Counting Mathods - Supplementary

COMP9020 Tutorial

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24T3 Week9, H18B & F15A

1 Tutorial Outline

1.1 Definition

Ask yourself these questions below to test your understanding.

- What are two *basic counting rules*?→ steps, classes
  - What is *the Inclusion/Exclusion Rule*?
  - How should we handle the *symmetric* case?
- What is the *permutation number* $(n)_k$?
  - What operation does it correspond to?
  - What is a *circular permutation*?
- What is the *combination number* $\binom{n}{k}$?
  - What operation does it correspond to?
  - What is its relationship to the permutation number?
  - What is the *"bars and stars"* method?

1.2 Brainstorming

The following questions are open and have no standard answers.

- What kinds of counting problems can we encounter in everyday life?
- What kinds of counting problems did we encounter in previous quizzes, and can you solve them on your own now?
- ......

2 Explanation of Tutorial Content

In this week's tutorial, I walked everyone through common counting methods using the *occupancy problem* as an example. However, it didn't go as smoothly as I'd hoped because I hadn't thought through some points thoroughly during my preparation, which led to a few incorrect conclusions.

So here, I'd like to make some corrections. My apologies—hopefully, this time everything is accurate.

Place k balls into n bins, where

-	Balls	Bins	Balls per Bin	Counting Number
$n \geq k$	✓	✓	≤ 1	$(n)_k$
-	✓	✓	Unrestricted	n^k
$n \geq k$	$\times$	✓	≤ 1	$\binom{n}{k}$
$n \geq k = \sum_{i=1}^{t} k_i$	$\star$	✓	≤ 1	$\binom{n}{k_1\ k_2\ \ldots\ k_t}$
-	$\times$	✓	Unrestricted	$\left(\binom{n}{k}\right)$
$k \geq n$	✓	$\times$	≥ 1	$\left\{ {n \atop k} \right\}$

Here,

- ✓ indicates ”distinguishable” and $\times$ indicates ”indistinguishable”.
- $\star$ means there are k_i balls are indistinguishable $(1 \leq i \leq t)$ respectively.

A few clarifications are needed here:

1. The *multinomial coefficient* $\binom{n}{k_1\ k_2\ \ldots\ k_t}$ and the *Stirling numbers of the second kind* $\left\{ {n \atop k} \right\}$ are not within the scope of our course material. I included them as extensions based on content that appeared in tutorials and quizzes.

2. Please don't just memorize the form—what's more important is understanding how the formula is derived.

The error regarding the Stirling numbers:

During the explanation for class F15A, a student pointed out that the Stirling numbers should correspond to the scenario where the balls are *distinguishable*, which is indeed my oversight.

For example, if we consider the *equivalence relations* generated by a set $A = \{1, 2, 3\}$, in my interpretation, cases like $[1], [2, 3]$ and $[2], [1, 3]$ would be distinct. However, if the balls are indistinguishable, these both represent the case where one bin has one ball and the other has two, which is not the intended outcome.[1]

I apologize for the oversight. Additionally, if the balls are also indistinguishable, this relates to a classic function in number theory—*the partition function* $p(n)$. While this goes beyond our course's scope, feel free to explore it if you're interested;)

[1] As for the “counterexample” I tried to argue in F15A class—where we treat $[1], [2, 3]$ and $[1], [3, 2]$ as the same case—since the balls are distinguishable, we can arrange them in sequence and proceed step-by-step. This way, such unordered cases won't occur. So this 'counterexample' attempts to disprove something that doesn't actually exist, making it invalid.