Pause, Reactivation, and Firm Dynamics

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September 29, 2024

Abstract

What is the macroeconomic implication of the option to pause and reactivate the operation of the production unit? To answer the question, we document the evidence of reactivation of the US establishments in data and study a model of heterogeneous firms with endogenous entry and exit. Temporarily inactive firms are inferred in the model that allows reactivation as in the data. In equilibrium, there is a smaller number of active firms than in an otherwise identical model without the option of pause and reactivation. Additional selections for active and inactive firms change the composition of the productivity distribution. In the long run, average productivity is almost unchanged, but aggregate productivity falls. The aggregate variables are reduced except for hours worked and fixed cost payment. The decline in the number of active firms explains most of the effect of introducing the option to pause and reactivate the operation.

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1 Introduction

Firms and establishments postpone their operation instead of exiting the market. On top of that, not every business opening is a "greenfield" startup meaning that some previously inactive businesses restart their operation. However, standard macroeconomic studies with firm dynamics consider entry as a startup and exit as a permanent exit that dissolves the firm. Although this can be regarded as a setting that provides a good approximation in aggregate, these studies are not free from the issue of overstating the role of startup and permanent exit. In order to fully assess the effect of entry and exit on the aggregate economy, the role of the option to pause and reactivate the operation should be examined. To this end, this paper aims to answer the following questions: does the option to pause and reactivate the operation matter in the aggregate economy? If it does, how and how much?

To answer the questions, we build a model of heterogeneous firms with endogenous entry and exit and employ a two-stage binary choice in the firm's problem. For reactivation to exist, pause and being temporarily inactive must have preceded. So, we infer the pausing and inactive firms in the model with the empirical evidence of reactivation in the US economy based on Business Dynamics Statistics (hereafter BDS). Entry and exit considering pause and reactivation would change the composition of the productivity distribution. On top of that, the model introduces temporarily inactive firms that can hold capital not utilized for productive activities and potentially deteriorates aggregate productivity. We analyze how these effects through the option of pause and reactivation shape the aggregate outcomes.

We document descriptive facts of reactivation of US establishments based on BDS. There exists business reactivation as a part of entrants over the sample period. Reactivation accounts for roughly 14 percent of entrants each year. On average, reactivated establishments are different from startups in that the average employment size by the reactivated establishment at the time of entry is smaller than that of startups. During the Great Recession, both startups and reactivation dropped, and they show reversal at the end of the recession. The share of reactivation among entrants rose to 22 percent and hover around 19 percent in recent sample periods. The composition of entrants has been changed.

Our model incorporates the option of pause and reactivation by a two-stage binary choice problem of continuation and operation. When it is calibrated to match the long-run properties of reactivation in data, the model produces such patterns well and provides predictions for pauses and firms sitting idle. The model infers 7 percent of total firms are inactive in the steady state. Unproductive and capital-small firms choose to pause their operation. Potential entrants with very low productivity signals can enter only as a startup. Above that level of the signal, reactivation allows more potential entrants to enter the market.

In matching the moments of reactivation, the difference in the number of potential entrants between reactivation and startups is important. Since the number of temporarily inactive firms is smaller than the number of potential startups, reactivation accounts for a smaller share of entrants than startups. For relative average size, the convexity of the labor demand function drives the result. The productivity distribution of startups is more dispersed, and the group of startups has more highly productive firms so that they prop up the average size.

To quantify the role of the option of pause and reactivation, we compare our full model to an otherwise identical reference model without the operation choice. In the full model, the number of active firms is reduced when two economies have the same total number of firms and aggregate exit rate. While the most unproductive firms are harder to be active next period, medium-productive firms are allowed to survive more easily in the full model economy. This change in composition affects productivity distribution little and the distribution of active firms is scaled down because of the smaller number of active firms. Aggregate variables are reduced in the full model economy except for hours worked and fixed costs. This mainly follows from the effect of change in the number of active firms. TFP is also compared across models. When we consider another reference economy without firms' entry and exit, the reference model with entry and exit features the highest TFP, and the full model economy lies between two reference models. This and aggregate values when compared across models imply that the option of pause and reactivation partially undo the result of the cleansing effect by selection in the reference model.

The possible extensions are discussed in the section after quantitative analysis. Capital adjustment costs should be considered to have a nontrivial distribution of inactive firms and to materialize the misallocation mechanism by idle capital. Other descriptive facts in the data are unexplained because the model and analysis now are focused on the long-run economy. Incorporating aggregate uncertainty in the model enables us to quantify the role of the option of pause and reactivation over business cycles.

This paper is related to the following strands of literature. First of all, this paper provides detailed entry and exit margins in firms' problems and studies how they influence the aggregate economy. Previous studies include Hopenhayn (1992), Lee and Mukoyama (2018), Clementi, Khan, Palazzo, and Thomas (2014), Smirnyagin (2018), and Ayres and

Raveendranathan (2021). While they have a single component for entry and exit, our paper considers the data-consistent entry and exit in addition to temporarily inactive firms in equilibrium.

Second, reactivation in the model adds cohort effect over business cycles because the fraction of reactivation in a cohort is time-varying in the model with aggregate uncertainty. Moreira (2016) and Sedláček and Sterk (2017) document that recession-born cohort starts small and stay small. This paper provides a new perspective on the composition of a cohort in the literature.

This paper employs the concept of idle capital that is held by temporarily inactive firms. Ottonello (2014) also considers unemployed capital considering search friction in the capital market. In contrast, idle capital in our model exists because of the firm's pausing decision without search friction. We see the role of idle capital in aggregate productivity. A new misallocation mechanism by temporarily inactive firms is a complementary study of Restuccia and Rogerson (2008).

The remainder of this paper is organized as follows. Section 2 provides empirical evidence about reactivation in the US economy. The model economy is followed in section 3. Section 4 analyzes the model economy and the role of pause and reactivation with the optimal policies. We do a quantitative analysis of steady state and inspect the core mechanisms with the comparison of models in section 5. Section 6 concludes.

2 Empirical Evidence

This paper focuses on the pause of operation, resultant establishment sitting idle, and its reactivation. Although temporary shutdown is often observable as an individual case, exit in publicly available data does not distinguish between businesses' permanent exit and temporary shutdown. The reactivated establishments are recorded as a part of birth or entry, and we can disentangle them from startups because they are different in terms of age. This section provides empirical evidence of the reactivated establishments of the US economy. The main findings of this section are summarized as follows.

Fact 1. Reactivated establishments account for roughly 14% of establishment entry each year

Fact 2. The average size of reactivated establishments is smaller than "greenfield" startups in the same cohort

Fact 3. The average employment share of reactivated establishments in the cohort hovers around 10%

Fact 4. Reactivation and startups co-move over sample periods before the Great Recession

Fact 5. The Great Recession features a reversal of the number of reactivated establishments and that of startups. This raises the fraction of reactivation to 22% after the recession and stayed around 19% onward.

Facts 1–3 are the long-run characteristics of reactivated establishments compared to the startups in the same cohort of entrants. *Facts 4–5* are associated with aggregate fluctuations. Especially, the Great Recession divides comovement between reactivated establishments and startups. The end of this section discusses potential explanations for the observed patterns in the data.

2.1 Data

The BDS database is a product of the U.S. Census Bureau. It is created based on the Longitudinal Business Database (LBD), covering longitudinal business establishments and firms that file payroll tax using Employment Identification Number (EIN). We use the annual information of the number of establishments and employment at birth by age for the period from 1984 to 2019 [12].

Entry, Exit, Temporary shutdown, and Reactivation BDS reports establishment entry (birth) and exit with annual frequency, and they are categorized with age. Entry and exit are determined by looking at the activeness of an establishment in the period including March 12 and the same observation a year ago. At period t, there is an entry when an establishment has positive employment at t but not in $t-1$. Likewise, an establishment is considered as an exit when it has zero employment at t but had positive employment

[1] The latest release of the BDS dataset includes a sample from 1978 to 2019. We choose the sample period from 1984 because of two reasons. BDS is left-censored data, so we do not know the age of the firm/establishment born before 1977. The reactivation is defined using the age category and the number of reactivated establishments is increasing in the early period of the sample because old firms/establishments start to be counted. Second, the Early period of the sample before 1984 contains a relatively large recession, then, selecting data from 1984 to 2006 excludes outliers and enables us to use sample moments to infer long-run population moments appealing to the Law of Large Numbers.

[2] We use establishments' reactivation based on BDS. BDS does not provide a firm's entry grouped by age. The way researchers identify firms' entry is by searching the number of firms with age 0. Therefore, we cannot distinguish between startups and reactivation in a unit of the firm.

in the previous period, $t-1$. This definition of entry and exit contains both startup and reactivation in entry and both permanent exit and temporary shutdown (pause) in an exit.

How can we disentangle reactivation from startup? Due to the categorization by age, the reactivation can be identified from entry. An establishment with age 0 at entry is a startup that has formed a business without any operation history before. Then, an establishment with an age greater than zero at the period of entry can be defined as a reactivated establishment. We use the following definition of startup and reactivation throughout the paper.

Definition 1 (Startup and Reactivation) *Startup* is a *de novo* establishment with positive employment at age 0. *Reactivation* is an entry with a history of activeness (age greater than 0)[3].

For the completeness, definition of permanent exit and temporary exit is provided and we use it throughout the paper.

Definition 2 (Permanent Exit and Temporary Exit) When an establishment *permanently exit*, it is dissolved. If an establishment *temporarily exit* (temporary shutdown), it ceases the operation.

One of the difficulties in explaining pause and resultant establishments sitting idle is identifying them in the data. Unfortunately, BDS does not individually report a temporary shutdown. This is also hard to be done because we need to track a longer horizon forward to check whether the now exiting establishment is going to reactivate the operation or not. So, it is difficult to check exiting establishment's mode of exit, for example, in 2019[4]. Therefore, we mainly use empirical facts from the reactivated establishments. Then, the standard model of heterogeneous firms with endogenous entry and exit is studied incorporating the option of pause and reactivation while targeting data moments of reactivation.

2.2 Descriptive facts

Population share of reactivated establishments We compare the number of reactivated establishments with the number of startups. Figure 1 displays the decomposed establish-

[3]The activeness is defined as payroll-activeness. Thus, if an establishment has positive employment, it is active. Here is the way BDS considers an establishment as being reactivated. When an establishment is active in year t, but was not active in $t-1$, the LBD looks up to 7 years in the past to determine whether the entrant is a reactivation.

[4]This also potentially leads to constantly changing the measure of permanent exit and temporary exit from each updated release of BDS data, especially for the recent sample period.

ment entry and the fraction of reactivated establishments in the cohort. On the left panel, the solid line is the number of reactivated establishments over the sample period. And the dashed line is for startups. Two characteristics arise by observing the figure. First, until the onset of the Great Recession, the number of reactivation and startups moved together, and it was also captured by the correlation coefficient, 0.61. Second, the Great Recession features a reversal of the two. The number of reactivated establishments soared to the highest level while the number of startups plunged to the lowest level in the sample.

On the right panel, the fraction of reactivated establishments is calculated as the share of reactivation among total entrants each year. Before the Great Recession, the population share hovers around 14%. During the Great Recession, it dropped slightly then rose to 22% for three years. The fraction does not come back to the pre-recession level and stays around 19%. Notably, a non-negligible share of entrants each year consists of the reactivated establishment. Therefore, this is a piece of indicative evidence for the necessity of understanding reactivation to fully characterize the composition of the entering cohort each year. This is also suggestive of the larger number of temporary shutdowns in exits than reactivation. Since not every temporary shutdown business comes back and reopens the next period, the share of temporary shutdowns can be larger than 14% among exits on average.

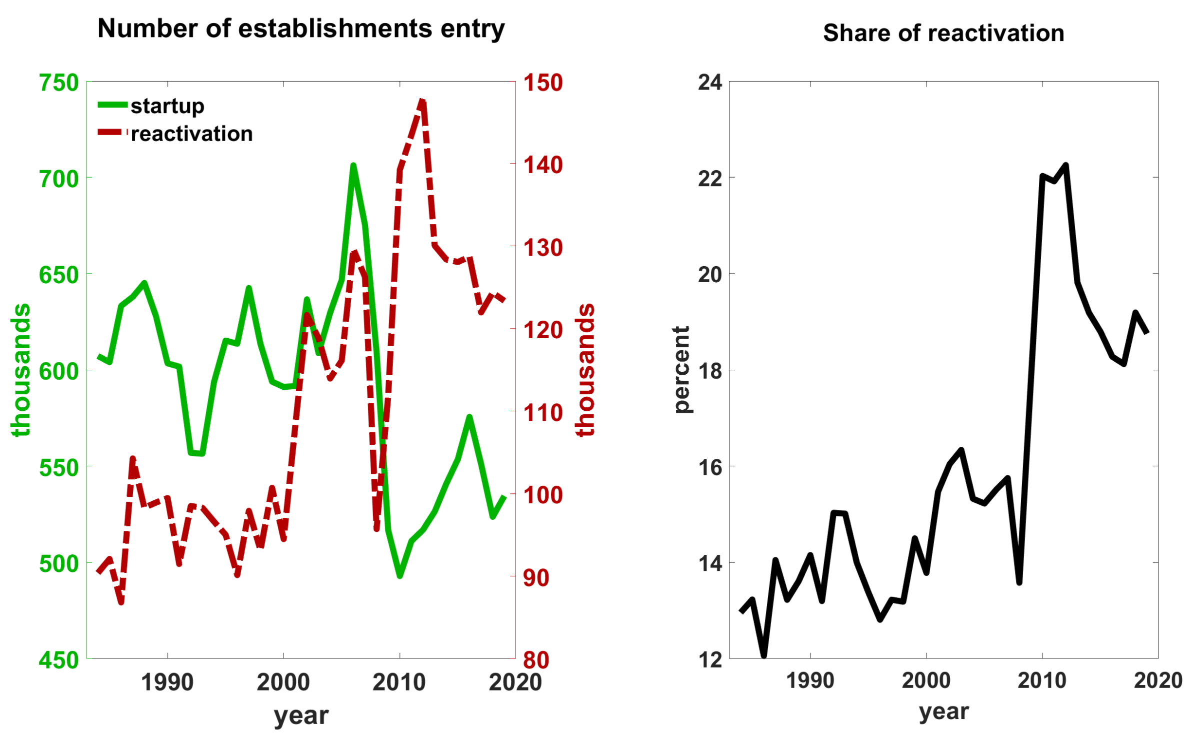

Figure 1: Decomposition of Establishment Births

Relative average size of reactivated establishments The quality of reactivated establishments may also be different from startups. One of the available information in BDS is the initial employment size when the establishment enters. The average size is calculated by the arithmetic mean of each group of entrants.

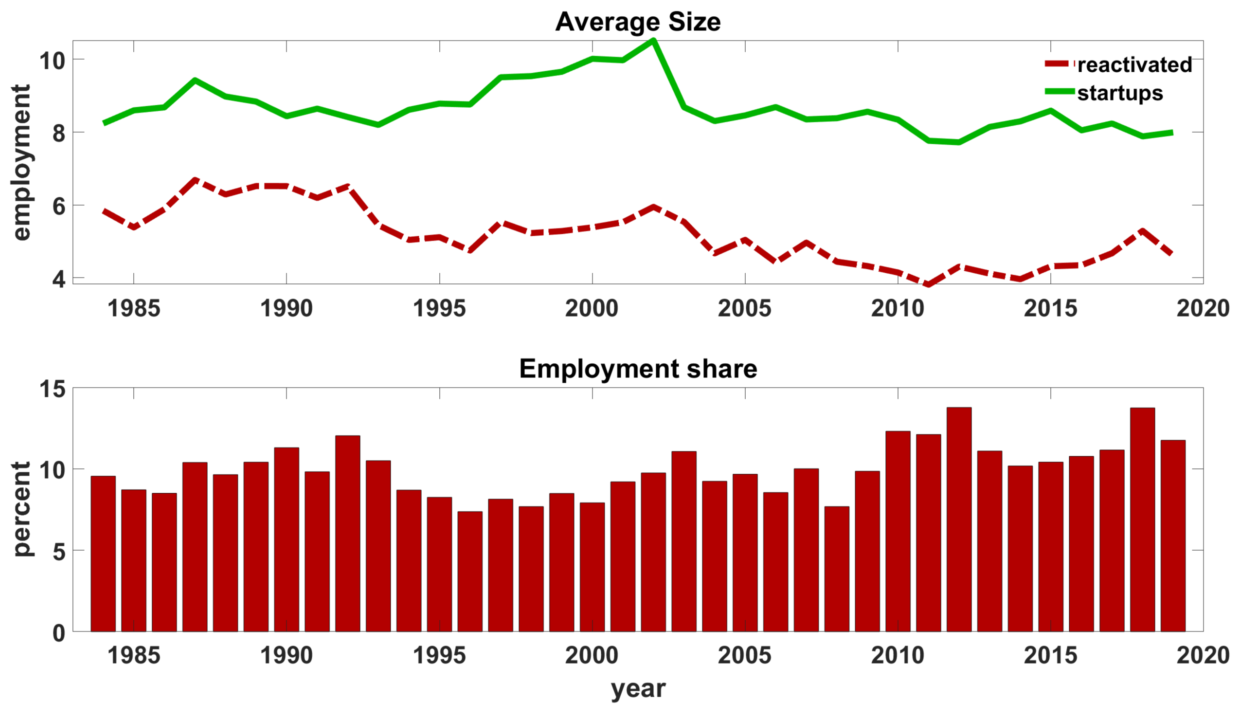

Figure 2: Average Size of Entrants and Employment Share of Reactivation

The top panel of Figure 2 shows the average size of reactivated establishments and startups. On average, the initial size of startups is 9. The reactivated establishments are smaller than startups over the sample period and steadily declined from 6 to 4 until the end of the Great Recession and it rose to 5 in the most recent year. Overall, the average size of reactivated establishments is about 62% of the average size of startups.

Employment share of reactivated establishments Lastly, we document the employment share of reactivated establishments in the cohort. Directly implied by the population share and small average size, the total employment share ends up small. At the bottom of Figure 2, the employment share of reactivated establishments in the entering cohort is presented. There are small fluctuations with upticks right at the end of each NBER recession period. On average, the total employment share hovers around 10%.

Evidence from other studies Chow et al. (2021) documents the population of reactivated establishments in LBD from 1978–2018 categorized by the number of years since they were last observed as active. In Table 1, most reactivated establishments were temporarily inactive for one or two years (2-year and 3-year types of reactivation). The weighted average of the number of years since they were last observed as active provides the average duration of being temporarily inactive. The number based on Table 1 is 2.0422 years. We may use this number for calibration later to characterize the reactivation.

Table 1: Establishment Reactivation 1978–2018

Type of Reactivation	2-year	3-year	4-year	5-year	6-year	7-year
Avg. Num. of Establishments per year	39,751	17,522	8,373	5,552	3,943	2,936
As a Pct. of Establishment Births	5.29	2.33	1.11	0.74	0.52	0.39

Source: Chow et al. (2021)

Potential explanations for *Fact 1–5* The empirical facts that hold steadily over the sample period can be considered as long-run properties of reactivated establishments. *Fact 1* could be explained by the difference between the number of temporarily inactive establishments and the number of potential startups. If the number of temporarily inactive establishments is smaller than the number of potential startups, then, the number of reactivated establishments can be limited. The second possible scenario is the case when the hurdle to enter the market is higher for reactivation than for a new startup. If this is the case, then even if both have the same number of potential entrants, reactivation will show less as a result.

Fact 2 also has multiple possible explanations. First, the larger average size of startups can result from a few high-productivity startups in the cohort while the group of reactivated establishments lacks such high-productivity peers. Alternatively, the difference in average size can stem from the average productivity gap while fixing the mass of high-productivity potential entrants the same. If startups are more productive on average, their initial employment is going to be larger than the reactivated establishment. Finally, *Fact 3* immediately holds when *Facts 1* and *2* are satisfied.

Facts 4–5 are related to business cycles. Temporarily inactive firms that are waiting for re-entry see that the expected value of the entry and the expected survival rate would be higher when the aggregate state is on the path of recovery or expansion. This predicts procyclical reactivation and a similar explanation would apply to startups, in turn, explains the comovement of reactivation and startups. For *Fact 5*, the reversal of reactivation and startup, we have two potential explanations. First, there was a large number of startups that entered just before the onset of the Great Recession (Figure 1). And the large exit at the recession may have contained a good amount of temporary exit. They came back as a form of reactivation. The second explanation is related to a financial shock. Financial shock hinders potential startups to get external finance to form the business. However, inactive firms hold capital already and they can even sell it off to resize the business. Hence, when it comes to financial shock, reactivation is easier to be implemented.

3 Model Economy

The option of pause and reactivation and its macroeconomic implication are studied in the model of heterogeneous firms with endogenous entry and exit. The model economy heavily depends on Clementi et al. (2014) and is extended to incorporate the option to pause and reactivate the operation for firms.

We consider capital heterogeneity for the analysis because we use both size and age information together in the previous data part[5]. The model without capital heterogeneity, for example, Hopenhayn (1992), has no role for an age when it is controlled by the size, vice versa. Capital heterogeneity is also important in that it allows the model to analyze the effect of the unemployed capital which arises because of firms with temporarily shut-down. If this pausing firms are not successfully regain their productivity as they grow, idle capital may harm the efficient allocations, in turn, aggregate productivity.

Time is discrete and the horizon is infinite. The economy consists of three decision makers: i) incumbent firms, ii) potential startups, and iii) representative households. Households supply labor and own all firms in the economy. Each period, there is a fixed measure of potential startups, and they differ across productivity signals. A part of them becomes startups by paying entry costs. There is a positive measure of price-taking firms in the economy. At the beginning of the period, incumbent firms are heterogeneous across capital stock, idiosyncratic productivity, and the operation history. After observing idiosyncratic productivity, a firm is able to operate by paying fixed cost of operation. If operating cost is not paid, then the firm should pause the operation. Pausing firm next faces continuation decision to stay in the economy. By paying the fixed cost of continuation, it can survive and chooses next period capital. If it decides not to pay continuation cost, then it is fully liquidated. In order to hire workers and produce, firms should pay

[5]Note that reactivation and startups are defined as different age groups at birth. We derive the characteristics of reactivated establishments from their relative size to startups

operating cost. The distribution of operating cost is different across operation history [6].

3.1 Incumbent firms

At the beginning of the period, an incumbent firm with pre-determined capital, $k \in \mathcal{K} \subset \mathbb{R}^+$ and operation history, $h \in \mathcal{H} = \{0, 1\}$, faces the realization of stochastic aggregate productivity, $z \in \mathcal{Z} \subset \mathbb{R}^+$, and idiosyncratic productivity, $\epsilon \in \mathcal{E} \subset \mathbb{R}^+$, which both follows Markov process. Hence, they are heterogeneous across the capital, operation history, and idiosyncratic productivity. The firm produces homogeneous goods exhibiting decreasing returns to scale technology with strictly increasing and strictly concave production function, $F(k, n)$. Thus, production is determined by $y = z\epsilon F(k, n)$. After the production, the active firm decides on continuation and operation for the next period by solving the two-stage binary choice problem. In the first stage, a firm can continue by paying the continuation cost, c_1^c where H is cumulative density function. Otherwise, a firm is dissolved, and we call it a permanent exit. Conditional on continuation, a firm faces the second stage that determines the next period's operation. In order to stay active and operate the next period, the operating cost, c_1^f must be paid. Or it can choose to pause the operation and becomes temporarily inactive next period. The timing of an active firm at period t is described in Figure 3. Fixed cost associated with continuation and operation choices are randomly drawn from a distribution with cumulative density function, $H_h^j(c_h^j)$ for $j = \{c, f\}$ and $h = 1, 2$ over bounded support, $[c_h^{j,L}, c_h^{j,U}] \subset \mathbb{R}^+$. We use subscript h for other functions and variables to identify the type of a firm.

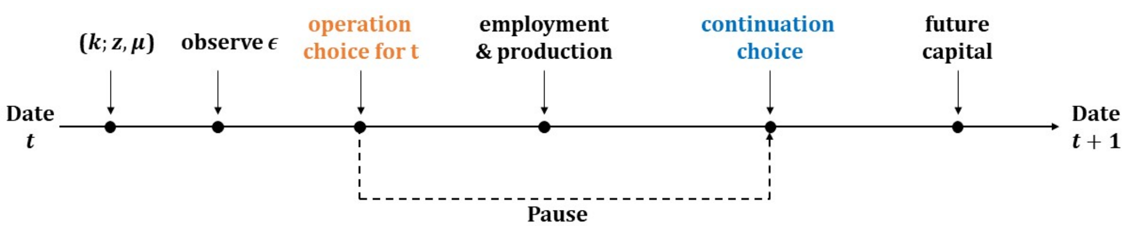

Figure 3: Timing of Incumbent Firm in Period t

The optimization problem of an incumbent firm can be described as follows. Given the current aggregate state, (z, μ), let $v^0(k, \epsilon, h; z, \mu)$ the value of an incumbent firm that starts the period with capital, k, idiosyncratic productivity, ϵ, and operation history, h, at

[6] The option of pause and reactivation is essentially friction for incumbent firms. We let this difference in distribution of operating cost and see if this is quantitatively consistent in the calibration step.

the beginning of the period;

$$v^0(k,\epsilon,h;z,\mu) = \int_{c^h_{f,L}}^{c^h_{f,U}} v^1(k,\epsilon,h,c^h_f;z,\mu) H^h_f(dc^h_f) \tag{1}$$

The value function $v^1(k,\epsilon,h,c^h_f;z,\mu)$ represents an active firm's expected discounted value just after observing its operating cost c^h_f;

$$v^1(k,\epsilon,h,c^h_f;z,\mu) = \max_{h'=\{0,1\}} \Bigg\{ (1-\delta_0)k + \int_{c_{c,L}}^{c_{c,H}} v^2(k,\epsilon,h,h'=0,c_c;z,\mu)H_c(dc_c), \tag{2}$$

$$-c^h_f + \pi(k,\epsilon,h;z,\mu) + (1-\delta)k + \int_{c_{c,L}}^{c_{c,U}} v^2(k,\epsilon,h,h'=1,c_c;z,\mu)H_c(dc_c) \Bigg\} \tag{3}$$

where the flow profit function is defined as

$$\pi(k,\epsilon;z,\mu) \equiv \max_n [z\epsilon F(k,n) - w(z,\mu)n], \tag{4}$$

and $v^2(k,\epsilon,h,h',c_c;z,\mu)$ is the expected discounted value just after observing its continuation cost, c_c. The second discrete decision is about the continuation. Let $d_{z'}(z,\mu)$ denote the discount factor each firm applies to its next-period value conditional on a realization of future aggregate productivity, z'. Taking the evolution of the firm distribution as given, $\mu' = \Gamma(z,\mu)$, the firm solves the following discrete choice problem.

$$v^2(k,\epsilon,h,h',c_c;z,\mu) = \max\Big\{0, -c_c + v^3(k,\epsilon,h,h';z,\mu)\Big\} \text{ where} \tag{5}$$

$$v^3(k,\epsilon,h,h'=1;z,\mu) = \max_{k'\in\mathcal{K}} \Big[-k' + \mathbb{E}_{z',\epsilon'}[d_{z'}(z,\mu)v^0(k',\epsilon',h'=1;z'\mu')|z,\epsilon]\Big] \tag{6}$$

$$v^3(k,\epsilon,h,h'=0;z,\mu) = \max_{k'\in\mathcal{K}} \Big[-k' + \mathbb{E}_{z',\epsilon'}[d_{z'}(z,\mu)v^0(k',\epsilon',h'=0;z',\mu')|z,\epsilon]\Big] \tag{7}$$

$v^3(k,\epsilon,h,h';z,\mu)$ represents the value of continuation.

Since a firm's history of operation affects the expected value, we let $g(k,\epsilon,h,h';z,\mu)$ denote the optimal decision rule for future capital solving 6 and 7. The decision of operation and continuation is described using the indicator function, χ_f and χ_c

$$\chi_f(k,\epsilon,h,c^h_f;z,\mu) = \begin{cases} 1 & \text{if } -c^h_f + \pi(k,\epsilon,h;z,\mu) + (\delta_0-\delta)k \\ & + \int_{c_{c,L}}^{c_{c,U}} [v^2(k,\epsilon,h,h'=0,c_c;z,\mu) \\ & -v^2(k,\epsilon,h,h'=1,c_c;z,\mu)]H_c(dc_c) \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

$$\chi_c(k,\epsilon,h,h',c_c;z,\mu) = \begin{cases} 1 & \text{if } -c_c + v^3(k,\epsilon,h,h';z,\mu) \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

3.2 Potential startups

There is a fixed measure, $M > 0$, of potential startups each period. Each potential startup draws a productivity signal, s, and observes the entry cost, c^e. Then, it chooses whether to pay an entry cost to become a startup next period. Potential startups that have decided to enter the next period choose a capital stock they start with at the beginning of the next period. By definition of startups, they are exempted from operating decision as they enter. Following functional equations describes the optimization problem for a potential startups identified by $(s, c_e; z, \mu)$.

$$v^{ps}(s,c_e;z,\mu) = \max\{0, -c_e + v^e(s;z,\mu)\} \tag{8}$$

$$v^e(s;z,\mu) = \max_{k'\in\mathcal{K}} \left[-k' + \mathbb{E}_{z',\epsilon'}[d_{z'}(z,\mu)v^{11}(k',\epsilon';z',\mu')|z,s]\right] \tag{9}$$

where

$$\begin{aligned} v^{11}(k',\epsilon',h'=1;z',\mu') &= \pi(k',\epsilon',1;z',\mu') + (1-\delta)k' \\ &\quad + \int_{c_{c,L}}^{c_{c,U}} v^2(k',\epsilon',h'=1,h''=1,c_c';z',\mu')H_c(dc_c) \end{aligned}$$

We let $g_e(s;z,\mu)$ denote the optimal decision rule for future capital solving 9. And entry decision is described using the indicator function χ_e

$$\chi_e(s,c_e;z,\mu) = \begin{cases} 1 & \text{if } -c_e + v^e(s;z,\mu) \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Timing of a potential startup is described in Figure 4

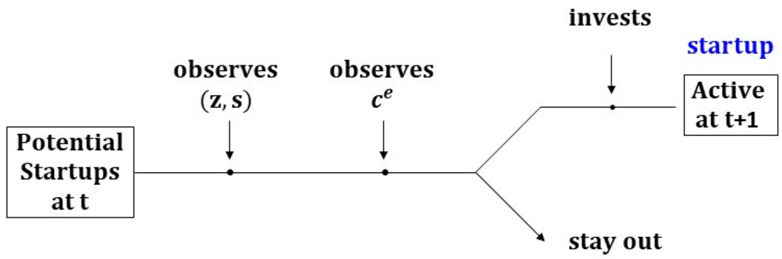

Figure 4: Timing of Potential Startup in Period t

3.3 Representative households

The economy is populated by a unit measure of infinitely lived, identical risk-averse households. Households choose consumption, c, hours worked, n^h, and the number of share purchases, $\lambda'(k', \epsilon')$ to maximize the lifetime expected utility. The problem is as follows,

$$W(\lambda; z, \mu) = \max_{c, n^h, \lambda'} \left[u(c, 1 - n^h) + \beta \mathbb{E}_{z'}[W(\lambda'; z', \mu')|z] \right] \tag{10}$$

subject to

$$c + \int_{\mathcal{K} \times \mathcal{E}} \rho_1(k', \epsilon'; z, \mu) \lambda'(d[k' \times \epsilon']) \leq w(z, \mu) n^h + \int_{\mathcal{K} \times \mathcal{E}} \rho_0(k, \epsilon; z, \mu) \lambda(d[k \times \epsilon])$$

where $\rho_1(k', \epsilon'; z, \mu)$ is ex-dividend price of share, $\rho_0(k, \epsilon; z, \mu)$ is dividend-inclusive price of share and $w(z, \mu)$ is real wage. Households also can access the full set of contingent claims, but it is not expressed in the budget constraint because the net supply is zero in equilibrium because the households are identical.

Let $C(\lambda; z, \mu)$ denote the household optimal consumption choice and let $N(\lambda; z, \mu)$ be its choice of hours worked. Finally, let $\Lambda(k', \epsilon', \lambda; z, \mu)$ be the number of shares purchased in firms that will begin the next period with k' units of capital stock and idiosyncratic productivity ϵ'.

3.4 Recursive competitive equilibrium

A *recursive competitive equilibrium* is a set of functions,

$$\left(w, d_{z'}, \rho_0, \rho_1, v^a, v^i, n, (g_h^p)_{h=1}^2, (g_h^f)_{h=1}^2, (\chi_h^c)_{h=1}^2, (\chi_h^f)_{h=1}^2, v^{ps}, g^e, \chi^e, W, C, N, \Lambda \right)$$

that solve firms' and households' problems and clear the markets for assets, labor, and output, as follows.

(i) v^a solves (1) - (7), and $(\chi_1^c, \chi_1^f, n, g_1^p, g_1^f)$ are associated policy functions for active firms.

(ii) v^i solves (**??**) - (**??**), and $(\chi_2^c, \chi_2^f, g_2^p, g_2^f)$ are associated policy functions for inactive firms.

(iii) v^{ps} solve (8) - (9), and χ^e and g^e are the associated policy functions for potential startups.

(iv) W solves (10), and (C, N, Λ) are the associated policy functions for households.

(v) $\Lambda(k', \epsilon', \mu; z, \mu) = \mu'(k', \epsilon'; z, \mu)$, for each $(k', \epsilon') \in \mathcal{K} \times \mathcal{E}$.

(vi) $N(\mu; z, \mu) = \int_{\mathcal{K}\times\mathcal{E}} n(k, \epsilon; z, \mu)\mu_1(d[k \times \epsilon])$.

(vii) $C(\mu; z, \mu) = \int_{\mathcal{K}\times\mathcal{E}} z\epsilon F(k, n(k, \epsilon; z, \mu))\mu_1(d[k \times \epsilon])$.

$$
\begin{aligned}
&- \int_{\mathcal{K}\times\mathcal{E}} \int_{c_1^{c,L}}^{c_1^{c,U}} \chi_1^c(\epsilon, c_1^c; z, \mu) \Bigg[c_1^c + \int_{c_1^{f,L}}^{c_1^{f,U}} \Bigg(\chi_1^f(\epsilon, c_1^f; z, \mu)[c_1^f + (g_1^f(\epsilon; z, \mu) - (1-\delta)k)] \\
&+ (1 - \chi_1^f(\epsilon, c_1^f; z, \mu))[g_1^p(\epsilon; z, \mu) - (1-\delta)k] \Bigg) H_1^f(dc_1^f) \Bigg] H_1^c(dc_1^c)\mu_1([k \times \epsilon]) \\
&+ \int_{\mathcal{K}\times\mathcal{E}} \int_{c_1^{c,L}}^{c_1^{c,U}} (1 - \chi_1^c(\epsilon, c_1^c; z, \mu))[(1-\delta)k] H_1^c(dc_1^c)\mu_1([k \times \epsilon]) \\
&- \int_{\mathcal{K}\times\mathcal{E}} \int_{c_2^{c,L}}^{c_2^{c,U}} \chi_2^c(s, c_2^c; z, \mu) \Bigg[c_2^c + \int_{c_2^{f,L}}^{c_2^{f,U}} \Bigg(\chi_2^f(s, c_2^f; z, \mu)[c_2^f + (g_2^f(s; z, \mu) - k)] \\
&+ (1 - \chi_2^f(s, c_2^f; z, \mu))[g_2^p(s; z, \mu) - k] \Bigg) H_2^f(dc_2^f) \Bigg] H_2^c(dc_2^c)\mu_2([k \times s]) \\
&+ \int_{\mathcal{K}\times\mathcal{E}} \int_{c_2^{c,L}}^{c_2^{c,U}} (1 - \chi_2^c(s, c_2^c; z, \mu)) k H_2^c(dc_2^c)\mu_2([k \times s]) \\
&- M \int_{\mathcal{S}} \int_{c^{e,L}}^{c^{e,U}} \chi^e(s, c^e; z, \mu)[c^e + g^e(s; z, \mu)] H^e(s; z, \mu) f_s(ds)
\end{aligned}
$$

where f_s is the probability density function of productive signal s.

(viii) Distribution of active firms evolves following $\mu_1' \equiv \Gamma_1(z, \mu_1, \mu_2)$ where

$$
\begin{aligned}
&\mu_1'(D_1, \epsilon_m) \\
&= \int_{\{(k,\epsilon_l)|g_1^f(k,\epsilon_l)\in D_1\}} \chi_1^c(\epsilon_l, c_1^c)\chi_1^f(\epsilon_l, c_1^f)\pi_{lm}^{\epsilon} H_1^c(dc_1^c) H_1^f(dc_1^f)\mu_1(d[k \times \epsilon_l]) \\
&+ \int_{\{(k,s_l)|g_2^f(k,s_l)\in D_1\}} \chi_2^c(s_l, c_2^c)\chi_2^f(s_l, c_2^f)\pi_{lm}^{\epsilon} H_2^c(dc_2^c) H_2^f(dc_2^f)\mu_2(d[k \times s_l]) \\
&+ M \int_{\{s_l|g^e(s_l)\in D_1\}} \pi_{lm}^{\epsilon} \int \chi^e(s_l, c^e) H^e(dc^e) f_s(ds_l)
\end{aligned}
$$

for all measurable open subset $(D_1, \epsilon_m) \subset (\mathcal{K} \times \mathcal{E})$.

(ix) Distribution of inactive firms evolves following $\mu_2' \equiv \Gamma_2(z, \mu_1, \mu_2)$ where

$$
\begin{aligned}
&\mu_2'(D_2, s_m) \\
&= \int_{\{(k,\epsilon_l)|g_1^p(k,\epsilon_l)\in D_2\}} \chi_1^c(\epsilon_l, c_1^c)(1 - \chi_1^f(\epsilon_l, c_1^f))\pi^s(s_m) H_1^c(dc_1^c) H_1^f(dc_1^f)\mu_1(d[k \times \epsilon_l]) \\
&+ \int_{\{(k,s_l)|g_2^p(k,s_l)\in D_2\}} \chi_2^c(s_l, c_2^c)(1 - \chi_2^f(s_l, c_2^f))\pi^s(s_m) H_2^c(dc_2^c) H_2^f(dc_2^f)\mu_2(d[k \times s_l])
\end{aligned}
$$

for all measurable open subset $(D_2, s_m) \subset (\mathcal{K} \times \mathcal{S})$.

(x) Total firm distribution is

$$
\mu(K, E) \equiv \mu_1(K, E) + \mu_2(K, E)
$$

for all open subset $(K, E) \subset (\mathcal{K} \times \mathcal{E})$ and it defines $\Gamma \equiv \Gamma_1 + \Gamma_2$.

4 Analysis

For general equilibrium analysis, we choose the equilibrium prices that are implied by households' optimality. We follow the same method of choosing equilibrium prices and redefining a firm's value function in the unit of marginal utility of consumption as in Khan and Thomas (2008) and Clementi et al. (2014). Therefore, general equilibrium is analyzed by solving the firm's problem.

4.1 Optimal decisions

4.1.1 labor demand and future capital choice

labor is demanded by any active firm. Because of DRS technology, labor demand is both increasing in current capital and idiosyncratic productivity and strictly concave in capital and strictly convex in productivity.

There are five optimal decision rules for future capital that narrow down to two. For each type of a firm there are two options for operation next period. Capital is adjusted either when the firm decides to operate next period or when it chooses to pause the operation. Without any real friction, the two-stage binary choice of continuation and operation is independent of the current level of capital. And the expected value of operation and expected value of pause are the same across the type of firm. The potential startup also chooses future capital as it enters the next period. The expected value of being a startup next period is the same as the expected value of operation for the firm. The decision rule for future capital is the same for active firm staying active, inactive firm that operates next period, and potential startup. The firm that chooses to pause the operation is also able to adjust the capital. Since the expected value of pause is the same across the type, they share the same optimal decision rule. When sitting idle, current capital has nothing to do with continuation and operation choices. So optimal level of capital for pausing firms is zero.

4.1.2 continuation, operation, and startup

For binary choices, we follow Clementi et al. (2014). We have binary choices about continuation, operation, and startup. They are all subject to a fixed cost that is randomly drawn from a distribution. We assume that fixed cost is a continuous random variable. The optimal policy of binary choice is the threshold policy. Therefore, for a fixed cost that is below the threshold cost, the firm would continue and operate. Similarly, a Potential startup would enter with a fixed cost at or below the threshold cost.

With threshold policy and fixed cost as a continuous random variable, we can compute the probability or fraction of continuation, operation, and startup by the cumulative distribution function of each distribution. This simplifies calculating expected value and aggregation by appealing to the Law of Large Numbers.

4.2 Pause and reactivation, entry and exit

As pointed out in the section on data, entry is not identical to startup and exit is not equal to permanent exit. The model economy now has two components in entry and exit by employing the option of pause and reactivation. We define entry and exit in the model as follows.

Definition 3 (Entry and Exit) *Entry* in period t is the sum of the number of startups and reactivation in t. *Exit* in period t is defined as a sum of permanent exit and pause from previously active firms. Both *entry* and *exit* are determined from the two-stage continuation and operation choice at the end of the previous period.

We look at entry and exit based on the active firm distribution. The outflows of the distribution are permanent exits and pauses. The inflows of the distribution are startup and reactivation. This difference with the definition in the standard model has an implication more than accounting.

4.2.1 misallocation

We suggest a new misallocation channel from the option of pause and reactivation. Note, however, the model we present now abstracts from capital adjustment cost so the following analysis is not be applied in the rest of the paper.

When a firm permanently exits by dissolution, the scrap value, here is current profit plus undepreciated capital, is going to be used by representative households for consumption and firms' investment. Since a firm can pause the operation, exit does not mean permanent exit with full liquidation. Firms sitting idle in equilibrium will hold capital if there is an adjustment cost of capital. These capitals not used for any production process will eventually reduce TFP.

Note also that not every pausing firm is going to re-enter the market. Some temporarily inactive firms decide to permanently exit, and they look like delaying exit decisions when they were active by choosing to pause instead of not choosing continuation. When these firms are dissolved, reallocation would happen, but it is delayed. Therefore, pause and reactivation behavior can potentially explain slow recovery from recession through misallocation.

5 Quantitative Analysis

This section provides the specific functional forms, calibration strategies to set the parameters, and numerical methods. Then, steady state analysis is provided. Lastly, we do some comparison exercises with the model without the option of pause and reactivation to shed light on the effect of pause and reactivation on the aggregate outcomes.

5.1 Functional forms

We use the period utility function that features indivisible labor supply stressed by Rogerson (1988) and Hansen (1985), $u(C, L) = \log C + \theta L$. For the production function, we employ Cobb-Douglas production function, $F(k, n) = k^{\alpha} n^{\nu}$ with $\alpha + \nu < 1$. The productivity signal is drawn from bounded Pareto distribution with the shape parameter, κ and the minimal and maximal value as ϵ_{min} and ϵ_{max}, respectively. We assume the fixed cost of continuation, operation, and entry follows a uniform distribution with different supports.

5.2 Calibration

The model period is a year to match the data frequency of BDS. By setting lower bound of support for uniform distributions as zero for every fixed cost, Only the upper bounds of the supports remain to be set. Also, By putting the realizations of signal on the same state space with the idiosyncratic productivity process, we do not need to set the scale parameters for the Pareto distribution.

For now, the remaining parameters are picked up from Clementi et al. (2014) except $\{\theta, c_1^{c,U}, c_1^{f,U}, c_2^{c,U}, c_2^{f,U}, c^{e,U}, M\}$ that directly affect the continuation and operation choices of firms and entry choice of potential startups, in turn, the aggregate exit rate. θ changes labor supply to match the total hours worked and M helps to set the total mass of firms in equilibrium. Table 2 provides a set of parameters adopted from Clementi et al. (2014).

α	ν	δ	β	ρ_{ϵ}	σ_{ϵ}	κ
0.26	0.6	0.069	0.96	0.757	0.182	5.2

Table 2: Parameter Set A

For calibrating the rest of the parameters, I ignore the identification issue deliberately for now and choose parameters to achieve moments that are close to the data[7]. Table 3 shows a group of parameters that are set to match the target moments.

[7] This is not an actual calibration. I chose some arbitrary numbers for the parameters in Table 3.

θ	$c_1^{c,U}$	$c_1^{f,U}$	$c_2^{c,U}$	$c_2^{f,U}$	$c^{e,U}$	M
2.5	0.09	0.08	0.048	0.02	0.35	0.6

Table 3: Parameters Set B

The matched moments given the set of parameters in Table 2 and 3 are provided in Table 4. The model can produce aggregate exit rate of 9.83%, aggregate employment 0.327, the total mass of firms 0.94, which are slightly lower than the data counterpart. Population share of reactivated firms among entrants 15.52%, and the relative average size of reactivated firms to startups 67.48%. The model predicts moments about reactivation a bit larger than that observed in data.

Moments	Data	Model
Exit rate	10.760	9.833
Employment	0.333	0.327
Total mass of firms	1.000	0.929
Fraction of reactivation	14.143	15.212
Relative size of reactivation	62.758	67.328
Employment share of reactivation	9.342	10.778

Table 4: Target Moments

5.3 Numerical methods

We describe the numerical methods used in computing the steady state of the model. As we subsume the households' optimization into firms' problems, equilibrium can be achieved by solving firms' problems with appropriate prices. The reformulation of the problem and equations for aggregation are provided in Appendix A. The challenge compared to the standard model of heterogeneous firms in the literature is that now we need to solve temporarily inactive and active firms' problems simultaneously. Thus, value function iteration for the active firm nests the value function iteration for the temporarily inactive firm. The other way also works . The main idea here is solving for the value function of two firms at the same time so that they are fixed points for each other. Here is the algorithm implemented to get the steady state.

1. Guess p

2. Guess V^a and V^i

3. Solve for inactive firm's problem using the value function iteration. This yields TV^i
4. Iterate 2–3 until $sup||TV^i - V^i||$ is sufficiently small; you get optimal decision rules for capital, continuation, and operation
5. Use fixed point V^i and proceed to solve for active firm's problem using the value function iteration. This yields TV^a
6. Iterate 2–5 until $sup||TV^a - V^a||$ is sufficiently small; you get optimal decision rules for capital, continuation, and operation
7. Given fixed point V^a, solve for potential startups problem; you get optimal decision of entry and future capital
8. Compute stationary distribution following the equations in RCE definition (viii) and (ix) such that $sup||T\mu_1 - \mu_1||$ and $sup||T\mu_2 - \mu_2||$ are sufficiently small.
9. Calculate aggregate variables using optimal decision rules and stationary distributions
10. Compute implied price ($p_{implied}$) from aggregate consumption
11. Repeat 1–10 with updated guess for price and stop if p is close enough to $p_{implied}$

We use an evenly spaced grid for physical capital. The idiosyncratic productivity is discretized using the Tauchen algorithm. We make productivity signals lie on the same grid points with idiosyncratic productivity and apply the same method to get discretized probability density function of bounded Pareto distribution. For searching equilibrium price, p, we use the bisection algorithm.

5.4 Steady state analysis

In this section, we explore the model in its steady state and inspect the mechanism worked in matching the moments of reactivation.

5.4.1 stationary distribution

Table 5 provides the measure of each type of firm in stationary distribution. Entrant is an active firm in the current period with no production (i.e., employment) in the previous period. In steady state, the share of entrants is the same as the entry rate, 9.83. Reactivated firms account for 1.5% of active firms. In the long run, the share of reactivated firms implies relatively less quantitative importance of them. However, like any entrants, they grow over the period and can have a meaningful impact on aggregates in the future. Also, in the short run, reactivated firms change the composition of the entering cohort, hence, the existence of reactivation in the entry can be meaningful over business cycles.

Our model generates temporarily inactive firms in equilibrium. Although we do not have direct observations about inactive firms in data, the model tells us what they look like given the information about reactivated firms that were previously inactive. Our model economy predicts a non-negligible measure of inactive firms in steady state (7% of all firms). 57% of them were also inactive in the previous period. The rest 43% are active firms that chose to pause in the previous period.

Stationary distribution implies that inflows and outflows of distribution must be equal. For active firm distribution, the inflows are startups, and reactivation and outflows are permanent exits and pauses. Note that the amount of exit by dissolution is not the same as the number of startups, which is different from the steady state result from standard heterogeneous firms with endogenous entry and exit studies. In this model, we have more startups than permanent exits by replacing a part of pausing firms.

	Active	Incumbent	Startups	Reactivated	Permanent Exit	Pause
measure	0.866	0.780	0.072	0.013	0.057	0.028
(%)	(93.13)	(90.17)	(8.33)	(1.50)	(6.63)	(3.20)
Total	**Inactive**	Stay inactive	New pause			
0.929	0.064	0.036	0.028			
(100.00)	(6.87)	(56.65)	(43.35)			

Note: numbers in blue denote the share of specific components in each distribution categorized by activeness.

Table 5: Active & Inactive Firm Distribution

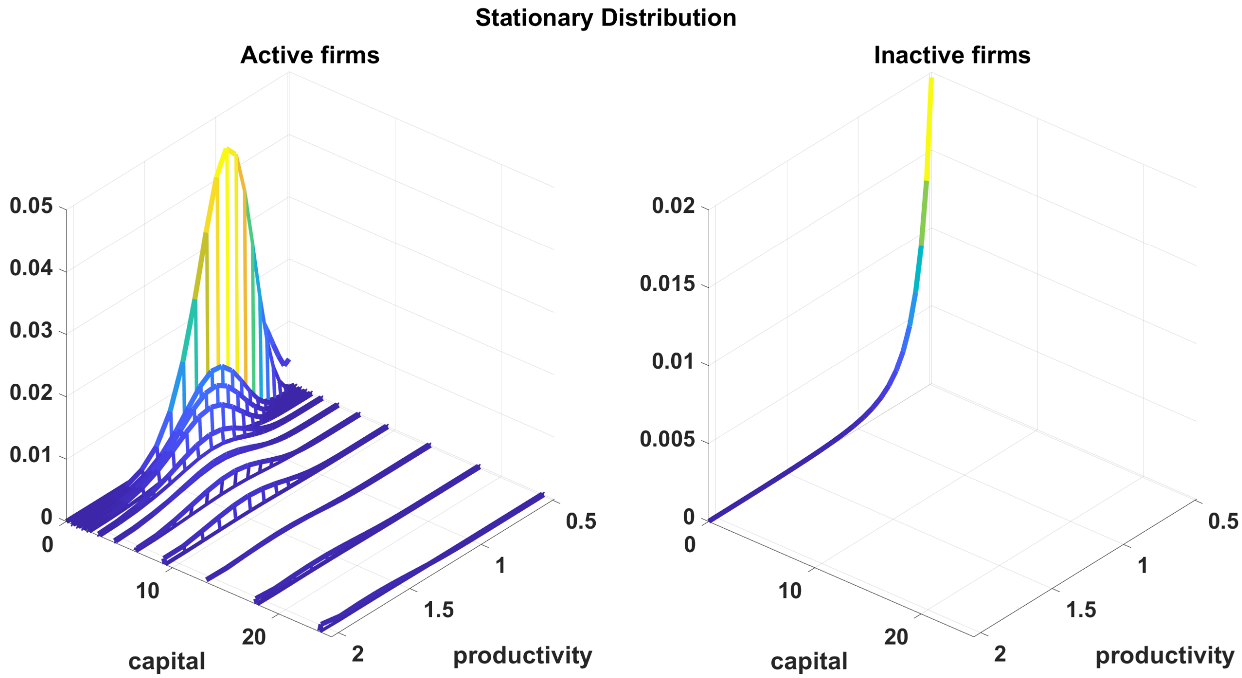

Figure 5: Active and inactive firm distribution

Figure 5 depicts the stationary distribution of active and inactive firms over capital and idiosyncratic productivity space. As the model abstracts from real friction in adjusting capital and the capital has no role in continuation/operation decision, inactive firms do not hold capital. And they are going to buy capital again as much as they need when re-entering the market later by observing the productivity signal.

Combining active and inactive firm distribution yields a stationary distribution of whole firms in the economy. In Figure 6, the uptick in the region of zero capital and productivity near 0.5 follows from the inactive firm distribution.

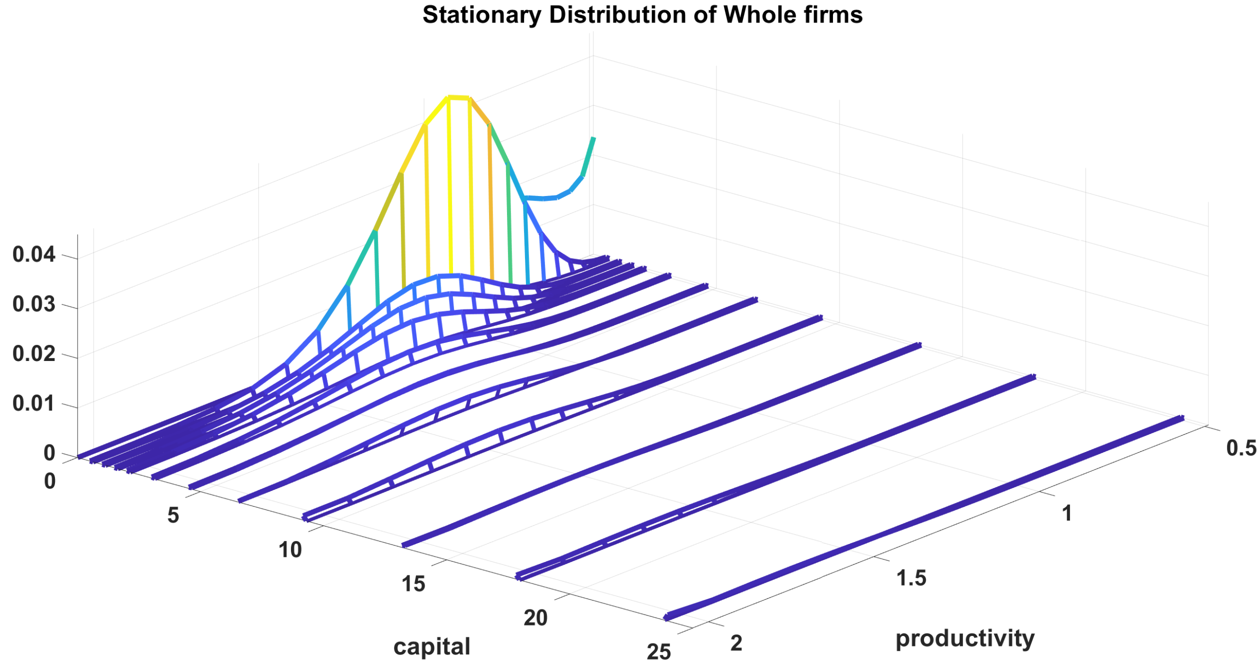

Figure 6: Active and inactive firm distribution

5.4.2 intertemporal decisions of firms and potential startup

As discussed in the analysis section, every type of firm and potential startup that chose to operate next period share the common optimal decision rule of future capital because of the same expected value of being active next period. For continuing firms when they choose to pause the operation, the future capital choice is zero since the current capital level has nothing to do with continuation and operation choice.

For now, we focus on the optimal decision of continuation and operation by the firms and choice of entry by potential startups. Since every fixed cost is randomly drawn from a distribution over continuous support. We are able to summarize the optimal decision by threshold policy. Then, we can compute the fraction of firms continuing, operating, or entering at each level of productivity (or productivity signal). Appealing to the Law of Large Numbers, we can also interpret this as the probability that a firm faces in state space. As the current capital does not affect the continuation and operation choices, these probabilities are the same for all current capital levels.

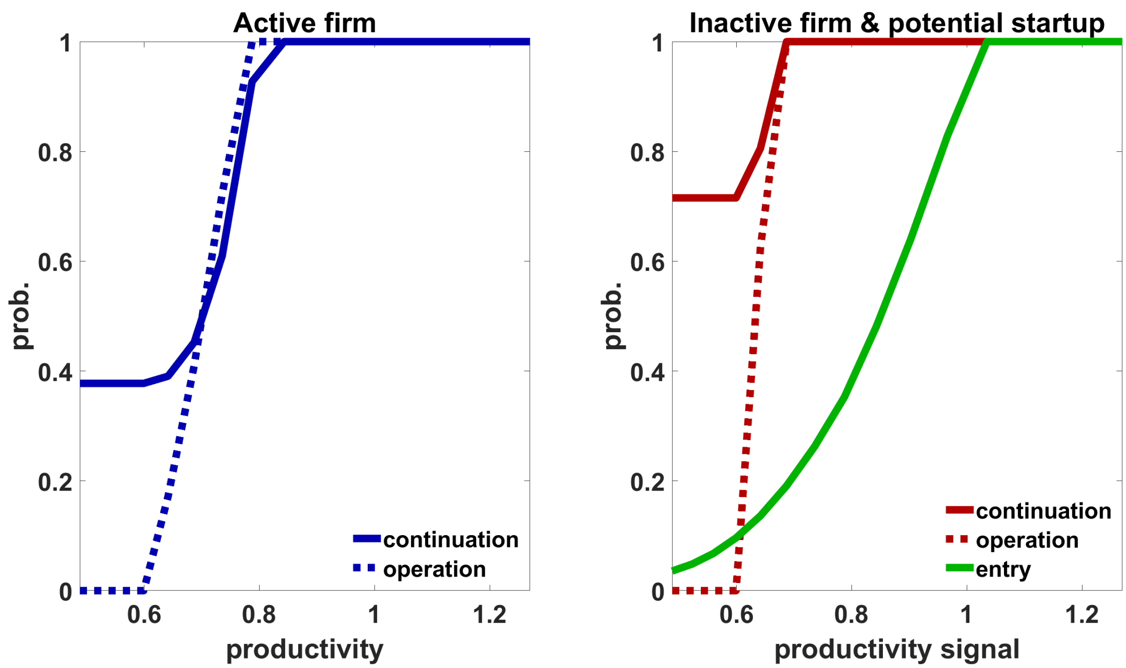

Figure 7: Probability of continuation, operation and entry

Figure 7 depicts these probabilities. On the left panel, it shows the probability of continuation in a solid line and that of operation in a dotted line for the active firm. Binary choice problem with fixed cost affects firms with relatively low productivity. In other words, firms with productivity higher than 0.9 can continue and operate in the next period for sure. Firms having productivity lower than 0.9 feature a nontrivial probability of continuation. For continuing firms, every firm with productivity at or below 0.6 postpones the operation next period. Continuing firms with productivity slightly higher than this face a nontrivial probability of operation.

The right panel of Figure 7 shows the probabilities for potential entrants, temporarily inactive firms, and potential startups. First, both are going to enter the market if the realization of a signal is sufficiently high. For the lower part of the signal, inactive firm and potential startups face a different process of entry. Especially, a firm with a signal at or below 0.6 will continue to be inactive next period. But some potential startups would enter. Thus, for the very low part of the signal, there are only potential startups who chose to enter the next period. For those with a higher signal than 0.6, inactive firms feature a much higher probability of entry than potential startups.

The option of pause and reactivation through a two-stage binary choice problem for continuation and operation has a direct implication for the composition of the productivity distribution. Since unproductive active firms can choose to pause on top of the permanent exit, the surviving firms can be even more productive next period when it is compared with the case involving only continuation problem. The productivity distribution of entrants is also affected. We can expect that startups' productivity distribution will be more dispersed because of more firms with low productivity.

5.4.3 matched moments

Given the parameters set in the previous section, the model is able to generate the fraction of reactivation among entrants, 15%, and the relative average size of reactivation to startups, 67%, which are close to the data moments. We had multiple potential explanations for facts in the data and now we inspect the mechanism that worked in the model.

First, the population share of reactivation is predicted about 15 percent. One of the factors leading to the result is the different number of potential entrants. The measure of potential startups, M, is 0.6, while the number of temporarily inactive firms, $\int_{\mathcal{K}\times\mathcal{E}} \mu_2(d[k \times \epsilon]) = 0.064$, which is smaller by an order of magnitude. Thus, even if both startups and dormant firms face the same entry probability (i.e., entry cost structure), the number of reactivations can be limited by the smaller number of potential entrants than a startup.

The other factor determining the population share follows from different entry barriers. Since the potential startups and temporarily inactive firms face different processes to enter the market, so is the probability of entry. Note that reactivation shows a higher fraction of being active. When I define entry ratio as a fraction of actual entry among potential entrants, the startup is 0.12 (=0.072/0.6) and reactivation is 0.20 (=0.013/0.064).

We do a counterfactual exercise that imposes the same entry structure with the inactive firm for potential startups. Table 6 shows the result. If potential startups face the combined probability of continuation and operation as their entry probability, there will be 0.122 startups which is even higher than the actual entry of startups in steady state. We confirm that reactivation allows more entry, but the small number of potential entrants reduces the population share of reactivation as observed in the data.

Potential entrants	Startup M (= 0.6)	Reactivation $\int_{\mathcal{K}\times\mathcal{E}} \mu_2(d[k \times \epsilon])$ (= 0.064)
Actual entry	0.072	0.013
Same entry probability (inactive)	0.122	0.013

Table 6: Counterfactual exercise

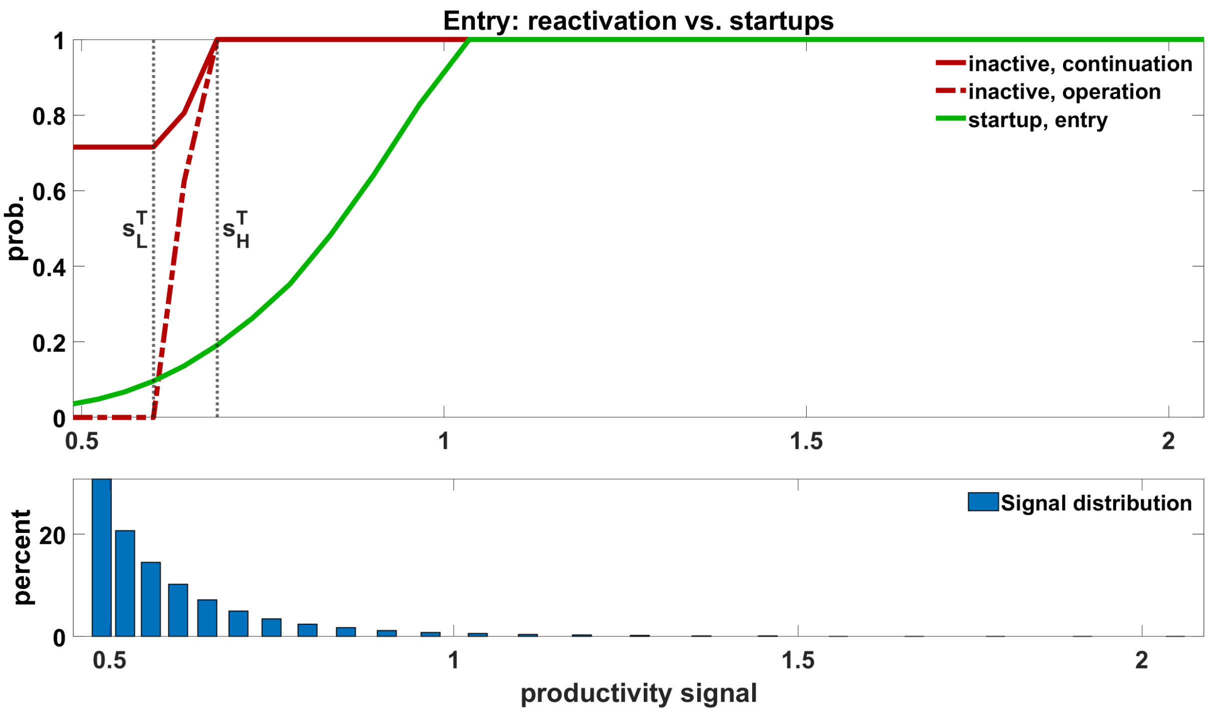

Figure 8: Probability of re-entry and new-entry

Before the discussion of the prediction of the model about the second fact of data, we show the composition of distribution is affected by different entry processes. The top panel of Figure 8 shows the probability of entry for both inactive firms and potential startups over productivity signal. The solid red line is the probability of continuation, and the dashed red line depicts the probability of operation. These are probabilities faced by a temporarily inactive firm. Three possible scenarios happen for continuing inactive firms. If it draws a productivity signal below the low threshold, s_L^T, it stays inactive the next period. When the productivity signal exceeds the high threshold, s_H^T, a firm is going to be reactivated regardless of a draw of continuation and operation costs. Firms with productivity signals between two thresholds, $s_L^T < s < s_H^T$ face a nontrivial probability of reactivation. And the probability is increasing in the signal. Potential startups only face one fixed cost, entry cost, and the probability of entry is also increasing in productivity signal. Compared to inactive firm, there is positive probability for $s < s_L^T$. And potential startups face a lower probability of entry for $s > s_L^T$ than the inactive firms. Overall, reactivation is easier for firms with signals above s_L^T, and highly unproductive potential entrants can only become entrants as startups.

Bottom panel of Figure 8 is signal distribution. Since there is a thicker mass on $s < s_L^T$ and only potential startups can enter with such a signal, the distribution of startups has more share in this region. The inactive firm allows more entry on $s_L^T < s$, which implies the distribution of reactivated firms will be more centered than startups.

The second moment we match in the model's steady state is the average size of reactivation relative to startup. Data shows the average size of reactivation is roughly 63% of startups and the model provides a bit higher number, 67%. Why is the average size of reactivation smaller than startups?

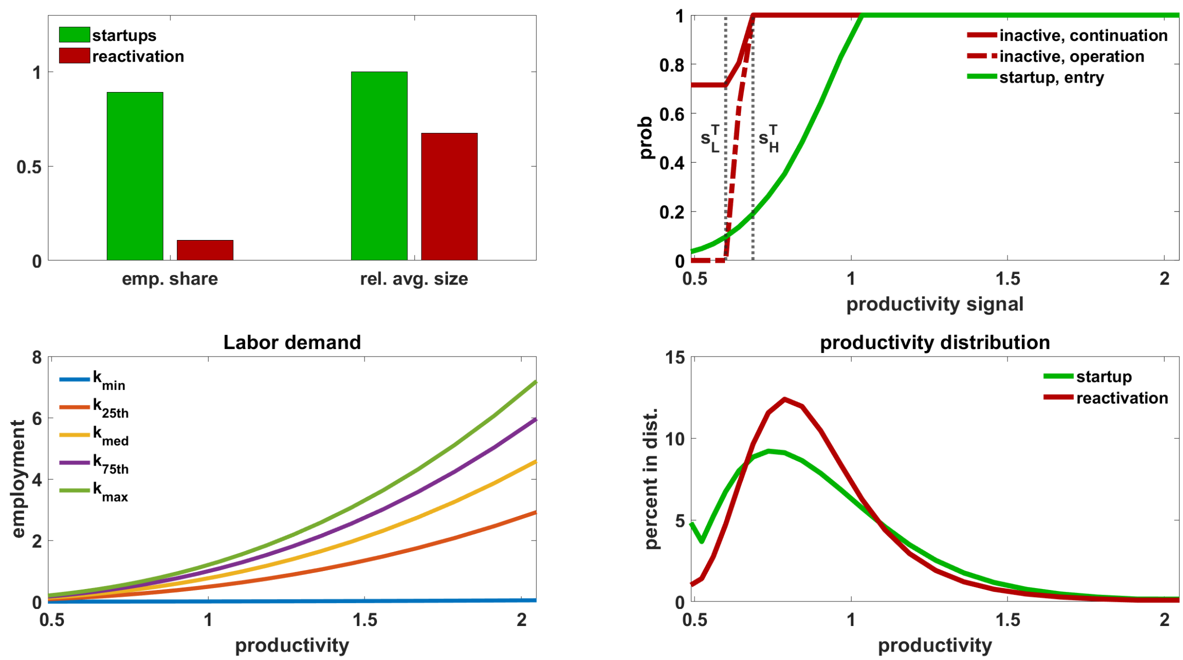

Figure 9: Relative average size of reactivation

As discussed in the previous part, potential startups can enter even if it draws a signal below s_L^T. This leads to more mass of less productive firms in distribution for startups next period. It can be seen at the bottom right panel of Figure 9 where the distribution is plotted in the unit of share in each distribution. Also, for those with a signal above s_L^T, the selection applies more strictly to potential startups, and it results in a more dispersed productivity distribution of startups showing more proportion of firms with high productivity than reactivation. Note that the bottom right panel of Figure 9 only shows changes in composition because it is normalized by the total mass of each distribution. Recall that the number of startups is more than five times larger than the number of reactivated firms. The composition effect can be amplified because of the difference in the absolute size of each entrant.

The bottom left panel of Figure 9 displays the labor demand function over productivity and plots with five different capital sizes. The labor demand is an increasing function of both capital and productivity. It is strictly concave in capital and strictly convex in idiosyncratic productivity. The group of startups has more mass of highly productive firms than the group of reactivations. Dispersed productivity distribution helps to produce larger employment on average for the group of startups because of the convexity of labor demand. Thus, the average size of reactivated firms is smaller than that of startups. Interestingly, the average productivity of the group of reactivations is higher than the group of startups by two percent. Because of convexity in labor demand, monotonicity of labor demand in productivity does not hold in aggregate. Employment share in entering cohort, as a by-product of *Fact* 1 and *Fact* 3, is higher for startups as depicted on the top left panel of Figure 9.

5.5 Model comparison: the option of pause and reactivation

This part explores the steady state of the model in terms of aggregate variables. Our full model is compared to the reference models without the option of pause and reactivation to shed light on the macroeconomic implication of having the option of pause and reactivation. We consider two reference models in this analysis. The first one, denoted by *REF 1*, is an otherwise identical model without the option of pause and reactivation. The second reference called *REF 2* is a model without entry and exit. The stock of firms in both reference models is the same as the steady state number of firms in the full model. For REF 1, we also match the same aggregate exit rate. To this end, we calibrate again the upper bound of continuation cost for active firm and measure of potential startups in REF 1. The model comparison in this section allows us to analyze the role of the option of pause and reactivation in aggregates qualitatively.

The stationary distribution and its components are summarized in Table 7 across models.

	Total	Active	Inactive	Startups	Reactivation	Permanent Exit	Pause
Full	0.929	0.866	0.064	0.072	0.013	0.057	0.028
	(100.00)	(93.13)	(6.87)	(8.33)	(1.50)	(6.63)	(3.20)
REF 1	0.929	0.929	-	0.091	-	0.091	-
	(100.00)	(100.00)	(-)	(9.83)	(-)	(9.83)	(-)
REF 2	0.929	0.929	-	-	-	-	-
	(100.00)	(100.00)	(-)	(-)	(-)	(-)	(-)

Note: numbers in blue denote the share of entry and exit components in active firms. Entry and exit are calculated based on the distribution of active firms since they are observable in data

Table 7: Stationary Distribution

The direct consequence of having the option of pause and reactivation in the model is that entry and exit have two components, which is also consistent with the definition of data in BDS. As a result, REF 1 overstates the entry and exit of startups and permanent exit, respectively. The model also predicts more pausing firms than reactivation. This holds when the temporarily inactive firms can decide to exit by dissolution and there are positive amounts of such delayed exits in equilibrium. The outflows of inactive firm distribution are reactivation and permanent exit while there is a sole inflow, new pause from active firms. Hence, the model may be built on a seemingly small portion of reactivated firms in data, the role of pause and reactivation can have a meaningful effect through pause and firms sitting idle.

One of the other differences between the full model and REF 1 is the existence of inactive firms. Matching the same aggregate exit rate, the full model has a smaller number of active firms than REF 1. Reference models exhibit the same number of active firms as the number of total firms.

	Y	C	I	N	K	FC	C/Y	I/Y	K/Y	TFP
Full	0.704	0.516	0.116	0.327	1.682	0.072	0.733	0.165	2.390	1.201
REF 1	0.714	0.525	0.119	0.326	1.723	0.070	0.736	0.167	2.415	1.213
REF 2	0.627	0.522	0.105	0.288	1.516	0.000	0.833	0.167	2.418	1.187

Table 8: Aggregate variables in steady state

Aggregate variables in steady state across the model are presented in Table 8. The reference model with entry and exit (REF 1) features a significant cleansing effect by selection and it is compared to the reference economy without entry and exit (REF 2). Since the economy is populated more productive firms on average, aggregate investment and capital are larger and there are more firms with higher labor demand. Aggregate output increases and consumption is also higher than the economy without entry and exit even though the fixed cost that is almost 10 percent of GDP is paid. TFP also rises by 2.19 percent in the long run.

To study the quantitative role of the option of pause and reactivation, the full model economy is compared to the reference economy with entry and exit. We have two effects working here. Although the total number of firms in the economy is the same, there is a smaller number of active firms in the full economy and here the effect stemming from this change in the number of active firms is called *scale effect*. Next, like the previous comparison between REF 1 and REF 2, the option of pause and reactivation also adds another layer of selection for firms, and it has two entry components that are not considered in the reference model. These changes in selection and components of entry and exit change the composition of the firm distribution. We call this as *composition effect*.

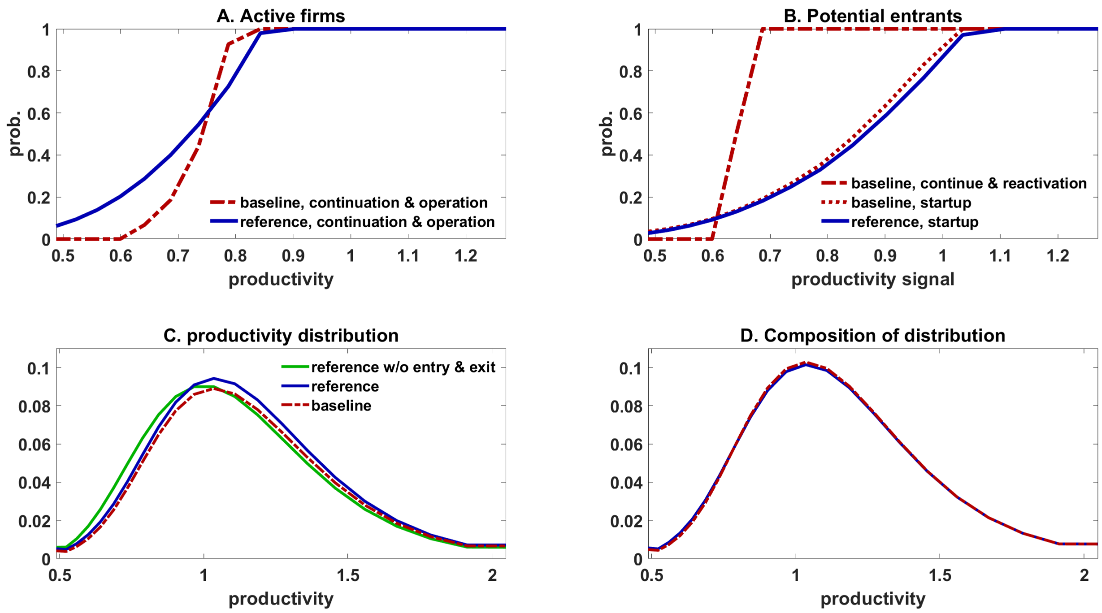

Figure 10: Model Comparison - productivity distribution

On panel D of Figure 10, the productivity distribution for both reference and full model is provided and they are normalized by the total mass of each distribution. We observe that the full model economy has more medium-productive firms and fewer low-productive firms in distribution. With the option of pause and reactivation, active firms in panel A allow more medium-productive firms and restrict low-productive firms to stay active. Likewise, Panel B implies that the full model economy's distribution would be more centered in the productivity space. However, this composition change does not look significant in that the average productivity of reference model is lowered only by 0.3 percent. Panel C of Figure 10 is productivity distribution at a steady state without normalization. As the full model exhibits a smaller number of active firms, the distribution looks scaled down. In stationary distribution, the scale effect is significant. Next, we see this holds for the result of aggregate variables as well.

In Table 8, the full model shows aggregate variables smaller than the reference model with entry and exit except for the aggregate hours worked and fixed cost. Each firm hires more workers and households also increase the labor supply so the equilibrium wage is lower in the full model. In aggregate, the fixed cost amounts to 10 percent of GDP and it restrains consumption. A single active firm features higher value and invests more but the scale effect mostly shapes the aggregate outcome. We explain this point again in the following Figure 11. Since the full model features a cleansing effect by the selection, it provides 1.18 percent higher TFP than REF 2, so are output, investment, employment, and capital. Interestingly, because of the high fixed cost, which is absent in REF 2, the aggregate consumption is even lower than REF 2, which reduces the households' value. TFP in the full model is 1 percent lower than REF 1. So, the option of pause and reactivation partly undo the cleansing effect in REF 1 by scaling down the active firms.

Figure 11 displays the continuation value, v^2, labor demand and optimal decision rule of future capital. The difference between the full and reference model follows from the scale effect and price changes in general equilibrium. The lower number of active firms and higher fixed cost reduces the aggregate consumption. Marginal utility of consumption rises, and equilibrium wage falls relative to the reference economy. Labor demand rises for all capital and productivity levels in response to a fall in the wage rate. This effect is more significant for firms with high productivity because of the convexity of labor demand. The firm value rises following the increase of current profit and a firm chooses higher future capital than one in the reference economy. As we can observe in the diagram of continuation value, however, the difference across models is small enough over low and medium productivity that future capital choice is the same in such regions.

As we can see in optimal policies for labor demand and future capital, the difference between models over low and medium productivity is quite small. The option of pause and reactivation brings composition effect on productivity distribution as in panel D of Figure 10, which is mainly affecting low and medium productivity regions. Therefore, the composition effect on the aggregate variable is negligible. And scale effect mostly explains the aggregates for the full model economy in Table 8.

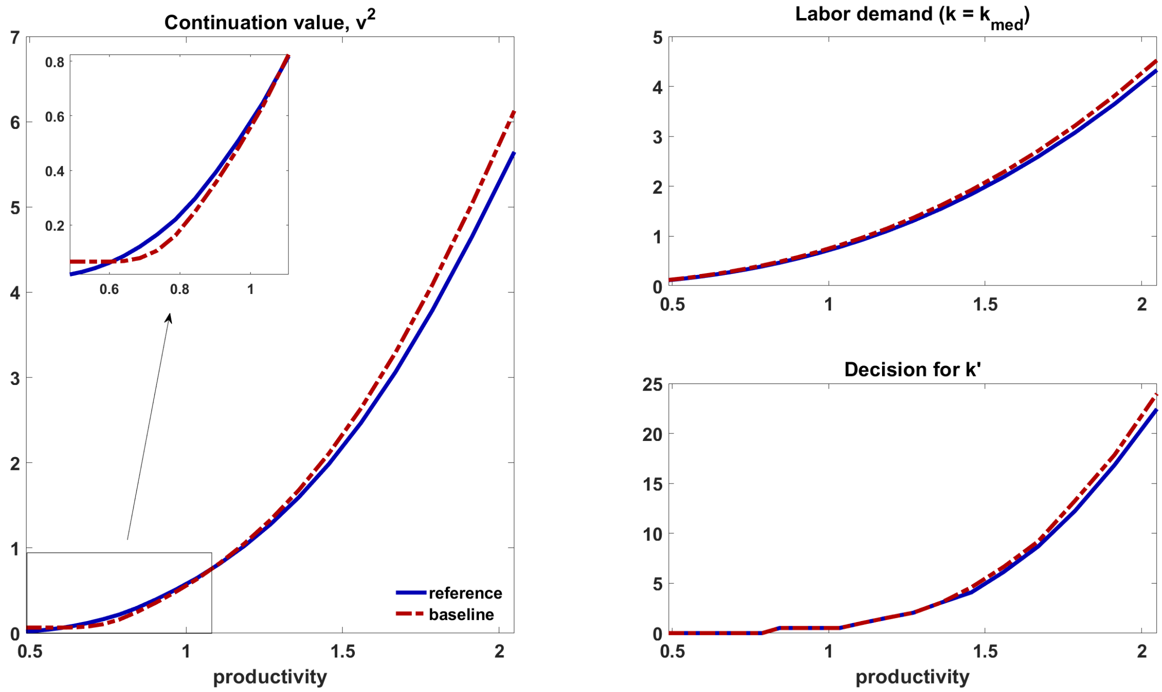

Figure 11: Model Comparison - value function and optimal decisions

5.6 Extension and discussion

We use this section to describe the plans for this research project with appropriate discussions.

5.6.1 adjustment cost of capital

One of the extensions from this paper is employing the capital adjustment cost in the model. Now, the model economy is not considering the real friction in adjusting the capital. Because of that, an active firm that pauses the operation next period sells off the capital. Therefore, in equilibrium, every inactive firm do not hold any capital at all since capital does not affect decisions of continuation and operation.

On top of matching the distribution of investment rates, the adjustment cost of capital makes temporarily inactive firms hold idle capital. This potentially generates a channel of resource misallocation which is unexplored in the literature. The idle capital that does not contribute to output lowers the TFP and the effect would be amplified when more capital-large firms pause the operation.

Additionally, for temporarily inactive firms when they decided to re-enter the market, holding capital can be an advantage for entry. Although there is a capital adjustment cost, their target capital might not be far from the current capital. So, it can reduce the cost of entry. However, startups should pay entire amount of future capital even though they are free from adjustment costs.

5.6.2 business cycles

The other extension is incorporating aggregate uncertainty in the model. Pause and reactivation behavior seems to be more relevant over business cycles. A recession creates more exits than usual, and the liquidation value of exit is typically lowered. This leads to a higher rate of pause among exits and provides more mass of temporarily inactive firms. These inactive firms potentially provide a mechanism that explains protracted recovery during the Great Recession through the misallocation channel discussed above.

Reactivation also can play a role in explaining recovery and/or time-varying cohort effect over business cycles. Moreira (2016) and Sedláček and Sterk (2017) show that cohorts born during recession start small and stay small. They seek the reason for time-varying cohort characteristics from the demand-side effect. With aggregate uncertainty, reactivation changes the composition of entrants over business cycles. By having capital adjustment cost and aggregate uncertainty, the reactivated firms can allow less productive firms to enter and the rising share of them in the entering cohort can drag down the resilience of the economy.

5.6.3 data and empirical studies

The major issue of this project is now data. We infer the pausing firms using the model that can explain the reactivation in data. This is imperfect in understanding how many actual permanent exits were in exits. Also, we need to track reactivated establishments or firms in order to understand the nature of reactivation. Do they grow faster than startups, or not? How many pausing and reactivated establishments are part of multi-plant firms? We do not provide any answers to these questions with limited access to the confidential LBD dataset.

Temporary shutdown of business gets attention recently because of COVID 19. However, the temporary shutdown, in this case, is too specific in the sense that it is related to federal or state policies fighting against the virus. What we are studying in this project is somewhat presented over the longer horizon of sample periods. Also, these are a higher frequency than the annual period that we are using currently. Other than covid-related studies, pause and reactivation behaviors are relatively unexplored in empirical studies.

6 Concluding Remarks

This paper studies the role of the option to pause and reactivate the operation in the aggregate economy. BDS provides evidence of reactivation of US establishments. Over the sample period, reactivation accounts for roughly 14% of the entering cohort. The average size of startups is 1.6 times of reactivated establishments. A model of heterogeneous firms with endogenous entry and exit is studied to understand the role of the option of pause and reactivation. The model incorporates a two-stage binary choice problem regarding continuation and operation.

When the model is calibrated to match the US aggregate exit rate, hours worked, and descriptive facts of reactivation, it predicts the population share of reactivation and their relative average size well. Also, reactivation requires temporarily inactive firms that were previously active and chose to pause the operation. In equilibrium, we have roughly 7 percent of inactive firms. Since the model has no real friction in adjusting capital, inactive firms do not hold capital. For a more realistic analysis, we will consider capital adjustment costs later with their quantitative role in misallocation and productivity.

Our full model economy is compared to reference economy without the option of pause and reactivation. Two effects are shaping the aggregate outcomes. Scale effect follows from a fall in the number of active firms. The composition effect on productivity distribution arises because the firm faces two-stage continuation and operation choice and two types of entrants provide rich selection processes than the reference model. Because of selection by entry and exit, the full model exhibits higher TFP than the reference economy without entry and exit. But the TFP is lower than reference with entry and exit because of the scale effect and it applies the same for other aggregate variables except hours worked and fixed cost.

All in all, the option of pause and reactivation reduces the number of active firms and changes the composition of the productivity distribution. What matters in aggregates, in the long run, is the scale effect and they partly undo the cleansing effect of entry and exit.

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A Reformulation of the Problem and Aggregation

Based on Clementi et al. (2014), we compute the equilibrium by solving firm's problem with the equilibrium implication of household' optimality conditions. Household's optimal labor supply implies $w(z, \mu) = D_2 u(C, 1 - N)/D_1(C, 1 - N)$ and firms' stochastic discount factor should be equal to the households intertemporal marginal rate of substitution of consumption, $d_{z'}(z, \mu) = \beta D_1 u(C', 1 - N')/D_1 u(C, 1 - N)$. Without loss of generality, we assign $p(z, \mu) = D_1 u(C, 1 - N)$ as an output price that firms and potential startups use to value current profits and payments.

We reformulate the firms' problem by expressing the value in units of marginal utility of consumption. At the beginning of the period,

$$V^a(k, \epsilon; z, \mu) = p\pi(k, \epsilon; z, \mu) + p(1 - \delta)k + \int V^1(\epsilon, c_1^c; z, \mu) H_1^c(dc_1^c)$$

After observing the continuation cost,

$$V^1(\epsilon, c_1^c; z, \mu) = \int \max\left\{0, -pc_1^c + V^2(\epsilon, c_1^f; z, \mu)\right\} H_1^f(dc_1^f)$$

After observing the operating cost,

$$V^2(\epsilon, c_1^f; z, \mu) = \max\left\{V^p(\cdot; z, \mu), -pc_1^f + V^f(\epsilon; z, \mu)\right\}$$

where

$$\begin{aligned} V^f(\epsilon; z, \mu) &= \max_{k' \in \mathcal{K}} \left[-pk' + \beta \mathbb{E}_{z', \epsilon'}[V^a(k', \epsilon'; z', \mu'|z, \epsilon]\right] \\ V^p(\cdot; z, \mu) &= \max_{k' \in \mathcal{K}} \left[-pk' + \beta \mathbb{E}_{z', s'}[V^i(k', s'; z', \mu'|z]\right] \end{aligned}$$

Value of inactive firm is written as

$$V^i(k, s; z, \mu) = pk + \int V^1(\epsilon, c_2^c; z, \mu) H_1^c(dc_2^c)$$

with

$$\begin{aligned} V^1(s, c_2^c; z, \mu) &= \int \max\left\{0, -pc_2^c + V^2(s, c_2^f; z, \mu)\right\} H_2^f(dc_2^f) \\ V^2(s, c_2^f; z, \mu) &= \max\left\{V^p(\cdot; z, \mu), -pc_2^f + V^f(s; z, \mu)\right\} \end{aligned}$$

A.1 Decision for continuation

For active firm, the optimal policy of continuation is threshold policy. Define $\widetilde{c_1^c}$ as the cost such that an active firm is indifferent to continuing, and define $\overline{c_1^c}$ as the resulting threshold cost confined to the support of H_1^c.

$$
\begin{aligned}
\widetilde{c_1^c}(k,\epsilon;z,\mu) &= \frac{1}{p}\int V^2(\epsilon, c_1^f; z,\mu) H_1^f(dc_1^f) \\
\overline{c_1^c}(k,\epsilon;z,\mu) &= \max\{c_1^{c,L}, \min\{\widetilde{c_1^c}(k,\epsilon;z,\mu), c_1^{c,U}\}\}
\end{aligned}
$$

Then the probability of continuation, $\alpha_1^c(k,\epsilon;z,\mu)$, and conditional expectation of the continuation cost that is going to be paid for an active firm with (k,ϵ) are

$$
\begin{aligned}
\alpha_1^c(k,\epsilon;z,\mu) &= H_1^c\left(\overline{c_1^c}(k,\epsilon;z,\mu)\right) \\
\psi_1^c(k,\epsilon;z,\mu) &= \int_{c_1^{c,L}}^{\overline{c_1^c}(k,\epsilon;z,\mu)} c_1^c H_1^c(dc_1^c)
\end{aligned}
$$

Likewise, for the inactive firm, threshold continuation cost, $\overline{c_2^c}$, probability of continuation and the expected cost paid are defined by

$$
\begin{aligned}
\widetilde{c_2^c}(k,s;z,\mu) &= \frac{1}{p}\int V^2(s, c_2^f; z,\mu) H_2^f(dc_2^f) \\
\overline{c_2^c}(k,s;z,\mu) &= \max\{c_2^{c,L}, \min\{\widetilde{c_2^c}(k,s;z,\mu), c_2^{c,U}\}\}
\end{aligned}
$$

and

$$
\begin{aligned}
\alpha_2^c(k,s;z,\mu) &= H_2^c\left(\overline{c_2^c}(k,s;z,\mu)\right) \\
\psi_2^c(k,s;z,\mu) &= \int_{c_2^{c,L}}^{\overline{c_2^c}(k,s;z,\mu)} c_2^c H_2^c(dc_2^c)
\end{aligned}
$$

A.2 Decision for operation

Again, since the operating decision is solving the discrete choice problem involving the operating cost, the optimal policy is threshold policy. Define $\widetilde{c_1^f}$ as the cost such that an active firm is indifferent to operating next period, and define $\overline{c_1^f}$ as the resulting threshold cost confined to the support of H_1^f.

$$\widetilde{c_1^f}(k,\epsilon;z,\mu) = \frac{1}{p}\left(V^f(\epsilon,;z,\mu) - V^p(\cdot;z,\mu)\right)$$
$$\overline{c_1^f}(k,\epsilon;z,\mu) = \max\{c_1^{f,L}, \min\{\widetilde{c_1^f}(k,\epsilon;z,\mu), c_1^{f,U}\}\}$$

The probability of operating next period, $\alpha_1^f(k,\epsilon;z,\mu)$, and the expected cost paid are defined by

$$\alpha_1^f(k,\epsilon;z,\mu) = H_1^f\left(\overline{c_1^f}(k,\epsilon;z,\mu)\right)$$
$$\psi_1^f(k,\epsilon;z,\mu) = \int_{c_1^{f,L}}^{\overline{c_1^f}(k,\epsilon;z,\mu)} c_1^f H_1^f(dc_1^f)$$

Again, the same logic applies to inactive firm's operating choice. Then, the threshold cost, $\overline{c_2^f}$, probability of operating next period and the expected cost paid are defined by

$$\widetilde{c_2^f}(k,s;z,\mu) = \frac{1}{p}\left(V^f(s,;z,\mu) - V^p(\cdot;z,\mu)\right)$$
$$\overline{c_2^f}(k,s;z,\mu) = \max\{c_2^{f,L}, \min\{\widetilde{c_2^f}(k,s;z,\mu), c_1^{f,U}\}\}$$

and

$$\alpha_2^f(k,s;z,\mu) = H_2^f\left(\overline{c_2^f}(k,s;z,\mu)\right)$$
$$\psi_2^f(k,s;z,\mu) = \int_{c_2^{f,L}}^{\overline{c_2^f}(k,s;z,\mu)} c_2^f H_2^f(dc_2^f)$$

A.3 Decision of entry by potential startup

Potential startups pay entry cost to become a startup next period. They will pay the entry cost if:

$$V^e(s;z,\mu) \geq pc^e \text{ where}$$
$$V^e(s;z,\mu) = \max_{k'\in\mathcal{K}}\left[-pk' + \beta\mathbb{E}_{z',\epsilon'}[V^a(k',\epsilon';z',\mu'|z,s]\right]$$

As the optimal decision of entry is threshold policy, we define threshold cost, probability of entry as a startup and expected cost paid by

$$
\begin{aligned}
\widetilde{c^e}(s;z,\mu) &= \frac{1}{p} V^e(s;z,\mu) \\
\overline{c^e}(s;z,\mu) &= \max\{c^{e,L}, \min\{\widetilde{c^e}(s;z,\mu), c^{e,U}\}\}
\end{aligned}
$$

and then,

$$
\begin{aligned}
\alpha^e(s;z,\mu) &= H^e\left(\overline{c^e}(s;z,\mu)\right) \\
\psi^e(s;z,\mu) &= \int_{c^{e,L}}^{\overline{c^e}(s;z,\mu)} c^e H^e(dc^e)
\end{aligned}
$$

A.4 Aggregation

Recall the distribution of total firm is the sum of distribution of active and inactive firms, $\mu = \mu_1 + \mu_2$. Given the probabilities of continuation and operation next period and entry as a startup in company with the conditional expectation of payment of fixed cost, labor demand, production and capital decision rules, we can do aggregation by weighted average over distribution.

Aggregate production and employment are

$$
\begin{aligned}
Y(z,\mu) &= \int_{\mathcal{K}\times\mathcal{E}} [y(k,\epsilon;z,\mu)]\, \mu_1(d[k\times\epsilon]) \\
N(z,\mu) &= \int_{\mathcal{K}\times\mathcal{E}} [n(k,\epsilon;z,\mu)]\, \mu_1(d[k\times\epsilon])
\end{aligned}
$$

Aggregate investment by both active and inactive firms is

$$
\begin{aligned}
I^c(z,\mu) = \int_{\mathcal{K}\times\mathcal{E}} \Bigg[& \alpha_1^c(k,\epsilon;z,\mu)\Bigg(\alpha_1^f(k,\epsilon;z,\mu)[g_1^f(k,\epsilon;z,\mu)-(1-\delta)k] \\
& + (1-\alpha_1^f(k,\epsilon;z,\mu))[g_1^p(k,\epsilon;z,\mu)-(1-\delta)k]\Bigg)\Bigg]\mu_1(d[k\times\epsilon]) \\
& - \int_{\mathcal{K}\times\mathcal{E}} \Bigg[\Bigg(1-\alpha_1^c(k,\epsilon;z,\mu)\Bigg)(1-\delta)k\Bigg]\mu_1(d[k\times\epsilon]) \\
& + \int_{\mathcal{K}\times\mathcal{E}} \Bigg[\alpha_2^c(k,s;z,\mu)\Bigg(\alpha_2^f(k,s;z,\mu)[g_2^f(k,s;z,\mu)-k] \\
& + (1-\alpha_2^f(k,s;z,\mu))[g_2^p(k,s;z,\mu)-k]\Bigg)\Bigg]\mu_2(d[k\times s]) \\
& - \int_{\mathcal{K}\times\mathcal{E}} \Bigg[\Bigg(1-\alpha_2^c(k,s;z,\mu)\Bigg)k\Bigg]\mu_2(d[k\times s])
\end{aligned}
$$

Aggregate investment by potential startups that pays entry cost is

$$
I^e(z,\mu) = M\int_{\mathcal{S}}\Bigg[\alpha^e(s;z,\mu)g^e(s;z,\mu)\Bigg]f_s(ds)
$$

Household consumption is

$$
C(z,\mu) = Y(z,\mu) - [I^c(z,\mu)+I^e(z,\mu)] - \Big[\Psi^c(z,\mu)+\Psi^f(z,\mu)+\Psi^e(z,\mu)\Big]
$$

where $\Psi^c(z,\mu), \Psi^f(z,\mu),$ and $\Psi^e(z,\mu)$ are the total costs paid with firm continuation, operation, and startup entry, respectively.

$$
\begin{aligned}
\Psi^c(z,\mu) &= \int_{\mathcal{K}\times\mathcal{E}} \psi_1^c(k,\epsilon;z,\mu)\mu_1(d[k\times\epsilon]) + \int_{\mathcal{K}\times\mathcal{S}} \psi_2^c(k,s;z,\mu)\mu_2(d[k\times s]) \\
\Psi^f(z,\mu) &= \int_{\mathcal{K}\times\mathcal{E}} \psi_1^f(k,\epsilon;z,\mu)\mu_1(d[k\times\epsilon]) + \int_{\mathcal{K}\times\mathcal{S}} \psi_2^f(k,s;z,\mu)\mu_2(d[k\times s]) \\
\Psi^e(z,\mu) &= M\int_{\mathcal{S}} \psi^e(s;z,\mu)f_s(ds)
\end{aligned}
$$

The model has two components for both entry and exit. Suppose current aggregate state is (z,μ). Then the number of startups next period is

$$
M\int_{\mathcal{S}}\alpha^e(s;z,\mu)f_s(ds)
$$

And the number of reactivated firms next period is calculated as

$$\int_{\mathcal{K}\times\mathcal{E}} \left[\alpha_2^c(k,s;z,\mu)\alpha_2^f(k,s;z,\mu)\right] \mu_2(d[k\times s])$$

The sum of startups and reactivation is entry in the next period. For exit, we only account the exits from active firms since they are observable. The number of permanent exiting firms is computed by

$$\int_{\mathcal{K}\times\mathcal{E}} \left[(1-\alpha_1^c(k,\epsilon;z,\mu))\right] \mu_1(d[k\times\epsilon])$$

And the number of pausing firms (or temporary shutdown) is

$$\int_{\mathcal{K}\times\mathcal{E}} \left[\alpha_1^c(k,\epsilon;z,\mu)\left(1-\alpha_1^f(k,\epsilon;z,\mu)\right)\right] \mu_1(d[k\times\epsilon])$$

Total exit next period is defined as the sum of the number of permanent exit and pause. Note that the exit decision is determined at the end of the period in the perspective of an individual firm. However, when we accounting in aggregate variable, we put them forward to the beginning of the next period. Therefore, in stationary economy, the mass of firms entering at the beginning of the next period is the same with the mass of firms exiting.