Urban Transit Scheduling: Framework, Review and Examples

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Abstract: A transit operational planning process includes four basic components performed usually in sequence: (1) network route design; (2) setting timetables; (3) scheduling vehicles to trips; and (4) assignment of drivers. This planning process is extremely cumbersome and complex and often creates confusion in its interrelationships among researchers and practitioners. The purpose of this work is to construct a framework and to provide an overview and examples of certain practical methodologies aimed at solving the transit scheduling problems. In the past 20 years, a considerable amount of effort has been invested in the computerization of the four components mentioned above. This is in order to provide more efficient, controllable, and responsive schedules. Nonetheless, despite the software used, no system is yet able to solve large scheduling problems, and manual intervention is necessary. There is a need to bridge the gap between the software system designers and the transit schedulers via the identification and organization of all the elements involved, including the current availability of data. This work emphasizes certain data needs along with examples for crystallizing and clarifying the transit scheduling undertaking. It is suggested that most of the scheduling tasks can be performed automatically, but it is preferable to undertake some of them through a conversational man-computer mode.

CE Database keywords: Urban areas; Public transportation; Scheduling.

Introduction

There is a saying by George Bernard Shaw: ''The person who behave sensibly is my tailor. He takes my measure anew every time he sees me. All the rest go on with their old measurements.'' A good public transport scheduling program is the one which has a tailor for frequent updates. However, only a few public transport properties actually update their scheduling data or are aware of the benefits they

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may have by this update. This paper overviews known and new methods for handling scheduling tasks in an efficient manner. This overview is accompanied with some examples used as an explanatory device for the prescribed methods.

Any public transport operational planning process includes four basic components performed usually in sequence: (1) network route design; (2) setting timetables; (3) scheduling vehicles to trips; and (4) assignment of drivers (crew). It is desirable for all four components to be planned simultaneously to exploit the system's capability to the greatest extent and to maximize the system's productivity and efficiency. However, this planning process is extremely cumbersome and complex, and therefore seems to require a separate treatment of each component, with the outcome of one fed as an input to the next component. In the past 20 years, a considerable amount of effort has been invested in the computerization of the four components mentioned previously, in order to provide more efficient, controllable, and responsive schedules. The best summary as well as the accumulative knowledge of this effort were presented in the second through the eighth international conferences on public transport scheduling, and appear in the books edited by Wren (1981), Rousseau (1985), Daduna and Wren (1988), Desrochers and Rousseau (1992), Daduna et al. (1995), Wilson (1999), and Voss and Daduna (2001). There are also a few commercially available software programs in the area of public transport scheduling, such as AUSTRICS (www.austrics.com.au), HASTUS (www.giro.ca), ILOG (www.ilog.co.uk), MERAKAS Ltd. (www2.omnitel.net/merakas), PTV (www.ptv.de), ROUTEMATCH (www.routematch.com), ROUREMATE (www.nemsys.it), ROUTELOGIC (www.routelogic.com), SCHEDULE MASTERS Inc. (www.smilink.com), SYSTRA (www.systra.com), TRAPEZE (www.trapezesoftware.com), and more. Nonetheless, some of the scheduling problems in those software programs are over simplified and are decomposed into subproblems.

This paper focuses on the three scheduling components: timetabling, vehicle scheduling, and crew scheduling while assuming that the public transport network is unchanged. The third scheduling component is usually divided into creating crew duties and crew rosters (rotation of duties among the drivers). The functional diagram of these scheduling elements appear in Fig. 1.

The timetable component in Fig. 1 is aimed to meet the general public transportation demand. The demand varies during the hours of the day, the day of the week, from one season to another, and even from one year to another. This demand reflects the business, industrial, cultural, educational, social, and recreational transportation needs of the community. It is the purpose of this component to set appropriate timetables for each transit route to meet the variation in public demand. Determination of timetables is performed on the basis of passenger counts and must comply with service frequency constraints. The vehicle scheduling component in Fig. 1 is to schedule vehicles to trips according to given timetables. A transit trip can be either planned to transport passengers along its route or to make a deadheading (DH) trip in order to connect efficiently two service trips. The scheduler's task is to list all daily chains of trips (some deadheading) for a vehicle, ensuring the fulfillment of the timetable requirements and the operator requirements (refueling, maintenance, etc.). The major objective of the scheduler's task is to minimize the number of vehicles required. The crew

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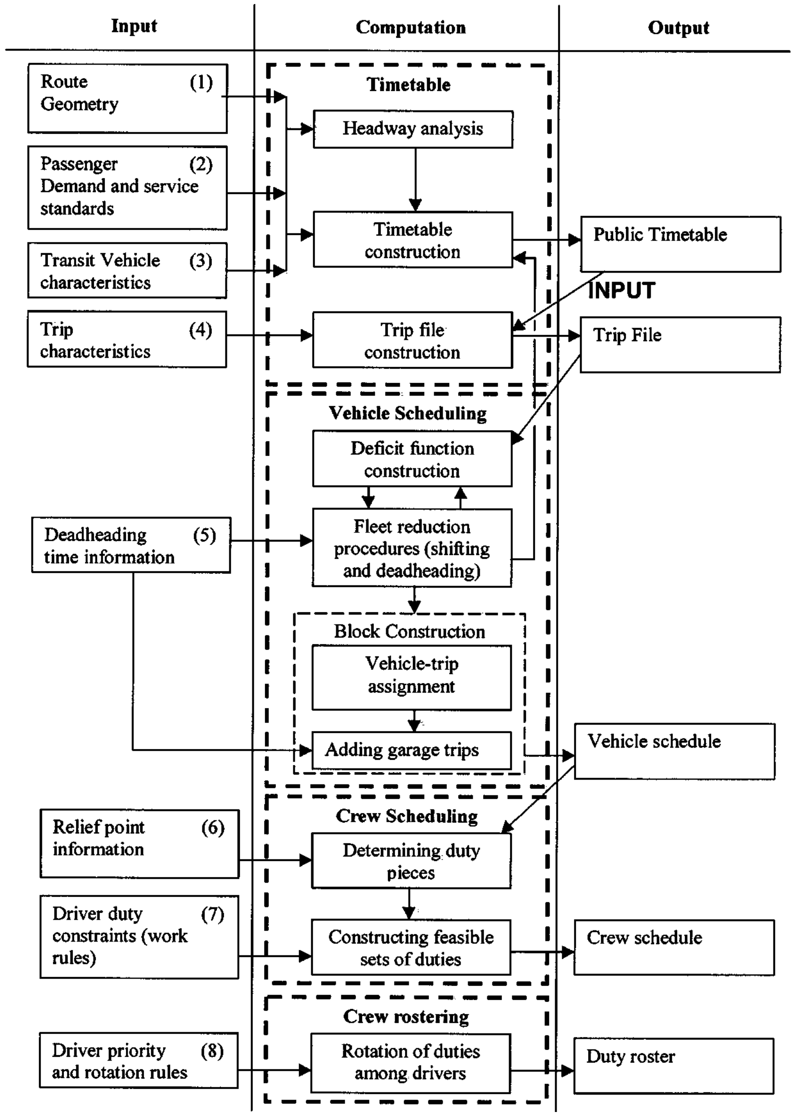

Fig. 1. Functional diagram of a public transport scheduling system (system architecture)

scheduling component in Fig. 1 is to assign drivers to the outcome of vehicle scheduling. This assignment must comply with some constraints, which usually are dependent on a labor contract. Finally, the crew rostering component in Fig. 1 usually refers to priority and rotation rules, rest periods, and drivers' preferences. Any transit company, which naturally wishes to utilize its resources more efficiently, has to deal with problems encountered by various pay scales (regular, overtime, weekends, etc.) and human-oriented dissatisfaction. All the components in Fig. 1 are very sensitive to internal and external factors, sensitivity that could easily lead toward an inefficient solution. In Fig. 1, the items under the input are numbered, and, respectively, their general descriptions are listed below, bearing in mind that their values differ by time of day and day of week.

1. Route geometry
   (*a*) Route number
   (*b*) Nodes, stops, and timepoints on a route
   (*c*) Pattern, sequence of nodes on a route
2. Passenger demand and service standards
   (*a*) Passenger loads between adjacent stops on a route
   (*b*) Load factor, desired number of passengers on board the transit vehicle
   (*c*) Policy headway, the inverse of the minimum frequency standard
3. Transit vehicle characteristics
   (*a*) Vehicle type
   (*b*) Vehicle capacity
   (*c*) Running time, vehicle travel time between stops and/or timepoints on a route
4. Trip characteristics
   (*a*) Trip layover (rest) time (maximum and minimum)
   (*b*) Trip departure time tolerances (maximum departure delay and maximum departures advance)
5. Deadheading time information
   (*a*) List of garages, name and location
   (*b*) List of trip start and end location
   (*c*) DH times from garage locations to each trip start location (pull-outs)
   (*d*) DH times from trip end locations to garage locations (pull-ins)
   (*e*) DH time matrix between all trip end and start locations
6. Relief point information
   (*a*) Relief point location (stops, trip start, and end points garages)
   (*b*) Travel times between relief points
7. Driver duty constraints (dependent on a labor contract)
   (*a*) Type of duty (early, late, split, full, tripper, etc.)
   (*b*) Duty length (maximum spread time)
   (*c*) Number of vehicle changes on duty
   (*d*) Meal breaks
   (*e*) Duty composition
   (*f*) Other work rules
8. Drivers priority and rotation rules
   (*a*) List of drivers by name and type (e.g., part time, full time, seniority)
   (*b*) Driver priority or equality rules
   (*c*) Work day on and off pattern

The complexity involved in the public transport operational planning process challenged researchers to develop automated computerized procedures, which led to a number of software packages available in the market. An overall view of such software is illustrated in Fig. 2. It is worth mentioning that the evaluation module of such a software package should be based on an external input related

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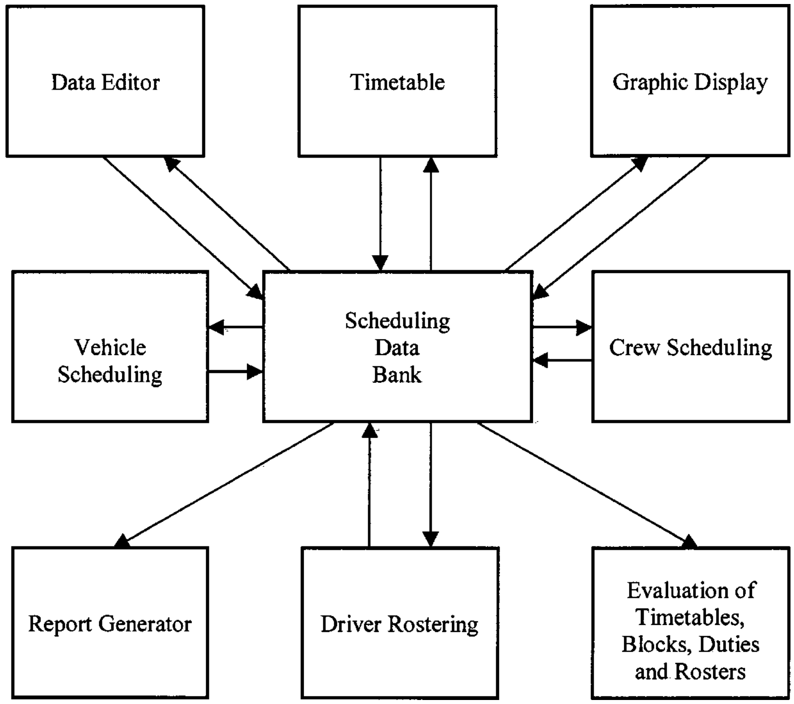

Fig. 2. Overall view of a public transport scheduling system

to cost coefficients and performance criteria. The cost coefficients include vehicle cost (fixed and variable), crew cost (fixed and variable), service benefit, and other costs. The performance criteria include measures of passenger service, measures for vehicle and crew schedules, and measures for duty rosters, and other criteria.

Timetables

Mathematical Programming Approaches

Mathematical programming methods for determining frequencies and timetables have been proposed by Furth and Wilson (1981), Koutsopoulos et al. (1985), Ceder and Stern (1984), and Ceder and Tal (1999). The objective in Furth and Wilson (1981) is to maximize the net social benefit, consisting of ridership benefit and wait timesaving, subject to constraints on total subsidy, fleet size, and passenger loading levels. Koutsopoulos et al. (1985) extended this formulation by incorporating crowding discomfort costs in the objective function and treating the time-dependent character of transit demand and performance. Their initial problem comprises a nonlinear optimization program relaxed by linear approximations. Ceder and Stern addressed the problem with an integer programming formulation and heuristic person-computer interactive procedure. The latter approach focuses on reconstructing timetables when the available vehicle fleet is restricted. Finally Ceder and Tal (1999) used mixed integer programming and heuristic procedures for constructing timetables with maximum synchronization. That is, the maximization of the number of simultaneous arrivals of vehicles to connection stops.

Passenger Data Based Approach

A public transport timetable is commonly constructed for given sets of derived frequencies. The basic criteria for the determination of frequencies are to provide adequate vehicle space to meet passenger demand and to assure a minimum frequency (maximum-policy headway) of service. Ceder (1984) described four methods for calculating the frequencies. Two methods are based on point-check (counting the passengers on board the transit vehicle at certain points), and two on ride-check (counting the passengers along the entire transit route). In point-check methods, the frequency is the division between passenger load at the maximum (max) load point (either the one across the day or in each hour) and the desired occupancy or load factor. In ride-check methods, the frequency is the division between the average or restricted-average passenger load and the desired occupancy. The average load is determined by the area under the load profile (in passenger-kilometer) divided by the route length (kilometer), and the restricted average is a higher value than the average one in order to ensure that in a certain percentage of the route length the load does not exceed the desired occupancy. This desired occupancy (or load factor) is the desired level of passenger load on each vehicle in each time period (e.g., number of seats).

In a follow-up study, Ceder (1986) analyzed optional ways for generating public timetables. This analysis allows for establishing a spectrum of alternative timetables based on three categories of options: (1) selection of type of headway; (2) selection of the frequency determination method for each period; and (3) selection of special requests. In Category (1), the headway (time interval between adjacent departures) can be equal or balanced. Equal headway refers to the case of evenly spaced headways and balanced headway and to the case of unevenly spaced headways but with an even average passenger load at the hourly maximum load point. In Category (2), it is possible to select for each time period one of the four frequency determination methods (two point-check and two ride-check) mentioned previously, or a given frequency by the scheduler. In Category (3), it is possible to request clock headways (departure times that repeat themselves in each hour, easy-to-memorize) and/or certain numbers of departures (usually for cases with limited resources).

The outcome of these analyses is a set of optional timetables in terms of vehicle departure times at all specified timepoints, using passenger load data. Each timetable is accompanied by two comparison measure that are used as an evaluation indicator in conjunction with resource saving. The first measure is the total required vehicle runs (departures), and the second is an estimate for the minimum required fleet size at the route level only.

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Vehicle Scheduling

Objectives

This second component in Fig. 1 determines, in an optimal manner, the construction of chains of trips or blocks (vehicle schedules). The main objective of this function is to construct vehicle blocks while either the existing number of vehicles (while minimizing the total DH kilometers and disruption to the timetable) or minimizing the number of vehicles required to carry out the schedule (the trip schedule).

In addition, the assignment of vehicle chains to garages should be determined in an efficient manner. The attainment of these objectives can be carried out through the interaction of the following subfunctions:

1. Trip characteristics study;
2. DH trip construction;
3. Intertrip DH trip insertions;
4. Timetable shifting;
5. Vehicle trip chaining; and
6. Garage chain assignment.

Exact Solution Approaches

The problem of scheduling vehicles in a multidepot scenario is known as the Multi-depot Vehicle Scheduling Problem (MDVSP). This problem is complex (NP-hard), and considerable effort is devoted to solve it in an exact way. A review and description of some exact solutions can be found in Desrosiers et al. (1995), Daduna and Paixao (1995), Löbel (1999), and Mesquita and Paixao (1999).

An example formulation of the MDVSP is as follows:

$$\text{objective function:}\ \min_{y}\left\{\sum_{i=1}^{n+1}\sum_{j=1}^{n+1} c_{ij}y_{ij}\right. \tag{1}$$

where i = event of ending a trip at time a_i; j = event of the start of a trip at time b_j; and $y_{ij}=1$, ending is connecting to start and 0 otherwise.

For $i=n+1$, then $y_{n+1j}=1$ if a depot supplies a vehicle for the jth trip. For $i=n+1$, $y_{i,n+1}=1$ if after the ith trip end the vehicle returns to a depot, and $y_{n+1,n+1}$ = number of vehicles remaining unused at a depot.

The cost function c_{ij} takes the form

$$c_{ij}=\left\{\begin{array}{lll} K; & i=n+1; & j=1,2,\ldots,n \\ O; & i=1,2,\ldots,n; & j=n+1 \\ L_{ij}+E_{ij}; & i,j=1,2,\ldots,n & \end{array}\right\} \tag{2}$$

where K = the saving incurred by reducing the fleet size by one vehicle; L_{ij} = direct DH cost from events i to j; and E_{ij} = cost of idle time of a driver between i and j.

This formulation appears in a similar form in Gavish et al. (1978) and covers the chaining of vehicles in a sequential order from the depot to the transit routes,

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alternating with idle time and DH trips and back to the depot. This is a zero-one integer programming problem that can be converted to a large-scale assignment problem. In addition, the assignment of vehicles from the depots to the vehicle schedule generated in the above chaining process can be formulated as a ''transportation problem'' known in the operations research literature.

Löbel (1999) is using a brunch-and-cut method for MDVSP with the generation of upper bounds and the use of Lagrangean relaxations and pricing. Mesquita and Paxiao (1999) are comparing, in this problem, the linear relaxation based on multicommodity network flow approach.

Heuristic Deficit Function Approach

Following is a description of the deficit function approach described by Ceder and Stern (1981) for assigning the minimum number of vehicles to allocate for a given timetable. A deficit function is simply a step function that increases by one at the time of each trip departure and decreases by one at the time of each trip arrival. Such a function may be constructed for each terminal in a multiterminal transit system. To construct a set of deficit functions, the only information needed is a timetable of required trips. The main advantage of the deficit function is its visual nature. Let $d(k,t,S)$ denote the deficit function for terminal k at time t for schedule S. The value of $d(k,t,S)$ represents the total number of departures minus the total number of trip arrivals at terminal k, up to and including time t. The maximal value of $d(k,t,S)$ over the schedule horizon $[T_1,T_2]$ is designated $D(k,S)$.

Let t_s^i and t_e^i denote the start and end times of trip i, $i \in S$. It is possible to partition the schedule horizon of $d(k,t,S)$ into sequence of alternating hollow and maximal intervals. The maximal intervals $[s_i^k, e_i^k]$, $i=1, \ldots, n(k)$ define the interval of time over which $d(k,t)$ takes on its maximum value. Note that S will be deleted when it is clear which underlying schedule is being considered. Index i represents the ith maximal intervals from the left, and $n(k)$ represents the total number of maximal intervals in $d(k,t)$. A hollow interval is defined as the interval between two maximal intervals. Hollows may consist of only one point, and if this case is not on the schedule horizon boundaries (T_1 or T_2), the graphical representation of $d(k,t)$ is emphasized by clear dot.

If the set of all terminals is denoted as T, the sum of $D(k)$ for all $k \in T$ is equal to the minimum number of vehicles required to service the set T. This is known as the fleet size formula. Mathematically, for a given fixed schedule S

$$D(S)=\sum_{k \in T} D(k)=\sum_{k \in T} \max_{t \in [T_1,T_2]} d(k,t) \tag{3}$$

where $D(S)$ = minimum number of buses to service the set T.

When DH trips are allowed, the fleet size may be reduced below the level described in Eq. (3). Ceder and Stern (1981) described a procedure based on the construction of a unit reduction DH chain (URDHC), which, when inserted into the schedule, allows a unit reduction in the fleet size. The procedure continues inserting URDHCs until no more can be included or a lower boundary on the minimum fleet in reached. The lower boundary $D_m(S)$ is determined from the

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overall deficit function defined as $g(t,S)=\Sigma_{k\in T}d(k,t,S)$, where $D_m(S) = \max_{t\in[T_1,T_2]} g(t,s)$. This function represents the number of trips simultaneously in operation. Initially, the lower bound was determined to be the maximum number of trips in a given timetable that are in simultaneous operation over the schedule horizon. Stern and Ceder (1983) improved this lower bound to $D_m(S')>D_m(S)$ based on the construction of a temporary timetable, S', in which each trip's arrival time is extended to the time of the first trip that may feasibly follow it in S. Further improvements of the lower bound on the fleet size made recently by Ceder (2002).

The deficit function theory was extended by Ceder and Stern (1982) to include possible shifting in departure times within bounded tolerances. Basically, the shifting criteria is based on a defined tolerance time $[t_s^i-\Delta_a^i, t_s^i+\Delta_d^i]$ where Δ_a^i is the maximum advance of the trip scheduled departure time (early departure), and Δ_d^i is the maximum delay allowed (late departure). The maximum interval is then compared with the appropriate tolerance time elements for establishing conditions in which it is possible to reduce the fleet size by one via certain shifts.

The algorithms of the deficit function theory are described in detail by Ceder and Stern (1981, 1982). However, it is worth mentioning the next terminal (NT) selection rule and the URDHC routines. The selection of the NT in attempting to reduce its maximal deficit function may rely on the basis of garage capacity violation, or on a terminal whose first hollow is the longest. The rationale here is to try to open the greatest opportunity for the insertion of the DH trip.

Once terminal k is selected, the algorithm searches to reduce $D(k)$ by shifting departure times (if allowed). Then all of the $d(k,t)$ values are updated, and the NT rule is again applied. When no more shiftings are possible, the algorithm searches for a URDHC from the selected terminal while considering possible blending between DH insertion and shiftings in departure times. In the URDHC routines, there are four rules: $R=0$ for inserting the DH trip manually in a conversational mode, $R=1$ for inserting the candidate DH trip that has the minimum travel time, $R=2$ for inserting a candidate DH trip whose hollow starts farthest to the right, and $R=3$ for inserting a candidate DH trip whose hollow ends farthest to the right. In the automatic mode $(R=1,2,3)$, if a DH trip cannot be inserted and the completion of a URDHC is blocked, the algorithm backs up to a DH candidate list and selects the next DH candidate on that list.

In the fixed schedule problem, the algorithm also terminates when $D(S)$ is equal to the improved lower bound. In the variable schedule problem (when shifting is allowed), the algorithm also uses this comparison, and if $D(S)$ is equal to the improved lower bound, the URDHC procedure (with shiftings) ceases and the shifting only mode applies. If the latter results in reducing $D(S)$, the URDHC procedure is again activated. The process terminates when $D(S)$ cannot be further reduced.

Finally, all of the trips, including those that were shifted and the DH trips, are chained together for constructing the vehicle schedule (blocks). Two rules can be applied for creating the chains: ‘‘first in first out’’ (FIFO) and a chain-extraction procedure described by Gertsbach and Gurevich (1977). The FIFO rule simply links the arrival time of a trip to the nearest departure time of another trip (at the same location) and continues to create a schedule until no connection can be

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made. The trips considered are deleted and the process continues. The chain-extraction procedure allows an arrival-departure connection for any pair within a given hollow (on each deficit function). The pairs considered are deleted, and the procedure continues. Both methods end with the minimum derived number of vehicles (blocks). The ideas of the deficit function approach led to design and schedule trip departure times for short-turn strategies in which some trips can start further down the route and/or end prior to its final stop (Ceder 1990, 1991).

Crew Scheduling and Rostering

Overview

The last two components of the public transport operational planning process (shown in Fig. 1) is the assignment of drivers to carry on vehicle schedules and the rotation of duties among drivers. The purpose of the assignment function is to determine a feasible set of driver duties in an optimal manner. The criteria for this determination is based on an efficient use of manpower resources while maintaining the integrity of any work rule agreements. The construction of the selected crew schedule is usually a result of the following subfunctions:

1. duty piece analysis;
2. work-rules coordination;
3. feasible duty construction; and
4. duty selection.

The duty piece analysis function divides or partitions each vehicle block at selected relief points into a set of duty pieces. These duty pieces are assembled in the feasible duty construction function. Other required information are travel times between relief points and a list of relief points designated as required duty stops and start locations.

Theoretically, each relief point may be used to split the vehicle block into new duty pieces. Usually, it is more efficient to use one or more of the following criteria for the selection of which relief points to include.

1. Minimum duty piece length;
2. Select a piece so that the next relief point selected is as close as possible to the maximum duty part time (maximum time before having a break);
3. Only a few (say two) relief points in each piece; and
4. Operator decisions.

In order to utilize any crew scheduling method, a list of work rules to be used in the construction of feasible driver duties is required. The work rules are the result of an agreement between the drivers (or their unions) and the public transport company (and/or public authorities).

The determination of different feasible sets of duties may be selected based on, for example, one or more of the following performance measures: number of duties (drivers), number of split duties, total number of changes, total duty hours, average duty length, total working hours, average working time, number of short duties, and costs.

Once the set of duties is established, it is common to group them into rosters. A roster is defined as a periodic duty assignment that guarantees that all the trips

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are covered for a certain number of consecutive days (a week, month, or any other period). Commonly a roster contains a subset of duties covering 6 consecutive days (called weeks). The length of a roster is typically between 30 days to 60 days (5 to 10 weeks). The usual rostering problem is to find a feasible set of rosters to cover all the duties in the minimum number of weeks, implying the minimization of the number of crews required (as explained in detail by Caprara et al. 1999).

Worth nothing is that in railways the vehicle scheduling component is obviously not important, and instead, it is required to build the work schedule of the train crews (drivers and conductors together).

Mathematical Approaches

Crew scheduling and rostering are widely treated in the books by Wren (1981), Rousseau (1985), Daduna and Wren (1988), Desrochers and Rousseau (1992), Daduna et al. (1995), Wilson (1999) and in the forthcoming book by Voss and Daduna (2001). Specific detailed studies can be found in the works by Bodin et al. (1983), Carraresi and Gallo (1984), Bianco et al. (1992), Caprara et al. (1999), and Freling et al. (1999).

The basic formulation of the crew scheduling problem is zero-one integer linear programming, called a set partitioning problem (SPP). In this SPP, the objective is to select a minimum set of feasible duties, such that each task is included in exactly one of these duties

$$\min \sum_{q \in Q} c_q x_q \tag{4}$$

$$\text{s.t.} \sum_{q \in Q(j)} X_q = 1, \quad \text{for all} \quad j \in J \tag{5}$$

$$x_q = (0,1), \quad \text{for all} \quad q \in Q \tag{6}$$

where c_q = cost of duty $q \in Q$; Q = set of all feasible duties; and $Q(j) \in Q$ = set of duties covering task $j \in J$. A binary zero-one variable x_q is used for indicating if duty q is selected in the solution or not. Constraint Eq. (6) ensures that each task will be covered by exactly one duty. An easier way to solve the crew scheduling problem is to relax constraint Eq. (6)

$$\sum_{q \in Q(j)} x_q \geqslant 1, \quad \text{for all} \quad j \in J \tag{7}$$

Eq. (7) represents a new problem called a set covering problem (SCP). This SCP is usually solved first, and the solution is changed to handle the SPP by deleting overlapping trips. This deletion process involves changes in the considered duties, and a crew member who is assigned to such a duty will make the trip as a passenger. Freling et al. (1999) explains that such a change affects neither the feasibility nor the cost of the considered duties.

A mixed (heuristic and exact) approach to solve the crew scheduling problem, mentioned in Bodin et al. (1983), is to (1) generate all feasible pieces of work (to be derived from the vehicle blocks); (2) establish and interval piece cost (based

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Table 1. Given Data for Example Problem

Time	Departure Time at the Route Departure Point		Average Observed Maximum Number of Passengers onboard Vehicles		TRAVEL TIME INCLUDING LAYOVER TIME (MIN.)				Desired Occupancy (Passengers) on each Vehicle	
					Service		Deadheading			
	A to B	B to A	A to B	B to A	A to B	B to A	A to B	B to A	A to B	B to A
6–7	6:20		15							
a.m.	6:40	6:30	30	22						
	6:50	6:45	47	38	60	50	40	35	50	50
7–8	7:05		58							
a.m.	7:15		65							
	7:25	7:10	79	52						
	7:30		90		75	60	45	40	65	50
	7:40	7:25	82	43						
	7:50		62							
		7:45		59						
8–9	8:00		75							
a.m.	8:10		68							
	8:20		55							
	8:35	8:25	80	23	70	60	45	40	65	50
	8:50	8:40	70	51						
		8:55		28						

Note: Minimum frequency: two vehicles per hour for all hours, both directions

on piece characteristics and past experience); (3) create an acyclic network from each block where nodes are the relief points and arcs represent the cost of each feasible piece in that block; (4) solve the shortest path problem in order to establish the best (minimum cost) pieces; and (5) solve a matching problem while using the two or three legal piece combination, and if the solution is not feasible, to reiterate part of the process by updating the piece combination cost and redo the shortest path (step 4) until the solution is feasible and satisfactory. Part of this shortest path approach is exhibited in the example problem in the next section.

Freling et al. (1999) provides several approaches for integrating vehicle and crew scheduling problems. They concluded that a combined formulation, with heuristics for the column generation (piece coverage) procedure, results in better solutions, especially for situations when it is not allowed for the crew to change vehicles during a duty. More integration consideration and mathematical treatments appear in Voss and Daduna (2001).

Finally, the duty rostering problem in different public transport agencies and countries is also well covered in the book by Voss and Daduna (2001). It is apparent that the duty rostering rules and priorities are dependent on the location and agency.

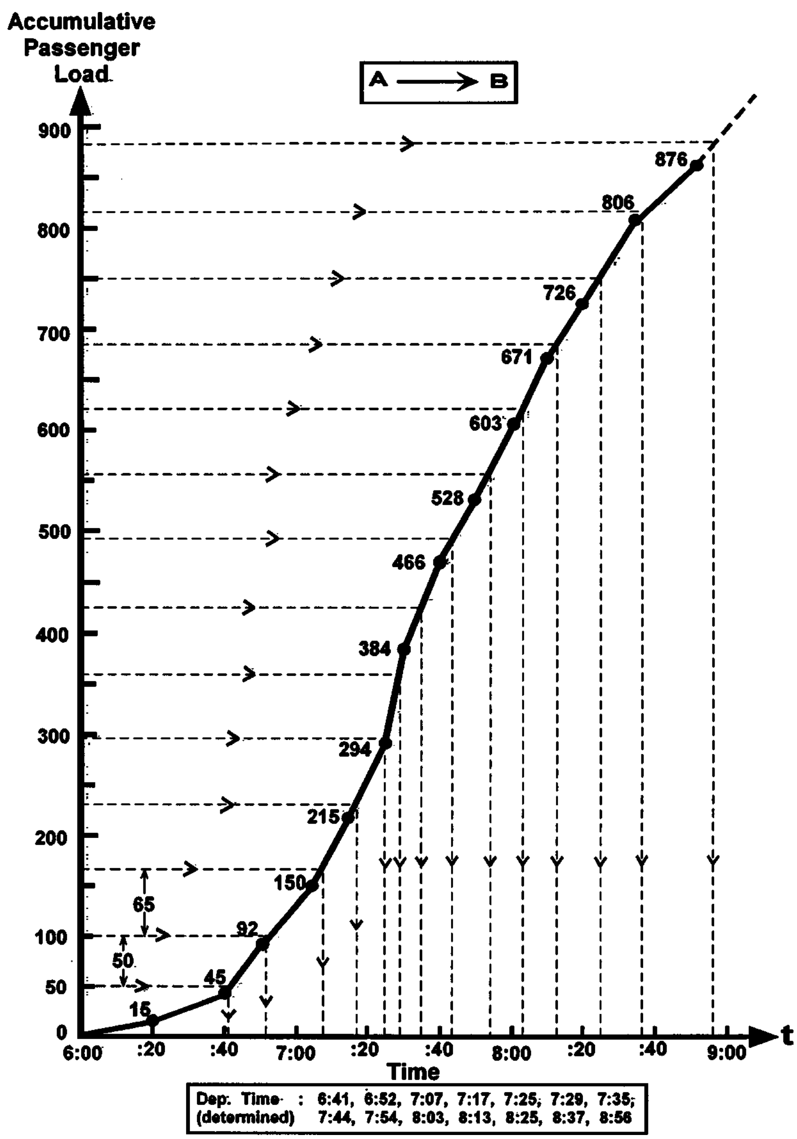

Fig. 3. Determination of balanced load departure times for the example problem, direction A–B

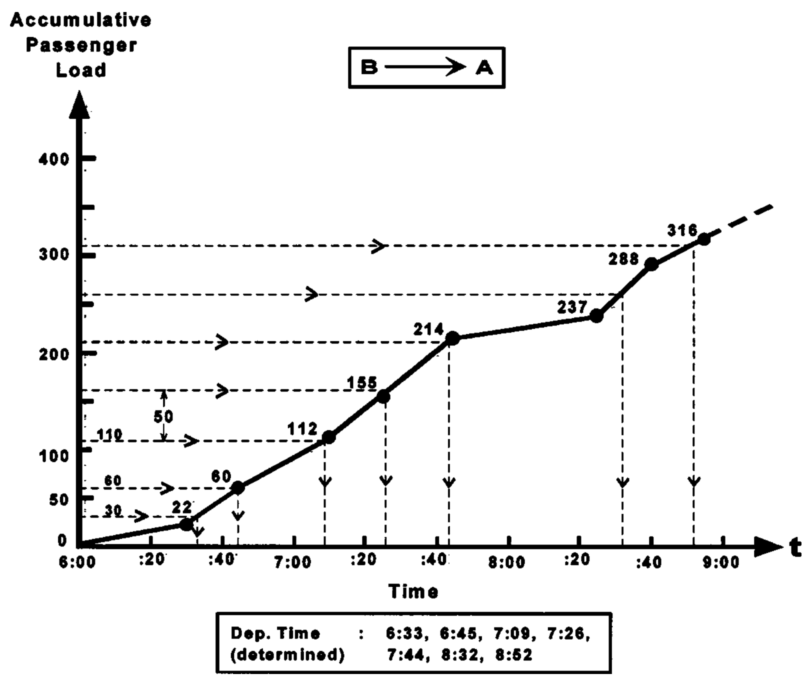

Fig. 4. Determination of balanced load departure times for the example problem, direction B–A.

Example

The example in this section is used as an explanatory device for describing scheduling procedures and considerations. Table 1 contains the necessary information and data for a 3-h example of a transit line from Points A to B and Points B to A. Point B can be perceived as the CBD that attracts the majority of the demand between 6–9 a.m. There are 14 and 8 departures for Points A to B and B to A, respectively. The average observed maximum (max) load on each trip, service and DH travel times, desired occupancies, and minimum frequency are all shown in Table 1.

In order to construct a balanced load timetable, these Table 1 data are used for running the accumulative load procedures described by Ceder (1986). The balanced load timetable is based on even, average loads on-board the public transport vehicles as opposed to even headways. Given that the maximum load is observed at the same stop for each direction, the accumulative load procedures determine the new departure times shown in Figs. 3 and 4. There are 14 new departures for direction A to B in Fig. 3 that are based on desired occupancies of 50 and 65 passengers. There are seven new departures for direction B to A in Fig. 4.

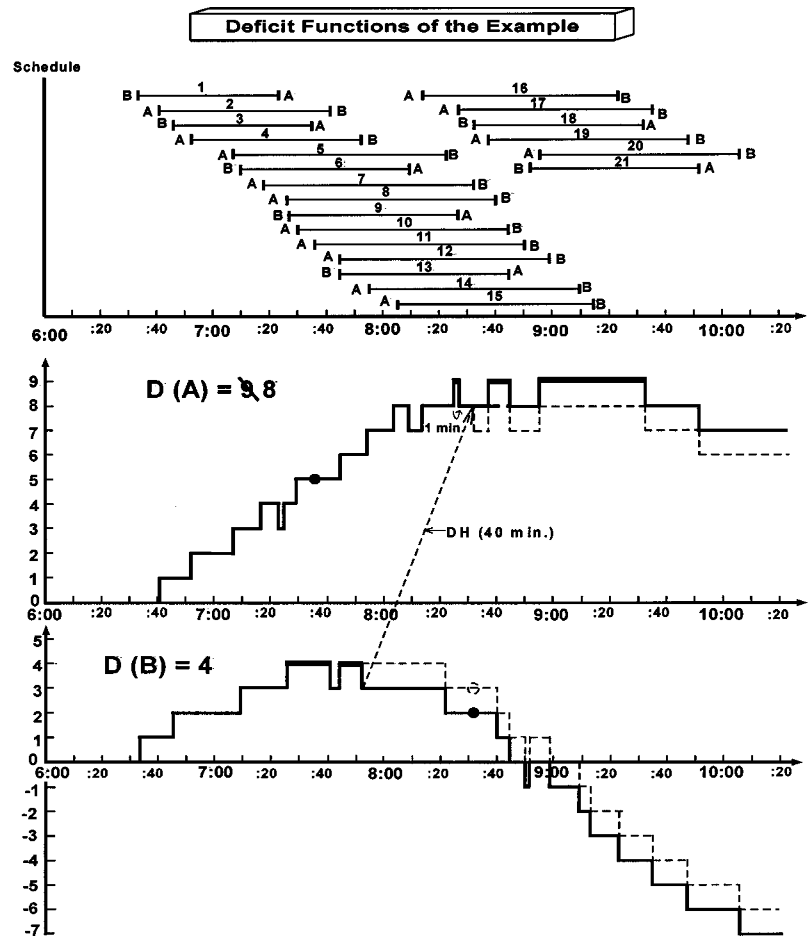

Fig. 5. Deficit function analysis for the example problem

Once the departure times are set at both route endpoints, the vehicle scheduling component can be integrated into the two-direction timetables. First, two deficit functions (DF) are constructed at A and B, as it is shown in Fig. 5. These DFs are based on the schedule of 21 trips (14 of A–B, 7 of B–A), presented with respect to their travel times in the upper part of Fig. 5. Second, the DF theory leads to save one vehicle at $d(\mathrm{A},t)$ through a shifting of trip No. 17 by 1 min forward (late departure), and inserting a DH trip from B to A (7:52 to 8:32 a.m). The total fleet required is then $8+4=12$ vehicles.

Having completed the deficit function procedure, the final efficient schedule can be set for both balancing the passenger loads and for the minimum fleet size required. The timetables at the route's endpoints appear in the upper part of Table 2 and in the blocks in its lower part. The blocks (vehicle schedules) are contracted

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Table 2. Timetable and Vehicle Schedule (Blocks) of Example Problem

A to B			B to A				
Trip number	Departure time	Arrival time	Trip number	Departure time	Arrival time	Block number	Trips in block (in sequence, via FIFO)
2	6:41	7:41	1	6:33	7:23	1	1-8-21
4	6:52	7:52	3	6:45	7:35	2	2-13-20
5	7:07	8:22	6	7:09	8:09	3	3-11
7	7:17	8:32	9	7:26	8:26	4	4-DH-19
8	7:25	8:40	13	7:44	8:44	5	5-18
10	7:29	8:44	DH[a]	7:52	8:32	6	6-16
11	7:35	8:50	18	8:32	9:32	7	7
12	7:44	8:59	21	8:52	9:52	8	9-17
14	7:54	9:09				9	10
15	8:03	9:13				10	12
16	8:13	9:23				11	14
17	8:26[b]	9:36				12	15
19	8:37	9:47					
20	8:56	10:06					

[a]Inserted deadheading trip.

[b]Trip 17 was shifted by 1 min (see Fig. 5).

from the timetable using the FIFO rule. The first block, for example, starts with Trip 1, which is linked with its first feasible connection at A, Trip 8 (7:23 links to 7:25) and Trip 21 at B (8:40 links to 8:52).

The blocks that appear in Table 2 are now subjected to a crew scheduling procedure. Given that both A and B are relief points, each block can be partitioned into alternative pieces covering all possible combinations. Fig. 6 shows how to partition the blocks into combinations of pieces. In addition, each piece is assigned an internal piece cost based on the piece characteristics (e.g., time of day, arrival and departure locations, type of vehicle required) and past experience (how much is the cost of a similar piece in past crew schedules).

The right hand side column of Fig. 6 contains the acyclic network of each block representing all the possible pieces and their internal costs. Then it is possible to apply a shortest path algorithm, such as the known Dijkstra algorithm (see any operations research literature on networks). The results of the Dijkstra procedure are emphasized in Fig. 6 along with the minimum piece cost to cover the whole block. These results are illustrated in Fig. 7 for each block. Given that each piece is eligible to be covered by one driver Fig. 7 shows that the 12 blocks require 16 drivers with a total cost of 95. However, there are more possibilities to match some pieces if it is both legal (from the labor agreement perspective) and can reduce the cost. Taking into account the deadheading times in Table 1 as a measure to move between A and B or B and A, there are only two possible matchings to examine: to connect the first piece of Block 1 with either the second piece of block 2 or with the second piece of Block 8 as is shown in Fig. 7. Both

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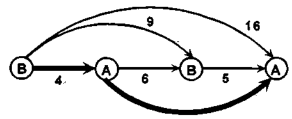
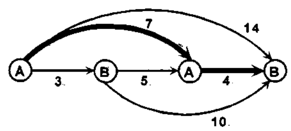
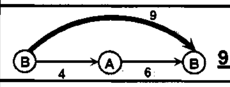
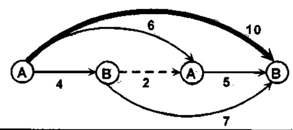
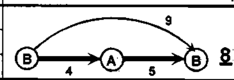
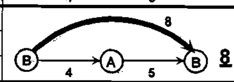
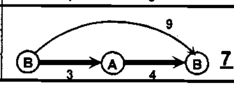
Block* No.	Segment (in bracket-trip No. or combination of trips)	Segment Cost	Shortest path Solution (best pieces emphasized)
1	B-A (1)	4	 12=shortest path
	A-B (8)	6	
	B-A (21)	5	
	B-A-B (1-8)	9	
	A-B-A (8-21)	8	
	B-A-B-A (1-8-21)	16	
2	A-B (2)	3	 11
	B-A (13)	5	
	A-B (20)	4	
	A-B-A (2-13)	7	
	B-A-B (13-20)	10	
	A-B-A-B (2-13-20)	14	
3	B-A (3)	4	 9
	A-B (11)	6	
	B-A-B (3-11)	9	
4	A-B (4)	4	 10
	B-A (DH)	2	
	A-B (19)	5	
	A-B-A (4-DH)	6	
	B-A-B (DH-19)	7	
	A-B-A-B (4-DH-19)	10	
5	A-B (5)	3	 8
	B-A (18)	5	
	A-B-A (5-18)	9	
6	B-A (6)	4	 8
	A-B (16)	5	
	B-A-B (6-16)	8	
8	B-A (9)	3	 7
	A-B (17)	4	
	B-A-B (9-17)	9	

*refers to blocks with more than one trip

Fig. 6. Partitioning of each block into minimum cost pieces

possible matchings do not require a DH time. Usually, if the best (minimum cost) feasible matching results in a cost reduction (less than 95), this matching is selected, and the number of drivers will be reduced to 15.

This simplified example demonstrates part of the complexities involved in the public transport scheduling undertaking. Furthermore, on top of the combinatorial problems inherent in scheduling tasks, there are human dissatisfaction issues, relating to the crews, that deserve attention and make this undertaking even more cumbersome.

Concluding Remarks

In practical public transport scheduling, schedulers attempt to create timetables and allocate vehicles and crews in the most efficient manner possible. However,

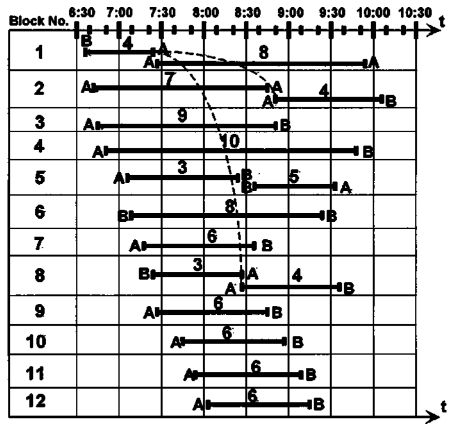


Fig. 7. The results of partitioning the blocks into best pieces

these scheduling tasks are time consuming and exacting, requiring the services of imaginative and experienced schedulers. The overview and methodologies presented in this paper suggest that most of the scheduling tasks can be performed automatically or in a conversational man-computer mode.

The transit operational planning process includes four basic components usually performed in sequence: (1) network route design, (2) setting timetables, (3) scheduling vehicles to trips, and (4) assignment of drivers. Only components 2, 3 and 4 are covered in this work. The first component of network route design is accompanied by three perspectives: user, operator, and the community interpreted by its objective functions. Components 3 and 4 refer to the tasks undertaken by the public transport operator and naturally use the operator's perspective. Component 2, however, is carried out by either the community or the operator and contains the user perspective (e.g., minimum vehicle frequency, even passenger load versus even vehicle headway). No doubt that public transport timetables (second component) are one of the predominant bridges between the operator (and/or the community) and the passengers. Therefore, more attention should be provided for the construction of a timetable in order to improve its correspondence with fluctuated passenger demand.

Finally, the adoption of new practical scheduling methods will undoubtedly produce more efficient timetables and vehicle and crew schedules that ultimately will result in saving in operational costs. Moreover, it will then be possible to better match public transport demand that varies systematically by season, day of the week, time of day, location, and direction of travel with the resultant public transport services. There is a saying by François Gautier, ''More important than the quest for certainty is the quest for clarity.'' Many elements in the implementation stages of public transport scheduling are uncertain, mainly due to traffic congestion, vehicle breakdowns, and crew behavior. It is important to emphasize the clarity of the scheduling processes in order to allow for immediate adjustments and smoothing of interferences.

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