

MPCP

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1 Model description

We consider a life-cycle consumption model with:

1. T periods of life
2. T_r periods of working
3. CRRA utility from consumption with age-varying parameter, σ_t
4. CRRA utility from joy-of-giving bequests with floor \underline{a}
5. Heterogeneous discount factors, $\beta_i \sim \mathcal{U}(\mu_\beta - \sigma_\beta, \mu_\beta + \sigma_\beta)$
6. Heterogeneous income levels, $\alpha_i \sim \mathcal{U}(\mu_\alpha - \sigma_\alpha + \iota\beta_i, \mu_\alpha + \sigma_\alpha + \iota\beta_i)$, correlated with discount factors
7. Fixed growth factor while working, G
8. Permanent/persistent shocks, ψ_t (permanent if $\rho = 1$)
9. Fully transitory shocks with left tail, $\tilde{\xi}_t$
10. Fixed retirement benefit ratio, ϕ
11. Top-tax rate for labor, τ , income above κ
12. Potential permanent tax-free bonus, $\chi_t \in \{0, 1\}$, of size Δ
13. Return factor, R , and no borrowing

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2 Recursive problem

The recursive problem is:

$$\begin{aligned}
 v_t(\alpha_i, \beta_i, \chi_t, p_t, m_t) &= \max_{c_t} \frac{c_t^{1-\sigma_t}}{1-\sigma_t} + \beta_i \left[\mathbb{1}_{t+1 < T} v_{t+1}(\bullet_{t+1}) + \mathbb{1}_{t+1=T} v \frac{(a_t + \underline{a})^{\sigma_{T_r}}}{1-\sigma_{T_r}} \right] \quad (1) \\
 \text{s.t.} \\
 \sigma_t &= \sigma_0 \omega^{\min\{t, T_r\}} \\
 a_t &= m_t - c_t \\
 p_{t+1} &= \psi_{t+1}^{\mathbb{1}_{t+1 < T_r}} p_t^\rho, \quad \log \psi_{t+1} \sim \mathcal{N}(-0.5\sigma_\psi^2, \sigma_\psi^2) \\
 G_{t+1} &= G^{\min\{t+1, T_r\}} \\
 \tilde{\xi}_{t+1} &= \begin{cases} \mu & \text{with prob. } \pi \\ \frac{\xi_{t+1} - \pi\mu}{\mu} & \text{else} \end{cases}, \quad \log \xi_{t+1} \sim \mathcal{N}(-0.5\sigma_\xi^2, \sigma_\xi^2) \\
 y_{t+1}^{\text{pre}} &= \alpha_i G_{t+1} \phi^{\mathbb{1}_{t+1 \geq T_r}} \tilde{\xi}_{t+1}^{\mathbb{1}_{t+1 < T_r}} p_{t+1} \\
 y_{t+1} &= y_{t+1}^{\text{pre}} - \tau \max\{y_{t+1}^{\text{pre}} - \kappa, 0\} \\
 m_{t+1} &= Ra_t + y_{t+1} + \chi_t \Delta \\
 \chi_{t+1} &= \chi_t \\
 a_t &\geq 0.
 \end{aligned}$$

3 Consumption functions

The optimal consumption function is denoted by

$$c_t^*(\alpha_i, \beta_i, \chi_t, p_t, m_t). \quad (2)$$

We can *equivalently* write it in terms of lagged assets,

$$\begin{aligned} c_t^*(\alpha_i, \beta_i, \chi_t, p_t, a_{t-1}, \tilde{\xi}_t) &= c_t^*(\alpha_i, \beta_i, \chi_t, p_t, m_t(\alpha_i, \chi_t, p_t, a_{t-1}, \tilde{\xi}_t)) \\ m_t(\alpha_i, \chi_t, p_t, a_{t-1}, \tilde{\xi}_t) &= Ra_{t-1} + y_t + \chi_t \Delta \\ G_t &= G^{\min\{t, T_r\}} \\ y_t^{\text{pre}} &= \alpha_i G_t \phi^{\mathbb{1}_{t \geq T_r}} \tilde{\xi}_t^{\mathbb{1}_{t < T_r}} p_t \\ y_t &= y_t^{\text{pre}} - \tau \max\{y_t^{\text{pre}} - \kappa, 0\}. \end{aligned} \quad (3)$$

4 MPC and MPCPs

Define $c_t^* \equiv c_t^*(\alpha_i, \beta_i, 0, p_t, m_t)$ and $m_t \equiv m_t(\alpha_i, 0, p_t, a_{t-1}, \tilde{\xi}_t)$.

We now consider four different kind of consumption responses to shocks:

1. The **Marginal Propensity to Consume (MPC)** is defined as

$$\text{MPC} = \lim_{\Delta \rightarrow 0} \frac{c_t^*(\alpha_i, \beta_i, 0, p_t, m_t + \Delta) - c_t^*}{\Delta}$$

Here Δ is received only in the current period.

2. The **Marginal Propensity to Consume out of a Permanent Shock (MPCP^{perm})** is defined as

$$\text{MPCP}^{\text{perm}} = \lim_{\Delta \rightarrow 0} \frac{c_t^*(\alpha_i, \beta_i, 1, p_t, a_{t-1}, \tilde{\xi}_t) - c_t^*}{m_t(\alpha_i, 1, p_t, a_{t-1}, \tilde{\xi}_t) - m_t}$$

Here χ_t switches from 0 to 1 so Δ is received in the current and all future periods.

3. The **Marginal Propensity to Consume out of a Persistent Shock (MPCP^{pers})** is defined as

$$\text{MPCP}^{\text{pers}} = \lim_{\Delta \rightarrow 0} \frac{c_t^*(\alpha_i, \beta_i, 0, p_t + \Delta, a_{t-1}, \tilde{\xi}_t) - c_t^*}{m_t(\alpha_i, 0, p_t + \Delta, a_{t-1}, \tilde{\xi}_t) - m_t}$$

Here the persistent income component is increased. The response of future income relative to current income depend on ρ , G and ϕ .

4. The **elasticity of consumption to a scaling of income and savings (MPCP^{scale})** is defined as

$$\text{MPCP}^{\text{scale}} = \lim_{\Delta \rightarrow 0} \frac{\log c_t^*(\alpha_i(1 + \Delta), \beta_i, 0, p_t, a_{t-1}(1 + \Delta), \tilde{\xi}_t) - \log c_t^*}{\log \alpha_i(1 + \Delta) - \log \alpha_i}$$

In the first period with $a_{-1} = 0$ this is just a scaling of the whole income path.

We are mainly interested in MPCP^{scale}.

We have the following observations:

1. For $\omega = 1$, $\underline{a} = 0$, and $\tau = 0$ we have $\text{MPCP}^{\text{scale}} = 1$
2. For $\omega = 1$ and $\underline{a} = 0$ we have $\text{MPCP}^{\text{scale}} \approx 1$
3. For $\omega < 1$ and/or $\underline{a} > 0$ we have $\text{MPCP}^{\text{scale}} < 1$
4. We generally have that $\text{MPCP}^{\text{scale}}$ is independent of ι

5 Data sets

Our baseline data set have $\omega = 1, \underline{a} = 0, \tau = 0, \rho < 1, \iota = 0$ and therefore $\text{MPCP}^{\text{scale}} = 1$.

We consider $2^4 - 1 = 15$ alternative data sets with the naming convention:

1. ***_non_homo:** $\omega < 1, \underline{a} > 0$: Big $\text{MPCP}^{\text{scale}} \downarrow$
2. ***_tau_homo:** $\tau > 0$: Small $\text{MPCP}^{\text{scale}} \downarrow$
3. ***_rho:** $\rho < 0$: $\text{MPCP}^{\text{scale}}$ unchanged
4. ***_iota:** $\iota > 0$: $\text{MPCP}^{\text{scale}}$ unchanged

Can we find a valid estimator for $\text{MPCP}^{\text{scale}}$ which always work?