Generative Adversarial Nets

Abstract.

Propose a new frame work for estimating generative models via an adversarial process
↳ Simultaniously train two models

Generative model G : captures data distribution.

Discriminative model D : estimates the probability that a sample came from the training data rather than G.

This framework corresponds to minmax two-player game.

There is no need for any Markov chains or unrolled approximate inference networks during either trainning or generation of samples.

1. Introduction.

The promise of deep learning is to discover rich, hierarchical models that represent probability distributions over the kinds of data encountered in AI applications.

지금까지 Deep learninge Discriminative model 이 많이서 성공적을 발전되었다. 그러나 Deep generative models 는 큰 성과를 보이지 못했다.

Due to the difficulty of approximating many intractable probabilistic computations that arise maximum likelihood estimation and related strategies.

and difficulty of leveraging the benefits of piecewise linear units in the generative context.

We propose a new generative model estimation procedure that sidesteps these difficulties.

- Difficulty of Deep generative models.
  - approximating many intractable probabilistic computation.
    - maximum likelihood estimation
    - related strategies.
  - leveraging the benefits of piecewise linear units
    - in generative context.

The generative model is pitted against an adversary

discrimitive model
: learns determine wether a sample is from the model distribution or the data distribution.

analogous to a team of counterfeiters.
위조범.

Competition in this game drives both teams to improve their methods until the counterfeiters indistinguishable from the genuine articles.

In this article, we explore the special case when the generative model generates samples by passing random noise through a multilayer perceptron, and the discrimnative model is also a multi layer perceptron.

No approximate inference or Markov chains are necessary

2. Related Work.

Deep generative model에 관한 초기까지의 연구들은 probability distribution function (확률분포함수)의 parametric specification을 최대로 적용했다.

가장 성공적인 예로 Hinton 교수의 Deep Boltzmann machine 이 있었다, 이러한 모델들은 log likelihood를 maximizing 함으로써 학습된다. 이러한 모델들은 일반적으로 intractable likelyhood function을 가지며 likelihood gradient를 구하기 위해 numerous approximations을 적용해야 했다.

이러한 어려움들을 해결하고자 "generative machines"가 개발되었다.

likelihood function을 정확하게 표현하지 않고, desired distribution으로부터 sample 생성 가능.

대표적인 예로 Bengio 교수의 Generative stochastic Networks 있다.

- numerous approximation이 아닌 정확한 backpropagation으로 학습가능.
- Markov chain을 사용

본 논문은 generative stochastic networks에서 Markov chain을 제거함으로써 generative machine의 Idea를 확장시켰다.

저자는 이전의 작업적 접근을 이용해 generative process의 derivative를 backpropagate 했다.

$$\lim_{\sigma \to 0} \nabla_x \mathbb{E}_{\epsilon \sim N(0, \sigma^2 I)} f(x+\epsilon) = \nabla_x f(x)$$

본 연구 바탕 사이에 유사한 연구들이 진행되었다.

① Kingma and Welling [18], Rezende et al. [23]

general stochastic backpropagation rules를 개발했다.

- finite variance의 Gaussian distribution을 통한 Backpropagate.
- mean과 미분가능한 covariance parameter로 Backpropagate 함.
- 본 논문에서 hyper parameter로 설정한 generator의 conditional variance를 학습가능.
- Variational autoencoders (VAEs)를 훈련시키기 위해 사용함.

VAEs

[Differentiable generator network
[Second neural network.
↳ recognition model : approximate inference

require differentiation through the hidden units
⇒ cannot have discrete latent variables.

GANs

[Generator Network
[Discriminator Network.

require differentiation through the visible units.
⇒ cannot model discrete data.

② The approach of using a discriminative criterion [29, 13]

· Deep generative model 이 다루기 어려운 criteria를 이용함.

· 이 방법들은 확률들의 비율을 포함하기 때문에 deep model로 근사하기 어렵다.

· Noise-contrasive estimation (NCE) [13]

- Fixed noise distribution으로 부터 data를 구별하기 쉽게 만들어주는 weight를 학습함으로써 generative model 훈련.
- 이전에 훈련된 model을 noise distribution으로 사용

⇒ quality 가 증가하는 일련의 model을 훈련

Informal competition 형태

- NCE는 discriminator가 noise distribution과 model distribution의 확률밀도의 비로 정의되기 때문에 이를 evaluate하고 backpropagate 할 수 있어야 한다.

③ Predictability minimization [26]

· 두개의 network가 경쟁하는 구조를 가진 선행연구들 중 본 연구와 가장 관련 깊은 연구.

· 한 network의 각각의 hidden unit은 나머지 모든 hidden unit들의 값이 주어졌을 때, 그 hidden unit의 값을 예측하는 second network의 output과 달라지게끔 훈련된다.

· 본 연구는 아홉의 세가지 측면에서 predictability minimization과 차이점을 가진다.

1) 경쟁구조가 유일한 training criterion 이며, 이 외에는 network를 훈련시키기에 충분함.

2) 경쟁의 성질이 다르다.

GANs 에서는 하나의 network는 다른 network의 input으로 작용하는 rich, high dimensional vector를 만들고, 다른 network이 어떻게 처리할지 모르는 input을 선택하려한다.

3) 학습과정의 특성이 다르다.

GANs는 optimization problem 보다는 minmax game의 가깝다.

한 agent는 maximize를 찾고 다른 하나는 minimize를 찾는 value function을 갖는다.

한편 GANs는 Adversarial example 개념 그리고 혼동될 수 있는데, Adversarial networks는 analysis tool 일뿐 generative model을 훈련시키기 위한 mechanism이 아니다.

3. Adversarial nets.

Adversarial modeling framework은 모델이 모두 MLP 일때 가장 적용하기 간단하다.

p_g : generator's distribution over data $\boldsymbol{x}$

$p_{\boldsymbol{z}}(\boldsymbol{z})$: Prior on input noise variables.

$G(\boldsymbol{z}; \theta_g)$: a mapping $\boldsymbol{z}$ to data $\boldsymbol{x}$ space.

differentiable function represented by a MLP with parameter θ_g.

$D(\boldsymbol{x}; \theta_d)$: probability that $\boldsymbol{x}$ came from the data rather than p_g.

outputs a single scalar.

D는 training example (real data)와 G가 만들어낸 Sample을 잘 구별하도록 훈련시키고, 동시에

G는 $\log(1 - D(G(\boldsymbol{z})))$를 최소화하도록 훈련시킨다.

즉, 대응된 value function $V(G, D)$에 대해 minmax game을 수행하게 한다.

$$\min_G \max_D V(D, G) = \mathbb{E}_{\boldsymbol{x} \sim p_{data}(\boldsymbol{x})}[\log D(\boldsymbol{x})] + \mathbb{E}_{\boldsymbol{z} \sim p_{\boldsymbol{z}}(\boldsymbol{z})}[\log(1 - D(G(\boldsymbol{z})))]$$

Algorithm 1 Minibatch stochastic gradient descent training of generative adversarial nets. The number of steps to apply to the discriminator, k, is a hyperparameter. We used $k = 1$, the least expensive option, in our experiments.

for number of training iterations **do**

  for k steps **do**

  - Sample minibatch of m noise samples $\{\boldsymbol{z}^{(1)}, \ldots, \boldsymbol{z}^{(m)}\}$ from noise prior $p_g(\boldsymbol{z})$.
  - Sample minibatch of m examples $\{\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(m)}\}$ from data generating distribution $p_{\text{data}}(\boldsymbol{x})$.
  - Update the discriminator by ascending its stochastic gradient:

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^{m} \left[\log D\left(\boldsymbol{x}^{(i)}\right) + \log\left(1 - D\left(G\left(\boldsymbol{z}^{(i)}\right)\right)\right) \right].$$

  end for

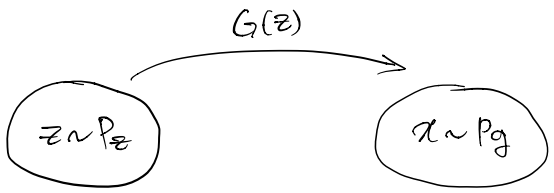
  - Sample minibatch of m noise samples $\{\boldsymbol{z}^{(1)}, \ldots, \boldsymbol{z}^{(m)}\}$ from noise prior $p_g(\boldsymbol{z})$.
  - Update the generator by descending its stochastic gradient:

$$\nabla_{\theta_g} \frac{1}{m} \sum_{i=1}^{m} \log\left(1 - D\left(G\left(\boldsymbol{z}^{(i)}\right)\right)\right).$$

end for

The gradient-based updates can use any standard gradient-based learning rule. We used momentum in our experiments.

4. Theoretical Results



Generator G는 확률분포 P_g를 정의한다.

충분한 capacity 와 training time이 주어졌을 때, Algorithm 1이 P_{data}를 잘 estimate 하는 방향으로 수렴하는 것을 증명하고자 한다. 이때 infinite capacity를 갖는 model 과 model 과 같은 non-parametric setting 에 대해 수렴성을 증명한다. (MLP 는 Parametric setting)

4.1 에서는 앞서 정의한 minimax game이 $P_g = P_{data}$ 의 global optimum을 갖는 것을,

4.2 에서는 Algorithm 1이 minimax game을 optimize 하는것을 증명한다.

4.1 Global Optimality of $P_g = P_{data}$.

임의의 Generator G에 대해

Proposition 1.

For G fixed, the optimal discriminator D is

$$D^*_G(x) = \frac{P_{data}(x)}{P_{data}(x) + P_g(x)}$$

Proof.

Algorithm 1. 에서 Discriminator 부분을 보면, 임의의 주어진 G에 대해, discriminator G는 $V(G,D)$를 maximize 하게끔 한다.

$$V(G,D) = \int_x P_{data}(x) \log(D(x))\,dx + \int_z P_z(z) \log(1-D(g(z)))\,dz$$

$$= \int_x P_{data}(x) \log(D(x)) + P_g(x) \log(1-D(x))\,dx$$

임의의 $(a,b) \in \mathbb{R}^2 \setminus \{0,0\}$ 에 대해 $a\log(y) + b\log(1-y)$ 는 $\frac{a}{a+b}$ 에서 maximum 을 갖는다.

↳ 이렇하면 함수형을.

즉, $Supp(P_{data}) \cup Supp(P_g)$ 상에서

$$D_G^*(x) = \frac{P_{data}(x)}{P_{data}(x) + P_g(x)}$$

본 논문에서는 이에 대해 "The discriminator does not need to be defined outside of $Supp(P_{data}) \cup Supp(P_g)$." 라고 설명하며 증명을 끝내었다.

⇒ 주어진 G에 대해 Algorithm 1 은 optimal discriminator $D_G^*(x) = \frac{P_{data}(x)}{P_{data}(x) + P_g(x)}$ 를 찾는다.

Proposition 1. 으로부터 minimax game 은 다음처럼 다시 쓸 수 있다.

$$\begin{aligned} C(G) &= \max_D V(G,D) \\ &= \mathbb{E}_{x\sim P_{data}}[\log D_G^*(x)] + \mathbb{E}_{z\sim P_z}[\log(1 - D_G^*(g(z))] \\ &= \mathbb{E}_{x\sim P_{data}}[\log D_G^*(x)] + \mathbb{E}_{x\sim P_g}[\log(1 - D_G^*(x))] \\ &= \mathbb{E}_{x\sim P_{data}}\left[\log \frac{P_{data}(x)}{P_{data}(x)+P_g(x)}\right] + \mathbb{E}_{x\sim P_g}\left[\log \frac{P_g(x)}{P_{data}(x)+P_g(x)}\right] \quad (4) \end{aligned}$$

Theorem 1.

The global minimum of the virtual training criterion $C(G)$ is acheived if and only if $P_g = P_{data}$. At that point, $C(G)$ acheives the value $-\log 4$.

Proof.

앞서 $P_g = P_{data}$ 이면 $D_G^*(x) = \frac{1}{2}$ $\therefore C(G) = -\log 4$.

이제 $P_g = P_{data}$ 에서 $C(G)$가 유일한 global optimal 임을 보이자.

(4) 번식의 양변에서 $-\log 4$를 빼면

$$C(G) + \log 4 = \mathbb{E}_{x\sim P_{data}}\left[\log \frac{P_{data}(x)}{P_{data}(x)+P_g(x)}\right] + \mathbb{E}_{x\sim P_g}\left[\log \frac{P_g(x)}{P_{data}(x)+P_g(x)}\right] + \log 4$$

$$= \mathbb{E}_{x\sim P_{data}}\left[\log \frac{P_{data}(x)}{\frac{P_{data}(x)+P_g(x)}{2}}\right] + \mathbb{E}_{x\sim P_g}\left[\log \frac{P_g(x)}{\frac{P_{data}(x)+P_g(x)}{2}}\right]$$

참고: $KL(P\|Q) = \sum_x P(x) \log \frac{P(x)}{Q(x)}$

$\therefore \quad C(G) = -\log 4 + KL\left(P_{data} \,\|\, \frac{P_{data}+P_g}{2}\right) + KL\left(P_g \,\|\, \frac{P_{data}+P_g}{2}\right)$

참고: $JSD(P\|Q) = \frac{1}{2} KL(P\|M) + \frac{1}{2} KL(Q\|M)$

$= -\log 4 + 2JSD(P_{data} \| P_g)$

2항의 두 확률 분포에 대해 JSD는 $P_{data} = P_g$ 일때 0 값을 갖고, 그 의미는 항상 0보다 큼.

$\therefore$ $C(G)$의 global minimum은 $P_g = P_{data}$ 일때 $-\log 4$ 이다.

4.2 Convergence of Algorithm 1.

Proposition 2.

If G and D have enough capacity, and at each step of Algorithm 1. the discriminator is allowed to reach its optimum given G, and P_g is updated so as to improve the criterion

$$E_{x\sim P_{data}}[\log D^*_G(x)] + E_{x\sim P_g}[\log(1 - D^*_G(x))]$$

then P_g converges to P_{data}.

Proof.

Proof. Consider $V(G, D) = U(p_g, D)$ as a function of p_g as done in the above criterion. Note that $U(p_g, D)$ is convex in p_g. The subderivatives of a supremum of convex functions include the derivative of the function at the point where the maximum is attained. In other words, if $f(x) = \sup_{\alpha\in\mathcal{A}} f_\alpha(x)$ and $f_\alpha(x)$ is convex in x for every α, then $\partial f_\beta(x) \in \partial f$ if $\beta = \arg\sup_{\alpha\in\mathcal{A}} f_\alpha(x)$. This is equivalent to computing a gradient descent update for p_g at the optimal D given the corresponding G. $\sup_D U(p_g, D)$ is convex in p_g with a unique global optima as proven in Thm 1, therefore with sufficiently small updates of p_g, p_g converges to p_x, concluding the proof. □

이 논문의 제일 인상적인 부분인듯. 결국 Convex니까 조금씩 update 하면 수렴한다는 뻔한 결론.

정확히 이해하지 않는 부분도 있고, 여러모로 나중에 조금더 신경써서 읽어야 할듯.

그나저나 논문에서도 언급했듯이 수학적 증명은 infinite capacity 인 P_g 에 대한 것으로, MLP를 사용한 경우 보장하지 않는다. 그러나 MLP가 실제적으로 충분한 성능을 보여주고, 간편하므로 MLP를 사용한다.

본 논문
비교한 것
이용한 내용

generator
discriminator.
중요한 내용

Generative Adversarial Nets

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Abstract

We propose a new framework for estimating generative models via an adversarial process, in which we simultaneously train two models: a generative model G that captures the data distribution, and a discriminative model D that estimates the probability that a sample came from the training data rather than G. The training procedure for G is to maximize the probability of D making a mistake. This framework corresponds to a minimax two-player game. In the space of arbitrary functions G and D, a unique solution exists, with G recovering the training data distribution and D equal to $\frac{1}{2}$ everywhere. In the case where G and D are defined by multilayer perceptrons, the entire system can be trained with backpropagation. There is no need for any Markov chains or unrolled approximate inference networks during either training or generation of samples. Experiments demonstrate the potential of the framework through qualitative and quantitative evaluation of the generated samples.

1 Introduction

The promise of deep learning is to discover rich, hierarchical models [2] that represent probability distributions over the kinds of data encountered in artificial intelligence applications, such as natural images, audio waveforms containing speech, and symbols in natural language corpora. So far, the most striking successes in deep learning have involved discriminative models, usually those that map a high-dimensional, rich sensory input to a class label [14, 20]. These striking successes have primarily been based on the backpropagation and dropout algorithms, using piecewise linear units [17, 8, 9] which have a particularly well-behaved gradient . Deep *generative* models have had less of an impact, due to the difficulty of approximating many intractable probabilistic computations that arise in maximum likelihood estimation and related strategies, and due to difficulty of leveraging the benefits of piecewise linear units in the generative context. We propose a new generative model estimation procedure that sidesteps these difficulties. [1]

In the proposed *adversarial nets* framework, the generative model is pitted against an adversary: a discriminative model that learns to determine whether a sample is from the model distribution or the data distribution. The generative model can be thought of as analogous to a team of counterfeiters, trying to produce fake currency and use it without detection, while the discriminative model is analogous to the police, trying to detect the counterfeit currency. Competition in this game drives both teams to improve their methods until the counterfeits are indistiguishable from the genuine articles.

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[‡] Sherjil Ozair is visiting Université de Montréal from Indian Institute of Technology Delhi
[§] Yoshua Bengio is a CIFAR Senior Fellow.
[1] All code and hyperparameters available at `http://www.github.com/goodfeli/adversarial`

This framework can yield specific training algorithms for many kinds of model and optimization algorithm. In this article, we explore the special case when the generative model generates samples by passing random noise through a multilayer perceptron, and the discriminative model is also a multilayer perceptron. We refer to this special case as *adversarial nets*. In this case, we can train both models using only the highly successful backpropagation and dropout algorithms [16] and sample from the generative model using only forward propagation. No approximate inference or Markov chains are necessary.

2 Related work

Until recently, most work on deep generative models focused on models that provided a parametric specification of a probability distribution function. The model can then be trained by maximizing the log likelihood. In this family of model, perhaps the most succesful is the deep Boltzmann machine [25]. Such models generally have intractable likelihood functions and therefore require numerous approximations to the likelihood gradient. These difficulties motivated the development of "generative machines"–models that do not explicitly represent the likelihood, yet are able to generate samples from the desired distribution. Generative stochastic networks [4] are an example of a generative machine that can be trained with exact backpropagation rather than the numerous approximations required for Boltzmann machines. This work extends the idea of a generative machine by eliminating the Markov chains used in generative stochastic networks.

Our work backpropagates derivatives through generative processes by using the observation that

$$\lim_{\sigma \to 0} \nabla_{\boldsymbol{x}} \mathbb{E}_{\epsilon \sim \mathcal{N}(0, \sigma^2 \boldsymbol{I})} f(\boldsymbol{x} + \epsilon) = \nabla_{\boldsymbol{x}} f(\boldsymbol{x}).$$

We were unaware at the time we developed this work that Kingma and Welling [18] and Rezende *et al.* [23] had developed more general stochastic backpropagation rules, allowing one to backpropagate through Gaussian distributions with finite variance, and to backpropagate to the covariance parameter as well as the mean. These backpropagation rules could allow one to learn the conditional variance of the generator, which we treated as a hyperparameter in this work. Kingma and Welling [18] and Rezende *et al.* [23] use stochastic backpropagation to train variational autoencoders (VAEs). Like generative adversarial networks, variational autoencoders pair a differentiable generator network with a second neural network. Unlike generative adversarial networks, the second network in a VAE is a recognition model that performs approximate inference. GANs require differentiation through the visible units, and thus cannot model discrete data, while VAEs require differentiation through the hidden units, and thus cannot have discrete latent variables. Other VAE-like approaches exist [12, 22] but are less closely related to our method.

Previous work has also taken the approach of using a discriminative criterion to train a generative model [29, 13]. These approaches use criteria that are intractable for deep generative models. These methods are difficult even to approximate for deep models because they involve ratios of probabilities which cannot be approximated using variational approximations that lower bound the probability. Noise-contrastive estimation (NCE) [13] involves training a generative model by learning the weights that make the model useful for discriminating data from a fixed noise distribution. Using a previously trained model as the noise distribution allows training a sequence of models of increasing quality. This can be seen as an informal competition mechanism similar in spirit to the formal competition used in the adversarial networks game. The key limitation of NCE is that its "discriminator" is defined by the ratio of the probability densities of the noise distribution and the model distribution, and thus requires the ability to evaluate and backpropagate through both densities.

Some previous work has used the general concept of having two neural networks compete. The most relevant work is predictability minimization [26]. In predictability minimization, each hidden unit in a neural network is trained to be different from the output of a second network, which predicts the value of that hidden unit given the value of all of the other hidden units. This work differs from predictability minimization in three important ways: 1) in this work, the competition between the networks is the sole training criterion, and is sufficient on its own to train the network. Predictability minimization is only a regularizer that encourages the hidden units of a neural network to be statistically independent while they accomplish some other task; it is not a primary training criterion. 2) The nature of the competition is different. In predictability minimization, two networks' outputs are compared, with one network trying to make the outputs similar and the other trying to make the

outputs different. The output in question is a single scalar. In GANs, one network produces a rich, high dimensional vector that is used as the input to another network, and attempts to choose an input that the other network does not know how to process. 3) The specification of the learning process is different. Predictability minimization is described as an optimization problem with an objective function to be minimized, and learning approaches the minimum of the objective function. GANs are based on a minimax game rather than an optimization problem, and have a value function that one agent seeks to maximize and the other seeks to minimize. The game terminates at a saddle point that is a minimum with respect to one player's strategy and a maximum with respect to the other player's strategy.

Generative adversarial networks has been sometimes confused with the related concept of "adversarial examples" [28]. Adversarial examples are examples found by using gradient-based optimization directly on the input to a classification network, in order to find examples that are similar to the data yet misclassified. This is different from the present work because adversarial examples are not a mechanism for training a generative model. Instead, adversarial examples are primarily an analysis tool for showing that neural networks behave in intriguing ways, often confidently classifying two images differently with high confidence even though the difference between them is imperceptible to a human observer. The existence of such adversarial examples does suggest that generative adversarial network training could be inefficient, because they show that it is possible to make modern discriminative networks confidently recognize a class without emulating any of the human-perceptible attributes of that class.

3 Adversarial nets

The adversarial modeling framework is most straightforward to apply when the models are both multilayer perceptrons. To learn the generator's distribution p_g over data $\boldsymbol{x}$, we define a prior on input noise variables $p_{\boldsymbol{z}}(\boldsymbol{z})$, then represent a mapping to data space as $G(\boldsymbol{z}; \theta_g)$, where G is a differentiable function represented by a multilayer perceptron with parameters θ_g. We also define a second multilayer perceptron $D(\boldsymbol{x}; \theta_d)$ that outputs a single scalar. $D(\boldsymbol{x})$ represents the probability that $\boldsymbol{x}$ came from the data rather than p_g. We train D to maximize the probability of assigning the correct label to both training examples and samples from G. We simultaneously train G to minimize $\log(1 - D(G(\boldsymbol{z})))$. In other words, D and G play the following two-player minimax game with value function $V(G, D)$:

$$\min_G \max_D V(D, G) = \mathbb{E}_{\boldsymbol{x} \sim p_{\text{data}}(\boldsymbol{x})}[\log D(\boldsymbol{x})] + \mathbb{E}_{\boldsymbol{z} \sim p_{\boldsymbol{z}}(\boldsymbol{z})}[\log(1 - D(G(\boldsymbol{z})))]. \tag{1}$$

In the next section, we present a theoretical analysis of adversarial nets, essentially showing that the training criterion allows one to recover the data generating distribution as G and D are given enough capacity, i.e., in the non-parametric limit. See Figure 1 for a less formal, more pedagogical 교육적인 explanation of the approach. In practice, we must implement the game using an iterative, numerical approach. Optimizing D to completion in the inner loop of training is computationally prohibitive, and on finite datasets would result in overfitting. Instead, we alternate between k steps of optimizing D and one step of optimizing G. This results in D being maintained near its optimal solution, so long as G changes slowly enough. The procedure is formally presented in Algorithm 1.

In practice, equation 1 may not provide sufficient gradient for G to learn well. Early in learning, when G is poor, D can reject samples with high confidence because they are clearly different from the training data. In this case, $\log(1 - D(G(\boldsymbol{z})))$ saturates. Rather than training G to minimize $\log(1 - D(G(\boldsymbol{z})))$ we can train G to maximize $\log D(G(\boldsymbol{z}))$. This objective function results in the same fixed point of the dynamics of G and D but provides much stronger gradients early in learning.

4 Theoretical Results

The generator G implicitly defines a probability distribution p_g as the distribution of the samples $G(\boldsymbol{z})$ obtained when $\boldsymbol{z} \sim p_{\boldsymbol{z}}$. Therefore, we would like Algorithm 1 to converge to a good estimator of p_{data}, if given enough capacity and training time. The results of this section are done in a non-parametric setting, e.g. we represent a model with infinite capacity by studying convergence in the space of probability density functions.

We will show in section 4.1 that this minimax game has a global optimum for $p_g = p_{\text{data}}$. We will then show in section 4.2 that Algorithm 1 optimizes Eq 1, thus obtaining the desired result.

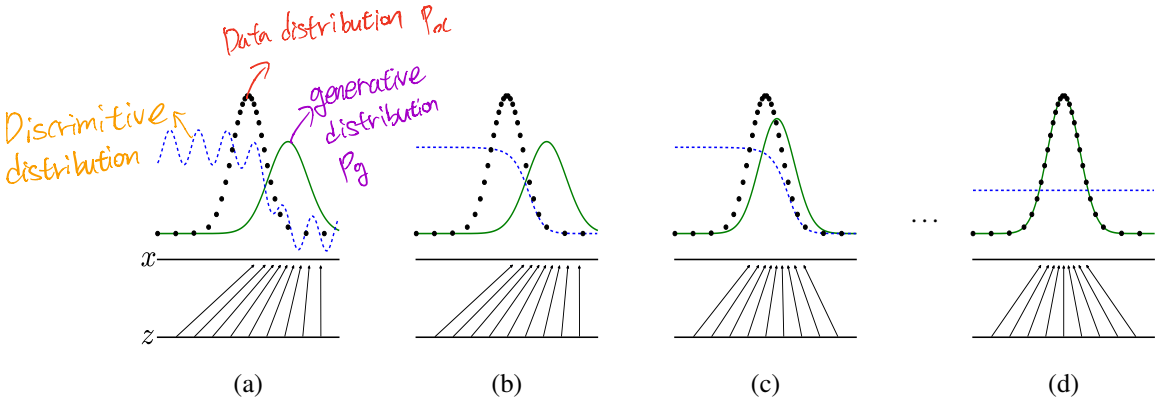

Figure 1: Generative adversarial nets are trained by simultaneously updating the **d**iscriminative distribution (D, blue, dashed line) so that it discriminates between samples from the data generating distribution (black, dotted line) $p_{\boldsymbol{x}}$ from those of the **g**enerative distribution p_g (G) (green, solid line). The lower horizontal line is the domain from which $\boldsymbol{z}$ is sampled, in this case uniformly. The horizontal line above is part of the domain of $\boldsymbol{x}$. The upward arrows show how the mapping $\boldsymbol{x} = G(\boldsymbol{z})$ imposes the non-uniform distribution p_g on transformed samples. G contracts in regions of high density and expands in regions of low density of p_g. (a) Consider an adversarial pair near convergence: p_g is similar to p_{data} and D is a partially accurate classifier. (b) In the inner loop of the algorithm D is trained to discriminate samples from data, converging to $D^*(\boldsymbol{x}) = \frac{p_{\text{data}}(\boldsymbol{x})}{p_{\text{data}}(\boldsymbol{x}) + p_g(\boldsymbol{x})}$. (c) After an update to G, gradient of D has guided $G(\boldsymbol{z})$ to flow to regions that are more likely to be classified as data. (d) After several steps of training, if G and D have enough capacity, they will reach a point at which both cannot improve because $p_g = p_{\text{data}}$. The discriminator is unable to differentiate between the two distributions, i.e. $D(\boldsymbol{x}) = \frac{1}{2}$.

Algorithm 1 Minibatch stochastic gradient descent training of generative adversarial nets. The number of steps to apply to the discriminator, k, is a hyperparameter. We used $k = 1$, the least expensive option, in our experiments.

for number of training iterations **do**

  for k steps **do**

  - Sample minibatch of m noise samples $\{\boldsymbol{z}^{(1)}, \ldots, \boldsymbol{z}^{(m)}\}$ from noise prior $p_g(\boldsymbol{z})$.
  - Sample minibatch of m examples $\{\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(m)}\}$ from data generating distribution $p_{\text{data}}(\boldsymbol{x})$.
  - Update the discriminator by ascending its stochastic gradient:

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^{m} \left[\log D\left(\boldsymbol{x}^{(i)}\right) + \log\left(1 - D\left(G\left(\boldsymbol{z}^{(i)}\right)\right)\right) \right].$$

  end for

  - Sample minibatch of m noise samples $\{\boldsymbol{z}^{(1)}, \ldots, \boldsymbol{z}^{(m)}\}$ from noise prior $p_g(\boldsymbol{z})$.
  - Update the generator by descending its stochastic gradient:

$$\nabla_{\theta_g} \frac{1}{m} \sum_{i=1}^{m} \log\left(1 - D\left(G\left(\boldsymbol{z}^{(i)}\right)\right)\right).$$

end for

The gradient-based updates can use any standard gradient-based learning rule. We used momentum in our experiments.

4.1 Global Optimality of $p_g = p_{\text{data}}$

We first consider the optimal discriminator D for any given generator G.

Proposition 1. *For G fixed, the optimal discriminator D is*

$$D^*_G(\boldsymbol{x}) = \frac{p_{data}(\boldsymbol{x})}{p_{data}(\boldsymbol{x}) + p_g(\boldsymbol{x})} \tag{2}$$

Proof. The training criterion for the discriminator D, given any generator G, is to maximize the quantity $V(G, D)$

$$V(G,D) = \int_{\boldsymbol{x}} p_{\text{data}}(\boldsymbol{x}) \log(D(\boldsymbol{x}))dx + \int_{z} p_{\boldsymbol{z}}(\boldsymbol{z}) \log(1 - D(g(\boldsymbol{z})))dz$$
$$= \int_{\boldsymbol{x}} p_{\text{data}}(\boldsymbol{x}) \log(D(\boldsymbol{x})) + p_g(\boldsymbol{x}) \log(1 - D(\boldsymbol{x}))dx \tag{3}$$

For any $(a, b) \in \mathbb{R}^2 \setminus \{0, 0\}$, the function $y \to a \log(y) + b \log(1 - y)$ achieves its maximum in $[0, 1]$ at $\frac{a}{a+b}$. The discriminator does not need to be defined outside of $Supp(p_{\text{data}}) \cup Supp(p_g)$, concluding the proof. □

Note that the training objective for D can be interpreted as maximizing the log-likelihood for estimating the conditional probability $P(Y = y|\boldsymbol{x})$, where Y indicates whether $\boldsymbol{x}$ comes from p_{data} (with $y = 1$) or from p_g (with $y = 0$). The minimax game in Eq. 1 can now be reformulated as:

$$\begin{aligned} C(G) =& \max_D V(G,D) \\ =& \mathbb{E}_{\boldsymbol{x}\sim p_{\text{data}}}[\log D_G^*(\boldsymbol{x})] + \mathbb{E}_{\boldsymbol{z}\sim p_{\boldsymbol{z}}}[\log(1 - D_G^*(G(\boldsymbol{z})))] \\ =& \mathbb{E}_{\boldsymbol{x}\sim p_{\text{data}}}[\log D_G^*(\boldsymbol{x})] + \mathbb{E}_{\boldsymbol{x}\sim p_g}[\log(1 - D_G^*(\boldsymbol{x}))] \\ =& \mathbb{E}_{\boldsymbol{x}\sim p_{\text{data}}}\left[\log \frac{p_{\text{data}}(\boldsymbol{x})}{P_{\text{data}}(\boldsymbol{x}) + p_g(\boldsymbol{x})}\right] + \mathbb{E}_{\boldsymbol{x}\sim p_g}\left[\log \frac{p_g(\boldsymbol{x})}{p_{\text{data}}(\boldsymbol{x}) + p_g(\boldsymbol{x})}\right] \end{aligned} \tag{4}$$

Theorem 1. *The global minimum of the virtual training criterion $C(G)$ is achieved if and only if $p_g = p_{data}$. At that point, $C(G)$ achieves the value $-\log 4$.*

Proof. For $p_g = p_{\text{data}}$, $D_G^*(\boldsymbol{x}) = \frac{1}{2}$, (consider Eq. 2). Hence, by inspecting Eq. 4 at $D_G^*(\boldsymbol{x}) = \frac{1}{2}$, we find $C(G) = \log \frac{1}{2} + \log \frac{1}{2} = -\log 4$. To see that this is the best possible value of $C(G)$, reached only for $p_g = p_{\text{data}}$, observe that

$$\mathbb{E}_{\boldsymbol{x}\sim p_{\text{data}}}[-\log 2] + \mathbb{E}_{\boldsymbol{x}\sim p_g}[-\log 2] = -\log 4$$

and that by subtracting this expression from $C(G) = V(D_G^*, G)$, we obtain:

$$C(G) = -\log(4) + KL\left(p_{\text{data}} \middle\| \frac{p_{\text{data}} + p_g}{2}\right) + KL\left(p_g \middle\| \frac{p_{\text{data}} + p_g}{2}\right) \tag{5}$$

where KL is the Kullback–Leibler divergence. We recognize in the previous expression the Jensen–Shannon divergence between the model's distribution and the data generating process:

$$C(G) = -\log(4) + 2 \cdot JSD\left(p_{\text{data}} \| p_g\right) \tag{6}$$

Since the Jensen–Shannon divergence between two distributions is always non-negative, and zero iff they are equal, we have shown that $C^* = -\log(4)$ is the global minimum of $C(G)$ and that the only solution is $p_g = p_{\text{data}}$, i.e., the generative model perfectly replicating the data distribution. □

4.2 Convergence of Algorithm 1

Proposition 2. *If G and D have enough capacity, and at each step of Algorithm 1, the discriminator is allowed to reach its optimum given G, and p_g is updated so as to improve the criterion*

$$\mathbb{E}_{\boldsymbol{x}\sim p_{data}}[\log D_G^*(\boldsymbol{x})] + \mathbb{E}_{\boldsymbol{x}\sim p_g}[\log(1 - D_G^*(\boldsymbol{x}))]$$

then p_g converges to p_{data}

Proof. Consider $V(G, D) = U(p_g, D)$ as a function of p_g as done in the above criterion. Note that $U(p_g, D)$ is convex in p_g. The subderivatives of a supremum of convex functions include the derivative of the function at the point where the maximum is attained. In other words, if $f(x) = \sup_{\alpha\in\mathcal{A}} f_\alpha(x)$ and $f_\alpha(x)$ is convex in x for every α, then $\partial f_\beta(x) \in \partial f$ if $\beta = \arg\sup_{\alpha\in\mathcal{A}} f_\alpha(x)$. This is equivalent to computing a gradient descent update for p_g at the optimal D given the corresponding G. $\sup_D U(p_g, D)$ is convex in p_g with a unique global optima as proven in Thm 1, therefore with sufficiently small updates of p_g, p_g converges to p_x, concluding the proof. □

In practice, adversarial nets represent a limited family of p_g distributions via the function $G(\boldsymbol{z}; \theta_g)$, and we optimize θ_g rather than p_g itself, so the proofs do not apply. However, the excellent performance of multilayer perceptrons in practice suggests that they are a reasonable model to use despite their lack of theoretical guarantees.

Model	MNIST	TFD
DBN [3]	138 ± 2	1909 ± 66
Stacked CAE [3]	121 ± 1.6	$\mathbf{2110 \pm 50}$
Deep GSN [5]	214 ± 1.1	1890 ± 29
Adversarial nets	$\mathbf{225 \pm 2}$	$\mathbf{2057 \pm 26}$

Table 1: Parzen window-based log-likelihood estimates. The reported numbers on MNIST are the mean log-likelihood of samples on test set, with the standard error of the mean computed across examples. On TFD, we computed the standard error across folds of the dataset, with a different σ chosen using the validation set of each fold. On TFD, σ was cross validated on each fold and mean log-likelihood on each fold were computed. For MNIST we compare against other models of the real-valued (rather than binary) version of dataset.

5 Experiments

We trained adversarial nets an a range of datasets including MNIST[21], the Toronto Face Database (TFD) [27], and CIFAR-10 [19]. The generator nets used a mixture of rectifier linear activations [17, 8] and sigmoid activations, while the discriminator net used maxout [9] activations. Dropout [16] was applied in training the discriminator net. While our theoretical framework permits the use of dropout and other noise at intermediate layers of the generator, we used noise as the input to only the bottommost layer of the generator network.

We estimate probability of the test set data under p_g by fitting a Gaussian Parzen window to the samples generated with G and reporting the log-likelihood under this distribution. The σ parameter of the Gaussians was obtained by cross validation on the validation set. This procedure was introduced in Breuleux *et al.* [7] and used for various generative models for which the exact likelihood is not tractable [24, 3, 4]. Results are reported in Table 1. This method of estimating the likelihood has somewhat high variance and does not perform well in high dimensional spaces but it is the best method available to our knowledge. Advances in generative models that can sample but not estimate likelihood directly motivate further research into how to evaluate such models. In Figures 2 and 3 we show samples drawn from the generator net after training. While we make no claim that these samples are better than samples generated by existing methods, we believe that these samples are at least competitive with the better generative models in the literature and highlight the potential of the adversarial framework.

6 Advantages and disadvantages

This new framework comes with advantages and disadvantages relative to previous modeling frameworks. The disadvantages are primarily that there is no explicit representation of $p_g(\boldsymbol{x})$, and that D must be synchronized well with G during training (in particular, G must not be trained too much without updating D, in order to avoid "the Helvetica scenario" in which G collapses too many values of $\mathbf{z}$ to the same value of $\mathbf{x}$ to have enough diversity to model p_{data}), much as the negative chains of a Boltzmann machine must be kept up to date between learning steps. The advantages are that Markov chains are never needed, only backprop is used to obtain gradients, no inference is needed during learning, and a wide variety of functions can be incorporated into the model. Table 2 summarizes the comparison of generative adversarial nets with other generative modeling approaches.

The aforementioned advantages are primarily computational. Adversarial models may also gain some statistical advantage from the generator network not being updated directly with data examples, but only with gradients flowing through the discriminator. This means that components of the input are not copied directly into the generator's parameters. Another advantage of adversarial networks is that they can represent very sharp, even degenerate distributions, while methods based on Markov chains require that the distribution be somewhat blurry in order for the chains to be able to mix between modes.

7 Conclusions and future work

This framework admits many straightforward extensions:

Figure 2: Visualization of samples from the model. Rightmost column shows the nearest training example of the neighboring sample, in order to demonstrate that the model has not memorized the training set. Samples are fair random draws, not cherry-picked. Unlike most other visualizations of deep generative models, these images show actual samples from the model distributions, not conditional means given samples of hidden units. Moreover, these samples are uncorrelated because the sampling process does not depend on Markov chain mixing. a) MNIST b) TFD c) CIFAR-10 (fully connected model) d) CIFAR-10 (convolutional discriminator and "deconvolutional" generator)

Figure 3: Digits obtained by linearly interpolating between coordinates in $\boldsymbol{z}$ space of the full model.

1. A *conditional generative* model $p(\boldsymbol{x} \mid \boldsymbol{c})$ can be obtained by adding $\boldsymbol{c}$ as input to both G and D.
2. *Learned approximate inference* can be performed by training an auxiliary network to predict $\boldsymbol{z}$ given $\boldsymbol{x}$. This is similar to the inference net trained by the wake-sleep algorithm [15] but with the advantage that the inference net may be trained for a fixed generator net after the generator net has finished training.
3. One can approximately model all conditionals $p(\boldsymbol{x}_S \mid \boldsymbol{x}_{\not S})$ where S is a subset of the indices of $\boldsymbol{x}$ by training a family of conditional models that share parameters. Essentially, one can use adversarial nets to implement a stochastic extension of the deterministic MP-DBM [10].
4. *Semi-supervised learning*: features from the discriminator or inference net could improve performance of classifiers when limited labeled data is available.
5. *Efficiency improvements:* training could be accelerated greatly by devising better methods for coordinating G and D or determining better distributions to sample $\mathbf{z}$ from during training.

This paper has demonstrated the viability of the adversarial modeling framework, suggesting that these research directions could prove useful.

	Deep directed graphical models	Deep undirected graphical models	Generative autoencoders	Adversarial models
Training	Inference needed during training.	Inference needed during training. MCMC needed to approximate partition function gradient.	Enforced tradeoff between mixing and power of reconstruction generation	Synchronizing the discriminator with the generator. Helvetica.
Inference	Learned approximate inference	Variational inference	MCMC-based inference	Learned approximate inference
Sampling	No difficulties	Requires Markov chain	Requires Markov chain	No difficulties
Evaluating $p(x)$	Intractable, may be approximated with AIS	Intractable, may be approximated with AIS	Not explicitly represented, may be approximated with Parzen density estimation	Not explicitly represented, may be approximated with Parzen density estimation
Model design	Models need to be designed to work with the desired inference scheme — some inference schemes support similar model families as GANs	Careful design needed to ensure multiple properties	Any differentiable function is theoretically permitted	Any differentiable function is theoretically permitted

Table 2: Challenges in generative modeling: a summary of the difficulties encountered by different approaches to deep generative modeling for each of the major operations involving a model.

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