Vehicle Scheduling for Public Mass Transit - An Overview

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Abstract: Vehicle scheduling always plays an important role in urban mass transit companies. Efficient schedules of a given timetable have a significant influence on the demand for vehicles and, hence, the demand for personnel. This means that the economic results of a urban mass transit company are closely linked to the planning activities in this area.

The objective of the paper is to show the state of the research in vehicle scheduling and its practical application in urban mass transit companies. First a brief outline of the historical developments is given. Subsequently, different retrictions for the vehicle scheduling problem are structured and explained. Some models and their mathematical formulation are described, whereby the questions of incorporating restrictions is an important issue. A depiction of solution possibilities offered by program systems for computer-aided scheduling in operational use and expieriences from urban mass transit companies complete the remarks.

1 Introduction

The vehicle scheduling is an important step in the planning process in public transit companies. This counts both for vehicle operations as for manpower planning. From the maximal number of blocks (during a certain time of the operating day) one gets a lower bound on the needed vehicles, consequently this strongly influences the fleet size. But this number of vehicles and the total duration of all blocks determines also in an important range the need in manpower.

In spite of these facts the application of computer technology in public mass transit did not originate with planning vehicle or duty schedules. Initially, during the sixties the use of computer-aided systems was focused only on processing timetable data, especially to produce timetable booklets and wall timetable. Because the timetable data has been available on storage devices, later the idea grew up, to make use of these data files for other planning problems. In this context the *vehicle scheduling problem* (VSP) came to the fore.

The results were at first not satisfying because the cost-to-performance ratio of available computers and also the the efficiency of the existing implementations of algorithms was not sufficient. But step by step, parallel to the extreme development on computer technology and efficient algorithms, computer-aided planning systems got more and more importance in public transit planning. Today there are many public mass transit companies which make

J. R. Daduna et al. (eds.), *Computer-Aided Transit Scheduling*

use of such systems (cf. Wren (1981), Rousseau (1985), Daduna / Wren (1988), Desrochers / Rousseau (1992)).

In the following we describe at first the structure and some charactaristics of a VSP, especially with respect to different situations in operational practice. Several models are presented, which can be solved using available algorithm. Finally, we give a short view on the use of commercial program systems and also on experiences in practice.

2 Problem structure

The basis for the VSP in public mass transit are the trips on service given by a timetable. As a vehicle can operate in a working period more than one trip, in relation to the time and space components of a timetable, this planning problem of assigning vehicles to trips can be presented as a combinatorial problem, with the following conditions (cf. Carraresi / Gallo (1984)):

- To each trip exactly one vehicle will be assigned
- Different technical and in-company restrictions have to be respected
- Accordingly to an objective function given in advance, a minimizing problem has to be solved

During the vehicle scheduling process, which is related only to the vehicles, at first we have the planning of the course of operations. In each case the restrictions are very different, or, as it is often said, different in each mass transit company. A range of basic charactaristics can be found out, which mainly determine the complexity of each planning problem (cf. Bodin / Golden (1981) and Bodin / Golden / Assad / Ball (1984)).

- **Number of depots**

  If it is a single-depot or a multi-depot problem do not result from the number of depots in a public transit company. If the attended transit area is free of intersections, i.e. from the beginning of the operational planning process it is fixed, which lines are served by which depot, the global problem can be splitted into a corresponding number of independent sub-problems. If the freedom of intersection is not given, it is a *multiple-depot vehicle scheduling problem* (MVSP), which consequently shows a higher degree of complexity (cf. Carraresi / Gallo (1984)). Indeed using a cluster first / schedule second proceedure, the freedom of intersection can be established by suitible constraints.

- **Assigning vehicles to a depot**

  The operating vehicles are normally assigned to a certain (origin or home) depot. Therefore each vehicle has to come back to this depot at the end of operation. In some cases interchanging of vehicles between different depots may be allowed.

- **Number of trips**

  A fixed number of trips is given by a timetable which must be carried out. The timetable is based on headways which may change from peak times to off-peak times during the day. In some cases trips may be shortened or departure times may be shifted within an acceptable range. Also some trips can be dropped if that leads to savings in the number of needed vehicles.

- **Assignment of lines / trips**

  Using an assignment of lines to depots in the MVSP (see above), a control of the distribution of the trips on service has to be done in a certain range. The number of the possible assignments shows the intersection-density of the depots. In addition, especially in the case of operating different types of vehicles (see below), the basic assignment of a line may be substituted partially by an explicit assignment of single trips.

- **Multiple types of vehicles and fleet size**

  From fleet size, a differentitation between type-independant and type-dependant constraints has to be done. The type-independent fleet size constraints are presented always when each trip done from a depot, can be operated with any typ of the available vehicles. Due to some in-company reasons different types of vehicles can be used in relation to specific transport demand (technical conditions, differenciation on service, different demand structures, etc.) so that we get type-dependent fleet size constraints.

  In a *single-depot vehicle scheduling problem* (SVSP) a violation of a type-dependant contstraints means that there is no feasible solution in relation to the given number of trips. In a MVSP the possibility exists to move capacities between the depots or to change the existing assignment of lines and / or trips. Indeed even here non-feasible solutions can be created, when alterations cannot be executed in a sufficient range. Type-dependent constraints increase the complexity for this kind of planning problems, even when in particular cases, trips which are assigned to a specific vehicle type can be done by other types in realtion to the need.

- **Manpower capacity**

  Also the question of the manpower availability can already produce restrictive consequences during the vehicle scheduling. Here the knowledge of vehicle types and lines especially have to be mentioned, because in greater traffic areas and / or by different vehicle-types each driver can be not appointed to each type of vehicle. If no feasible solution can be found, a redistribution between the depots has to be done. In addition here we have to consider, that translocations of manpower or vehicle capacities between the depots are often related to in-company problems.

- **In-company restrictions**

  The in-company restrictions result either of technical limitations, or agreements between the mangement and the employee' representatives. Here especially the buffer-times are to be mentioned, which have to be taken into account by operating trips following each other. Furthermore in-company objectives in relation to the structure of the blocks have to be respected, which will be included using weights in the model building for the vehicle scheduling procedure.

For interurban scheduling problems there is a need to keep the length of the blocks within a certain range. In this case the block corresponds to a duty. This is done in order to guarantee that the driver starts and finishes his duty at the same depot.

This description of the reality, in which a vehicle scheduling problem has to be solved, shows the main topics, without claiming completeness. The mathematical approach to be adopted depends on which of those facts have to be explicitly included in the model. In some cases, a simplification on one, or more, of these aspects can be considered with significant impact on the complexity of the resulting mathematical model (cf. Carraresi / Gallo (1984), Daduna / Mojsilovic (1988)).

Based on the given problem, to build up cost-efficient vehicle schedules, two basic objectives can be formulated:

- Minimization of the operating vehicles
- Minimization of the non-productive times (deadhead times, layovers, etc.)

These two objectives are not complementary in any case, so that a determination of a lexicographic order for these objectives is necessary.

As the number of vehicles is determined normally by a short-term peak load, vehicle scheduling mainly has to be for these periods. Even if, for example to a block only one trip is assigned, one vehicle and one driver is needed, independently of the related operating cost. The main objective is consequently the minimization of the operating vehicles, especially during peak hours, because out of this fact better results can be reached in relation to savings in the operating cost.

In some cases, a different objective function may be used. If only a fixed number of vehicles will be available to operate a given timetable, the objective should be to find out a schedule, minimizing the number of trips which are not performed.

3 Vehicle scheduling models

Based on the complex structure of the VSP in practice different models and solution procedures have been developed during the last 20 years (cf. Daduna / Wren (1988), Desrochers / Rousseau (1992)). In this section we give a rough description of three main groups of these models, the *basic vehicle scheduling problem* (BSVP), the *vehicle scheduling problem with a fixed number of vehicles* (p-VSP) and the *multiple-depot vehicle scheduling problem* (MVSP). In addition some special models of the VSP are mentioned which become more and more important from a practical point of view.

3.1 Basic vehicle scheduling problems

Let us recall that the BVSP, consists of assigning the minimal number of vehicles to trips such that all of the trips are operated by vehicles stationed at a single depot and the overall operational costs are minimized. To mathematically describe the BVSP, let $I = \{1, 2, .., n\}$

be the index set for the trips to be operated within a certain planning period. Each trip $i \in I$ is characterized by the following:

- Departure time and starting point, respectively, t_i^1 and p_i^1;
- Arrival time and ending point, respectively, t_i^2 and p_i^2.

Two trips $i,j \in I$ are said compatible if $t_i^2 + d(p_i^2, p_j^1) \leq t_j^1$, where $d(p_i^2, p_j^1)$ stands for the travelling time from the ending point of the trip i to the starting point of the trip j. Sometimes, an additional condition is used for allowing compability between trips. It is the case when the dead-heading trip from p_i^2 to p_j^1 plus the waiting time at p_j^1 is too long, that is, when $t_j^1 - t_i^2$ is greater than or equal to a certain time limit. Now, one may consider the graph $G = (V, A)$, illustratedd in figure 1, with

- A set of vertices $V = I \cup \{n+1\}$, the n trips and the depot;
- A set of arcs linking vertices corresponding to compatible trips and linking trips from and to the depot, that is, $A = \left\{(i,j) : t_i^2 + d(p_i^2, p_j^1) \leq t_j^1\right\} \cup \left\{(n+1,i) : i \in I\right\} \cup \left\{(i,n+1) : i \in I\right\}$.

Note that removing the vertex relative to the depot, the remaining graph is acyclic. Therefore, a circuit in G necessarily passes by the depot, i.e. the vertex index $n+1$, and corresponds to a feasible service for a vehicle.

Now, a cost can be assigned to each arc of G as follows. For $i,j \in I$, c_{ij} is the cost incurred if a vehicle performs the trip j immeditely after trip i; this value refers to the operational cost associated with the dead-heading trip from i to j and, in general, is proportional to distantce between p_j^1 and p_i^2. Also, it may account for some other aspects measuring the similarity between those trips (belonging to the same line, requiring the same type of vehicle, ...). For the arcs linking the depot to and from the trips, the cost can include a fixed value incurred by using a vehicle.

In graph theoretical terms, the BVSP can be described as determining the least cost set of circuits in G that covers each trip. If the fixed cost associated to the using of vehicles is large enough, that corresponds to finding the minimum number of vehicles necessary to operate all of the trips and the corresponding service with minimal operational cost.

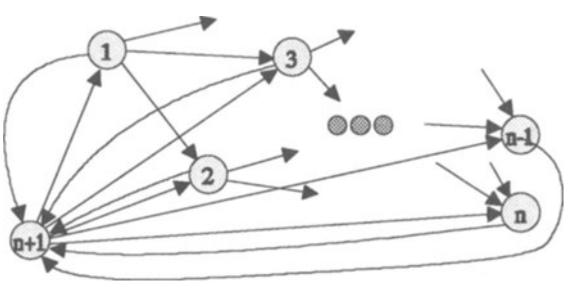

Fig. 1: Compability network for the BVSP

Several mathematical models have been presented for the BVSP corresponding to different underlying graphs. The graph defined above straigthforwardly suggests a minimum cost network flow model. This is defined on an expanded network (illustrated in Figure 2), where each trip-node $i \in I$ of G is duplicated into a node i^1 corresponding to the origin of the trip and another one i^2 refering to the destination of the same trip. A single arc is created from i^1 to i^2, with cost equal to 0, a lower and an upper bound for the quantity flowing that arc equal to 1.

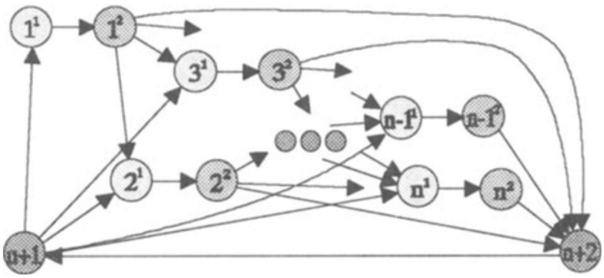

Fig. 2: Network flow model for the BVSP

Also, the depot is splited into two nodes (n+1 and n+2), corresponding to departures and arrivals, with a zero cost arc from n+2 to n+1. The lower bound for flow along that arc is equal to zero while the upper bound is set equal to n. A network flow model for the BVSP was first used by Bodin / Golden / Assad / Ball (1983) and, considering the expanded network, the complexity of the corresponding algorithm is $O((n+1)^3)$.

A different graph theoretical representation is obtained by considering a bipartite graph $G' = (V_1 \cup V_2, A)$, where $V_1 = \{1^1, 2^1, ..., n^1, n+1\}$, $V_2 = \{1^2, 2^2, ..., n^2, n+2\}$ and $A = \{(i^1,j^2) : i$ and j are compatible trips$\} \cup \{(n+1, i^2) : i \in I\} \cup \{(i^1, n+2) : i \in I\}$. The cost associated to each arc (i^1,j^2) is the operational cost for performing the trip j immediately after the trip i. The costs for the arcs linking the depot-nodes to and from the trip-nodes are defined as for the network flow problem. Finally, there is a zero cost arc from n+1 to n+2.

Then, the BVSP can be formulated as transportation problem with supplies and demands equal to 1 for the nodes related to trips, and equal to n for the depot nodes. In figure 3, we give an example of a transportation network associated to a BVSP.

Solving the BVSP as a transportation problem (Gavish / Shilfer (1978)), requires an algorithm of complexity $O((n+1)^3)$, too. Taking into account that this transportation problem is very close to an assignment problem, actually can be seen as a quasi-assignment problem, Paixão / Branco (1987) presented a specialized algorithm with complexity $O(n^3)$.

The BVSP can, also, be solved as a pure assignment problem making use of an algorithm with the complexity O(n 3) (see Orloff (1976) or Mojsilovic (1983)). In this case, the underlying bipartite graph G" = $(V_1' \cup V_2', A_1 \cup A_2)$ has vertex subsets $V_1' = \{1^1, 2^1, ..., n^1\}$ and

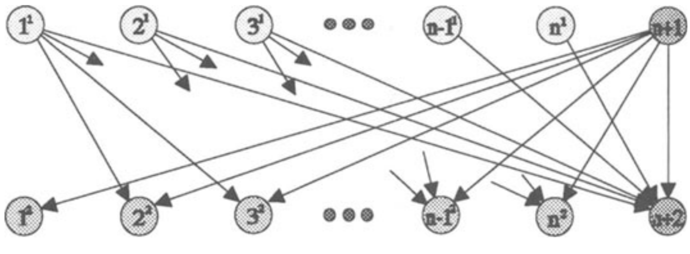

Fig. 3: Transportation model network for the BVSP

$V_2' = \{1^2, 2^2, ..., n^2\}$, with two types of arcs, $A_1 = \{(i^1 j^2) : i$ and j are compatible trips$\}$ and $A_2 = (I \times I) \backslash A_1$. The costs for the arcs in A_1 are defined as above while the cost for an arc $(i^1 j^2)$, with i and j not compatible, is given by $c_{i,n+1}\ c_{n+1,j}$ added by the fixed cost incurred by using a vehicle. The figure 4 illustrates the resulting complete bipartite graph with the arcs of A_2 represented by dotted arrows.

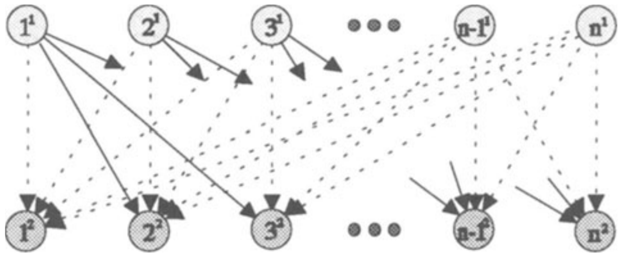

Fig. 4: Assignment model network for the BVSP

3.2 Vehicle scheduling problems with a fixed number of vehicles

In some practical situations, one aims to schedule a fixed number of vehicles, say p. That occurs namely in three different cases:

(i) When the BVSP is handled in a two-phase proccess. Firstly, one determines $p*$, the minimum number of vehicles needed to operate all of the trips. Secondly, the number of vehicles to operate is fixed equal to $p*$ and one aims to minimize the overall operational costs;

(ii) When a fleet larger than $p*$ is available and one wants to use all of them;

(iii) When the fleet size may be not large enough to operate all planned trips and the possibility of some trips being not performed is considered.

In the cases (i) and (ii), the p-VSP, can be dealt with by the models described above except the last one (pure assignment model). For the most general case, (iii), where one allows that some trips may be not performed, only the quasi-assignment model can be used. When one deals with the p-VSP, either in case (i) or (ii), the network flow model can be easily applied by setting equal to p the lower and upper bound for the flow through the arc $(n+2,n+1)$.

Also, the transportation / quasi-assignment model can be used by removing the arc from n^1 to n^2 in the corresponding network and fixing the supply and demand for the depot nodes equal to p.

As mentioned above, the assignment model described for BSVP can not deal with a fixed number of vehicles situation. However, a different assignment model may be used by replicating the depot nodes as many times as the number of vehicles. That is, as illustrated in figure 5, nodes $n+1^k, n+2^k, ..., n+p^k$ (with $k = 1,2$), are created with arcs $(n+l^1,i^2)$ and $(i^1,n+l^2)$ for $l = 1, 2, ..., p$ and $i \in I$. The problem is equivalent to determining the minimum cost matching in the expanded graph.

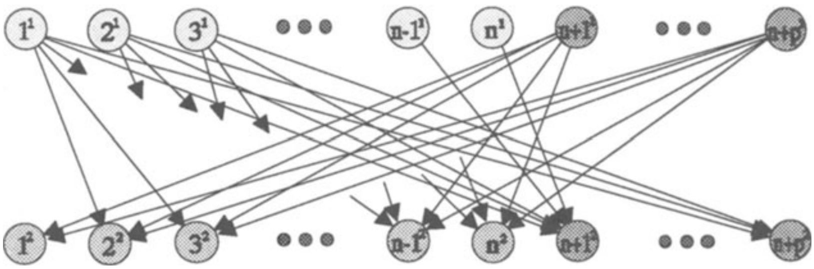

Fig. 5: Matching model network for the p-VSP

Note that for certain values of p the resulting problem may be not feasible since the number of available vehicles in these cases may be not enough for operating all planned trips. Hence, one may consider penalties associated to trips not assigned to a vehicle. That means, for the quasi-assignment model, that one use a penalty-cost for each one of the arcs (i^1,i^2), $i = 1,2, ..., n$, added to the network presented in the figure 3 while the modifications described for the cases (i) and (ii) are also made (see fig. 6).

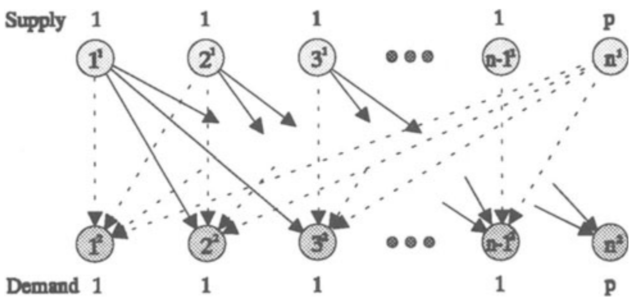

Fig. 6: Quasi-assignment model network for the p-VSP

This can not be dealt with by the other models except the matching model where, likewise for the quasi-assignment model, additional arcs linking the nodes relative to the same trip may be considered with a cost equal to penalty paid for not performing the corresponding trip.

Table 1 summarize what was said about the models and problems mentioned in this section and in the previous one.

Model	Author(s)	Complexity	BSVP	p-VSP	p-VSP / constraints
Assignment model	Orloff (1976) and Mojsilovic (1983)	$O(n^3)$	✔		
Transportation model	Gavish / Schweitzer / Shilfer (1978) and Gavish / Shilfer (1978)	$O((n+1)^3)$	✔	✔	
Network flow model	Carraresi / Gallo (1984) and Luedtke (1985)	$O((n+1)^3)$	✔	✔	
Quasi-assignment model	Paixão / Branco (1987) and (1988)	$O(n^3)$	✔	✔	✔
Matching model	Bertossi / Carraresi / Gallo (1987)	$O((n+p)^3)$	✔	✔	✔

Tab. 1: Basic vehicle scheduling problems

3.3 Multiple-depot vehicle scheduling problems

When in the presence of more than one depot one has the MVSP where the trips must be performed by vehicles stationed at D different depots with quantities m_k for $k = 1, 2, ..., D$. One looks for the minimum number of vehicles needed to operate a sequence of pairwise compatible trips returning to its origin depot, in such way that the overall operational costs are minimized too.

The models described for the single depot case can be somehow extended in order to cope with the multiple depot situation . However, due to the fact that the MVSP is NP-hard when $D \geq 2$ (Bertossi / Carraresi / Gallo (1987)), no polynomial time optimal algorithms are known for this problem. Hence, several heuristics have been proposed for the MVSP (see Bodin / Rosenfield / Kydes (1978), Ceder / Stern (1981), Smith / Wren (1981), El-Azm (1985), Bertossi / Carraresi / Gallo (1987), Lamatsch (1992), Mesquita / Paixão (1992)). Another solution approach (Daduna / Mojsilovic (1988) is based on a model with a specific structure of the arc costs to solve the MVSP using an assignment algorithm.

Branch-and-bound algorithms have been presented by Carpaneto / Dell'Amico / Fischetti / Toth (1989) and Ribeiro / Soumis (1991), in order to solve the MVSP to optimality. The first is based on the computation of lower bounds by an additive procedure while the second uses column generation for dealing with linear relaxation of an integer multi-commodity flow formulation.

The multi-commodity flow model for the MVSP is a straightforward extension of the network flow model for the single depot case, where one considers as many replica of the network as the number of depots. That is, for each depot k a $G^k = (V_k, A_k)$ with

$$V_k = \{\, i^1_k,\ i^2_k \text{ for } i = 1, 2, \ldots, n \,\} \cup \{\, (n+1)_k,\ (n+2)_k \,\} \text{ and}$$
$$A = \{\, (i^2_k, j^1_k) : t^2_i + d(p^2_i, p^1_j) \le t^1_j \,\} \cup \{\, (i^1_k, i^2_k) : i \in I \,\} \cup \{\, ((n+1)_k, j^1_k) : i \in I \,\} \cup$$
$$\{\, (i^2_k, (n+2)_k) : i \in I \,\} \cup \{\, ((n+2)_k, (n+1)_k) \,\}$$

The number of vehicles m_k available at the k-th depot is the upper bound on the quantity flowing in the arc $((n+2)_k, (n+1)_k)$ with cost $c^k_{n+2,n+1} = 0$. For the arcs linking the depot nodes to and from trip nodes, the costs may differ accordingly to the depot. The figure 7 shows an example for two depots where one sub-network is represented by full lines and the other by dotted lines. The shadow circles identify the arcs that are envolved by the constraints expliciting that, for each $i = 1, 2, \ldots, n$, only one arc (i^1_k, i^2_k) is allowed to have a positive flow. Dropping those constraints, the problem reduces to the single depot case.

Finally, note that if the vehicle are allowed to return to a depot different from its origin depot, the the problem can besolved as a single depot instance.

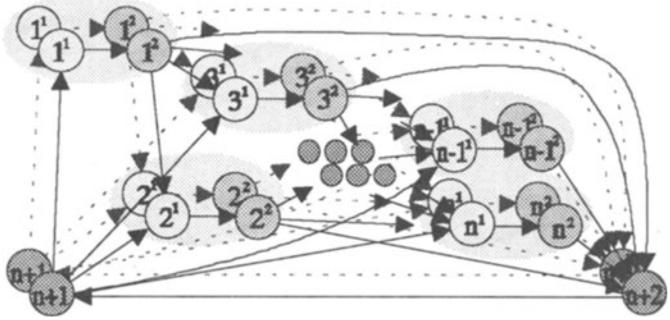

Fig. 7: Example of the multi-commodity flow model for the MVSP

3.4 Modified vehicle scheduling problems

For some special problem structures a modified VSP may be solved. Examples are outlined which based on operational restrictions. The modifications we take into consideration are fuzzy data structures (trip shifting), additional constraints, and extensions of the VSP.

- **Vehicle scheduling problems with trip shifting**

  Solving a VSP the trips are normally defined by a fixed timetable. In some cases trips may be shifted within a given range which leads to a modified VSP, the *vehicle scheduling problem with trip shifting* (VSP-TS) (cf. Ceder / Stern (1985), Daduna (1988), Hamer / Séguin (1992), Daduna / Mojsilovic / Schütze (1993), Daduna / Völker (1995)). In such problems two different planning steps, timetable constructing and vehicle scheduling, are connected. The objective is to reduce the number of blocks, especially during peak hours, using the increased degree of freedom in the combinatorial process.

- **Vehicle scheduling problems with multiple types of vehicles**

  The trips normally must not be served by an unique type of vehicle, because the demand on each line or trip may be different. Therefore from an economic point of view the offered capacity should correspond to the expected demand. Starting out from this situation a *vehicle scheduling model with multiple types of vehicles* (VSP-MT) is formulated which includes the different cost of vehicles (cf. Ceder (1993), Costa / Branco / Paixão (1993)). The objective is to determine a solution minimizing the operational cost and assigning to each trip a vehicle with an adequate capacity.

  A similar approach is used for example in the HOT system (cf. Daduna / Mojsilovic (1988)). In this program system different types of vehicles are assigned to lines or trips. These assignments are not included in the mathematical formulation as an additional constraint but they are implicitly taken into consideration in constructing the model respectively the cost matrix, which represents the basis for the combinatorial process.

- **Vehicle scheduling problems with maximum time constraint**

  The *vehicle scheduling problem with maximum time constraint* (VSP-TC) describes a VSP with an additional constraint (cf. Freling / Paixão (1993)). In this case the constraint may arise for example from technical restrictions (fuel capacity, etc.) or legal and in-company restrictions.

- **Vehicle and crew scheduling problems**

  In a *vehicle and crew scheduling problem* (VCSP) two different steps of the planning process are solved simultanously (cf. Ball / Bodin / Dial (1983), Tosini / Vercellis (1988), Patrikalakis / Xerocostas (1992)). The VCSP mainly appears in interurban transit where normally a block is corresponding with exact one duty. In some aspects this problem is similar to the VSP-TC, if the time constraint for a block results from the maximum length of a duty.

For future research especially the VSP-TS and the VSP-MT are of most interest, because in practice these problems become more and more important from the view of economic planning of operations.

4 Practical experiences

Introducing computer-aided planning systems in practice the different possible applications of vehicle scheduling-methods are always a constant point of controversy. On the one hand they provoke total dismissal, partially on the grounds that users of computer-aided planning systems are against automatically generated blocks. On the other hand they extremely widen the scope of expectation, which is, after all, based on manual planning procedures being transferred to automated operations and in which the human element only plays an indirect role. Unbiased discussions have proved that neither of these arguments is tenable.

The most important point is to understand that the planning process in vehicle scheduling can't be solved efficiently without human capacities. It is obviously impossible, not only today but also in the future, to generate in each case automatically blocks without any alterations. In practice on the one hand normally not all specific (technical) restrictions can't be taken into account formulating the model. On the other hand there are independencies which result from the actual situation of the combinatorial process.

Therefore, the vehicle scheduling process in general must be divided into three main steps which are carried out one after the other: a preparations phase, an automated scheduling step and interactive alterations.

- **Preparations phase**

  At first the *actual* vehicle scheduling problem must be defined by the user in detail (*day type*, constructing *giant trips* as a fixed combination of trips, etc.). The result of this step is a defined data basis for the model building procedure.

- **Automated scheduling process**

  Automated scheduling firstly involves setting up a (mathematical) model within the given in-company (and legal) restrictions. Making use of specific OR-methods (see section 3), the blocks are constructed (exactly solved based on optimization algorithms or with pleased results based on efficient heuristic). The result of this automated process must be a complete vehicle schedule, in which all existing restrictions are taken into account. This schedule is primarily a working basis for the user, whose job it is mainly to check in detail when and at what different points it is possible to make manual improvements within the existent restrictions.

- **Interactive alterations**

  Within a planning tool both, graphic and alpha-numerical functions, should be available for schedule alterations based on a man-machine-approach. The application of these functions must be independently from or combined with each other. It is of great importance for the acceptance of such planning tools that the user has a choice here, which enables him to adapt the working method to his own individual concept.

Efficient solution procedures for the VSP which are in practical use for many years are offered especially within the *HOT system* (cf. Mojsilovic (1983), Daduna / Mojsilovic (1988), Daduna / Mojsilovic / Schütze (1993), Völker / Schütze (1993), Daduna / Völker (1995), the *HASTUS system* (cf. Blais / Rousseau (1988), Blais / Lamont / Rousseau (1990), Hamer / Séguin (1992)), and also the *BUSMAN system* (cf. Wren / Chamberlain (1988), Chamberlain / Wren (1992)). The results show distinct savings for the number of blocks (during peak hours) in comparision to manual planning.

These quantitative improvements which are undoubtedly attained making use of such systems cannot on the whole be reconstructed explicitly. The reason for this situation is on the one hand the complexity, especially based on different timetable structures and in-company restrictions which are underlying the VSP in practice. On the other hand the working conditions in mass transit companies make no allowance for parallel processing to get exact results to draw a comparision between manual and computer-aided planning. But besides these quantitative aspects we must take into consideration also qualitative improvements (reduction in the (planning) time to solve a VSP, possibility for simulation of different data structures, etc.). All in all, there are considerable benefits from applying such systems, whether for arranging planning procedures or in respect of attained results.

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