Climate policies under wealth inequality

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Abstract

One of the keys to resolve the planet's climate problem is cooperation. One of the obstacles we faced trying to reach a consensus was the conflicting policies between rich and poor countries. In this paper, we will reproduce the work of Vítor Vasconcelos, Francisco Santos, Jorge Pacheco, and Simon Levin from their article *Climate policies under wealth inequality* [3]. We will implement a Threshold Public Good Game that will try to simulate the actual situation of the quest for cooperation. Players will be divided into two categories, rich and poor, the amount of players in each of those categories is asymmetrical as is currently the case in our countries. Each player has an initial endowment that represents the resources of the players, and this endowment can be used to reach the objective. Each player can decide to cooperate by contributing to reach the objective or to defect without contributing. If the objective is not reached, players will face the risk of losing a part of their endowment. Players will exhibit a certain amount of homophily that represents the influence between players of the same category. Our model shows that a higher fraction of the rich contribute. Rich countries will compensate for the defection of the poor, while poor countries will be more inclined to contribute if rich countries also contribute.

Introduction

Climate Change is probably the biggest challenge humanity is addressing during the XXIst century. Whereas the consequences of an atmospheric temperature augmentation are increasing both in frequency and strength, actions to deal with these emergencies at the opposite seem way more slower. In fact, according to IPCC [2] and climatologists, efforts made so far to address climate change remain way insufficient. This situation can be studied with an evolutionary game theory model. In this model, players will represent countries and own a certain endowment that they can use to contribute to a common goal. This endowment represents the actions a country can take to reduce its environmental print, as reducing its greenhouse gases emissions, funding the fight. The goal represents the threshold of actions required to limit Climate Change. If this goal is not reached, the players will be exposed to the risk of losing their endowment. In this game, each player will have the choice to contribute to the common goal or to wait for other players to do it in their place. This situation might be represented by a game theory model where the actors are the countries. In this game, each player will act according to his own best interest. If it is an evidence that all countries will prefer to avoid the Climate Change disaster, each country will have an interest not to contribute and let the others take all the responsibilities. We will explain how the cooperation between countries to deal with Climate Change can be seen as a Threshold Public Good Game (PGG), or more specifically a Collective Risk Dilemma (CRD), and then use our model to study how players decide to contribute or not in different situations. This article is based on the work of Vítor Vasconcelos, Francisco Santos, Jorge Pacheco, and Simon Levin [3] (hereafter the *original article*. Our objective is to reexplain their model of the PGG and reproduce their calculations to aim to get the same results. The experiments described in the section `Experiment` are the same experiment as the one in the original article and in the *Supporting Information Text* [1], and the results described in the section `Results`, as well as the conclusions we draw are ours. The code used to make our experiment can be found in a public repository on Github. [1]

Methods

Evolutionary Game Theory model

Threshold Public Good game The model used to simulate this situation is the Threshold Public Good game. In this kind of game, players can collaborate in order to reach a common goal. For this purpose, players receive a certain endowment (represented as a numeric value) which represents the resources that a player can use to reach the common objective. In order to represent the wealth disparities among the countries, the population will be composed of two kinds of players: the rich and the poor players, differentiated by a greater endowment for rich players. A ratio of rich - poor players of 80% - 20% will be considered as it reflects the fact that 20% of the wealthiest countries in the world have

[1] `https://github.com/Jepsiko/learning_dynamics_group_19`

the same gross domestic product as the other 80%. Players have therefore two strategies to play the game:

- Collaboration: the player allocates a part of its resources to the pursuit of the common objective
- Defection: the player does not allocate anything to the pursuit of the objective

In other words, each player can either contribute to the common objective or hope for the others to do it for him. The contribution is defined as a fraction of a player's endowment. This fraction is constant for all players so each collaborator will contribute the same percentage of its endowment. The goal is considered complete only if the threshold is reached. If the goal is not reached, players will face the risk of losing the remaining of their endowments. The risk perception factors indicates how much players are aware of the likelihood of this consequence. Each player knows the actions taken by other players and this can have an influence on its own behavior. A homophily parameter comprised between 0 and 1 describes this phenomenon: if the homophily is close to 0 players can be influenced by other players indistinctly, if the homophily is close to 1, players will be more – only – influenced by players of the same wealth class.

Threshold Public Good game in practice In practice, the population has a finite size $Z = Z_P + Z_R$ and it is assumed $Z_P \geq Z_R$ with a ratio poor players - rich players of 80% - 20%. From this population, sub-groups of size N will be randomly constituted to play the game (groups, therefore, contain variable amounts of rich and poor players). Each player receives a certain endowment b for which it will be assumed $b_R > b_P$. A fraction c of this endowment can be used to contribute to the common goal. In that way, rich players and poor players contribute the same fraction of their total resources. Each player can make the choice to collaborate (C) and use this fraction for the common goal or defect and keep it for itself (D). The goal is defined in function of the fraction $0 < c < 1$, the average endowment of the population $\bar{b} = (b_R Z_R + b_P Z_P)\frac{1}{Z} = 1$ and an integer $0 < M \leq N$ which aims to make the threshold reachable even if not every single player contributes to it. The risk perception factor is denoted by $r \in [0, 1]$, the homophily factor by $h \in [0, 1]$. Finally, decisions are subject to errors with a small frequency, which aims to represent stochastic effects and which can lead to behavioral mutations and is noted $\mu = \frac{1}{Z}$.

Payoff For a sub-group of players, the payoff is computed as follows (value is given for rich/poor players respectively). For defectors:

$$\Pi^D_{R/P}(j_R, j_P) = b_{R/P}\{\Theta\left(c_R j_R + c_P j_P - Mc\bar{b}\right) + (1-r)\left[1 - \Theta\left(c_R j_R + c_P j_P - Mc\bar{b}\right)\right]\} \tag{1}$$

And for cooperators

$$\Pi^C_{R/P} = \Pi^D_{R/P} - c_{R/P} \tag{2}$$

where j_R and j_P respectively represent the number of rich and poor players in each group, c_R and c_P the amount of rich and poor contributors, $Mc\bar{b}$ is the goal to reach and Θ is the Heaviside function defined as follows: $\Theta(x) = 1$ if $x \geq 0$, 0 otherwise. In other words, the Heaviside function is equal to 1 if the total contribution of the Cs is greater or equal to the goal to reach, or 0 in the other case.

Fitness The fitness of a player adopting a given strategy is associated with the average payoff of that same strategy in the entire population. In the following, we will consider configurations the game $i = \{i_R, i_P\}$ where i_R and i_P are respectively the total number of rich and poor cooperators. The average payoff of a given strategy in a specific configuration $i = \{i_R, i_P\}$, is computed with a multivariate hypergeometric sampling (without replacement):

$$f^C_R(i) = \binom{N-1}{Z-1}^{-1} \sum_{j_R=0}^{N-1} \sum_{j_P=0}^{N-1-j_R} \binom{i_R - 1}{j_R}\binom{i_P}{j_P} \times \binom{Z - i_R - i_P}{N - 1 - j_R - j_P} \Pi^C_R(j_R + 1, j_P) \tag{3}$$

$$f^C_D(i) = \binom{N-1}{Z-1}^{-1} \sum_{j_R=0}^{N-1} \sum_{j_P=0}^{N-1-j_R} \binom{i_R}{j_R}\binom{i_P}{j_P} \times \binom{Z - 1 - i_R - i_P}{N - 1 - j_R - j_P} \Pi^D_R(j_R, j_P) \tag{4}$$

$$f^C_P(i) = \binom{N-1}{Z-1}^{-1} \sum_{j_R=0}^{N-1} \sum_{j_P=0}^{N-1-j_R} \binom{i_R}{j_R}\binom{i_P - 1}{j_P} \times \binom{Z - i_R - i_P}{N - 1 - j_R - j_P} \Pi^C_R(j_R, j_P + 1) \tag{5}$$

$$f^P_D(i) = \binom{N-1}{Z-1}^{-1} \sum_{j_R=0}^{N-1} \sum_{j_P=0}^{N-1-j_R} \binom{i_R}{j_R}\binom{i_P}{j_P} \times \binom{Z - 1 - i_R - i_P}{N - 1 - j_R - j_P} \Pi^C_R(j_R, j_P) \tag{6}$$

with C/D related to strategies and R/P related to wealth class.

Evolution of the system The probability that a player starts using the strategy of another player is given by the Fermi function:

$$\frac{1}{1 + e^{\beta(f^X_k - f^Y_l)}} \tag{7}$$

where $X, Y \in C, D$ and the parameter β controls the intensity of selection. The global evolution of the system (evolution of the distributions of the strategies among players) depends only on the current state and can therefore be described as a Markov process over two-dimensional space which probability distribution is computed as follows:

$$p_i(t+\tau) - p_i(t) = \sum_{i'} (T_{ii'} p_{i'}(t) - T_{i'i} p_i(t)) \quad (8)$$

where $T_i' i$ are the transition probabilities and can be formulated as a function of $T_k^{X \to Y}$, which gives, for a certain configuration $i = \{i_R, i_P\}$, the probability of an individual in the subpopulation $k \in \{R, P\}$ to change from strategy $X \in \{C, D\}$ to a different strategy $Y \in \{C, D\}$. This formula is influenced by both the current subpopulation k and the other subpopulation l ($l = R$ when $k = P$ and $l = P$ when $k = R$).

$$\begin{aligned} T_k^{X \to Y} = \frac{i_k^X}{Z}((1 \\ - \mu)[\frac{i_k^Y}{Z_k - 1 + (1-h)Z_l}(\frac{1}{1 + e^{\beta(f_k^X - f_l^Y)}})^{-1} \\ + \frac{(1-h)i_l^Y}{Z_k - 1 + (1-h)Z_l}](\frac{1}{1 + e^{\beta(f_k^X - f_l^Y)}})^{-1} \\ + \mu) \end{aligned} \quad (9)$$

Gradient The gradient gives the direction the population will most likely follow for each possible configuration i which can be expressed using the probability at each step to increase or decrease by one the number of cooperators denoted respectively by $T_{i,k}^{+}$ and $T_{i,k}^{-}$. Thus, we have that $T_{i,k}^{+} = T_k^{D \to C}$, $T_{i,k}^{-} = T_k^{C \to D}$. The gradient for each configuration i, ∇_i, is a vector expressed by:

$$\nabla_i = \{T_{i,R}^{+} - T_{i,R}^{-}, T_{i,P}^{+} - T_{i,P}^{-}\} \quad (10)$$

Average group achievement The fraction of time a group succeeds in achieving the goal Mcb is given by the following formula :

$$\eta_G = \sum_i (\bar{p}_i a_G(i)) \quad (11)$$

where i is a configuration of the population $i = \{i_R, i_P\}$ (in index of the sum, it designates every possible population configuration), a_G is the fraction of groups having configuration i that manage to reach a total of $M c \bar{b}$ contributions and $\bar{p}_i$ is the stationary distribution of the population.

Timescale Separation: Games among the Rich and among the Poor

To assess what game the rich play in the presence of the poor (and inversely the poor in the presence of the rich), we will observe the evolution of each subpopulation assuming that the number of Cs in the other subpopulation is constant.

Experiment

We carried out two different experiments that will help us understand the link between the parameters and the willingness of players to cooperate.

Our first experiment will show the evolution of the gradient of selection for one category while the other category is fixed. If we want to see how the gradient of selection evolves for rich people when the state of the poor population is fixed, Z_p and (i_P/Z_p) are kept constant. We will calculate the evolution of the gradient of selection for rich and poor when $Z_P = Z_R$ or $Z_P = 4Z_R$ and $b_R > b_P$ or $b_R >> b_P$, and when the fraction of C_P or C_R is fixed at 90%, 50% and 10%. This experiment reproduce the one on the Fig. S1 of the SI text[1].

The second experiment will show us the gradient of selection(∇) for each configuration of the population. We will calculate this gradient of selection for different value of homophily(h) (0.0, 0.7, 1.0) and for different value of perception of risk (r) (0.2, 0.3). This experiment is made twice with different parameters to reproduce the Fig. S2 of the SI text[1] with $Z_P = Z_R$ (Figure 2), and the Fig. 2 of the article itself with $Z_P = 4Z_R$ (Figure 3).

For both experiments, we will use the same parameters as the one used in the original article. The authors of this article did not explain the motivation of such parameters, so they must be taken for what it's worth. These parameters are described in the caption below each figure.

Results

Gradient evolution

The result of the first experiment is given by the Figure 1. It shows us the evolution of the gradient of rich and poor while the composition of the other population is fixed(Z_P and the fraction Cs (i_P/Z_P).

Rich countries tend to a gradient of 0.0 when $Z_P = Z_R$ no matter of the fraction of C_p or if the initial endowment b_R is $>$ or $>>$ than b_p. When $Z_p = 4Z_R$, The gradient tend to a lower value when $b_R > b_P$. For this second configuration of the population, the lower the fraction of poor cooperators will be, the more the rich will want to become defectors.

When we evolve to poor population and keep the population constant, we see that the lower the fraction of C_R will be, the lower the trend of the gradient will be. The speed at which poor countries go from cooperator to defectors stay the same in every configuration, the only thing that change is the initial value of the gradient, more countries will go from defector to cooperator at the beginning of the experiment when the fraction of C_R is high.

Evolutionary dynamics

Despite we don't have the stationary distribution, the arrow representing the gradient of selection allows us to draw some observations.

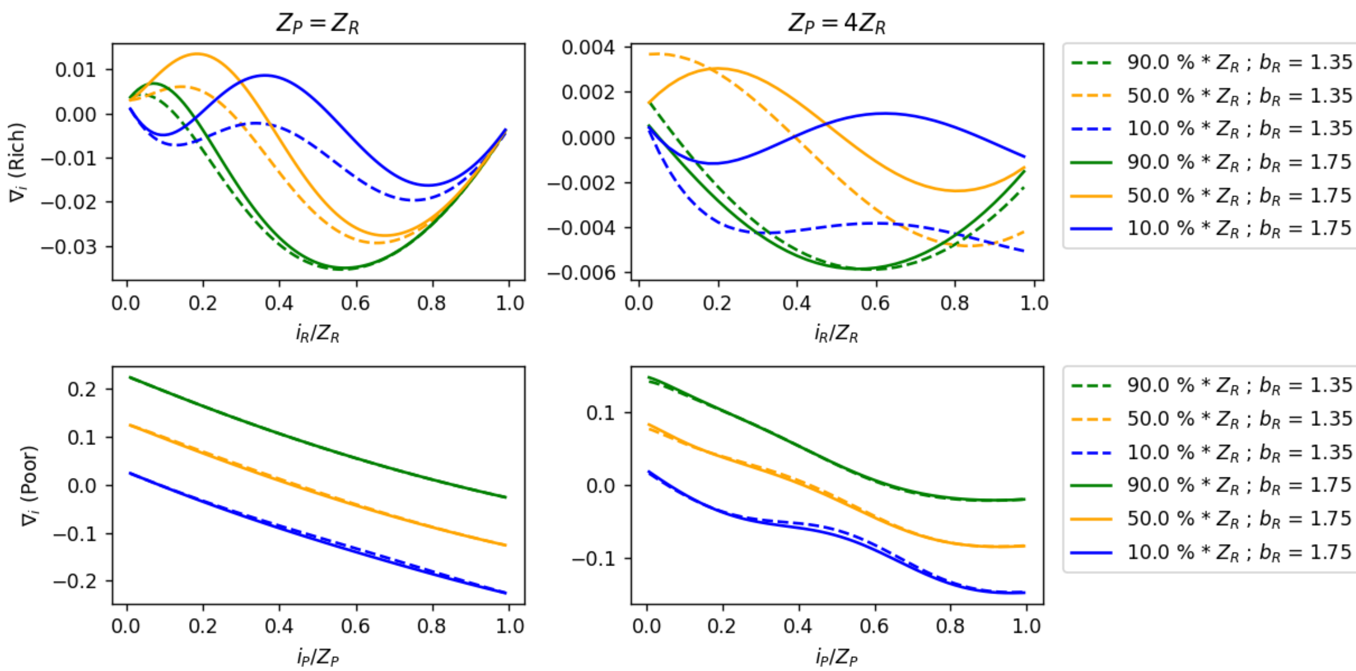

Figure 1: Evolution of the Gradient of selection of one subpopulation (rich or poor), while the number of Cs in the other subpopulation is constant. Above (below) : the evolution of the gradient of selection for the rich (poor), while the fraction f of Cs $\frac{i_P}{Z_P}$ ($\frac{i_R}{Z_R}$) is constant for three value of f (0.1, 0.5, 0.9). On the left, $Z_P = Z_R$, while on the right $Z_P = 4Z_R$. For each case we consider two different two different initial endowments for rich and poor. For the continuous line, $b_R = 1.35 > b_P$, while for the dotted line $b_R = 1.75 >> b_P$. The average endowment $\bar{b} = (b_R Z_R + b_P Z_P)\frac{1}{Z}$ is always equal to 1. Other parameters: $Z = 200$, $h = 0$, $r = 0.3$, $N = 10$, $M = 3$, $\beta = 10$, $\mu = 1/Z$, $c_R = 0.1 b_R$, and $c_P = 0.1 b_P$

Risk perception plays a crucial role here. We see in Fig. 2 that when r is low (0.2) rich and poor countries will not cooperate. When homophily is lower than 1, even if countries tend to defect, more rich countries will cooperate before defecting. When homophily = 1, the cooperation will collapse. When the risk perception is higher (0.3), rich and poor countries will be more inclined to cooperate. Here too, homophily will have an adverse impact on cooperation. When homophily = 1, only some rich countries will continue to cooperate even though poor countires defect.

The fact that rich countries cooperate more when homophily is high can be seen very well in Fig. 3. When r = 0.3, the more homophily is close to 0, the less the poor cooperate, the rich compensate the lack of contribution by cooperating more.

Discussion

During our attempt to reproduce the results of the paper, we stumble across a problem concerning the transition probabilities $T_{ii'}$ necessary to compute the stationary probability distributions $\bar{p}$ as it is not defined explicitly using the formula $T_k^{X \to Y}$. Despite we did not manage to calculate the stationary distributions, and consequently neither the average group achievement, we have obtained very similar results to the original article. For the experiment on the evolution of the gradient of selection of one subpopulation with the other subpopulation staying constant, we got identical curves for the poor (both for $Z_P = Z_R$ and $Z_P = 4Z_R$, and for b_R being both $>$ and $>>$ than b_P). For the case of the rich, we got a difference only when $b_R >> b_P$. For the second experiment, the curves are exactly the same. We can therefore draw the same conclusions.

Conclusion

We have reproduced the model and the experiment described in the article of Vasconcelos et al. *Climate policies under wealth inequality*. This model represents the international cooperation in the fight against Climate Change as a Public Good Game where countries can choose to cooperate by engaging a fraction of their endowment, or defect and letting the other take the responsibility. In this model, countries are divided into two subpopulations of rich and poor. The aim was to observe how countries will cooperate or not based on each other actions. In particular, we observed the influence of two parameters, the perception of risk and the homophily, both described in the section `Methods` . The first

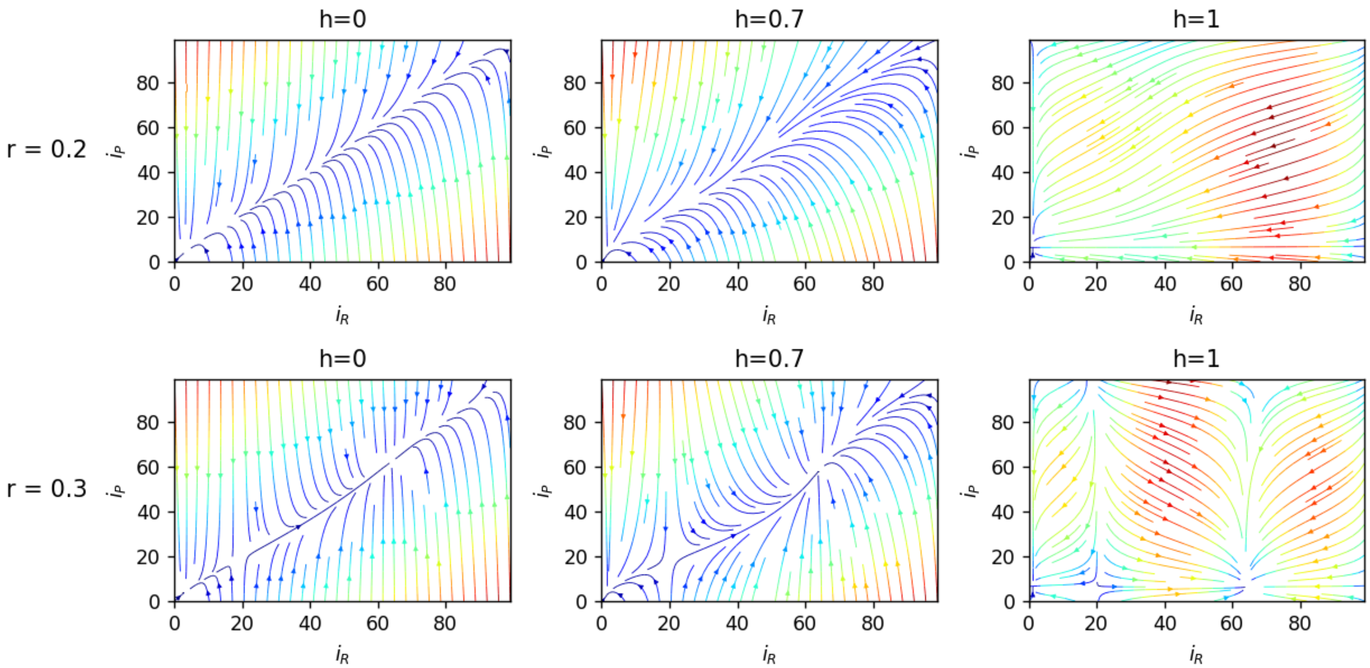

Figure 2: Evolutionary dynamics for all possible configuration of the population, with the same number of rich and poor. This experiment correspond to the Fig. S2 of the SI from the original article[1]. The gradient goes from dark blue (low) to red (high). The six panels correspond to the combination of two different value of the perception risk r (0.2 and 0.3), and three different values of the homophily h (0, 0.7, 1). Other parameters: $Z = 200$, $Z_P = Z_R$, $M = 3c\bar{b}$, $N = 6$, $\beta = 5$, $\mu = 1/Z$, $c_R = 0.1b_R$, $c_P = 0.1b_P$, $b_R = 1.7$, and $b_P = 0.3$.

experiment allowed us to study the evolution of the subpopulation independently to the other. The second experiment consisted in modelizing the evolutionary dynamics for all possible configurations of the population. The results we have obtained were very similar to the results presented in the original article. With these results, we could observe that cooperation is crucial to save the planet from dramatic climate change, poor or rich can't reach the objective alone. Two of the main obstacle we encountered were a high homophily and a small perception of risk, those two parameters limit the cooperation. Therefore, we could conclude that make people be aware of the real risk of Climate Change and limit the homophily behavior (in our real world, it means that countries from the global North and the global South must inspire and be inspired by each others) could help us to save the planet.

References

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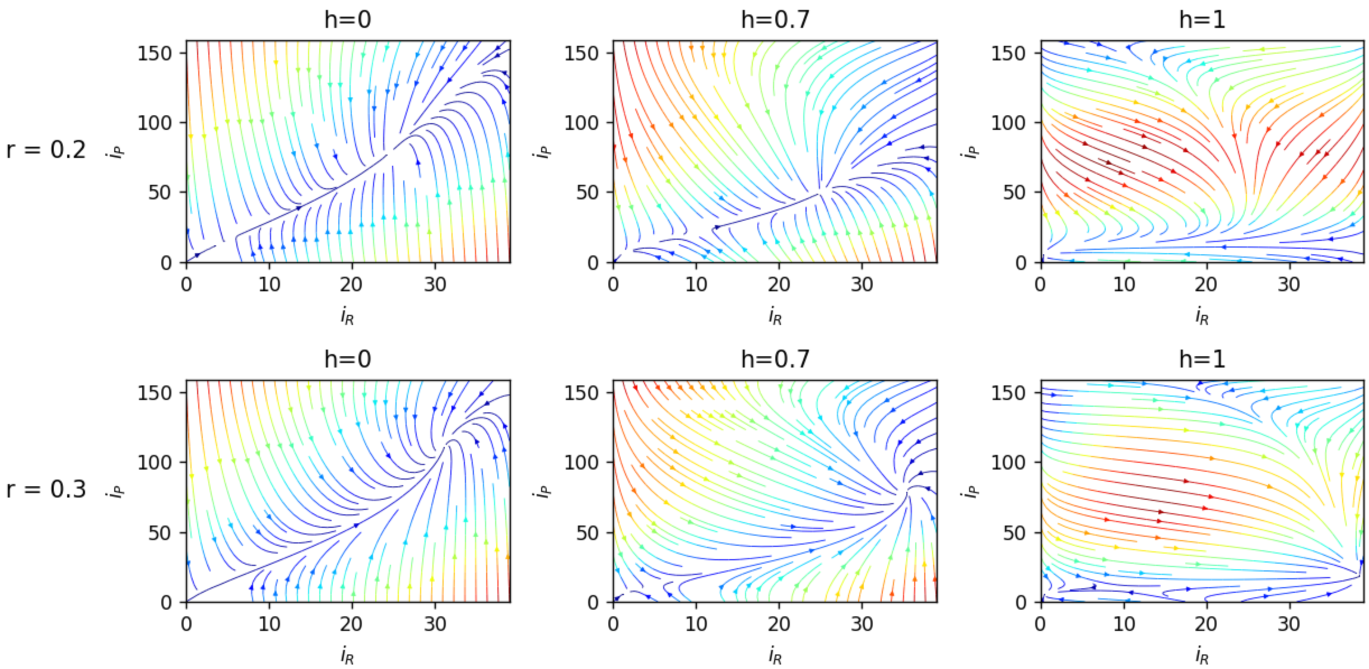

Figure 3: Evolutionary dynamics for all possible configuration of the population, with 20% of rich and 80% of poor. This experiment correspond to the Fig. S2 of the the original article itself [3] See Figure 2 for more details. Other parameters : $h(0, 0.7, 1)$, $r(0.2, 0.3)$, $Z = 200$, $Z_R = 40$, $Z_P = 160$, $c = 0.1$, $N = 6$, $M = 3c\bar{b}$, $\beta = 5$, $\mu = 1/Z$, $c_R = 0.1b_R$, $c_P = 0.1b_P$, $b_P = 0.625$, $b_R = 2.5$.