NONPARAMETRIC AND SEMIPARAMETRIC METHODS IN R

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Abstract. The R environment for statistical computing and graphics (R Development Core Team (2008)) offers practitioners a rich set of statistical methods ranging from random number generation and optimization methods through regression, panel data, and time series methods, by way of illustration. The standard R distribution ('base R) comes preloaded with a rich variety of functionality useful for applied econometricians. This functionality is enhanced by user supplied packages made available via R servers that are mirrored around the world. Of interest in this chapter are methods for estimating nonparametric and semiparametric models. We summarize many of the facilities in R and consider some tools that might be of interest to those wishing to work with nonparametric methods who want to avoid resorting to programming in C or Fortran but need the speed of compiled code as opposed to interpreted code such as Gauss or Matlab by way of example. We encourage those working in the field to strongly consider implementing their methods in the R environment thereby making their work accessible to the widest possible audience via an open collaborative forum.

1. Introduction

Unlike their more established parametric counterparts, many nonparametric and semiparametric methods that have received widespread theoretical treatment have not yet found their way into mainstream commercial packages. This has hindered their adoption by applied researchers, and it is safe to describe the availability of modern nonparametric methods as fragmented at best, which can be frustrating for users who wish to assess whether or not such methods can add value to their application. Thus, one frequently heard complaint about the state of nonparametric kernel methods concerns the lack of software along with the fact that implementations in interpreted environments such as Gauss are orders of magnitude slower than compiled implementations written in C or Fortran. Though many researchers may code their methods, often using interpreted environments such as Gauss, it is fair to characterize much of this code as neither designed nor suited as tools for general purpose use as they are typically written solely to demonstrate 'proof of concept'. Even though many authors are more than happy to circulate such code (which is of course appreciated!), this often imposes certain hardships on the user including 1) having to purchase a (closed and proprietary) commercial software package and 2) having to modify the code substantially in order to use it for their application.

The R environment for statistical computing and graphics (R Development Core Team (2008)) offers practitioners a range of tools for estimating nonparametric, semiparametric, and of course parametric models. Unlike many commercial programs, which must first be purchased in order to evaluate them, you can adopt R with minimal effort and with no financial outlay required. Many

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nonparametric methods are well documented, tested, and are suitable for general use via a common interface structure (such as the 'formula' interface) making it easy for users familiar with R to deploy these tools for their particular application. Furthermore, one of the strengths of R is the ability to call compiled C or Fortran code via a common interface structure thereby delivering the speed of complied code in a flexible easy to use environment. In addition, there exist a number of R 'packages' (often called 'libraries' or 'modules' in other environments) that implement a variety of kernel methods, albeit with varying degrees of functionality (e.g., univariate versus multivariate, the ability/inability to handle numerical and categorical data and so forth). Finally, R delivers a rich framework for implementing and making code available to the community.

In this chapter we outline many of the functions and packages available in R that might be of interest to practitioners, and consider some illustrative applications along with code fragments that might be of interest. Before proceeding further, we first begin with an introduction to the R environment itself.

2. The R Environment

What is R? Perhaps it is best to begin with the question "what is S"? S is a language and environment designed for statical computing and graphics which was developed at Bell Laboratories (formerly AT&T, now Lucent Technologies). S has grown to become the de-facto standard among econometricians and statisticians, and there are two main implementations, the commercial implementation called 'S-PLUS', and the free, open-source implementation called 'R'. R delivers a rich array of statistical methods, and one of its strengths is the ease with which 'packages' can be developed and made available to users for free. R is a mature open platform that is ideally suited to the task of making ones method available to the widest possible user base free of charge.

In this section we briefly describe a handful of resources available to those interested in using R, introduce the user to the R environment, and introduce the user to the `foreign` package that facilitates importation of data from packages such as SAS, SPSS, Stata, and Minitab, among others.

2.1. **Web sites.** A number of sites are devoted to helping R users, and we briefly mention a few of them below.

http://www.R-project.org/: This is the R home page from which you can download the program itself and many R packages. There are also manuals, other links, and facilities for joining various R mailing lists.

http://CRAN.R-project.org/: This is the 'Comprehensive R Archive Network,' "a network of ftp and web servers around the world that store identical, up-to-date, versions of code and documentation for the R statistical package." Packages are only put on CRAN when they pass a rather stringent collection of quality assurance checks, and in particular are guaranteed to build and run on standard platforms.

http://cran.r-project.org/web/views/Econometrics.html: This is the CRAN 'task view' for computational econometrics. "Base R ships with a lot of functionality useful for computational econometrics, in particular in the stats package. This functionality is complemented by many packages on CRAN, a brief overview is given below." This provides an excellent summary of both parametric and nonparametric packages that exist for the `R` environment.

http://pj.freefaculty.org/R/Rtips.html: This site provides a large and excellent collection of `R` tips.

2.2. **Getting started with `R`.** A number of well written manuals exist for `R` and can be located at the `R` web site. This section is clearly not intended to be a substitute for these resources. It simply provides a minimal set of commands which will aid those who have never used `R` before.

Having installed and run `R`, you will find yourself at the > prompt. To quit the program, simply type `q()`. To get help, you can either enter a command preceded by a question mark, as in `?help`, or type `help.start()` at the > prompt. The latter will spawn your web browser (it reads files from your hard drive, so you do not have to be connected to the Internet to use this feature).

You can enter commands interactively at the `R` prompt, or you can create a text file containing the commands and execute all commands in the file from the `R` prompt by typing `source ("commands.R")`, where `commands.R` is the text file containing your commands. Many editors recognize the `.R` extension providing useful interface for the development of `R` code. For example, GNU Emacs is a powerful editor that works well with `R` and also LaTeX (http://www.gnu.org/software/emacs/emacs.html).

When you quit by entering the `q()` command, you will be asked whether or not you wish to save the current session. If you enter `Y`, then the next time you run `R` *in the same directory* it will load all of the objects created in the previous session. If you do so, typing the command `ls()` will list all of the objects. For this reason, it is wise to use different directories for different projects. To remove objects that have been loaded, you can use the command `rm(`*objectname*`)` or `rm(list=ls())` will remove all objects in memory.

2.3. **Importing data from other formats.** The `foreign` package allows you to read data created by different popular programs. To load it, simply type `library(foreign)` from within `R`. Supported formats include

read.arff: Read Data from ARFF Files
read.dbf: Read a DBF File
read.dta: Read Stata Binary Files
read.epiinfo: Read Epi Info Data Files
read.mtp: Read a Minitab Portable Worksheet
read.octave: Read Octave Text Data Files
read.S: Read an S3 Binary or data.dump File
read.spss: Read an SPSS Data File
read.ssd: Obtain a Data Frame from a SAS Permanent Dataset, via read.xport
read.systat: Obtain a Data Frame from a Systat File

read.xport: Read a SAS XPORT Format Library

The following code snippet reads the Stata file 'wage1.dta' (Wooldridge (2002)) and lists the names of variables in the data frame.

```
R> library(foreign)
R> mydat <- read.dta(file="wage1.dta")
R> names(mydat)
 [1] "wage"     "educ"     "exper"    "tenure"   "nonwhite" "female"
 [7] "married"  "numdep"   "smsa"     "northcen" "south"    "west"
[13] "construc" "ndurman"  "trcommpu" "trade"    "services" "profserv"
[19] "profocc"  "clerocc"  "servocc"  "lwage"    "expersq"  "tenursq"
```

Clearly `R` makes it simple to migrate data from one environment to another.

Having installed `R` and having read in data from a text file or supported format such as a Stata binary file, you can then install packages via the `install.packages()` command, as in `install.packages("np")` which will install the `np` package (Hayfield & Racine (2008)).

3. Some Nonparametric and Semiparametric Routines Available in R

Table 1 summarizes some of the nonparametric and semiparametric routines available to users of `R`. As can be seen, there appears to be a rich range of nonparametric implementations available to the practitioner. However, upon closer inspection many are limited in one way or another in ways that might frustrate applied econometricians. For instance, some nonparametric regression methods admit only one regressor, while others admit only numerical data types and cannot admit categorical data that is often found in applied settings. Table 1 is not intended to be exhaustive, rather, it ought to serve to orient the reader to a subset of the rich array of nonparametric methods that currently exist in the `R` environment. To see a routine in action, you can type `example("funcname",package="pkgname")` where `funcname` is the name of a routine and `pkgname` is the associated package and this will run an example contained in the help file for that function. For instance, `example("npreg",package="np")` will run a kernel regression example from the package `np`.

Table 1. An illustrative summary of R packages that implement nonparametric methods.

Package	Function	Description
`ash`	*ash1*	Computes univariate averaged shifted histograms
	ash2	Computes bivariate averaged shifted histograms
`car`	*n.bins*	Computes number of bins for histograms with different rules
`gam`	*gam*	Computes generalized additive models using the method described in Hastie & Tibshirani (1990)
`GenKern`	*KernSec*	Computes univariate kernel density estimates
	KernSur	Computes bivariate kernel density estimates
`Graphics`	*boxplot*	Produces box-and-whisker plot(s)
(base)	*nclass.Sturges*	Computes the number of classes for a histogram
	nclass.scott	Computes the number of classes for a histogram
	nclass.FD	Computes the number of classes for a histogram
`KernSmooth`	*bkde*	Computes a univariate binned kernel density estimate using the fast Fourier transform as described in Silverman (1982)
	bkde2D	Compute a bivariate binned kernel density estimate as described in Wand (1994)
	dpik	Computes a bandwidth for a univariate kernel density estimate using the method described in Sheather & Jones (1991)
	dpill	Computes a bandwidth for univariate local linear regression using the method described in Ruppert, Sheather & Wand (1995)
	locpoly	Computes a univariate probability density function, bivariate regression function or their derivatives using local polynomials
`ks`	*kde*	Computes a multivariate kernel density estimate for 1- to 6-dimensional numerical data
`locfit`	*locfit*	Computes univariate local regression and likelihood models
	sjpi	Computes a bandwidth via the plug-in Sheather & Jones (1991) method
	kdeb	Computes univariate kernel density estimate bandwidths
`MASS`	*bandwidth.nrd*	Computes Silverman's rule-of-thumb for choosing the bandwidth of a univariate Gaussian kernel density estimator
	hist.scott	Plot a histogram with automatic bin width selection (Scott)
	hist.FD	Plot a histogram with automatic bin width selection (Freedman-Diaconis)
	kde2d	Computes a bivariate kernel density estimate
	width.SJ	Computes the Sheather & Jones (1991) bandwidth for a univariate Gaussian kernel density estimator
	bcv	Computes biased cross-validation bandwidth selection for a univariate Gaussian kernel density estimator
	ucv	Computes unbiased cross-validation bandwidth selection for of a univariate Gaussian kernel density estimator
`np`	*npcdens*	Computes a multivariate conditional density as described in Hall, Racine & Li (2004)
	npcdist	Computes a multivariate conditional distribution as described in Li & Racine (forthcoming)
	npcmstest	Conducts a parametric model specification test as described in Hsiao, Li & Racine (2007)
	npconmode	Conducts multivariate modal regression
	npindex	computes a multivariate single index model as described in Ichimura (1993), Klein & Spady (1993)
	npksum	Computes multivariate kernel sums with numeric and categorical data types
	npplot	Conducts general purpose plotting of nonparametric objects
	npplreg	computes a multivariate partially linear model as described in Robinson (1988), Racine & Liu (2007)
	npqcmstest	Conducts a parametric quantile regression model specification test as described in Zheng (1998), Racine (2006)
	npqreg	Computes multivariate quantile regression as described in Li & Racine (forthcoming)
	npreg	Computes multivariate regression as described in Racine & Li (2004), Li & Racine (2004)
	npscoef	Computes multivariate smooth coefficient models as described in Li & Racine (2007*b*)
	npsigtest	Computes the significance test as described in Racine (1997), Racine, Hart & Li (2006)
	npudens	Computes multivariate density estimation as described in Parzen (1962), Rosenblatt (1956), Li & Racine (2003)
	npudist	Computes multivariate distribution functions as described in Parzen (1962), Rosenblatt (1956), Li & Racine (2003)
`stats`	*bw.nrd*	Univariate bandwidth selectors for gaussian windows in *density*
	density	Computes a univariate kernel density estimate
(base)	*hist*	Computes a univariate histogram
	smooth.spline	Computes a univariate cubic smoothing spline as described in Chambers & Hastie (1991)
	ksmooth	Computes a univariate Nadaraya-Watson kernel regression estimate described in Wand & Jones (1995)
	loess	Computes a smooth curve fitted by the loess method described in Cleveland, Grosse & Shyu (1992) (1-4 numeric predictors)

3.1. **Nonparametric Density Estimation in R.** Univariate density estimation is one of the most popular exploratory nonparametric methods in use today. Readers will no doubt be intimately familiar with two popular nonparametric estimators, namely the univariate histogram and kernel estimators. For an in-depth treatment of kernel density estimation we direct the interested reader to the wonderful monographs by Silverman (1986) and Scott (1992), while for mixed data density estimation we direct the reader to Li & Racine (2003) and the references therein. We shall begin with an illustrative *parametric* example.

Consider any random variable X having probability density function $f(x)$, and let $f(\cdot)$ be the object of interest. Suppose one is presented with a series of independent and identically distributed draws from the unknown distribution and asked to model the density of the data, $f(x)$.

For this example we shall simulate $n = 500$ draws but immediately discard knowledge of the true data generating process (DGP) pretending that we are unaware that the data is drawn from a mixture of normals ($N(-2, 0.25)$ and $N(3, 2.25)$ with equal probability). The following code snippet demonstrates one way to draw random samples from a mixture of normals.

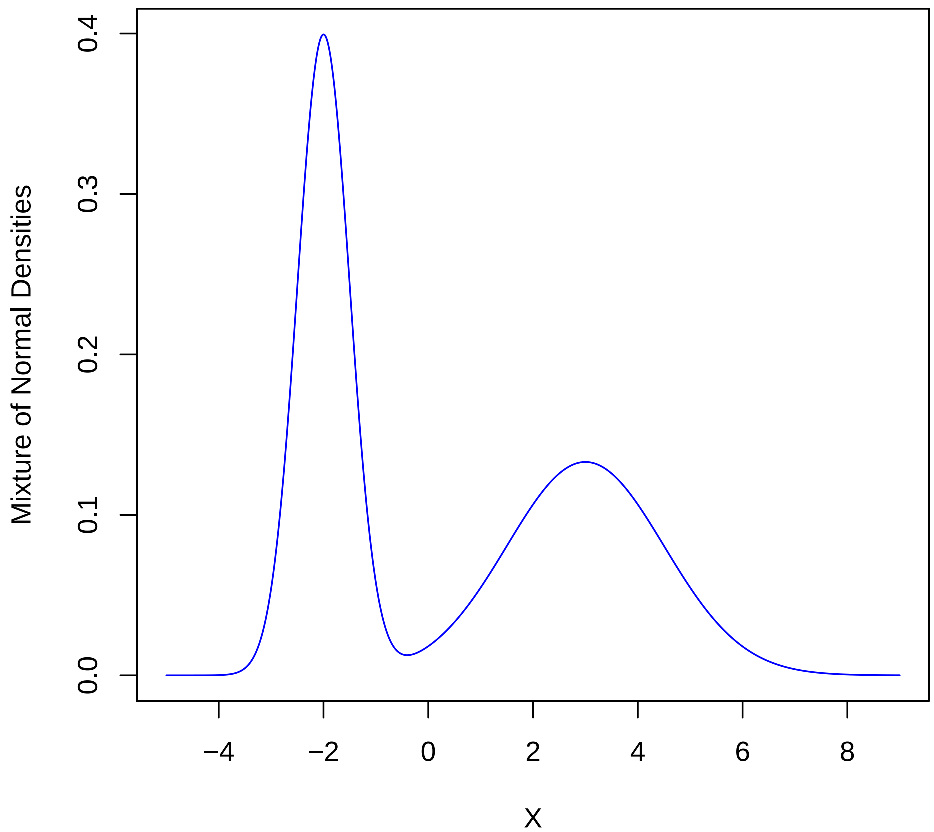
```
R> library(np)
```

```
Nonparametric Kernel Methods for Mixed Datatypes (version 0.20-3)
```

```
R> set.seed(123)
R> n <- 250
R> x <- sort(c(rnorm(n,mean=-2,sd=0.5),rnorm(n,mean=3,sd=1.5)))
```

The following figure plots the true DGP evaluated on an equally spaced grid of $1,000$ points.

```
R> x.seq <- seq(-5,9,length=1000)
R> plot(x.seq,0.5*dnorm(x.seq,mean=-2,sd=0.5)+0.5*dnorm(x.seq,mean=3,sd=1.5),
+       xlab="X",
+       ylab="Mixture of Normal Densities",
+       type="l",
+       main="",
+       col="blue",
+       lty=1)
```



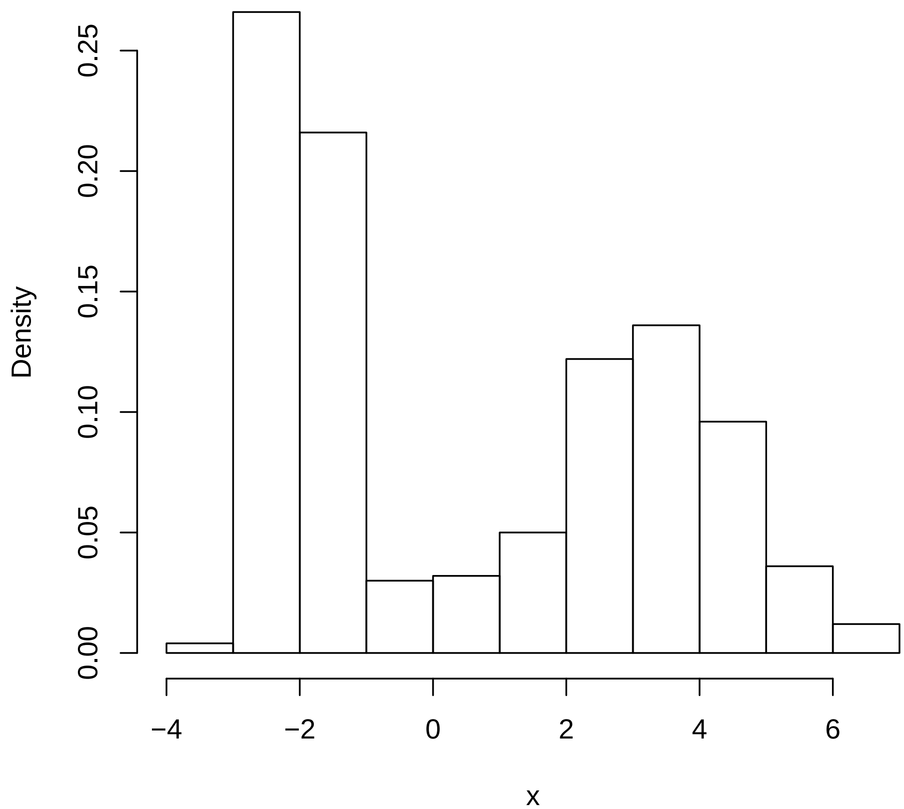
Suppose one naïvely presumed that the data is drawn from, say, the normal parametric family (not a mixture thereof), then tested this assumption using the Shapiro-Wilks test. The following code snipped demonstrates how this is done in R.

```
R> shapiro.test(x)

        Shapiro-Wilk normality test


data:  x
W = 0.87, p-value < 2.2e-16
```

Given that this popular parametric model is flatly rejected by this dataset, we have two choices, namely 1) search for a more appropriate parametric model or 2) use more flexible estimators. For what follows, we shall presume that the reader has found themselves in just such a situation. That is, they have faithfully applied a parametric method and conducted a series of tests of model adequacy that indicate that the parametric model is not consistent with the underlying DGP. They then turn to more flexible methods of density estimation. Note that though we are considering density estimation at the moment, it could be virtually any parametric approach that we have been discussing, for instance, regression analysis and so forth.

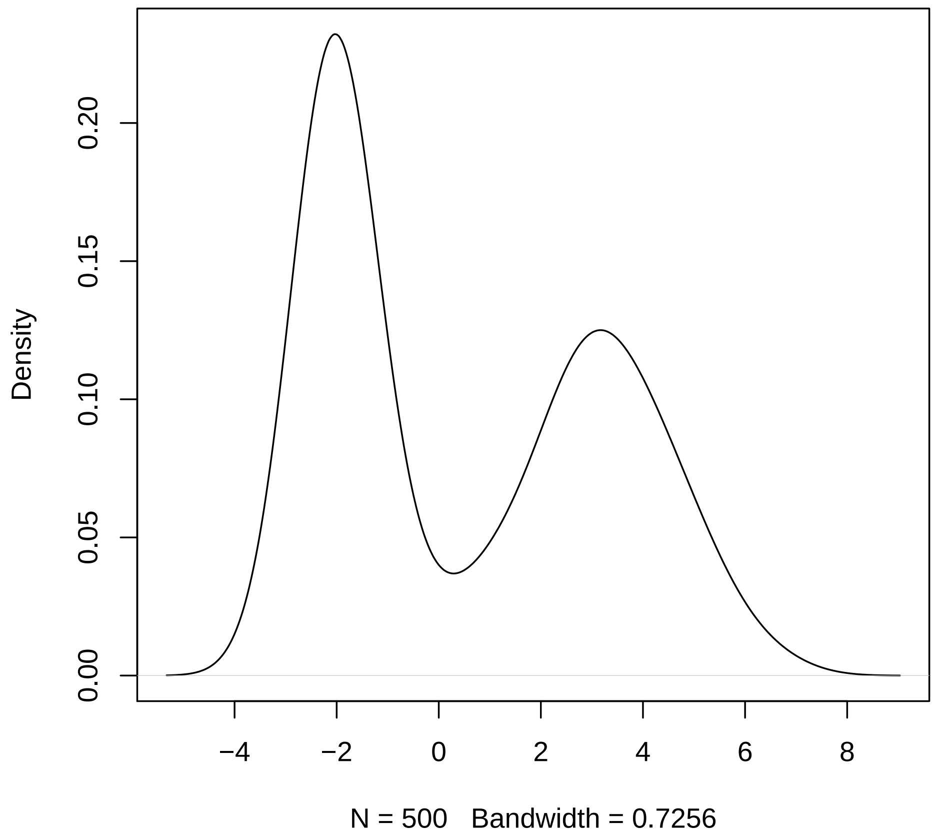
If one wished to examine a histogram one could use the following code snippet,

```
R> hist(x,prob=TRUE,main="")
```



Of course, though consistent, the histogram suffers from a number of drawbacks hence one might instead consider a smooth nonparametric density estimator such as the univariate Parzen kernel estimator (Parzen (1962)). A univariate kernel estimator can be obtained using the `density` command that is part of `R` base. This function supports a range of bandwidth methods (see `?bw.nrd` for details) and kernels (see `?density` for details). The default bandwidth method is Silverman's 'rule of thumb' (Silverman (1986, page 48, eqn (3.31))), and for this data we obtain the following:

```
R> plot(density(x),main="")
```

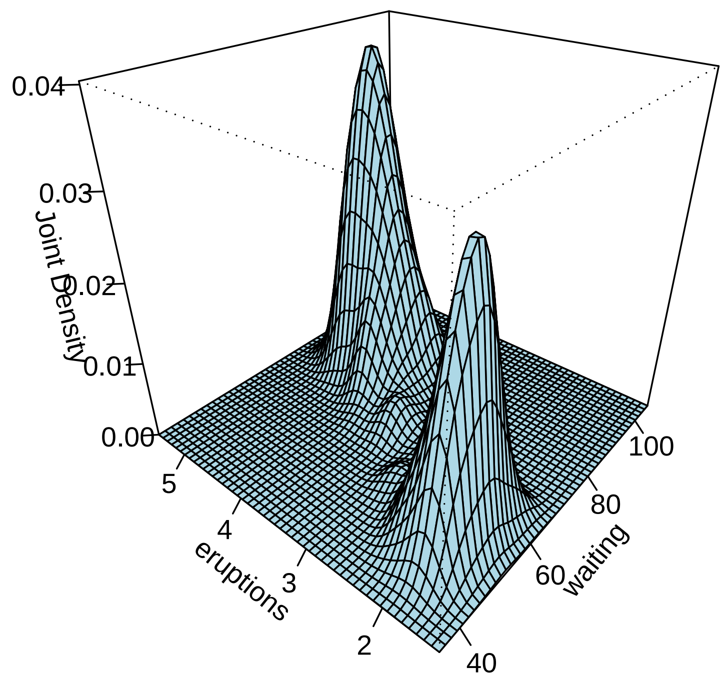


The `density` function in `R` has a number of virtues. It is extremely fast computationally speaking as the algorithm disperses the mass of the empirical distribution function over a regular grid and then uses the fast Fourier transform to convolve this approximation with a discretized version of the kernel and then uses a linear approximation to evaluate the density at the specified points. If one wishes to obtain a univariate kernel estimate for a large sample of data then this is definitely the function of choice. However, for a bivariate (or higher dimensional) density estimate one would require alternative `R` routines. The function `bkd2dD` in the `KernSmooth` package can compute a two-dimensional density estimate as can `kde2d` in the **MASS** package and **kde** in the `ks` package though neither package implements a data-driven two-dimensional bandwidth selector. The `np` package, however, contains the function `npudens` that computes multivariate density estimates, is quite flexible, and admits data-driven bandwidth selection for an arbitrary number of dimensions and for both numeric and categorical data types. As the method does not rely on Fourier transforms and approximations it is nowhere near as fast as the `density` function[1], however, it is much more

[1]To be specific, bandwidth selection is nowhere near as fast though computing the density itself is comparable once the bandwidth is supplied.

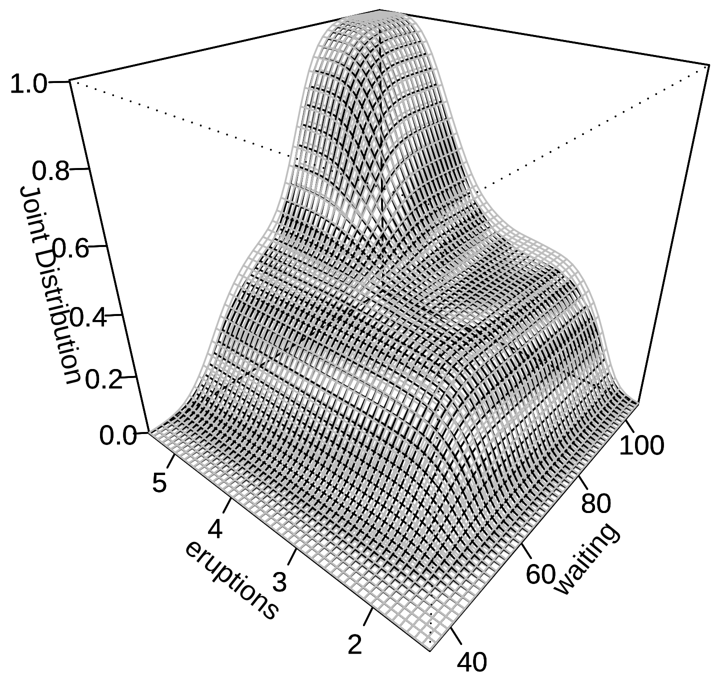
flexible. The default method of bandwidth selection is likelihood cross-validation, and the following code snippet demonstrates this function using the “Old Faithful” dataset. The Old Faithful Geyser is a tourist attraction located in Yellowstone National Park. This famous dataset containing $n = 272$ observations consists of two variables, eruption duration (minutes) and waiting time until the next eruption (minutes).

```
R> data("faithful",package="datasets")
R> fhat <- npudens(~waiting+eruptions,data=faithful)
R> plot(fhat,view="fixed",xtrim=-0.1,theta=310,phi=30,main="")
```



For dimensions greater than two, one can plot “partial density surfaces” that plot one-dimensional slices of the density holding variables not on the axes constant at their median/modes (these can be changed by the user - see `?npplot` for details). One can also plot asymptotic and bootstrapped error surfaces, the CDF and so forth as the following code snippet reveals.

```
R> plot(fhat,cdf=TRUE,plot.errors.method="asymptotic",
+       view="fixed",xtrim=-0.1,theta=310,phi=30,main="")
```



3.2. **Kernel Density Estimation with Numeric and Categorical Data.** Suppose that we were facing a mix of categorical and numeric data and wanted to model the joint density[2] function. When facing a mix of categorical and numeric data, traditionally researchers using kernel methods resorted to a 'frequency' approach. This approach involves breaking the numeric data into subsets according to the realizations of the categorical data ('cells'). This of course will produce consistent estimates. However, as the number of subsets increases, the amount of data in each cell falls leading to a 'sparse data' problem. In such cases, there may be insufficient data in each subset to deliver sensible density estimates (the estimates will be highly variable). In what follows we consider the method of Li & Racine (2003) that is implemented in the `np` package via the `npudens` function.

By way of example we consider Wooldridge's (2002) 'wage1' dataset ($n = 526$), and model the joint density of two variables, one numeric ('lwage') and one categorical ('numdep'). 'lwage' is the logarithm of average hourly earnings for an individual. 'numdep' the number of dependents

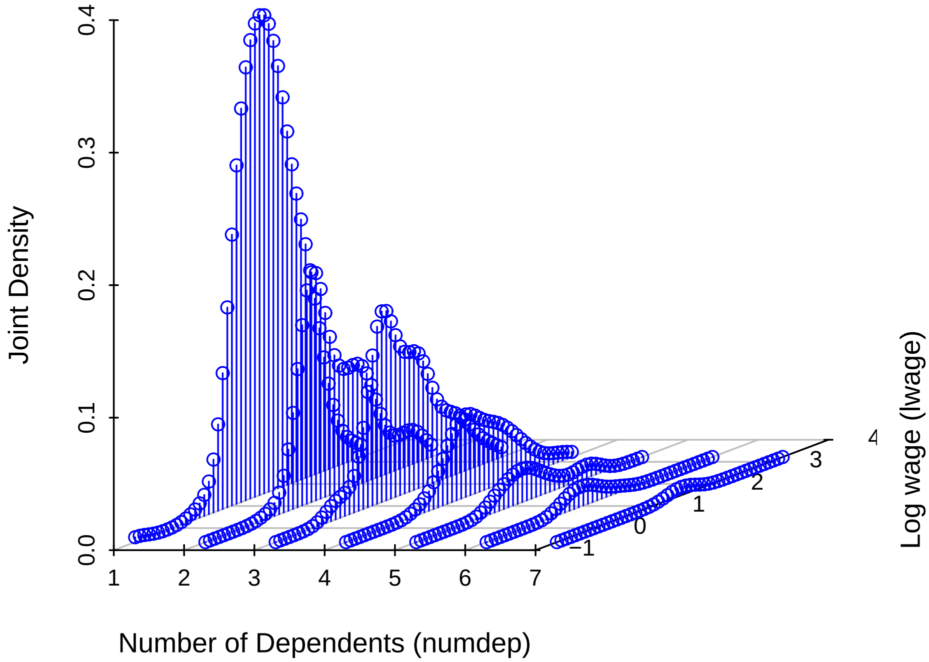
[2]The term 'density' is appropriate for distribution functions defined over mixed categorical and numeric variables. It is the measure defined on the categorical variables in the density function that matters.

$(0, 1, \dots)$. We use likelihood cross-validation to obtain the bandwidths. Note that this is indeed a case of 'sparse' data, and the traditional approach would require estimation of a nonparametric univariate density function based upon only two observations for the last cell ($c = 6$).

TABLE 2. Summary of numdep ($c = 0, 1, \dots, 6$)

c	n_c
0	252
1	105
2	99
3	45
4	16
5	7
6	2

```
R> library(scatterplot3d)
R> data("wage1")
R> attach(wage1)
R> bw <- npudensbw(~lwage+ordered(numdep),tol=.1,ftol=.1,data=wage1)
R> numdep.seq <- sort(unique(numdep))
R> lwage.seq <- seq(min(lwage),max(lwage),length=50)
R> wage1.eval <- expand.grid(numdep=ordered(numdep.seq),lwage=lwage.seq)
R> fhat <- fitted(npudens(bws=bw,newdata=wage1.eval))
R> f <- matrix(fhat,length(unique(numdep)),50)
R> scatterplot3d(wage1.eval[,1],wage1.eval[,2],fhat,
+              ylab="Log wage (lwage)",
+              xlab="Number of Dependents (numdep)",
+              zlab="Joint Density",
+              angle=15,box=FALSE,type="h",grid=TRUE,color="blue")
R> detach(wage1)
```



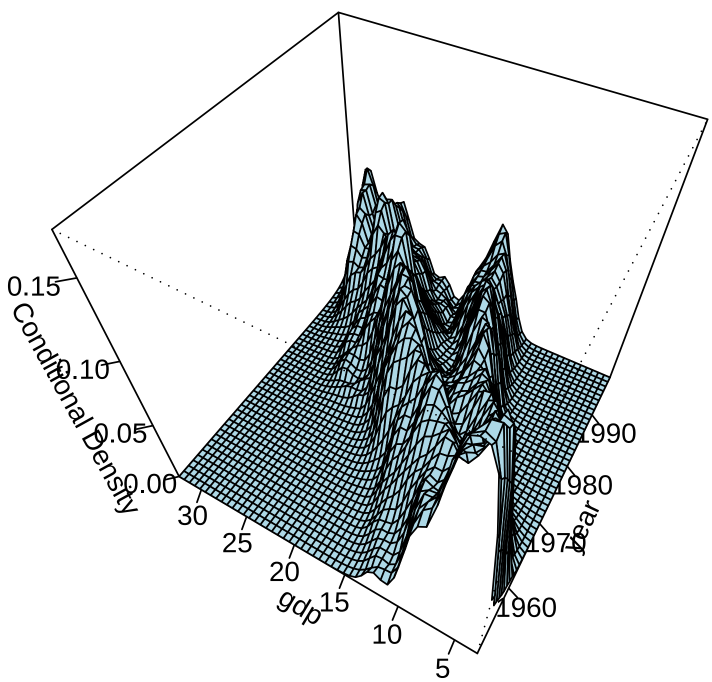
3.3. **Conditional Density Estimation.** Conditional density functions underlie many popular statistical objects of interest, though they are rarely modelled directly in parametric settings and have perhaps received even less attention in kernel settings. Nevertheless, as will be seen, they are extremely useful for a range of tasks, whether directly estimating the conditional density function, modelling count data (see Cameron & Trivedi (1998) for a thorough treatment of count data models), or perhaps modelling conditional quantiles via estimation of a conditional CDF. And, of course, regression analysis (i.e., modelling conditional means) depends directly on the conditional density function, so this statistical object in fact implicitly forms the backbone of many popular statistical methods.

We consider Giovanni Baiocchi's Italian GDP growth panel for 21 regions covering the period 1951-1998 (millions of Lire, 1990=base). There are 1,008 observations in total, and two variables, 'gdp' and 'year'. We treat gdp as numeric and year as ordered[3]. The code snippet below plots

[3]It is good practise to classify your variables according to their data type in your data frame. This has already been done hence there is no need to write `ordered(year)`.

the estimated conditional density, $\hat{f}(\text{gdp}|\text{year})$ based upon likelihood cross-validated bandwidth selection.

```
R> data("Italy")
R> attach(Italy)
R> fhat <- npcdens(gdp~year)
R> plot(fhat,view="fixed",main="",theta=300,phi=50)
```

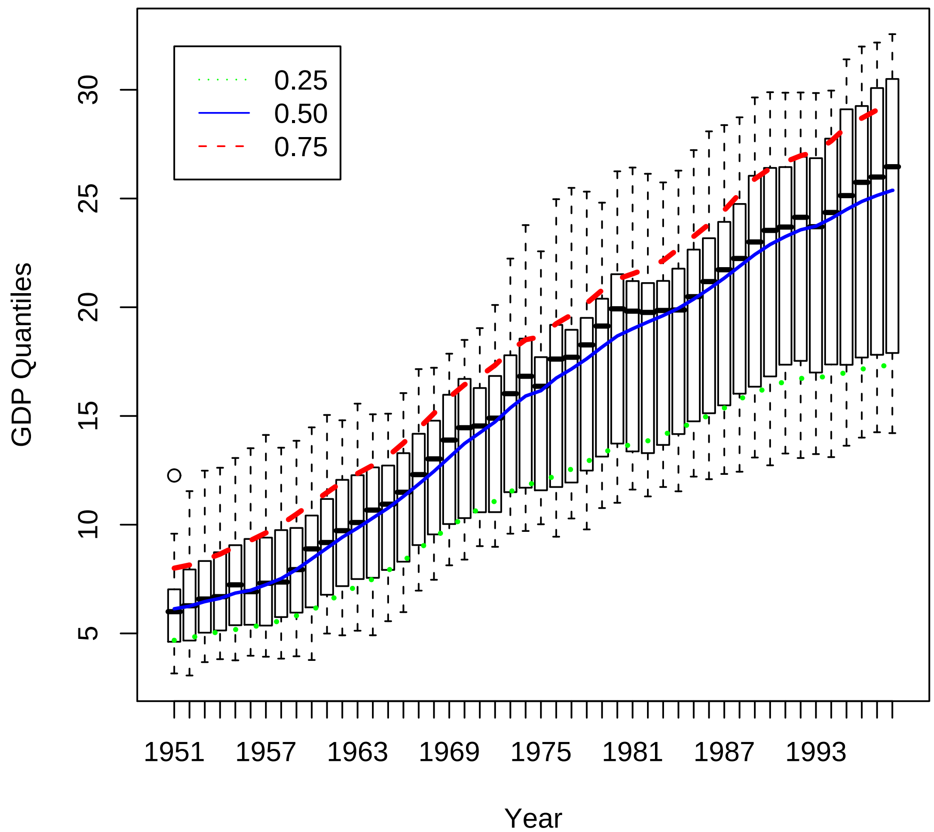


It is clear that the distribution of income has evolved from a unimodal one in the early 1950s to a markedly bimodal one in the 1990s. This result is robust to bandwidth choice, and is observed whether using simple rules-of-thumb or data-driven methods such as least-squares or likelihood cross-validation. The kernel method readily reveals this evolution which might easily be missed were one to use parametric models of the income distribution. For instance, the (unimodal) log-normal distribution is a popular parametric model for income distributions, but is incapable of revealing the multi-modal structure present in this dataset.

3.4. **Kernel Estimation of a Conditional Quantile.** Estimating regression functions is a popular activity for applied economists. Sometimes, however, the regression function is not representative of the impact of the covariates on the dependent variable. For example, when the dependent variable is left (or right) censored, the relationship given by the regression function is distorted. In such cases, conditional quantiles above (or below) the censoring point are robust to the presence of censoring. Furthermore, the conditional quantile function provides a more comprehensive picture of the conditional distribution of a dependent variable than the conditional mean function

We consider the method described in Li & Racine (forthcoming) that is implemented in the `npqreg` function in the `np` package.

```
R> bw <- npcdensbw(gdp~ordered(year), tol=.1, ftol=.1)
R> model.q0.25 <- npqreg(bws=bw, tau=0.25)
R> model.q0.50 <- npqreg(bws=bw, tau=0.50)
R> model.q0.75 <- npqreg(bws=bw, tau=0.75)
R> plot(ordered(year), gdp,
+      main="",
+      xlab="Year",
+      ylab="GDP Quantiles")
R> lines(ordered(year), model.q0.25$quantile, col="green", lty=3, lwd=3)
R> lines(ordered(year), model.q0.50$quantile, col="blue", lty=1, lwd=2)
R> lines(ordered(year), model.q0.75$quantile, col="red", lty=2,lwd=3)
R> legend(ordered(1951), 32, c("0.25", "0.50", "0.75"),
+        lty=c(3, 1, 2), col=c("green", "blue", "red"))
R> detach(Italy)
```



3.5. **Binary Choice and Count Data Models.** We define a conditional mode by

$$m(x) = \max_y g(y|x). \tag{1}$$

In order to estimate a conditional mode $m(x)$, we need to model the conditional density. Let us call $\hat{m}(x)$ the estimated conditional mode, which is given by

$$\hat{m}(x) = \max_y \hat{g}(y|x), \tag{2}$$

where $\hat{g}(y|x)$ is the kernel estimator of $g(y|x)$. By way of example, we consider modelling low birthweights (a binary indicator) using this method.

For this example, we use data on birthweights taken from the `R` MASS library (Venables & Ripley (2002)), and compute a parametric Logit model and a nonparametric conditional mode model. We then compare their confusion matrices[4] and assess their classification ability. The

[4]A 'confusion matrix' is simply a tabulation of the actual outcomes versus those predicted by a model. The diagonal elements contain correctly predicted outcomes while the off-diagonal ones contain incorrectly predicted (confused) outcomes.

outcome is an indicator of low infant birthweight (0/1). The method can handle unordered and ordered multinomial outcomes without modification. This application has $n = 189$ and 7 regressors.

Variables are defined as follows:

(1) 'low' indicator of birth weight less than 2.5kg
(2) 'smoke' smoking status during pregnancy
(3) 'race' mother's race ('1' = white, '2' = black, '3' = other)
(4) 'ht' history of hypertension
(5) 'ui' presence of uterine irritability
(6) 'ftv' number of physician visits during the first trimester
(7) 'age' mother's age in years
(8) 'lwt' mother's weight in pounds at last menstrual period

Note that all variables other than age and lwt are categorical in nature in this example.

```
R> data("birthwt",package="MASS")
R> attach(birthwt)
R> model.logit <- glm(low~factor(smoke)+
+                     factor(race)+
+                     factor(ht)+
+                     factor(ui)+
+                     ordered(ftv)+
+                     age+
+                     lwt,
+                     family=binomial(link=logit))
R> cm <- table(low, ifelse(fitted(model.logit)>0.5, 1, 0))
R> ccr <- sum(diag(cm))/sum(cm)
R> cm
```

```
low   0   1
  0 119  11
  1  34  25
```

```
R> bw <- npcdensbw(factor(low)~factor(smoke)+
+                factor(race)+
+                factor(ht)+
+                factor(ui)+
+                ordered(ftv)+
+                age+
+                lwt)
R> model.np <- npconmode(bws=bw)
R> model.np$confusion.matrix
```

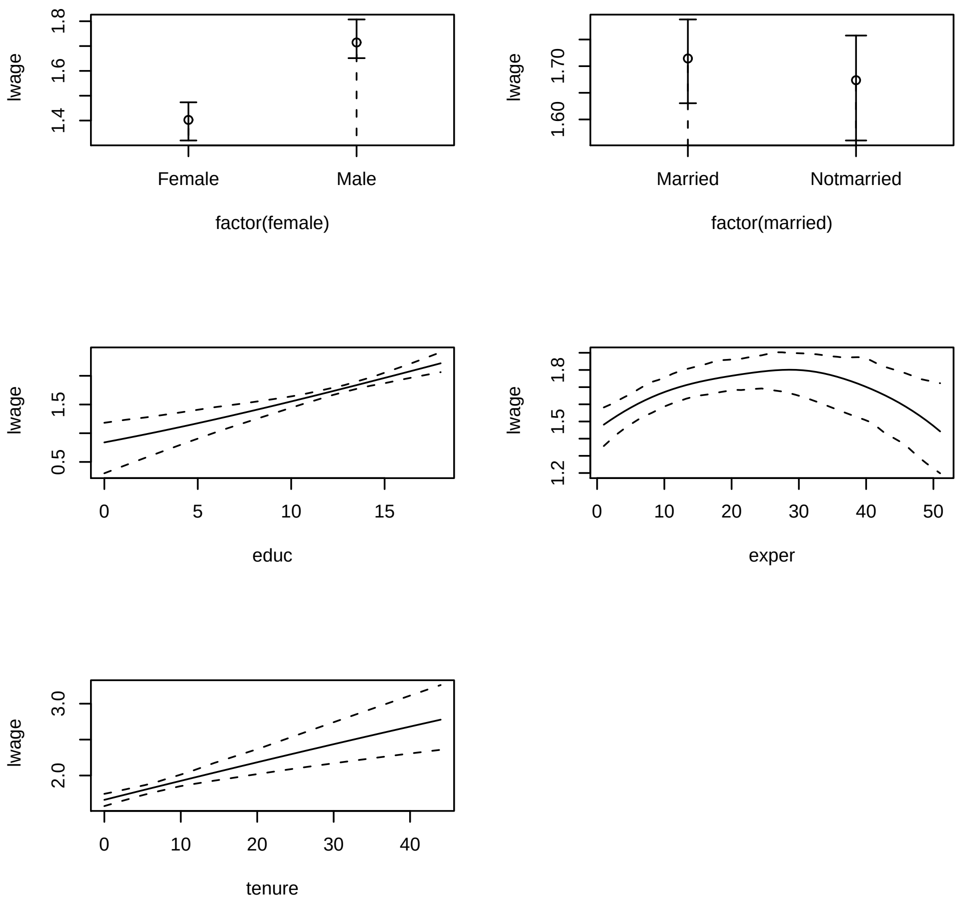
```
      Predicted
Actual   0   1
     0 127   3
     1  27  32

R> detach(birthwt)
```

3.6. **Regression.** One of the most popular methods for nonparametric kernel regression was proposed by Nadaraya (1965) and Watson (1964) and is known as the 'Nadaraya-Watson' estimator (also known as the 'local constant' estimator), though the 'local polynomial' estimator (Fan (1992)) has emerged as a popular alternative.

For what follows, we consider an application taken from Wooldridge (2003, pg. 226) that involves multiple regression analysis with both numeric and categorical data types.

We consider modelling an hourly wage equation for which the dependent variable is log(wage) (lwage) while the explanatory variables include three numeric variables, namely educ (years of education), exper (the number of years of potential experience), and tenure (the number of years with their current employer) along with two categorical variables, female ('Female'/'Male') and married ('Married'/'Notmarried'). For this example there are $n = 526$ observations. We use Hurvich, Simonoff & Tsai's (1998) AIC$_c$ approach for bandwidth selection.

```
R> attach(wage1)
R> #bw.all <- npregbw(formula=lwage~factor(female)+
R> #                 factor(married)+
R> #                 educ+
R> #                 exper+
R> #                 tenure,
R> #                 regtype="ll",
R> #                 bwmethod="cv.aic",
R> #                 data=wage1)
R>
R> model.np <- npreg(bws=bw.all)
R> plot(model.np,
+      plot.errors.method="bootstrap",
+      plot.errors.boot.num=100,
+      plot.errors.type="quantiles",
+      plot.errors.style="band",
+      common.scale=FALSE)
R> detach(wage1)
```

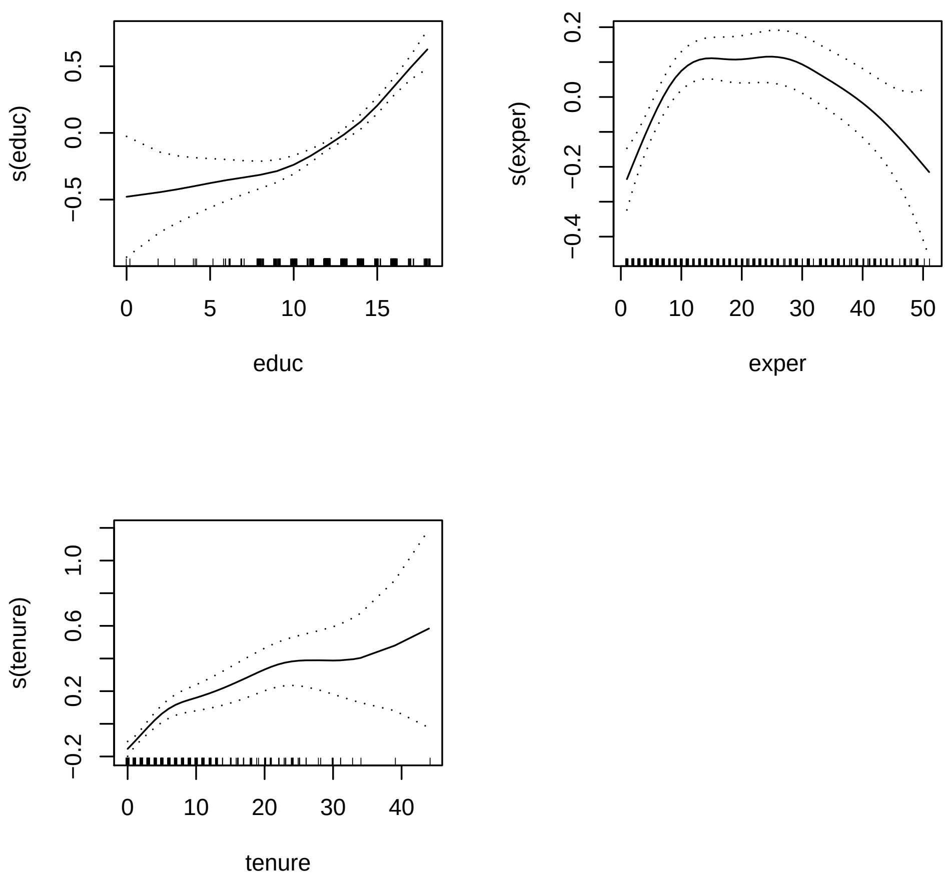


3.7. **Semiparametric Regression.** Semiparametric methods constitute some of the more popular methods for flexible estimation. Semiparametric models are formed by combining parametric and nonparametric models in a particular manner. Such models are useful in settings where fully nonparametric models may not perform well, for instance, when the curse of dimensionality has led to highly variable estimates or when one wishes to use a parametric regression model but the functional form with respect to a subset of regressors or perhaps the density of the errors is not known. We might also envision situations in which some regressors may appear as a linear function (i.e., linear in variables) but the functional form of the parameters with respect to the other variables is not known, or perhaps where the regression function is nonparametric but the structure of the error process is of a parametric form.

Semiparametric models can best be thought of as a compromise between fully nonparametric and fully parametric specifications. They rely on parametric assumptions and can therefore be misspecified and inconsistent, just like their parametric counterparts.

3.8. **Generalized Additive Models.** Generalized additive models are popular in applied settings, though one drawback is that they do not support categorical variables. The following code snipped considers the `wage1` dataset and uses three numeric regressors.

```
R> options(SweaveHooks = list(multifig = function() par(mfrow=c(2,2))))
R> library(gam)
R> attach(wage1)
R> model.gam <- gam(lwage~s(educ)+s(exper)+s(tenure))
R> plot(model.gam,se=T)
R> detach(wage1)
```



3.9. **Partially Linear Models.** The partially linear model is one of the simplest semiparametric models used in practise, and was proposed by Robinson (1988) while Racine & Liu (2007) extended the approach to handle the presence of categorical covariates. Suppose that we again consider the wage1 dataset from Wooldridge (2003, pg. 222), but now assume that the researcher is unwilling to presume the nature of the relationship between exper and lwage, hence relegates exper to the nonparametric part of a semiparametric partially linear model. The following code snipped considers a popular parametric specification followed by a partially linear one.

```
R> model.lm <- lm(formula=lwage~factor(female)+
+                 factor(married)+
+                 educ+
+                 tenure+
+                 exper+
+                 I(exper^2),
+                 data=wage1)
R> summary(model.lm)

Call:
lm(formula = lwage ~ factor(female) + factor(married) + educ +
    tenure + exper + I(exper^2), data = wage1)

Residuals:
    Min      1Q  Median      3Q     Max
-1.8185 -0.2568 -0.0253  0.2475  1.1815

Coefficients:
                           Estimate Std. Error t value Pr(>|t|)
(Intercept)                0.181161   0.107075    1.69    0.091 .
factor(female)Male         0.291130   0.036283    8.02  6.9e-15 ***
factor(married)Notmarried -0.056449   0.040926   -1.38    0.168
educ                       0.079832   0.006827   11.69  < 2e-16 ***
tenure                     0.016074   0.002880    5.58  3.9e-08 ***
exper                      0.030100   0.005193    5.80  1.2e-08 ***
I(exper^2)                -0.000601   0.000110   -5.47  7.0e-08 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.401 on 519 degrees of freedom
Multiple R-squared: 0.436,        Adjusted R-squared: 0.43
F-statistic: 66.9 on 6 and 519 DF,  p-value: <2e-16

R> bw <- npplregbw(formula=lwage~factor(female)+
+                  factor(married)+
+                  educ+
+                  tenure|exper,
```

```
+                 data=wage1)
R> model.pl <- npplreg(bw)
R> summary(model.pl)

Partially Linear Model
Regression data: 526 training points, in 5 variable(s)
With 4 linear parametric regressor(s), 1 nonparametric regressor(s)

              y(z)
Bandwidth(s): 2.05

               x(z)
Bandwidth(s): 4.194
              1.353
              3.161
              0.765

                factor(female) factor(married)   educ tenure
Coefficient(s):           0.29         -0.0372 0.0788 0.0166

Kernel Regression Estimator: Local-Constant
Bandwidth Type: Fixed

Residual standard error: 0.155
R-squared: 0.449

Continuous Kernel Type: Second-Order Gaussian
No. Continuous Explanatory Vars.: 1
```

3.10. **Index Models.** A semiparametric single index model is of the form

$$Y = g(X'\beta_0) + u, \tag{3}$$

where Y is the dependent variable, $X \in \mathbb{R}^q$ is the vector of explanatory variables, β_0 is the $q \times 1$ vector of unknown parameters, and u is the error satisfying $E(u|X) = 0$. The term $x'\beta_0$ is called a 'single index' because it is a scalar (a single index) even though x is a vector. The functional form of $g(\cdot)$ is unknown to the researcher. This model is semiparametric in nature since the functional form of the linear index is specified, while $g(\cdot)$ is left unspecified.

Ichimura (1993), Manski (1988) and Horowitz (1998, pp. 14–20) provide excellent intuitive explanations of the identifiability conditions underlying semiparametric single index models (i.e., the set of conditions under which the unknown parameter vector β_0 and the unknown function $g(\cdot)$ can be sensibly estimated), and we direct the reader to these references for details.

We consider applying Ichimura (1993)'s single index method which is appropriate for numeric outcomes, unlike that of Klein & Spady (1993) outlined below. We again make use of the wage1 dataset found in Wooldridge (2003, pg. 222). Table **??** presents a summary of the analysis.

```
R> attach(wage1)
R> model.lm <- lm(formula=lwage~factor(female)+
+                factor(married)+
+                educ+
+                tenure+
+                exper+
+                expersq)
R> summary(model.lm)

Call:
lm(formula = lwage ~ factor(female) + factor(married) + educ +
    tenure + exper + expersq)

Residuals:
    Min      1Q  Median      3Q     Max
-1.8185 -0.2568 -0.0253  0.2475  1.1815

Coefficients:
                           Estimate Std. Error t value Pr(>|t|)
(Intercept)                0.181161   0.107075    1.69    0.091 .
factor(female)Male         0.291130   0.036283    8.02  6.9e-15 ***
factor(married)Notmarried -0.056449   0.040926   -1.38    0.168
educ                       0.079832   0.006827   11.69  < 2e-16 ***
tenure                     0.016074   0.002880    5.58  3.9e-08 ***
exper                      0.030100   0.005193    5.80  1.2e-08 ***
expersq                   -0.000601   0.000110   -5.47  7.0e-08 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.401 on 519 degrees of freedom
Multiple R-squared: 0.436,       Adjusted R-squared: 0.43
F-statistic: 66.9 on 6 and 519 DF,  p-value: <2e-16

R> bw <- npindexbw(formula=lwage~factor(female)+
+                  factor(married)+
```

```
+                  educ+
+                  exper+
+                  expersq+
+                  tenure,
+                  data=wage1)
R> model <- npindex(bw)
R> summary(model)
```

```
Single Index Model
Regression Data: 526 training points, in 6 variable(s)

      factor(female) factor(married)   educ   exper   expersq tenure
Beta:              1         -0.0611 0.0434 0.0179 -0.00041 0.0101
Bandwidth: 0.0553
Kernel Regression Estimator: Local-Constant

Residual standard error: 0.152
R-squared: 0.463

Continuous Kernel Type: Second-Order Gaussian
No. Continuous Explanatory Vars.: 1
```

```
R> detach(wage1)
```

We again consider data on birthweights taken from the R MASS library (Venables & Ripley (2002)), and compute a single index model (the parametric Logit model and a nonparametric conditional mode model results are reported in *Conditional Density Estimation*). The outcome is an indicator of low infant birthweight (0/1) and so Klein & Spady's (1993) approach is appropriate. The confusion matrix is presented to facilitate a comparison of the models.

```
R> bw <- npindexbw(formula=low~
+                  factor(smoke)+
+                  factor(race)+
+                  factor(ht)+
+                  factor(ui)+
+                  ordered(ftv)+
+                  age+
+                  lwt,
+                  method="kleinspady",
+                  data=birthwt)
R> model.index <- npindex(bws=bw, gradients=TRUE)
R> summary(model.index)
```

```
Single Index Model
Regression Data: 189 training points, in 7 variable(s)

      factor(smoke) factor(race) factor(ht) factor(ui) ordered(ftv)     age
Beta:             1       0.0508      0.363      0.184      -0.0505 -0.0158
           lwt
Beta: -0.00145
Bandwidth: 0.0159
Kernel Regression Estimator: Local-Constant

Confusion Matrix
      Predicted
Actual   0   1
     0 119  11
     1  22  37

Overall Correct Classification Ratio:  0.825
Correct Classification Ratio By Outcome:
    0     1
0.915 0.627

McFadden-Puig-Kerschner performance measure:  0.808

Continuous Kernel Type: Second-Order Gaussian
No. Continuous Explanatory Vars.: 1
```

3.11. **Smooth Coefficient (Varying Coefficient) Models.** The smooth coefficient model is given by

$$
\begin{aligned}
Y_i &= \alpha(Z_i) + X_i'\beta(Z_i) + u_i \\
&= (1 + X_i') \begin{pmatrix} \alpha(Z_i) \\ \beta(Z_i) \end{pmatrix} + u_i \\
&= W_i'\gamma(Z_i) + u_i
\end{aligned} \tag{4}
$$

where X_i is a $k \times 1$ vector and where $\beta(z)$ is a vector of unspecified smooth functions of z.

Suppose that we once again consider the wage1 dataset from Wooldridge (2003, pg. 222), but now assume that the researcher is unwilling to presume that the coefficients associated with the numeric variables do not vary with respect to the categorical variables female and married. The following code snipped presents a summary from the linear and smooth coefficient specification.

```
R> attach(wage1)
R> expersq <- exper^2
R> model.lm <- lm(formula=lwage~factor(female)+
+               factor(married)+
+               educ+
+               tenure+
+               exper+
+               expersq)
R> summary(model.lm)

Call:
lm(formula = lwage ~ factor(female) + factor(married) + educ +
    tenure + exper + expersq)

Residuals:
    Min      1Q  Median      3Q     Max
-1.8185 -0.2568 -0.0253  0.2475  1.1815

Coefficients:
                           Estimate Std. Error t value Pr(>|t|)
(Intercept)                0.181161   0.107075    1.69    0.091 .
factor(female)Male         0.291130   0.036283    8.02  6.9e-15 ***
factor(married)Notmarried -0.056449   0.040926   -1.38    0.168
educ                       0.079832   0.006827   11.69  < 2e-16 ***
tenure                     0.016074   0.002880    5.58  3.9e-08 ***
exper                      0.030100   0.005193    5.80  1.2e-08 ***
expersq                   -0.000601   0.000110   -5.47  7.0e-08 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.401 on 519 degrees of freedom
Multiple R-squared: 0.436,        Adjusted R-squared: 0.43
F-statistic: 66.9 on 6 and 519 DF,  p-value: <2e-16

R> bw <- npscoefbw(formula=lwage~
+                educ+
+                tenure+
+                exper+
+                expersq|factor(female)+factor(married))
```

```
R> model.scoef <- npscoef(bw,betas=TRUE)
R> summary(model.scoef)
Smooth Coefficient Model
Regression data: 526 training points, in 2 variable(s)

              factor(female) factor(married)
Bandwidth(s):        0.00175           0.134

Bandwidth Type: Fixed

Residual standard error: 0.147
R-squared: 0.479

Unordered Categorical Kernel Type: Aitchison and Aitken
No. Unordered Categorical Explanatory Vars.: 2
R> ## You could examine the matrix of smooth coefficients, or compute the average
R> ## coefficient for each variable. One might then compare the average with the
R> ## OLS model by way of example.
R>
R> colMeans(coef(model.scoef))
Intercept      educ    tenure     exper   expersq
 0.340220  0.078649  0.014296  0.030052 -0.000595
R> coef(model.lm)
              (Intercept)       factor(female)Male factor(married)Notmarried
                 0.181161                 0.291130                -0.056449
                     educ                   tenure                     exper
                 0.079832                 0.016074                  0.030100
                  expersq
                -0.000601
R> detach(wage1)
```

3.12. **Panel Data Models.** The nonparametric and semiparametric estimation of panel data models has received less attention than the estimation of standard regression models. Data panels are samples formed by drawing observations on N cross-sectional units for T consecutive periods yielding a dataset of the form $\{Y_{it}, Z_{it}\}_{i=1,t=1}^{N,T}$. A panel is therefore simply a collection of N individual time series that may be short ("small T") or long ("large T").

The nonparametric estimation of time series models is itself an evolving field. However, when T is large and N is small then there exists a lengthy time series for each individual unit and in such cases one can avoid estimating a panel data model by simply estimating separate nonparametric models for each individual unit using the T individual time series available for each. If this situation applies, we direct the interested reader to Li & Racine (2007*a*, Chapter 18) for pointers to the literature on nonparametric methods for time series data.

When contemplating the nonparametric estimation of panel data models, one issue that immediately arises is that the standard (parametric) approaches that are often used for panel data models (such as first-differencing to remove the presence of so-called 'fixed effects') are no longer valid unless one is willing to presume additively separable effects, which for many defeats the purpose of using nonparametric methods in the first place.

A variety of approaches have been proposed in the literature, including Wang (2003), who proposed a novel method for estimating nonparametric panel data models that utilizes the information contained in the covariance structure of the model's disturbances, Wang, Carroll & Lin (2005) who proposed a partially linear model with random effects, and Henderson, Carroll & Li (2006) who consider profile likelihood methods for nonparametric estimation of additive fixed effect models which are removed via first differencing. In what follows, we consider direct nonparametric estimation of fixed effects models.

Consider the following nonparametric fixed effects panel data regression model,

$$Y_{it} = g(X_{it}) + u_{it}, \quad i = 1, 2 \ldots, N,\ t = 1, 2, \ldots, T,$$

where $g(\cdot)$ is an unknown smooth function, $X_{it} = (X_{it,1}, \ldots, X_{it,q})$ is of dimension q, all other variables are scalars, and $E(u_{it}|X_{i1}, \ldots, X_{iT}) = 0$.

We say that panel data is 'poolable' if one can 'pool' the data, by in effect, ignoring the time series dimension, that is, by summing over both i and t without regard to the time dimension thereby effectively putting all data into the same pool then directly applying the methods in, say, *Regression*. Of course, if the data is not poolable this would obviously not be a wise choice.

However, to allow for the possibility that the data is in fact *potentially* poolable, one can introduce an *unordered* categorical variable, say $\delta_i = i$ for $i = 1, 2, \ldots, N$, and estimate $E(Y_{it}|Z_{it}, \delta_i) = g(Z_{it}, \delta_i)$ nonparametrically using the mixed categorical and numeric kernel approach introduced in *Density and Probability Function Estimation*. Letting $\hat{\lambda}$ denote the cross-validated smoothing parameter associated with δ_i, then if $\hat{\lambda} = 1$, one gets $g(Z_{it}, \delta_i) = g(Z_{it})$ and the data is thereby pooled in the resulting estimate of $g(\cdot)$. If, on the other hand, $\hat{\lambda} = 0$ (or is close to 0), then this effectively estimates each $g_i(\cdot)$ using only the time series for the ith individual unit. Finally, if $0 < \hat{\lambda} < 1$, one might interpret this as a case in which the data is partially poolable.

We consider a panel of annual observations for six U.S. airlines for the fifteen year period 1970 to 1984 taken from the Ecdat `R` package (Croissant (2006)) as detailed in Greene (2003, Table F7.1, page 949)). The variables in the panel are airline ('airline'), year ('year'), the logarithm of total cost in $1,000 ('lcost'), the logarithm of an output index in revenue passenger miles ('loutput'), the logarithm of the price of fuel ('lpf'), and load factor, i.e., the average capacity utilization of the fleet ('lf'). We treat 'airline' as an ordered factor and 'year' as an ordered factor and use a local linear estimator with Hurvich et al.'s (1998) AIC$_c$ approach.

```
R> library(plm)
[1] "kinship is loaded"
R> library(Ecdat)
R> data(Airline)
R> model.plm <- plm(log(cost) ~ log(output) + log(pf) + lf,
+           data = Airline,
+           model = "within",
+           index=c("airline","year"))
R> summary(model.plm)
Oneway (individual) effect Within Model

Call:
plm(formula = log(cost) ~ log(output) + log(pf) + lf, data = Airline,
    model = "within", index = c("airline", "year"))

Balanced Panel: n=6, T=15, N=90

Residuals :
   Min. 1st Qu.  Median 3rd Qu.    Max.
-0.1560 -0.0352 -0.0093  0.0349  0.1660

Coefficients :
            Estimate Std. Error t-value   Pr(>|t|)
log(output)   0.9193     0.0299   30.76    < 2e-16 ***
log(pf)       0.4175     0.0152   27.47    < 2e-16 ***
lf           -1.0704     0.2017   -5.31 0.00000011 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Total Sum of Squares: 39.4
Residual Sum of Squares: 0.293
Multiple R-Squared: 0.993
F-statistic: 3604.81 on 81 and 3 DF, p-value: 0.00000644
R> attach(Airline)
R> lcost <- as.numeric(log(cost))
R> loutput <- as.numeric(log(output))
R> lpf <- as.numeric(log(pf))
R> lf <- as.numeric(lf)
```

```
R> bw <- npregbw(lcost~loutput +
+               lpf +
+               lf +
+               ordered(year) +
+               factor(airline),
+               regtype="ll",
+               bwmethod="cv.aic",
+               ukertype="liracine",
+               okertype="liracine")
R> summary(bw)
```

```
Regression Data (90 observations, 5 variable(s)):

Regression Type: Local-Linear
Bandwidth Selection Method: Expected Kullback-Leibler Cross-Validation
Formula: lcost ~ loutput + lpf + lf + ordered(year) + factor(airline)
Bandwidth Type: Fixed
Objective Function Value: -8.9e+15 (achieved on multistart 2)

Exp. Var. Name: loutput         Bandwidth: 0.0961 Scale Factor: 0.160
Exp. Var. Name: lpf             Bandwidth: 0.326  Scale Factor: 0.768
Exp. Var. Name: lf              Bandwidth: 0.0129 Scale Factor: 0.469
Exp. Var. Name: ordered(year)   Bandwidth: 0.489    Lambda Max: 1
Exp. Var. Name: factor(airline) Bandwidth: 0.155    Lambda Max: 1

Continuous Kernel Type: Second-Order Gaussian
No. Continuous Explanatory Vars.: 3

Unordered Categorical Kernel Type: Li and Racine
No. Unordered Categorical Explanatory Vars.: 1

Ordered Categorical Kernel Type: Li and Racine
No. Ordered Categorical Explanatory Vars.: 1
```

```
R> detach(Airline)
```

3.13. **Rolling your own Functions.** The `np` package contains the function `npksum` that computes kernel sums on evaluation data, given a set of training data, data to be weighted (optional), and a bandwidth specification (any bandwidth object).

`npksum` exists so that you can create your own kernel objects with or without a variable to be weighted (default Y=1). With the options available, you could create new nonparametric tests or even new kernel estimators. The convolution kernel option would allow you to create, say, the least squares cross-validation function for kernel density estimation.

`npksum` uses highly-optimized C code that strives to minimize its 'memory footprint', while there is low overhead involved when using repeated calls to this function (see, by way of illustration, the example below that conducts leave-one-out cross-validation for a local constant regression estimator via calls to the 'R' function 'nlm', and compares this to the 'npregbw' function).

`npksum` implements a variety of methods for computing multivariate kernel sums (p-variate) defined over a set of possibly numeric and/or categorical (unordered, ordered) data. The approach is based on Li and Racine (2003) who employ 'generalized product kernels' that admit a mix of numeric and categorical data types.

Three classes of kernel estimators for the numeric data types are available: fixed, adaptive nearest-neighbor, and generalized nearest-neighbor. Adaptive nearest-neighbor bandwidths change with each sample realization in the set, x[i], when estimating the kernel sum at the point x. Generalized nearest-neighbor bandwidths change with the point at which the sum is computed, x. Fixed bandwidths are constant over the support of x. `npksum` computes $\sum_j W_j' Y_j K(X_j)$, where A_j represents a row vector extracted from A. That is, it computes the kernel weighted sum of the outer product of the rows of W and Y. In the examples from `?npksum`, the uses of such sums are illustrated.

`npksum` may be invoked either with a formula-like symbolic description of variables on which the sum is to be performed or through a simpler interface whereby data is passed directly to the function via the 'txdat' and 'tydat' parameters. Use of these two interfaces is mutually exclusive.

Data contained in the data frame 'txdat' (and also 'exdat') may be a mix of numeric (default), unordered categorical (to be specified in the data frame 'txdat' using the 'factor' command), and ordered categorical (to be specified in the data frame 'txdat' using the 'ordered' command). Data can be entered in an arbitrary order and data types will be detected automatically by the routine (see 'np' for details).

Data for which bandwidths are to be estimated may be specified symbolically. A typical description has the form 'dependent data explanatory data', where 'dependent data' and 'explanatory data' are both series of variables specified by name, separated by the separation character '+'. For example, `y1 ~ x1 + x2` specifies that `y1` is to be kernel-weighted by `x1` and `x2` throughout the sum. See below for further examples.

A variety of kernels may be specified by the user. Kernels implemented for numeric data types include the second, fourth, sixth, and eighth order Gaussian and Epanechnikov kernels, and the uniform kernel. Unordered categorical data types use a variation on Aitchison and Aitken's (1976) kernel, while ordered data types use a variation of the Wang and van Ryzin (1981) kernel (see 'np' for details).

The following example implements leave-one-out cross-validation for the local constant estimator using the `npksum` function and the `R` `nlm` function that function carries out a minimization of a function using a Newton-type algorithm.

```
R> n <- 100
R> x1 <- runif(n)
R> x2 <- rnorm(n)
R> x3 <- runif(n)
R> txdat <- data.frame(x1, x2, x3)
R> tydat <- x1 + sin(x2) + rnorm(n)
R> ss <- function(h) {
+
+   if(min(h)<=0) {
+
+     return(.Machine$double.xmax)
+
+   } else {
+
+     mean <-  npksum(txdat,
+                     tydat,
+                     leave.one.out=TRUE,
+                     bandwidth.divide=TRUE,
+                     bws=h)$ksum/
+                       npksum(txdat,
+                              leave.one.out=TRUE,
+                              bandwidth.divide=TRUE,
+                              bws=h)$ksum
+
+     return(sum((tydat-mean)^2)/length(tydat))
+
+   }
+
+ }
R> nlm.return <- nlm(ss, runif(length(txdat)))
R> bw <- npregbw(xdat=txdat, ydat=tydat)
R> ## Bandwidths from nlm()
R>
R> nlm.return$estimate

[1]   0.223   0.460 896.936

R> ## Bandwidths from npregbw()
R>
R> bw$bw
```

```
[1]       0.223       0.460 1602195.268
R> ## Function value (minimum) from nlm()
R>
R> nlm.return$minimum
[1] 0.953
R> ## Function value (minimum) from npregbw()
R>
R> bw$fval
[1] 0.953
```

4. Summary

The R environment for statistical computing and graphics (R Development Core Team (2008)) offers practitioners a rich set of statistical methods ranging from random number generation and optimization methods through regression, panel data, and time series methods, by way of illustration. The standard R distribution ('base R) comes preloaded with a rich variety of functionality useful for applied econometricians. This functionality is enhanced by user supplied packages made available via R servers that are mirrored around the world. We hope that this chapter will encourage users to pursue the R environment should they wish to adopt nonparametric or semiparametric methods, and we wholeheartedly encourage those working in the field to strongly consider implementing their methods in the R environment thereby making their work accessible to the widest possible audience via an open collaborative forum.

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