Income-based Unsecured Credit and Business Cycles

Jeongwoo Moon
The Ohio State University

September 29, 2024

Abstract

This paper studies the quantitative importance of income-based borrowing limits on the aggregate consumption in the economy with household heterogeneity and incomplete market. The type of borrowing constraint does not change the aggregate quantities and price at a steady state, thus, the role of income-based borrowing limit is muted in a steady state. Contrarily, perfect foresight exercise with negative TFP shock provides discernible changes in the fluctuations of aggregate consumption. Deleverage effect followed by negative TFP shock amplifies aggregate consumption dynamics under the income-based borrowing limit. This credit channel broadens our understanding of TFP shock as a traditional source of the business cycle by generating procyclical consumer credit without default risk in the model. The credit channel features nonlinearity. The amplification of aggregate consumption is enlarged when i) the earnings are highly persistent, ii) the risk of earnings is countercyclical, and iii) the recession is large.

1 Introduction

The onset of the Great Recession, the role of the credit market on the aggregate fluctuations is more highlighted than before. Moreover, researches on the business cycle have focused on the financial sector and credit environment by which the economic agents are influenced. This paper focuses on the unsecured credit market and studies the quantitative importance of borrowing limits for unsecured credit in the fluctuations of aggregate consumption. The purpose of this paper is not, for now, to provide the best prediction on the business cycle based on US data, but to examine the model components to understand which elements are essential to generate particular predictions on the data using model-based analysis.

Among many characteristics of the unsecured credit market, this paper pays attention to the credit limits that an individual faces when she takes a loan. Based on the Survey of Consumer Finances, an individual faces a tighter unsecured credit limit when the labor income is low. The unsecured credit constraint which is commonly assumed in the literature, however, is uniform across households and does not take into account the empirical patterns of unsecured credit limits seriously when it is adopted as an exogenous borrowing constraint[1]. This paper argues that having a realistic borrowing constraint is important to understand the dynamics of aggregate consumption. For example, the decline of aggregate consumption during the Great Recession is mostly explained by the low-wealth group of people who features a higher marginal propensity to consume than wealth-rich people (Krueger et al. (2016)). And people who are wealth-poor and low-income (unlucky) are mostly constrained by credit limits and affected by the credit shock (Guerrieri and Lorenzoni (2017)). This implies that changes in the nature of credit limits are important to understand aggregate consumption. Therefore, I examine how much the shape of exogenous borrowing constraint changes the fluctuation of aggregate consumption, which has been relatively less explored in the literature of incomplete market models[2].

Exogenous borrowing limit that is identical for all households has an issue regarding

[1] In the default risk literature, the fact that unsecured credit limit is increasing in income is considered as an equilibrium result, so-called endogenous borrowing limits. However, this paper concentrates on exogenous borrowing constraints.

[2] For the studies of consumption inequality, the type of borrowing constraints is examined but not for the aggregate fluctuations. For example, Storesletten et al. (2004a) studies consumption inequality in the OLG model with expected wage tomorrow being borrowing constraint for today. They show that the types of borrowing constraint does not change the inequality of consumption at a steady state. Chatterjee et al. (2007) provide consumption inequality across different type of borrowing constraints implied by the model and show that the default risk model exhibits the result that is between the economy without credit (Bewley economy) and the economy with credit and full commitment to repay (Aiyagari economy).

the cyclical property of unsecured credit market. Standard models like Aiyagari (1994) and Krusell and Smith (1998) predict countercyclical consumer credit. Consumers insure themselves against aggregate uncertainty using credit. The consumption smoothing motive plays a critical role in predicting the countercyclical consumer credit. However, consumer credit is known to be procyclical as documented in Nakajima and Ríos-Rull (2019). In standard models, both credit shock and risk-free bond that has fixed net supply are required to explain the procyclicality of consumer credit as in Guerrieri and Lorenzoni (2017). I consider income-based borrowing limits and see if the model can predict procyclical consumer credit.

This paper aims to answer the following questions: How much does exogenous income-based borrowing limit change the aggregate consumption dynamics? Where do changes come from? Does exogenous income-based borrowing limit generate procyclical consumer credit? To this end, I take the empirical facts about unsecured credit limits from the Survey of Consumer Finances and specify the borrowing limits that are able to capture the key patterns observed in the data. In order to analyze the quantitative importance of the type of borrowing limits, I compare the model predictions across different borrowing limits in the dynamic general equilibrium model with heterogeneous households and incomplete market. A perfect foresight exercise with TFP shock is implemented to study the consumption smoothing and transitional dynamics of other aggregate variables.

The main result of this paper implies the changing role of income-based borrowing limit between steady states and transitional dynamics. I introduce a channel of TFP shock that depends on the type of borrowing limits and it is able to alter the aggregate fluctuations, which does not occur at a steady state. Calibrated to target the postwar US economy, the steady states are almost the same in terms of aggregate variables across the type of borrowing limits. This allows us to use the identical borrowing limit to approximate the unsecured credit market and this has been widely used in the literature because it simplifies the analysis. However, this is not the case for the transitional dynamics with TFP shock. Negative TFP shock tightens the credit limits when the households face income-based borrowing limits by lowering the real wage rate. The credit channel of TFP shock amplifies the response of aggregate consumption to TFP shock, which leads to discernible changes from the Aiyagari economy and it generates procyclical consumer credit. In order to understand the credit channel of TFP shock, alternative specifications for earning processes and large recessions are considered. The credit channel features nonlinearity in the sense that the amplification in the response of aggregate consumption is increasing in the persistence of earnings, the variance of earnings, and the size of the shock.

This paper lies on the strand of literature that studies the credit limit of heterogeneous households and its aggregate implication. In a steady state, borrowing limit and its equilibrium outcome are studied in Huggett (1993) and Aiyagari (1994). The transitional dynamics of a credit crunch are then investigated by Guerrieri and Lorenzoni (2017). Alonso (2018) compares the credit shock effect across different credit conditions in partial equilibrium.

This study is not the first one that incorporates income-based borrowing limits for unsecured credit. Storesletten et al. (2004a) finds that the borrowing constraint does not change the consumption inequality much in steady state. In the literature of default risk, the papers such as Chatterjee et al. (2007) and Abrahám and Cárceles-Poveda (2010) document endogenous borrowing limit that is dependent on income as an equilibrium result. This paper assumes an exogenous borrowing limit without default risk in the model and examines its effect on aggregate fluctuations.

In terms of saving technology, this paper follows the Neoclassical framework that uses physical capital as a financial claim. Some main articles of many can be listed as Aiyagari (1994), Krusell and Smith (1998), Chatterjee et al. (2007), Nakajima and Ríos-Rull (2019) and Krueger et al. (2016). On top of that, this paper follows the notion that the total factor productivity is a major source of business cycle like in the standard real business cycle studies[3]. The propagation mechanism and related properties are studied under the income-based borrowing constraint in the perfect foresight exercise.

The remainder of this paper is structured as follows. In section 2, the empirical facts about credit card limits in the Survey of Consumer Finances are discussed and I specify the borrowing limits for the model economy. Section 3 describes the baseline model economy and explains the underlying mechanisms that determine the steady state. The calibration and computation methods are covered in section 4. In section 5, the quantitative analysis of steady state and transitional dynamics with TFP shock is provided. Section 6 discusses alternative specifications to understand the credit channel of TFP shock regarding its nonlinearity. Finally, I conclude in section 7.

2 Unsecured Credit Limit

This section discusses empirical facts about unsecured credit limits and how they can be adopted in the model economy. In order to understand how unsecured credit limits are different across income levels and what determines the limit, previous studies are based on the Survey of Consumer Finances (hereafter, SCF). SCF contains survey data about the financial conditions of families in the US economy. For the unsecured credit limit,

[3]e.g. Kydland and Prescott (1982), King et al. (1988) and Hansen (1985).

in particular, previous studies looked at *Question x414* which asks the maximum amount one can borrow on all credit card accounts.

With the cross-sectional data of 2004 SCF, Abrahám and Cárceles-Poveda (2010) shows that credit card limit is increasing in labor income, hence, people with higher income have a better ability to obtain unsecured credit than people with low income. And if it is expressed as a proportion of labor income, the credit limit gets tighter as labor income rises. Figure 1 is the borrowing limits in 2004 SCF taken from Abrahám and Cárceles-Poveda (2010). Solid lines depict the average value of credit limits within decile from income distribution and dotted lines are borrowing limits predicted by the regression.[4]

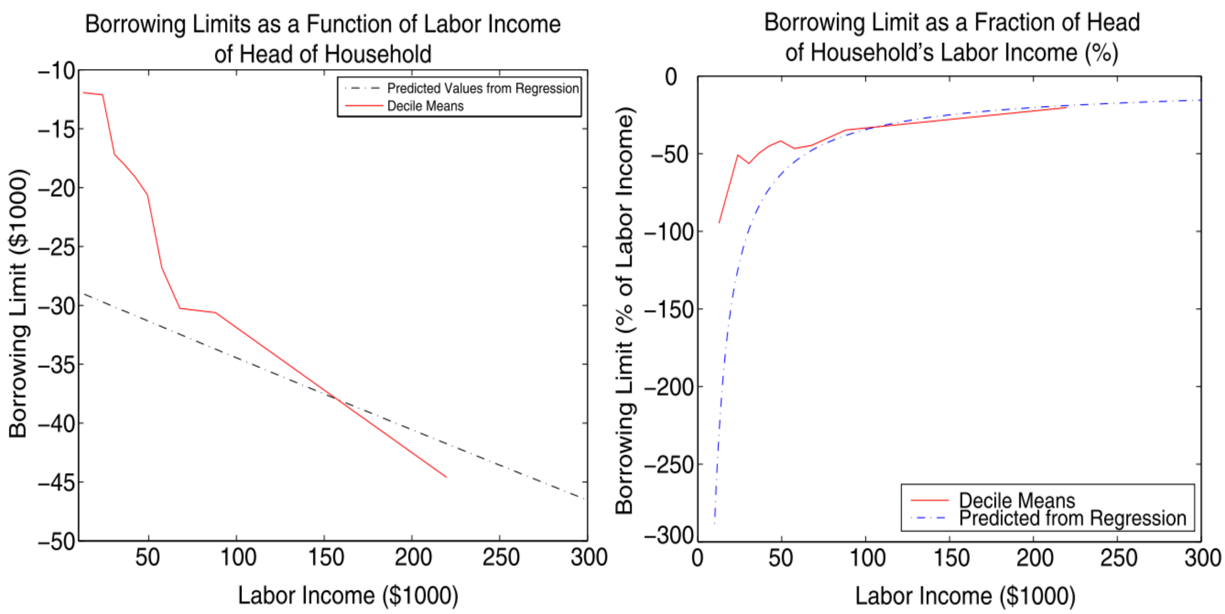

Figure 1: Borrowing limits in 2004 SCF (Abrahám and Cárceles-Poveda (2010))

The fact that an unsecured credit limit depends on labor income also holds in SCF with different sample periods. Choi et al. (2018) examines the role of income, net worth, demographic characteristics, and financial conditions in the determination of unsecured credit limit. Based on the credit card limit data from SCF between 2001 and 2016, the regression analysis shows that current income significantly increases the credit limits[5].

[4]Regression is used to explain the credit limit using income, age, gender, and education as regressors. For the details, see Abrahám and Cárceles-Poveda (2010)

[5]The dependency of the unsecured credit limit on labor income is not a unique feature of the US economy. Del Rio and Young (2006) studies determinants of unsecured borrowing limit for the UK economy.

Based on the previous studies, the borrowing constraint in the model economy should satisfy the following empirical facts:

1. Income is a factor that explains the unsecured credit limit (state-dependency)
2. Unsecured credit limit is increasing in labor income
3. Unsecured credit limit as a fraction of labor income gets tighter as labor income rises

In the model economy, households differ in their labor productivity, $\theta \in \Theta \subset \mathbb{R}_{++}$, which is assumed to be a Markov process. Heterogeneous households are able to save/borrow with a productive asset, a, to insure themselves against idiosyncratic risk. Borrowers face an exogenous borrowing limit, $\phi \in \mathbb{R}_{+}$, which indicates the maximum amount of the loan they can take,

$$a' \geq -\phi \tag{1}$$

where $a' \in A \subset \mathbb{R}$ is the decision rule for saving and the negative value of a' represents a decision for debt holdings tomorrow. Prime on the letter denotes the variable at a period later.

Reflecting the empirical facts from SCF, the borrowing limit is assumed to be a function of labor income, $\phi : \Theta \to \mathbb{R}_{+}$. One way to achieve this is allowing the households to borrow in proportion to the expected income tomorrow with the real wage as given today[6]. Then, the borrowing limit is specified as follows, [7]

$$\phi(\theta) \equiv \phi_1 w \mathbb{E}[\theta'|\theta] \tag{2}$$

where $\phi_1 \in \mathbb{R}_{++}$ and $w \in \mathbb{R}_{++}$ are a multiplier and equilibrium real wage rate, respectively. The last term expresses that an individual forms an expectation for the labor productivity tomorrow conditional on today's realization. From now on, this borrowing limit is denoted by *income-based* borrowing limit. The state-independent and uniform borrowing limit is also considered to understand the role of income-based borrowing limit by comparison,

$$\phi(\theta) = \phi \equiv \phi_2 \tag{3}$$

[6]For households to expect their labor income tomorrow, the future wage should be predicted given current information. Since the model economy in this paper does not have aggregate uncertainty, the wage is assumed to be fixed at equilibrium but as we extend the model with aggregate uncertainty, prediction for future wage should be considered.

[7]Note that this is not the only specification possible to resemble the patterns observed in the data. Also, by considering the infinite horizon model and stochastic labor productivity that can change every period, the form I assumed in this paper may not be the best description for individual household who does not experience frequent changes in the credit limit.

where $\phi_2 \in \mathbb{R}_{++}$ and it is labeled as *identical* borrowing limit, hereafter.[8]

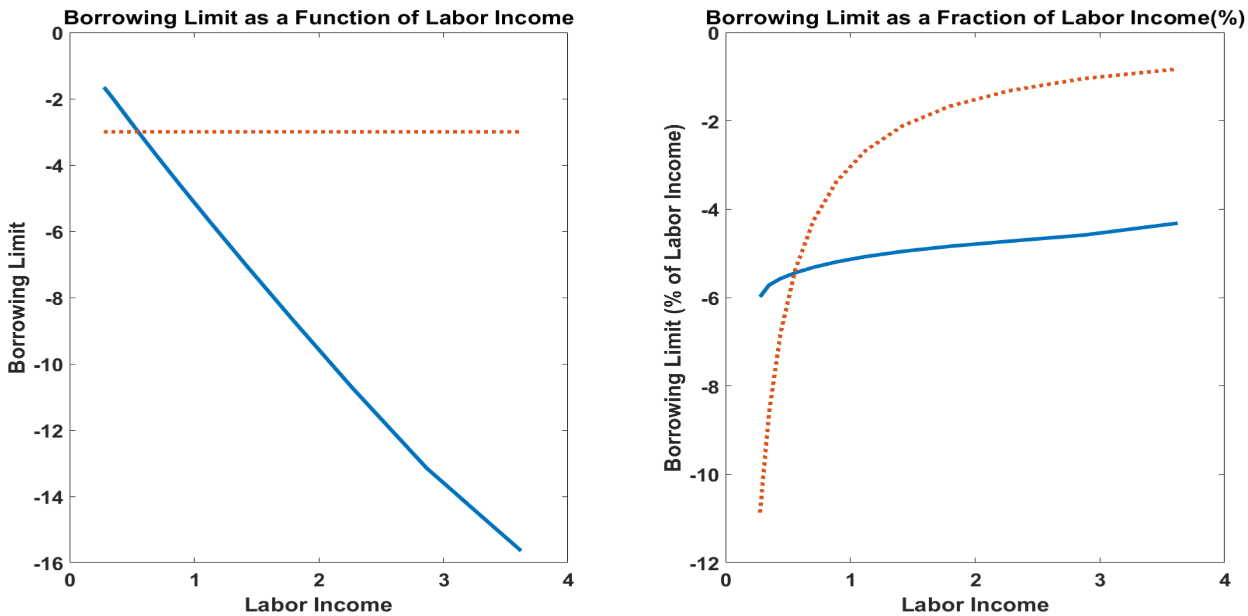

Figure 2: Income-based (solid line) and identical (dotted line) borrowing limits

Figure 2 plots the same graphs with Figure 1 but based on the borrowing limits in (2) and (3).[9] The income-based borrowing limit is able to capture the key patterns observed in data but identical borrowing limit cannot. These two borrowing limits do not necessarily intersect each other and it depends on the parameter values. However, it is interesting to note that if the borrowing limits are set like in Figure 2, there are low-income households who face stricter borrowing limits than those with identical borrowing limit. At the same time, households can have looser limits by drawing higher labor productivity today. Therefore, *ceteris paribus*, aggregate savings and quilibrium interest rate are possible to be altered by changes in the nature of borrowing limits. Or, if the effects of two groups of households with changes in borrowing limit exactly counteract, the equilibrium would be unchanged.

[8] Identical borrowing limit is more widely used in the literature of heterogeneous households with incomplete market. For example, Huggett (1993), Aiyagari (1994), Krusell and Smith (1998), Krueger et al. (2016), and Guerrieri and Lorenzoni (2017)

[9] $\phi_1 = 5$, $\phi_2 = 3$, $w = 1$, $\rho = 0.9$ and $\sigma_\theta = 0.5$ are used to calculate the graph. Continuous process for labor productivity in logs, $\log \theta' = \rho \log \theta + \epsilon'$, $\epsilon' \sim N(0, \sigma_\epsilon^2)$, is discretized into 12 states using Tauchen algorithm.

3 Model Economy

3.1 Model components

The role of income-based borrowing limit is studied in the dynamic general equilibrium model of heterogeneous households with incomplete market. The main features of the model are based on Aiyagari (1994) and the baseline model adopts the income-based borrowing constraint instead of identical borrowing constraint. For the purpose of comparison, the model economy with identical borrowing constraint which is exactly the same with Aiyagari (1994) is considered and is denoted as Aiyagari economy, henceforth.

Consider an infinite horizon economy populated by a continuum of heterogeneous households who face uninsurable idiosyncratic earning risk. The only asset traded is a capital that yields return, $1 + r_{t+1}$, for saving. If the households borrow a unit of consumption goods today, they have to repay $1 + r_{t+1}$ tomorrow, and debt is restricted by exogenous borrowing limit $\phi(\theta_t)$. There is no default risk and bankruptcy protection in the model, thus, all of the debt is going to be repaid a period later with full commitment.

Households' preferences are represented by the lifetime utility function

$$\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t u(c_t)\right]$$

where period utility function $u(\cdot)$ is strictly concave, strictly increasing and twice-continuously differentiable. c_t is consumption at date t and $\beta \in (0,1)$ is the subjective discount factor and it is invariant over lifetime. The households' budget constraint is

$$c_t + a_{t+1} \leq (1 + r_t)a_t + w_t\theta_t n_t$$

where a_t is capital holdings, r_t is the real interest rate ($r_t = r_t^k - \delta$ and r_t^k is rental rate of capital), w_t is real wage rate and n_t is the labor supply. θ_t is an idiosyncratic labor productivity which follows an AR(1) process in logs: $\log(\theta_t) = \rho_\theta \log(\theta_{t-1}) + \epsilon_t$, $\epsilon_t \sim N(0, \sigma_\epsilon^2)$. Note that labor supply is always 1 since the households do not value the leisure. For the simplicity, price of consumption goods is normalized to 1 and so the other prices are expressed as relative price. Households are able to borrow capital but is bounded below by the exogenous limit $\phi(\theta_t)$ which is a function of current labor productivity,

$$a_{t+1} \geq -\phi(\theta_t)$$

As discussed before, income-based borrowing limit is set to be $\phi(\theta_t) = \phi_1 w_t \mathbb{E}[\theta_{t+1}|\theta_t]$ to resemble the empirical facts about unsecured credit limit observed in SCF. Given prices and initial conditions $\{c_0, a_1\}$, households maximize lifetime utility by choosing consumption and saving/borrowing profiles, $\{c_t, a_{t+1}\}_{t=1}^{\infty}$ subject to budget constraint and borrowing constraint at each date.

There is a representative firm that produces consumption goods using aggregate capital, K_t, which depreciates at rate δ and aggregate labor, N_t, with constant return to scale technology, $Y_t = z_t F(K_t, N_t)$. The uppercase letters are used to denote the aggregate variables. The representative firm solves the following static profit maximization problem,

$$\max_{K_t, N_t} z_t F(K_t, N_t) - r_t^k K_t - w_t N_t$$

Since the households are heterogeneous, the equilibrium should be defined with joint distribution over individual states. Let $\mu_t(a_t, \theta_t)$ denote the joint distribution over wealth, a_t, and current labor productivity, θ_t at date t. Households' optimal decision on capital holdings and stochastic process for $\{\theta_t\}$ determines $\mu_{t+1}(a_{t+1}, \theta_{t+1})$ as follows,

$$\mu_{t+1}(a_{t+1}, \theta_{t+1}) = \int_{\Theta} \int_{\{a_t \in [-\phi(\theta_t), \infty] \mid g(a_t, \theta_t) = a_{t+1}\}} \mu_t(a_t, \theta_t) f(\theta_{t+1}|\theta_t) da_t d\theta_t, \ \ \forall\, a_{t+1}, \theta_{t+1}$$

where $f(\theta_{t+1}|\theta_t)$ is density function that describes the transition probabilities from θ_t to θ_{t+1} by Markov process assumption.

Since the problem described above features recursiveness and for the ease of application to computation, I write a functional equation for the households' problem and define recursive competitive equilibrium. The relevant state variables in the economy are capital holdings and labor productivity, (a, θ) and there is not an aggregate uncertainty in the model economy. Time subscript is omitted and prime on the letter is used to denote the variable at a period later.

Definition 1. *A recursive competitive equilibrium is a set of functions $g(a, \theta)$, values $V(a, \theta)$, prices w and r and joint distribution $\mu(a, \theta)$ such that*

1. *Given w and r, the households with (a, θ) solve*

$$V(a, \theta) = \max_{c, a'} \left[u(c) + \beta \int_{\Theta} V(a', \theta') f(\theta'|\theta) d\theta \right]$$

subject to

$$c + a' \leq (1 + r)a + w\theta$$
$$a' \geq -\phi(\theta)$$

and $g(a, \theta)$ is the associated decision rule

2. *Prices are competitively determined*

$$r = zD_1F(K, N) - \delta$$
$$w = zD_2F(K, N)$$

3. *Capital and labor markets clear*

$$\int_{\Theta}\int_{A} a\mu(a, \theta)dad\theta = K$$

$$\int_{\Theta}\theta\int_{A}\mu(a, \theta)dad\theta = N$$

4. *Invariant stationary distribution is determined as follows,*

$$\mu(a', \theta') = \int_{\Theta}\int_{\{a\in[-\phi(\theta),\infty]\ |\ g(a,\theta)=a'\}}\mu(a, \theta)f(\theta'|\theta)dad\theta, \ \ \forall\ a' \text{ and } \theta'$$

where $D_1F(\cdot, \cdot)$ denotes the partial derivative of $F(\cdot, \cdot)$ with respect to the first variable and $D_2F(\cdot, \cdot)$ is for the second variable.

3.2 Mechanisms working in incomplete market

Before the quantitative analysis of the effect of income-based borrowing limits on the aggregate economy, it is useful to discuss how equilibrium result is determined. This model economy also exhibits the features of incomplete market for equilibrium result, which implies that this is also the model with precautionary savings. There are two channels that determine the aggregate values at equilibrium.

First of all, consider the optimality conditions of households' problem which imply the Euler equation. For each θ,

$$Du(c) \geq \beta\mathbb{E}_{\theta'}[Du(c')(1 + r')|\theta]$$

where $Du(\cdot)$ represents a derivative of utility function and $\mathbb{E}_{\theta'}$ denotes expectation for θ'. Note that the Euler equation holds with equality when individuals are not constrained by borrowing limit. Depending on the realization of labor productivity and existing wealth, individuals occasionally face binding borrowing constraint. If the borrowing is restricted, households cannot consume as much as they would when they are unconstrained. Thus, those facing borrowing limit have a higher marginal propensity to consume than the other part of the population. We will see later that these households are important to explain the fluctuations in aggregate consumption. On top of that, households would save more with a precautionary motive not to be close to the borrowing limit. This precautionary saving motive exists independent of the equilibrium interest rate. Therefore, I call this effect as *partial equilibrium effect*.

There is another channel that affects households' optimal consumption/saving choices and in turn aggregate results. The effect comes from the changes in equilibrium interest rate which I label as *general equilibrium effect* to point out the difference with the previous channel. If interest is lowered, the intertemporal substitution effect implies less saving and more consumption because the price of current consumption falls. And wealth effect implies both consumption and saving decreases. The change in consumption is ambiguous because of the counteracting effect of substitution and wealth effect but the decrease in saving with a fall in equilibrium interest rate is straightforward. Combining the effects through the previous channel determines the aggregate capital and interest rate at equilibrium.

Because of market incompleteness, the steady state value of aggregate capital is larger and the interest rate is lower than the economy with a complete market. This is both theoretically and quantitatively correct as documented in Aiyagari (1994), which confirms the partial equilibrium effect dominates the equilibrium effect. I check this fact later after calibrating the model and solving for a steady state.

3.3 Changes in borrowing limits

Since the purpose of this paper is to investigate the role of income-based borrowing limits, it is useful to think about the effect of changes in borrowing limits on the aggregate economy. Suppose there is a change in the borrowing limits, from identical limits to income-based limits. At an individual level, the borrowing limit can be either stricter or looser than before depending on states, (a, θ) as depicted in Figure 2. Both partial equilibrium effect and general equilibrium effect play a key role in shaping the aggregate result. Guerrieri and Lorenzoni (2017) analyzes these two effects in terms of deleverage scenario (i.e. tightening the identical borrowing limit) and documents that wealth-poor households are affected mainly by partial equilibrium effect and wealth-rich households are more sensitive to changes in interest rate.

Change in borrowing limits can lead to a rise of aggregate savings if there are large enough people who have a small portion of wealth and draw low labor productivity.

They would experience tighter borrowing limits and the partial equilibrium effect would be prevalent in aggregate. On the other hand, aggregate savings can also fall if there are large enough populations who are wealth-rich or face looser borrowing limits. Therefore, a steady state is determined by combining the initial distribution, the shape of borrowing limits, and how much people with different wealth levels are sensitively responding to changes in borrowing limits.

4 Calibration and Computation

For the quantitative analysis, households' preference and production technology of representative firm are assumed to have specific functional forms. The instantaneous utility function $u(c)$ is assumed to exhibit constant relative risk aversion with the following form,

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

where σ represents the coefficient of relative risk aversion. Output is produced through Cobb-Douglas production function,

$$zF(K, N) = zK^{\alpha}N^{1-\alpha} \tag{4}$$

with z as the total factor productivity and $\alpha \in (0, 1)$ implies the share of aggregate capital in the production.

The model period is taken to be one year and the parameters are set to describe the US economy. Table 1 summarizes calibrated parameters for steady state. The subjective

Parameter	Explanation	Value	Source/target
β	discout factor	0.96	Aiyagari (1994)
σ	Coefficient of relative risk aversion	3	Aiyagari (1994)
α	Share of capital in production	0.36	Aiyagari (1994)
δ	Depreciation rate	0.08	Aiyagari (1994)
ρ_θ	Persistence of labor productivity	0.9136	Floden and Lindé (2001)
σ_θ	Standard deviation of labor productivity	0.5076	Floden and Lindé (2001)
ϕ_1	Multiplier on income-based borrowing limit	8.6601	debt-to-GDP = 0.18
ϕ_2	Identical borrowing limit	3.9289	debt-to-GDP = 0.18

Table 1: Calibration of Parameters

discount factor, β, is set to target the annual interest rate of 4% as if the economy were assumed to exhibit a complete market. The coefficient of relative risk aversion is taken to be 3. The share of capital in the production function is 0.36 which implies the labor share is 0.64 in production. Capital depreciates at a constant rate which is 0.08. These parameters are followed from Aiyagari (1994) to be consistent with aggregate features of the postwar US economy.

The exogenous and continuous stochastic process for idiosyncratic labor productivity has two parameters to be determined and they are taken from Floden and Lindé (2001). The parameter that determines the persistence, ρ, is set to be 0.9136 and the standard deviation of labor productivity is 0.5076 that is consistent with the estimate of the variance of the shock, σ_ϵ^2, that is reported as 0.0426 on Table 4 of Floden and Lindé (2001).

In order to compare the equilibrium of two economies with different borrowing limits, the steady state should be determined by achieving the same target. In particular, since this paper analyzes the role of different types of borrowing limits for households' unsecured credit, the economies are assumed to share the same feature in terms of consumer credit. By doing so, I am able to compare the steady state results by inspecting the differences in policy functions and stationary joint distribution over wealth and labor productivity. On top of that, it helps to compare the transitional dynamics that start with the steady states exhibiting the same aggregate indebtedness. To this end, I set ϕ_1 and ϕ_2 to target the same debt-to-GDP ratio. I match the debt in the model to consumer credit, which was 18% of GDP in 2006 as documented in Guerrieri and Lorenzoni (2017).

For the computational purpose, the continuous stochastic process for idiosyncratic labor productivity is discretized by the Tauchen algorithm with 12 states. The steady state is solved using the value function iteration with endogenous grid method which is developed by Carroll (2006). I use a grid for capital that has denser for lower values and fewer points as the capital is close to the upper bound. For the stationary distribution, an evenly spaced but finer grid is used and the policy function is approximated with linear interpolation. For both solving for the steady state and perfect foresight exercise, the following steps are used.

1. guess interest rate r and initial value function
2. solve for the optimal decision rules using endogenous grid method with the Euler equation
3. evaluate the value function and iterate until it converges
4. find a stationary joint distribution with the previous optimal decision rule for saving
5. compute the market clearing condition and debt-to-GDP ratio

6. check whether both the market clearing condition for aggregate capital and calibration target are satisfied. If they do not hold, update interest rate and ϕ_1 (or ϕ_2) and back to 2 until they hold

For the perfect foresight exercise, I follow the method used in Guerrieri and Lorenzoni (2017). Similar steps are taken but solve for the decision rules backward over $t = T, \ldots, 1$ where T is the end of simulation periods. Then, I solve forward the distribution for each t starting with stationary distribution at $t = 1$. Finally, update the interest rates until the market clearing condition for capital holds. [10]

5 Quantitative Analysis

5.1 Steady state

This part describes the steady state of the baseline model and inspects the underlying mechanism. Steady state level of interest rate, aggregate capital, and borrowing limits are presented in Table 2.

Model	r	K	debt	$\phi(\theta)$
Baseline	1.5779	9.0574	0.4338	[3.7145, 37.7626]
Aiyagari	1.5783	9.0568	0.4337	3.9289

Note: $r^* = 4.17$ *and* $K^* = 6.23$ *in complete market*

Table 2: Steady states results

Based on Table 2, both economies with incomplete market show that partial equilibrium effect dominates the general equilibrium effect, which leads aggregate capital to be accumulated more than that of the complete market. By targeting the debt-to-GDP ratio as 0.18, baseline economy shows a wide range of borrowing limit which depends on current labor productivity and identical borrowing limit lies between bounds of the income-based borrowing limit. The baseline model has slightly larger aggregate capital than the economy with an identical borrowing limit so the equilibrium interest rate is a little bit lowered. However, the difference is minuscule. For example, the aggregate capital of the baseline economy is 0.007% larger than the Aiyagari economy. Therefore, aggregate savings and equilibrium interest rates are almost identical between the two economies. Since aggregate labor is fixed, aggregate production is solely determined by aggregate capital.

[10] I choose $T = 120$ and the first 60 periods are used to present in figures.

By targeting the same debt-to-GDP ratio, debt levels in two economies are also the same as observed in Table 2. From the aggregate resource constraint, aggregate consumption has little difference between the two economies. Therefore, the income-based constraint has little effect on steady state results compared to an identical borrowing constraint.

Where do these similar results come from? Does this happen by chance in aggregates? This similarity between the two economies also applies to stationary wealth distribution. In Figure 3, the upper panel shows the entire distribution over wealth summing over labor productivity and the red dotted line shows the distribution for the baseline economy. Two economies have almost the same distribution all over the wealth level in steady state. The bottom panel magnifies a part of the distributuion for indebted households. With an income-based borrowing limit, some people have more debt than identical borrowing limit but fewer individuals at identical borrowing limit, right above $k = -4$. This is straightforward that high-income households now can borrow more than low-income households. However, combined with policy functions discussed below, differences in distribution for households close to the borrowing limit do not affect steady state aggregate capital and equilibrium interest rate much.

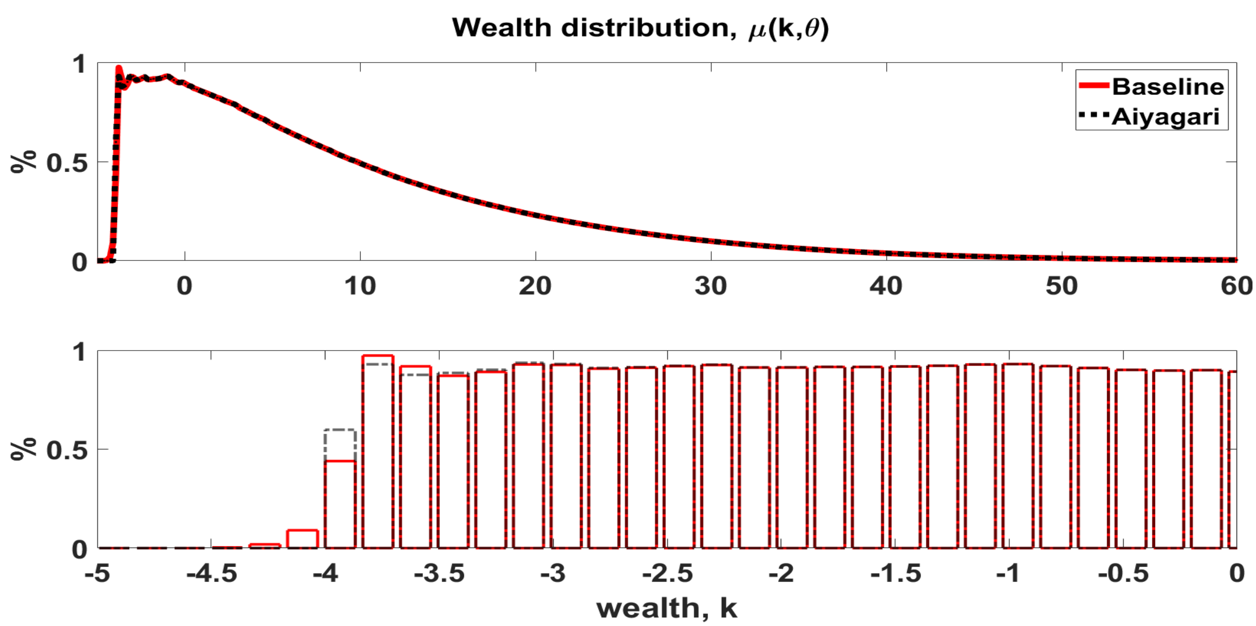

Figure 3: Wealth distribution

Let's look at the optimal consumption and decision rule at the household level. Figure 4 shows consumption function (left panel) and saving rule (right panel) across different labor productivities, $\{\theta_1, \theta_7, \theta_{12}\}$ among $\{\theta_1, \theta_2, \ldots, \theta_{12}\}$ where θ_1 is the lowest level of labor productivity. The solid line is for the Aiyagari economy and the dotted line is from the baseline economy. x-axis represents the current capital holdings or wealth level of the households. Based on the Euler equation, the households at the borrowing limit exhibit the greater marginal utility of consumption today than discounted present value of marginal utility of consumption tomorrow. Occasionally binding borrowing limit makes low productivity households feature higher marginal propensity to consume over wealth when they have low wealth. This appears in large curvature of consumption function for wealth-poor and low-productive households.

As confirmed by the right panel of the Figure 4, households save when they are poor and at the same time when they draw high labor productivity. This saving pattern which is consistent with the downward sloping graph occurs because of precautionary saving motive and consumption smoothing motive stemmed from the uninsurable income risk and risk-averse households.

Policy functions show little difference between the two economies. Consumption functions overlap exactly for the area where households are populated. For households close to income-based borrowing limit, more saving is done by low productive individuals and less saving is done by middle productive individuals but again such differences are somewhat offsetting each other and distribution shows there are not many people who have such low wealth. Therefore, the type of borrowing limit, particularly, income-based borrowing limit does not change the aggregate results at a steady state compared with an identical borrowing limit. In other words, an identical borrowing limit, when it is specified in the model of heterogeneous households with the incomplete market and without default risk, approximates the unsecured credit market with almost no changes in terms of aggregate variables at a steady state.

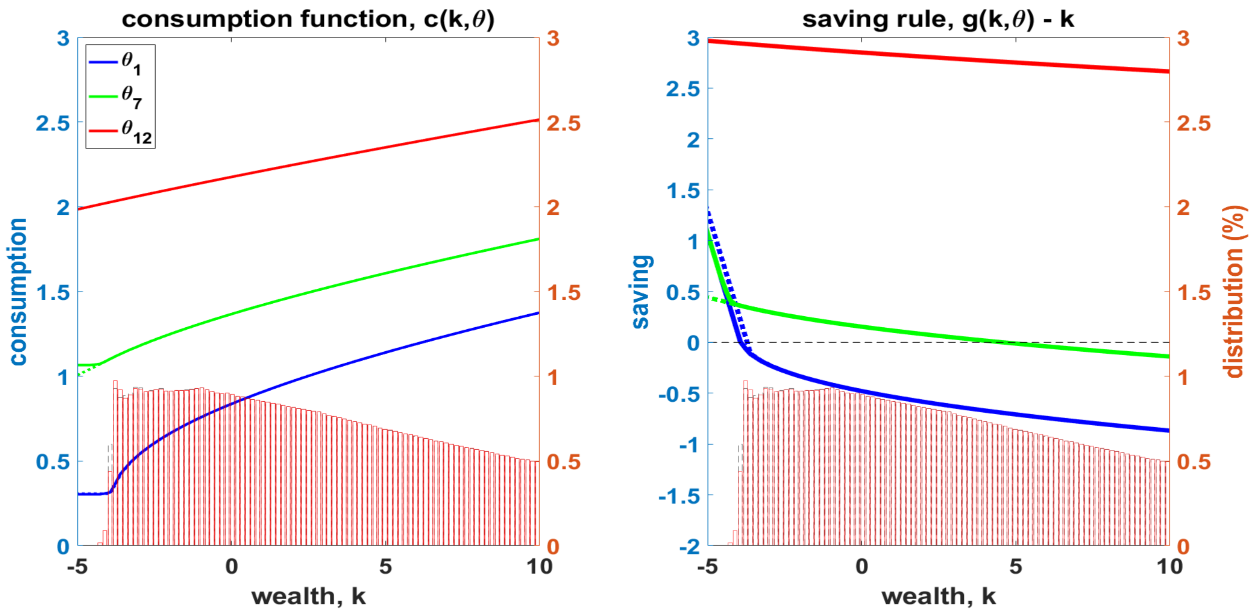

Figure 4: Consumption function (left) and saving rule (right)

5.2 Transitional dyanmics

Departed from steady state, transitional dynamics are studied to understand the role of income-based borrowing limits in aggregate fluctuations. Here and for the next section, I consider changes in total factor productivity and analyze the transitional dynamics. The aggregate productivity, z, follows AR(1) process in logs: $\log z' = \rho_z \log z + \eta'$ with $\eta' \sim N(0, \sigma_\eta^2)$ setting $\rho_z = 0.909$ and $\sigma_\eta = 0.014$ followed by Khan and Thomas (2013). Consider the economy experiences a single and negative 1 standard deviation shock at $t = 1$. The transitional dynamics are depicted in Figure 5.

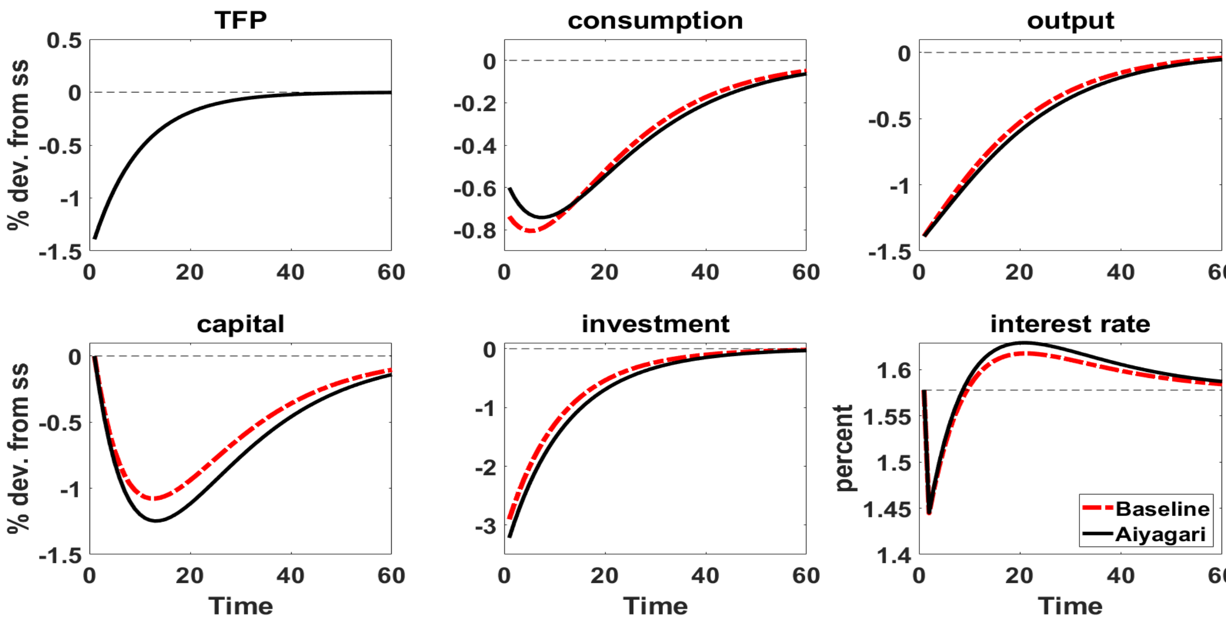

Figure 5: Impulse response to persistent negative TFP shock

In the baseline model, the amplification in consumption response is observed. Compared to the Aiyagari economy, the aggregate consumption from the steady state is reduced more by 0.13 percentage point at the impact date and the size of recession (at the trough) is enlarged more by 0.06 percentage point. In other words, the consumption in baseline economy shows 23% larger fall than the case of Aiyagari economy at impact date, which is larger difference than steady state. For other variables, the baseline model exhibits more stable responses. A relatively small fall in investment leads to less drop in aggregate capital and then aggregate output shows faster recovery. The interest rate has little difference by fall in TFP but less fluctuating in recovery path.

Since the baseline model has almost the same steady state result as the Aiyagari economy, the amplification of consumption and reduction in capital responses should not come from the differences in consumption/saving functions and/or stationary wealth distribution at the impact date. Then, where does it come from? The gap in transitional dynamics is occurred by type of the borrowing constraints. In the baseline model, the income-based borrowing limit is changing over time because TFP affects the real wage rate and borrowing limits of all households are affected.

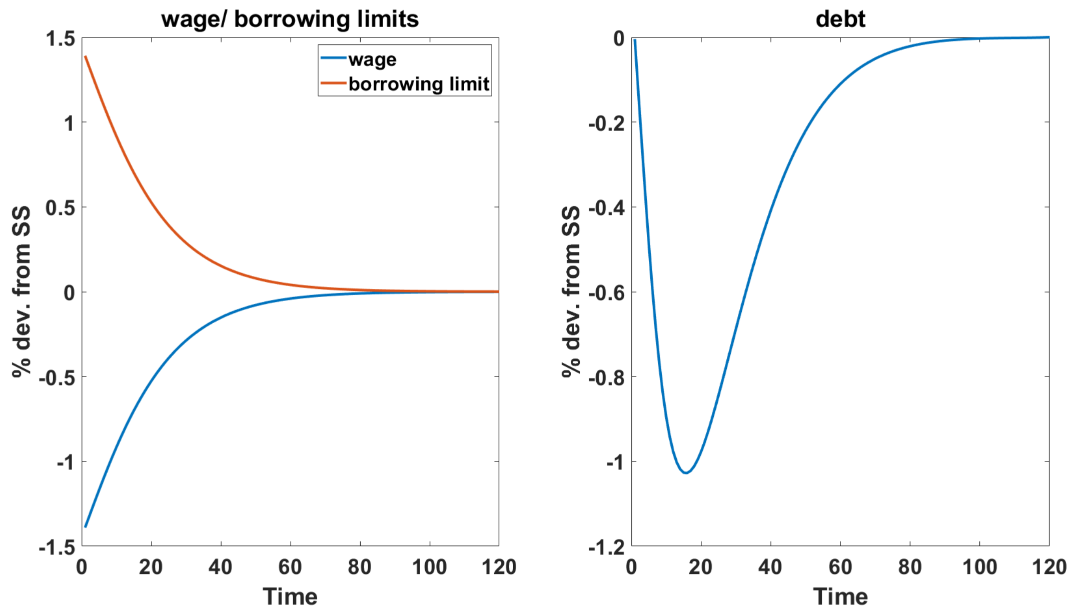

Figure 6: Dynamics of wage, borrowing limit and debt

Figure 6 shows dynamics of wage, borrowing limits, and aggregate debt in baseline economy. Negative TFP shock lowers the real wage rate and conversely raises the borrowing limit meaning tightening the credit limit. The tighter credit limit for all households triggers stronger precautionary saving and so the aggregate debt is showing fall over time, which generates procyclical consumer credit as observed in US data[11]. This temporary deleveraging effect boosts the precautionary saving motive of the economy and households reduce consumption to save more. Therefore, income-based borrowing limits introduce a credit channel of TFP shock that amplifies the fluctuations of aggregate consumption and helps to explain the procyclicality of consumer credit.

[11] For procyclicality of consumer credit, see Nakajima and Ríos-Rull (2019)

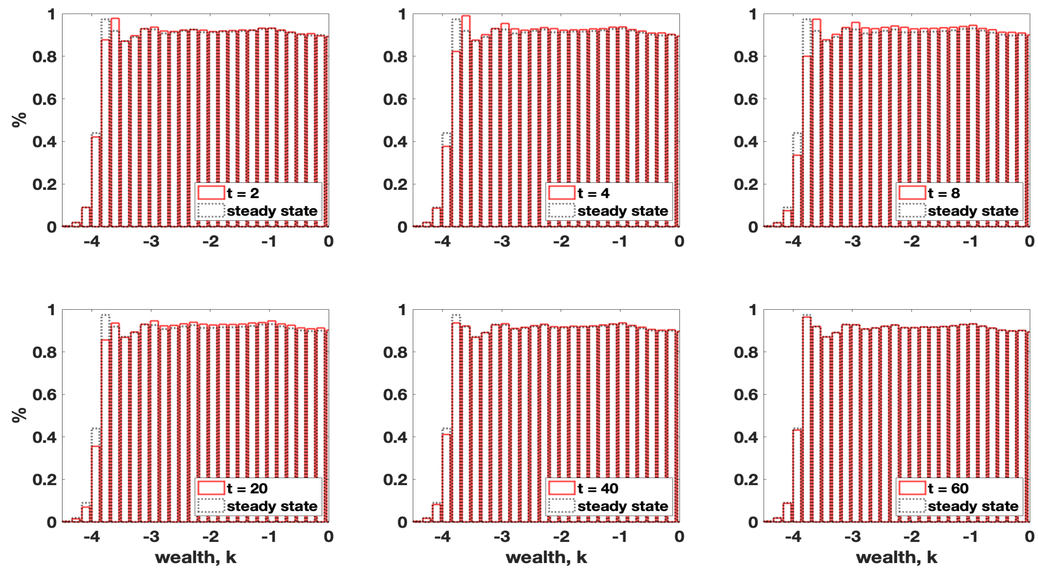

Figure 7: Evolution of wealth distribution for indebted households

Time-varying feature of borrowing limits under income-based borrowing limit can also be confirmed in Figure 7. Figure 7 shows the evolution of wealth distribution for the part of indebted households. As time goes from $t = 2$ to $t = 8$, the households at near $k = -4$ reducing their loan size and we can observe more people are in less debt than $k = -4$. As time passes further, people are taking back their initial loan and aggregate debt is going back to steady state level which was depicted on the right panel in Figure 6.

The other thing to note in Figure 7 is the population who are most indebted, mostly $k < -4$. Population with debt over -4 shows little changes. These people are middle-income households. They are able to take a loan that is larger than the Aiyagari economy but not much responsive to negative TFP shock. Changes in distribution occur mostly in households with low income. They take tightened borrowing limits seriously and decrease consumption to save. Therefore, the credit channel of TFP shock is effective on low-income households and amplification in consumption response can be accounted for by these low-income households. In line with Krueger et al. (2016), I suggest this channel can provide an additional explanation for the large decline of consumption during the Great Recession by low-income (and with low wealth) households.

Note that if the asset were not productive, the deleveraging effect of negative TFP shock would change the previous impulse responses. In the baseline model, a decrease in output is mitigated because tightening of the credit limit raises the aggregate capital and that capital is used for production which provides extra insurance on top of individual savings. However, if the asset is a non-contingent, risk-free bond with fixed net supply, the aggregate savings is not affected but the equilibrium interest rate would fluctuate with more volatility. Consumption would be amplified more because of a stronger precautionary motive. Furthermore, if the households value leisure, the output can drop further. This way of setting provides recession under credit crunch as discussed in Guerrieri and Lorenzoni (2017). However, I adopt the productive asset, capital, for the baseline economy to be comparable with the Aiyagari economy. And at least for aggregate consumption, there is the only difference in the magnitude of responses but the underlying mechanism which amplifies the consumption is the same across the assets.

6 What Affects the Credit Channel?

Amplification in consumption under the income-based borrowing limit can depend on how we calibrate the model. This section studies the factors that change the effectiveness of the credit channel discussed above by considering the following cases: 1) different specification for earning process and 2) large TFP shocks

6.1 Earning process and the shape of credit limits

The shape of the income-based borrowing limit is determined by the specification of earning process. Less persistent earnings flatten the credit limit over income level. And high earning risk widens the range of limit. In the baseline model, the credit limit is set to resemble the empirical facts about unsecured credit limits given an earning process. Having different earning processes, we can test other possible sets of borrowing limits but maintaining their main empirical features. To this end, $\rho = 0.3$ is used for the case of low persistence and the high risk case considers $\sigma_\epsilon = 0.2828$ which is twice as risky compared to the one in the baseline model.

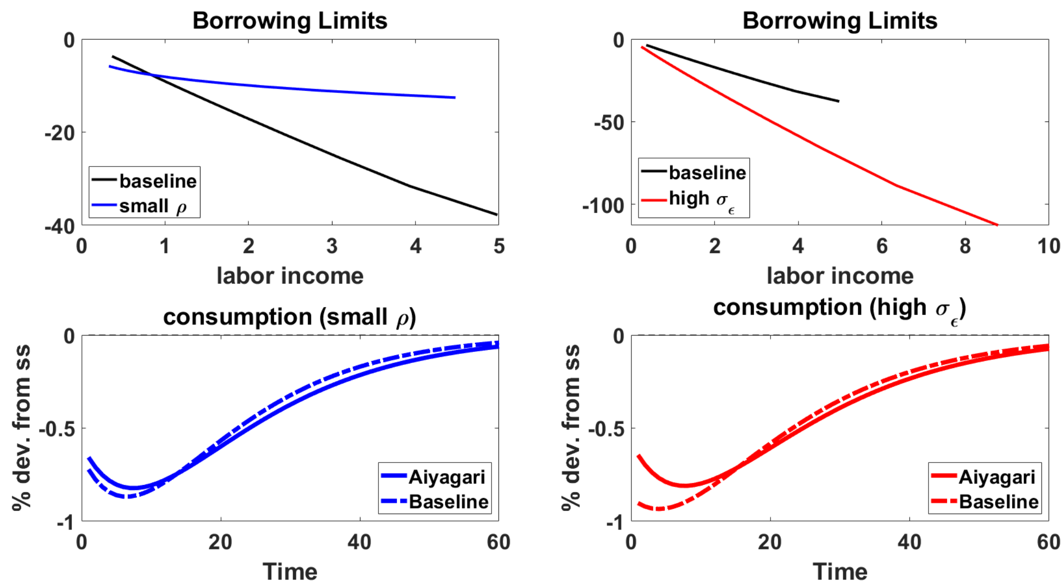

Figure 8: Impulse response to persistent negative TFP shock: alternative earning processes

Borrowing Limit	impact date (%p)	trough (%p)
baseline	-0.13	-0.06
small ρ	-0.07	-0.05
high σ_ϵ	-0.26	-0.12

Table 3: Changes in aggregate consumption by income-based credit limit: alternative earning processes

Figure 8 shows borrowing limits and impulse response of consumption to negative TFP shock. On the left panel, the case of small persistence is considered. Less persistent labor productivity creates less variance for expected labor productivity tomorrow conditional on today's productivity, $\mathbb{E}[\theta'|\theta]$, and so there are small differences in credit limit across labor income. Hence, the borrowing limit in the graph looks flatter than the baseline economy. Since a flatter borrowing limit implies weak state-dependency, there are less constrained and indebted households. The solid line shows the consumption response for the Aiyagari economy and the dotted line represents the case of the income-based borrowing limit. The impulse responses of consumption are not much different from each other. At impact date, it is reduced more by 0.07 percentage point and 0.05 percentage point at a trough.

On the right panel of Figure 8, borrowing limits and the response of aggregate consumption are depicted for the case of higher risk. A large value for σ_ϵ means a large variance of labor productivity. This widens the range for credit limits over labor income. As documented in Table 3, the consumption is amplified more than the baseline case. This result has a clear implication for aggregate fluctuations over the business cycle based on the countercyclicality of earning risk observed in Storesletten et al. (2004b). I do not consider this into the model economy for now but the perfect foresight result suggests the credit channel of TFP shock with income-based borrowing limit can be notable especially in a recession.

6.2 Size of the shocks

The effect of the size of shocks on the response of aggregate consumption is examined to see the nonlinearity of the credit channel. To this end, two large TFP shocks are considered comparing with the baseline result in Figure 5: 5 standard deviation shock and 10 standard deviation shock. Figure 9 depicts the dynamics of TFP and aggregate consumption responses given the shocks (blue lines for $5\sigma_\eta$ and red lines for $10\sigma_\eta$). The dotted line represents the response of aggregate consumption for the economy with an income-based borrowing limit.

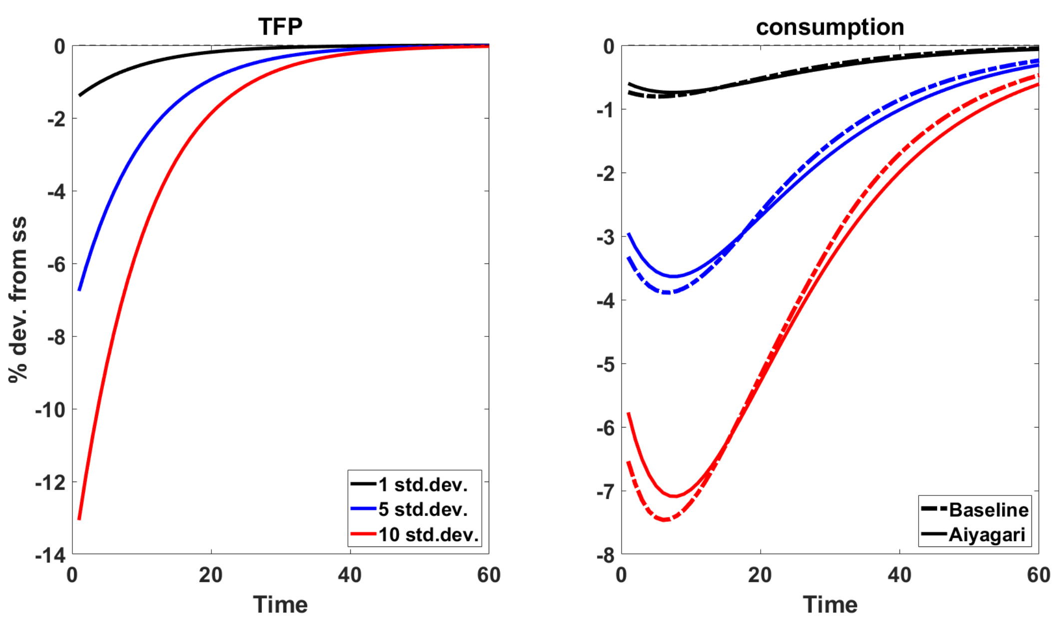

Figure 9: Impulse response to persistent negative TFP shock: size of the shocks

TFP (std.dev.)	impact date (%p)	trough (%p)
1	-0.13	-0.06
5	-0.37	-0.25
10	-0.77	-0.37

Table 4: Changes in aggregate consumption by income-based credit limit: large TFP shocks

As discussed in the previous section, the negative TFP shock amplifies the size of consumption drops through the credit channel by tightening the credit limit for all households. In Table 4, the size of reduction in consumption is increasing in the size of TFP shock both at impact date and at a trough. Therefore, the credit channel of TFP shock has nonlinearity in its effect on aggregate consumption. Furthermore, the state-dependency of borrowing limit becomes more significant in understanding consumption dynamics when the economy is hit by a large shock.

7 Concluding Remarks

This paper studies the quantitative importance of the income-based borrowing limit for unsecured credit on nondurable consumption. Motivated by empirical facts about credit card limits observed in SCF, a proportion of expected wage tomorrow is taken as borrowing limit in the model to resemble the three key patterns in the data. In order to study the role of income-based borrowing limit in the model with household heterogeneity and incomplete market, the model economy is taken from Aiyagari (1994) replacing the identical borrowing limit with income-based borrowing limit. Calibrated to match the postwar US economy, the steady state is rarely affected by employing the income-based borrowing limit. Thus, the identical borrowing limit which is widely used in the literature simplifies the analysis and approximates the unsecured credit market with almost no cost for the steady state analysis.

Departed from steady state, changes in aggregate consumption smoothing are analyzed by transitional dynamics of the economy with TFP shock. A single and negative shock in persistent TFP reduces the aggregate consumption and its response is amplified under the income-based borrowing limit. The amplification is followed from the credit channel of TFP shock which exists only with an income-based borrowing limit. Negative TFP shock has an adverse effect on the real wage rate and it tightens the borrowing limit. This deleverage effect also generates procyclical consumer credit. Following the same mechanism discussed in Guerrieri and Lorenzoni (2017), tighter credit condition along the transitional path leads to stronger precautionary saving motive and so aggregate investment, capital, and output reduce less by negative TFP shock than they would be with an identical borrowing limit. Particularly, the difference in aggregate consumption across borrowing limits is not negligible compared to the difference of steady state results.

The credit channel of TFP shock is further analyzed considering alternative specifications for earning processes to change the shape of the borrowing limit and large recessions. The first experiment considers a less persistent earning process. The income-based borrowing limit becomes flatter over labor income. Then the aggregate consumption is less amplified. In contrast, if households face a higher risk of earning process, the borrowing limit shows steeper and widens the range of credit limit. In response to negative TFP shock, the consumption is reduced more than the baseline model does. This result implies that countercyclical earning risk is able to add another source of amplification in fluctuations of consumption over the business cycle. The credit channel of TFP shock is also studied under large recessions. With large negative TFP shocks, the amplification in aggregate consumption response is increasing in the size of shocks, which exhibits nonlinearity. Thus, the income-based borrowing limit should be carefully considered in the model to understand the consumption dynamics when the economy is hit by a large shock.

This project helps to understand the role of income-based borrowing limit, especially along the transitional path by introducing the credit channel of TFP shock. However, there are still unresolved caveat and issues which should be covered later for a more accurate quantitative analysis. I would conclude by providing a further avenue for future studies. First, the patterns of credit limit found in previous studies are limited to look at the cross-sectional picture of the economy at a specific time. But we also need to look at the credit limit of each individual by tracking it over time for extra validation. Second, the model economy is based on Aiyagari (1994) which assumes full commitment to repay the debt. This may sound strict assumption in that it cannot capture the default and bankruptcy behavior of unsecured consumer credit. In the perspective of default risk literature, the income-based borrowing limit is an equilibrium result implied by the model with default risk. For the purpose of analyzing aggregate fluctuations and quantitative relevance of borrowing limit, the model should consider aggregate uncertainty like Krusell and Smith (1998). Then the stochastic general equilibrium of the model economy in this paper can be compared with Nakajima and Ríos-Rull (2019).

References

Abrahám, A. and Cárceles-Poveda, E. (2010). Endogenous trading constraints with incomplete asset markets. *Journal of Economic Theory*, 145(3):974–1004.

Aiyagari, S. R. (1994). Uninsured idiosyncratic risk and aggregate saving. *The Quarterly Journal of Economics*, 109(3):659–684.

Alonso, C. (2018). Hard vs. soft financial constraints: Implications for the effects of a credit crunch. *Journal of Macroeconomics*, 58:198–223.

Carroll, C. D. (2006). The method of endogenous gridpoints for solving dynamic stochastic optimization problems. *Economics letters*, 91(3):312–320.

Chatterjee, S., Corbae, D., Nakajima, M., and Ríos-Rull, J.-V. (2007). A quantitative theory of unsecured consumer credit with risk of default. *Econometrica*, 75(6):1525–1589.

Choi, K. J., Koo, H. K., Lim, B. H., and Yoo, J. (2018). The determinants of unsecured credit constraint. *Available at SSRN 2657352*.

Del Rio, A. and Young, G. (2006). The determinants of unsecured borrowing: evidence from the bhps. *Applied financial economics*, 16(15):1119–1144.

Floden, M. and Lindé, J. (2001). Idiosyncratic risk in the united states and sweden: Is there a role for government insurance? *Review of Economic dynamics*, 4(2):406–437.

Guerrieri, V. and Lorenzoni, G. (2017). Credit crises, precautionary savings, and the liquidity trap. *The Quarterly Journal of Economics*, 132(3):1427–1467.

Hansen, G. D. (1985). Indivisible labor and the business cycle. *Journal of monetary Economics*, 16(3):309–327.

Huggett, M. (1993). The risk-free rate in heterogeneous-agent incomplete-insurance economies. *Journal of economic Dynamics and Control*, 17(5-6):953–969.

Khan, A. and Thomas, J. K. (2013). Credit shocks and aggregate fluctuations in an economy with production heterogeneity. *Journal of Political Economy*, 121(6):1055–1107.

King, R. G., Plosser, C. I., and Rebelo, S. T. (1988). Production, growth and business cycles: I. the basic neoclassical model. *Journal of monetary Economics*, 21(2-3):195–232.

Krueger, D., Mitman, K., and Perri, F. (2016). Macroeconomics and household heterogeneity. In *Handbook of Macroeconomics*, volume 2, pages 843–921. Elsevier.

Krusell, P. and Smith, Jr, A. A. (1998). Income and wealth heterogeneity in the macroeconomy. *Journal of political Economy*, 106(5):867–896.

Kydland, F. E. and Prescott, E. C. (1982). Time to build and aggregate fluctuations. *Econometrica: Journal of the Econometric Society*, pages 1345–1370.

Nakajima, M. and Ríos-Rull, J.-V. (2019). Credit, bankruptcy, and aggregate fluctuations.

Storesletten, K., Telmer, C. I., and Yaron, A. (2004a). Consumption and risk sharing over the life cycle. *Journal of monetary Economics*, 51(3):609–633.

Storesletten, K., Telmer, C. I., and Yaron, A. (2004b). Cyclical dynamics in idiosyncratic labor market risk. *Journal of political Economy*, 112(3):695–717.