MPCP

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1 Model description

We consider a life-cycle consumption model with:

1. T periods of life
2. T_r periods of working
3. CRRA utility from consumption with age-varying parameter, σ_t
4. CRRA utility from joy-of-giving bequests with floor $\underline{a}$
5. Heterogeneous discount factors, $\beta_i \sim \mathcal{U}(\mu_\beta - \sigma_\beta, \mu_\beta + \sigma_\beta)$
6. Heterogeneous income levels, $\alpha_i \sim \mathcal{U}(\mu_\alpha - \sigma_\alpha + \iota\beta_i, \mu_\alpha + \sigma_\alpha + \iota\beta_i)$, correlated with discount factors
7. Fixed growth factor while working, G
8. Permanent/persistent shocks, ψ_t (permanent if $\rho = 1$)
9. Fully transitory shocks with left tail, $\tilde{\xi}_t$
10. Fixed retirement benefit ratio, ϕ
11. Top-tax rate for labor, τ , income above κ
12. Potential permanent tax-free bonus, $\chi_t \in \{0, 1\}$, of size Δ
13. Return factor, R, and no borrowing

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2 Recursive problem

The recursive problem is:

$$
\begin{aligned}
v_t(\alpha_i, \beta_i, \chi_t, p_t, m_t) &= \max_{c_t} \frac{c_t^{1-\sigma_t}}{1-\sigma_t} + \mathbb{1}_{t+1<T} \beta_i v_{t+1}(\bullet_{t+1}) + \mathbb{1}_{t+1=T} \beta_i \nu \frac{(a_t + \underline{a})^{\sigma_{T_r}}}{1-\sigma_{T_r}} \qquad (1) \\
&\text{s.t.} \\
\sigma_t &= \sigma_0 \omega^{\min\{t, T_r\}} \\
a_t &= m_t - c_t \\
p_{t+1} &= \psi_{t+1}^{\mathbb{1}_{t+1<T_r}} p_t^{\rho}, \;\; \log \psi_{t+1} \sim \mathcal{N}(-0.5\sigma_\psi^2, \sigma_\psi^2) \\
G_{t+1} &= G^{\min\{t+1, T_r\}} \\
\tilde{\xi}_{t+1} &= \begin{cases} \mu & \text{with prob. } \pi \\ \frac{\xi_{t+1}^{\mathbb{1}_{t+1<T_r}} - \pi\mu}{\mu} & \text{else} \end{cases}, \;\; \log \xi_{t+1} \sim \mathcal{N}(-0.5\sigma_\xi^2, \sigma_\xi^2) \\
y_{t+1}^{\text{pre}} &= \phi^{\mathbb{1}_{t+1<T_r}} \alpha_i G_{t+1} \tilde{\xi}_{t+1} p_{t+1} \\
y_{t+1} &= y_{t+1}^{\text{pre}} - \tau \max\left\{ y_{t+1}^{\text{pre}} - \kappa, 0 \right\} \\
m_{t+1} &= R a_t + y_{t+1} + \chi_t \Delta \\
\chi_{t+1} &= \chi_t \\
a_t &\geq 0.
\end{aligned}
$$

3 Consumption functions

The optimal consumption function is denoted by

$$c_t^{\star}(\alpha_i, \beta_i, \chi_t, p_t, m_t). \tag{2}$$

We can equivalently write it in terms of lagged assets,

$$\begin{aligned}
c_t^{\star}(\alpha_i, \beta_i, \chi_t, p_t, a_{t-1}, \tilde{\xi}_t) =& c_t^{\star}(\alpha_i, \beta_i, \chi_t, p_t, m_t(\alpha_i, \chi_t, p_t, a_{t-1}, \tilde{\xi}_t)) \\
m_t(\alpha_i, \chi_t, p_t, a_{t-1}, \tilde{\xi}_t) =& R a_{t-1} + y_t + \chi_t \\
& G_t = G^{\min\{t,T_r\}} \\
& y_t^{\text{pre}} = \alpha_i G_t \tilde{\xi}_t p_t \\
& y_t = y_t^{\text{pre}} - \tau \max\left\{y_t^{\text{pre}} - \kappa, 0\right\}.
\end{aligned} \tag{3}$$

4 MPC and MPCPs

Define $c_t^\star \equiv c_t^\star(\alpha_i, \beta_i, 0, p_t, m_t)$ and $m_t \equiv m_t(\alpha_i, 0, p_t, a_{t-1}, \tilde{\xi}_t)$.

We now consider four different kind of consumption responses to shocks:

1. The **Marginal Propensity to Consume (**MPC**)** is defined as

$$\text{MPC} = \lim_{\Delta \to 0} \frac{c_t^\star(\alpha_i, \beta_i, 0, p_t, m_t + \Delta) - c_t^\star}{\Delta}$$

   Here Δ is received only in the current period.

2. The **Marginal Propensity to Consume out of a *Permanent* Shock (MPCP$^{\text{perm}}$)** is defined as

$$\text{MPCP}^{\text{perm}} = \lim_{\Delta \to 0} \frac{c_t^\star(\alpha_i, \beta_i, 1, p_t, a_{t-1}, \tilde{\xi}_t) - c_t^\star}{m_t(\alpha_i, 1, p_t, a_{t-1}, \tilde{\xi}_t) - m_t}$$

   Here χ_t switches from 0 to 1 so Δ is received in the current and all future periods.

3. The **Marginal Propensity to Consume out of a *Permanent* Shock (MPCP$^{\text{perm}}$)** is defined as

$$\text{MPCP}^{\text{perm}} = \text{MPCP}^{\text{pers}} = \lim_{\Delta \to 0} \frac{c_t^\star(\alpha_i, \beta_i, 0, p_t + \Delta, a_{t-1}, \tilde{\xi}_t) - c_t^\star}{m_t(\alpha_i, 0, p_t + \Delta, a_{t-1}, \tilde{\xi}_t) - m_t}$$

   Here the persistent income component is increased. The response of future income relative to current income depend on ρ, G and ϕ.

4. The **elasticity of consumption to a scaling of income and savings (MPCP$^{\text{scale}}$)** is defined as

$$\text{MPCP}^{\text{scale}} = \lim_{\Delta \to 0} \frac{\log c_t^\star(\alpha_i(1+\Delta), \beta_i, 0, p_t, a_{t-1}(1+\Delta), \tilde{\xi}_t) - \log c_t^\star}{\log \alpha_i(1+\Delta) - \log \alpha_i}$$

We have mainly interested in MPCP$^{\text{scale}}$. We have the following observations:

1. For $\omega = 1$, $\underline{a} = 0$, and $\tau = 0$ we have $\text{MPCP}^{\text{scale}} = 1$
2. For $\omega = 1$ and $\underline{a} = 0$ we have $\text{MPCP}^{\text{scale}} \approx 1$
3. For $\omega < 1$ and/or $\underline{a} > 0$ we have $\text{MPCP}^{\text{scale}} < 1$
4. We generally have that $\text{MPCP}^{\text{scale}}$ is independent of ι

5 Data sets

We consider the following four data sets:

1. **homotheticity_simple:** $\omega = 1$, $\underline{a} = 0$, $\tau > 0$, $\iota = 0$: $\text{MPCP}^{\text{scale}} \approx 1$
2. **homotheticity_full:** $\omega = 1$, $\underline{a} = 0$, $\tau > 0$, $\iota > 0$: $\text{MPCP}^{\text{scale}} \approx 1$:
3. **non_homotheticity_simple:** $\omega < 1$, $\underline{a} > 0$, $\tau > 0$, $\iota = 0$: $\text{MPCP}^{\text{scale}} < 1$
4. **non_homotheticity_full:** $\omega < 1$, $\underline{a} > 0$, $\tau > 0$, $\iota > 0$: $\text{MPCP}^{\text{scale}} < 1$

Can we find a valid estimator for $\text{MPCP}^{\text{scale}}$?