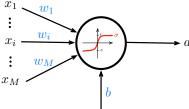
PERCEPTRON MULTICOUCHES (MULTILAYER PERCEPTRON)

Jérémie Cabessa Laboratoire DAVID, UVSQ

Le **percepton** est un neurone qui agit comme un classifieur binaire. Sa dynamique est donnée par:

$$a = \sigma \left(\boldsymbol{w}^T \boldsymbol{x} + b \right)$$

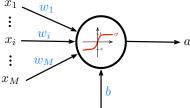
- $\mathbf{x} = (x_1, \dots, x_M) \in \mathbb{R}^M$ sont les inputs;
- $\mathbf{w} = (w_1, \dots, w_M) \in \mathbb{R}^M$ sont les poids synaptiques;
- $b \in \mathbb{R}$ est le biais;
- $ightharpoonup \sigma: \mathbb{R} \to \mathbb{R}$ est une fonction d'activation sigmoïdale.
- a est l'activation du neurone



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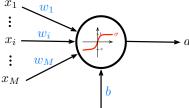
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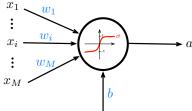
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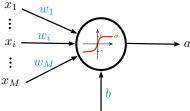
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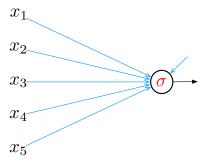
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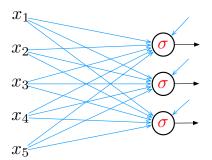
PERCEPTRON: 1 NEURONE

$$a = \sigma\left(\boldsymbol{w}^T\boldsymbol{x} + b\right)$$
 où $\boldsymbol{w} = (w_1, \dots, w_M)$



PERCEPTRON: 1 COUCHE

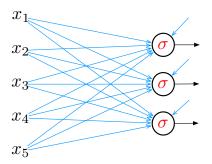
$$a_i = \sigma_i (\mathbf{w_i}^T \mathbf{x} + b_i)$$
 où $\mathbf{w_i} = (w_{i1}, \dots, w_{iM}), i = 1, 2, 3.$



Perceptron: 1 couche

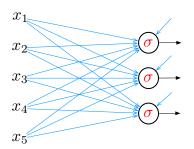
$$\boldsymbol{a} = \boldsymbol{\sigma} \left(\boldsymbol{W} \boldsymbol{x} + \boldsymbol{b} \right) \quad \text{où} \quad \boldsymbol{W} = \begin{pmatrix} \cdots \boldsymbol{w_1}^T \cdots \\ \cdots \boldsymbol{w_2}^T \cdots \\ \cdots \boldsymbol{w_3}^T \cdots \end{pmatrix} = \begin{pmatrix} w_{11} & \cdots & w_{1M} \\ w_{21} & \cdots & w_{2M} \\ w_{31} & \cdots & w_{3M} \end{pmatrix}, \ \boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

et $\sigma = (\sigma_1, \sigma_2, \sigma_3)$ appliquée composante par composante.



PERCEPTRON: 1 COUCHE

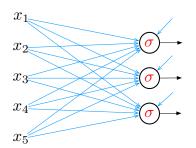
$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \pmb{\sigma} \left[\begin{pmatrix} w_{11} & w_{12} & w_{13} & w_{14} & w_{15} \\ w_{21} & w_{22} & w_{23} & w_{24} & w_{25} \\ w_{31} & w_{32} & w_{33} & w_{34} & w_{35} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \right]$$



 $\blacktriangleright w_{ij}$: poids de l'input j vers neurone i (et non de i vers j).

PERCEPTRON: 1 COUCHE

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \pmb{\sigma} \left[\begin{pmatrix} w_{11} & w_{12} & w_{13} & w_{14} & w_{15} \\ w_{21} & w_{22} & w_{23} & w_{24} & w_{25} \\ w_{31} & w_{32} & w_{33} & w_{34} & w_{35} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \right]$$



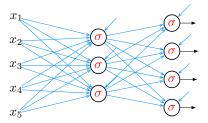
▶ w_{ij} : poids de l'input j vers neurone i (et non de i vers j).



PERCEPTRON: 2 COUCHES

$$egin{array}{lcl} a^{[1]} & = & \sigma \left(W^{[1]} x + b^{[1]}
ight) & := & \sigma \left(z^{[1]}
ight) \ a^{[2]} & = & \sigma \left(W^{[2]} a^{[1]} + b^{[2]}
ight) & := & \sigma \left(z^{[2]}
ight) \end{array}$$

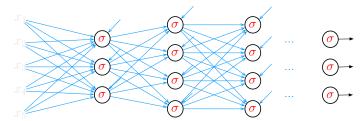
$$\begin{array}{llll} \text{O\`{u}} & \boldsymbol{W^{[i]}} = \begin{pmatrix} \cdots \boldsymbol{w_{1}^{[i]^{T}}} & \cdots \\ \cdots & \cdots & \cdots \\ \cdots \boldsymbol{w_{l_{i}}^{[i]^{T}}} & \cdots \end{pmatrix} = \begin{pmatrix} w_{11}^{[i]} & \cdots & w_{1l_{i-1}}^{[i]} \\ \cdots & \cdots & \cdots \\ w_{l_{i}1}^{[i]} & \cdots & w_{l_{i}l_{i-1}}^{[i]} \end{pmatrix} & \text{et} & \boldsymbol{b} = \begin{pmatrix} b_{1}^{[i]} \\ \vdots \\ b_{l_{i}}^{[i]} \end{pmatrix} & i = 1, 2 \end{array}$$



MULTI LAYER PERCEPTRON (MLP)

$$egin{cases} m{a^{[0]}} &= m{x} \ m{z^{[l]}} &= m{W^{[l]}}m{a^{[l-1]}} + m{b^{[l]}}, \ m{a^{[l]}} &= m{\sigma}\left(m{z^{[l]}}
ight) \end{cases}$$

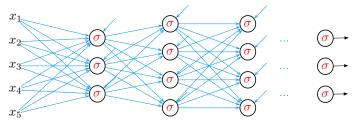
- lacksquare x est l'input et $a^{[L]}$ est l'output
- $m{W}^{[l]} \in \mathbb{R}^{\lambda_l} imes \mathbb{R}^{\lambda_{l-1}}$ et $m{b}^{[l]} \in \mathbb{R}^{\lambda_l}$, où λ_l taille de la couche l
- $lackbox{$|w_{ij}^{[l]}$: poids du neurone j (couche $l-1$) vers neurone i (couche l)}$



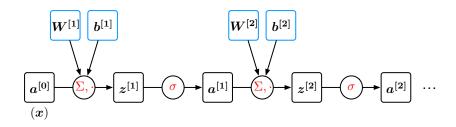
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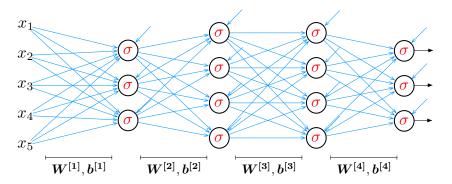


MLP: REPRÉSENTATION GRAPHIQUE



carrés = variables ronds = opérations

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```
Algorithm 1: MLP: forward pass
```

```
\begin{aligned} & \mathsf{Data:} \; \mathsf{dataset} = \{(x_i, y_i) : i = 1, \dots, N\} \\ & \mathsf{Inputs:} \; \mathsf{MLP} = \left\{ \begin{pmatrix} \boldsymbol{W}^{[l]}, \boldsymbol{b}^{[l]} \end{pmatrix} : l = 1, \dots, L \right\} \\ & \mathsf{predictions} = [] \\ & \mathsf{for} \; i = 1 \; to \; N \; \mathsf{do} \\ & & | \; a^{[0]} = x_i \\ & \mathsf{for} \; l = 1 \; to \; L \; \mathsf{do} \\ & & | \; z^{[l]} = \boldsymbol{W}^{[l]} a^{[l-1]} + b^{[l]} \\ & & | \; a^{[l]} = \sigma \left( z^{[l]} \right) \\ & \mathsf{end} \\ & \mathsf{predictions.append}(\boldsymbol{a}^{[L]}) \end{aligned}
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predictions = []
for i = 1 to N do
      a^{[0]} = x_i
      for l=1 to L do
                                                                                          // forward pass
             z^{[l]} = W^{[l]}a^{[l-1]} + b^{[l]}
           oldsymbol{a^{[l]}} = oldsymbol{\sigma}\left(oldsymbol{z^{[l]}}
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          oldsymbol{a^{[l]}} = oldsymbol{\sigma}\left(oldsymbol{z^{[l]}}
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      end
      predictions.append(a^{[L]})
end
return predictions
```

- lacksquare Soit le dataset $S = \{(\boldsymbol{x_i}, \boldsymbol{y_i}) : i = 1, \dots, N\}$
- On peut *paralléliser* la forward pass en passant les data "batch par batch" (batches de taille $B = 32, 64, \dots$).
- Le i-ème batch $B_i=(X_i,Y_i)$ est composé de B inputs et outputs $m{x_k}$ et $m{y_k}$ alignés en deux matrices:

$$egin{aligned} X_i = egin{pmatrix} dots & dots & \ldots & dots \ x_1 & x_2 & \cdots & x_B \ dots & dots & \ldots & dots \end{pmatrix} ext{ et } Y_i = egin{pmatrix} dots & dots & \ldots & dots \ y_1 & y_2 & \cdots & y_B \ dots & dots & \ldots & dots \end{pmatrix} \end{aligned}$$

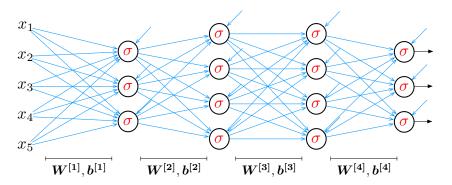
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$$\begin{cases} A^{[0]} = X_i \\ Z^{[l]} = W^{[l]}A^{[l-1]} \oplus b^{[l]}, \\ A^{[l]} = \sigma(Z^{[l]}), \quad l = 1, \dots, L \end{cases} \quad l = 1, \dots, L$$



```
Algorithm 2: MLP: forward pass (batched)
```

 $\begin{aligned} \mathbf{Data:} \ & \ \mathsf{dataloader} = \{ \boldsymbol{B_i} = (\boldsymbol{X_i}, \boldsymbol{Y_i}) : i = i, \dots, nb_batches \} \\ & \ \mathsf{Inputs:} \ \mathsf{MLP} = \left\{ \left(\boldsymbol{W}^{[l]}, \boldsymbol{b}^{[l]} \right) : l = 1, \dots, L \right\} \\ & \ \mathsf{predictions} = [] \\ & \ \mathsf{for} \ i = 1 \ to \ nb_batches \ \mathsf{do} \\ & \ A^{[0]} = \boldsymbol{X_i} \\ & \ \mathsf{for} \ l = 1 \ to \ L \ \mathsf{do} \\ & \ A^{[l]} = \boldsymbol{W}^{[l]} A^{[l-1]} + \boldsymbol{b}^{[l]} \end{aligned}$

```
predictions.append(A<sup>12</sup>
```

return concat(predictions)

```
Data: dataloader = \{B_i = (X_i, Y_i) : i = i, \dots, nb\_batches\}
Inputs: MLP = \{(W^{[l]}, b^{[l]}) : l = 1, \dots, L\}

predictions = []
for i = 1 to nb\_batches do // loop over batches
\begin{vmatrix} A^{[0]} = X_i \\ \text{for } l = 1 \text{ to } L \text{ do} \\ Z^{[l]} = W^{[l]}A^{[l-1]} + b^{[l]} \\ A^{[l]} = \sigma(Z^{[l]}) \\ \text{end} \\ \text{predictions.append}(A^{[L]}) \end{aligned}
```

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```

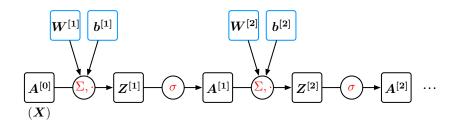
return concat(prediction

```
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```

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Inputs: MLP = \{(W^{[l]}, b^{[l]}) : l = 1, ..., L\}
predictions = []
for i = 1 to nb batches do
                                                              // loop over batches
     A^{[0]} = X_i
     for l=1 to L do
                                                                    // forward pass
         Z^{[l]} = W^{[l]}A^{[l-1]} + b^{[l]}
        A^{[l]} = \sigma\left(Z^{[l]}
ight)
     end
     predictions.append(A^{[L]})
end
return concat(predictions)
```

MLP: REPRÉSENTATION GRAPHIQUE (BATCHED)



carrés = variables ronds = opérations

BIBLIOGRAPHIE



Fleuret, F. (2022). Deep Learning Course.

