Bank Deposit Mix
and
Aggregate Implications for Financial Stability

Jeongwoo Moon*
The Ohio State University

November 16, 2024

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Abstract

This paper examines the implications of banks' deposit mix and liquidity risk across the bank size distribution for financial stability and macro-prudential policy. U.S. banks mix savings and time deposits, with the share of savings deposits increasing in bank size. I incorporate this in a macroeconomic model of banking industry dynamics, where heterogeneous banks choose their deposit mix and asset portfolio under withdrawal risk in savings deposits. Repayment of withdrawals incentivizes banks to hold securities. Costly time deposits protect against withdrawals and enable banks to substitute loans for securities. Withdrawal risk varies by bank size, explaining the balance sheet composition of assets and deposits, as observed in data. A higher share of savings deposits coming from the low withdrawal risk reduces the average funding cost, expands the banking sector, and increases loans and output in the steady state but raises short-term financial instability. When liquidity requirements are introduced, large banks are the most responsive; they increase demand for securities and cut loan supply, leading to an unintended output cost associated with a less concentrated banking sector in the long run.

*I am especially grateful to Kyle Dempsey, Gabriel Mihalache, and Julia Thomas for their advice and support throughout this project. I would also like to thank David Berger, Dean Corbae, Pablo D'Erasmo, Alessandro Dovis, Rafael Guntin, Aubhik Khan, Pok-Sang Lam, Sean McCrary, Robert Shimer, as well as participants at the OSU Macro workshop, 88th Annual Midwest Economic Association Meetings, 19th Economics Graduate Student Conference by Washington University in St. Louis, and 11th Northeast Ohio Economics Workshop by Cleveland State University and Federal Reserve Bank of Cleveland for their helpful discussion and comments.
Email: moon.448@osu.edu. Website: www.jeongwoomoon.com

1 Introduction

Deposits make up about 70 percent of liabilities of commercial banks in the U.S. Deposits are prone to sudden withdrawals though, posing liquidity risk due to the maturity mismatch between assets and liabilities.[1] Effective liquidity risk management requires maintaining sufficient liquid assets and diversifying funding sources. Failure to do so can lead to capital losses and insolvency, threatening financial stability.

Banks primarily issue two types of deposit products; savings deposits and time deposits.[2] Deposit products vary in their interest costs and liquidity, which is based on how easily they can be withdrawn. It suggests that the composition of a bank's deposits can affect its profitability and stability of funding flows. Due to the highly concentrated banking industry in the U.S. and the varying asset and deposit profiles across the bank size distribution (Corbae and D'Erasmo (2021), d'Avernas, Eisfeldt, Huang, Stanton, and Wallace (2023)), banks can exhibit heterogeneous deposit mixes and liquidity risks. This paper addresses two key questions. First, how do variation in banks' deposit mix and liquidity risk across bank size distribution impact financial stability? Second, what are the implications for macro-prudential policies?

This paper examines the deposit mix and balance sheet composition of U.S. commercial banks of different sizes using call report data. Based on these empirical findings, I develop a macroeconomic model of heterogeneous banks that considers liquidity risk and portfolio choices in both assets and liabilities. Withdrawal risk on savings deposits incentivizes banks to hold securities and issue time deposits, balancing profitability and stability. This risk varies by bank size, with large banks facing lower withdrawal risk. Changes in withdrawal risk influence banks' deposit mix and liquidity risk, affecting lending decisions. The equilibrium in the deposit market impacts the banking sector's size through the banks' funding cost and depositors' income effect, while the loan market equilibrium affect industry concentration by competitive loan rate. A banking sector reliant on time deposits due to higher withdrawal risk incurs higher funding costs and sector shrinkage in the steady state but can enhance financial stability in the short run by maintaining loan supplies. The liquidity coverage ratio, as a macro-prudential policy, affects banks differently based on size. Large banks, with their lower withdrawal risk, are more responsive to regulation, leading to unintended consequences such as reduced loan supply, output costs, and decreased industry concentration in the long run.

[1] Liquidity risk arises from the mistmatch of maturity between assets and liabilities as in Diamond and Dybvig (1983).

[2] Typical example of time deposits is a certificate of deposit (CD). Savings deposits include savings accounts, money market deposit accounts (MMDA), and demand deposits, such as checking accounts, which are mainly used for transaction purposes.

Banks in the U.S. issue both savings and time deposits. Time deposits generally offer higher interest rates than savings deposits, with an average spread of 2.5 percentage points. The stability of deposits on the balance sheet can be assessed by examining deposit flows, measured by the net growth rate. Over time, savings deposits exhibit fluctuations that are 1.1 percentage points greater than those of time deposits. Therefore, there is a tradeoff between profitability and stability in the composition of deposit portfolios. Additionally, deposits in large banks show greater stability compared to smaller banks. On the banks' balance sheet, the share of savings deposits increase in bank size; large banks allocate over 80 percent of their total deposits to savings deposits. Conversely, smaller banks increasingly rely on time deposits. Among the bottom 90 percent of banks, 40 percent of total deposits consist of time deposits. Considering the free withdrawal feature of savings deposits, banks strategically choose a deposit mix, balancing the interest cost of deposits with the stability of funding flows and its pattern varies by bank size.

To study the impact of deposit mix and liquidity risk across bank size distribution on financial stability and macro-prudential policy motivated by data facts, I develop a macroeconomic model of banking industry dynamics where heterogeneous banks choose their deposit mix and asset portfolio under withdrawal risk in savings deposits. In the model, banks are heterogeneous in terms of net worth and the size of their deposit base. A bank invests in liquid securities and illiquid loans using net worth, savings, and time deposits, and equity issunace, subject to a capital requirement. In the middle of each period, a random, idiosyncratic fraction of savings deposits is withdrawn, and the bank must repay the withdrawal by re-balancing its asset portfolio. A fire-sale penalty on loans sold before maturity provides an incentive for a bank to invest in securities to meet withdrawal requests. Time deposits do not face withdrawal risk, but they generally bear higher interest rates. Therefore, the bank chooses the optimal deposit mix, balancing the marginal benefits and costs of holding securities and issuing time deposits. The deposit mix decision affects the bank's total funding base and therefore its lending decision. Additionally, a realized withdrawal shock impacts the bank's future net worth. The use of assets to fulfill withdrawals entails discounted asset sales and foregone interest income; therefore realized net worth is reduced by the withdrawal.

Using specified functional forms, I calibrated a set of parameters to align the model-generated moments with aggregate moments in the U.S. banking industry and deposit markets over the sample periods. Motivated by the empirical pattern above, withdrawal risk varies with bank size. Withdrawal shocks at the bank level are not directly observable and are inferred from balance sheet data on securities and savings deposits. The sale of securities to meet withdrawal demands accompanies foregone interest income. Therefore, as the bank experiences larger withdrawal shocks, the ratio of market value of securities to the stock of savings deposits declines. I use these inferred withdrawal shocks to estimate kernel densities across different size classes. Large banks face a smaller and more concentrated withdrawal risk over savings deposits than small banks. Notably, medium-sized banks' withdrawal risk resembles small banks, despite their asset size resembling large banks.

With calibrated parameters and embedded withdrawal shock process, the model successfully captures qualitative patterns in bank balance sheets, considering both assets and deposits across different bank sizes. For large banks, the withdrawal risk per unit of deposits is small, allowing them to issue deposits at lower interest rates by favoring savings deposits. Additionally, low withdrawal risk lets large banks hold fewer securities given the same amount of savings deposits compared to their smaller counterparts. Conversely, small banks prioritize time deposits to mitigate relatively higher withdrawal risk and allocate more resources to securities, safeguarding their capital. This explains why large banks, which have a higher share of savings deposits, tend to have a lower proportion of securities in their asset portfolios compared to small banks.

To examine how both the level and size dependence of liquidity risk shape aggregate outcome, I compare steady-state results between the baseline economy and an alternative economy called the uniform risk economy, where banks face the same withdrawal risk as small banks. An increased share of time deposits due to an increased withdrawal risk for all banks raises the share of loans in the bank-level asset portfolio. However, the increased average funding cost and reduced deposit base shrink the overall banking sector, leading to a contraction in output because of the reduced availability of credit for firms. Large banks, in particular, supply fewer loans, while small and medium-sized banks take advantage of the increased equilibrium loan rates to expand their loan supplies. Large banks experience the most significant changes in withdrawal risk and respond by issuing more time deposits compared to their smaller counterparts. This requires them to offer higher interest rates on time deposits, which in turn reduces their net interest margins the most. As a result, the banking sector becomes less concentrated in the uniform risk economy.

I then study how these forces affect short-run dynamics, by simulating an adverse aggregate shock to bank net worth. Banks substitute savings deposits with time deposits to maintain loan supply and recover net worth. A large dependence on time deposits in the banking sector of the uniform risk economy improves financial stability as banks can accumulate net worth faster with a smaller drop in loan supply and a less severe recession than the baseline economy. Both economies reduce the share of savings deposits to limit the impact of net worth shocks on loan supply. This lowers funding costs and raises equilibrium loan rates, increasing net interest margins. In the baseline economy, the shift to time deposits is more pronounced, leading to higher overall funding costs due to increased interest rates on these deposits. The higher returns on deposits reduce the banking sector due to the income effect on depositors, outweighing the rise in net interest margins and explaining the different responses to the net worth shock. Therefore, a large dependence on time deposits is costly in the steady state but can be a source of financial stability in the short run.

Lastly, I investigate the impact of deposit mix and liquidity risk across bank size distribution on macro-prudential policy by introducing liquidity requirement in the model. The liquidity requirement considered is the Basel III liquidity coverage ratio, which regulates the ratio of a bank's liquid assets to total deposits outflows. Through the lens of the model, the regulation is particularly effective for large banks, prompting them to increase their demand for securities and reduce their loan supply. The resulting higher equilibrium loan rates allow other banks to supply more loans and expand. Small and medium-sized banks issue more time deposits to finance loans, reducing expected withdrawal risk. In the steady state, there is an unintended output cost due to the reduced loan supply, as the advantage of low withdrawal risk for large banks is not fully considered in the liquidity coverage ratio, leading to a less concentrated banking sector. This suggests that implementing a size-dependent liquidity coverage ratio, with more tailored constraints for large banks, could potentially reduce the output cost.

Related literature

This paper contributes to several strands of macroeconomic and finance literature. First, extensive research has explored the role of banks in macroeconomics. For instance, Gertler and Kiyotaki (2010) investigates the real effects of financial intermediaries during crises. Gertler and Kiyotaki (2015) develops a model that delivers an equilibrium bank run over business cycles. Deposits in banking problems are often assumed to be an exogenous process (Corbae and D'Erasmo (2021), Rıos-Rull, Takamura, and Terajima (2020)) or one-period single type of debt (Gertler and Kiyotaki (2015)). In contrast, this paper provides a model where the deposit market and deposits for individual banks are endogenously determined with a realistic deposit mix within banks and deposit market power to analyze their implications for financial stability and macro-prudential policies.

Furthermore, the interplay between realistic deposit structures and market dynamics has been a subject of study in both finance and macro-finance. Jermann and Xiang (2023) models deposit withdrawals for non-maturing saving deposits and studies debt dilution in the context of banking. Supera (2021), which is the most closely related to this paper, focuses on the secular trend in bank deposits and explains the decline of firm entry rates by the falling share of time deposits with monetary policy. Drechsler, Savov, and Schnabl (2021) explores deposit market power, revealing that banks respond insensitively to monetary policy regarding deposit rates. This paper stands apart by modeling a macroeconomic framework of heterogeneous banks with funding mix choices and market power to analyze the distributional impact of deposit structures and macro-prudential policies within the banking industry and its broader macroeconomic implications.

Thirdly, numerous studies have focused on models emphasizing banking industry dynamics and exploring their aggregate implications across various contexts. Corbae and D'Erasmo (2021) investigates capital requirement regulation within a concentrated banking industry. Similarly, Rıos-Rull et al. (2020) examines how countercyclical capital requirements impact aggregate dynamics with a model economy based on industry dynamics. Dempsey and Faria-e Castro (2022) studies the role of the relationship between borrower and lender and its aggregate consequences. Building on this literature, this paper contributes by incorporating realistic deposit mix choices observed in data into a dynamic model of the banking industry. This approach allows for the study of the role of bank deposits in banks' life cycles and their aggregate effects. Furthermore, the framework of this paper serves as a laboratory for studying the impact of liquidity regulations within the banking industry equilibrium, considering the specific characteristics of bank deposit mix.

The subsequent sections of this paper are structured as follows: In Section 2, I present empirical evidence about bank deposits and their distribution within the U.S. economy. Following this, Section 3 introduces the model economy. Section 4 analyzes the role of withdrawal risk in bank capital and the household problem to inspect the core implications of the funding mix for banks and households. Section 5 outlines a quantitative approach for aligning our model with empirical data. In Section 6, empirical validation of the model is provided. The quantitative analysis is detailed in Section 7. Finally, Section 8 concludes.

2 Bank Deposits in the U.S. Economy

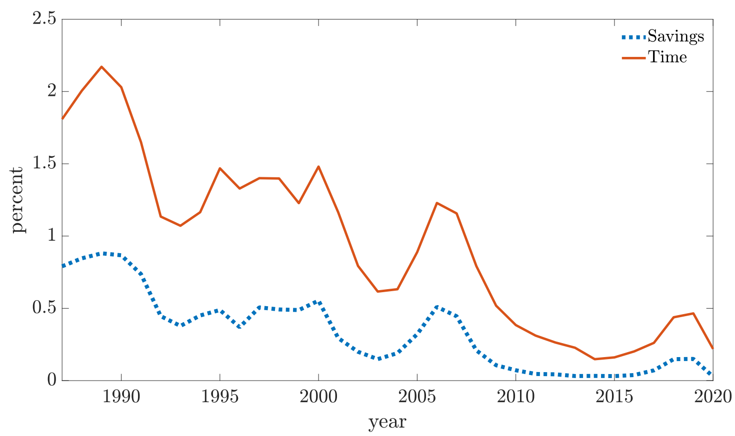
This section provides empirical facts on the deposit products and their distribution across U.S. commercial banks. The data used in this analysis is Consolidated Reports of Condition and Income (so-called Call Report) and focuses on U.S. commercial banks located within the 50 states and the District of Columbia. The dataset covers information about the bank's balance sheet and income statement and we restrict our sample periods from 1984 Q1 to 2021 Q2. Here is the summary of the empirical facts:

1. The average interest cost of savings deposits is lower than time deposits.
2. The flow of savings deposits is more volatile than time deposits.
3. The variance of flows decreases in bank size.
4. The share of savings deposits in the balance sheet increases in bank size.

2.1 Deposit products: trade-off between cost and stability

In essence, the key distinctions between savings deposits and time deposits arise from their contract structures. Savings deposits are non-maturing and typically allow unrestricted withdrawals. Conversely, time deposits have a fixed maturity specified in the contract, and early withdrawals incur penalties. While various factors differentiate these deposit types, it remains essential for a profitable banking operation to secure favorable borrowing costs for profitability and ensure a steady funding stream as assets tend to feature longer terms. As a result, our focus centers on deposit rates and deposit flows.

First, Savings deposit pays a lower interest cost than time deposits. The effective (ex-post) interest rate for both deposits is calculated by total interest expense divided by the total balance for each type of deposit. Figure 1 shows the asset-weighted average deposit rates of each deposit type over sample periods. The effective interest rates for time deposits are always higher than savings deposits, while the spread fluctuates over time.



Note: This figure is constructed using Consolidated Reports of Condition and Income (Call Report) that covers U.S. commercial banks located within the 50 states and the District of Columbia. The sample period ranges from 1987 to 2020. Every aggregation is asset-weighted. Savings deposits include both transaction and non-transaction accounts of savings deposit added demand deposits.

Figure 1: Interest Rates of Deposit Products

Second, the degree of free withdrawal and the presence of maturity can influence deposit flows. For instance, a depositor needing quick access to funds would withdraw from savings deposits to avoid early withdrawal penalties associated with time deposits, which add an outflow of savings deposits for a bank. Consequently, savings deposits exhibit greater volatility in both inflows and outflows compared to time deposits. The median quarterly outflow of savings deposits is 3.8 percent, which is higher than the 2.9 percent outflow for time deposits. Additionally, savings deposits have an inflow rate of 4.9 percent, surpassing the 3.5 percent inflow rate for time deposits.[3] [4]

		Large	Medium	Small
Outflows	Savings (freq.)	0.88	1.36	1.73
		(0.50)	(0.65)	(0.72)
	Time	0.50	0.98	1.05
		(0.50)	(0.71)	(0.73)
	Total	1.14	1.74	2.11
		(0.50)	(0.66)	(0.72)
Inflows	Savings (freq.)	1.22	1.88	2.21
		(0.50)	(0.63)	(0.68)
	Time	0.52	1.28	1.39
		(0.50)	(0.74)	(0.75)
	Total	1.25	2.34	2.75
		(0.01)	(0.69)	(0.73)

Note: "freq." in parenthesis represents the proportion of outflows or inflows that exceed the median outflow or inflow of large banks within each size group. Savings deposits include both transaction and non-transaction accounts of savings deposit added demand deposits. *Large* is Top 0.1% banks in asset size. *Medium* is Top 10% excluding Top 0.1% banks. *Small* contains banks in bottom 90% of asset percentile.

Table 1: Flows of Bank Deposits

2.2 Deposit mix over size distribution

Table 1 shows the median inflows and outflows of deposits for bank size classes. We categorize the banks into three groups: $\{large,\ medium,\ small\}$. Banks with consolidated assets in the top 0.1% in each period fall into large banks. Between top 0.1% and top 10%, there are medium-sized banks. And the bottom 90% consists of small banks [5]. Across all categories

[3]The growth rates are used as a proxy for the flow of deposits since the direct observation of the flow of deposits is not available. Jermann and Xiang (2023) also discusses the lack of data for the flow of deposits. The quarter-by-quarter growth rate of deposit stock is not a perfect measure for flows but it still enables us to compare the relative volatility of two deposit products in the bank's balance sheet.

[4]Savings deposits in both inflows and outflows exhibits a larger spread of the middle 50 percents as well. The interquartile range for savings deposits is 7.2 for inflows and 5.1 for outflows, while the range for time deposits is 6.4 for inflows and 4.0 for outflows.

[5]For example, in the first quarter of 2021, JPMorgan Chase Bank, Bank of America, Wells Fargo Bank, Citibank, and U.S. bank fall into *large* group. The threshold asset size for the top 10% is $1,923 million so we have fewer banks than the Fed reports as large commercial banks that have consolidated assets of $300 million or more. The size classes of banks are somewhat loosely defined but it provides a summarizing picture of asset market shares. As of the first quarter of 2021 (last period of the sample), *large* banks account for 48% of total assets in the banking sector and *medium* banks take 45% and then *small* banks explains the rest, 7%. Among 4,215 banks, only five banks are *large*, and the number of *medium* and *small* is 417 and 3,793, respectively.

of deposits, those held in large banking institutions exhibit a higher degree of stability compared to other bank sizes. By evaluating the median flow values specific to large banks, we determine the proportion of instances where the magnitude of flows is greater than or equal to this median benchmark for large banks. It has been observed that this ratio inversely correlates with the size of the bank, diminishing as the bank size increases. Although there are statutory similarities in deposit contracts across different sizes of banks, the flow of deposits behaves differently.

Due to the inherent non-maturity characteristic of savings deposits, banks face constraints in repurchasing these deposits. Consequently, Table 1 suggests that smaller banks may be more vulnerable to substantial withdrawals of savings deposits [6]. Conversely, larger banks benefit from their size, with the Law of Large Numbers contributing to the stability of their deposits. Savings deposits, which are typically susceptible to runs, are actually more secure in larger institutions. This study does not delve into the underlying reasons for the reduced outflow rates in large banks. However, one plausible explanation is that large U.S. banks have achieved growth by extending their branch networks across multiple states, rather than concentrating within limited areas, leading to a more diversified deposit base compared to smaller, local banks. This observation aligns with the findings from the exogenous deposit capacity process estimated by Corbae and D'Erasmo (2021).

Heterogeneity in deposit flows across different asset classes implies that an additional unit of deposits can have a disparate impact on the overall deposit flows of large versus small banks. Then, it can be the case that different sizes of banks would exhibit different optimal levels of deposit mix. Indeed, it is observed that the share of savings deposits in the balance sheet rises in bank size. In Figure 2, there is a (both time- and within-size bin) average pattern of the share of savings deposits in individual bank's balance sheets across different size classes. Savings deposit is the major type of deposit for very large banks in the top 0.1%. For them, time deposit takes up slightly less than 20 percent of their balance sheet. In contrast, within smaller banks—specifically those in the lower 90th percentile—time deposits constitute 2/5 of the balance sheet's deposits. This represents a proportion approximately double that of larger banking institutions.

The composition of deposits is crucial for a bank's operations for two primary reasons: profitability and funding stability. As indicated in Figure 2, smaller banks tend to incur higher interest expenses due to the predominance of time deposits, which carry higher interest rates as shown in Figure 1[7]. This reliance on time deposits can erode profitability unless offset by substantial returns on assets. Furthermore, a bank's funding capacity, particularly

[6] Deposit outflows typically occur due to two factors: (i) depositor-initiated withdrawals and (ii) strategic balance sheet reductions by the bank itself. In a later chapter, we discuss the identification of withdrawal from outflow of deposits.

[7] We confirm this relationship in scatter plots and regression analysis. Details are in Appendix C.

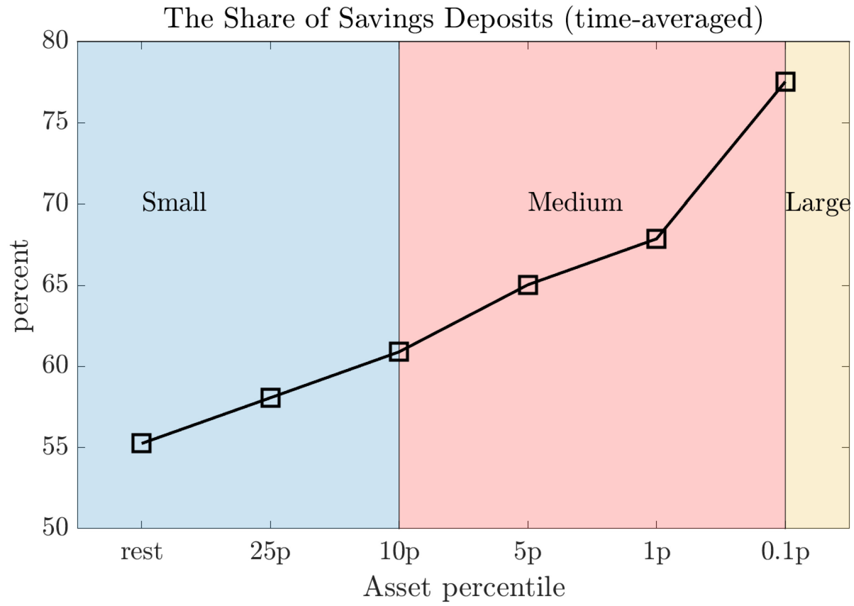


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Figure 2: Share of Savings Deposit in the Balance Sheet by Bank Size

for long-term loans, is influenced by its deposit mix, as detailed in Table 1. Variations in the stability of deposit flows can, therefore, have a significant impact on the bank's lending abilities.

3 Model Economy

This section describes the macroeconomic model of the banking industry with heterogeneous banks to explain the empirical patterns documented earlier and to study their aggregate implications on the aggregate economy. Banking industry equilibrium is modeled applying Hopenhayn (1992) to the banking sector of the economy. Therefore, an individual bank's lifecycle is embedded in the model and the banking industry equilibrium features a distribution of heterogeneous banks with endogenously determined bank capital and size of assets and deposits.

The model economy is populated by a representative household, a represetative firm, a continuum of heterogeneous banks, and government. Banks engage in liquidity transformation, utilizing their net worth, equity, and two types of deposits to finance loans and securities. Banks have market power over their deposits based on Drechsler et al. (2021) and the markets for loans and securities are perfectly competitive. Firm produce consumption goods with decreasing returns to scale production technology with labor as a sole input. The firm borrows from banks to finance a fraction of working capital (wage bill). A representative household valuing both consumption and liquidity chooses consumption and saving with an endowment. Bank deposits deliver bank-specific liquidity services and household allocates deposits across bank distribution given the deposit contracts offered by banks. Households own banks and firm. The government supplies securities as a form of government bonds which carry a risk-free interest rate and are funded through a lump-sum tax on the households.

3.1 Representative household

There is a continuum of identical (equivalently, representative) infinitely-lived households in the economy. Household's cash on hand at the beginning of the period consists of wage income (wN_H), stock of deposits ($D_S + D_T$), profit from banks (Π_B), profit from firm (Π_F) and lump-sum tax (τ), households consume (C) and make bank deposits to save. Households can save to bank deposits by allocating deposits across a continuum of banks, $j \in (0,1)$. Since the bank deposit market is assumed to be monopolistically competitive, households are offered a bank-specific deposit price for both savings and time deposits, $\{q_{S,j}, q_{T,j}\}$. Then, the total amount of savings to bank deposit is $\int \left\{q_{S,j}d'_{S,j} + q_{T,j}d'_{T,j}\right\} \mathrm{d}j$. Therefore, the budget constraint of the household at the beginning of the period is given by

$$C + \int \left\{q_{S,j}d'_{S,j} + q_{T,j}d'_{T,j}\right\} \mathrm{d}j \leq wN_H + D_S + D_T + \Pi_B + \Pi_F - \tau$$

where consumption good is a numeraire and $D_S = \int d_{S,j}\mathrm{d}j$ and $D_T = \int d_{T,j}\mathrm{d}j$ are total amount of deposits pre-determined from previous period. Profits from banks and firm are defined as net dividend payouts based on the ownership.

On top of the consumption over the lifetime, households value aggregate liquidity from the bank deposits. Liquidity service and consumption forms a CES composite, Z that enters into the isoelastic utility function, $U(Z)$,

$$Z = \left(\lambda_C^{\frac{1}{\eta}} C^{\frac{\eta-1}{\eta}} + (1-\lambda_C)^{\frac{1}{\eta}} L_H^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}$$

where η is the elasticity of substitution between consumption and liquidity service. λ_C implies the share of consumption goods in the CES composite. Aggregate liquidity comes from the aggregation of bank-specific liquidities (L_j) which we assume to be imperfectly substitutable. Then, the aggregate liquidity is

$$L_H = \left(\int \delta_j^{\frac{1}{\nu}} L_j^{\frac{\nu-1}{\nu}} \mathrm{d}j\right)^{\frac{\nu}{\nu-1}}$$

where ν is the elasticity of substitution between liquidity services of banks. δ_j plays as a weight for bank-specific liquidity in the CES aggregator. This reflects the deposit base or liquidity capacity of a bank which is unrelated with banks' deposit rates and it can be considered to summarize the geographical availability of bank branches and/or depositors' inertia created by the relationship with bankers or search costs. In the bank's problem, δ_j is an idiosyncratic state that indirectly affects the scale of the operation. Within a bank, households make a portfolio choice between savings deposits and time deposits given discount prices of deposits, $\{q_{S,j}, q_{T,j}\}$, set by a bank. Both as a form of CES composite deliver a bank-specific liquidity service, L_j.

$$L_j = \left(\lambda_S^{\frac{1}{\epsilon}} (d'_{S,j})^{\frac{\epsilon-1}{\epsilon}} + (1-\lambda_S)^{\frac{1}{\epsilon}} (d'_{T,j})^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}$$

where ϵ is the elasticity of substitution between saving and time deposits and λ_S reflects the share of savings deposit in the liquidity for bank j.

Denote μ as the distribution of banks and $\mathcal{Q} \equiv \{q_{S,j}, q_{T,j}\}_{j=0}^1$ as a set of deposit prices from banks. The value of a representative household is given by

$$V^H(D_S, D_T, \mu, \mathcal{Q}) = \max_{Z,C,L_H,\{L_j\},\{d'_{S,j}\},\{d'_{T,j}, N_H\}} U(Z) + \beta \mathbb{E}\left[V^H(D'_S, D'_T, \mu', \mathcal{Q}')\right] \quad \text{s.t.} \tag{1}$$

$$C + \int \left\{q_{S,j} d'_{S,j} + q_{T,j} d'_{T,j}\right\} \mathrm{d}j \leq wN_H + D_S + D_T + \Pi_B + \Pi_F - \tau \tag{2}$$

$$Z = \left(\lambda_C^{\frac{1}{\eta}} C^{\frac{\eta-1}{\eta}} + (1-\lambda_C)^{\frac{1}{\eta}} L_H^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \tag{3}$$

$$L_H = \left(\int \delta_j^{\frac{1}{\nu}} L_j^{\frac{\nu-1}{\nu}} \mathrm{d}j\right)^{\frac{\nu}{\nu-1}} \tag{4}$$

$$L_j = \left(\lambda_S^{\frac{1}{\epsilon}} (d'_{S,j})^{\frac{\epsilon-1}{\epsilon}} + (1-\lambda_S)^{\frac{1}{\epsilon}} (d'_{T,j})^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \tag{5}$$

3.2 Representative firm

There is a representative firm produce consumption goods with decreasing returns to scale production technology with labor as a sole input,

$$Y = F(N_F) = \overline{Z} N_F^{\alpha}$$

where Z is total factor productivity and $\alpha \in (0,1)$. In each period, the firm is subject to working capital constraint such that the firm should pay a fraction of wage bill by borrowing using bank loans,

$$\kappa \overline{w} N_F \leq L'_F$$

The firm borrows one-period loans from banks at price q_ℓ and repay the debt at the beginning of the next period. Thus, the firm's profit at each period is determined as

$$\Pi_F = \overline{Z} N_F^\alpha - \overline{w} N_F + q_\ell L'_F - L_F$$

The firm is owned by household and so the profit each period is paid out to household as a form of dividend. The recursive formulation of the firm problem is

$$\begin{aligned} V^F(L_F; \mu, \mathcal{Q}) = \max_{N_F, L'_F} \; & \overline{Z} N_F^\alpha - \overline{w} N_F + q_\ell L'_F - L_F + \beta \mathbb{E}[V^F(L'_F; \mu', \mathcal{Q}')] \\ \text{subject to} \quad & \kappa \overline{w} N_F \leq L'_F \end{aligned} \tag{6}$$

3.3 Heterogeneous banks

At the beginning of each period, an incumbent bank starts the operation with maturing loans (ℓ), government bonds (a), outstanding savings deposits (d_S), and time deposits (d_T) in the balance sheet. From the balance sheet identity, the bank's net worth at the beginning of the period (n) is

$$n = \ell + a - d_S - d_T.$$

To earn profit, banks invest in one-period assets. Both loans ($\tilde{\ell}'$) and government bonds ($\tilde{a}'$) which return one at the beginning of the next period can be purchased at the discount price q_ℓ and q_a, respectively in the perfectly competitive markets. Banks use net worth and source funds through deposits ($\tilde{d}'_S + \tilde{d}'_T$) and equity issuance ($e > 0$). Equity issuance incurs a non-pecuniary cost and it is valued in $\psi(e)$. The cost of equity issuance induces a bank to issue deposits and helps to produce a non-trivial capital structure for the bank. Bank deposits are non-defaultable one-period debts and fully insured in the case of bank failure. Each bank has market power over deposits so deposit contracts offered by banks feature discounted prices and the quantity of deposits. The contract of savings deposit is $\{q_S, \tilde{d}'_S\}$ and contract of time deposit is $\{q_T, \tilde{d}'_T\}$. Bank pays a dividend ($\mathcal{D}$) and values it and it is assumed that a bank does not issue equity while dividend payouts are positive. Then, the flow budget at the beginning of the period is

$$q_\ell \tilde{\ell}' + q_a \tilde{a}' + \mathcal{D} \leq n + q_S \tilde{d}'_S + q_T \tilde{d}'_T + e \tag{7}$$

In the deposit market, each bank has market power over its deposits. It enables a bank to set interest rates for deposits given the household's demand functions. Banks are heterogeneous with respect to liquidity capacity (δ). It is an idiosyncratic state of a bank and it scales the household's demand functions, in turn, determines the scale of the bank's balance sheet. The sizes of assets and deposits are endogenously determined in the model. Yet, the size type of the bank in terms of liquidity capacity helps us to match the model to data. Later, we show that the classification of bank size classes is evident with estimated withdrawal shocks. AR(1) term creates uncertainty and it helps to generate heterogeneity within each group of banks with the same types.

The main difference between savings and time deposits from the perspective of the bank is the degree of exposure to the withdrawal risk. Since both deposits are one-period debt, it is assumed that there is a withdrawal shock in the middle of a period for savings deposits. In contrast, a time deposit is free from the withdrawal risk. So, a bank is obligated to service the withdrawal of savings deposits in the middle of each period before assets mature. The withdrawal risk in savings deposits makes a bank face a liquidity risk. We model the withdrawal as an idiosyncratic shock, $\theta \in (0,1)$ that is i.i.d. with a cumulative density function, $F(\theta|\delta)$ where $\partial \mathrm{E}(\theta|\delta)/\partial\delta < 0$ and $\partial \mathrm{Var}(\theta|\delta)/\partial\delta < 0$. As we make the distribution of withdrawal shock depend on the liquidity capacity, this captures the lower outflows in saving deposits for large banks relative to other banks as documented in data.

Withdrawal of saving deposits, $\theta\tilde{d}'_S$, is serviced by selling the assets at the market prices. We assume that loans are illiquid and government bonds are liquid in the asset market in the sense that selling loans accompanies a discount and the value of loans is partially recovered, $\omega q_\ell \tilde{\ell}'$ where $1-\omega \in (0,1)$ is a fraction of loss with a discount. Whereas, the value of government bonds (securities) is recovered at the current market price without any discount, $q_a\tilde{a}'$. This gives an incentive for a bank to hold securities in normal times although their returns are lower than loans.

Meeting the withdrawal request is assumed to follow a pecking-order decision as in Supera (2021). Securities come first to serve the withdrawal and if the amount of withdrawal exceeds the current market value of securities in the balance sheet, the bank starts selling loans with a discount. The model analysis in the next section shows that the pecking-order decision is the optimal for a particular order of asset prices. Let ι denote the policy function of the sale of loans ($\iota = 1$, otherwise $\iota = 0$). The pecking-order decision can be expressed as

$$\begin{cases} \iota = 0, & \text{if } \ \theta\tilde{d}'_S \leq q_a\tilde{a}' \\ \iota = 1, & \text{if } \ \theta\tilde{d}'_S > q_a\tilde{a}' \end{cases}$$

Following the realization of the withdrawal shock, the bank's assets are re-balanced. Then, the bank's balance sheet for the next period is determined with re-balanced assets and remaining savings deposits as $\{\ell', a', d'_S, d'_T\}$. The balance sheet next period implies bank's net worth for the next period satisfies

$$n' = \ell' + a' - d'_S - d'_T$$

Re-balanced assets and remaining liabilities differ by the size of the withdrawal shock and deposit mix. Based on the pecking-order decision, the change in the balance sheet is summarized as follows,

i) $\iota = 0$ when $\theta \tilde{d_S}' \leq q_a \tilde{a}'$,

$$\{\ell', a', d'_S, d'_T\} = \begin{cases} \ell' &= \tilde{\ell}' \\ a' &= \tilde{a}' - \theta \tilde{d_S}'/q_a \\ d'_S &= (1-\theta)\tilde{d_S}' \\ d'_T &= \tilde{d_T}' \end{cases}$$

ii) $\iota = 1$ when $\theta \tilde{d_S}' > q_a \tilde{a}'$,

$$\{\ell', a', d'_S, d'_T\} = \begin{cases} \ell' &= \tilde{\ell}' - (\theta \tilde{d_S}' - q_a \tilde{a}')/(\omega q_\ell) \\ a' &= 0 \\ d'_S &= (1-\theta)\tilde{d_S}' \\ d'_T &= \tilde{d_T}' \end{cases}$$

While the bank operates, the bank is regulated at the time of investment decision is made such that it must hold sufficient bank capital as a fraction of risk-weighted assets. The capital requirement constraint is

$$q_\ell \tilde{\ell}' + q_a \tilde{a}' - q_S \tilde{d}'_S - q_T \tilde{d}'_T \geq \chi \omega_\ell q_\ell \tilde{\ell}'$$

where χ is the capital requirement and ω_ℓ is risk-weight specified by the regulation.

The objective of the bank is to maximize the expected present discounted value of future net dividend payouts with a valuation function, $\psi(e)$. Since the representative household owns the bank, future values are discounted using the same discount factor as the household. Before the beginning of the next period, every incumbent bank faces an exogenous exit shock, $1-\pi \in (0,1)$, which prevents a bank from accumulating a large enough capital to stop issuing the deposits. The exited banks are replaced with the entrants of the same mass with zero net worth.

The value of an incumbent bank is given by

$$V^0(n, \delta; \mu, \mathcal{Q}) = \max_{\tilde{\ell}', \tilde{a}', q_S, q_T, e, \mathcal{D}} \mathcal{D} - e - \psi(e) + \mathbb{E}_\theta \left[V^1(\tilde{\ell}', \tilde{a}', \tilde{d_S}', \tilde{d_T}', \delta, \theta) \right] \tag{8}$$

subject to

$$q_\ell \tilde{\ell}' + q_a \tilde{a}' + \mathcal{D} \leq n + q_S \tilde{d_S}' + q_T \tilde{d_T}' + e$$

$$q_\ell \tilde{\ell}' + q_a \tilde{a}' - q_S \tilde{d_S}' - q_T \tilde{d_T}' \geq \chi q_\ell \tilde{\ell}'$$

$$\mathcal{D} + e = \mathbf{1}\{\mathcal{D} \geq 0\} \mathcal{D} + (1 - \mathbf{1}\{\mathcal{D} \geq 0\}) e$$

$$\tilde{d_S}' = \tilde{d_S}'(q_S, q_T, \delta; \mu, \mathcal{Q}) \;\; \& \;\; \tilde{d_T}' = \tilde{d_T}'(q_S, q_T, \delta; \mu, \mathcal{Q})$$

and

$$V^1(\tilde{\ell}', \tilde{a}', \tilde{d_S}', \tilde{d_T}', \delta, \theta) = \beta \pi \mathbb{E} V^0(n', \delta'; \mu', \mathcal{Q}') \tag{9}$$

subject to

$$n' = \ell' + a' - d'_S - d'_T$$

$$\ell' = \tilde{\ell}' - \iota(\tilde{a}', \tilde{d}'_S, \theta)(\theta \tilde{d}'_S - q_a \tilde{a}')/(\omega q_\ell)$$

$$a' = (1 - \iota(\tilde{a}', \tilde{d}'_S, \theta))(\tilde{a}' - \theta \tilde{d}'_S / q_a)$$

$$d'_S = (1 - \theta) \tilde{d}'_S$$

$$d'_T = \tilde{d}'_T$$

$$\iota(\tilde{a}', \tilde{d}'_S, \theta_T) = \begin{cases} 0\,, & \text{if } \;\theta \tilde{d}'_S \leq q_a \tilde{a}' \\ 1\,, & \text{if } \;\theta \tilde{d}'_S > q_a \tilde{a}' \end{cases}$$

where the value function, V^0, is defined over the individual state $\{n, \delta\}$. μ is the distribution of banks and $\mathcal{Q} = \{q_{S,j}, q_{T,j}\}_{j=0}^{1}$ is a set of deposit prices offered by banks in the economy. $\mathcal{Q}$ is now the aggregate state variable because each bank has a market power in the deposit market and competes with each other to attract deposits from the household.

3.4 Distribution of banks

The distribution of heterogeneous banks is defined over $m \equiv \{n, \delta\}$. The evolution of the distribution is summarized as a functional operator $T\mu$ and it is given by

$$
\begin{aligned}
\mu'(\hat{n}', \delta') &= T\mu \\
&= \pi \Pi_\delta(\delta'|\delta) \Bigg[\int \mathbf{1} \Big\{ g_\ell(m) = \tilde{\ell}', g_a(m) = \tilde{a}', \tilde{d}_S' = \tilde{d}_S'(g_{q_S}(m), g_{q_T}(m)), \\
&\tilde{d}_T' = \tilde{d}_T'(g_{q_S}(m), g_{q_T}(m)) \Big\} \times \Pi_\theta \mathbf{1} \left\{ n' = \hat{n}' | \theta, g_\iota \right\} \mu(m) \mathrm{d}m \Bigg] \\
&+ \mathbf{1} \left\{ \hat{n}' = 0 \;\&\; \delta' = \min(\delta) \right\} (1 - \pi) \mu
\end{aligned} \tag{10}
$$

for all $\{\hat{n}', \delta'\}$. Π_δ and Π_θ are the density of δ and θ, respectively. $\mathbb{1}\{\cdot\}$ is an indicator function that produces 1 when the statement is true. $\{g_\ell, g_a, g_{q_S}, g_{q_T}\}$ are the optimal policy functions for loans, securities, savings deposit price, and time deposit price. The stationary distribution at the steady state satisfies, $\mu = T\mu$.

3.5 Government

The government in this economy levies lump-sum tax (τ, transfer if negative) from the representative household to elastically supply the government bonds to banks. Define $A_B \equiv \int a'(n, \delta') d\mu(n, \delta')$ as the aggregate demand for government bonds. The equation for the balanced budget is $q_a' A_B' + \tau = A_B$. The required tax to meet the aggregate demand is given by

$$
\begin{aligned}
\tau(A_B, A_B', q_a') &= A_B - q_a' A_B' \\
&= \int a'(n, \delta') d\mu(n, \delta') - q_a' \int a''(n, \delta') d\mu'(n, \delta')
\end{aligned}
$$

In the stationary equilibrium, $A_B' = A_B$ and $q_a = q_a'$,

$$
\tau(A_B, q_a) = (1 - q_a) A_B \tag{11}
$$

3.6 Timing

In any period, the following stages occur.
Stage 1. Given the distribution of the bank μ, exogenous exit shock $(1 - \pi)$ is realized.

- Entrants replace exited banks with net worth, $n_e = 0$.
- Incumbent banks start operating with pre-determined net worth.

Stage 2.

- Banks: given their net worth and liquidity capacity, banks strategically allocate resources by choosing loans, securities, deposits, and determining dividend/equity issuance, $(\tilde{\ell}', \tilde{a}', \tilde{d}'_S, \tilde{d}'_T, e)$, expecting the withdrawal risk during the interim period.

- Households: given the cash-on-hand ($\equiv w + D_S + D_T + \Pi_B + \Pi_F - \tau$), households choose consumption and make deposits across banks.

- Firm: hire labor to produce and demand loans after repaying outstanding loans.

Stage 3. Withdrawal shock θ is realized and banks carry out re-balancing assets to meet the withdrawal request.
Stage 4. At the start of the next period, assets and deposits mature thereby net worth is determined.

The same timing assumption of the model is depicted in Figure 3.

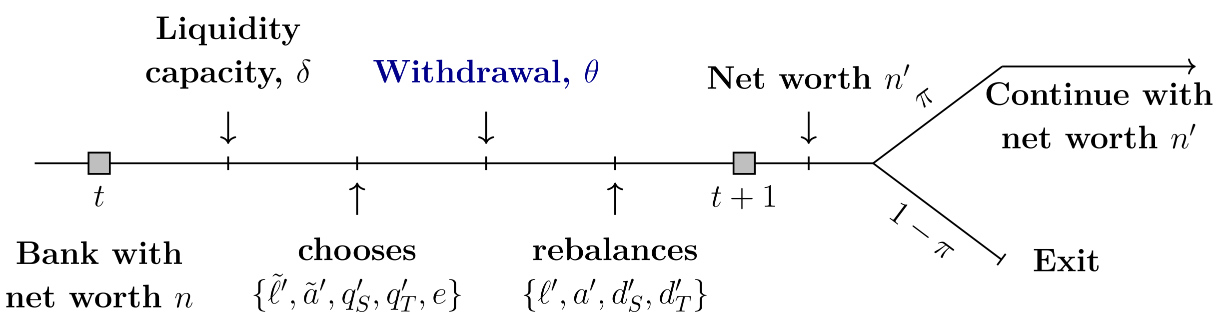


Figure 3: Timing of the bank problem

3.7 Definition of equilibrium

A stationary recursive equilibrium is a set of functions: 1) value functions, $\{V^0, V^1, V^H, V^F\}$, 2) policy functions, $\{g_\ell, g_a, g_{d_S}, g_{d_T}, g_e, \iota, \{d'_{S,j}\}, \{d'_{T,j}\}, C, Z, L, \{L_j\}, L'_F, N\}$, 3) Distribution, μ, and prices and aggregates, $\{L_B, A_B, D'_{S,B}, D'_{T,B}, w, \Pi_B, \Pi_F, \{P_{L,j}\}, P_L, P_Z, q_\ell, q_a\}$ such that

(i) $\{g_\ell, g_a, g_{q_S}, g_{q_T}, g_e, \iota\}$ solves the bank problem, 8 and 9.

(ii) Given deposit rate offers $\mathcal{Q}$, $\left\{\{d'_{S,j}\}, \{d'_{T,j}\}, C, L, \{L_j\}, Z\right\}$ solves the household problem, 1.

(iii) Firm solves 6 with associated policy functions, $\{L'_F, N\}$

(iv) Distribution of banks (μ) evolves following 10 and stationary distribution satisfies $\mu = T\mu$.

(v) Deposit market clears:

- savings deposits: $D_S = \int \tilde{d}'_S(g_{q_S}(n,\delta), g_{q_T}(n,\delta))\mathrm{d}\mu(n,\delta)$
- time deposits: $D_T = \int \tilde{d}'_T(g_{q_S}(n,\delta), g_{q_T}(n,\delta))\mathrm{d}\mu(n,\delta)$

(vi) Loan market clears: $L'_F = \int \tilde{\ell}'(n,\delta)\mathrm{d}\mu(n,\delta)$

(vii) Labor market clears given wage rate, $\bar{w}$ and the equilibrium hours worked is determined by firm's optimality condition.

(viii) Government's budget satisfies as in 11 that sustains the aggregate demand for the government bond at price q_a.

(ix) Loan market clears with a perfectly elastic demand at q_ℓ.

4 Model Implications

This section analyzes model implications. First, we see how withdrawal risk in savings deposits is related to bank capital. We discuss the optimality of the pecking-order decision rules. Next, in the household's problem. We show that the liquidity premium on deposits arises endogenously because the household values bank liquidities in the utility function. Finally, we describe the deposit demand function of the household.

4.1 The effect of withdrawal risk on the bank capital

At the beginning of the period, the bank's choices of assets and deposits define the maximal net worth for the next period, $\overline{n}'$,

$$\overline{n}' = \tilde{\ell}' + \tilde{a}' - \tilde{d}'_S - \tilde{d}'_T$$

Note that this is a hypothetical net worth that is attainable in scenarios where withdrawal shocks are absent. After observing the withdrawal shock, the realized net worth for the next period affected by re-balanced assets is

$$n'_0 \equiv n'|_{\iota=0} = \tilde{\ell}' + \tilde{a}' - \tilde{d}'_S - \tilde{d}'_T - \left(1 - \frac{1}{q_a}\right)\theta\tilde{d}'_S$$
$$n'_1 \equiv n'|_{\iota=1} = \tilde{\ell}' + \frac{q_a}{\omega q_\ell}\tilde{a}' - \tilde{d}'_S - \tilde{d}'_T - \left(1 - \frac{1}{\omega q_\ell}\right)\theta\tilde{d}'_S$$

Proposition 1 *The realized net worth after the withdrawal of saving deposits is smaller than the maximal net worth.*

Proof. see Appendix A. □

The loss in net worth because of withdrawal shock includes foregone interest income from the assets as they are sold earlier than their maturity. The loss can be large when the illiquid loans are used or the risk-free rate is high. Thus, in the low interest rate economy, the bank would not face much net worth loss. However, the loss can be great and the liquidity risk is elevated as the risk-free rate rises. When a bank faces a large withdrawal in savings deposits, of course, raises the loss more.

4.1.1 Is it optimal to follow the pecking-order decision rule?

The bank in the model economy is assumed to follow the pecking-order decision rule to satisfy the withdrawal request on savings deposits: government bonds come first to meet the withdrawal and if the current market value of government bonds is insufficient, loans are used with a partial recovery. This pecking-order decision rule implies a threshold policy for selling the loans. Given the choices $\{\tilde{a}', \tilde{d}'_S\}$ and the realization of withdrawal shock, θ, banks start selling loans only if $\theta \tilde{d}_S > q_a \tilde{a}'$. Therefore, we can define a threshold value of withdrawal shock as $\overline{\theta}(\tilde{a}', \tilde{d}'_S; q_a) \equiv \frac{q_a \tilde{a}'}{\tilde{d}'_S}$. The pecking-order decision rule can be described as

$$\iota(\tilde{a}', \tilde{d}_S, \theta) = \begin{cases} 0, & \text{if } \theta \leq \overline{\theta}(\tilde{a}', \tilde{d}'_S; q_a) \\ 1, & \text{if } \theta > \overline{\theta}(\tilde{a}', \tilde{d}'_S; q_a) \end{cases}$$

In the interim period after observing the withdrawal shock, paying out the withdrawal request by liquid government bonds reduces the net worth smaller than selling out illiquid loans to preserve the bank net worth next period if $\omega q_\ell < q_a$. In other words, selling the liquid assets first following the pecking-order rule can be optimal because both assets are one-period and capital requirement is already considered at the moment of choosing $\{\tilde{\ell}', \tilde{a}', \tilde{d}'_S, \tilde{d}'_T\}$. Therefore, the optimal policy of repaying the withdrawal of saving deposits is such that maximizes the net worth of the next period since the value function at the beginning of the next period is increasing in net worth [8]. To minimize the loss in the bank capital next period, it is optimal to hold sufficiently large securities so that banks can fully insure the risk of selling loans with discounts so there is a precautionary motive for purchasing liquid assets.

Corollary 1 *Loss in bank capital is greater for a unit increase in withdrawal of savings deposits when loans are used than meeting withdrawal using only securities,* $\left|\frac{\partial n'_0}{\partial \theta \tilde{d}'_S}\right| < \left|\frac{\partial n'_1}{\partial \theta \tilde{d}'_S}\right|$

[8]However, the pecking-order decision rule can be suboptimal in a more general environment. Later in this part, we are going to come back and check the optimality of the pecking order decision in the full-blown model (with different timing of capital requirement and/or long-term assets).

Proof. see Appendix A □

4.2 Endogenous liquidity premium and bank deposit demand

4.2.1 Liquidity service in utility function

The households value liquidity services in the utility function. Liquidity service consists of CES composite of savings and time deposits. The Euler equations from the household problem are

$$q_{S,j}\Omega = \beta\mathbb{E}[\Omega'] + \frac{\partial U}{\partial d'_{S,j}}$$
$$q_{T,j}\Omega = \beta\mathbb{E}[\Omega'] + \frac{\partial U}{\partial d'_{T,j}}$$

where Ω is the Lagrange multiplier for the budget constraint. The second term on the right-hand side of the Euler equations shows an additional value directly delivered to the utility function. Given that $\frac{\partial U}{\partial d'_{S,j}} > 0$ and $\frac{\partial U}{\partial d'_{T,j}} > 0$ and focusing on the stationary economy where $\Omega' = \Omega$, liquidity premium exists and so $q_{S,j} > \beta$ and $q_{T,j} > \beta$. Considering the ratio of two marginal utilities, we can compare the relative size of liquidity premium for each type of deposit.

$$\frac{\frac{\partial U}{\partial d'_{S,j}}}{\frac{\partial U}{\partial d'_{T,j}}} = \frac{\frac{\partial U}{\partial Z}\frac{\partial Z}{\partial L}\frac{\partial L}{\partial L_j}\frac{\partial L_j}{\partial d'_{S,j}}}{\frac{\partial U}{\partial Z}\frac{\partial Z}{\partial L}\frac{\partial L}{\partial L_j}\frac{\partial L_j}{\partial d'_{T,j}}} = \frac{\frac{\partial L_j}{\partial d'_{S,j}}}{\frac{\partial L_j}{\partial d'_{T,j}}} = \frac{\lambda_S^{\frac{1}{\epsilon}}(d'_{S,j})^{-\frac{1}{\epsilon}}}{(1-\lambda_S)^{\frac{1}{\epsilon}}(d'_{T,j})^{-\frac{1}{\epsilon}}}$$

If we have very large λ_S, then $q_{S,j} > q_{T,j}$ is available. If either type of deposit brings a larger marginal effect on liquidity service, then the liquidity premium is higher, which makes the discount price further from the risk-free price, β.

4.2.2 Bank-level deposit demand and allocating over distribution

The solution to the household problem in the stationary economy provides the deposit demand function for bank j as a function of cash-on-hand and discount prices of deposits.

$$d'^{*}_{S,j}(q_{S,j}, q_{T,j}, L_j) = \lambda_S(q_{S,j} - \beta)^{-\epsilon}\, \overline{q}_j(q_{S,j}, q_{T,j})L_j \tag{12}$$
$$d'^{*}_{T,j}(q_{S,j}, q_{T,j}, L_j) = (1-\lambda_S)(q_{T,j} - \beta)^{-\epsilon}\, \overline{q}_j(q_{S,j}, q_{T,j})L_j \tag{13}$$

where $\overline{q}_j(q_{S,j}, q_{T,j}) \equiv (\lambda_S(q_{S,j} - \beta)^{1-\epsilon} + (1-\lambda_S)(q_{T,j} - \beta)^{1-\epsilon})^{\frac{\epsilon}{1-\epsilon}}$.

Liquidity service from bank j is

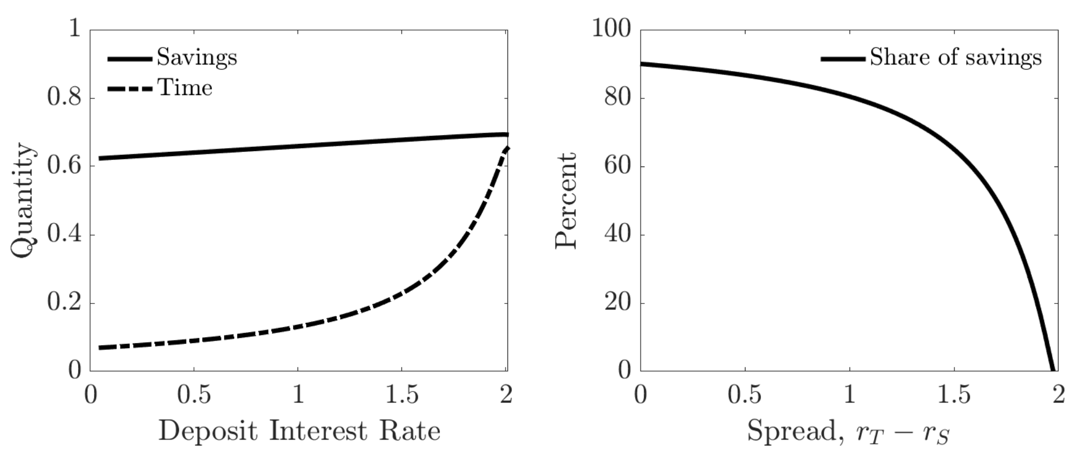
$$L_j^*(P_{L,j}, P_L, L; \delta_j) = \delta_j \left(\frac{P_{L,j}}{P_L}\right)^{-\nu} L \tag{14}$$

where $P_{L,j}$ is the price index for liquidity service from bank j and P_L is the price for aggregate liquidity as follows,

$$P_{L,j} = \left[\lambda_S(q_{S,j} - \beta)^{-\epsilon} q_{S,j} + (1 - \lambda_T)(q_{T,j} - \beta)^{-\epsilon} q_{T,j}\right] \overline{q}_j(q_{S,j}, q_{T,j}) \tag{15}$$

$$P_L = \left[\int \delta_j P_{L,j}^{1-\nu} \mathrm{d}j\right]^{\frac{1}{1-\nu}} \tag{16}$$

Note that bank-specific liquidity decreases in relative price, $P_{L,j}/P_L$ because banks compete for deposits and depositors optimally allocate bank-specific liquidity service based on its relative price. The bank distribution and a set of deposit prices determined by banks affect how much households allocate liquidity services across banks. Therefore, $\mathcal{Q} = \{q_{S,j}, q_{T,j}\}$ becomes a relevant aggregate state variable. Liquidity capacity, δ_j, scales L_j, and then each deposit demand level. This helps a bank to become large without charging a higher deposit rate than other banks [9].



Notes: For the expositional purpose, the Left panel fixes the interest rate for time deposit at 4% and the right panel sets the saving deposit rate at 0.04%. This figure uses $\{\lambda_S, \beta, \epsilon, \nu, \delta, L, P_L\} = \{0.9, 1/1.01, 0.2, 0.1, 1.0, 0.65, 1.05\}$.

Figure 4: Household deposit demand function

[9] Going back to data, we do not observe large banks in the U.S. attract depositors by offering high interest rates for deposits. Rather they put markdown on deposits with a stronger deposit market power as documented in Drechsler et al. (2021). The liquidity capacity term, hence, models attributes from large banks that attract customers and prevent turnover. For example, relationship, depositor's inertia (habit), expansions of branches, etc.

Figure 4 depicts a household's deposit demand function for an individual bank. Upper-left and bottom-right panel shows that deposit demand rises in their own interest rates. Since savings and time deposits are imperfectly substitutable, time deposit demand falls as the savings deposit rate increases as in the bottom-left panel. Likewise, savings deposit demand decreases in time deposit rate which can be found in the upper-right panel.

5 Mapping the Model to the Data

To quantitatively study the role of bank deposit structure in the aggregate economy, specific functional forms are assumed and we assign parameter values. Later, a set of parameters is going to be internally estimated for the calibration. For now, all parameters are externally set.

5.1 Withdrawal shock inferred from data

5.1.1 Model-based approach

Withdrawal of deposits is not directly observed in the data and so we recover and estimate the withdrawal shock from the bank's deposit flows. Before we use our model to recover the withdrawal shock, we need an assumption about the timing of observation of data because the model economy has interim period withdrawal shock and then the bank's balance sheets between the beginning and the end of the period are different. We assume that data reflects the end-of-period balance sheet of the bank, which enables us to get the information about realized withdrawal shock.

At the end of the period, for any withdrawal shock, $\theta \in \Theta$, the remaining balances for securities and savings deposits are

$$a' = \tilde{a}' - \frac{\theta \tilde{d}'_S}{q_a}$$
$$d'_S = (1-\theta)\tilde{d}'_S$$

From the model, we know that securities are demanded to insure against withdrawal shock and the realization of withdrawal shock changes the balances of securities and savings deposits. Thus, we consider their ratio to retrieve the realized withdrawal shock, a'/d'_S. Then,

$$\begin{aligned}\frac{a'}{d'_S} &= \frac{\tilde{a}' - \dfrac{\theta \tilde{d}'_S}{q_a}}{(1-\theta)\tilde{d}'_S} \\ \Rightarrow \frac{q_a a'}{d'_S} &= \frac{q_a \tilde{a}' - \theta \tilde{d}'_S}{(1-\theta)\tilde{d}'_S} \\ \Rightarrow (1-\theta)\frac{q_a a'}{d'_S} &= \frac{q_a \tilde{a}'}{\tilde{d}'_S} - \theta \end{aligned} \tag{17}$$

Let's call the ratio, $q_a a'/d'_S$, as operational liquidity ratio (OLR). Then the equation 17 relates operational liquidity ratios between the beginning and the end of the period to the realized withdrawal shock. Securities in the model are demanded to prevent the sale of loans provided that the recovery value of loans is strictly smaller than the market value of securities and the bank's value function is weakly increasing in bank capital. The optimal securities demand implies that

$$\begin{aligned} q_a \tilde{a}' &\geq \theta \tilde{d}'_S \\ \Rightarrow q_a \tilde{a}' &= \overline{\theta} \tilde{d}'_S \end{aligned} \tag{18}$$

where $\overline{\theta} \equiv \max \Theta$. Combining 17 and 18 yields

$$\begin{aligned} (1-\theta)\frac{q_a a'}{d'_S} + \theta &= \overline{\theta} \\ \Rightarrow \frac{\overline{\theta} - \theta}{1-\theta} &= \frac{q_a a'}{d'_S} \end{aligned} \tag{19}$$

Equation 19 can be used to recover θ given $\overline{\theta}$ and the operational liquidity ratio.

5.1.2 Withdrawal shock and kernel density estimation

For implementing the identification method using 19, let's re-express it again for observation level (bank(i), time(t)) subscript.

$$\frac{\overline{\theta}_{it} - \theta_{it}}{1-\theta_{it}} = \frac{q_{a,t} a_{it}}{d_{S,it}} \tag{20}$$

Based on 19, θ_{it} is identified into two steps. First, the upper bound of withdrawal shock is the largest value of the operational liquidity ratio. As a limit case, if $\theta_{it} = 0$, $\dfrac{q_{a,t} a_{it}}{d_{S,it}} = \overline{\theta}_{it}$. Note that the operational liquidity ratio is decreasing in θ_{it}. Then, we can think of the largest operational liquidity ratio as the case where a bank chooses large enough securities to cover the worst withdrawal shock but the actual realization is zero. Suppose that banks in the same period have the same upper bound, $\overline{\theta}_{it} = \overline{\theta}_t$. Then $\overline{\theta}_t = \max\left\{\frac{q_{a,t}a_{it}}{d_{S,it}}\right\}$. Next, we can directly use 20 to retrieve $\{\theta_{it}\}$ [10].

$$\theta_{it} = \frac{\overline{\theta}_t - q_{a,t}a_{it}/d_{S,it}}{1 - q_{a,t}a_{it}/d_{S,it}}$$

Finally, given the full set of withdrawal shock $\{\theta_{it}\}$, the cumulative density function, $F(\theta)$, is estimated using the kernel density estimation. In practice, we argue that the withdrawal shock distribution differs across the size classes. As in the previous part of the paper, we categorize the banks into three groups: large (top 0.1%), medium (top 10% - top 0.1%), and small (bottom 90%), $g = \{\ell, m, s\}$. Then, we have $\overline{\theta}_{gt}$ and the rest of the procedure is the same.

In reality, banks can hold liquid assets for other reasons. So, it may be unnatural to assume $q_a a'/d'_S = 0$ in any period. In other words, It would be more appropriate to think that it is not very likely that a bank faces $\theta = \overline{\theta}$. From a distribution perspective, $\overline{\theta}$ is the right-tail. So, we do not expect a meaningful non-zero measure there. Hence, we exclude the observation of $\theta = \overline{\theta}$ after computing the upper bound in the first step. Kernel density estimation is used to figure out the distribution of withdrawal shock, $F(\theta_g)$ [11]. The estimated kernel density shows a very small right-tail probability. We compare kernel density estimates with uniform distribution in Figure 5.

5.2 Functional forms

We assume the household's utility function is isoelastic and it is $U(Z) = \frac{Z^{1-\gamma}}{1-\gamma}$. Bank values dividend payout and equity issuance using $\psi(e)$. Dividend is valued linearly and equity issuance brings non-pecuniary cost. We assume the following functional form:

$$\psi(e) = \overline{\psi}e$$

As documented in Dempsey and Faria-e Castro (2022), this specification is widely used in dynamic corporate finance studies and helps to prevent a bank from issuing equity when the capital requirement is slack.

[10] $q_{a,t} = \frac{1}{1+r_{a,t}}$ holds and we use Federal Funds Rates with quarterly frequency for $r_{a,t}$.

[11] We use the Epanechnikov kernel function which is optimal in a mean squared error sense among many kernel functions. The bandwidth is a free parameter. The optimal width is selected to minimize the mean integrated squared error in normal densities. The kernel density estimates are evaluated at evenly-spaced grids with the minimum and the maximum value of $\{\theta_{it}\}$ as the lower and upper bounds for each asset group, g.

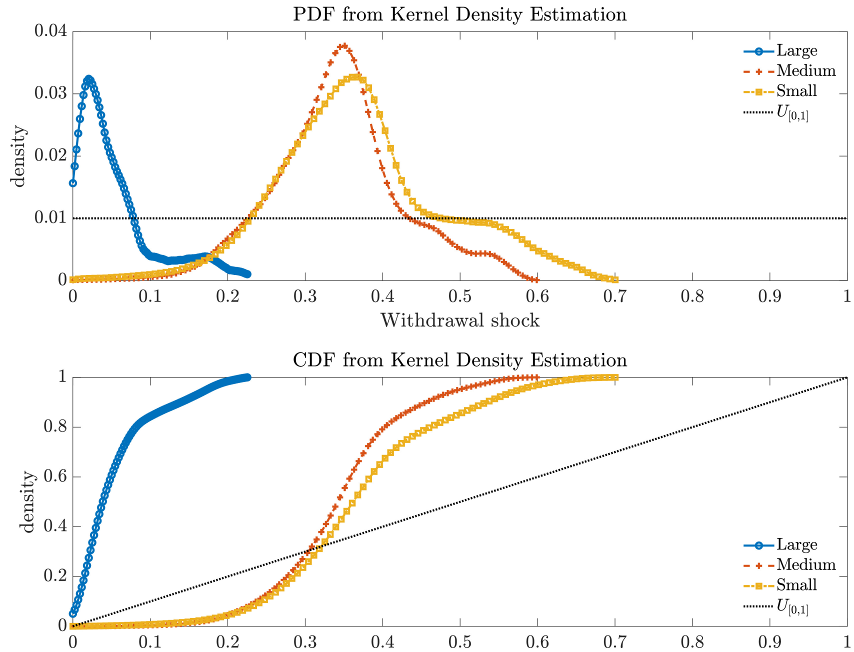


Figure 5: Kernel Density Estimates: PDF and CDF

Liquidity capacity follows

$$\delta = \delta_i \delta_p$$
$$\delta_p' = \mu_\delta + \rho_\delta \delta_p + \epsilon_\delta, \quad \epsilon_\delta \sim N(0, \sigma_{\epsilon,i}^2)$$

where δ_i is the permanent component of liquidity capacity and δ_p is a persistent component following AR(1) process. The shock has a type-specific term, $\sigma_{\epsilon,i}$, which helps to explain the size variations within each group of banks. Permanent component of liquidity capacity has three values, $\delta_i = \{\delta_\ell, \delta_m, \delta_s\}$ with subscript $\{\ell, m, s\}$ denotes large, medium, and small-sized banks. Permanent type of liquidity capacity captures the difference in near-permanent scale of each bank so that we can separate banks with economy-wide branches, regional banks, and community banks, for example.

5.3 Parameterization

5.3.1 External parameters

The model period is a quarter. A household's risk-aversion coefficient γ is set to 2 which is only used for solving the non-stationary economy because there is no uncertainty in the household's problem in the solution of the stationary equilibrium. Household's time discount factor, β is set to match with 4% of the annual risk-free interest rate. Then, we have $\beta = q_a = 1/(1+r_f)$. The capital requirement χ is 8% which is in line with current regulation for large bank holding companies in the U.S.. Bank exit probability is equal to the average quarterly bank failure rate, $1 - \pi = 0.72\%$. The loan price is set to target the annual loan rate is equal to 5%. The wage rate, $\overline{w}$, is normalized to 1. For the AR(1) term of bank liquidity capacity (δ_p), the coefficient targets the average bank deposit retention rate which is 84%. Deposit retention is considered as household weighing aggregate liquidity with the same δ_p. Thus, the average of diagonal element of transition matrix for δ_p is set to be 0.84. For the permanent type of bank, we make it δ_i have a mean equals to one. To this end, $p_\ell \delta_\ell + p_m \delta_m + p_s \delta_s = 1$ must hold. The fraction of each type of banks is based on the definition of large, medium, and small banks we used in data, $\{0.001, 0.099, 0.9\}$. The labor share in the production function; α is set to 0.75. The coefficient for working capital constraint, κ, is 3.0. The total factor productivity, $\overline{Z}$, is normalized to 1.

5.3.2 Internally determined parameters

There are parameters that are jointly determined by targeting the moments. The equity issuance cost, $\overline{\psi}$, helps to target the rate of equity issuance and net dividend payouts over bank net worth. Share of consumption in Z targets aggregate consumption-deposit rato. Other elasticities of substitution and share parameters are determined to match spread between savings and time deposits, variation of deposits and the interest rates, and the share of savings deposits in banking sector. The parameters for liquidity capacity process are calibrated to taget relative size of different type of banks and variations of deposits within asset classes. All parameter values used in the model are summarized in Table 2.

5.4 Solving the model

The banking industry equilibrium is solved by clearing the deposit market and loan market. Since the banks have market power over both types of deposits, households observe all deposit prices that banks offer and allocate deposits across bank distribution. This requires mapping the distribution of banks defined over net worth and liquidity capacity to the space of deposit price dimension. The household's bank-level deposit demand can be computed given the distribution of deposit rates offered by all banks. Appendix D contains the details of the computational algorithm to solve for the model.

6 Model Validations and Mechanism in Steady State

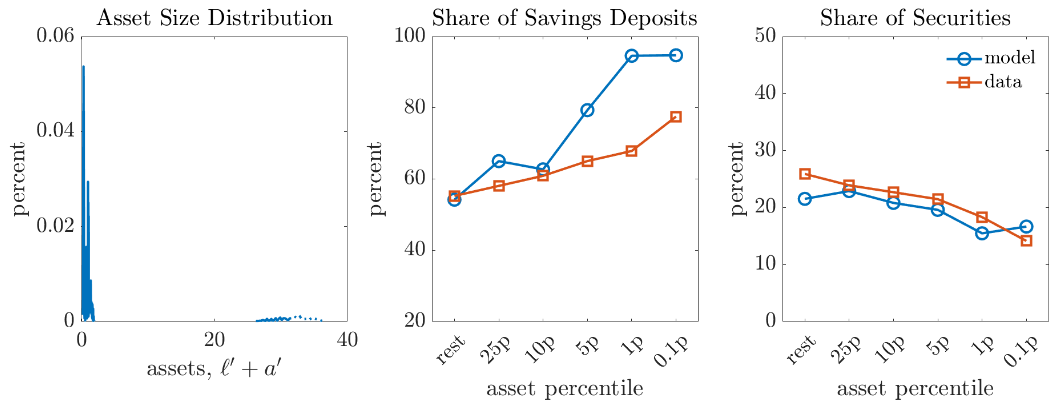
	Description	Value	Target	Model	Data
External Parameters					
χ	Capital requirement	0.08	U.S. bank regulation		
ω	Recovery rate for loans	0.8			
π	Survival probability	0.9928	Failure rate, 0.72%		
q_ℓ	Loan price	0.9756	Annual loan rate, 5%		
q_a	Gov't bond price	0.9901	Annual risk-free rate, 4%		
ρ_δ	AR(1) coefficient for δ_p	0.9127	Bank deposit retention rate, 84%		
p_l	fraction $\rho_i = \ell$ of banks	0.001	By definition		
p_m	fraction $\rho_i = m$ of banks	0.099	"		
p_s	fraction $\rho_i = s$ of banks	0.9	"		
α	labor share	0.75	standard		
κ	coeff. working capital	3.0	Debt to GDP		
$\overline{Z}$	Total factor productivity	1.0	normalization		
$\overline{w}$	wage rate	1.0	normalization		
γ	Risk aversion	2.0	standard		
Internal Parameters					
$\overline{\psi}$	Equity issuance cost	0.5	Bank leverage	0.926	0.877
λ_C	Share of consumption in Z	0.25	Average NIMs	0.042	0.018
λ_S	Share of savings deposits in L_j	0.9	Equity issuance / net worth	0.000	0.011
η	Elasticity of subst. of C and L	0.5	Net dividend/ net worth	0.139	0.058
ν	Elasticity of subst. of L_j	1.1	Consumption-deposit ratio	0.326	0.320
ϵ	Elasticity of subst. of $d_{S,j}$ and $d_{T,j}$	4.5	Spread between d_S and d_T	0.011	0.024
$\sigma_{\delta,g}$	SD of liquidity cap. ($g = \{s, m, \ell\}$)	{0.07,0.1,0.05}	SD of deposits	0.267	2.375
δ_g	Type of liquidity cap.	{0.941, 1.523, 1.843}	SD of deposit rates	0.0003	0.167
			Avg. share of savings deposits	0.558	0.667
			SD of deposits within $g = \ell$	0.211	0.621
			SD of deposits within $g = m$	0.365	1.347
			SD of deposits within $g = s$	0.138	0.738
			Size ratio, ℓ *to* m	1.383	1.235
			Size ratio, m *to* s	1.763	1.546

Table 2: Parameters and Targeted Moments

6.1 Empirical validations of the model

This section provides empirical validations of the model. Although the model is not fully calibrated, our model qualitatively matches the patterns of bank balance sheets across size classes. Figure 6 shows the bank balance sheet composition by the level of total assets, $\ell' + a'$. Based on the left panel of Figure 6, bank size distribution over assets is right-skewed and long right-tail.

In the middle panel of Figure 6, the model shows that asset-small banks hold a larger share of time deposits in the balance sheet which is consistent with the data. And this share is falling as banks become large. Recall that withdrawal of savings deposit reduces the bank capital next period. And holding securities is required not to sell loans in the middle of the period. Investment in loans that banks can raise their future net worth is feasible with a higher share of time deposits. As banks become large, they substitute time deposits for more savings deposits. Since the interest rate for savings deposits is lower than time deposits, it is desirable. Moreover, the type-specific withdrawal risk contributes to this pattern. For example, medium-sized banks face a lower and less dispersed withdrawal risk for a unit of



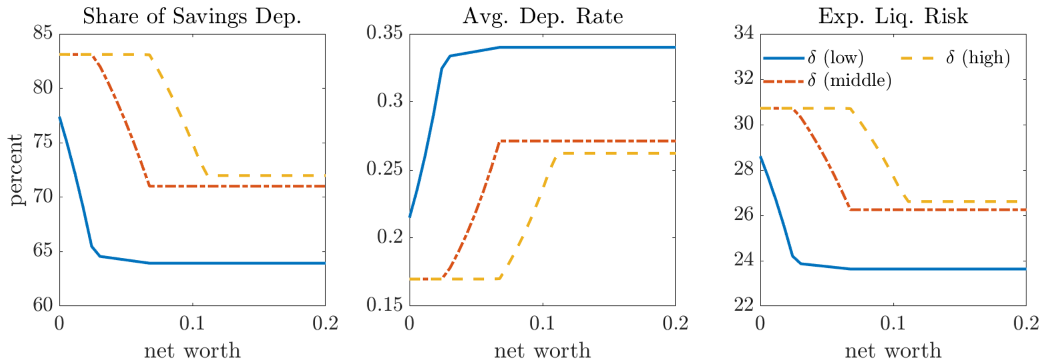
Notes: Asset distribution (left), $\tilde{\mu}(\ell' + a')$, is derived by mapping bank states to the choice set of assets, (ℓ', a'). Values for the share of savings deposits and securities in the balance sheet are calculated as a weighted average over asset distribution and expressed on asset percentile as in the data.

Figure 6: Asset Distribution and Bank Balance Sheet Composition by Size

savings deposit than small banks. This enables medium-sized banks to increase the share of savings deposits to reduce the average funding cost while maintaining or even reducing the securities holding. The share of securities in the asset side of the balance sheet is falling in size on the right panel of Figure 6. Despite a large share of savings deposits, a lower withdrawal risk for a unit of savings deposits in a large bank reduces an incentive to invest in securities. This pattern is also empirically observed, which features the same higher share of securities among small banks and a lower share in large banks.

6.2 Mechanism in the steady state

Bank deposit mix choice affects the liquidity risk of the bank and it changes investment decision. Distribution of heterogeneous liquidity risk and investment of banks shape the aggregate outcomes in the steady state. Firstly, endogenous liquidity risk of a bank stems from two components in the model, the withdrawal risk and deposit mix. Figure 7 shows the deposit mix choice by the share of savings deposits, the average interest rates of deposits, and expected liquidity risk. I measure the expected liquidity risk as the expected proportion of withdrawal in total deposits, $\mathbb{E}\left[\theta \mid \delta\right] \frac{d'_S}{d'_S + d'_T}$. This shows tradeoff between stability of funds and funding cost. The expected liquidity risk is increasing in the share of savings deposits. But the average interest rate for deposits is decreasing in the share of savings deposits. When the bank has a low net worth, keeping a high net interest margin by lowering the average interest rate with deposit market power is the optimal to accumulate bank capital. However, the bank has to face a higher expected liquidity risk because of a higher share of savings deposits. The expected liquidity risk is lessen in banks with large net worth by issuing more time deposits.



Notes: Share of savings deposits is $d'_{S,j}/D'_j$, Avg. Dep. Rate is the average interest rates for deposits (annualized), and the Exp. Liq. risk is the expected liquidity risk that measures expected proportion of withdrawal in total deposits. This figure is plotted for medium-sized banks.

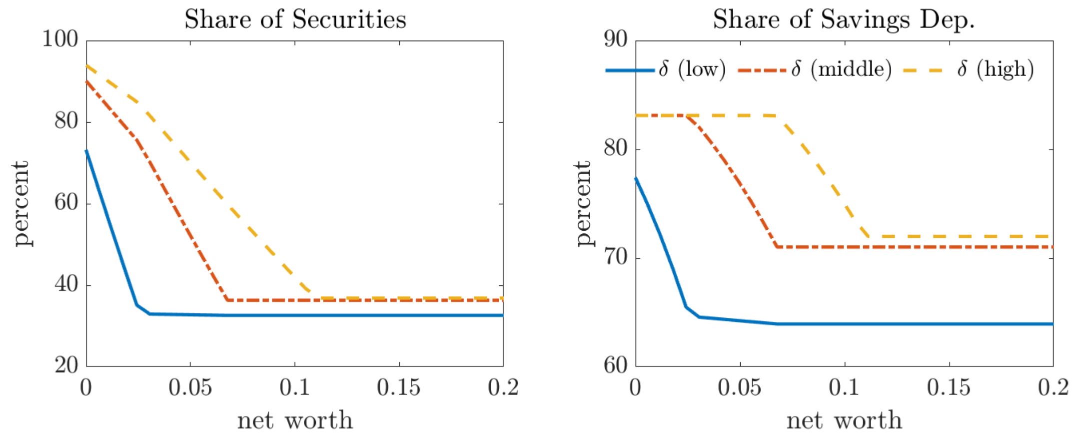
Figure 7: Deposit Mix, Interest Rates, and Liquidity Risk

Deposit mix and endogenous expected liquidity risk also affect the asset portfolio of the bank. Figure 8 depicts the relationship between deposit mix and asset portfolio. The share of securities in the asset portfolio is increasing in the share of savings deposits. The bank has to hold more securities to insure against withdrawal risk when the share of savings deposits is high. This is because the bank has to sell securities to cover the withdrawal of savings deposits without a firesale of loans.

Banks optimally insure against withdrawal risk by holding securities and issuing time deposits across different level of net worth. When the bank's net worth is small, the bank has to hold more securities to insure against withdrawal risk. The bank with a large net worth can bear a higher funding cost so that it issues more time deposits to reduce the proportion of deposits being exposed to the withdrawal risk. The bank can hold fewer securities to insure against withdrawal risk.

Deposit mix choice changes the asset portfolio. The bank with a higher share of savings deposits has to hold more securities to insure against withdrawal risk. As the bank grows in net worth, the bank can issue more time deposits and it further reduces the share of securities in the asset portfolio. In Figure 8, the slope of share of securities is steeper as the bank starts issuing more time deposits. Therefore, bank can not only reduce the share of deposits being exposed to the withdrawal risk but also reduce the amount of securities to insure against withdrawal risk. In other words, the bank can hold more loans in the asset portfolio.

For the aggregate outcome in the steady state, the liquidity risk is heterogeneous across banks and its distribution is important. The stationary distribution of banks is defined



Notes: Share of securities is $a'_j/(\ell'_j + a'_j)$ and the share of savings deposits is $d'_{S,j}/D'_j$.

Figure 8: Deposit Mix and Asset Portfolio

over net worth and liquidity capacity. The distribution of liquidity risk is endogenously determined by the distribution of bank's net worth and liquidity capacity. In Figure 7, the expected liquidity risk is decreasing in net worth. And the withdrawal shock distribution from Figure 5 differs across the size classes that is affected by liquidity capacity. Then, the model have a distribution of heterogeneous liquidity risk across banks. Figure 9 shows the distribution of expected liquidity risk. Given the withdrawal shock distribution and bank's optimal policy of deposit mix, roughly 25-30 percent of total deposits are expected to be withdrawn in the middle of a period. Whereas, large bank group has a lower expected liquidity risk only 10 percent of total deposits are expected to be withdrawn. This implies that banks face different level of liquidity risk per unit of deposits. Then, combining the result from the Figure 8 provides that heterogeneity of liquidity risk and its distribution result in aggregate asset portfolio of the banking sector.

7 Aggregate Role of Bank Deposit Mix

Banks mix deposit products and liquidity risk is endogenously determined. Given the heterogeneity in liquidity risk, the bank's optimal choice of deposit mix affects the asset portfolio. To understand the aggregate role of the bank's deposit mix choice, I consider an alternative economy with banks facing the same withdrawal risk. The alternative economy is called uniform risk economy. The uniform risk economy is the one where the banks face the same withdrawal risk as the small banks. The optimal choice of deposit mix is changed as the underlying withdrawal risk associated with savings deposit is changed. Aggregate variables of the baseline economy is then compared with these alternative economies.

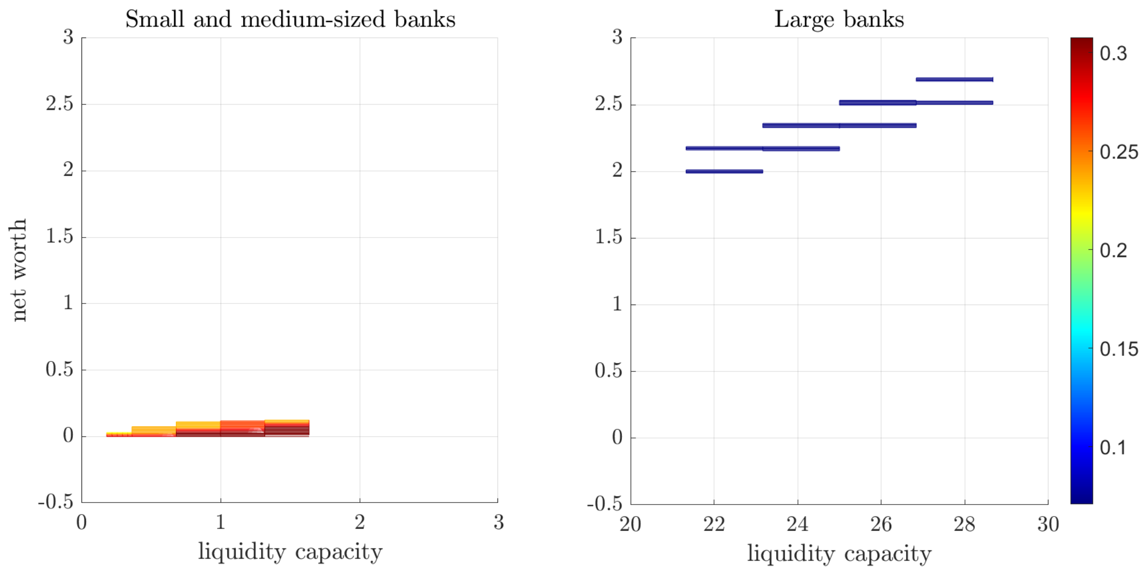


Figure 9: Distribution of Expected Liquidity Risk

7.1 Deposit mix and aggregate outcomes in the long run

Deposit mix does two things in the economy. First, it determines the average funding cost as two deposits have different interest rates. Second, it affects the liquidity risk of the bank by choosing the share of deposits exposed to the withdrawal shock. Alternative economy with uniform risk puts a higher withdrawal risk for large and medium-sized banks than the baseline economy. By comparing the baseline with uniform risk economy, deposit mix is associated with a change in the volume and composition of assets, and the change in distribution of credit creation.

Large and medium-sized banks would have an incentive to issue time deposits and demand more securities to insure against increased withdrawal shock. Table 3 shows the increased time deposit issuance in aggregate in the uniform risk economy. For a bank, a rise in the share of time deposits is associated with a lower expected liquidity risk and helps them to reduce the share of securities in the asset portfolio. Figure 10 shows that the small and medium-sized banks' liquidity risk is lowered. The large banks' liquidity risk is increased but given that uniform risk economy puts the same withdrawal risk with small banks for large, the liquidity risk is reduced as large banks issue more time deposits.

A large issuance of time depoists in the uniform risk economy is associated with a higher equilibrium interest rate for time deposits, which increases the average funding cost for banks and it lowers the bank's net worth in steady state. At the same time, household save less because of income effect from higher return of savings. As a result, the balance sheet of banking industry shrinks. In Table 3, aggregate loan is reduced relative to the baseilne economy because large banks face a higher liquidity risk and the balance sheet of banking

Economy		**Baseline**	**Uniform Risk**	
Unit		*level*	*level*	*%Δ*
Loans	L	0.934	0.925	-1.0
Securities	A	0.185	0.150	-19.1
Savings	D_S	0.781	0.366	-53.2
Time	D_T	0.254	0.627	146.7
Output	Y	0.417	0.414	-0.7
Labor	N	0.311	0.308	-1.0
Net Dividend	$\mathbb{D} - E$	0.0059	0.0056	-6.1
Tax	τ	0.00199	0.00198	-0.5
Interest rates				
Savings	r_S	0.1	0.1	0
Time	r_T	0.77	0.92	15
Loan	r_ℓ	2.55	2.90	35

Notes: %Δ denotes the percentage change from the baseline economy. For interest rates, the percentage change is in basis points.

Table 3: Aggregate Outcomes: Baseline vs. w/ Uniform Withdrawal Risk

sector is reduced. Therefore, the loan rate is increased by 35 basis points which reduces the available funding for the firm. Hence, the aggregate output is reduced in the uniform risk economy. A larger dependence on time deposits in the uniform risk economy helps banks to reduce the liquidity risk but it also increases the funding cost and decreases the deposit base.

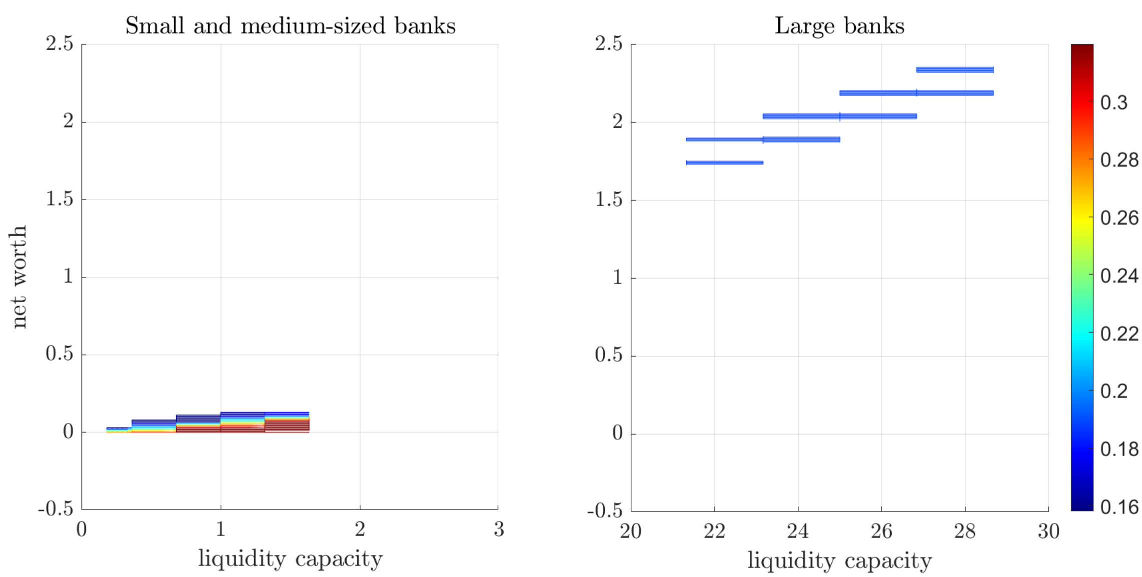


Figure 10: Distribution of Expected Liquidity Risk: Uniform Risk

Based on Table 4, the large banks reduces the share of savings deposits the most. They need to issue more time deposits not only because of the chagne in the withdrawal risk but also to maintain te size. Since they need to invest in loans to get a high enough return to accumulate net worth, large banks face a higher funding cost and a larger dependence on time depoits than other banks. This further lowers the net worth of large banks, so, large bank's share in the asset market is reduced as in Table 5.

	Baseline			Uniform Risk		
	small	medium	large	small	medium	large
Loans	0.836	0.815	0.854	0.881	0.848	0.865
Savings deposits	0.422	0.614	0.946	0.297	0.413	0.355

Table 4: Balance Sheet Composition by Size Groups

	Baseline			Uniform Risk		
	small	medium	large	small	medium	large
Assets	0.143	0.351	0.503	0.158	0.391	0.452
Deposits	0.141	0.354	0.504	0.156	0.391	0.453

Table 5: Share of Assets and Deposits by Size Groups

There is an equilibrium effect on loan market which has an implication for the concentration of the banking industry. Decrease in loan supply by large banks put an increasing force for loan rates. Small and medium-sized banks lend out more with given the increased loan rate. This incentivizes small and medium-sized banks issue more time deposits further to finance more loans. As these banks lowers the share of savings depoits in the balance sheet, demand for securities is reduced, which also explains why the securities in the uniform risk economy is smaller than the baseline. Relative to a large change of deposit product composition in the deposit market, the equilibrium volume of loans is not much changed. Nontheless, there is a change in distribution of credit creation. The large banks reduce the share and medium-banks mostly fill the gap by increasing the loan supply. Therefore, the bank's asset market becomes less concentrated.

7.2 Withdrawal risk, deposit mix, and financial stability

To understand the role of deposit mix and heteroegneous liquidity riks on the aggregate economy in the short-run, I consider the impulse response of the aggregate economy to an aggregate shock to bank net worth. The shock is a 10% decrease in the net worth of all banks. The impulse response is shown in Figure 11.

The adverse shock to bank net worth proportionally decrease the deposit issuance and asset creation in the banking sector. The shock is transmitted to the real economy through the credit channe. The loan rate is increased by more than 100 basis points and the output is droppbed by 2.5 percent from the steady state. Mostly by the increase in the loan rate, net interest margins of banks are increased after the shock which helps banks to recover net worth over time.

The impulse response of the balance sheet composition is shown in Figure 12. In response to adverse net worth shock, banks issue more time deposits while reducing the savings deposits. This helps to decrease the share of deposits exposed to withdrawal risk. The lowered share of savings deposits allows banks to lower the share of securities in the asset portfolio. Since loans are yielding a higher return than securities, bank can recover the net worth more effectively with loan investment. Loans are not much responsive to adverse net worth shock.

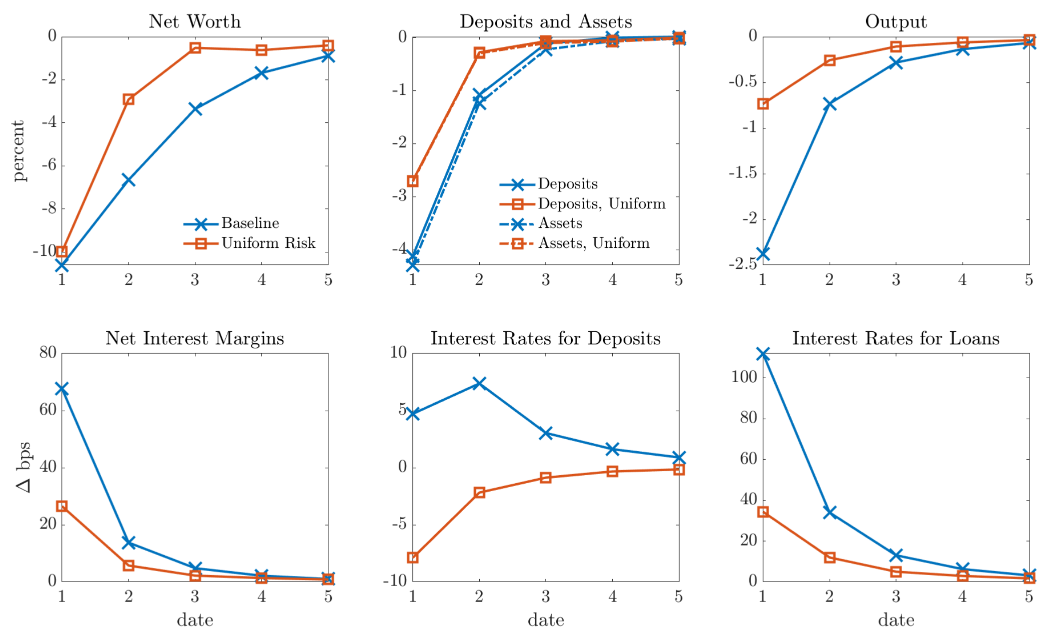


Figure 11: Impulse Response to Aggregate Shock to Bank Net Worth

Comparing the baseline economy with the uniform risk economy, baseline economy shows a slower recovery in the net worth although net interest margins are higher. This occurs mainly because of a larger decline in loan supply in the baseline economy. Note that uniform risk economy has a higher withdrawal riks overall and the banking sector features a large issuance of time deposits in the steady state. Thus, the same size of net worth shock is less severe in the uniform risk economy, especially, medium-sized banks face a lower liquidity risk than the baseline economy and have a larger role in supplying the loans. This change in the distribution of credit creation also helps the loan market not being affected a lot by adverse net worth shock, which eventually results in a smaller decline in the output.

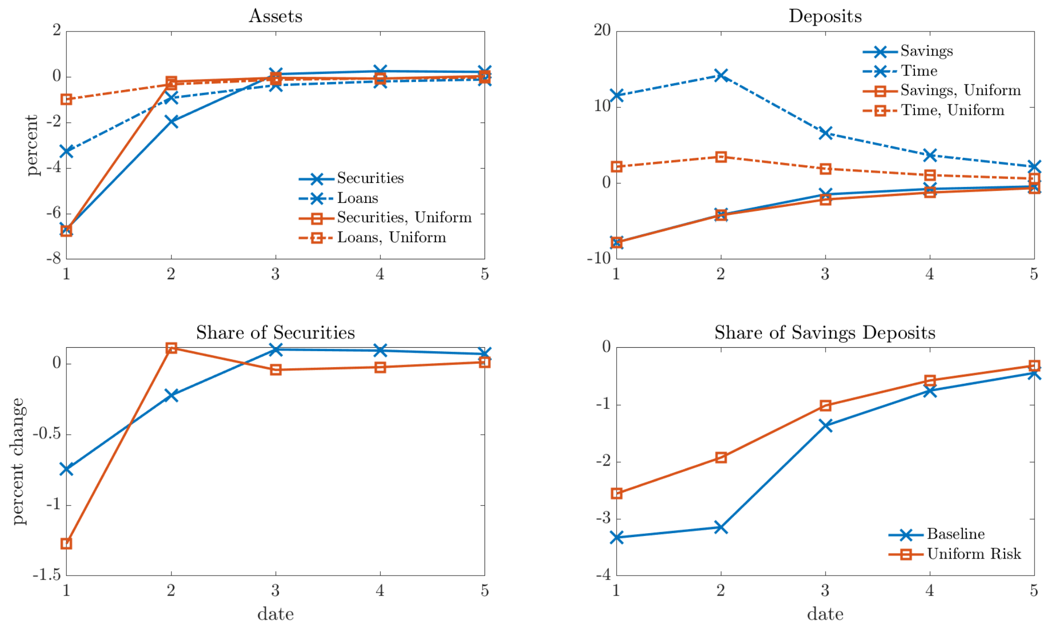


Figure 12: Impulse Response to Aggregate Shock to Bank Net Worth, Balance Sheet Composition

Interestingly, the role of deposit mix and heterogeneous liquidity risk can vary over time. A large dependence on time deposits given a high withdrawal risk in the economy can be a source of financial stability in the short-run. The bank can recover the net worth more effectively with loan investment. However, the large dependence on time deposits can be a source of financial instability in the long-run. The increased funding cost and the reduced deposit base lead to a smaller balance sheet of the banking sector and a lower loan supply.

7.3 Liquidity Requirement in the Banking Sector

The liquidity requirement is a regulation that requires banks to hold a certain amount of liquid assets to meet the withdrawal demand. The liquidity coverage ratio (LCR) is a regulation that requires banks to hold high-quality liquid assets (HQLA) to cover the net cash outflows over a 30-day period. The LCR is a part of Basel III, which is a comprehensive set of reform measures designed to strengthen the regulation, supervision, and risk management within the banking sector. The LCR is designed to improve the banking sector's ability to absorb shocks arising from financial and economic stress, whatever the source, thus reducing the risk of spillover from the financial sector to the real economy.

Distribution of heterogeneous banks and their deposit mix choice has direct implications of different degree of liquidity risk that each bank faces. In this paper, I further discuss the effect of introducing the liquidity coverage ratio in the banking sector. I extend the model so that individual bank faces liquidity requirement at the time of investment decision.

A bank faces the following liquidity requirement constraint:

$$\varrho\left(q_S\tilde{d}'_S + q_T\tilde{d}'_T\right) \leq q_a\tilde{a}' + \pi(\theta = \overline{\theta})$$

where ϱ is the liquidity coverage ratio. The model period is a quarter, therefore, this value for the model is 1/3. Define $\pi(\theta = \overline{\theta})$ as the expected cash flow at the beginning of the next period when the largest withdrawal is realized, which is the stress scenario in the model. The cash flow is the sum of interest income net of interest cost and it also differs depending on the bank's choice of the sale of loans in the middle of a period. Hence, in the case when a bank hold enough securities to sell to fulfull the withdrawal demand, the cash flow is

$$\begin{aligned}\pi = &\left(\frac{1}{q_\ell} - 1\right) q_\ell\tilde{\ell}' + \left(\frac{1}{q_a} - 1\right)\left(q_a\tilde{a}' - \overline{\theta}\tilde{d}'_S\right) \\ &- \left(\frac{1}{q_S} - 1\right)(1-\overline{\theta})q_S\tilde{d}'_S - \left(\frac{1}{q_T} - 1\right) q_T\tilde{d}'_T\end{aligned}$$

In the case when a bank does not hold enough securities to sell, the cash flow is

$$\begin{aligned}\pi_t = &\left(\frac{1}{q_{\ell,t}} - 1\right)\left\{q_{\ell,t}\tilde{\ell}_{t+1} - \left(\overline{\theta}_x\tilde{d}_{S,t+1} - q_{a,t}\tilde{a}_{t+1}\right)\big/\omega\right\} \\ &- \left(\frac{1}{q_{S,t}} - 1\right)(1-\overline{\theta}_x)q_{S,t}\tilde{d}_{S,t+1} - \left(\frac{1}{q_{T,t}} - 1\right) q_{T,t}\tilde{d}_{T,t+1}\end{aligned}$$

In this extended model, I ask the following questions: can the current liquidity coverage ratio requirement be effective in reducing the liquidity risk in the banking sector? How does the liquidity requirement affect the bank's balance sheet composition and the aggregate economy?

7.3.1 Who is affected by the liquidity requirement?

The liquidity coverage ratio requirement sets a minimum level of high-quality liquid assets. The effect on the bank's demand for liquid assets can be different as banks with different size type face different withdrawal risk. The right-panel of Figure 13 shows the share of securities in the balance sheet by the size of banks. Compared with the baseline economy, the liquidity requirement raises the large banks holding of securities. This implies that the volume of loan supply by large banks is reduced. On the other hand, the liquidity requirement is not binding for small and medium-sized banks. These banks can increase the loan supply to replace the large banks. To finance more loans, small and medium-sized banks issue more time deposits, which is reflected in the decline in the share of savings deposits in the balance sheet on the left-panel of Figure 13. Therefore, the liquidity requirement reduces the deposits that are exposed to the withdrawal risk by indirectly affecting the deposit mix choice of small and medium-sized banks.

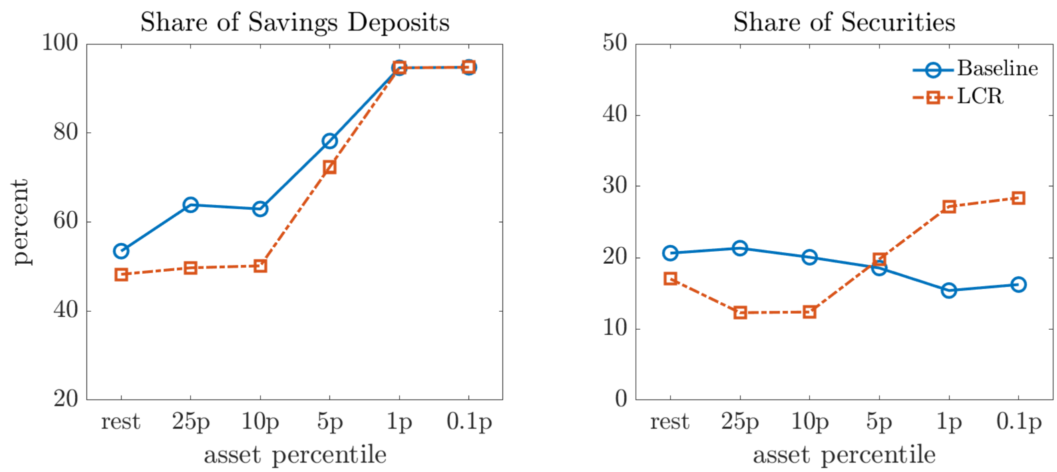


Figure 13: Balance Sheet Composition: Baseline vs. Liquidity Coverage Ratio

Small and medium-sized banks reduces the expected liquidity risk by issuing less savings deposits. Left-panel of Figure 14 shows the changed liquidty risk compared with the Figure 9.

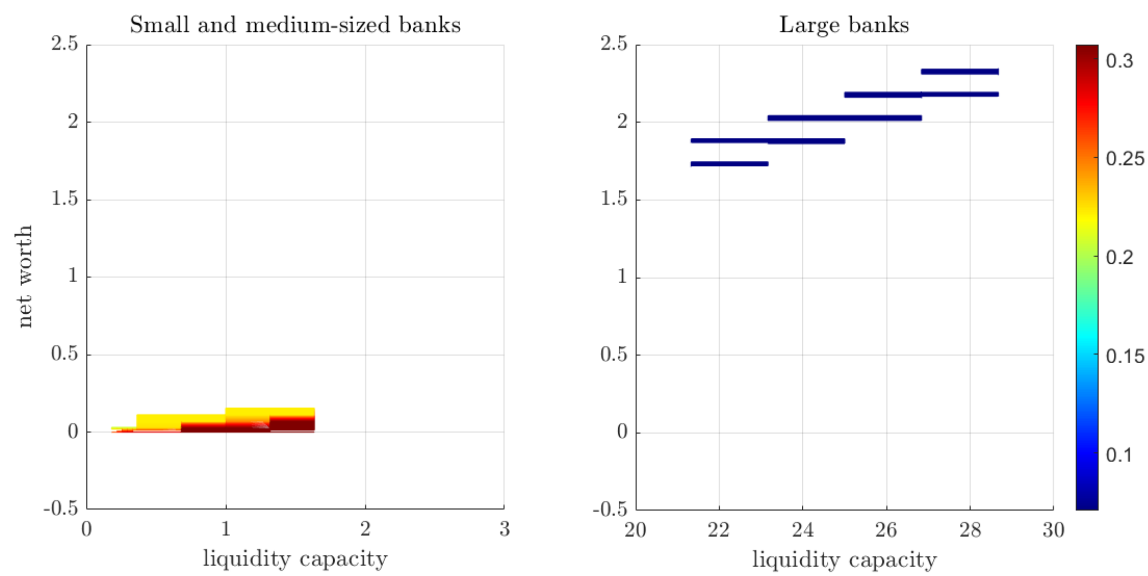


Figure 14: Distribution of Expected Liquidity Risk, LCR

7.3.2 How much does the liquidity requirement affect the aggregate economy?

The implementation of liquidity requirement is to enhance the resilience of banking industry to shocks. This can be effectively achieved by requiring banks to hold enough liquid assets to prepare for short term obligations in liabilities with stress scenario. Here, I provide alternative long-run effect of liquidity coverage ratio which is less explored by the literature and policy markers.

Since the LCR directly influence the bank's investment of liquid assets, the liquidity regulation has an long run implication for industry concentration. Large banks lose the asset share by 1.7 percent and medium-sized banks gain the asset share by 2.1 percent. As a result, the banking industry is less concentrated.

	Baseline			LCR		
	Small	Medium	Large	Small	Medium	Large
Assets	0.143	0.351	0.503	0.142	0.372	0.486
Deposits	0.141	0.354	0.504	0.140	0.365	0.495

Table 6: Share of Assets and Deposits of Size Groups: Baseline vs. LCR

In the steady state, LCR lowers the large banks' role as a loan supplier replacing this with medium-sized banks. However, the aggregate loan supply is reduced by 4.1 percent. Large banks are more efficient in supplying loans because a unit of savings deposit is less risky than other banks. Letting medium-sized banks to supply more loans is not enough to offset the reduction in loan supply by large banks because medium-sized banks face a higher liquidity risk.

Economy		**Baseline**	**LCR**	
Unit		*level*	*level*	$\%\Delta$
Loans	L	0.93	0.90	-4.1
Securities	A	0.19	0.22	18.4
Savings	D_S	0.78	0.72	-7.4
Time	D_T	0.25	0.30	18.1
Output	Y	0.417	0.404	-3.1
Labor	N	0.311	0.299	-4.1
Net Dividend	$\mathbb{D} - E$	0.0059	0.0089	49.8
Tax	τ	0.0019	0.0020	1.5
Interest rates				
Savings	r_S	0.1	0.1	0
Time	r_T	0.77	0.82	5
Loan	r_ℓ	2.55	4.02	147

Notes: $\%\Delta$ denotes the percentage change from the baseline economy. For interest rates, the percentage change is in basis points.

Table 7: Aggregate Outcomes: Baseline vs. w/ LCR

The economy has more liquid assets with a rise in securities demand among large banks.

Small and medium-sized banks issue more time deposits and the banking sector has 18 percent more of time deposits. A lower supply of loans in the economy props up the interest rate for loans by 147 basis point. Hence, the available funds for firm are reduced and the interest cost is high. The aggregate output is reduced by 3.1 percent as a higher loan rate pushes down the labor demand of the firm. The liquidity coverage ratio, in the long run, has an adverse effect on the aggregate economy by counteracting the advantage of the low withdrawal risk of large banks.

The long-run output cost provides a room for the optimal liquidity coverage ratio and size-dependent liquidity requirement. Particualrly, the withdrawal risk of large banks is lower than other banks and it is advantageous for the economy to let large banks to hold less liquid assets. One of candidates of size-dependent liquidity coverage ratio is adjusting stress scenario for each bank. A moderate stress scenario for large banks and a severe stress scenario for small and medium-sized banks can be a solution to reduce the long-run output cost.

8 Concluding Remarks

This paper studies the effect of heterogeneous bank deposit mix and liquidity risk on financial stability in the aggregate economy. Bank deposits are the main source of funds for banks to operate liquidity transformation. Most banks mix different types of deposit products rather than issuing a single type. Savings deposits are relatively volatile but pay lower interest rates than time deposits. Large banks issue savings deposits which take up more than 80% of total deposits in the balance sheet, whereas the dependence on time deposits is increasing as banks become smaller. The bottom 90% holds 40% of the total deposits in time deposits on the balance sheet. Looking at the outflows of savings deposits, large banks face smaller outflows on average and less frequent large outflows than small banks. With the free withdrawal of savings deposits, banks optimally choose a deposit mix considering the tradeoff between the interest cost of deposits and the degree of stability in funding flows.

I build an macroeconomic model with banking industry dynaimcs and heterogeneous banks with liquidity risk. Savings deposits are a cheaper source of external finance for banks at the expense of withdrawal risk in the middle of a period. Withdrawal of savings deposits incurs asset re-balancing and potentially reduces the bank capital and this provides an incentive for a bank to demand liquid assets. Banks can avoid it by issuing costly time deposits. Loans are illiquid in the sense that they accompany large discounts when sold before maturity. Therefore, banks match liquidity across assets and liabilities.

The model successfully generates qualitative features of bank balance sheet patterns for deposit composition and asset composition across different sizes. Withdrawal risk depending on the size of bank is important to produce a declining share of securities and a rising share of savings deposits. A steady-state comparison with a uniform withdrawal risk economy is used to study the model mechanism. An increased share of time deposits because of an increased witdrawal risk makes banks raise the share of loans in the asset portfolio. However, increased average funding cost and reduced deposit base downsize the banking sector. Large banks extend loans less in the uniform risk economy and small and medium-sized banks can take advantage of increased loan rate to increase the loan supply in equilibrium. Nonetheless, the shrinkage of banking sector results in a decrease in loans overall and output by firm is also reduced. In steady state, there happens a chage in distribution of credit creation and the banking industry becomes less concentrated.

To further study the role of deposit mix and heterogeneous bank liquidity in short-run dynamics, I consider the perfect foresight with an adverse aggregate shock to bank net worth. A large dependence on time deposits of the banking sector improves the financial stability as banks can recover net worth faster with smaller drop in loans and less severe recession occurring. A large time deposit issuance in the steady state would be costly but can be a source of financial stability in the short run.

Lastly, I provide a policy implication of the deposit mix and heterogeneous bank liquidity risk considering the bank liquidity regulation proposed by Basel III. Liquidity coverage ratio is affecting large banks the most by raising the demand for securities. Small and medium-sized banks already demand enough securities preparing the potential withdrawal risk and so the effect of the current liquidity coverage ratio is limited. In steay state, the liquidity coverage ratio reduces the loan supply by counteracting the advantage of the low witdrawal risk of large banks. Small and medium-sized banks can increase the loan supply in equilibrium but the aggreagte loan supply is reduced with a higher loan rate. Therefore, the liquidity coverage ratio has an adverse effect on the aggregate economy by reducing the loan supply and output in the long-run and the banking industry becomes less concentrated.

The model provides a framework to study the effect of deposit mix and liquidity risk on financial stability. The model can be extended to study the optimal liquidity coverage ratio and size-dependent liquidity requirement. The model can also be used to study the effect of monetary policy on the bank balance sheet and the aggregate economy.

References

Corbae, D., & D'Erasmo, P. (2021). Capital buffers in a quantitative model of banking industry dynamics. *Econometrica*, *89*(6), 2975–3023.

d'Avernas, A., Eisfeldt, A. L., Huang, C., Stanton, R., & Wallace, N. (2023). The deposit business at large vs. small banks. *National Bureau of Economic Research*.

Dempsey, K., & Faria-e Castro, M. (2022). A quantitative theory of relationship lending. *FRB St. Louis Working Paper*(2022-33).

Diamond, D. W., & Dybvig, P. H. (1983). Bank runs, deposit insurance, and liquidity. *Journal of political economy*, *91*(3), 401–419.

Drechsler, I., Savov, A., & Schnabl, P. (2021). Banking on deposits: Maturity transformation without interest rate risk. *The Journal of Finance*, *76*(3), 1091–1143.

Gertler, M., & Kiyotaki, N. (2010). Financial intermediation and credit policy in business cycle analysis. In *Handbook of monetary economics* (Vol. 3, pp. 547–599). Elsevier.

Gertler, M., & Kiyotaki, N. (2015). Banking, liquidity, and bank runs in an infinite horizon economy. *American Economic Review*, *105*(7), 2011–2043.

Hopenhayn, H. A. (1992). Entry, exit, and firm dynamics in long run equilibrium. *Econometrica: Journal of the Econometric Society*, 1127–1150.

Jermann, U., & Xiang, H. (2023). Dynamic banking with non-maturing deposits. *Journal of Economic Theory*, *209*, 105644.

Rıos-Rull, J.-V., Takamura, T., & Terajima, Y. (2020). Banking dynamics, market discipline and capital regulations. *cit. on*, 25–27.

Supera, D. (2021). Running out of time (deposits): Falling interest rates and the decline of business lending. *Investment and Firm Creation.*

A Proofs

A.1 Proof of Proposition 1

Proof. Consider first the case that only government bonds are enough to serve the withdrawal request.

$$\begin{aligned}
n_0' &= \tilde{\ell}' + \tilde{a}' - \frac{\theta \tilde{d}_S'}{q_a} - (1-\theta)\tilde{d}_S' - \tilde{d}_T' \\
&= \tilde{\ell}' + \tilde{a}' - \tilde{d}_S' - \tilde{d}_T' - \left(\frac{1}{q_a} - 1\right)\theta \tilde{d}_S' \\
&= \overline{n}' - \left(\frac{1}{q_a} - 1\right)\theta \tilde{d}_S' < \overline{n}' \quad \text{if} \quad q_a < 1
\end{aligned}$$

where $\overline{n}' \equiv \tilde{\ell}' + \tilde{a}' - \tilde{d}_S' - \tilde{d}_T'$ defines the maximal net worth in the next period that a bank can achieve with a balance sheet choice, $\{\tilde{\ell}', \tilde{a}', \tilde{d}_S', \tilde{d}_T'\}$.

The difference in net worth comes from the foregone interest income associated with the government bonds that are used for serving the withdrawal before its maturity. The loss is positive when the net risk-free interest rate is greater than zero.

Second, when loans are additionally needed to be sold with a discount

$$\begin{aligned}
n_1' &= \tilde{\ell}' - \frac{\theta \tilde{d}_S' - q_a \tilde{a}'}{\omega q_\ell} - (1-\theta)\tilde{d}_S' - \tilde{d}_T' \\
&= \tilde{\ell}' + \tilde{a}' - \tilde{d}_S' - \tilde{d}_T' - \left(\frac{1}{\omega q_\ell} - 1\right)\theta \tilde{d}_S' + \left(\frac{q_a}{\omega q_\ell} - 1\right)\tilde{a}' \\
&< \overline{n}' - \left(\frac{1}{\omega q_\ell} - 1\right) q_a \tilde{a}' + \left(\frac{q_a}{\omega q_\ell} - 1\right)\tilde{a}' \quad (\because \quad \theta d_S' > q_a a') \ \& \ \text{if} \ \ \omega q_\ell < 1 \\
&= \overline{n}' + q_a \tilde{a}' - \tilde{a}' = \overline{n}' - (1-q_a)\tilde{a}' < \overline{n}' \quad \text{if} \quad q_a < 1
\end{aligned}$$

□

A.2 Proof of Corollary 1

Proof. Recall the net worth in the next period, n_0' and n_1' are from the Proof of Proposition 1.

$$\begin{aligned}
n_0' &= \overline{n}' - \left(\frac{1}{q_a} - 1\right)\theta \tilde{d}_S' \\
n_1' &= \tilde{\ell}' + \tilde{a}' - \tilde{d}_S' - \tilde{d}_T' - \left(\frac{1}{\omega q_\ell} - 1\right)\theta \tilde{d}_S' + \left(\frac{q_a}{\omega q_\ell} - 1\right)\tilde{a}'
\end{aligned}$$

An extra unit of withdrawal has an adverse effect on the future net worth by $\frac{\partial n_0'}{\partial(\theta \tilde{d}_S')}$ and $\frac{\partial n_1'}{\partial \theta \tilde{d}_S'}$.

$$\frac{\partial n_0'}{\partial(\theta \tilde{d}_S')} = -\frac{1}{q_a}$$
$$\frac{\partial n_1'}{\partial(\theta \tilde{d}_S')} = -\frac{1}{\omega q_\ell}$$

Thus, $\left|\frac{\partial n_0'}{\partial(\theta \tilde{d}_S')}\right| < \left|\frac{\partial n_1'}{\partial(\theta \tilde{d}_S')}\right|$ if and only if $\omega q_\ell < q_a < 1$. $\square$

B Derivations

B.1 Household's optimal policies

Suppose μ is the Lagrange multiplier for 2. Consider the optimality conditions for the inner-most CES composite first (i.e., the allocation of type of deposits in bank j). First-order conditions are

$$
\begin{aligned}
[C] :& Z^{-\gamma} Z^{\frac{1}{\eta}} \lambda_C^{\frac{1}{\eta}} C^{-\frac{1}{\eta}} = \mu \\
[d'_{S,j}] :& Z^{-\gamma} Z^{\frac{1}{\eta}} (1-\lambda_C)^{\frac{1}{\eta}} L^{-\frac{1}{\eta}} L^{\frac{1}{\nu}} \delta_j^{\frac{1}{\nu}} L_j^{-\frac{1}{\nu}} L_j^{\frac{1}{\epsilon}} \lambda_S^{\frac{1}{\epsilon}} (d'_{S,j})^{-\frac{1}{\epsilon}} + \beta \mathbb{E}\left[\frac{\partial V^H(D'_S, D'_T; \mu'_B)}{\partial d'_{S,j}}\right] = q_{S,j}\mu \\
[d'_{T,j}] :& Z^{-\gamma} Z^{\frac{1}{\eta}} (1-\lambda_C)^{\frac{1}{\eta}} L^{-\frac{1}{\eta}} L^{\frac{1}{\nu}} \delta_j^{\frac{1}{\nu}} L_j^{-\frac{1}{\nu}} L_j^{\frac{1}{\epsilon}} (1-\lambda_S)^{\frac{1}{\epsilon}} (d'_{T,j})^{-\frac{1}{\epsilon}} + \beta \mathbb{E}\left[\frac{\partial V^H(D'_S, D'_T; \mu'_B)}{\partial d'_{T,j}}\right] = q_{T,j}\mu
\end{aligned}
$$

Envelope condition is $\frac{\partial V^H(D_S,D_T;\mu'_B)}{\partial d_{S,j}} = \frac{\partial V^H(D_S,D_T;\mu'_B)}{\partial d_{T,j}} = \mu$. In the stationary economy, $\mu' = \mu$ holds and combining the first-order conditions for deposits lead to

$$
\begin{aligned}
\left(\frac{\lambda_S}{1-\lambda_S}\right)^{\frac{1}{\epsilon}} (d'_{S,j})^{-\frac{1}{\epsilon}} \left(\frac{q_{T,j}-\beta}{q_{S,j}-\beta}\right) &= (d'_{T,j})^{-\frac{1}{\epsilon}} \\
\Rightarrow d'_{T,j} &= \frac{1-\lambda_S}{\lambda_S}\left(\frac{q_{T,j}-\beta}{q_{S,j}-\beta}\right)^{-\epsilon} d'_{S,j} \qquad (21)
\end{aligned}
$$

Consider the total expenditure for $d'_{S,j}$ and $d'_{T,j}$ for bank j and denote it as $E_j \equiv q_{S,j}d'_{S,j} + q_{T,j}d'_{T,j}$. Using the equation 21,

$$
\left(q_{S,j} + \frac{1-\lambda_S}{\lambda_S}\left(\frac{q_{T,j}-\beta}{q_{S,j}-\beta}\right)^{-\epsilon} q_{T,j}\right) d'_{S,j} = E_j \qquad (22)
$$

Then, we can solve for $d'_{S,j}$ and $d'_{T,j}$ as function of $\{q_{S,j}, q_{T,j}, E_j\}$ and model parameters

$$
\begin{aligned}
d'^{*}_{S,j}(q_{S,j}, q_{T,j}, E_j) &= \frac{\lambda_S}{\lambda_S q_{S,j} + (1-\lambda_S)\left(\frac{q_{T,j}-\beta}{q_{S,j}-\beta}\right)^{-\epsilon} q_{T,j}} E_j \\
&= \frac{\lambda_S (q_{S,j}-\beta)^{-\epsilon}}{\lambda_S (q_{S,j}-\beta)^{-\epsilon} q_{S,j} + (1-\lambda_S)(q_{T,j}-\beta)^{-\epsilon} q_{T,j}} E_j \qquad (23) \\
d'^{*}_{T,j}(q_{S,j}, q_{T,j}, E_j) &= \frac{(1-\lambda_S)\left(\frac{q_{T,j}-\beta}{q_{S,j}-\beta}\right)^{-\epsilon}}{\lambda_S q_{S,j} + (1-\lambda_S)\left(\frac{q_{T,j}-\beta}{q_{S,j}-\beta}\right)^{-\epsilon} q_{T,j}} E_j \\
&= \frac{(1-\lambda_S)(q_{T,j}-\beta)^{-\epsilon}}{\lambda_S (q_{S,j}-\beta)^{-\epsilon} q_{S,j} + (1-\lambda_S)(q_{T,j}-\beta)^{-\epsilon} q_{T,j}} E_j \qquad (24)
\end{aligned}
$$

To achieve the price index of liquidity service from the bank j, substitute the equations 23 and 24 into 5.

$$L_j = \frac{(\lambda_S(q_{S,j}-\beta)^{1-\epsilon} + (1-\lambda_S)(q_{T,j}-\beta)^{1-\epsilon})^{\frac{\epsilon}{\epsilon-1}}}{\lambda_S(q_{S,j}-\beta)^{-\epsilon}q_{S,j} + (1-\lambda_S)(q_{T,j}-\beta)^{-\epsilon}q_{T,j}} E_j$$

Define the price index as $P_{L,j} \equiv E_j|_{L_j=1}$. Then,

$$P_{L,j} = \frac{\lambda_S(q_{S,j}-\beta)^{-\epsilon}q_{S,j} + (1-\lambda_S)(q_{T,j}-\beta)^{-\epsilon}q_{T,j}}{(\lambda_S(q_{S,j}-\beta)^{1-\epsilon} + (1-\lambda_S)(q_{T,j}-\beta)^{1-\epsilon})^{\frac{\epsilon}{\epsilon-1}}}$$

Using the price index, we can re-express the optimal deposit demand functions as

$$\begin{aligned}
d'^*_{S,j}(q_{S,j}, q_{T,j}, E_j) &= \frac{1}{P_{L,j}} \left(\lambda_S(q_{S,j}-\beta)^{1-\epsilon} + (1-\lambda_S)(q_{T,j}-\beta)^{1-\epsilon}\right)^{\frac{\epsilon}{1-\epsilon}} \lambda_S(q_{S,j}-\beta)^{-\epsilon} E_j \\
d'^*_{T,j}(q_{S,j}, q_{T,j}, E_j) &= \frac{1}{P_{L,j}} \left(\lambda_S(q_{S,j}-\beta)^{1-\epsilon} + (1-\lambda_S)(q_{T,j}-\beta)^{1-\epsilon}\right)^{\frac{\epsilon}{1-\epsilon}} (1-\lambda_S)(q_{T,j}-\beta)^{-\epsilon} E_j
\end{aligned}$$

Substituting these back to 5 confirms that $P_{L,j}$ is indeed the price index for the aggregate liquidity service for bank j, $P_{L,j}L_j = E_j = q_{S,j}d'_{S,j} + q_{T,j}d'_{T,j}$. And the deposit demand function is expressed with bank-level liquidity service.

$$d'^*_{S,j}(q_{S,j}, q_{T,j}, L_j) = \lambda_S(q_{S,j}-\beta)^{-\epsilon} \left(\lambda_S(q_{S,j}-\beta)^{1-\epsilon} + (1-\lambda_S)(q_{T,j}-\beta)^{1-\epsilon}\right)^{\frac{\epsilon}{1-\epsilon}} L_j \tag{25}$$

$$d'^*_{T,j}(q_{S,j}, q_{T,j}, L_j) = (1-\lambda_S)(q_{T,j}-\beta)^{-\epsilon} \left(\lambda_S(q_{S,j}-\beta)^{1-\epsilon} + (1-\lambda_S)(q_{T,j}-\beta)^{1-\epsilon}\right)^{\frac{\epsilon}{1-\epsilon}} L_j \tag{26}$$

Now, let's consider the CES composite in the middle (i.e., allocation of liquidity services across banks). Using the price index for liquidity service for bank j, we can re-write the household's budget constraint as

$$C + \int \{P_{L,j}L_j\} \, \mathrm{d}j \leq w + D_S + D_T + \Pi - \tau$$

Let's consider the first-order condition for L_j.

$$Z^{-\gamma} Z^{\frac{1}{\eta}} (1-\lambda_C)^{\frac{1}{\eta}} L^{-\frac{1}{\eta}} L^{\frac{1}{\nu}} \delta_j^{\frac{1}{\nu}} L_j^{-\frac{1}{\nu}} = P_{L,j}\mu$$

Note that allocating liquidity service over the distribution of banks is a static choice, so the optimality condition does not involve continuation value. Considering the same first-order condition for bank $i \neq j$, we have

$$\left(\frac{\delta_j}{\delta_i}\right)^{\frac{1}{\nu}} L_j^{-\frac{1}{\nu}} = \left(\frac{P_{L,j}}{P_{L,i}}\right) L_i^{-\frac{1}{\nu}}$$
$$\Longrightarrow L_i = \left(\frac{\delta_i}{\delta_j}\right)\left(\frac{P_{L,i}}{P_{L,j}}\right)^{-\nu} L_j \tag{27}$$

Define $E_L \equiv \int P_{L,i} L_i \mathrm{d}i = \int E_i \mathrm{d}i$ to be the total expenditure for deposits across all banks. Substituting 27 into the definition of E_L yields

$$E_L = \int P_{L,i}\left(\frac{\delta_i}{\delta_j}\right)\left(\frac{P_{L,i}}{P_{L,j}}\right)^{-\nu} L_j \mathrm{d}i = \left(\int \delta_i P_{L,i}^{1-\nu}\mathrm{d}i\right)\frac{P_{L,j}^{\nu} L_j}{\delta_j}$$
$$\Longrightarrow L_j = \frac{\delta_j}{P_{L,j}^{\nu}} \cdot \frac{E_L}{\int \delta_i P_{L,i}^{1-\nu}\mathrm{d}i} \tag{28}$$

use 28 and back into 4:

$$L = \left(\int \delta_j^{\frac{1}{\nu}} L_j^{\frac{\nu-1}{\nu}}\right)^{\frac{\nu}{\nu-1}} = \left[\int \delta_j^{\frac{1}{\nu}} \delta_j^{\frac{\nu-1}{\nu}} P_{L,j}^{1-\nu}\mathrm{d}j\right]^{\frac{\nu}{\nu-1}} \frac{E_L}{\int \delta_i P_{L,i}^{1-\nu}\mathrm{d}i}$$

The price index can be defined as $P_L \equiv E_L | L = 1$. Then,

$$P_L = \left[\int \delta_j P_{L,j}^{1-\nu}\mathrm{d}j\right]^{\frac{1}{1-\nu}}$$

so, $P_L L = E_L = \int P_{L,i} L_i \mathrm{d}i = \int E_i \mathrm{d}i = \int \left\{q_{S,i} d'_{S,i} + q_{T,i} d'_{T,i}\right\} \mathrm{d}i$. Using this,

$$L_j^*(P_{L,j}, P_L, L) = \delta_j P_{L,j}^{-\nu} \frac{E_L}{P_L^{1-\nu}} = \delta_j \left(\frac{P_{L,j}}{P_L}\right)^{-\nu} L \tag{29}$$

Now the budget constraint can be re-expressed as

$$C + P_L L \leq w + D_S + D_T + \Pi - \tau$$

Note that the choice of liquidity service is a static decision as if L is another type of consumption goods. Household's intertemporal choice is already considered in the inner CES problem and the outer problem cares only about allocating the available resources to consumption and aggregate liquidity service.

The first-order conditions are

$$
\begin{aligned}
[C]: &\ \ Z^{-\gamma} Z^{\frac{1}{\eta}} \lambda_C^{\frac{1}{\eta}} C^{-\frac{1}{\eta}} = \mu \\
[L]: &\ \ Z^{-\gamma} Z^{\frac{1}{\eta}} (1-\lambda_C)^{\frac{1}{\eta}} L^{-\frac{1}{\eta}} = P_L \mu
\end{aligned}
$$

Combining them together yields

$$
\begin{aligned}
P_L \left(\frac{\lambda_C}{1-\lambda_C}\right)^{\frac{1}{\eta}} C^{-\frac{1}{\eta}} &= L^{-\frac{1}{\eta}} \\
\Rightarrow L &= \left(\frac{1-\lambda_C}{\lambda_C}\right) P_L^{-\eta} C
\end{aligned}
$$

As the utility function is monotonically increasing in both consumption and total liquidity, budget constraint binds at the optimum. Substituting back to the budget constraint leads to

$$
\left(1 + \left(\frac{1-\lambda_C}{\lambda_C}\right) P_L^{1-\eta}\right) C = w + D_S + D_T + \Pi - \tau
$$

To simplify the notation, let $\overline{w} \equiv w + \Pi - \tau$. We can solve for the optimal consumption and liquidity service as functions of prices and $(\overline{w}, D_S, D_T)$.

$$
\begin{aligned}
C &= \frac{\lambda_C}{\lambda_C + (1-\lambda_C) P_L^{1-\eta}} (\overline{w} + D_S + D_T) \\
L &= \frac{(1-\lambda_C) P_L^{-\eta}}{\lambda_C + (1-\lambda_C) P_L^{1-\eta}} (\overline{w} + D_S + D_T)
\end{aligned}
$$

By substituting to the CES aggregator, Z, the price index, $P_Z \equiv (\overline{w} + D_S + D_T)|_{Z=1}$, is derived.

$$
P_Z = (\lambda_C + (1-\lambda_C) P_L^{1-\eta})^{\frac{1}{1-\eta}}
$$

Using this price index, demand functions for consumption and liquidity can be re-expressed as

$$
\begin{aligned}
C &= \frac{1}{P_Z} \lambda_C \left(\frac{1}{P_Z}\right)^{-\eta} (\overline{w} + D_S + D_T) \\
L &= \frac{1}{P_Z} (1-\lambda_C) \left(\frac{P_L}{P_Z}\right)^{-\eta} (\overline{w} + D_S + D_T)
\end{aligned}
$$

Similar to the inner CES problem, substituting these demand functions into 3 again confirms

$P_Z Z = \overline{w} + D_S + D_T$. Then, optimal consumption and liquidity demand functions are

$$C(P_Z, Z) = \lambda_C \left(\frac{1}{P_Z}\right)^{-\eta} Z$$
$$L(P_L, P_Z, Z) = (1-\lambda_C)\left(\frac{P_L}{P_Z}\right)^{-\eta} Z$$

B.2 Firm's loan demand and production

Suppose λ is the Lagrange multiplier for the working capital constraint. The first-order conditions for the firm's problem are

$$[L_F'] : \ \lambda = -q_\ell - \beta \mathbb{E}\left[\frac{\partial V^F(L_F'; \mu', \mathcal{Q}')}{\partial L_F'}\right]$$
$$[N] : \ \kappa \overline{w} \lambda = \alpha \overline{Z} N^{1-\alpha} - \overline{w}$$

Envelop condition is $\frac{\partial V^F(L_F;\mu,\mathcal{Q})}{\partial L_F} = \lambda = -1$. In the stationary economy, $\lambda' = \lambda$, the first-order condition for loans can be re-expressed as

$$\lambda = \beta - q_\ell$$

The optimal labor demand is

$$N = \left[\frac{\overline{Z}\alpha/\overline{w}}{1+\kappa\lambda}\right]^{\frac{1}{1-\alpha}}$$

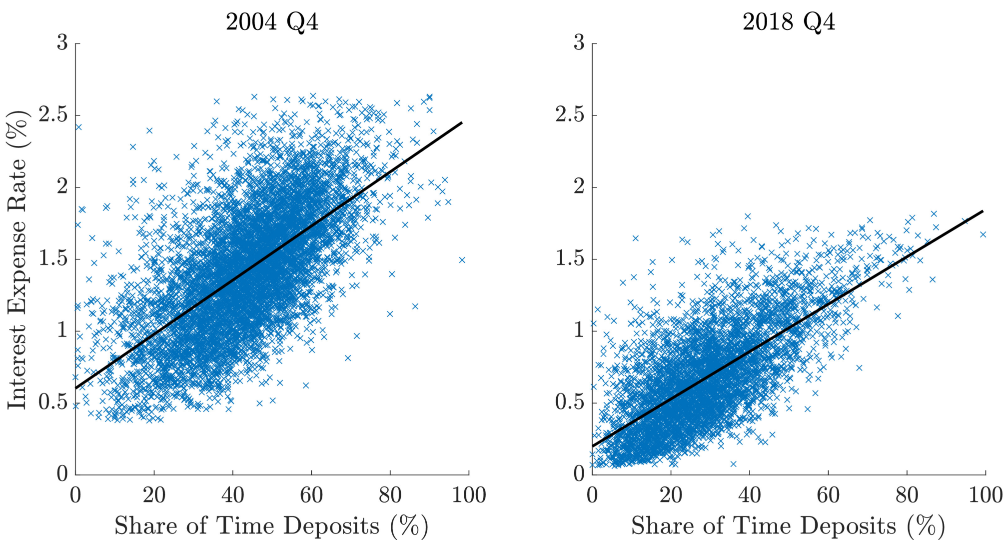
As long as the loan rate is greater than risk-free rate, the working capital constraint is binding at the optimum. The optimal loan demand is then

$$L_F = \kappa \overline{w} N = \kappa \overline{w} \left[\frac{\overline{Z}\alpha/\overline{w}}{1+\kappa\lambda}\right]^{\frac{1}{1-\alpha}}$$

C Regression analysis: share of time deposits and profitability

C.1 Deposit share and bank profitability

A higher dependence on time deposits among small-sized banks leaves a question about the adverse effect on their profitability given that sourcing funds using time deposits is more expensive as documented earlier. Although deposits are the primary source of funding for the banking business, banks can borrow in other forms of liabilities or accumulate capital to compensate for such higher costs associated with time deposits. Scatter plots in Figure 15 show the relationship between the share of time deposits in the balance sheet and the interest expense rates for two specific sample periods. They show a clear positive correlation implying that banks with a higher share of time deposits also face a higher interest expense rate. This correlation is confirmed again with the regression analysis with a full sample range



Note: This figure is constructed using Consolidated Reports of Condition and Income (Call Report) that covers U.S. commercial banks located within the 50 states and the District of Columbia. The sample period ranges from 1987 to 2020. Every aggregation is asset-weighted. Scatter plot is provided for two periods (2004 Q4 and 2018 Q4), and this pattern holds for other periods).

Figure 15: The share of time deposits and the interest expense rate

controlling bank size, other liabilities, and equity ratio.

The relationship between the share of time deposits in the individual bank's balance sheet and its profitability is studied with a regression analysis. Consider the following specification,

$$Y_{b,t} = \alpha_0 + \beta X_{b,t} + \gamma \mathbf{Z}_{b,t} + \alpha_b + \alpha_t + \epsilon_{b,t} \tag{30}$$

where the subscript b is the name of the bank. For the profitability, we use bank interest expense rate ($= \frac{\text{interest expense}}{\text{asset}}$) which is denoted as $Y_{b,t}$ in the regression equation. $X_{b,t}$ is the share of time deposits. $\mathbf{Z}_{b,t}$ is the vector of control variables. For the controls, we use bank equity ratio, non-deposit liability ratio, total assets, and lagged variable of X and $\mathbf{Z}$. Regression also includes bank and time-fixed effects. Clustered standard errors are computed in the following Table 8. Each column of the table differs by the control variables. We consider the second column without lagged control variables as the baseline result. $\beta = 0.0229$ implies that a 1%p rise in the share of time deposits raises the interest expense rate by 2.3bps. This number is comparable with the coefficient for the equity ratio. By this regression result, we can conclude that a 1%p drop in equity ratio can be equivalently compensated by lowering the time deposit share by 1.55%p. Positive and statistically significant $\hat{\beta}$ shows that after controlling the size of the banks and their alternative funding sources, banks with a higher share of time deposits on average face a larger interest expense rate. We interpret this as an unavoidable cost of time deposits even with alternative options of funding, which leads us to consider the potential advantage of holding time deposits regarding managing the flow of liabilities and/or credit supply.

	(1) Model 1	(2) Model 2	(3) Model 3
Time deposit share	0.0251***	0.0229***	0.0147***
	(0.0007)	(0.0006)	(0.0006)
Asset	-0.0004	-0.0002	0.0061*
	(0.0003)	(0.0003)	(0.0025)
Equity ratio		-0.0356***	-0.0325***
		(0.0014)	(0.0028)
Other liabilities		0.0088***	0.0027***
		(0.0005)	(0.0005)
Lag_Time deposit share			0.0088***
			(0.0005)
Lag_asset			-0.0064*
			(0.0026)
Lag_equity ratio			-0.0027
			(0.0023)
Lag_other liabilities			0.0074***
			(0.0006)
Constant	2.0634***	2.4853***	2.4085***
	(0.0302)	(0.0236)	(0.0253)
Bank FE	Yes	Yes	Yes
Time FE	Yes	Yes	Yes
N	1239626	1239626	1221149

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 8: Regression analysis

D Numerical algorithm

D.1 Steady state

Let's denote j to summarize state variables for a bank. To solve for the industry equilibrium with deposit market equilibrium,

1. Assume $g^0_{q_S}(j),\ g^0_{q_S}(j),\ \mu^0(j),\ L^0$
   - start with guessed $P^0_{L,j},\ P^0_L,\ L^0_j$ from 15, 16, and 14
2. Solve for the bank's problem to get $g^1_\ell(j),\ g^1_a(j),\ g^1_{q_S}(j),\ g^1_{q_T}(j)$ facing 12 and 13
3. Compute the stationary distribution, $\mu^1(j)$ by 10
4. Calculate
   - implied $P^1_{L,j}, P^1_L, L^1_j$ using $g^1_{q_S}(j),\ g^1_{q_T}(j),$ and, $\mu^1(j)$
   - D_S, D_T, and $\overline{w}$ based on the bank's choices and distribution
   - L^1 from household's problem
5. Iterate until we get $||L^0 - L^1||$ and $||P^0_L - P^1_L||$ to be small enough.
   - iteration with updates on $g^0_{q_S}(j),\ g^0_{q_T}(j),\ \mu^0(j),\ L^0$

D.2 Transitional dynamics

Suppose the total length of transition period is N_T and t denotes the date for each period.

1. Guess the initial set of L^0_t and $P^0_{L,t}$ containing N_T number of L and P_L for each t
2. start with value function at the terminal period, $v_{N_T} = v^0_{N_T}$
3. back solve for bank's optimal policies from $t = N_T$ to $t = 1$
   - produces $\{g_{\ell,t},\ g_{a,t},\ g_{S,t},\ g_{T,t}\}$
   - value function, v^1_t
4. forward solve to update the distribution from $t = 1$ to $t = N_T$
   - start with guessed $\mu_1 = \mu^0_1$
   - produces $\mu_t(n, \delta)$
   - implied prices and bank-level liquidity are computed, $P^1_{L,t}$ and $L^1_{j,t}$

- $D_{S,t}$, $D_{T,t}$, and $\overline{w}_t$ are calculated
- L_t^1 from the household's problem

5. check the convergence of $||L_t^0 - L_t^1||$ and $||P_{L,t}^0 - P_{L,t}^1||$ and iterate with updated L_t^0, $P_{L,t}^0$, v_t^0, and μ_t^0

E Data description

E.1 Call Reports

Bank-level data used in the paper is from Consolidated Reports of Condition and Income (so-called Call Reports). The sample is restricted to include U.S. commercial banks (domestic only) located within the 50 states and the District of Columbia. The sample period covers from 1984 Q1 to 2021 Q2.

All variables are deflated with the Consumer Price Index (CPI) to exclude the effect of trends in the economy's price level. To rule out unusual variations in the bank's balance sheet arising from bank entry and exit, I exclude the first and the last observations of each bank. The following table shows the definition of the variables used in the paper.

Name	Variable	Coverage
Assets	RCFD3368	1978/12-
Securities	RCFD0390	1984/03-1993/12
	RCFD1754 + RCFD1773	1994/03-
Liabilities	RCFD2948	1976/03-
Demand Deposits	RCON2210	1976/03-
Time Deposits	RCON2604 + RCON6648	1984/03-2009/12
	RCONJ474 + RCONJ473 + RCON6648	2010/03-
Savings Deposits	RCON2215 + RCON2389	1984/03-
	RCON2215 + RCON0352 + RCON6810	1987/03-
Deposits	Savings Deposits + Time Deposits	
Equity	RCFD3210	1976/03-
Interest Expense, Time	RIAD4174 + RIAD4512	1987/03-1996/12
	RIADA517 + RIADA518	1997/03-2016/12
	RIADHK03 + RIADHK04	2017/03-
Interest Expense, Savings	RIAD4508 + RIAD4509 + RIAD4511	1987/03-
	RIAD4508 + RIAD0093	2001/03-

Note: Definition of variables follow Drechsler et al. (2021) except the fact that savings deposits here includes demand deposits and interest expense excludes the deposits in foreign offices.

Table 9: Variable definition and coverage

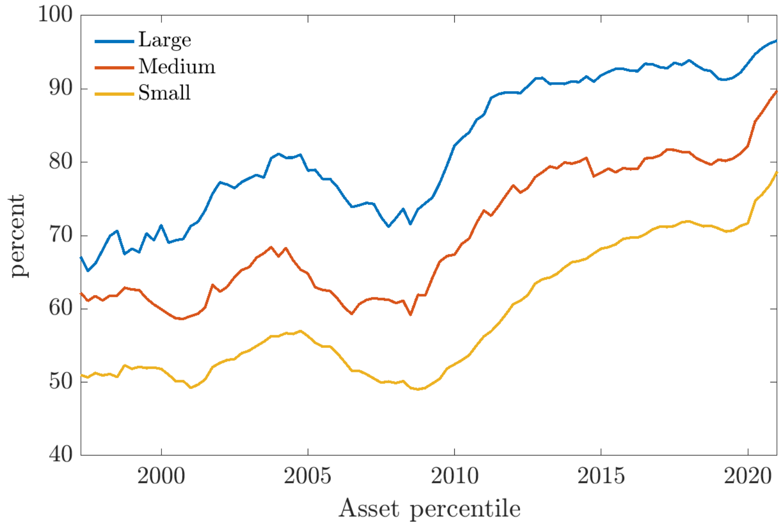
Ex-post interest rates for savings deposits and time deposits are calculated by interest expense divided by total volume of each deposit products and they are annualized.

E.2 Flow of Funds

E.3 Definition of variables

F Extra figures and tables

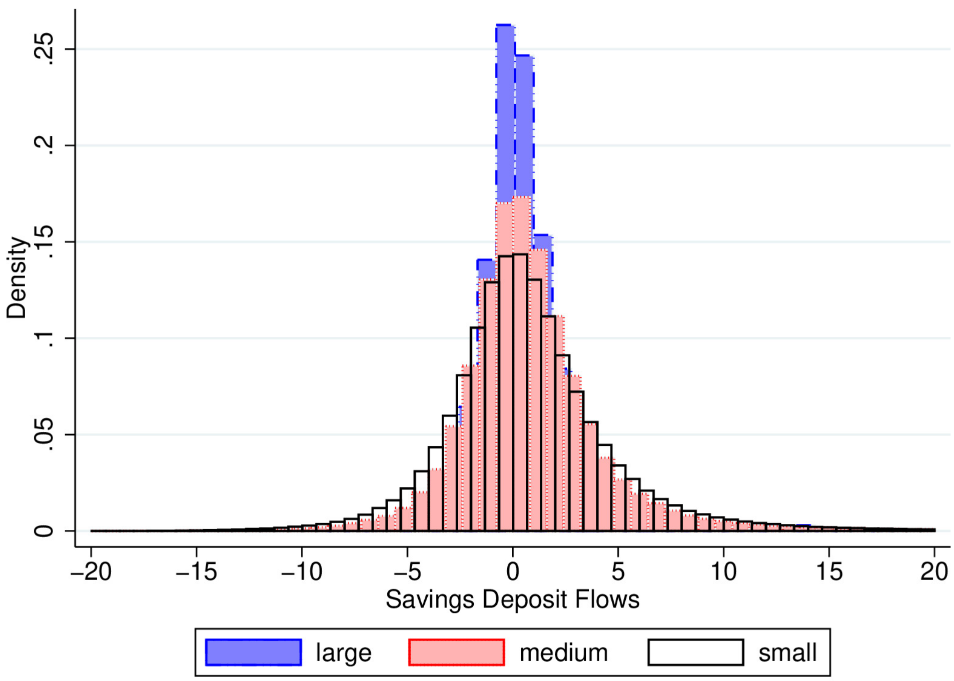
Figure 16 shows a time series of the share of savings deposits in the balance sheet for each size group. This confirms that the time-average pattern provided in Figure 2 consistently holds for each period in the sample.



Note: This figure is constructed using Consolidated Reports of Condition and Income (Call Report) that covers U.S. commercial banks located within the 50 states and the District of Columbia. The sample period ranges from 1984 Q1 to 2021 Q2. Every aggregation is asset-weighted. Savings deposits include both transaction and non-transaction accounts of savings deposit added demand deposits. *Large* is Top 0.1% banks in asset size. *Medium* is Top 10% excluding Top 0.1% banks. *Small* contains banks in bottom 90% of asset percentile.

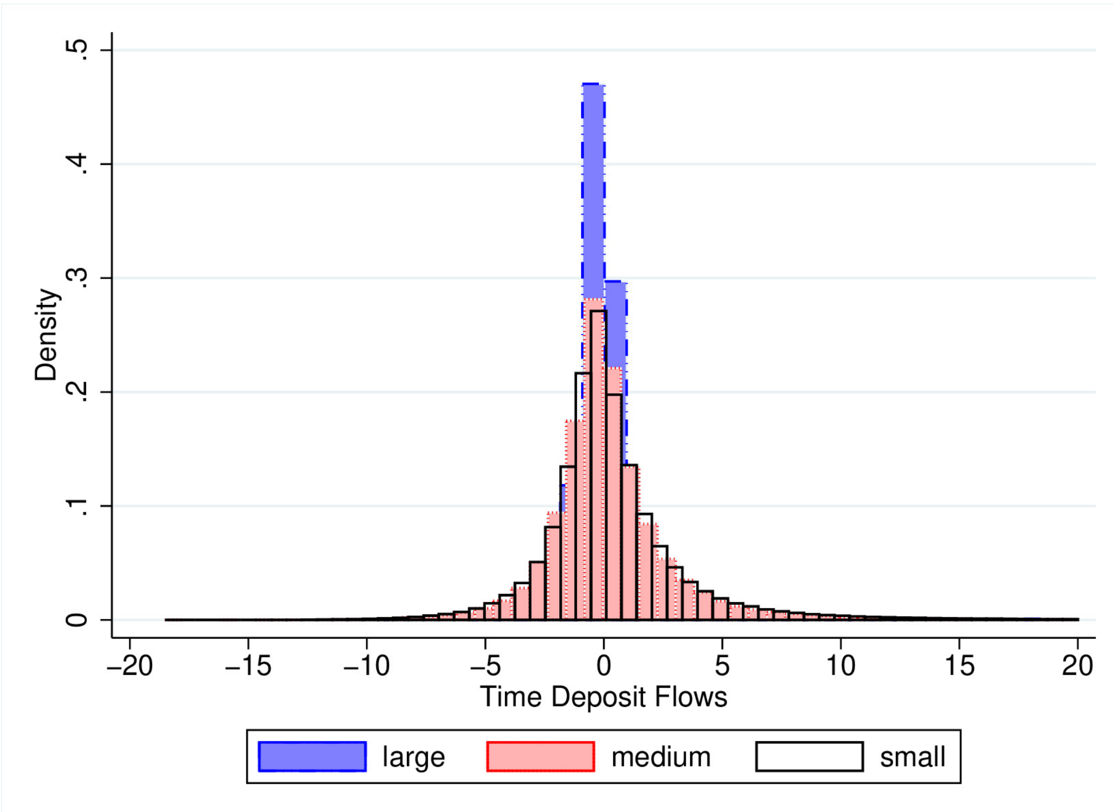
Figure 16: The share of savings deposit, time-series

Figure 17 and 18 display histograms of flows of savings and time deposits, respectively. By comparing between two figures, we see that the flows in savings deposits exhibit a larger variance than time deposits for all size classes. For each type of deposit product, there are clear differences in the frequency of a specific flow. For large banks, both savings and time deposits are relatively more stable and the flows are mostly bounded by the size of 5 percent. However, medium- and small-sized banks are facing large inflows and outflows more often.



Note: This figure is constructed using Consolidated Reports of Condition and Income (Call Report) that covers U.S. commercial banks located within the 50 states and the District of Columbia. The sample period ranges from 1984 Q1 to 2021 Q2. Deposits used to construct flow variables are deflated with the CPI index. Savings deposits include both transaction and non-transaction accounts of savings deposit added demand deposits. *Large* is Top 0.1% banks in asset size. *Medium* is Top 10% excluding Top 0.1% banks. *Small* contains banks in bottom 90% of asset percentile.

Figure 17: Savings deposit flow, histogram



Note: This figure is constructed using Consolidated Reports of Condition and Income (Call Report) that covers U.S. commercial banks located within the 50 states and the District of Columbia. The sample period ranges from 1984 Q1 to 2021 Q2. Deposits used to construct flow variables are deflated with the CPI index. Savings deposits include both transaction and non-transaction accounts of savings deposit added demand deposits. *Large* is Top 0.1% banks in asset size. *Medium* is Top 10% excluding Top 0.1% banks. *Small* contains banks in bottom 90% of asset percentile.

Figure 18: Savings deposit flow, histogram