Investigation of the Trade-Off Between Which-Way Information and Fringing in Single-Photon Interferometer

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Abstract

I study the relationship between the appearence of fringing on a camera lens in a modified Mach-Zehnder interferometer in which the two paths are polarized orthogonally and the angle at which an analyzing polarizer is placed in front of the lens. The fringe depth, a measure of intensity of fringing, is measured as analyzer angle is varied in separate cases of a dense light beam and a beam attenuated to produce single-photon interference. I show that in the single-photon case the pattern of the aforementioned relationship displays characteristics consistent with the expected behavior of the non-attenuated case. A regression analysis of the data produces a mathematical model consistent with the elimination of fringing corresponding to an absolute knowledge of which-way information, and fringing which is most apparent when which-way knowledge is entirely eliminated. While this study cannot confirm that the non-attenuated beam case conforms to this same expected behavior, a few suggestions are offered to obtain data that would show a conformity to theoretical prediction.

I. INTRODUCTION

Light is commonly considered as a countinuous transfer of energy through space, easily thought of as an electromagnetic wave. Quantum mechanics dictates that while many approximations are valid using this classical treatment, it is sometimes more useful to treat it as a particle called a photon, which acts as a packet of energy.

This so-called *wave-particle duality* creates some interesting questions when considering the results of previous experiments which investigated the wave nature of light. Among the most notable experiments to consider is Young's double slit experiment, in which interference of light wavefronts much like that seen in waves of water travelling through two slits in a wall are observed exhibiting fringing patterns on a screen due to superposition and Huygen's principle. One may consider whether this interference pattern still exists when a beam is attenuated to the point of only allowing one photon into an interferometer at a time. The answer to this question is "yes." Philosophical interpretations vary, with a common interpretation being a vague but effective explanation of "the photon interferes with itself."

Interestingly, in the case of an interferometer with two potential paths of travel, having the knowledge of which path the photon travelled will eliminate the fringing, much like covering one of the slits in Young's classical experiment eliminates fringing.

A paper of interest by Schneider and LaPuma[1] points out that the original explanation for this, often attributed to Neils Bohr, attributes the desctruction of interference to a momentum kick imparted on photons in one path which collapses the wave function. However, the authors offer a simple experiment to show that this disruption of momentum is not necessary for the disruption of the inteference, and that any physical characteristic which is indicative of which-way information is enough to eliminate interference.

An exploration of this phenomena can be achieved by making use of a Mach-Zehndnder interferometer, where a beam from a laser is split into two paths **A** and **B** with slightly different lengths. The beams are recombined at another beam splitter and sent to a viewing screen, which in this case is a camera. Schneider and LaPuma[1] suggest placing a polarizer in **A** which is set orthogonal to a polarizer in **B**. With the laser polarized exactly between the two angles, so each path gets equal transmission, an analyzing polarizer is also placed in front of the camera after recombination. This analyzer can be set to polarize parallel to the original laser, which will allow photons from both paths to be transmitted, or can be varied in the direction of polarization of one of the paths, encoding an increasing amount of which-way information as the analyzer angle approach the polarization angle of **A** or **B**.

The above setup can be used in the non-attenuated case, as well as for attenuating a beam to the single-photon case. The analyzing polarizer can be considered an "eraser," which can give insight into the nature of the tradeoff between which-way knowledge and the appearance of interference fringes, and one can search for a periodic relationship between the two as the analyzer's angle is varied. This relationship and this method of exploring it are the focus of this study.

II. APPARATUS

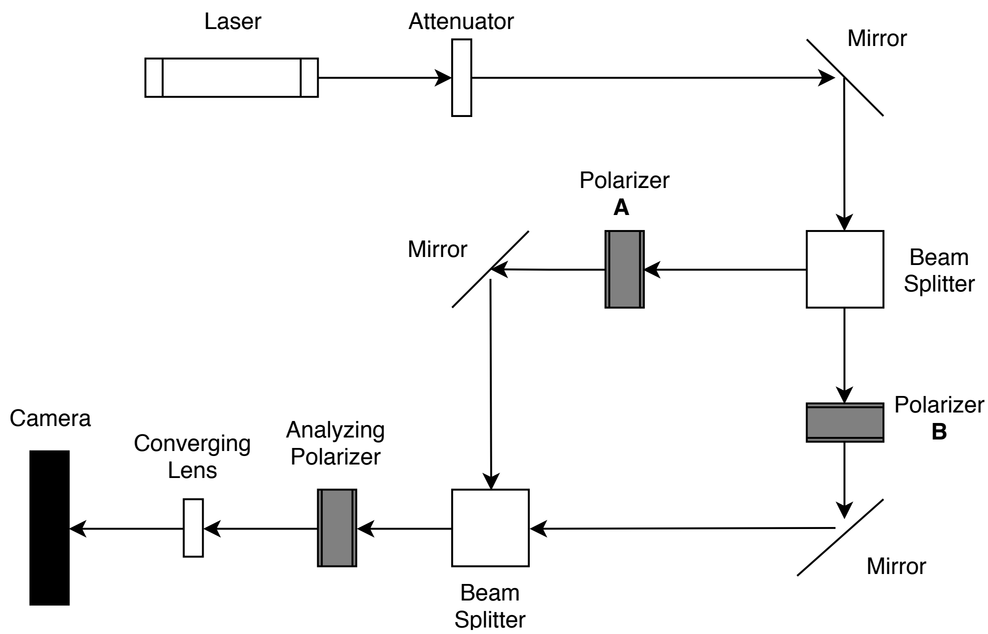

FIG. 1. A modified Mach–Zehnder interferometer with two polarizers **A** and **B**, and an analyzing polarizer which will yield which-way information when **A** and **B** are polarized orthogonally.

The apparatus for this experiment is derived from a Mach–Zehnder interferometer, which uses a collimated beam (in this case, a laser) as its light source. This beam is polarized to 135° so that equal transmission through polarizers **A** and **B** will be possible, and the output of the laser is 1.5 mW at a wavelength of 633 nm. As seen in Figure 1, the beam passes through an attenuator and is sent via a mirror to a beam splitter. From this beam splitter, the two beams travel slightly different distances before recombining at the second beam splitter with a phase difference. Each of these two paths has a polarizer (**A** is set to 90°, and **B** is set to 0°) and is then bounced off a mirror to the second beam splitter. This apparatus only makes use of one of the exit paths from the second beam splitter, because only one point of observation is necessary to perceive the desired effects. On this path is an analyzing polarizer which will be varied to change erase or uncover which-way information, followed by a converging lens to send the beams to the CMOS sensor which takes 6.66 x 5.32mm images. The image of the beam is captured and analyzed on a computer. The converging lens is only present in the apparatus when data is being taken for the single-photon interference case.

III. METHODS

The procedure for the experiment comprises of recording images from the camera as the analyzing polarizer is varied in 15° intervals from 0° to 90° first in the case of a beam where many photons are in the apparatus between the beam splitters, and second in the case of a beam attenuated so that only one photon passes between the beam splitters at a time, and determining whether these cases exhibit the predicted trade-off between which-way knowledge and interference.

The level of attenuation **A** used for the single photon case must be enough so that the time between successive photon transmissions is greater than the time of travel between the two beam splitters. That is,

$$t_t \geq t_b \tag{1}$$

t_b is simply the distance of the path between the two beam splitters. This distance was measured along path **B** and is (0.64 ± 0.05)m. t_t is the inverse of the rate of photons leaving the attenuation filter. In other words,

$$t_t = \frac{E_p}{A \cdot P} \tag{2}$$

Where E_p is the Energy of a photon from the laser, A is the attenuation coefficient, and P is the power of the laser. Using Planck's equation $E_p = h \cdot \nu = \frac{h \cdot c}{\lambda}$, we can substitute the rightmost side into Eq. (2) in place of E_p. Substituting this into Eq. (1), we have:

$$\frac{h \cdot c}{A \cdot P \cdot \lambda} \geq \frac{d}{c} \tag{3}$$

Solving for A gives:

$$A \leq \frac{c^2 \cdot h}{P \cdot \lambda \cdot d} \tag{4}$$

Using $d = (0.64 \pm 0.05)$m and $P = 1.5\,\text{mW}$ in Eq. (4), we obtain a minimum attenuation coefficient of $A = 9.8 \cdot 10^{-8} \pm (5 \cdot 10^{-9})$. For the single photon case, an attenuation coefficient of $A = 10^{-8}$ is used to ensure that the power is decreased beyond the necessary amount.

To obtain more general information about the setup of the apparatus, I removed the converging lens a calculated the angle between the beams at the camera by measuring the fringe separation with polarizers **A** and **B** removed according to $\Delta = \frac{\lambda}{2 \cdot \sin(\frac{\theta}{2})}$ and determined the angle between the beams to be $(\theta = 0.158 \pm (2.15 \cdot 10^{-7}))^{\circ}$. One could potentially take measurements like this to develop more precise predictions for the behavior of the intensity at any point on the CMOS sensor under the effects of both interference and Malthus' law.

In order to correctly measure the periodicity of the relationship between which-way information and interference appearance, the polarizers **A** and **B** were set to 90° and 0°, respectively. To verify that the instruments were working properly, I moved polarizer **A** to be directly behind polarizer **B** (see Figure 1 for reference) and blocked the path where **A** was originally located with a sheet of paper. No image displayed on the camera when the laser was powered on, so the orthogonality of the polarization after these two devices was confirmed.

Polarizer **A** was returned to its original position, the paper removed, and the attenuation filter was set to $A = 10^{-4}$ so as not to saturate our camera for the high density beam case. The converging lens was not present in the apparatus for this set of data. The analyzing polarizer was set to 0°, and the camera's image was recorded. I took the same steps to obtain data with the polarizer set at 15°, 30°, 45°, 60°, 75°, and 90° while keeping the camera's settings unchanged.

The attenuation filter is then set to $A = 10^{-8}$ and the converging lens placed in the apparatus to focus the low-powered beam so that it's visible on the camera. An exposure time of 2 seconds (the camera's maximum) was chosen to maximize the intensity of the image. Images were recorded with the analyzer at each of the same angles as in the previous, high-density beam case.

IV. ANALYSIS

Finding an appropriate way to analyze the data was difficult. The goal is to investigate the varied analyzing polarizer angle's impact on the appearance of fringing, but how do we quantify the appearance of fringing? It was suggested to process each of the captured images in a program called ImageJ by assessing the gray value, a measure of intensity of digital graphics, along a line perpendicular to the alternating fringes. In this case, we drew vertical lines, because the interference produces horizontal lines (see Figure 3). From here, one can obtain maximum and minimum values for light intensity in units of grey value. For each image, I generated a grey value intensity plot as just mentioned, and I excluded the outer edges of the beam where intensity drops off due to what seems like the slight misalignment of the beam (see Figure 2). The apparence of fringes can be described by a quantity I will

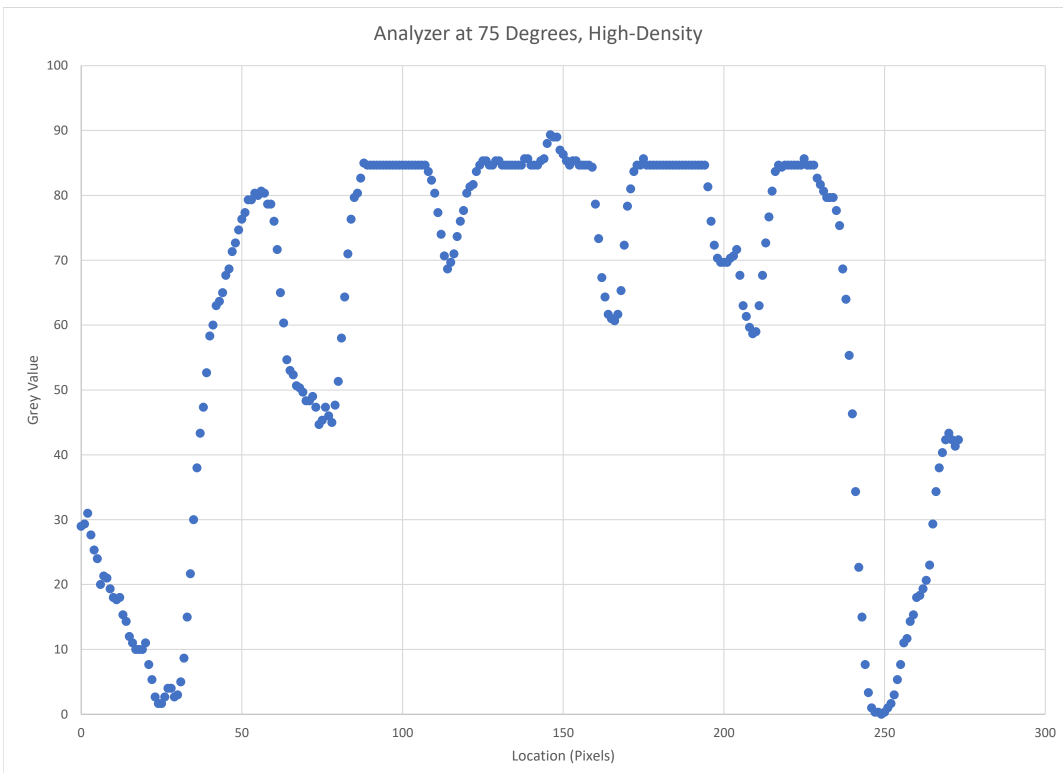

FIG. 2. The intensity profile in the high-density beam case with the analyzing polarizer set to 75°. Notice that between pixel 50 and pixel 225, the beam is displaying fringing, while outside those limits, there is a sharp dropoff because the plot profile begins near the edge of the beam.

call *fringe depth* in units of gray value. This will be defined as the maximum intensity value minus a minimum intensity value. Given any maximum gray value, it is not possible to

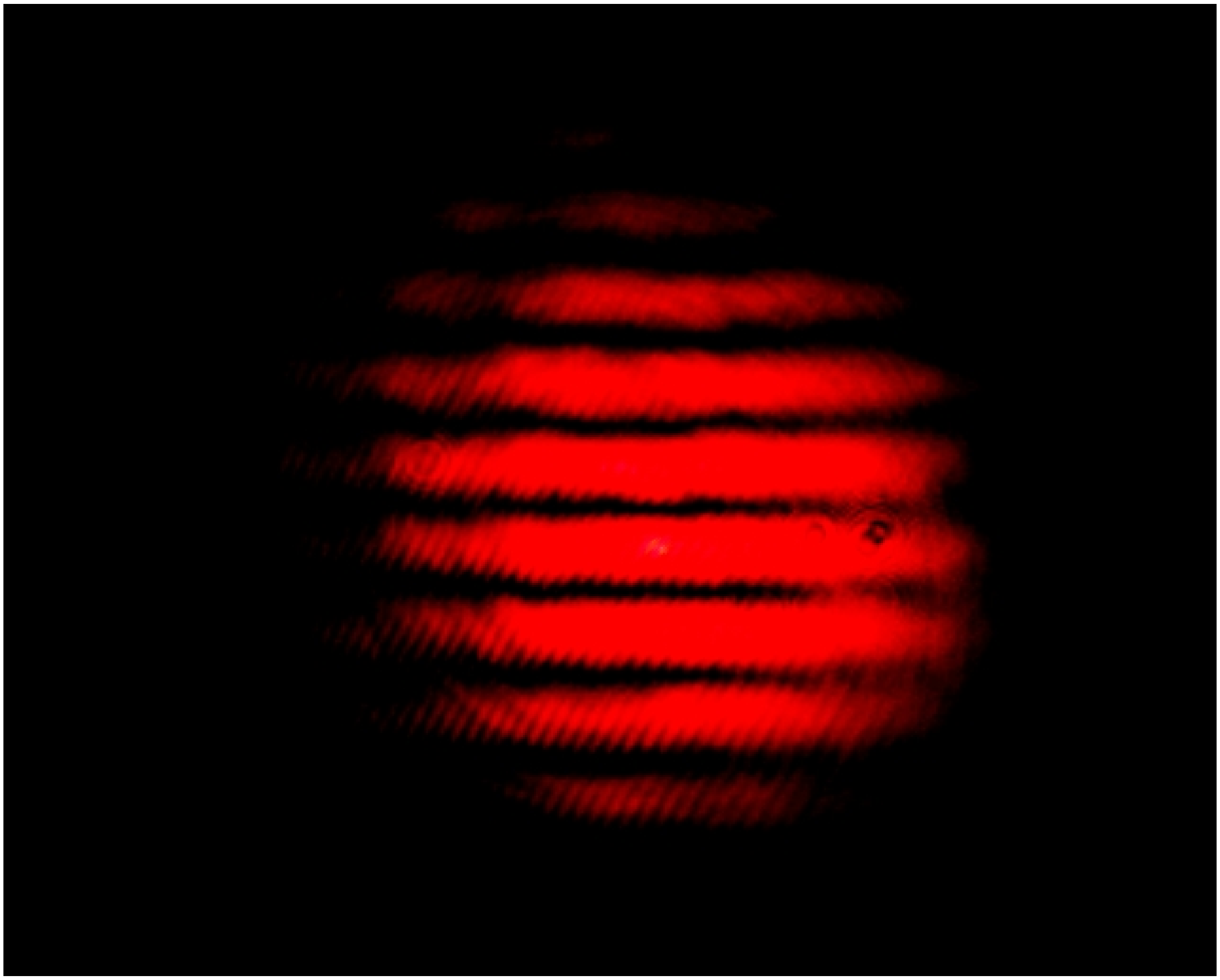

FIG. 3. The fringing pattern with the analyzing polarizer at 45° in the high-density case.

guarantee that the global minimum on the intensity profile will be the adjacent extremum to the maximum. To address this, I took the average of the local minima on either side of the global maximum, where I defined a local minimum as a point from which grey values are non-decreasing in both directions until an increase of 5% of the maximum is achieved. This ensured I wasn't choosing what appeared to be natural fluctuations in gray value as minima, and upon visual inspection, appeared to choose only points corresponding to visible fringes in the images as minima.

The duration of the experiment was not very long, so I decided to use a method of estimating the fluctuation of the gray values without collecting a data set of hundreds of pictures. Separately, for the single photon and high-density cases, I took an intensity profile parallel to the fringes with the analyzing polarizer at 45° and measured the standard deviation in gray value from this, and applied that uncertainty to my data. This uncertainty is slightly reduced in the case of minima because it is a mean measurement. As you will see in the results and conclusion sections, I believe that the camera was saturated in the high-density case, which resulted in very low fluctuation along the peak of the fringes and an underestimated deviation.

The error in the angle of polarization and the analyzing polarizer is assumed to be negligent, as I tested their alignments in the previous section, and the inclusion of an uncertainty in this dimension would lead to painstaking calculations which I believe to be unlikely to significantly affect the results. If there is an error in the angle of polarization, it would produce a small phase shift in the data.

The fringe depth for each analyzing polarizer angle was plotted along with its error and a curve was fitted for the single-photon and high-density cases separately.

In particular, I was investigating whether the nature of the relationship could be fitted to squared-sinusoidal curve with a period of 180° because the CMOS camera is assumed to be detecting superimposed Malthus' law obediences. If the phenomena obeys prediction, a maximum of interference occurs when the analyzing polarizer is polarizing along the bisector of the angles of **A** and **B**, in this case, when the analyzer is at odd multiples of 45°. Following this period and these location of miniima and maxima, the resulting phase shift of the relationship should be zero. The form of the predicted model for fringe depth F in units of gray value is:

$$F = a \cdot \sin^2\left(\frac{2\pi}{p_r}\theta_r + c_r\right) + d \tag{5}$$

Where a, p, c, d are parameters denoting amplitude, period, phase shift, and amplitude offset. θ_r is the analyzing polarizer angle. The subscript r denotes the parameter in *radians*, while the lack of subscript will refer to the parameters' values in *degrees*. I added the amplitude offset to account for any nonzero base value of fringe depth generated by light pollution, which is assumed to be constant but with its own natural fluctuations.

V. RESULTS

In the case of the high-density beam of light, there was not a satisfying fit to a curve of the form in Eq. (5). The model yields a χ^2_{red} value of **8.5336**, indicating a poor fit. To understand this, I looked for an explanation in the data, including looking at minima and

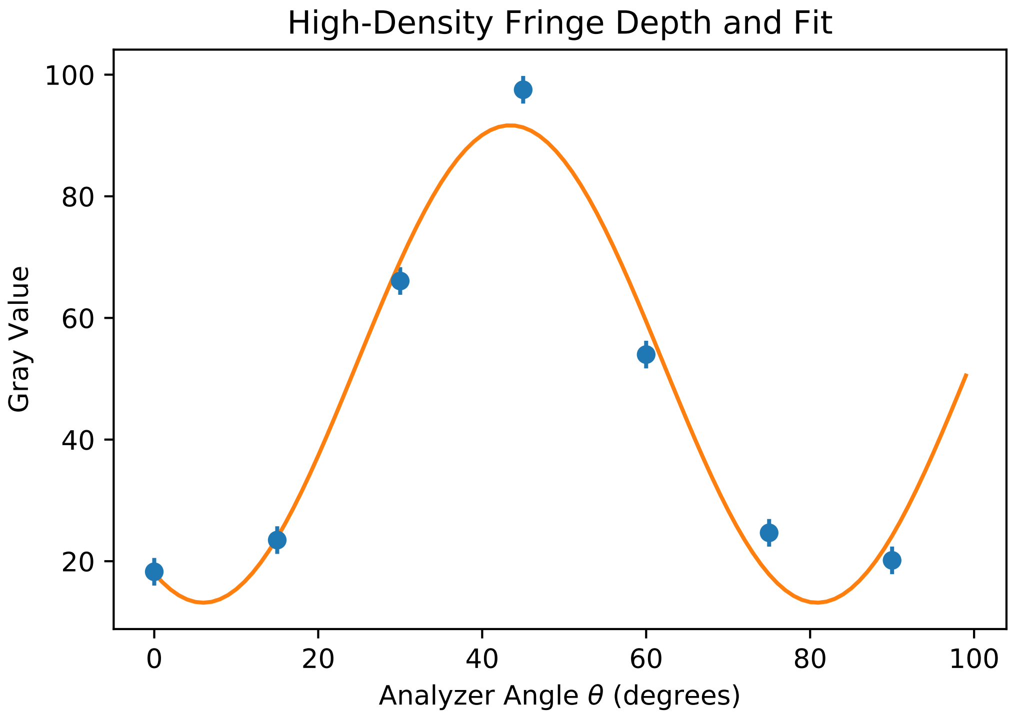

FIG. 4. The high-density case exhibits a poor squared-sinusoidal relationship, with values at 0° and 90° that appear too high. Values for the parameters are found in Table I

maxima in Figure 5.

In addition to my concern about the data at 45°, I also am concerned about the data at the 0° and 90° angles, because it appears that the fringe depth may have been given an artificial minimum due to my assumptions rather than the parameter d. I stated earlier my belief that the camera may have been unknowingly saturated during this phase of data collection, and this will have affected the 0° and 90° cases. Without the presence of visible fringes, the gray value across the beam, if the camera is saturated, has virtually no variation, and the minima, according to my definition, will necessarily be found at the edges of the beam, which may not even be due to fringing but a slight misalignement in the beam. Therefore, I believe the fringe depth data at 0° and 90° to be higher than their actual values. This effect of forcing minima to the edges is illustrated in Figure 6.

The effect that this possible overestimation of fringe depth in the full which-way information case has on the goodness of fit is that I believe a squared-sinusoidal fit as in Eq. (5) will underestimate p so that the full which-way cases at 0° and 90° are fit to the curve on

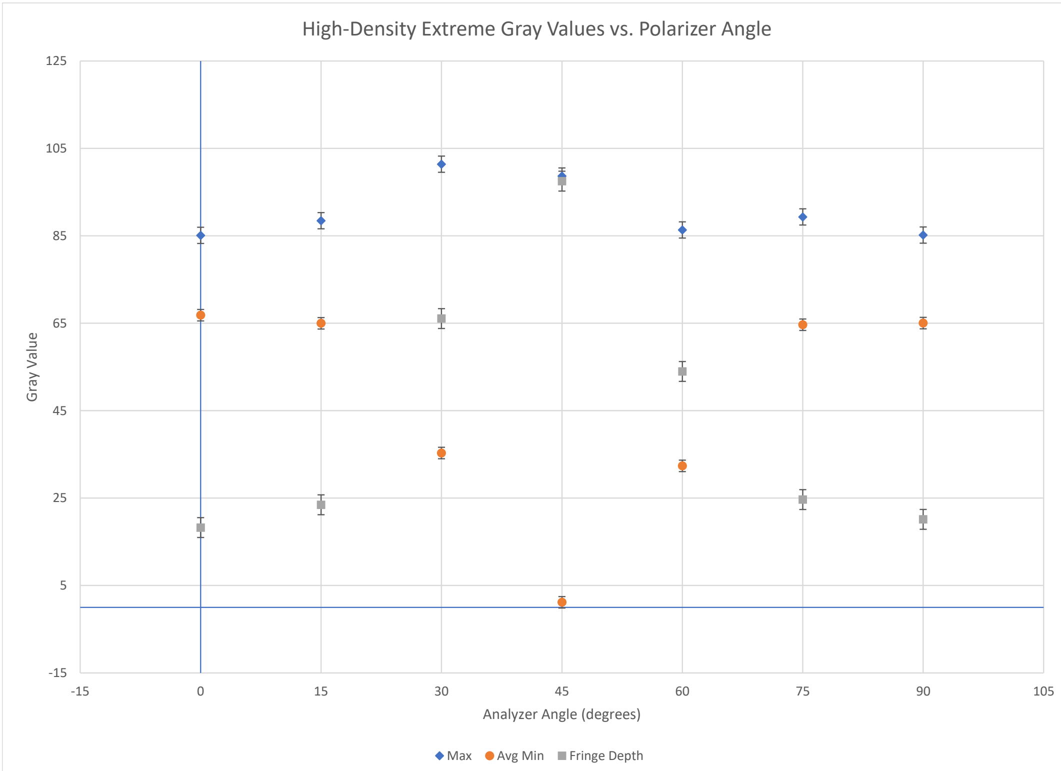

FIG. 5. The maxima and averaged minima for the high-density beam case along with their fringe depths. in the case of 45°, the entire profile seems darker. This could be due to the placement of the drawn line for intensity profiling, or could be due to a hidden camera setting compensating for saturation. Either way, the minimum value here is so low that we see almost no light pollution, indicating that for this image, the gray value is low due to error, and fringe depth may be underestimated.

the wrong side of a minimum. It would also generate an artificially high value for d by increasing the minimum fringe depth.

The fit parameters generated according to residual minimization reflect this possible error. The period for the high-density case was $151.2796 \pm 6.6491°$. The known period of 180° is over 4 standard deviations away from the period our data generated, and the phase of $-14.3563 \pm 5.5603°$ leaves the assumed phase of 0° almost 3 standard deviations away. This data is not conforming to the curve form, but most notably, these are the two parameters which should be accurate no matter what periodic function form we chose to fit to.

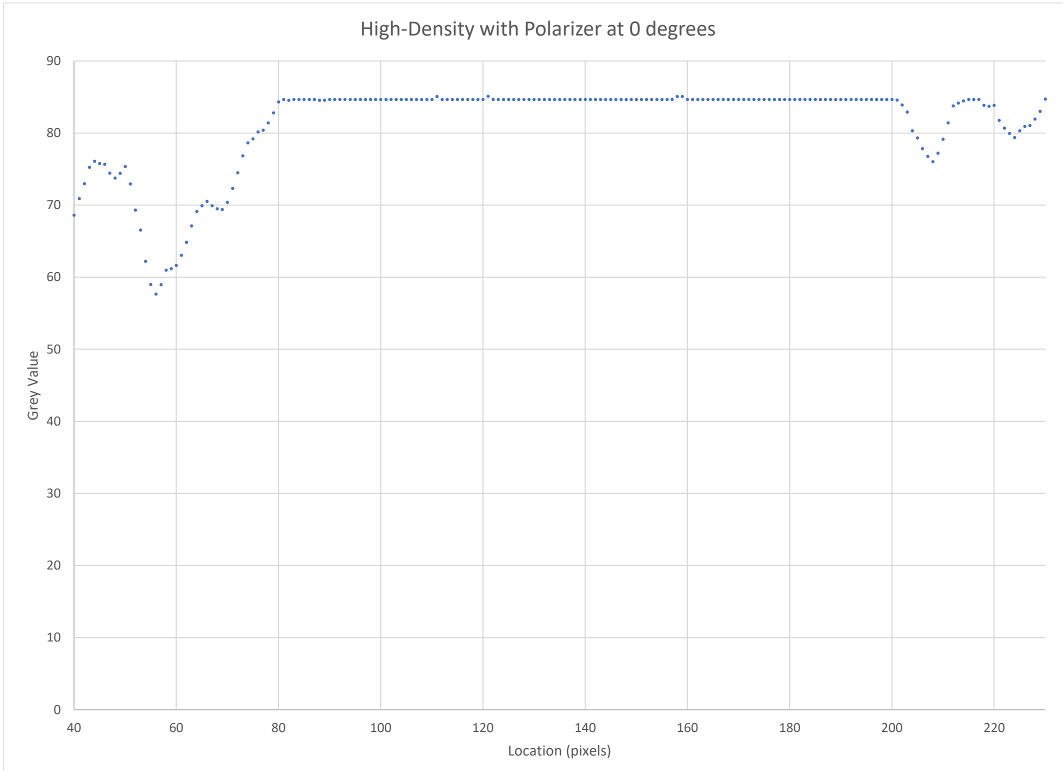

FIG. 6. The camera seems as though it may be saturated in this case. On the right side, it's unclear whether the dip between pixels 200 and 220 is a fringe or another phenomena, but the minimum on the left side which fits my definition is at pixel 56, and this is unlikely to be a real fringe.

TABLE I. Curve-Fit Parameters

Case	Amplitude a (gray value)	Period p (deg)	Phase c (deg)	Offset d (gray value)
High-Density	78.5031 ± 7.414	151.2796 ± 6.6491	-14.3563 ± 5.5603	13.1753 ± 3.7541
Single-Photon	26.9380 ± 2.1935	196.6764 ± 17.8301	13.6295 ± 7.3613	5.9155 ± 1.9343

The single-photon case generated a squared-sinusoidal relationship with a low χ^2_{red} value of **0.7734**. This suggests a very good fit. Upon closer analysis, the fit parameters are suggesting that this relationship does obey the known characteristics we should have seen in the high-density case. According to Table I, our data has a period $p = 196.6764 \pm 17.8301$,

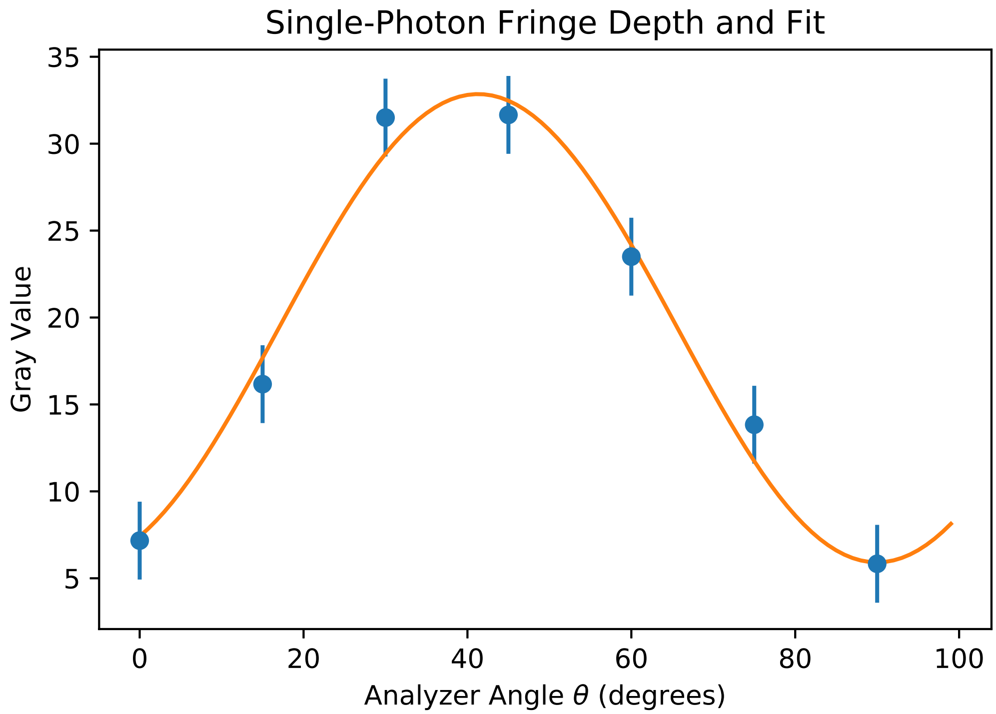

FIG. 7. The single-photon case exhibits a relationship much like the prediction, showing strong squared-sinusoidal resemblence and a period of about the right length. The fringe depths at angles 0° and 90° are not showing the same problems as in the high-density case, nor are there signs of camera saturation, so I believe that this set of data is not affected by the concerns mentioned in the high-density case

which is within one standard deviation of the expected 180° result. Though a 95% confidence interval is from $p = 161.729°$ to $p = 231.623°$, I believe this interval could be narrowed if more data was taken. The phase of the relationship did not generate exactly what I expected. I expected a value of c close to 0, and instead the results produced a value of $13.6295 \pm 7.3613°$, leaving the expected 0 at a Z-score of $Z = 1.8515$, making me wonder if our analyzing polarizer was somehow off by a constant value. The amplitude is not of much interest here, because factors, mainly camera settings, made it nearly impossible to predict what gray value we'd be observing as intensity is changed by itself. It's worth noting that the value for d in the single-photon case was near 0 as expected, but is definitely not exactly 0, as I also expected due to some possible light pollution and natural fluctuation in intensity leading to the appearance of "fringe depth" at analyzing angles 0° and 90° even when there are no visible fringes.

VI. CONCLUSION

I believe the results described in the section above to be supporting evidence that single photon interference has the same periodic relationship with which-way information that higher density photon beam interference has been confirmed to have, though our test of the high-density case yielded what I believe to be erroneous data, and our single-photon case has a nonzero phase shift, which I believe could be corrected by taking more data.

ACKNOWLEDGMENTS

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[1] Mark B. Schneider and Indhira A. LaPuma, "A simple experiment for discussion of quantum interference and which-way measurement," Am. J. Phys. 70 (3), 266–271 (2002)