W8 Complement

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Q2 in PS5

Regarding the second question of Problem Set 5, based on Seb's answer, you need to consider both the 'general' and 'particular' ways, making this a challenging problem. In fact, we first need to consider the properties of deductive validity and subset relations.

'Subset Relation' is a partial order.

We can easily observe the reflexivity, antisymmetry, and transitivity of subset relations, from which we conclude that subset relations are a partial order. (Please review the *definitions of subsets*, *definition of equality of sets* and *transitivity of subsets*.)

Proposition: Let X be a nonempty set and $\mathcal{P}(X)$ the power set of X. Define the "subset" relation S on $\mathcal{P}(X)$ as follows: For every $A, B \in \mathcal{P}(X)$, $A \, S \, B \iff A \subseteq B$.

Proof:

Let $\mathcal{A}$ be the universe of all sets, and the subset relation $\subseteq$ be defined on $\mathcal{A}$.

- S is reflexive: For all $A \in \mathcal{P}(X)$, $A \subseteq A$, therefore $A \, S \, A$.
- S is antisymmetry: Assume $A \subseteq B$ and $B \subseteq A$, then by definition of equality of sets, $A = B$.
- S is transitive: Assume $A \, S \, B$ and $B \, S \, C$. So $A \subseteq B$ and $B \subseteq C$. Then by transitivity of subsets, $A \subseteq C$, and $A \, S \, C$.

Thus the "subset" relation S is a partial relation. $\square$

“Deductive Validity” have similar properties as 'Subset Relation'.

In a model, we consider the set P_{re}, which consists of all elements in the domain such that make the premises true (this should actually be the intersection of the sets corresponding to these premises' predicates), and the set C_{on}, which consists of all elements in the domain such that make the conclusion true.

If the argument is valid, P_{re} should be a subset of C_{on} (because we know that any elements $x \in P_{re}$, x satisfying all the predicate of premises, from deductive validity, x should also holds the predicate of conclusion, therefore $x \in C_{on}$). As a result, all properties of deductive validity can be mapped (or be functioned) to the subset relationship, making it clear that they share the same properties of relation.

Summary:

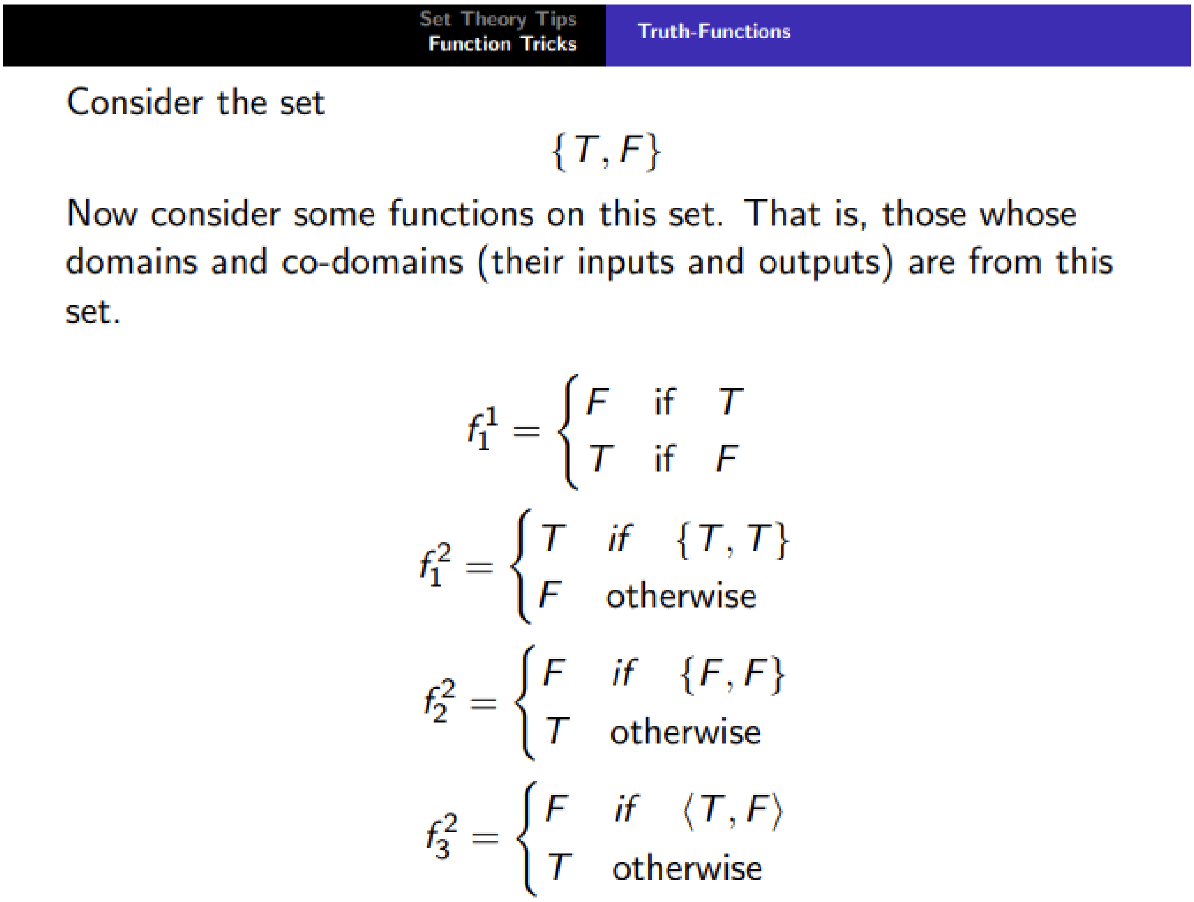
In the analysis above, we first explored the properties of these two relationships, attempting to find their commonalities. During this process, we discovered that the concept of 'Deductive Validity' can be described in terms of set and 'Subset Relation', thus establishing a connection between the two. Therefore, the answer can be summarized as follows:

> In general, both are partial orders. In particular, the subset relation tracks satisfaction in models for valid arguments.

In fact, you don't need to write a lot; as long as your reasoning is reasonable and comprehensive, you can score well. (Many people overlook the properties of relationships, which might be due to the course structure. However, by the time this problem set was released, our lecture had already covered relationship properties. Additionally, please note that all the concepts we study in this course are interconnected; please avoid thinking of them in isolation.)

Q5 in PS5

For the last question of Problem Set 5, I have to say the problem statement isn't very clear. Seb wants to examine the relationships between the three functions **he defined in the lecture slides**.



In fact, f_1^1, f_2^2, and f_3^2 correspond to $\sim$, $\vee$, and $\rightarrow$ respectively. Therefore, using the equivalence $p \rightarrow q \equiv \sim p \vee q$ we can easily reach the answer, which is $f_3^2(p, q) = f_2^2(f_1^1(p), q)$ $\square$

Note:

In this question, I also initially misunderstood the problem. However, I interpreted it as

> "All binary logical functions f_3^2 can be represented using a specific unary logical function f_1^1 and a specific binary logical function f_2^2."

I have already discussed this proof in the tutorial, but this is not the original intent of the question, so I won't elaborate further here. If you're interested, you can study the related proof.