Lappeenranta University of Technology
BM40A0702 Pattern Recognition and Machine Learning

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3-D DIGIT RECOGNITION

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CONTENTS

LIST OF ABBREVIATIONS

SVD	Singular value decomposition
3-D	3-dimensional

1 INTRODUCTION

1.1 Problem statement

The aim of this practical assignment was to developed a pattern recognition system for hand-written digits. The system should be able to recognize and classify digits correctly based on the 3-dimensional (3-D) location of the time series. The 3-D location data have been obtained using a LeapMotion sensor and digits are written as free hand strokes in the air [1]. Fig 1 shows two example digits in two dimensions. This work presents singular value decomposition (SVD) based feature extraction technique and a classifier that performs a Bayesian classification with a Gaussian distribution model to identify handwritten 3-D digits.

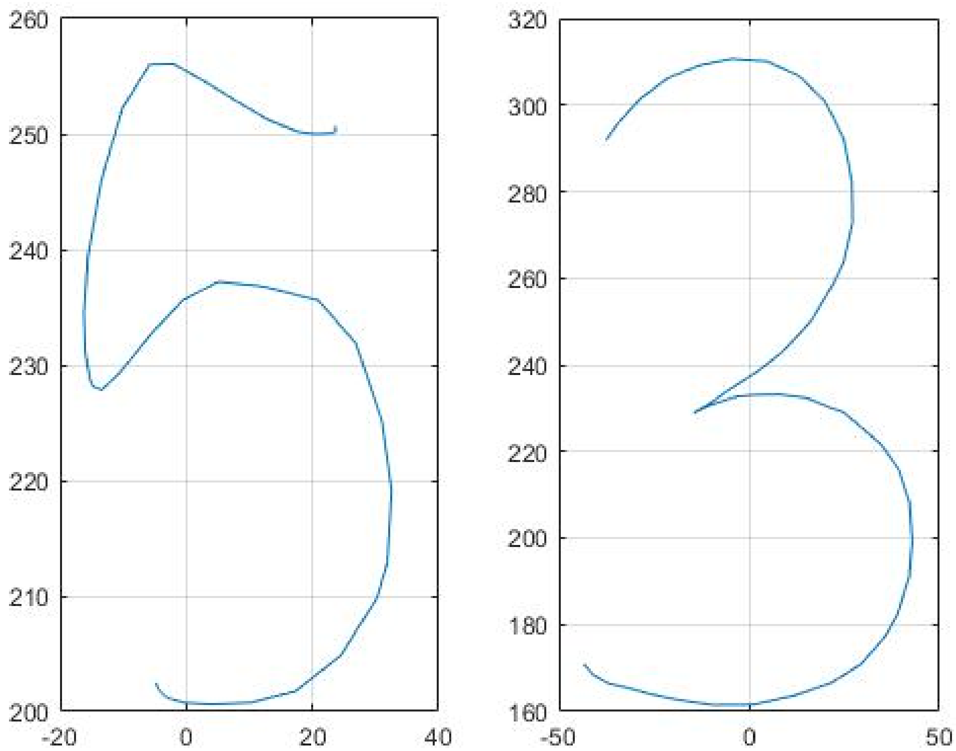

Figure 1. Two example digits in 2-dimensions.

2 DATA AND METHODS

2.1 Data

The provided training dataset contains location information of each stroke and the experiments have been stored into separated Matlab files. The strokes were digits 0 to 9 and there were 100 different experiments for each digit. In the provided dataset, each stroke contains three columns but not necessary same number of rows.

2.2 Data Preprocessing

The purpose of the data preprocessing is to prepare the data for feature extraction. In this work the data preprocessing was implemented as follows: firstly, each digit in the training dataset was divided into x, y, and z coordinates. The next step was to linearly interpolate the data so that each digit contains the same number of rows and then center x, y, and z coordinates. After that, each digit was reshaped into one column vector size of 300×1. Finally, column vectors were put into the matrix $\mathbf{X}$ and the average digit was computed and subtracted from each column vector of $\mathbf{X}$.

2.3 Feature Extraction

In this work SVD on mean subtracted data is applied to extract the most relevant features from the data. Feature extraction in this work was based on an idea presented in [2]. The SVD is a matrix factorization that exists every matrix $m \times n$ [3] .

$$\mathbf{X} = \mathbf{U}\mathbf{\Sigma}\mathbf{V} \tag{1}$$

The columns of $\mathbf{U}$ are arranged in order of importance from the most important to least important that most describe correlation among columns of $\mathbf{X}$ [4]. These column vectors of $\mathbf{U}$ define a new coordinate system and by taking inner product between each column vector of $\mathbf{X}$ and column vectors of $\mathbf{U}$, column vectors of $\mathbf{X}$ can be projected into space spanned by column vectors of $\mathbf{U}$. By using r first column vectors of matrix $\mathbf{U}$, r-dimensional feature space can be obtained. In this work $r = 10$, so the dimension of the feature space is 10. A new digit vector that is not part of the training data set can be projected similarly to this space by taking inner product.

2.4 Bayesian Classification

Classification of digits in this work was based on Bayesian classification with a Gaussian distribution model. Assume that a single r-dimensional feature vector is denoted as $\mathbf{x} = [x_1, x_2, ..., x_\mathrm{r}]^T$. The probability that $\mathbf{x}$ belongs to the class ω_k can be expressed as a posteriori probability $P(\omega_\mathrm{k}|\mathbf{x})$. The posteriori probability can be calculated as

$$P(\omega_k|\mathbf{x}) = \frac{p(\mathbf{x}|\omega_i)P(\omega_i)}{p(\mathbf{x})} \tag{2}$$

where $p(\mathbf{x}|\omega_i)$ is class conditional probability density function and $p(\mathbf{x})$ prior probability. By using discriminant functions and taking natural logarithm, following expression can be obtained

$$P(\omega_k|\mathbf{x}) = g_i(\mathbf{x}) = \ln p(\mathbf{x}|\omega_i) + \ln P(\omega_i). \tag{3}$$

By assuming that class-conditional probability densities are multivariate Gaussians the following equation can be obtained

$$g_i(\mathbf{x}) = \frac{1}{2}(\mathbf{x} - \mu_i)^T \mathbf{\Sigma}_i^{-1}(\mathbf{x} - \mu_i) - \frac{1}{2}\ln|\mathbf{\Sigma}_i| + \ln P(\omega_i) \tag{4}$$

as shown in [5]. The class of specific unknown sample can then be found by selecting the class that corresponds to the maximal discriminant value. Before one can perform classification, means (μ_i), covariances (Σ_i), and prior probabilities ($P(\omega_i)$) have to be estimated for the each class from training data.

3 EXPERIMENTS

3.1 Evaluation criteria

In this practical assignment, the classification performance was measured simply by using classification accuracy. Classification accuracy is the rate of correct classifications and it is defined as

$$ACC = \frac{T}{N} \tag{5}$$

where T is number of correctly classified samples and N is the total number of samples.

3.2 Description of experiments

The provided training dataset was divided into a training set of 700 digits and a test set of 300 digits by random selection. In order to obtain reliable information on the classification accuracy, this procedure was repeated 100 times.

3.3 Results

The average classification accuracy is shown in table 1. Results indicate that SVD based feature extraction and Bayesian classifier allow accurate classification of handwritten 3D digits despite a very simple implementation. In addition to quite an accurate classification result, classification is also performed fast with this feature extraction method and classifier. Fig. 2 shows confusion matrix from the experiment described above. Based on the confusion matrix the most significant issue was the misclassification of four and nine.

Table 1

Classifier	Accuracy
Bayesian	0.965

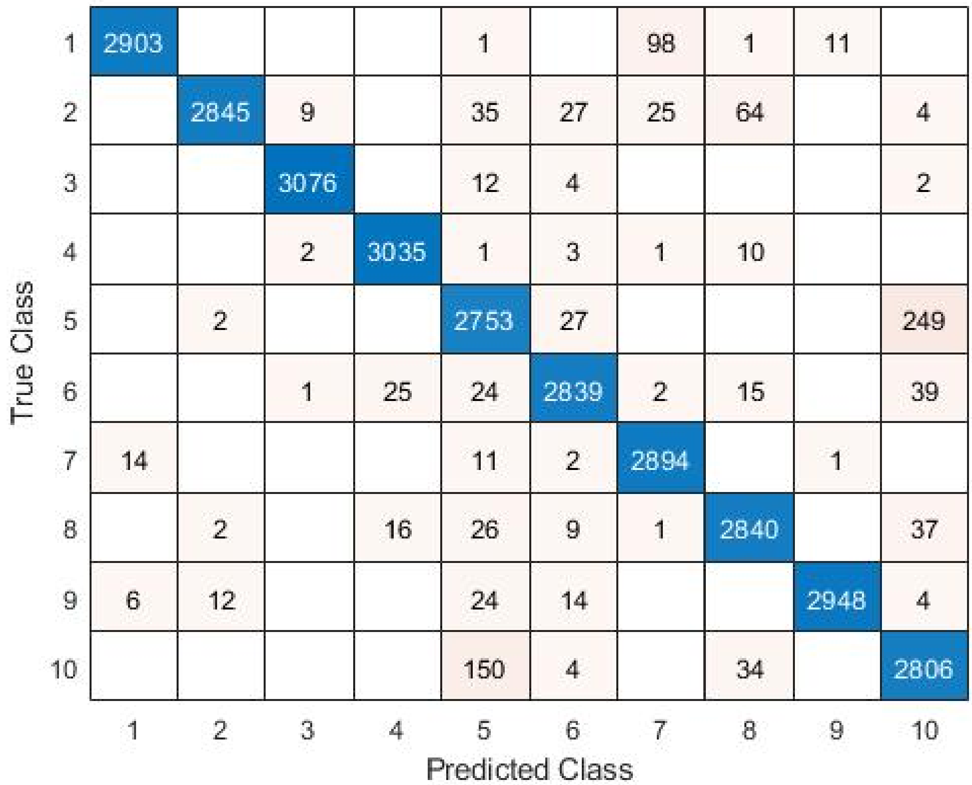

Figure 2. Confusion matrix. Due to class labeling, in the predicted class and the True class axes, 1 corresponds to 0 and 2 corresponds to 1, and so on.

4 CONCLUSION

In this practical assignment, pattern recognition system was developed to recognize hand written digits based on the 3-D location of the time series. Regarding simple implementation, the results show that the developed SVD based feature extraction technique and Bayesian classifier allow accurate (0.965) and quick classification of handwritten 3D digits. Although classification accuracy is high with this proposed method, it should be pointed out that the result could be even better if a more sophisticated classifier would be used. The second thing that should be considered more closely in this approach is the dimension of the feature space.

REFERENCES

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