CSE 480 – Final Exam Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

This exam is open book, and open note. You are welcome to use the internet to learn more about these topics generally, but please don’t search and find the answer to the exact question here. I trust you to use proper judgment and understand what this means. If you do use the internet, please cite your sources.

You are welcome to use additional paper if you feel you need to, but I have tried to be generous in the space allowed to accommodate what I anticipate are reasonable answers.

Have fun!

1. a) In your own words, briefly define each of the following classes. (I am looking for a sentence or less for each).

* EXPSPACE

The class of languages where, in order to decide whether a string belong to them, it takes a Turing machine **exponentially as much space** **as the length of the string itself** in order to do so.

* EXPTIME

The class of languages where, in order to decide whether a string belong to them, it takes a Turing machine **exponentially as many steps of operation** **as the length of the string itself** in order to do so.

* L

The class of languages that are **decidable by a deterministic Turing machine in Logarithmic Space.**

* P

The class of languages that are **decidable by a deterministic Turing machine in Polynomial Time.**

* PSPACE

The class of languages that are **decidable by a deterministic Turing machine in Polynomial Space.**

* NL

The class of languages that are **decidable by a non-deterministic Turing machine in Logarithmic Space**

* NP

The class of languages that are **decidable by a non-deterministic Turing machine in Polynomial Time.**

* NPSPACE

The class of languages that are **decidable by a non-deterministic Turing machine in Polynomial Space**

b) Draw a Venn Diagram showing the known relationships among the above classes.

c) List any containment relationships that have been shown to be proper (i.e., where we have shown that two classes cannot be equal).

2. Describe the P vs NP issue. For example: What does this issue refer to? Why is it so consequential? What do most current researchers think about the issue?

3. Define “mutual friends” in a graph to be a group of three nodes where all three are connected. *FRIENDS* = {<*G*> | *G* is a graph than contains a group of mutual friends}. Show that *FRIENDS* .

4. Consider the problem of scheduling final exams on campus. Assume you have:

* *F* – The set of all final exams.
* *S* – A list of the final exams for each individual student (i.e., this is a list of sets, where each set is a subset of F, namely the set of finals that a particular student is taking).
* *h* – The number of final exam spots available on campus that can be scheduled.

*FINALS* = {<*F, S, h*> | Where all finals *F* can be scheduled in *h* slots such that no student is taking two exams at the time time}. Show that *FINALS* .

5. Please see Problem 7.28 on Page 325 in the textbook. Show that this language, *PUZZLE*, is NP-Complete.

6. Define *BALANCED-BRACES* to be the language of strings over the alphabet *{ () }* (in other words, opening and closing parentheses) where the parentheses are properly balanced/nested. In other words, the strings “( )” or “( ( ) ( ( ) ) ) ( )” or “( ( ) ( ) )” are in the language, but “) (“ or

“( ( )” or “( ) ) ( )” are not. Show that *BALANCED-BRACES* .

7. Karl has developed a **non-deterministic** algorithm that can run in space. His algorithm is tight and clean and doesn’t show any clear ways for improvement. His brother, Kevin, claims to have a **deterministic** solution for the same problem that runs in *PSPACE*.

What do you think about the possibility of Kevin’s claim?

8. Recall that a coloring of a graph is an assignment of colors to its nodes so that no two adjacent nodes are assigned the same color, and that *3COLOR* = {<*G*> | *G* is colorable with 3 colors}.

Assume you had an oracle that could decide the *3COLOR* problem. What impact would this oracle have on solving the *FINALS* problem discussed above? Explain how you might make use of the oracle.

9. Consider the language *CATS* that contains the binary representation of .PNG files that contain a picture of a cat. Emily has developed a probabilistic algorithm that determines whether a given picture contains a cat, but the algorithm is not great. When the picture contains a cat, it only accurately predicts it with 60% accuracy. When the picture does not contain a cat, it accurately rejects it 58% of the time.

A) Is *CATS* ? Explain.

B) Is *CATS* ? Explain.

C) Explain how you could reduce the error of Emily’s algorithm so that it is substantially more accurate.

10. Which theorem from the second half of the semester did you find the most interesting? Why?