \mathbf{s}_{jt}^e by group g , but we do allow these expectations to vary by group in practice. See Remark 4.

Given estimates of $\hat{\beta}_g$ and $\hat{\phi}_g$, we can analyze how the compositions of neighborhoods might evolve under different counterfactual values of \mathbf{s}_t^e . At $\mathbf{s}_t^e = \tilde{\mathbf{s}}$, the counterfactual demand of group g households for neighborhood j is equal to

$$N_{gjt}(\tilde{\mathbf{s}}) = f_{gjt}(\tilde{\mathbf{s}}; \hat{\beta}_g, \hat{\phi}_g) \quad (4)$$

from which we obtain

$$s_{gjt}(\tilde{\mathbf{s}}) = \frac{N_{gjt}(\tilde{\mathbf{s}})}{\sum_{g'} N_{g'jt}(\tilde{\mathbf{s}})} \quad (5)$$

Calculating equation (5) for each group g yields the matrix-valued function $\mathbf{s}_t(\tilde{\mathbf{s}})$, whose j th column is $\mathbf{s}_{jt}(\tilde{\mathbf{s}})$ with generic element $s_{gjt}(\tilde{\mathbf{s}})$. By considering any counterfactual value of $\tilde{\mathbf{s}}$, we can identify $\mathbf{s}_t(\cdot)$ by simulation. This function defines a dynamic system that fully characterizes the evolution of neighborhood-level demographics (and thus segregation) from any initial state in the absence of future exogenous shocks starting from t . By repeatedly evaluating $\mathbf{s}_t(\cdot)$ starting from $\tilde{\mathbf{s}}$, we construct the *simulated trajectory* $\mathbb{T}_t(\tilde{\mathbf{s}})$, which is a sequence whose first element is $\mathbb{T}_t^0(\tilde{\mathbf{s}}) = \tilde{\mathbf{s}}$, and whose subsequent elements are recursively defined as $\mathbb{T}_t^r(\tilde{\mathbf{s}}) = \mathbf{s}_t(\mathbb{T}_t^{r-1}(\tilde{\mathbf{s}}))$.

We define a *steady state* as follows:

Definition 1. State \mathbf{s}^* is a *steady state* if $\mathbb{T}_t(\tilde{\mathbf{s}})$ converges to \mathbf{s}^* for some $\tilde{\mathbf{s}}$.¹⁰

The limit point of the simulated trajectory $\mathbb{T}_t(\mathbf{s}_t)$ (if it converges) tells us how neighborhoods would look in the long-run in the absence of any external shocks from period t onward. This serves as the relevant benchmark when considering the long-run effects of counterfactual policies on segregation as we do in Section 5.

Remark 1. There is an intimate connection between assumptions on expectations and the endogenous feedback loop. Note that $\mathbf{s}_t(\mathbf{s}_t^e) = \mathbf{s}_t$ by construction since observed choices in t are made when $\tilde{\mathbf{s}} = \mathbf{s}_t^e$. Thus, assuming households perfectly forecast the compositions (i.e., $\mathbf{s}_t^e = \mathbf{s}_t$) implies $\mathbf{s}_t(\mathbf{s}_t) = \mathbf{s}_t$; that is, it implies that data are observed in steady state and there is no feedback loop. More generally, the trajectory of convergence towards the steady state is likely affected by expectations, so it is crucial

¹⁰This notion of “steady state” in this paper has been sometimes referred to as “equilibrium” in the theoretical literature on the dynamics of segregation (Schelling (1969, 1971); Becker and Murphy (2000)). We view “steady state” as a more appropriate term because neighborhoods are understood to be always in Perfect Bayesian Equilibrium in our setup. Indeed, equation (2) can be thought of as an aggregation of best responses by households in a game of incomplete information.

that we avoid strong assumptions on the formation of households' expectations if we want to study the dynamics of segregation.

3.1 Identification

Below we formalize a model of neighborhood choice that gives rise to equation (2) and impose restrictions that allow for identification and feasible estimation of the function $f_{gjt}(\cdot; \beta_g, \phi_g)$ with available data.

3.1.1 A Dynamic Model of Neighborhood Choice

At any period t , household i faces the dynamic optimization problem

$$\max_{j_{i\tau} \in \mathbb{J}} \mathbb{E}_t \left[\sum_{\tau=t}^{\mathcal{T}} \delta^{\tau-t} \cdot u(j_{i\tau}, \mathbf{b}_{i\tau}) | j_{it}, \mathbf{b}_{it} \right], \quad (6)$$

where $j_{i\tau}$ and $\mathbf{b}_{i\tau}$ are the choice and state variables of household i in period τ respectively, \mathbb{J} is each household's choice set, $u(\cdot)$ is their flow indirect utility function, \mathcal{T} is their time horizon, and δ is their inter-temporal discount factor.

We define the value function as $V(\mathbf{b}_{it}) = \max_{j \in \mathbb{J}} v(j, \mathbf{b}_{it})$, where the choice-specific value function is written as

$$v(j, \mathbf{b}_{it}) = u(j, \mathbf{b}_{it}) + \int \delta \cdot V(\mathbf{b}_{it+1}) dF_b(\mathbf{b}_{it+1} | j, \mathbf{b}_{it}). \quad (7)$$

$F_b(\mathbf{b}_{it+1} | j, \mathbf{b}_{it})$ is the expected distribution of the state variable in $t+1$ conditional on the choice and the state variable from t .

Assumption 1. (Additive Separability, Logit Error, Conditional Independence)

1. $u(j, \mathbf{b}_{it}) = u(j, \mathbf{x}_{it}) + \epsilon_{ijt}$ where ϵ_{ijt} is the j th element of $\boldsymbol{\epsilon}_{it}$.
2. ϵ_{ijt} is i.i.d. extreme value type I.
3. $F_x(\mathbf{x}_{it+1} | j, \mathbf{x}_{it}, \boldsymbol{\epsilon}_{it}) = F_x(\mathbf{x}_{it+1} | j, \mathbf{x}_{it})$ where $F_x(\cdot)$ is the cumulative density function of x .

Assumption 1 implies

$$v(j, \mathbf{b}_{it}) = u(j, \mathbf{x}_{it}) + \underbrace{\int \delta \cdot \bar{V}(\mathbf{x}_{it+1}) f_x(\mathbf{x}_{it+1} | j, \mathbf{x}_{it})}_{v(j, \mathbf{x}_{it})} + \epsilon_{ijt}, \quad (8)$$

where $\bar{V}(\cdot)$ is the *integrated value function*.¹¹

At the beginning of period t , households observe the state variable \mathbf{b}_{it} and choose (a) whether or not to move, and upon deciding to move (b) an option in $\mathbb{J} = \{0, \dots, J\}$. Options $j \in \{1, \dots, J\}$ correspond to residing in neighborhood j . Option $j = 0$ corresponds to the outside option of residing outside of the city.¹² Following Bayer et al. (2016), we simplify notation and index the option of staying in the same home in any neighborhood as option $J + 1$.

Following Bayer et al. (2016), we impose the following restriction on the moving cost parameters to ensure that the identification and estimation of cumulative utilities is feasible with available data:

Assumption 2. *Group g households who decide to move incur a fixed moving cost ϕ_g irrespective of their neighborhoods of origin (j_{it-1}) and destination (j_{it}).*¹³

Under this assumption, we can specify $v(j, \mathbf{x}_{it})$ on the right side of equation (8) as

$$v(j, \mathbf{x}_{it}) = \mathbf{1}(j \in \{0, \dots, J\}) \cdot (v_{gjt} - \phi_g) + \mathbf{1}(j = J + 1) \cdot v_{gjt} \quad (9)$$

where $\mathbf{1}(\cdot)$ is the indicator function, and v_{gjt} , the average of the moving cost-free component of $v(j, \mathbf{x}_{it})$ over all households of group g , is defined as

$$v_{gjt} = \beta'_g \mathbf{s}_{jt}^e + \xi_{gjt}. \quad (10)$$

This definition is without loss of generality, as equation (10) simply projects v_{gjt} separately by group into a component dependent on the expected socioeconomic composition of the neighborhood and a remainder.¹⁴ We define the vector of state variables

¹¹This function, defined as $\bar{V}(\mathbf{x}_{it}) = \int V(\mathbf{x}_{it}, \boldsymbol{\epsilon}_{it}) dG_{\boldsymbol{\epsilon}}(\boldsymbol{\epsilon}_{it})$, is the unique solution to the integrated Bellman equation $\bar{V}(\mathbf{x}_{it}) = \int \max_{j \in \mathbb{J}} \left(u(j, \mathbf{x}_{it}) + \epsilon_{ijt} + \delta \cdot \sum_{\mathbf{x}_{it+1}} \bar{V}(\mathbf{x}_{it+1}) f_x(\mathbf{x}_{it+1} | j, \mathbf{x}_{it}) \right) dG_{\boldsymbol{\epsilon}}(\boldsymbol{\epsilon}_{it})$, where $G_{\boldsymbol{\epsilon}}(\boldsymbol{\epsilon}_{it})$ is the extreme value type I cumulative density function. See Aguirregabiria and Mira (2010).

¹²As in Bayer et al. (2016), we only observe data on homeowners, so in our application, $j = 0$ also corresponds to the outside option of renting within the city.

¹³Allowing for heterogeneity in moving costs by group and neighborhood of origin is infeasible because there are not enough households of certain groups residing in certain neighborhoods during the whole sample period. Because the observed distribution of residential choices varies greatly by socioeconomic group, some of the heterogeneity in moving costs by neighborhood may be incorporated into our estimates of ϕ_g .

¹⁴Note that we do not assume that \mathbf{s}_{jt}^e enters linearly in equation (10). Rather, any non-linearity will be embedded in ξ_{gjt} , which will have implications for Assumption 3. This allows us to interpret $\beta'_g \mathbf{s}_{jt}^e$ as the best linear approximation of the (potentially non-linear) relationship between \mathbf{s}_{jt}^e and

$\mathbf{x}_{it} = (j_{it-1}, \mathbf{s}_t^e, \boldsymbol{\xi}_{g_{it}})$, where g_i is the demographic group to which i belongs and $\boldsymbol{\xi}_{g_{it}}$ is the vector whose j -th element is $\xi_{g_{it}jt}$.¹⁵

Remark 2. Standard dynamic discrete choice approaches often parametrically specify the transition probability $f_{\mathbf{x}}(\mathbf{x}_{it+1}|j, \mathbf{x}_{it})$ from equation (8) and assume \mathbf{x} is observed or estimable by the econometrician.¹⁶ We want to avoid such assumptions in our context. Because $\mathbf{x}_{it} = (j_{it-1}, \mathbf{s}_t^e, \boldsymbol{\xi}_{g_{it}})$, these assumptions would restrict how \mathbf{s}_t^e and $\boldsymbol{\xi}_{g_{it}}$ transition over time, which in turn would restrict simulated trajectories (Remark 1). Avoiding these assumptions implies, for instance, that the time horizon (\mathcal{T}) and the inter-temporal discount factor (δ) may vary across groups, and neither needs to be observed or identified.¹⁷

3.1.2 Instrumental Variables

In order to identify β_g , we impose the following exclusion restriction, which implicitly restricts what is included in $\boldsymbol{\xi}_{g_{it}}$ (see Remark 3):

Assumption 3. $Cov(\xi_{g_{jt}}, \mathbf{s}_{jt-T} | \text{inflows}_{jt-1}, \dots, \text{inflows}_{jt-T'}) = 0$ for some $T > T' \geq 1$.

where $\text{inflows}_{j\tau} = (\text{inflows}_{1j\tau}, \dots, \text{inflows}_{Gj\tau})$, and $\text{inflows}_{gj\tau}$ is the number of households of group g who moved to a new house in neighborhood j in period τ . Under Assumption 3, we can identify β_g using \mathbf{s}_{jt-T} as an instrumental variable (IV) for \mathbf{s}_{jt}^e once we control for a flexible function of inflows in intermediate periods between t and $t - T$. In words, our exclusion restriction states that “no information that was relevant to decision-making in $t - T$ or before (i.e., correlated to \mathbf{s}_{jt-T}) and irrelevant to inflows in $t - 1, \dots, t - T'$ (i.e., uncorrelated to $\text{inflows}_{g'jt-1}, \dots, \text{inflows}_{g'jt-T'}$ for all g') is relevant to inflows in t (i.e., correlated to $\boldsymbol{\xi}_{jt}$).”

Our identification strategy exploits an asymmetry in equation (10): while \mathbf{s}_{jt}^e is

v_{gjt} . In the estimation section we show how we can weaken Assumption 3 in practice by adding flexible controls, and in Footnote 35 we also discuss a variety of robustness checks that we conducted.

¹⁵Note that different households of the same group are allowed to differ from each other only via their previous choice (j_{it-1}) and ϵ_{it} . This restriction can be weakened if additional data is available (e.g., Berry, Levinsohn and Pakes (1995)). See also Remark 4, where we argue that in practice \mathbf{s}_t^e is allowed to vary by group.

¹⁶See, e.g., Aguirregabiria and Mira (2010) for a great survey of the literature. To facilitate comparison with standard approaches, we use their notation whenever possible.

¹⁷These parameters may be embedded inside \mathbf{s}_t^e and $\boldsymbol{\xi}_{g_{it}}$ since they are part of v_{gjt} (see equations (6) and (10)).

(the expectation of) a *stock* variable, v_{gjt} is only a *flow* variable.¹⁸ To illustrate this, we consider the simplest case with $T = 2$. As before, let the superscript “ e ” refer to expectations of the corresponding variable taken by households just before their decisions are made in t . We can decompose a generic scalar element of \mathbf{s}_{jt}^e , s_{gjt}^e , into expected inflows and expected stayers as follows:

$$\begin{aligned} s_{gjt}^e &= \frac{N_{gjt}^e}{\sum_{g'} N_{g'jt}^e} \\ &= \frac{\text{inflows}_{gjt}^e + \text{stayers}_{gjt}^e}{\sum_{g'} (\text{inflows}_{g'jt}^e + \text{stayers}_{g'jt}^e)} \end{aligned} \quad (11)$$

$$= \frac{\text{inflows}_{gjt}^e + \pi_{gjt}^e \cdot N_{gjt-1}}{\sum_{g'} (\text{inflows}_{g'jt}^e + \pi_{g'jt}^e \cdot N_{g'jt-1})} \quad (12)$$

$$= \frac{\text{inflows}_{gjt}^e + \pi_{gjt}^e \cdot (\text{inflows}_{gjt-1} + \pi_{gjt-1} \cdot N_{gjt-2})}{\sum_{g'} (\text{inflows}_{g'jt}^e + \pi_{g'jt}^e \cdot (\text{inflows}_{g'jt-1} + \pi_{g'jt-1} \cdot N_{g'jt-2}))} \quad (13)$$

Equation (11) follows from the accounting identity $N_{gjt} = \text{inflows}_{gjt} + \text{stayers}_{gjt}$, where stayers_{gjt} are those who decided to stay in the same house in period t . Equation (12) follows from substituting $\pi_{gjt}^e \cdot N_{gjt-1}$ for stayers_{gjt}^e into the previous equation, where π_{gjt}^e is the expected probability (as of t) that a group g household stays in the same home in neighborhood j from $t-1$ to t . Finally, equation (13) follows from substituting $\text{inflows}_{gjt-1} + \pi_{gjt-1} \cdot N_{gjt-2}$ for N_{gjt-1} into the previous equation.¹⁹

Note that the IV ($s_{gjt-2} = \frac{N_{gjt-2}}{\sum_{g'} N_{g'jt-2}}$) and the endogenous variable of interest (s_{gjt}^e) are potentially correlated, since both are functions of $\mathbf{N}_{jt-2} = (N_{1jt-2}, \dots, N_{Gjt-2})'$. Based on equation (13), s_{gjt-2} may be correlated to s_{gjt}^e via the following three paths that correspond to the first, second and third terms of equation (13):

1. $\text{Cov}(\text{inflows}_{g'jt}^e, s_{gjt-2}) \neq 0$, i.e., some residents are expected to move into j in t for reasons that are correlated to s_{gjt-2} .
2. $\text{Cov}(\pi_{g'jt}^e \cdot \text{inflows}_{g'jt-1}, s_{gjt-2}) \neq 0$, i.e., some residents who moved into j in $t-1$ for reasons that are correlated to s_{gjt-2} are expected to remain in t .
3. $\pi_{g'jt}^e \cdot \pi_{g'jt-1} \neq 0$, i.e., some residents of j in $t-2$ are expected to remain there in t even if what originally led them to reside there no longer affects future inflows

¹⁸The term “flow” here is in contrast to “stock”, so it means something different from the term “flow” used elsewhere in the paper.

¹⁹For simplicity, the superscript “ e ” is dropped in $t-1$ or before since as of t households might already know these values from the past (although that is not assumed, see Remark 3).

(i.e., even if $\text{Cov}(\text{inflows}_{g'jt}^e, s_{gjt-2}) = \text{Cov}(\pi_{g'jt}^e \cdot \text{inflows}_{g'jt-1}, s_{gjt-2}) = 0$).

We would like to exploit the third path for identification of β_g since it does not affect households who move in t and is therefore uncorrelated to ξ_{gjt} . However, it is confounded by the first and second paths. By flexibly controlling for $\text{inflows}_{g'jt-1}$ for all g' , we eliminate the second path. Finally, once the second path is eliminated, the first path is also eliminated by Assumption 3. In this simpler example our exclusion restriction can be restated as “no information that was relevant to decision-making in $t - 2$ or before (i.e., correlated to s_{jt-2}) and irrelevant to inflows in $t - 1$ (i.e., uncorrelated to $\text{inflows}_{g'jt-1}$ for all g') is relevant to inflows in t (i.e., correlated to ξ_{jt}).” More generally, for a given choice of T' , increasing T weakens the three paths of correlation (and weakens Assumption 3) and hence represents a tradeoff of instrument relevance (third path) for validity (eliminating the first and second paths).²⁰

Intuitively, our IV leverages only variation in neighborhood compositions due to *mismatched* households: those who moved long ago and for whom the reasons for their original move are no longer relevant. If neighborhood amenities or expectations change after $t - T$, some households no longer find themselves residing in their most desired neighborhood, but moving costs have “locked” them into their current home. Because expectations may continue to evolve after households move, mismatch may accumulate between each household’s current neighborhood and its newly most desired neighborhood. Only when mismatch exceeds moving costs do households re-sort to their most desired neighborhood, in turn resetting their mismatch to zero. In a context where moving costs are sufficiently high and amenities or expectations change sufficiently over time, a great deal of mismatch may have accumulated at any moment in the data, which our IV strategy exploits. These mismatched households affect the current composition of the neighborhood, but because the *reasons* why they are there do not plausibly affect inflows today, we can take their impact on s_{jt}^e to be exogenous.

Remark 3. The restriction on ξ_{gjt} in Assumption 3 reflects the idea that inflows in t do not use past information (from $t - T$ or before) in a more sophisticated manner than inflows of some group in $t - 1, \dots, t - T'$. This is a restriction on the *relative* level of sophistication, not on the *absolute* level, so it is consistent with many formulations of expectations, ranging from the narrowly myopic households of Schelling (1969) to

²⁰We performed a series of robustness checks to compare estimates for $T = 13, \dots, 36$ for $T' = 12$; we found that the relevance of the IV remained strong as T grew, yet the estimates did not change. See Figures 13 and 14 in the appendix.

highly sophisticated households. For instance, consider households with rational expectations who use their information set in the best way possible (their forecast errors are orthogonal to their information set). Let those making decisions in t form their expectations with information from the last τ periods, while those making decisions in $t - T'$ form their expectation with information from the last τ' periods. Then Assumption 3 is compatible with any values of τ and τ' provided that $\tau \leq \tau' + T'$. In particular, households in t are allowed to be somewhat more sophisticated than those in $t - T'$ (i.e., $\tau > \tau'$ is allowed).²¹ Note that $(\text{inflows}_{jt-1}, \dots, \text{inflows}_{jt-T'})$ or any other past information is not required to be in the information set of group g households in t since we use only the variation in \mathbf{s}_{gt-T} that is orthogonal to it in order to identify β_g .²² Of course, we also require \mathbf{s}_{t-T} to be correlated to \mathbf{s}_t^e conditional on $(\text{inflows}_{jt-1}, \dots, \text{inflows}_{jt-T'})$ to guarantee the relevance of the IV. While this imposes further restrictions on expectations, this is a testable assumption.

3.1.3 Interpretation of β

The coefficient matrix β captures the various responses of households of different socioeconomic status to their expectations of the socioeconomic compositions of neighborhoods. We do not separately identify whether the response is mediated through a change in the flow utility or the continuation value associated with a neighborhood choice. To see this, consider equation (8), and define u_{gjt} (the flow utility) and CV_{gjt} (the continuation value of households if they choose neighborhood j in t) as the averages of $u(j, \mathbf{x}_{it})$ and $\int \delta \cdot \bar{V}(\mathbf{x}_{it+1}) f_{\mathbf{x}}(\mathbf{x}_{it+1} | j, \mathbf{x}_{it})$ across all households of group g

²¹Although at first our identification strategy might look similar to strategies used in the production function literature, such as the “proxy variable” literature (e.g., Olley and Pakes (1996)) and the “dynamic panel” literature (e.g., Arellano and Bond (1991)), there are important differences. In our setup, there is a distinct asymmetry between the outcome variable (v_{gjt}) and the main explanatory variable (\mathbf{s}_{jt}): while v_{gjt} reflects the decisions of those who are choosing a new house in t , \mathbf{s}_{jt} reflects the decisions of many other households as well (e.g., households of all groups who made their choices in the past) which may have been mediated by moving costs and different information sets. We exploit this asymmetry to build an identifying assumption that relates the information set of decision makers of one group in t with the information sets of all past decision makers. This only requires us to be restrictive with the information set of the household in t *relative* to the information set of the households in the past; we do not have to impose *absolute* restrictions on their information set. In the context of the production function literature, identification exploits absolute restrictions in the information set of decision makers (e.g., firms) at the time of their decision. See Akerberg (2020) for an illuminating discussion of the identifying assumption made in this literature.

²²Our IV approach is very different from the shift-share IV approach (e.g., Bartik (1991)). Although both IVs use past shares, ours assumes exogeneity of them only conditional on the valuations in intermediate periods, whereas shift-share IVs assume exogeneity of them unconditionally as discussed by Goldsmith-Pinkham, Sorkin and Swift (2020).

respectively. Then we can write $v_{gjt} = u_{gjt} + CV_{gjt}$. For each g and g' , we identify $\beta_{g,g'} = \frac{\partial v_{gjt}}{\partial s_{g'jt}^e} = \frac{\partial u_{gjt}}{\partial s_{g'jt}^e} + \frac{\partial CV_{gjt}}{\partial s_{g'jt}^e}$, i.e. the total marginal effect of an expected increase in g' share on the group g valuation of that neighborhood.

We focus on this reduced-form effect because it allows us to study many aspects of segregation without imposing additional assumptions required for this decomposition (see Remark 2). As Manski (2004) argues, choice data alone is insufficient to separately identify expectations and preferences. For instance, suppose a neighborhood is expected to increase its poor share, and we observe rich households responding to it by reducing their demand for that neighborhood. From choice data alone, we could not conclude that they responded to prejudice against poor households (a preference) as opposed to a signal that the neighborhood would become less desirable to them in the future for some other reason (an expectation), or both. While this would prevent us from identifying, say, households' willingness to pay to avoid residing close to poor neighbors, it would not restrict us from analyzing how households sort into or out of a neighborhood in response to an increase in the poor share since this is fundamentally related to households' choices and not their preferences *per se*. Hence, we impose fewer restrictions on expectations, which entails fewer restrictions on simulated trajectories (Remark 1). Of course, this limits the types of counterfactuals we can consider.

The parameter β includes any type of discriminatory sorting on the basis of the socioeconomic composition of neighbors, including pure socioeconomic animus (or affinity) and statistical discrimination. It is useful to elaborate on what may constitute "statistical discrimination" in the context of neighborhood sorting. In the example above, suppose rich households inferred from the expected increase in poor share in t that the quality of the neighborhood school will decline in the future. A response to that expectation would qualify as statistical discrimination.²³ Thus, β includes not only sorting on the basis of expected changes in socioeconomic compositions *per se*, but also sorting on the basis of expected future changes in other amenities due to expected changes in socioeconomic compositions. Only the effects of expected future amenities that are affected by s_{jt}^e are included in β ; the rest are included in ξ per equation (10).²⁴

Moreover, β may also reflect supply-driven discrimination. For instance, suppose we

²³In a world of complete information, households would not use the neighbors' attributes to predict other amenities in the future, as they would be able to know their values directly. Incomplete information leads them to use such information. See Fang and Moro (2011) for a survey of models of statistical discrimination.

²⁴Relatedly, we also made sure to specify only pre-determined controls in order to avoid shutting off any mediation effect from s_{jt}^e to v_{gjt} .

found that Black households responded positively to an increase in the Black share. This would be possible even if Blacks exhibited no demand-driven discrimination, whether taste-based or statistical. Indeed, the same pattern could alternatively be explained by Black households simply facing obstacles to residing in neighborhoods without Blacks because of discrimination on the part of, say, the mortgage market (e.g., Ladd (1998)) or real estate agents (e.g., Ondrich, Ross and Yinger (2003)). Using the language of Christensen and Timmins (2019), in this example supply-driven discrimination would “steer” Blacks toward Black neighborhoods, which would lead us to find that Blacks respond positively to Black share even if there was no demand-driven discrimination on their part.

Therefore, β reflects the overall *ability* of households to discriminate, i.e. sort on the basis of the expected socioeconomic composition of the neighborhood for whatever reason. This ability is affected by both demand and supply considerations, and frictions (other than moving costs, which are explicitly modelled separately in the paper) may restrict or enhance this ability, so they show up in β .

Finally, note that although moving costs (ϕ) are separate from β in the model, this only applies to moves in period t . The expected costs of future moves (in response to unforeseen changes in neighborhood characteristics) are not included in ϕ , but they do impact CV_{gjt} ; hence, by the argument above, they are loaded into β . As a result, β may also contain a component related to the interaction between the anticipated possibility of forecast errors and expected future moving costs. Indeed, households may recognize that they are unable to perfectly predict the future socioeconomic compositions of neighborhoods and any associated effects on other neighborhood amenities, and this may necessitate a future (costly) move.

3.2 Estimation and Simulation

Our empirical approach unfolds in three stages: we first estimate v_{gjt} and ϕ_g for all g, j and t (stage 1) and then we estimate the effect of \mathbf{s}_{jt}^e on v_{gjt} (stage 2). Finally, we use these estimates to simulate the evolution of the demographic compositions of neighborhoods under different counterfactuals (stage 3).

Stage 1: Estimation of v_{gjt} and ϕ_g

This stage follows closely from Bayer et al. (2016). First, we use the choices of only those who moved in period t to estimate the cumulative utilities v_{gjt} . Having decided

to move, household i solves the following optimization problem:

$$\max_{j \in \{0, \dots, J\}} v_{gijt} - \phi_{g_i} + \epsilon_{ijt} \quad (14)$$

Following Assumption 1, the choice-specific probabilities are

$$\begin{aligned} P(j_{it} = j \mid j \notin \{J+1\}, j_{it-1}) &= \frac{\exp(v_{gijt} - \phi_g)}{\sum_{j'=0}^J \exp(v_{gij't} - \phi_g)} \\ &= \frac{\exp(v_{gijt})}{\sum_{j'=0}^J \exp(v_{gij't})} \end{aligned} \quad (15)$$

Because moving costs are assumed to not vary by the neighborhood of origin or destination (Assumption 2), they cancel out. Following Berry (1994), we estimate \hat{v}_{gjt} for $j \in \{0, \dots, J\}$ as

$$\hat{v}_{gjt} = \log(\text{inflows}_{gjt}) - \log(\text{inflows}_{g0t}). \quad (16)$$

Next, we consider the choice of whether or not to stay in the same home to identify the moving cost parameter ϕ_g . For household i who resided in j last period, the probability of choosing option $J+1$ (not moving) is

$$\begin{aligned} P(j_{it} = J+1 \mid j_{it-1} = j) &= P(v_{gijt} + \epsilon_{iJ+1t} > v_{gij't} - \phi_{g_i} + \epsilon_{ij't}, \forall j' \mid j_{it-1} = j) \\ &= \frac{\exp(v_{gijt})}{\sum_{j'=0}^J \exp(v_{gij't} - \phi_{g_i}) + \exp(v_{gijt})} \end{aligned} \quad (17)$$

where the first line must hold for all $j' = 0, \dots, J$, and the second line follows from the logit formula (Assumption 1). The data analog to $P(j_{it} = J+1 \mid j_{it-1} = j)$ is simply $\frac{\text{stayers}_{gijt}}{N_{gijt-1}}$, or the proportion of group g_i households residing in neighborhood j in $t-1$ who decided to stay in the same home in the following period. Hence, equation (17) yields the J moment restrictions

$$h_j(\phi_g; \hat{\mathbf{v}}_{\mathbf{g}t}) = \frac{\text{stayers}_{gjt}}{N_{gjt-1}} - \frac{\exp(\hat{v}_{gjt})}{\sum_{j'=0}^J \exp(\hat{v}_{gjt} - \phi_g) + \exp(\hat{v}_{gjt})} \quad (18)$$

By plugging in our estimates of \hat{v}_{gjt} from equation (16) into equation (17), we can

estimate ϕ_g by GMM using moment conditions (18).

Stage 2: Estimation of β_g

Following equation (10), we decompose \hat{v}_{gjt} , which was estimated in the first stage, as

$$\hat{v}_{gjt} = \beta'_g \mathbf{s}_{jt} + \Lambda_g(\hat{\mathbf{v}}_{jt-1}, \dots, \hat{\mathbf{v}}_{jt-T'}) + \gamma_{gt} + \check{\xi}_{gjt} \quad (19)$$

where $\Lambda_g(\cdot)$ is a flexible function²⁵, γ_{gt} is a group-period fixed effect, and $\check{\xi}_{gjt}$ is an error term that includes all remaining unobserved determinants of \hat{v}_{gjt} . The parameters of interest, β_g , represent the effects of \mathbf{s}_{jt} on \hat{v}_{gjt} . We use \mathbf{s}_{jt-T} as an Instrumental Variable (IV) for \mathbf{s}_{jt} to estimate β_g via Two-Stage Least Squares.

Because we do not observe v_{gjt} or \mathbf{s}_{jt}^e , we use \hat{v}_{gjt} and \mathbf{s}_{jt} , respectively, as proxies for them. Subtracting equation (10) from equation (19) and rearranging yields the following error decomposition:

$$\check{\xi}_{gjt} = (\xi_{gjt} - \Lambda_g(\hat{\mathbf{v}}_{jt-1}, \dots, \hat{\mathbf{v}}_{jt-T'}) - \gamma_{gt}) + \beta'_g(\mathbf{s}_{jt}^e - \mathbf{s}_{jt}) + (\hat{v}_{gjt} - v_{gjt}) \quad (20)$$

The first term corresponds to the determinants of households' cumulative utilities that are due to amenities other than \mathbf{s}_{jt}^e (ξ_{gjt} from equation (10)) and that are orthogonal to $(\hat{\mathbf{v}}_{jt-1}, \dots, \hat{\mathbf{v}}_{jt-T'})$ and γ_{gt} . Assumption 3 implies that the IV is uncorrelated to this term.²⁶ The second term corresponds to forecast errors in households' expectations. Note that v_{gjt} reflects choices made with the same information set that was used to form \mathbf{s}_{jt}^e . Hence, forecast errors $(\mathbf{s}_{jt}^e - \mathbf{s}_{jt})$ cannot affect decisions in t .²⁷ Finally, the third term corresponds to any error in estimation of households' cumulative utilities that arose from the first stage. Assumptions 1 and 2 imply that the first stage estimates are consistent, thus $\hat{v}_{gjt} - v_{gjt}$ will be uncorrelated to our IV.

²⁵In practice, we specify Λ_g as cubic B-splines of $\hat{v}_{g'jt-\tau}$ for each g' and τ . There are four knots for each element. Note that each of the coefficients of these variables are allowed to vary by g .

²⁶Equation (16) implies a direct relationship between inflows $_{g'jt-\tau}$ and $\hat{v}_{g'jt-\tau}$: $\hat{v}_{g'jt-\tau} = \log \text{inflows}_{g'jt-\tau} - \log \text{inflows}_{g'0t-\tau}$. Since $-\log \text{inflows}_{g'0t-\tau}$ does not vary across neighborhoods, it is absorbed by γ_{gt} . Thus, Assumption 3 implies that controlling *flexibly* for $\mathbf{v}_{jt-1}, \dots, \mathbf{v}_{jt-T'}$ and γ_{gt} in equation (19) yields consistent estimates of β_g . In practice, controlling flexibly for inflows $_{jt-\tau} = (\text{inflows}_{1jt-\tau}, \dots, \text{inflows}_{Gjt-\tau})$ instead yields estimates that are statistically indistinguishable from our main estimates.

²⁷More specifically, any variation in \mathbf{s}_{jt} that affects v_{gjt} must do so through \mathbf{s}_{jt}^e . It follows that any relevant IV of \mathbf{s}_{jt} would affect v_{gjt} only through \mathbf{s}_{jt}^e and not through $\mathbf{s}_{jt}^e - \mathbf{s}_{jt}$.

Stage 3: Identifying the Trajectory Toward Steady State by Simulation

Once we obtain estimates of \hat{v}_{gjt} , $\hat{\phi}_g$ and $\hat{\beta}_g$, we can identify by simulation how the demographic composition of each neighborhood will evolve from any initial state in the absence of external shocks. Consider, for instance, period t as our starting point.²⁸ We denote the population distribution of the entire city with group-specific population vectors $\mathbf{N}_{gt} = (N_{g1t}, \dots, N_{gJt})$, which imply share vectors $\mathbf{s}_{gt} = (s_{g1t}, \dots, s_{gJt})$ that comprise the aggregate demographic composition matrix \mathbf{s}_t . For any given counterfactual matrix of expected compositions of neighborhoods $\tilde{\mathbf{s}} = (\tilde{\mathbf{s}}_1, \dots, \tilde{\mathbf{s}}_J)$, response parameters $\tilde{\beta}$, and moving costs $\tilde{\phi}$, we write the counterfactual expected valuation of neighborhood j by group g households as

$$v_{gjt}(\tilde{\mathbf{s}}) = \tilde{\beta}'_g \tilde{\mathbf{s}}_j + \hat{\xi}_{gjt} \quad (21)$$

where $\hat{\xi}_{gjt} = \hat{v}_{gjt} - \hat{\beta}'_g \mathbf{s}_{jt}$. This equation is the simulation-analog to equation (10). It simply removes the part of the valuation due to \mathbf{s}_{jt} (from the data) and adds the part due to $\tilde{\mathbf{s}}_j$ (from the counterfactual). Note that equation (21) implies that we hold constant $\boldsymbol{\xi} = \hat{\boldsymbol{\xi}}_t$ in the simulation.²⁹

We simulate counterfactual group-specific valuations for all neighborhoods from $\tilde{\mathbf{s}}$ as

$$\begin{aligned} N_{gjt}(\tilde{\mathbf{s}}) = & N_{gjt} \left(\frac{\exp(v_{gjt}(\tilde{\mathbf{s}}))}{\exp(-\tilde{\phi}_g) + \sum_{j'=1}^J \exp(v_{gj't}(\tilde{\mathbf{s}}) - \tilde{\phi}_g) + \exp(v_{gjt}(\tilde{\mathbf{s}}))} \right) + \\ & + \sum_{k=1}^J N_{gkt} \left(\frac{\exp(v_{gjt}(\tilde{\mathbf{s}}) - \tilde{\phi}_g)}{\exp(-\tilde{\phi}_g) + \sum_{j'=1}^J \exp(v_{gj't}(\tilde{\mathbf{s}}) - \tilde{\phi}_g) + \exp(v_{gkt}(\tilde{\mathbf{s}}))} \right) \end{aligned} \quad (22)$$

²⁸The time subscripts in equations (21)-(23) refer only to the period at which the simulation begins; that is, all time-indexed functions and variables are fixed in all iterations of the simulation.

²⁹From equation (10), $\xi_{gjt} = v_{gjt} - \beta'_g \mathbf{s}_{jt}^e$, which implies that $\hat{\xi}_{gjt} - \xi_{gjt} = (\hat{v}_{gjt} - v_{gjt}) + (\hat{\beta}'_g \mathbf{s}_{jt} - \beta'_g \mathbf{s}_{jt}^e)$. Under Assumptions 1-3, $\hat{\xi}_{gjt}$ is asymptotically equivalent to $\xi_{gjt} + \beta'_g (\mathbf{s}_{jt} - \mathbf{s}_{jt}^e)$. The term $\beta'_g (\mathbf{s}_{jt} - \mathbf{s}_{jt}^e)$ can be understood as an unobserved amenity of neighborhood j from the perspective of group g households. Like ξ_{gjt} , this unobserved amenity does not by itself generate endogenous sorting because $\mathbf{s}_{jt} - \mathbf{s}_{jt}^e$ is merely a forecast error: households sorted on the basis of \mathbf{s}_{jt}^e , but they may have misestimated how attractive neighborhood j would seem to households of other groups. See Remark 4 for a discussion of how we allow for expectations to vary by group in practice.

The first term on the right-hand side of equation (22) corresponds to the simulated number of households who resided in neighborhood j and remained in their house, incurring no moving costs. The second term represents the simulated number of households who resided in neighborhood k and then moved to neighborhood j next period (households with $k = j$ moved houses within neighborhood j). By implementing this simulation simultaneously for all neighborhoods, we incorporate all endogenous feedback that spills over from one neighborhood to another. Because our simulation holds fixed all factors that affect households' propensity to choose the outside option, $v_{g0t}(\tilde{\mathbf{s}})$ is constant, so $v_{g0t}(\tilde{\mathbf{s}}) = v_{g0t} = 0$, where the second equality follows from the normalization in period t .³⁰

Once we obtain $N_{gjt}(\tilde{\mathbf{s}})$, we can calculate

$$s_{gjt}(\tilde{\mathbf{s}}) = \frac{N_{gjt}(\tilde{\mathbf{s}})}{\sum_{g'} N_{g'jt}(\tilde{\mathbf{s}})} \quad (23)$$

$s_{gjt}(\tilde{\mathbf{s}})$ is the $(g, j)^{\text{th}}$ element of the function $\mathbf{s}_t(\tilde{\mathbf{s}})$, which characterizes the evolution of the demographic compositions of all neighborhoods in the absence of shocks. By repeatedly evaluating $\mathbf{s}_t(\cdot)$ starting from $\tilde{\mathbf{s}}$, we can construct the simulated trajectory $\mathbb{T}_t(\tilde{\mathbf{s}})$ using equations (21), (22) and (23). Given a sufficiently fine grid, a tolerance μ and a time threshold \bar{t} , the state \mathbf{s}^* for which $\|\mathbb{T}_t^\tau(\tilde{\mathbf{s}}) - \mathbf{s}^*\| < \mu$ for all $\tau > \bar{t}$ is interpreted as a steady state (Definition 1). We can in principle identify all steady states by conducting a grid search of all possible counterfactual states $\tilde{\mathbf{s}}$ and computing simulated trajectories $\mathbb{T}_t(\tilde{\mathbf{s}})$ for each counterfactual.³¹

Remark 4. To simplify notation, we write \mathbf{s}_t^e as if it does not depend on the group who forms the expectation; however, we do not impose this restriction on expectations across groups when we estimate β . Because $\Lambda_g(\cdot)$ and γ_{gt} vary by g , the variation used

³⁰In our simulation we keep the total number of households in each group across the Bay Area constant at the level observed in t . More generally, if we wanted to consider a counterfactual where group g experienced net population growth due to, say, migration into the Bay Area, we could re-weight $\sum_{j=1}^J N_{gjt}(\tilde{\mathbf{s}}) = (1 + \psi_g) \cdot \sum_{j=1}^J N_{gjt}$, where ψ_g corresponds to the growth rate.

³¹One might be concerned that the linear specification in equation (19) does not allow for flexibility in the simulated trajectories to the steady state. In previous work (Caetano and Maheshri (2017)), we show that a simple linear specification already provides substantial flexibility in the trajectories even in a far more restrictive context (e.g., with one dimensional $\tilde{\mathbf{s}}$, with $\tilde{s}_{j't} = s_{j't}$ for all $j' \neq j$, and with myopic households: $s_t^e = s_{t-1}$). Relatedly, it is possible that for some initial state $\tilde{\mathbf{s}}$, the trajectory does not converge (e.g., it may oscillate). This did not occur for any of the counterfactuals that we considered in our analysis.

to identify β_g – variation in s_{jt-T} that is orthogonal to $\Lambda_g(\cdot)$ and γ_{gt} – is generally different than the variation used to identify $\beta_{g'}$ since these groups may form expectations differently. Moreover, we could have allowed different groups of households to form different expectations in the simulation stage, but doing so would presume that the data were not observed in steady state, since all households must have the same expectations in steady state (see Remark 1). Thus, we consider only counterfactuals where all groups share expectations in our application.

4 Empirical Results

Our empirical analysis covers eight socioeconomic groups – all combinations of four races and two income groups – each of whom are allowed to respond heterogeneously to unobserved amenities as well as to four endogenous amenities – the shares of Blacks, Hispanics, and Asians (relative to Whites) and the share of the poor (relative to the rich).³²

In Table 2, we present estimates of the responses to the socioeconomic compositions of neighborhoods (β_g), and the moving costs (ϕ_g) for households of each group. The endogenous amenities s_{jt} are instrumented by s_{jt-13} in equation (19), and $\Lambda_g(\hat{v}_{jt-1}, \dots, \hat{v}_{jt-12})$ is specified as a cubic B-spline of each element $v_{g'jt-\tau}$ for all g' and τ .³³

Since White (poor) share is the omitted race (income) amenity, the responses $\beta_{g,g'}$ are interpreted as the response of group g to a marginal increase in $s_{g'jt}^e$ relative to a marginal increase in the share of White (rich) neighbors. We find that households of each group respond positively to neighbors of the same race and to neighbors of the same income. Poor White households are more responsive to poor neighbors than rich White households, but this pattern is reversed for minority households. Hispanics respond most positively to neighbors of their own race, followed by Asians and Blacks. Interestingly, not all responses are reciprocated: e.g., Blacks respond negatively to Hispanics, but Hispanics show little response to Blacks. There is also heterogeneity in the interaction between race and income. e.g., rich Asians respond less intensely than poor Asians to same-race neighbors, but the opposite is true for Blacks and Hispanics.

³²We lack sufficient data to precisely estimate β if we allowed each of the eight groups to respond to race and income in an unrestricted, non-separable way (i.e., 8×7 instead of 8×4 estimates of $\beta_{g,g'}$). With more data this could be implemented.

³³For each of the 8 groups g' and 12 lags τ , there are 4 knots of each element $\hat{v}_{g'jt-\tau}$, which yields a total of $8 \cdot 12 \cdot 4 = 384$ control variables. We allow the coefficients of each of these control variables to vary by g .

Altogether, these heterogeneous responses may give rise to complex dynamics.

Table 2: Responses to the Race and Income Compositions of Neighborhoods (β) and Moving Costs (ϕ)

	White		Black		Hispanic		Asian	
	Rich	Poor	Rich	Poor	Rich	Poor	Rich	Poor
Responses to:								
Black Share	-2.83*** (0.40)	-2.25*** (0.45)	3.45*** (0.43)	2.41*** (0.42)	0.49 (0.40)	-0.12 (0.43)	-1.22*** (0.43)	-0.83** (0.41)
Hispanic Share	-4.57*** (0.67)	-1.18 (0.91)	-1.42** (0.66)	-3.22*** (0.69)	11.41*** (0.81)	8.85*** (0.81)	-0.25 (0.77)	0.02 (0.79)
Asian Share	-0.49 (0.48)	-3.97*** (0.63)	0.34 (0.52)	-1.17*** (0.49)	-0.81 (0.61)	-1.97*** (0.62)	5.05*** (0.67)	7.37*** (0.68)
Poor Share	-0.69*** (0.24)	3.60*** (0.42)	-2.35*** (0.37)	0.44 (0.29)	-1.67*** (0.39)	0.17 (0.34)	-2.94*** (0.37)	1.42*** (0.40)
Moving Costs	28.65*** (0.02)	28.70*** (0.02)	26.96*** (0.05)	28.18*** (0.03)	27.50*** (0.03)	28.71*** (0.01)	28.03*** (0.02)	28.30*** (0.01)
R^2	0.79							
Num. of Observations	147,840							

Notes: This specification includes group-month fixed effects and control variables $\Lambda_g(\hat{v}_{jt-1}, \dots, \hat{v}_{jt-12})$ (see Footnote 33) as well as $s_{g'jt-13}$ for each of the four socioeconomic shares as instrumental variables. White is the omitted racial share and rich is the omitted income share. All standard errors clustered by group-month. * - 90% significance, ** - 95% significance, *** - 99% significance. The p-values for both the Cragg-Donald and the Kleinbergen-Paap weak identification tests are less than 0.001, which implies a strong first stage.

Although we estimate mostly statistically significant discriminatory responses, the key takeaway is that these responses are small in comparison to our estimates of moving

costs, which are an order of magnitude larger.³⁴ This suggests that substantial amenity mismatch may accumulate since many households may be locked into a neighborhood that is no longer their most preferred neighborhood. Although our estimates of moving costs are generally statistically different from each other, they are similar in magnitude across all socioeconomic groups (the maximum variation in these moving costs is less than 10% of the estimates).

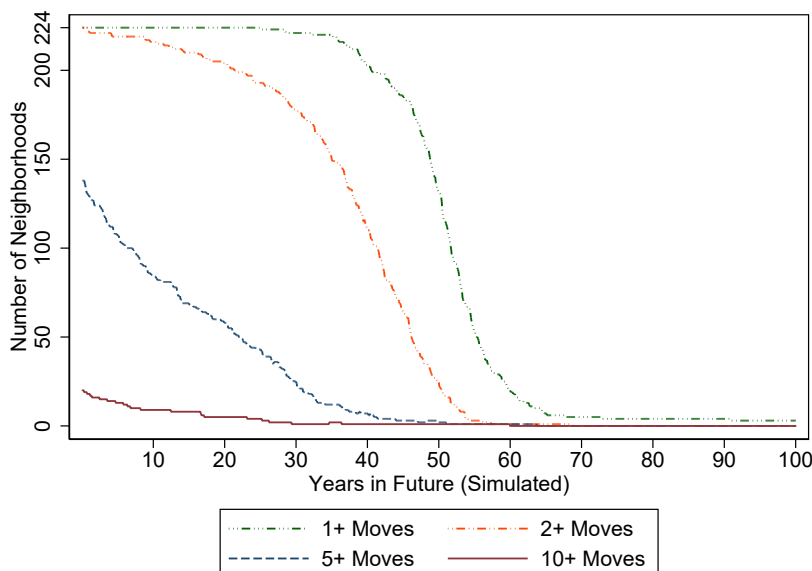
In the Appendix, we present raw OLS estimates of β (Table 5). These are much larger in magnitude than our IV estimates since there are many confounding reasons why similar households would choose similar neighborhoods (e.g., they tend to value other amenities more similarly), all of which would bias the OLS estimates upward in magnitude. The OLS bias is most pronounced for the within-group parameter estimates, as expected. We also report estimates of β for different values of T (the period corresponding to our IV) in Figures 13 and 14. Larger values of T weaken Assumption 3 resulting in an IV that is more likely to be valid. We find that all 12 parameter estimates change very little for $T = 13, \dots, 36$.³⁵

With these estimates, we simulate how the socioeconomic compositions of neighborhoods would evolve holding $\xi_{g'jt}$ constant at $\hat{\xi}_{g'jt}$ for all g' (i.e., in the absence of future external shocks). We focus on the counterfactual $\tilde{\mathbf{s}} = \mathbf{s}_t$ where t refers to the final month of our sample, November 2004, and conduct the simulation until convergence. In Figure 2, we present a graph of the number of neighborhoods that experience at least 1, 2, 5 or 10 simulated moves that change their socioeconomic composition. If, for instance, a rich White homeowner simply left a neighborhood, that would count as one change (one outflow). If instead they were replaced by another rich White homeowner, that would count as zero changes. If they were replaced by a homeowner of a different race or income level, that would count as two changes (one outflow plus one inflow). We

³⁴As discussed in Kennan and Walker (2011), household-level moving costs in such discrete choice frameworks can be interpreted as also including ϵ (defined in Assumption 1), so they may vary substantially across households. This can explain why some households would move even with such large gaps between β and ϕ . Thus, moving costs *conditional on moving* are far less prohibitive than the moving cost estimates shown in Table 2. As a robustness check, we allowed for moving costs to vary by both group and year, but we found little heterogeneity over time.

³⁵We also performed two other types of robustness checks in order to assess whether controlling flexibly for $\mathbf{v}_{jt-1}, \dots, \mathbf{v}_{jt-12}$ was sufficient to absorb confounders: (1) We re-estimated β under different specifications of $\Lambda_g(\cdot)$ (linear rather than cubic B-spline, different number of knots, and using inflows $g'_{jt-\tau}$ instead of $v_{g'jt-\tau}$) and obtained similar estimates for all flexible specifications; (2) we further controlled for average neighborhood prices $P_{jt-1}, \dots, P_{jt-12}$ and obtained similar estimates of all β coefficients. We also attempted to increase the value of T' ; this check turned out to be uninformative as it yielded imprecise estimates since each additional lag increased the numbers of controls in $\Lambda_g(\cdot)$ dramatically.

Figure 2: Number of Neighborhoods In Flux (Simulated)



Notes: Figure shows the number of neighborhoods with at least one, two, five or ten moves that change their socioeconomic composition (out of a total of 224 neighborhoods). Simulation begins in November 2004.

describe neighborhoods experiencing such changes to their socioeconomic compositions as “in flux.”

Initially, and for decades to follow, nearly all neighborhoods are in flux. From this, we conclude that the Bay Area is not observed to be in steady state.³⁶ Despite substantial moving costs, the amenities of the neighborhoods where households are observed to reside are sufficiently unattractive to some households that most neighborhoods experience turnover. Over time, changes in the socioeconomic compositions of these neighborhoods feedback and also spill over to other neighborhoods, which in turn changes their relative attractiveness to homeowners of all socioeconomic groups. This process is slow, as it takes 60-70 years for the Bay Area to approximate steady state.³⁷

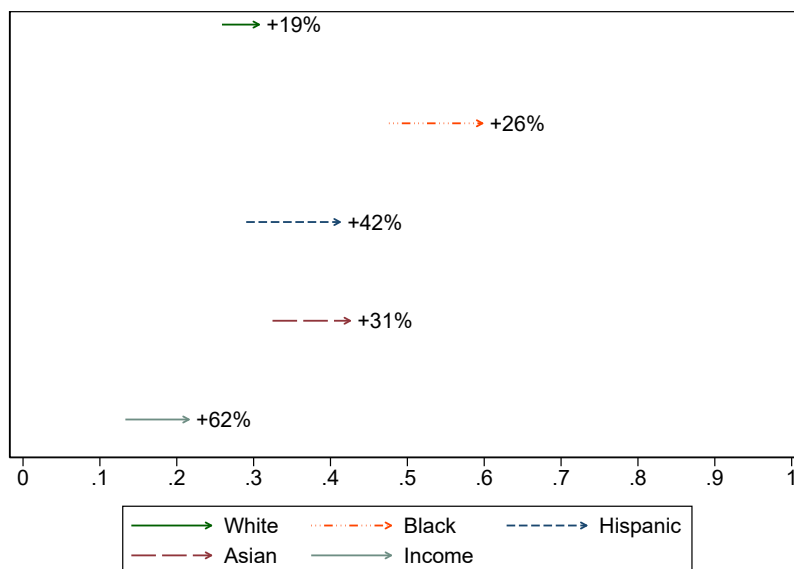
The outcome of this pattern of sorting is a change in the levels of segregation in the Bay Area. In Figure 3, we present the long-run change in the dissimilarity index for each

³⁶This also implies that neighborhoods are not observed at “tipping points” since they correspond to an unstable steady state. Hence, small deviations in our simulation due to, say, estimation error, should leave our long-run conclusions effectively unchanged, which we confirmed empirically.

³⁷It takes 153 years for all Bay Area neighborhoods to experience no moves (see Appendix Figure 15).

race (pooling income groups) and for each income group (pooling races) across all Bay Area neighborhoods. Over time, all races experience modest increases in segregation. White households experience the smallest increase in segregation in both absolute and relative (19%) terms. Black households start off more segregated than all other races and remain so throughout the simulation. Hispanic homeowners experience the largest absolute and relative (42%) increases in segregation, followed by Asians, who experience a 31% increase in segregation. Homeowners are least segregated by income initially and remain so throughout the simulation despite the largest relative increase (62%).

Figure 3: Steady State Changes in Race and Income Segregation (Simulated)



Notes: The arrows represent the changes in simulated Dissimilarity Indices for households of each race and income from November 2004 to steady state in the absence of exogenous shocks. Numbers correspond to the relative change in dissimilarity. A Black dissimilarity index of, say, 0.60, means that 60% of Black homeowners would have to be relocated in order to generate an equal distribution of Blacks across all Bay Area neighborhoods.

5 Determinants of Long-Run Segregation

In this section we study the roles of discrimination, moving costs, incomplete information and the initial allocations of households in explaining the long-run changes

in segregation that we found in Section 4. We weigh the importance of these determinants by leveraging the various moving parts of our framework to simulate several relevant counterfactuals. This ensures that we allow for complex sorting patterns to emerge that would otherwise be difficult to predict but are nonetheless integral to the dynamic process of segregation. Indeed, the discriminatory responses that we estimate may not necessarily increase segregation as one may expect. For instance, rich White homeowners fleeing a neighborhood that is becoming more Black will, all else constant, increase not only the Black share of neighbors, but also the Hispanic and Asian shares of neighbors. That in turn may lead to further inflows of not only Blacks, but also Hispanics and Asians.³⁸ The complexity of this sorting pattern grows over time not only because all groups continue to respond endogenously to each of these changes in a given neighborhood, but also because they respond to concomitant changes in other neighborhoods. Because we identify the baseline trajectory as well as the counterfactual trajectory, we can estimate the effect of a counterfactual change for any elapsed time from the moment the change took effect (November 2004). Although we mostly focus on reporting the long-run (i.e., steady state) effects of these counterfactuals, we also show plots of the dynamics of the neighborhoods over time in order to provide glimpses of these counterfactual trajectories.

5.1 The Roles of Discriminatory Responses: Race and Income

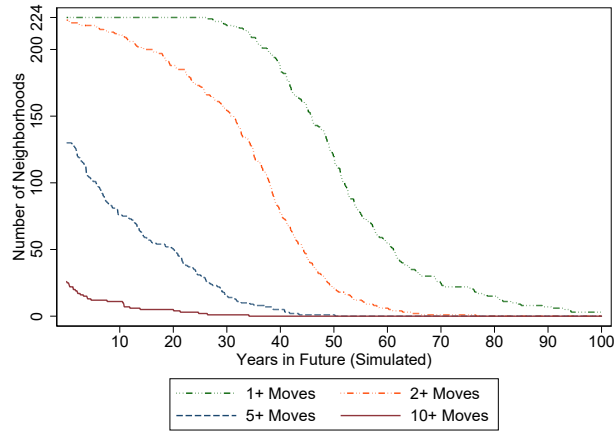
Our estimates of $\hat{\beta}$ reveal systematic discriminatory responses of homeowners of all socioeconomic groups. To isolate their roles in explaining the patterns of segregation dynamics presented in Figures 2-3, we consider a series of counterfactuals in which households are either “race-blind”, i.e., unresponsive to the racial composition of their neighbors, “income-blind”, i.e., unresponsive to the income composition of their neighbors, or both race- and income-blind. As shown in Figure 4, discriminatory responses have little qualitative effect on segregation dynamics, though they do slightly slow down the process of arriving at steady state.

We present the simulated increase in segregation under each of these counterfactuals in Figure 5. Baseline effects from Figure 3 are reproduced in light gray. This makes

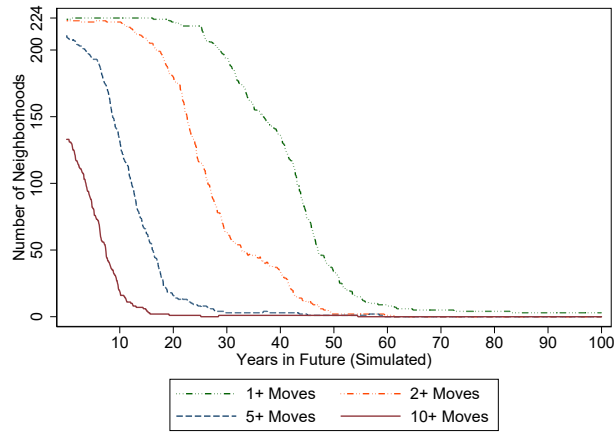
³⁸As shown in Table 2, households tend to respond more positively to an increase in the share of same-race households than negatively to an increase in the shares of other races.

Figure 4: Number of Neighborhoods In Flux - No Discrimination (Simulated)

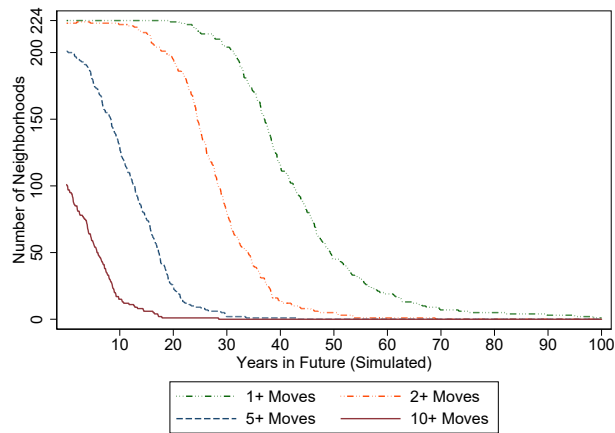
(a) No Racial Discrimination



(b) No Income Discrimination



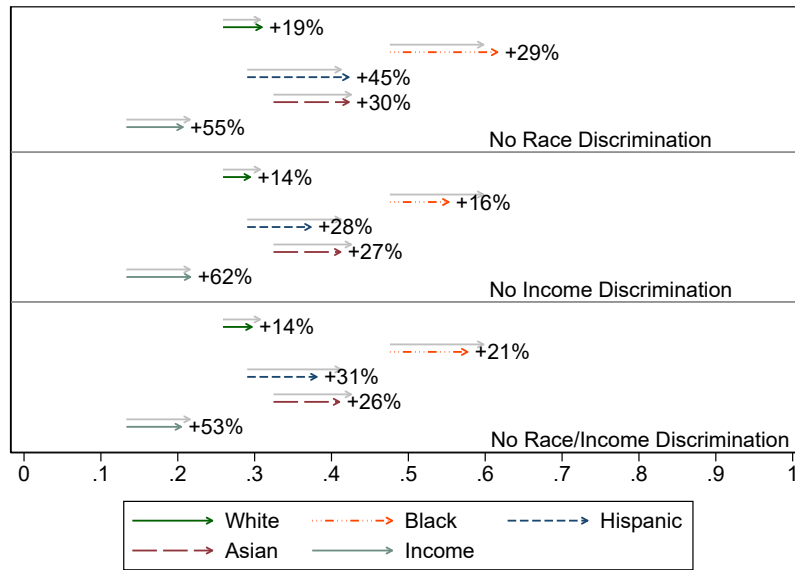
(c) No Racial or Income Discrimination



Notes: Each panel shows the number of neighborhoods with at least one, two, five or ten moves that change their socioeconomic composition (out of a total of 224 neighborhoods) under a different counterfactual. Simulation begins in November 2004.

it apparent that removing discrimination, either by race-blinding households, income-blinding households, or both, has little impact on segregation (with the exception of Black and Hispanic segregation, for which income discrimination does seem to matter more). In contrast, the bottom panel shows that sorting on the basis of amenities other than the socioeconomic compositions of neighborhoods (ξ) explains most of the long-run changes in segregation.³⁹

Figure 5: Steady State Changes in Race and Income Segregation - No Discrimination (Simulated)



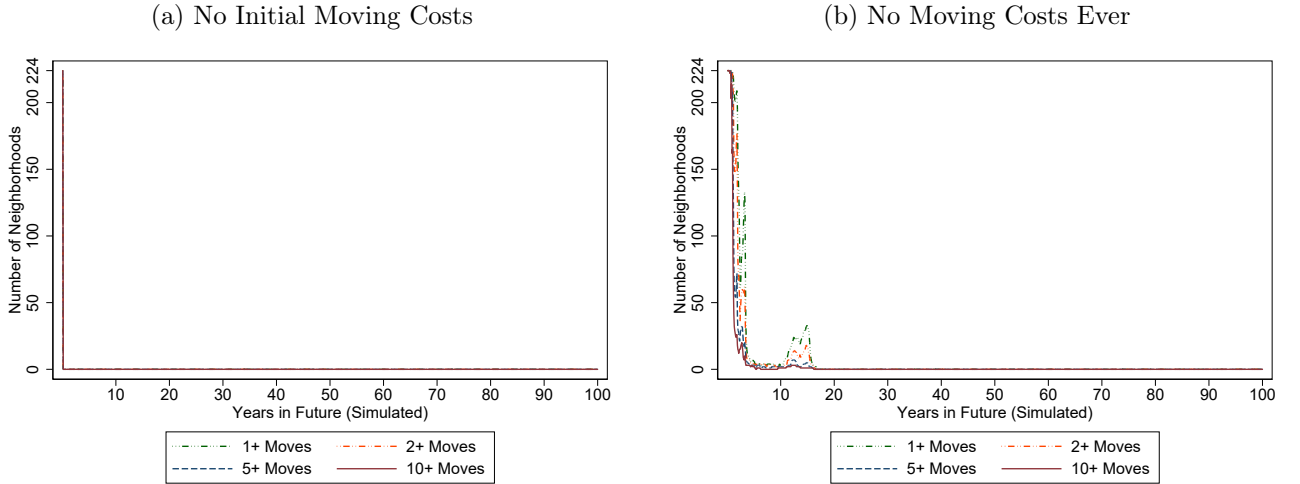
Notes: For each panel representing a different counterfactual, the arrows represent changes in simulated Dissimilarity Indices for households of each race and income from November 2004 to the steady state in the absence of exogenous shocks. Numbers in parentheses correspond to the relative change in dissimilarity. Gray arrows correspond to the baseline changes shown in Figure 3. A Black dissimilarity index of, say, 0.60, means that 60% of Black homeowners would have to be relocated in order to generate an equal distribution of Blacks across all Bay Area neighborhoods.

³⁹We refrain from considering counterfactual changes in ξ because we are unable to estimate the causal relationship between a specific amenity included in ξ and residential decisions, which is an actual policy parameter. In principle, large variation in ξ might lead to a completely different trajectory due to the possibility of multiple steady states. However, small variation in ξ should affect our long-run conclusions only to the extent that it perturbed the location of the steady state by slightly altering the trajectory since we found that the data were not observed near a tipping point (Footnote 21). We verified this empirically.

5.2 The Role of Moving Costs

The gradual declines of Figures 2 and 4 suggest that moving costs play an important role in the dynamics of segregation. To explore this further, we consider a counterfactual in which all homeowners enjoy a one-time moving-cost amnesty at the beginning of the simulation. As shown in the first panel of Figure 6, the Bay Area converges to a steady state instantaneously. In the first period, the lack of moving costs allows households to eliminate their mismatch (per their *ex ante* expectation in t). However, this does not imply that there is no mismatch in t or in further periods, as forecast errors may lead households to reside in neighborhoods that are suboptimal. Nevertheless, this mismatch is insufficient to overcome moving costs which are restored in future periods because the elements of β are small in magnitude relative to the elements of ϕ .

Figure 6: Number of Neighborhoods In Flux - No Moving Costs (Simulated)



Notes: Each panel shows the number of neighborhoods with at least one, two, five or ten moves that change their socioeconomic composition (out of a total of 224 neighborhoods) under a different counterfactual. Simulation begins in November 2004.

In order to illustrate this point, consider a different counterfactual where we eliminate moving costs both today and in every future period by permanently setting $\phi = \mathbf{0}$. We do not consider this counterfactual to be particularly sensible because households would have to be repeatedly surprised by the future elimination of moving costs every

time it occurs.⁴⁰ However, this exercise is valuable as it allows us to gauge the role of incomplete information, which is hidden in our context but would perhaps play a more important role in a context with much lower moving costs or much higher discrimination than the one we encounter in our sample. The dynamics of this second counterfactual are shown in the second panel of Figure 6. While convergence is still much faster than in the baseline case with moving costs, it is not instantaneous, owing to the fact that forecast errors would still trigger further moves (which would remain costless in this scenario), leading to the feedback loop discussed in Schelling (1969).⁴¹

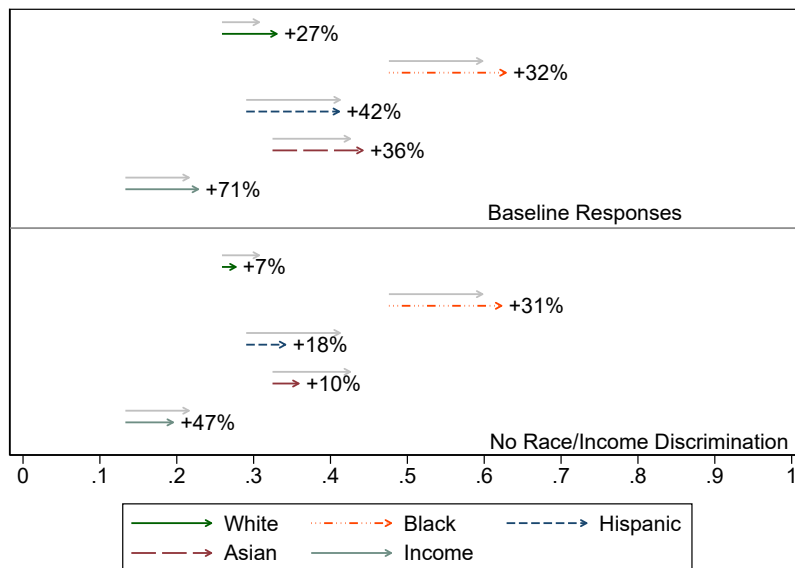
We explore the interaction between moving costs and segregation in Figure 7, which is the analog to Figure 3 when moving costs are eliminated at the beginning of the simulation, but resume in the future at the levels households expected during the sample period. This counterfactual in the top panel (which maintains baseline discriminatory responses) leads to higher segregation levels across the board relative to the baseline effects shown in the gray arrows. When we further shut off discriminatory responses (lower panel), the resulting increases in segregation are much less pronounced.

A comparison of the two panels of Figure 7 reveals that discrimination plays an important role in a world without moving costs. For instance, in such a world, discriminatory sorting would be responsible for a 24% increase in Hispanic segregation ($42-18=24$). In reality, discrimination is much less important than sorting towards other amenities (as shown in the bottom panel of Figure 5); because moving is costly, only a few households sort at a time, which reduces the scope for socioeconomic changes to trigger future moves. In Figure 16 in the appendix we show analogous results for the counterfactual with no moving costs ever. As expected, segregation would increase substantially in that counterfactual (first panel) but much less under no discriminatory responses (second panel), highlighting a greater role for the endogenous feedback loop. Because moving costs after the first adjustment are still zero in this counterfactual, the

⁴⁰This “surprise” must occur because β also contains the marginal effects of s_{jt} on the continuation values and was estimated using data generated in a world with expectations of non-zero future moving costs (see Section 3.1.3). Note that we would encounter a related issue if we instead decided to separately identify the flow utility and the continuation value components of β . Doing so would require us to assume households are forward looking and anticipate future moving costs in a very specific way, but if this assumption was invalid, it would lead to misestimation of the simulated trajectories. For instance, in practice households may discount the future differently depending on their socioeconomic group.

⁴¹If we also set $\beta = \mathbf{0}$ in the second counterfactual, then we obtained instantaneous convergence to steady state again. This is expected, since in this case we removed not only all scope for discrimination, we also ensured that any forecast errors of socioeconomic compositions would not trigger *ex-post* changes in the valuations of neighborhoods, so there is no scope for further mismatch.

Figure 7: Steady State Changes in Race and Income Segregation - No Initial Moving Costs (Simulated)



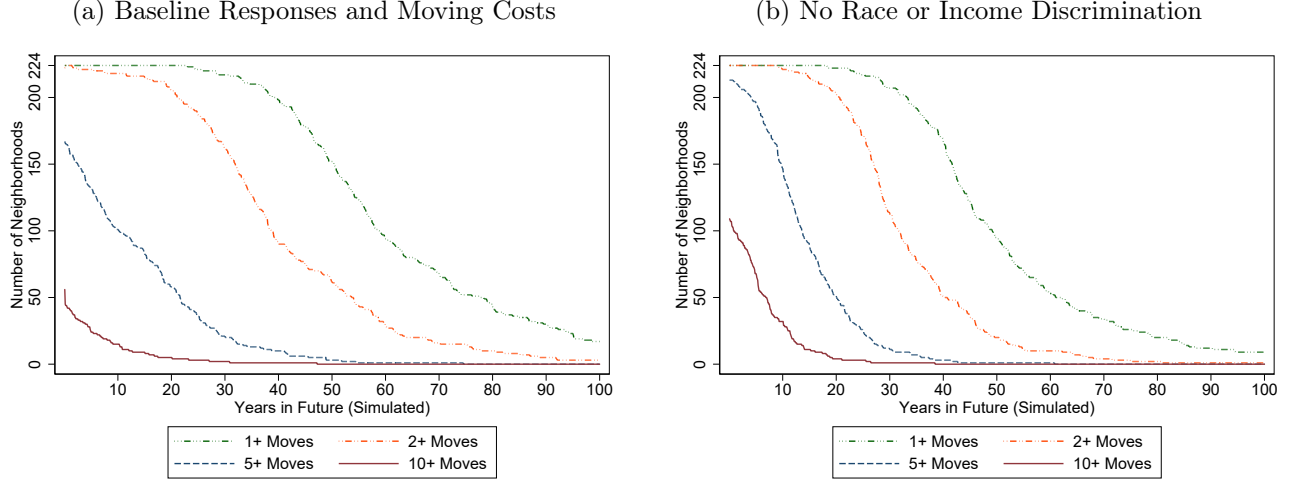
Notes: For each panel representing a different counterfactual, the arrows represent changes in simulated Dissimilarity Indices for households of each race and income from November 2004 to steady state in the absence of exogenous shocks. Numbers in parentheses correspond to the relative change in dissimilarity. Gray arrows correspond to the baseline changes shown in Figure 3. A Black dissimilarity index of, say, 0.60, means that 60% of Black homeowners would have to be relocated in order to generate an equal distribution of Blacks across all Bay Area neighborhoods.

feedback loop is more intense, and the importance of discrimination relative to sorting on the basis of other amenities increases. Thus, we conclude that frictions, especially moving costs, disproportionately mitigate the role of discrimination on segregation, leaving more space for other amenities to play a larger role.

5.3 The Role of the Initial Allocation of Households

We now consider a counterfactual that changes the initial allocation of households across neighborhoods; in particular, we re-allocate households so that all neighborhoods have the exact same initial socioeconomic compositions (mimicking a policy that generates full integration of all race and income groups). The first panel of Figure 8 plots the number of neighborhoods in flux after the full integration policy. As compared with the benchmark in Figure 2, this re-arrangement of households takes longer to reach steady

Figure 8: Number of Neighborhoods In Flux - Full Integration (Simulated)

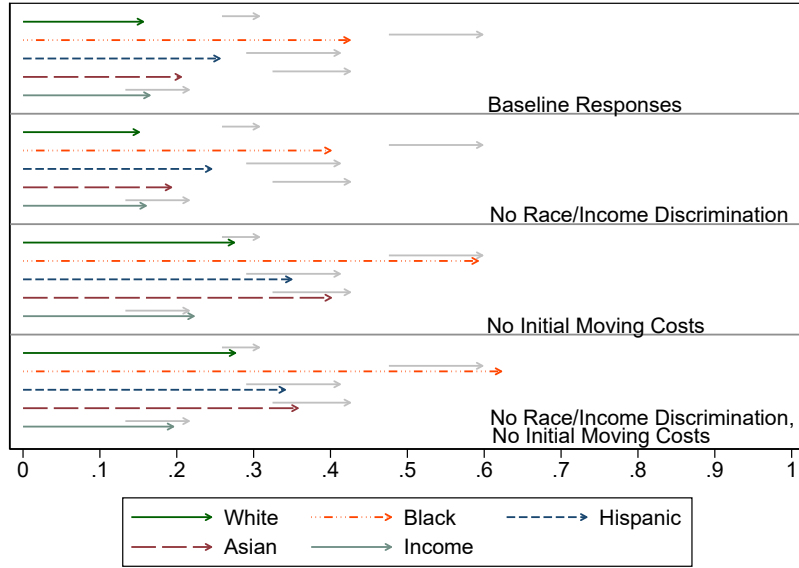


Notes: Each panel shows the number of neighborhoods with at least one, two, five or ten moves that change their socioeconomic composition (out of a total of 224 neighborhoods) under a different counterfactual. Simulation begins in November 2004.

state. This is intuitive, as this policy likely leads to a major misalignment that takes longer to undo because of moving costs. Eliminating discrimination, as in the second panel of Figure 8, seems to speed up convergence only slightly.

We explore the relationship between initial socioeconomic compositions and segregation dynamics in Figure 9 under four counterfactuals. When starting in a fully integrated Bay Area, all dissimilarity indexes are zero by assumption. The top panel shows that this integration policy would reduce segregation in steady state (relative to the baseline counterfactual shown in the gray arrows). This is evidence of multiple steady states since initial conditions matter in the long run. With multiple steady states, it is perhaps not surprising that segregation increases less under a full integration policy since moving costs prevent sorting by many households. The similarity of the effects in the top two panels implies that eliminating discrimination has very little impact on segregation. However, when moving costs are eliminated in a one time amnesty, segregation increases to levels that are closer to the levels of the gray arrows. This simply reflects the fact that without moving costs, the initial allocation of households is largely irrelevant since they can be reallocated costlessly. In the bottom panel, we eliminate both initial moving costs and discrimination. Households still reallocate

Figure 9: Steady State Changes in Race and Income Segregation - Full Integration (Simulated)



Notes: For each panel representing a different counterfactual, the arrows represent changes in simulated Dissimilarity Indices for households of each race and income from November 2004 to steady state in the absence of exogenous shocks. Numbers in parentheses correspond to the relative change in dissimilarity. Gray arrows correspond to the baseline changes shown in Figure 3. A Black dissimilarity index of, say, 0.60, means that 60% of Black homeowners would have to be relocated in order to generate an equal distribution of Blacks across all Bay Area neighborhoods.

costlessly, but they converge to a steady state that is a bit less segregated than in the third panel.

It is worth mentioning that the initial allocation of households is not entirely irrelevant even when moving costs are initially eliminated. The reason for this is that the friction from forecast errors still exists. Differences in the initial allocations of households result in different expectations over the socioeconomic compositions of neighborhoods in t , which in turn result in different choices in the aggregate. This explains why the steady state under no initial moving costs starting from observed neighborhood compositions (the first panel of Figure 7) differs from the steady state under no initial moving costs starting from fully integrated neighborhoods (the third panel of Figure 9). This also explains why when we eliminate discrimination, the initial conditions become irrelevant (compare the second panel of Figure 7 with the fourth panel of Figure 9).

We conclude that moving costs are extremely important in explaining both the long-run level and speed of increases in segregation, as they restrict sorting by many households to their most desired neighborhoods. We would expect this in an environment where moving costs are large relative to discrimination as there is limited scope for changes in the race and income compositions of neighborhoods to trigger a cascade of new moves. This dampening of the feedback loop discussed in Schelling (1969) gives other amenities a more prominent role in explaining residential sorting and segregation.

6 House Prices and Segregation

6.1 Identification and Estimation

In this section, we explicitly incorporate neighborhood house prices into our framework to explore their impacts on segregation. We identify and estimate v_{gjt} and ϕ_g exactly as described in Section 3, so we restrict our discussion to the later stages after they are estimated. We augment our main regression, equation (19), with a price term

$$v_{gjt} = \boldsymbol{\theta}'_g \mathbf{s}_{jt}^e + \alpha_g P_{jt} + \xi_{gjt}^P, \quad (24)$$

where P_{jt} represents the average price of neighborhood j in period t , and α_g represents the effect of an increase in P_{jt} on the average cumulative (indirect) utility of group g households for neighborhood j in t . Note that $\xi_{gjt}^P = \xi_{gjt} - \alpha_g P_{jt}$, where ξ_{gjt} is the error from equation (10).

To close the model, it is now necessary to add a price equation:

$$P_{jt} = \boldsymbol{\rho}' \mathbf{s}_{jt}^e + \eta_{jt}, \quad (25)$$

where η_{jt} is an error term incorporating the component of prices not explained linearly by \mathbf{s}_{jt}^e . Equation (25) is written without loss of generality since η_{jt} is merely the remainder of the equation. Below we impose restrictions on η in order to interpret $\boldsymbol{\rho}' \mathbf{s}_{jt}^e$ as the best linear approximation of the (potentially non-linear) effect of \mathbf{s}_{jt}^e on P_{jt} . We use this equation only to model how households expect prices to change for counterfactual values of \mathbf{s}_{jt}^e .⁴²

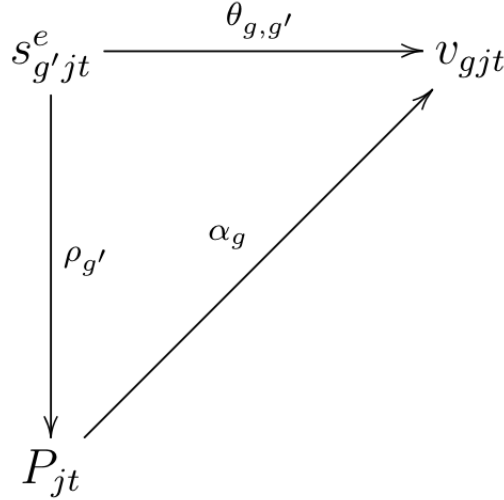
⁴²We do not impose the assumptions required to interpret $\boldsymbol{\rho}$ as the average marginal willingness to pay for \mathbf{s}_{jt}^e as in hedonic regressions (e.g., Rosen (1974)).

Note that the parameters of equations (24) and (25) are related to the parameters of equation (10) as follows:

$$\beta_{g,g'} = \theta_{g,g'} + \rho_{g'} \cdot \alpha_g \quad (26)$$

where $\beta_{g,g'}$, $\theta_{g,g'}$ and $\rho_{g'}$ denote the g' -th element of the vectors $\boldsymbol{\beta}_g$, $\boldsymbol{\theta}_g$ and $\boldsymbol{\rho}$, respectively. Figure 10 describes their relationship graphically where each arrow represents a direct causal effect that is captured by a parameter. The *total* effect of $s_{g'jt}^e$ on v_{gjt} ($\beta_{g,g'}$ from equation (10)) is simply equal to the direct effect ($\theta_{g,g'}$) plus the indirect effect mediated through a change in price. This indirect effect arises through the interaction of two effects: the effect of $s_{g'jt}^e$ on P_{jt} ($\rho_{g'}$) and the effect of P_{jt} on v_{gjt} (α_g).

Figure 10: Different Channels of Causality from $s_{g'jt}^e$ to v_{gjt}



Notes: Each arrow in this graph represents a direct causal relationship. According to equation (26), the *total* effect of $s_{g'jt}^e$ on v_{gjt} ($\beta_{g,g'}$) is equal to the *direct* ($\theta_{g,g'}$) plus the *indirect* ($\rho_{g'} \cdot \alpha_g$) effects of $s_{g'jt}^e$ on v_{gjt} , where the indirect channel of causality is mediated via prices, P_{jt} .

For intuition, it is useful to explicitly describe a hypothetical sequence of events that might be expected to occur within a single period t . Consider a change in $s_{g'jt}^e$ at the very beginning of period t . This may affect the desired inflows of households of different groups differently as reflected in $\theta_{g,g'}$. Next, P_{jt} may adjust to accommodate excess supply or demand in the neighborhood as reflected in $\rho_{g'}$, a reduced-form parameter

that incorporates both supply and demand elasticities. As prices begin to adjust, the desired inflows of each group may respond differently according to α_g . This process may iterate a number of times so that by the end of t we observe P_{jt} and inflows $_{gjt}$ for all g .⁴³ For each g , the reduced-form coefficient $\beta_{g,g'}$ incorporates the effect of $\mathbf{s}_{g'jt}^e$ on v_{gjt} taking into account all of the adjustment processes that households might *expect* to occur during t , such as this one. We make no assumptions on the particulars of the process by which $\mathbf{s}_{g'jt}^e$ causes v_{gjt} . We simply posit that whatever households expect the process to be, it can be decomposed into one price channel, $\rho_{g'} \cdot \alpha_g$ and a second residual channel $\theta_{g,g'}$.⁴⁴

In our main analysis, we simply identified β_g . This allowed us to characterize segregation without imposing additional assumptions required to disentangle the direct and indirect effects of \mathbf{s}_{jt} on v_{gjt} . We intentionally selected the set of endogenous amenities parsimoniously by focusing on the two *primitive* endogenous dimensions along which households sort: race and income. However, that specification was not equipped to study the potential role of prices in explaining segregation. To do so, we must separately identify $\theta_{g,g'}$, α_g and $\rho_{g'}$ for all g and g' , which requires an additional assumption.

Assumption 4. For some $T > T' \geq 1$,

1. $COV(\xi_{gjt}^P, \mathbf{s}_{jt-T} | \text{inflows}_{jt-1}, \dots, \text{inflows}_{jt-T'}) = 0$.
2. $COV(\xi_{gjt}^P, P_{jt-T} | \text{inflows}_{jt-1}, \dots, \text{inflows}_{jt-T'}) = 0$.

Following Assumption 4, we use \mathbf{s}_{jt-T} and P_{jt-T} as IVs for \mathbf{s}_{jt} and P_{jt} to estimate θ_g and α_g in the Two Stage Least Squares (2SLS) regression

$$v_{gjt} = \theta'_g \mathbf{s}_{jt} + \alpha_g P_{jt} + \Lambda_g(\hat{\mathbf{v}}_{jt-1}, \dots, \hat{\mathbf{v}}_{jt-T'}) + \gamma_{gt} + \tilde{\xi}_{gjt}^P, \quad (27)$$

where $\tilde{\xi}_{gjt}^P = (\xi_{gjt}^P - \Lambda_g(\hat{\mathbf{v}}_{jt-1}, \dots, \hat{\mathbf{v}}_{jt-T'}) - \gamma_{gt}) + \beta'_g(\mathbf{s}_{jt}^e - \mathbf{s}_{jt}) + (\hat{v}_{gjt} - v_{gjt})$.⁴⁵

Assumption 4.1 is a clear analog to Assumption 3. It simply replaces ξ_{gjt} with ξ_{gjt}^P , so it further assumes that a subcomponent of the unobservable ξ_{gjt} , namely ξ_{gjt}^P , is also

⁴³Importantly, these inflows must be self-consistent with \mathbf{s}_{jt}^e , otherwise households would have had different expectations about the socioeconomic composition in t . This hypothetical sequence of events which households may expect to happen within t resembles how steady state is assumed to be achieved in each period in the simulations from Bayer, McMillan and Rueben (2004a).

⁴⁴Note, in particular, that there is no need to assume that supply equals demand within a given period.

⁴⁵See the discussion around equation (20) for an explanation of why \mathbf{s}_{jt-T} is uncorrelated to $\tilde{\xi}_{gjt}^P - \xi_{gjt}^P$. An analogous argument implies P_{jt-T} is also uncorrelated to $\tilde{\xi}_{gjt}^P - \xi_{gjt}^P$.

uncorrelated to the IV \mathbf{s}_{jt-T} . Assumption 4.2 is also similar in spirit to Assumption 3, but there are key differences. It states that no unobservable affecting inflow decisions in t should be correlated to P_{jt-T} once we condition on $\text{inflows}_{jt-1}, \dots, \text{inflows}_{jt-T'}$. It is useful to discuss which *additional* source of variation (independent of \mathbf{s}_{jt-T}) the IV P_{jt-T} exploits. Consider two neighborhoods j and j' that are otherwise identical in $t - T$, except for the fact that j has a higher level of one amenity that is liked (in different intensities) by households of all groups. These different intensities may lead to $\mathbf{s}_{jt-T} \neq \mathbf{s}_{j't-T}$, which contributes to the relevance of the IV \mathbf{s}_{jt-T} as discussed in Section 3. However, the common component to the valuation of this amenity across all groups would not lead to $\mathbf{s}_{jt-T} \neq \mathbf{s}_{j't-T}$ yet would lead to $P_{jt-T} > P_{j't-T}$ because of an excess demand for neighborhood j relative to neighborhood j' , since all groups tend to like this amenity.⁴⁶ Of course, only those amenities that do not affect inflow decisions in $t - 1, \dots, t - T'$ are exploited for identification since we control for $\text{inflows}_{jt-1}, \dots, \text{inflows}_{jt-T'}$.

Why would P_{jt-T} be correlated with P_{jt} conditional on $\text{inflows}_{jt-1}, \dots, \text{inflows}_{jt-T'}$? In Section 3.2, we discussed that the relevance of \mathbf{s}_{jt-T} as an IV for \mathbf{s}_{jt} arises from an asymmetry on the demand-side between \mathbf{s}_{jt} , a stock variable that depends on decisions from the past, and v_{gjt} , a flow variable that depends only on inflow decisions in period t . The relevance of P_{jt-T} as an IV for P_{jt} arises from a second asymmetry: while the outcome variable v_{gjt} depends on amenities from neighborhood j , the endogenous variable P_{jt} depends on amenities of all neighborhoods because they all compete with each other in the housing market (Berry, Levinsohn and Pakes (1995)). To the extent that these amenities may persist over time, P_{jt-T} might be correlated to P_{jt} . The idea underlying Assumption 4.2 is that by controlling for inflows from $t - 1$ to $t - T'$, the component of ξ_{gkt-T}^P that is correlated to ξ_{gjt-T}^P will plausibly not persist to t . Still, some of ξ_{gkt-T}^P may persist to t , thereby affecting P_{jt} via competition across neighborhoods.

Assumptions 3 and 4 allow us to estimate β , θ and α ; we can also use them to estimate ρ via 2SLS using \mathbf{s}_{jt-T} as an IV for \mathbf{s}_{jt} in the equation

$$P_{jt} = \rho' \mathbf{s}_{jt} + \Gamma(\mathbf{v}_{jt-1}, \dots, \mathbf{v}_{jt-T'}) + \lambda_t + \check{\eta}_{jt} \quad (28)$$

where $\check{\eta}_{gjt} = (\eta_{gjt} - \Gamma(\hat{\mathbf{v}}_{jt-1}, \dots, \hat{\mathbf{v}}_{jt-T'}) - \lambda_t) + \rho'(\mathbf{s}_{jt}^e - \mathbf{s}_{jt})$ and $\Gamma(\cdot)$ is a flexible function specified analogously to $\Lambda_g(\cdot)$. To see this, note that Assumptions 3 and

⁴⁶More formally, let A_{jt-T} be the amenity in this example. Then $A_{jt-T} - \mathbb{E}[A_{jt-T} | \mathbf{s}_{jt-T}]$ is the common component of this amenity that would not affect \mathbf{s}_{jt-T} yet would affect P_{jt-T} .

4.1 imply $COV(\eta_{jt}, \mathbf{s}_{jt-T} | \text{inflows}_{jt-1}, \dots, \text{inflows}_{jt-T'}) = 0$, where η_{jt} is the error from equation (25).⁴⁷

With estimates of $\hat{\phi}$, $\hat{\theta}$, $\hat{\alpha}$ and $\hat{\rho}$, and counterfactuals $\tilde{\phi}$, $\tilde{\theta}$, $\tilde{\alpha}$ and $\tilde{\rho}$, we augment the simulation procedure in a straightforward manner. As before, we simulate the trajectory $\mathbb{T}_t(\tilde{\mathbf{s}}) = \{\mathbb{T}_t^0(\tilde{\mathbf{s}}), \mathbb{T}_t^1(\tilde{\mathbf{s}}), \dots\}$ where $\mathbb{T}_t^0(\tilde{\mathbf{s}}) = \tilde{\mathbf{s}}$, and $\mathbb{T}_t^r(\tilde{\mathbf{s}}) = \mathbf{s}(\mathbb{T}_t^{r-1}(\tilde{\mathbf{s}}))$. However, now the function $\mathbf{s}_t(\cdot)$ changes slightly relative to what we had in Section 3. We replace equation (21) with

$$v_{gjt}(\tilde{\mathbf{s}}) = \tilde{\theta}'_g \tilde{\mathbf{s}}_j + \tilde{\alpha}_g P_{jt}(\tilde{\mathbf{s}}) + \hat{\xi}_{gjt}^P, \quad (29)$$

where $\hat{\xi}_{gjt}^P = \hat{v}_{gjt} - \hat{\theta}'_g \mathbf{s}_{jt} - \hat{\alpha}_g P_{jt}$, and we add one more equation:

$$P_{jt}(\tilde{\mathbf{s}}) = \tilde{\rho}' \tilde{\mathbf{s}}_j + \hat{\eta}_{jt}, \quad (30)$$

where $\hat{\eta}_{jt} = P_{jt} - \hat{\rho}' \mathbf{s}_{jt}$.

Thus, instead of simulating the trajectory using equations (21), (22) and (23), we simulate it using equations (29), (30), (22) and (23).

Remark 5. Just like β and θ , α incorporates two components: $\alpha_g = \frac{\partial v_{gjt}}{\partial P_{jt}} = \frac{\partial u_{gjt}}{\partial P_{jt}} + \frac{\partial CV_{gjt}}{\partial P_{jt}}$. The flow utility component ($\frac{\partial u_{gjt}}{\partial P_{jt}}$) should be negative since households prefer to pay lower prices for their house, all else constant. However, the sign of the continuation value component ($\frac{\partial CV_{gjt}}{\partial P_{jt}}$) is theoretically ambiguous since it depends on whether the price of a neighborhood today signals disproportionate expected future appreciation than an otherwise comparable neighborhood, which would have consequences for homeowners' expected wealth.⁴⁸

⁴⁷By substituting equation (25) into equation (24), we obtain $v_{gjt} = \theta'_g \mathbf{s}_{jt}^e + \alpha_g \rho' \mathbf{s}_{jt}^e + \eta_{jt} \alpha_g + \xi_{gjt}^P$. Comparing this to equation (10), we conclude that $\xi_{gjt} = \eta_{jt} \alpha_g + \xi_{gjt}^P$. It follows that Assumptions 3 and 4.1 imply $COV(\eta_{jt}, \mathbf{s}_{jt-T} | \text{inflows}_{jt-1}, \dots, \text{inflows}_{jt-T'}) = 0$.

⁴⁸The buying and selling of a house may impact household wealth. Despite its undeniable importance when studying the behavior of homeowners, we do not explicitly model the effects of moving on wealth, and we do not allow for household heterogeneity by wealth either. In our context, doing so would substantially increase the number of groups of households that we would need to consider and would render our analysis infeasible since there are not enough households of each race and income level to study their decisions by wealth levels. Note, however, that wealth is partially incorporated in our analysis since our parameters are allowed to vary by group, and these groups may have different wealth on average.

6.2 Empirics

We compute monthly neighborhood prices by averaging the sales prices of all observed transactions. The average neighborhood price in our sample is \$329,000 with a standard deviation of \$232,000. There is considerable appreciation over our sample period, as the average price rises from \$248,000 in 1990 to \$564,000 in 2004 (all prices in constant November 2004 dollars). For practical purposes, we impose one additional restriction on α , namely that these parameters vary only by income, not by race and income (e.g., rich Whites and rich Blacks have the same α_g .) We do so only because we cannot obtain precise estimates of α otherwise.

We present estimates of θ and α in Table 3. Compared to Table 2, we conclude that our estimates of responses to socioeconomic compositions with and without prices (θ and β respectively) are quite similar, which suggests a limited role for prices in influencing segregation dynamics. Households of both income groups respond negatively to higher neighborhood prices, but the poor are over four times more price-sensitive than the rich.

We present estimates of ρ in Table 4. They imply that a 10 percentage point increase in the expected Black share of a neighborhood, all else constant, leads to a reduction in average price of \$19,400. This effect is over twice as large for the same increase in the expected Hispanic or poor shares of a neighborhood and roughly the same size for the same increase in the expected Asian share of a neighborhood. We should not interpret these estimates as households' marginal willingness to pay for their neighbors. Rather, we simply use these estimates to simulate how households expect neighborhood prices to change depending on endogenous changes in the expected socioeconomic composition.⁴⁹

⁴⁹All IV estimates of θ , α and ρ are robust to our choice of T . See Figures 17-20 in the appendix, which are analogous Figures 13 and 14 for β . Larger values of T imply weaker Assumptions 3 and 4, since, all else constant, amenities from $t - T$ or before would be less likely to affect inflows in t . Hence, the stability of our estimates is evidence in favor of our exclusion restrictions.

Table 3: Responses to the Socioeconomic Compositions (θ) and Prices (α) of Neighborhoods

	White		Black		Hispanic		Asian	
	Rich	Poor	Rich	Poor	Rich	Poor	Rich	Poor
Responses to:								
Black Share	-2.95*** (0.40)	-2.64*** (0.45)	3.36*** (0.43)	2.08*** (0.42)	0.35 (0.40)	-0.54 (0.43)	-1.29*** (0.44)	-1.16** (0.41)
Hispanic Share	-4.75*** (0.67)	-2.09 (0.92)	-1.63** (0.67)	-4.17*** (0.69)	11.23*** (0.82)	8.01*** (0.82)	-0.46 (0.78)	-0.92 (0.80)
Asian Share	-0.52 (0.48)	-4.21*** (0.63)	0.26 (0.52)	-1.44*** (0.48)	-0.77 (0.61)	-2.18*** (0.62)	4.98*** (0.67)	7.09*** (0.69)
Poor Share	-0.86*** (0.26)	2.80*** (0.44)	-2.52*** (0.38)	-0.37 (0.30)	-1.84*** (0.39)	-0.63* (0.36)	-3.12*** (0.38)	0.61*** (0.41)
Response to Price [†] (Millions)	-0.39** (0.19)	-1.71*** (0.20)	-0.39** (0.19)	-1.71*** (0.20)	-0.39** (0.19)	-1.71*** (0.20)	-0.39** (0.19)	-1.71*** (0.20)
R^2	0.79							
Num. of Observations	147,840							

Notes: This table shows 2SLS estimates of θ_g and α_g from equation (27). The specification includes group-month fixed effects and control variables $\Lambda_g(\hat{v}_{jt-1}, \dots, \hat{v}_{jt-12})$ (see Footnote 33). As instrumental variables, we use $s_{g'jt-13}$ for all g' as well as P_{jt-13} . White is the omitted racial share and rich is the omitted income share. All standard errors clustered by group-month. * - 90% significance, ** - 95% significance, *** - 99% significance. The p-values for both the Cragg-Donald and the Kleibergen-Paap weak identification tests are less than 0.001, which implies a strong first stage. [†] α_g is allowed to vary only by income groups in this specification.

Table 4: Implicit Price of $s_{jt}^e(\rho)$

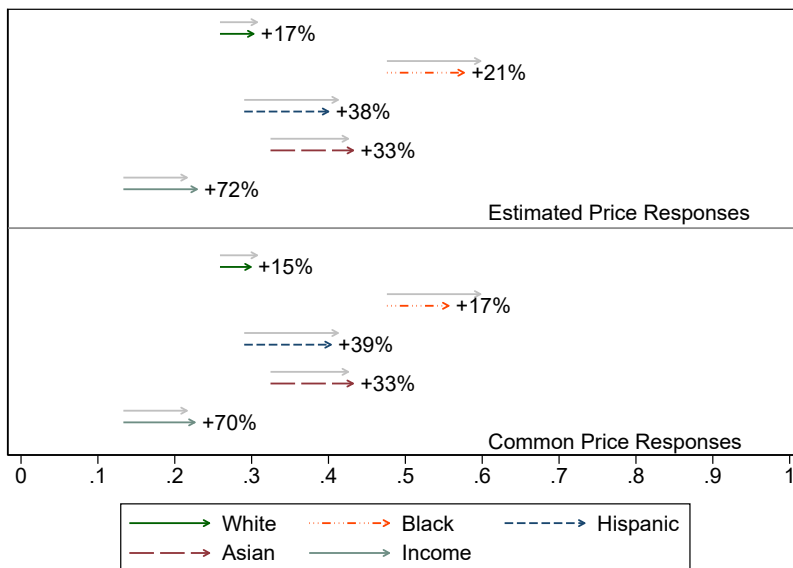
	Black Share	Hispanic Share	Asian Share	Poor Share
Price (Millions)	-0.201*** (0.010)	-0.565*** (0.018)	-0.165*** (0.020)	-0.472*** (0.017)
R^2	0.62			
Num. Obs.	36,960			

Notes: This table shows 2SLS estimates of ρ from equation (28). The specification includes month fixed effects and control variables $\Gamma(\hat{v}_{jt-1}, \dots, \hat{v}_{jt-12})$ (see Footnote 33). As instrumental variables, we use $s_{g'jt-13}$ for all g' . White is the omitted racial share and rich is the omitted income share. All standard errors clustered by month. * - 90% significance, ** - 95% significance, *** - 99% significance. The p-values for both the Cragg-Donald and the Kleibergen-Paap weak identification tests are less than 0.001, which implies a strong first stage.

In Figure 11, we present simulated trajectories of segregation levels when price is explicitly incorporated into the analysis. Comparing the main arrows in the top panel with the light gray arrows representing our baseline findings, we conclude that the results are essentially unchanged. Indeed, the results of all counterfactual analyses carried out in Section 5 are almost identical when prices are explicitly accounted for. If we were conducting these simulations with the true values of β , θ , α and ρ (i.e., not estimated) then this would be expected because of the identity expressed in equation (26). Because we estimate these parameters, our findings could in principle have been different under violations of Assumptions 3, 4, or even our restriction that α_g can only vary across income levels. In light of this, we view the similarity of our findings as reassuring evidence in support of our identification strategy.

The explicit inclusion of prices presents an opportunity to consider a new counterfactual whereby all households have an identical response to prices (i.e., α_g is the same for all g). Specifically, we set α for each group to be equal to the population weighted average of $\hat{\alpha}_g$ across all g . A comparison of the bottom panel and the top panel of Figure 11 allows us to infer the role of heterogeneous price responses on segregation. It is clear that this heterogeneity has a very small effect on segregation on the same order of magnitude as race and income discrimination.

Figure 11: Trajectories of Segregation Levels by Race and by Income - Explicit Price Responses



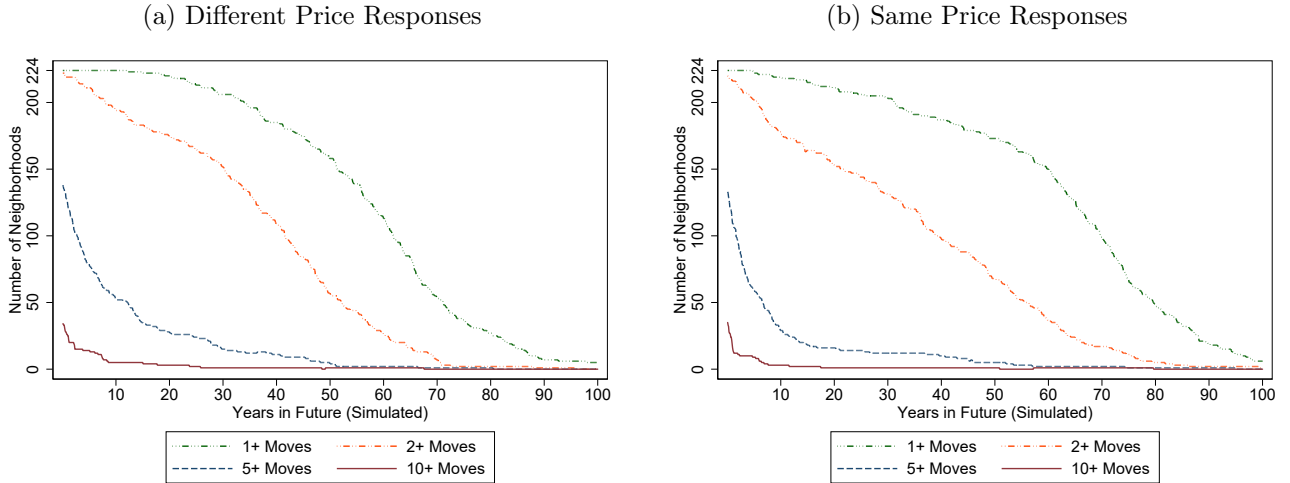
Notes: For each panel representing a different counterfactual, the arrows represent changes in simulated Dissimilarity Indices for households of each race and income from November 2004 to steady state in the absence of exogenous shocks. Numbers in parentheses correspond to the relative change in dissimilarity. Light gray arrows correspond to the baseline changes shown in Figure 3. A Black dissimilarity index of, say, 0.60, means that 60% of Black homeowners would have to be relocated in order to generate an equal distribution of Blacks across all Bay Area neighborhoods. The rich dissimilarity index is identical to the poor dissimilarity index.

In Figure 12, we compare the turnover of neighborhoods in the baseline case (first panel) versus in the counterfactual case where everyone has the same price response (second panel). As expected, the first panel of Figure 12 is very similar to Figure 2, which only implicitly incorporates prices into the analysis. Moreover, the first and second panels of Figure 12 look very similar, suggesting that heterogeneous price responses have little impact on neighborhood dynamics as well.

7 Conclusion

Neighborhoods constantly evolve: their amenities are not static and their residents are in flux. Theoretical models of segregation tend to attribute this evolution to endoge-

Figure 12: Number of Neighborhoods In Flux (Simulated)



Notes: Figure shows the number of neighborhoods with at least one, two, five or ten moves that change their socioeconomic composition (out of a total of 224 neighborhoods). The second panel is under the counterfactual where α_g is equal to the population weighted average of $\hat{\alpha}$. Simulation begins in November 2004.

nous changes in neighborhood residents arising from discrimination, while disaggregated models of residential choice tend to attribute this evolution to exogenous changes in other amenities. In this paper, we develop an empirical framework that synthesizes these two approaches and provides new perspectives on how the aggregate phenomenon of segregation arises from the accumulation of disaggregate residential choices.

We use this framework to study the determinants of race and income segregation in the San Francisco Bay Area from 1990 to 2004. By delineating the interconnected roles of socioeconomic discrimination, other neighborhood amenities, incomplete information, moving costs, initial allocations of households across neighborhoods, and heterogeneity in price-sensitivity, we explore the underlying forces that drive segregation through counterfactual analyses. We find that while discrimination and heterogeneous price responses matter for segregation, they are much less important than sorting on the basis of other amenities. This is in large part due to frictions, primarily moving costs (although incomplete information also plays a discernible role). These frictions prevent much desired sorting from occurring, which weakens the feedback loop generated by endogenous discriminatory sorting. The interplay of all of these forces contribute to a metropolitan area that is on a gradual path to further segregation.

An important caveat in our analysis is that we do not observe the socioeconomic composition of renters over time. This may be less damaging to our conclusions if the

aspects of the expected composition of neighborhoods that are most relevant to sorting decisions are the ones proxied by the actual composition of homeowners (e.g., different allocations of local public goods spending depending on the socioeconomic composition of local taxpayers). However, this may be a concern in neighborhoods with lower rates of homeownership if the aspects of the expected composition of neighborhoods that are most relevant to sorting decisions are the compositions of the people that *use* public goods and, at the same time, landlords' socioeconomic status is a poor predictor of tenants' socioeconomic status. In any case, because renters face relatively lower moving costs than homeowners, we would expect to find patterns of segregation somewhere in between our baseline findings and our counterfactual findings without moving costs. Future research with access to better data is needed to address these issues.

Richer data would also provide opportunities to study socioeconomic segregation at finer levels; for instance, we could consider more income groups or we could disaggregate Asians into Chinese-Americans, Korean-Americans, etc. Ultimately, we view our framework as a platform for the empirical analysis of determinants of segregation that can be directly adapted to various contexts. The use of this framework to study sorting along different demographic dimensions (e.g., race, income, partisanship, education) in different settings (e.g., neighborhoods, schools, virtual communities, physical venues) could prove valuable in revealing the importance of different cleavages in our society.

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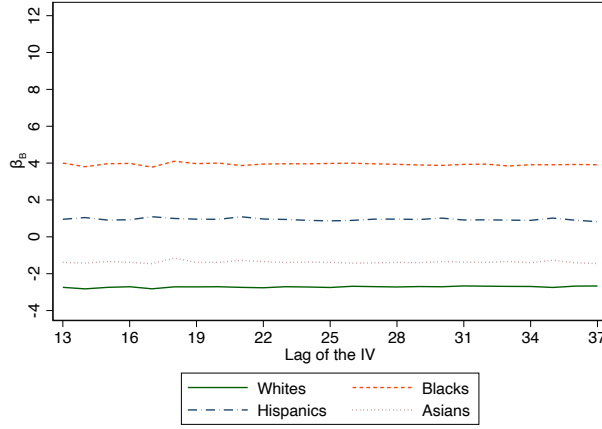
A Online Appendix: Tables and Figures

Table 5: OLS Estimates of Responses to the Race and Income Compositions of Neighborhoods (β)

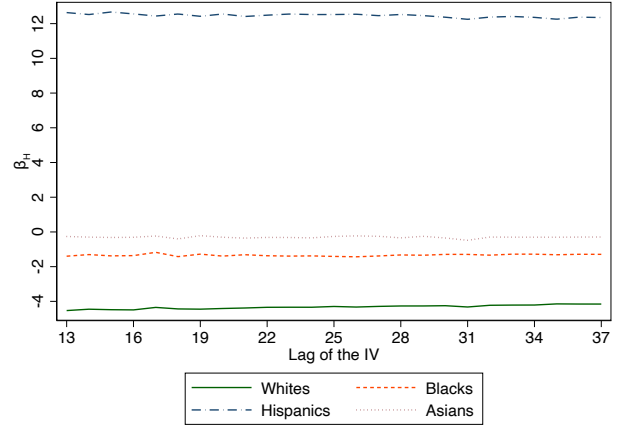
	White		Black		Hispanic		Asian	
	Rich	Poor	Rich	Poor	Rich	Poor	Rich	Poor
Responses to:								
Black Share	-12.25*** (0.35)	-7.32*** (0.52)	13.47*** (0.43)	16.35*** (0.43)	-0.81*** (0.33)	8.49*** (0.35)	-4.81*** (0.37)	1.02*** (0.34)
Hispanic Share	-17.69*** (0.51)	2.66*** (0.53)	8.18*** (0.47)	8.86*** (0.38)	35.92*** (0.67)	43.64*** (0.53)	-1.76*** (0.46)	16.25*** (0.48)
Asian Share	-5.63*** (0.30)	-9.99*** (0.38)	-0.44 (0.34)	-2.64*** (0.28)	-0.21 (0.35)	-3.54*** (0.37)	24.24*** (0.56)	26.63*** (0.61)
Poor Share	-7.01*** (0.35)	1.28*** (0.39)	-7.52*** (0.30)	1.11*** (0.27)	-12.77*** (0.30)	-2.60*** (0.35)	-16.17*** (0.37)	-2.91*** (0.34)
R^2	0.38							
Num. of Observations	147,840							

Notes: This specification includes only group-month fixed effects as controls. White is the omitted racial share and rich is the omitted income share. All standard errors clustered by group-month. * - 90% significance, ** - 95% significance, *** - 99% significance.

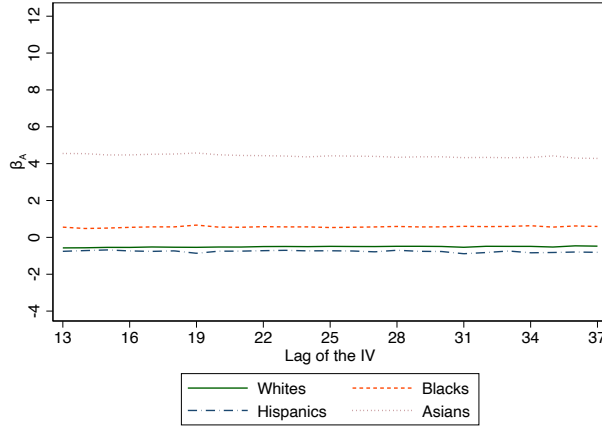
Figure 13: Responses of Rich Households of Different Races to Race and Income Compositions for Different Values of T



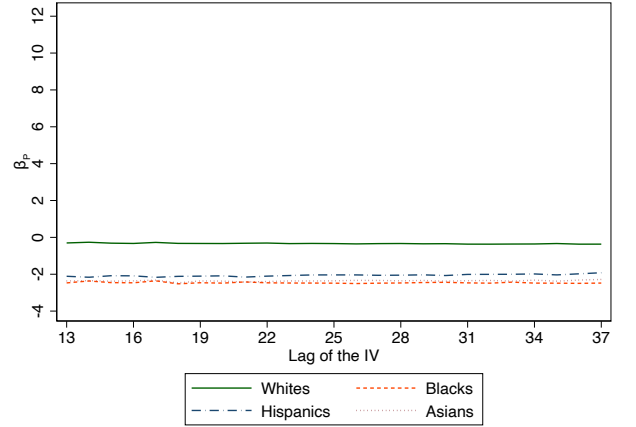
(a) Responses to Black Share



(b) Responses to Hispanic Share



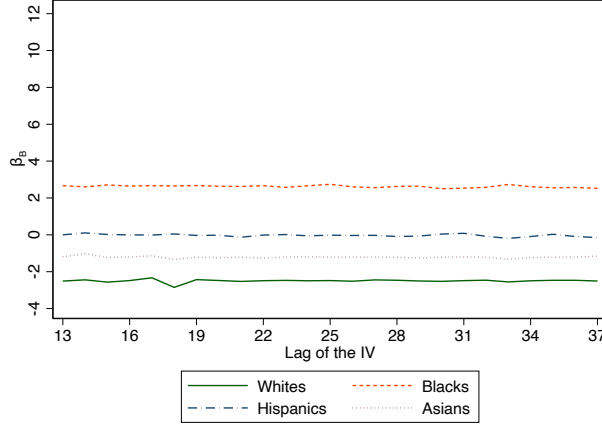
(c) Responses to Asian Share



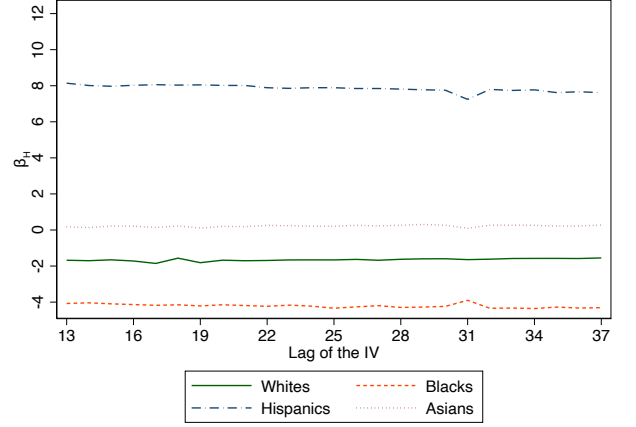
(d) Responses to Poor Share

Notes: Each panel shows $\hat{\beta}_{g,g'}$ for all g' for different values of T , the lag of the Instrumental Variable, $s_{j,t-T}$. In all panels of this figure g represents rich Whites, Blacks, Hispanics and Asians. We set $T' = 12$ in all cases. For comparison, we keep the sample constant in all cases, so the first 37 months of the sample are not used in the estimation of β for all values of T .

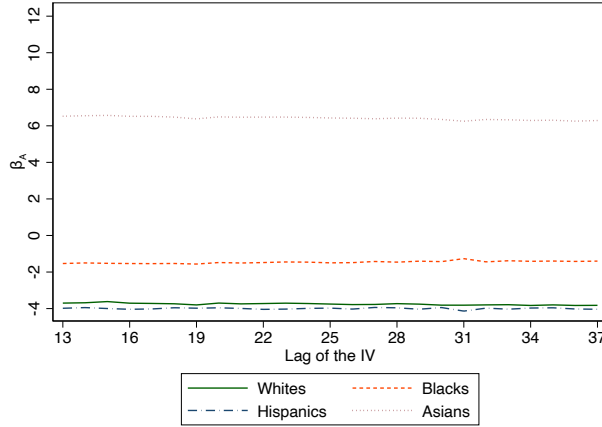
Figure 14: Responses of Poor Households of Different Races to Race and Income Compositions for Different Values of T



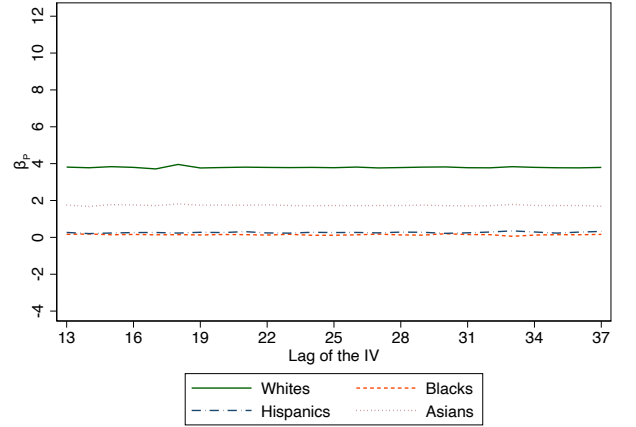
(a) Responses to Black share



(b) Responses to Hispanic Share



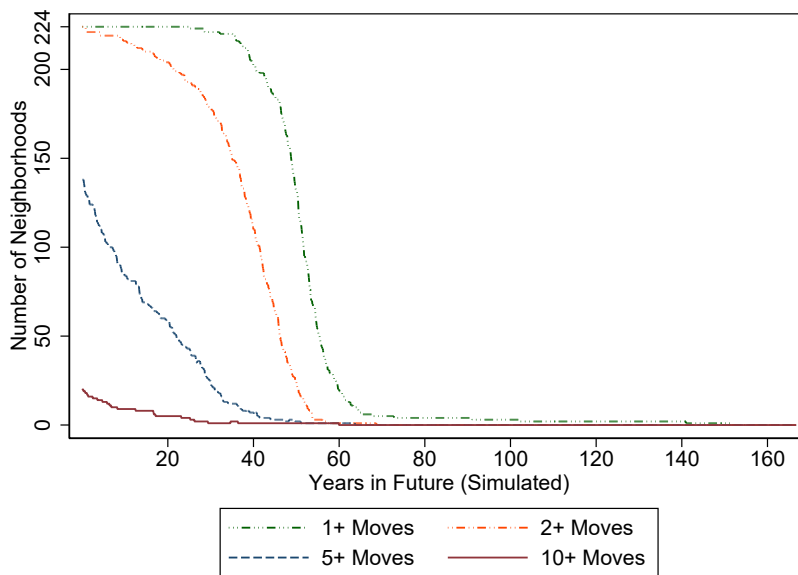
(c) Responses to Asian Share



(d) Responses to Poor Share

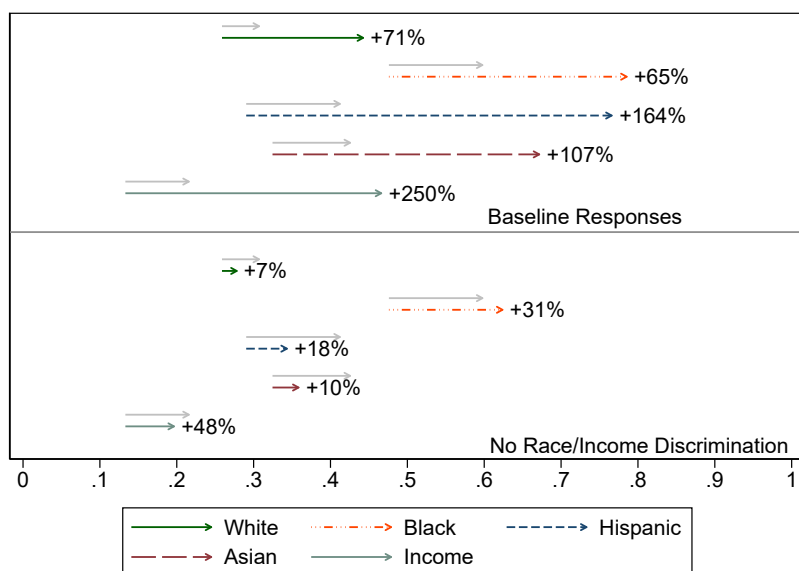
Notes: Each panel shows $\hat{\beta}_{g,g'}$ for all g' for different values of T , the lag of the Instrumental Variable, s_{jt-T} . In all panels of this figure g represents poor Whites, Blacks, Hispanics and Asians. We set $T' = 12$ in all cases. For comparison, we keep the sample constant in all cases, so the first 37 months of the sample are not used in the estimation of β for all values of T .

Figure 15: Number of Neighborhoods In Flux (Simulated)



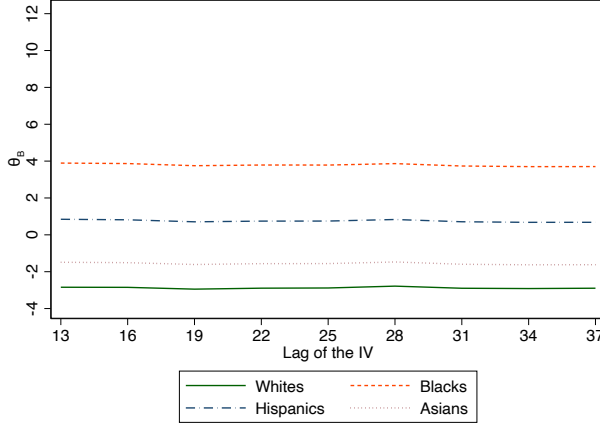
Notes: Figure shows the number of neighborhoods with at least one, two, five or ten moves that change their socioeconomic composition (out of a total of 224 neighborhoods). Simulation begins in November 2004 and continues for 200 years.

Figure 16: Steady State Changes in Race and Income Segregation - No Moving Costs Ever (Simulated)

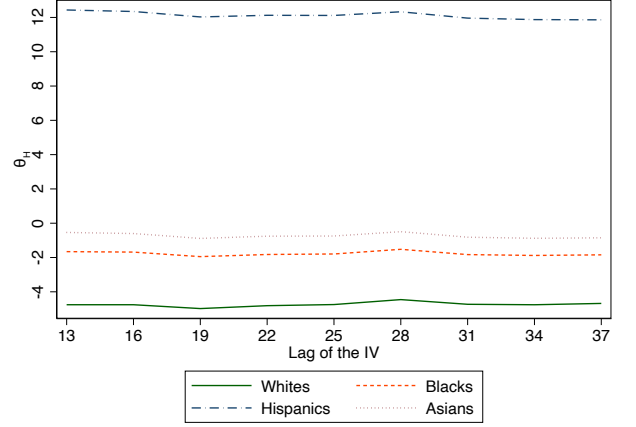


Notes: For each panel representing a different counterfactual, the arrows represent changes in simulated Dissimilarity Indices for households of each race and income from November 2004 to the steady state in the absence of exogenous shocks. Numbers in parentheses correspond to the relative change in dissimilarity. Gray arrows correspond to the baseline changes shown in Figure 3. A Black dissimilarity index of, say, 0.60, means that 60% of Black homeowners would have to be relocated in order to generate an equal distribution of Blacks across all Bay Area neighborhoods.

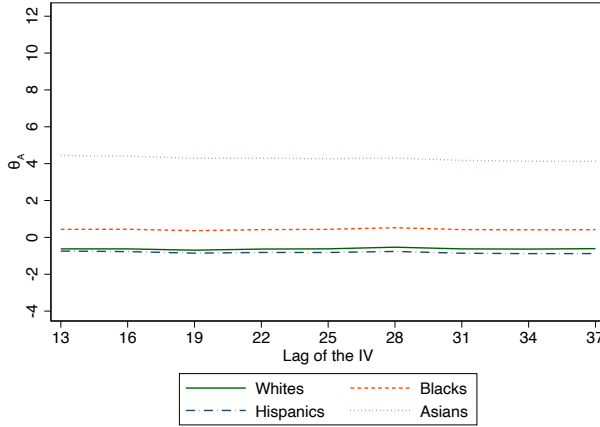
Figure 17: Responses of Rich Households of Different Races to Race and Income Compositions ($\theta_{g,g'}$) for Different Values of T : Price Explicitly Included



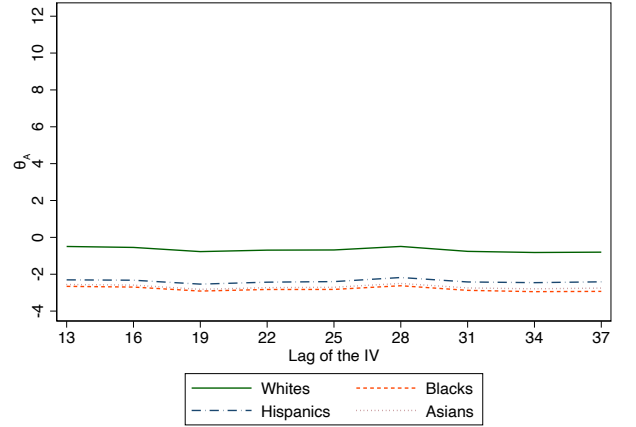
(a) Responses to Black Share



(b) Responses to Hispanic Share



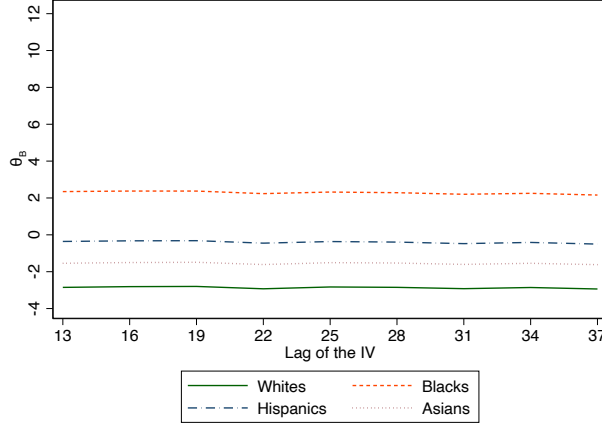
(c) Responses to Asian Share



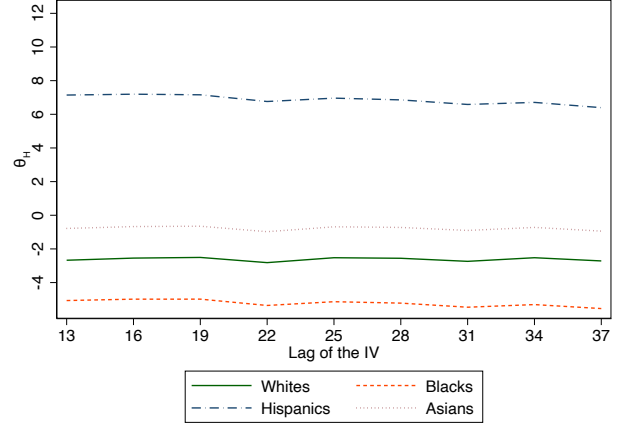
(d) Responses to Poor Share

Notes: Each panel shows $\hat{\theta}_{g,g'}$ for all g' for different values of T , the lag of the Instrumental Variables, s_{jt-T} and P_{jt-T} . In all panels of this figure g represents rich Whites, Blacks, Hispanics and Asians. We set $T' = 12$ in all cases. For comparison, we keep the sample constant in all cases, so the first 37 months of the sample are not used in the estimation of θ for all values of T .

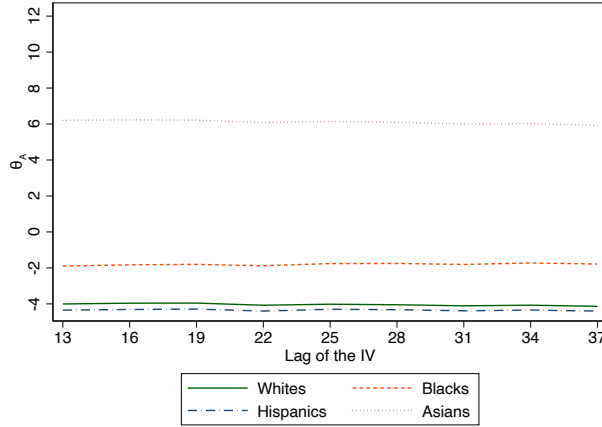
Figure 18: Responses of Poor Households of Different Races to Race and Income Compositions ($\theta_{g,g'}$) for Different Values of T : Price Explicitly Included



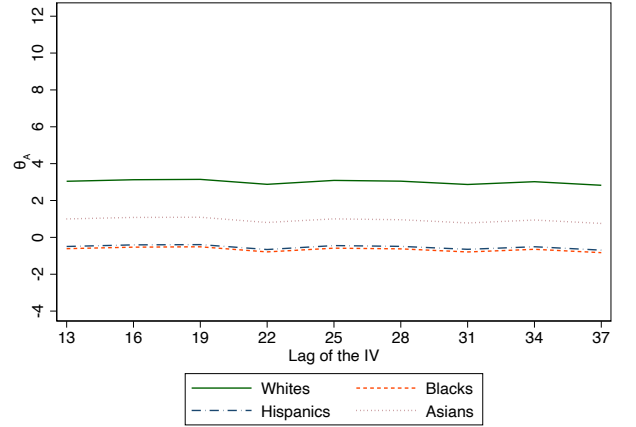
(a) Responses to Black share



(b) Responses to Hispanic Share



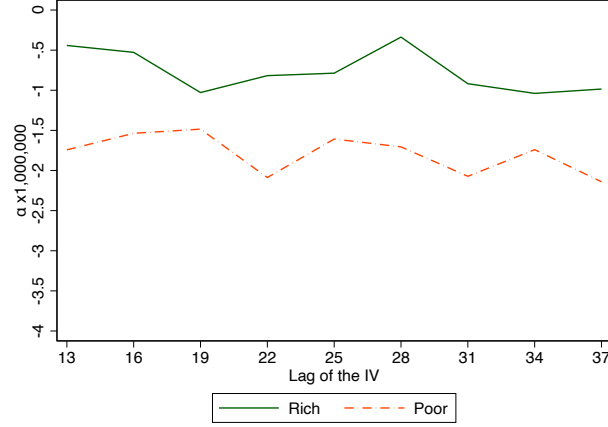
(c) Responses to Asian Share



(d) Responses to Poor Share

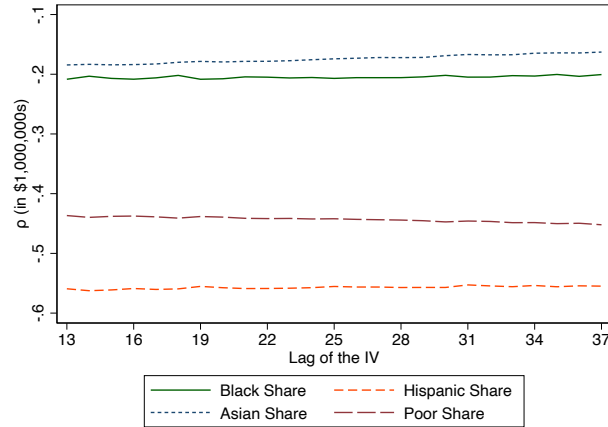
Notes: Each panel shows $\hat{\theta}_{g,g'}$ for all g' for different values of T , the lag of the Instrumental Variables, s_{jt-T} and P_{jt-T} . In all panels of this figure g represents poor Whites, Blacks, Hispanics and Asians. We set $T' = 12$ in all cases. For comparison, we keep the sample constant in all cases, so the first 37 months of the sample are not used in the estimation of θ for all values of T .

Figure 19: Responses of Rich and Poor Households to Prices (α_g) for Different Values of T : Price Explicitly Included



Notes: The plot shows $\hat{\alpha}_g$ for $g \in \{\text{rich}, \text{poor}\}$ for different values of T , the lag of the Instrumental Variables, s_{jt-T} and P_{jt-T} . We set $T' = 12$ in all cases. For comparison, we keep the sample constant in all cases, so the first 37 months of the sample are not used in the estimation of α for all values of T .

Figure 20: Implicit Price of Race and Income Compositions ($\rho_{g'}$) for Different Values of T



Notes: The plot shows $\hat{\rho}_{g'}$ for each g' for different values of T , the lag of the Instrumental Variable, s_{jt-T} . We set $T' = 12$ in all cases. For comparison, we keep the sample constant in all cases, so the first 37 months of the sample are not used in the estimation of ρ for all values of T .