Chapter 3. Syntax Analysis I
(Top down parsing)

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3.1 Introduction

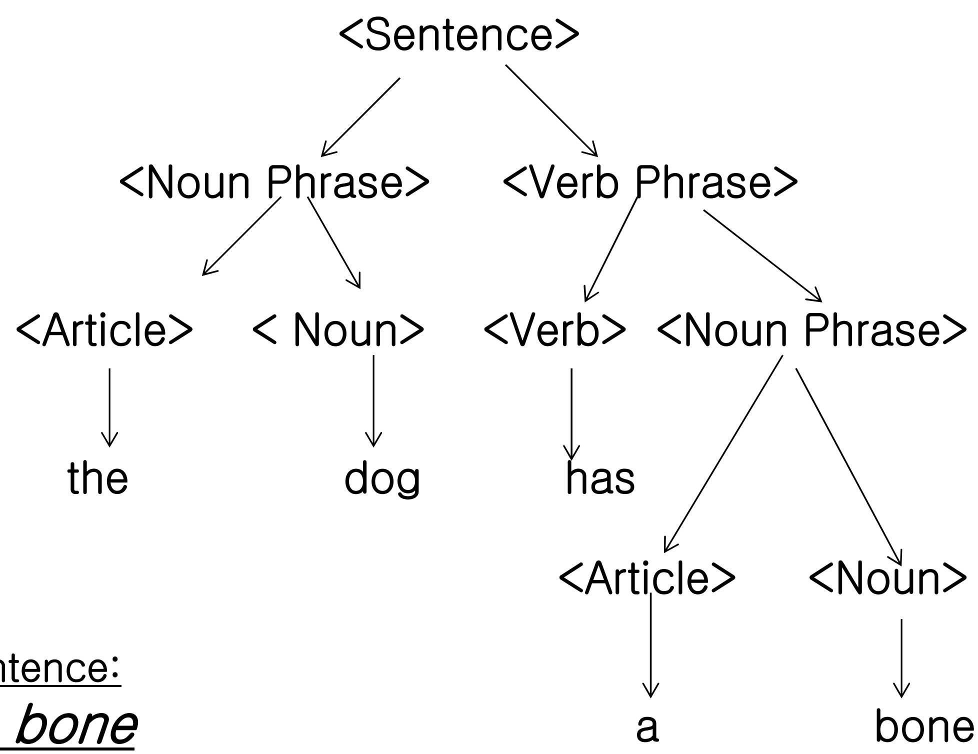
Syntax Analysis is a phase where the structure of source code is recognized and constructed using a finite set of rules called productions.

Ex. of Rules for a subset of English

<Sentence> -> <Noun Phrase> <Verb Phrase>
<Noun Phrase> -> <Article> <Noun>
<Verb Phrase> -> <Verb> <Noun Phrase>
<Noun> -> dog, cat, bone, school
<Article> -> the, a, an
<Verb> -> goes, has

Terms:
Production, Non-terminal Symbols, Terminal symbols

Sentence: The dog has a bone



However, the 2^{nd} sentence:
The dog goes a bone
has no meaning!

BNF NOTATION: Backus-Naur(Normal) Form

A Specific notation to describe the productions.
Ex.
<Expression> ::= <Expression> + <Term> | <Expression> - <term> | <Term>

<Term> ::= <Term> * <Factor> | <Term> / <Factor> | <Factor>

<Factor> ::= <Identifier> | <Number> | (<Expression>)

Extended BNF: Allows additional notations; such as
{ } 0 or more times,
[ ] optional

<Number> ::= <Digit> { <Digit> }
<Identifier> ::= <Letter> { <Letter> | <Digit> }
<Factor> ::=   [-] <Primary>

3.2 Grammar

a) Def: Grammar $G = (T, N, S, R)$ where

T is a finite set of terminal symbols

N is a finite set of non-terminal symbols

$S \in N$ is a unique Starting symbol

R is a finite set of productions of the form $\alpha \rightarrow \beta$ where

α, β are strings of terminal ad non-terminal symbols

Def: The language of grammar G is the set of all sentences that can be generated by G and it is written $L\{G\}$

Ex. of a Grammar

G = (

T = {id, +, -, *, /, (,) },

N = {<Expr>, <Term>, <Factor> }

S = <Expr>,

R = { 1. <Expr> -> <Expr> + <Term>

2. <Expr> -> <Expr> - <Term>

3. <Expr> -> <Term>

4. <Term> -> <Term> * <Factor>

5. <Term> -> <Term> / <Factor>

6. <Term> -> <Factor>

7. <Factor> -> id

8. <Factor> -> (<Expr>)

}

)

b) Derivation

Def: Derivation is replacing one non–terminal symbol at a time in order to recognize a sentence

Ex. Source code: *a / (c – d)*

<Expr> => <term>
=> <Term> / <Factor>
⇒ <Factor> / <Factor>
⇒ id(a) / <Factor>
⇒ a / (<Expr>)
⇒ a / (<Expr> – <Term>)
⇒ a / (<Term> – <Term>)
⇒ a / (<Factor> – <Term>)
⇒ a / (c – <Term>)
⇒ a / (c – <Factor>)
⇒ a / (c – d)

In general, we say <Expr> $\overset{*}{=>}$ a / (c–d)

- Sentential Form: is a string of symbols (N or T) appearing in various steps in derivation

- Left Most Derivation(LMD) : Replace the Left most Non-terminal in each step

- Right Most Derivation(RMD): Replace the Right most non-terminal in each step

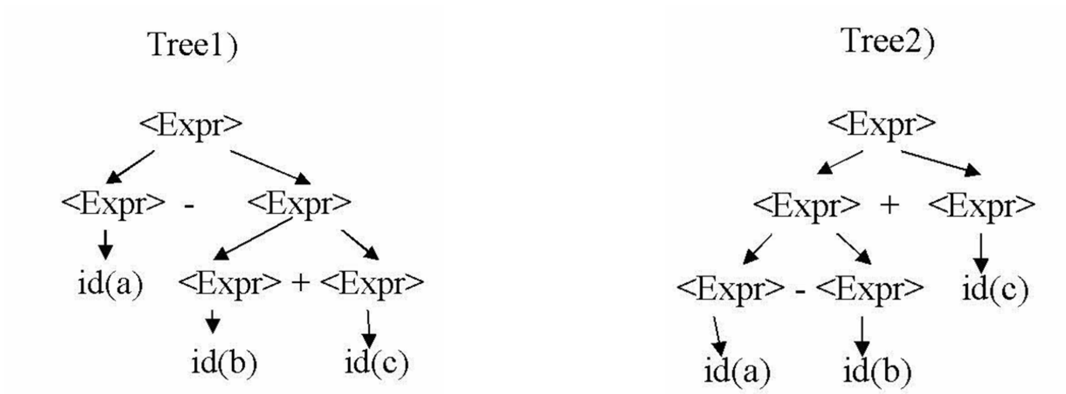
- Def: $L(G) \underset{G}{=} \{\omega \mid S \Rightarrow^* \omega\}$

c)Ambiguity
Def: A grammar is ambiguous if there are two different parse trees for some words in L(G)

Ex: Assume Productions

<Expr> -> <Expr> + < Expr>
<Expr> -> <Expr> - <Expr>
<Expr> -> id

Consider a string a−b+c



2 different parse trees => the grammar is ambiguous

Def: A language for which NO UNAMBIGUOUS grammar exists is called
Inherently ambiguous language

G1

G2

G3

….

Gn

L(G) = { s1, s2, s3,………s∞ }

All G1, G2,….,Gn are ambiguous

NOTICE:
Most Programming languages are ambiguous

Ex. if-then-else statement (Dangling-Else Problem)
Consider the following:

```
If (x > y) then
   If ( x >  z ) then
        a = b
else a = c;
```

Q: Where does "else" belong?
A: Belongs to the closest "if" => impose an outside rule!

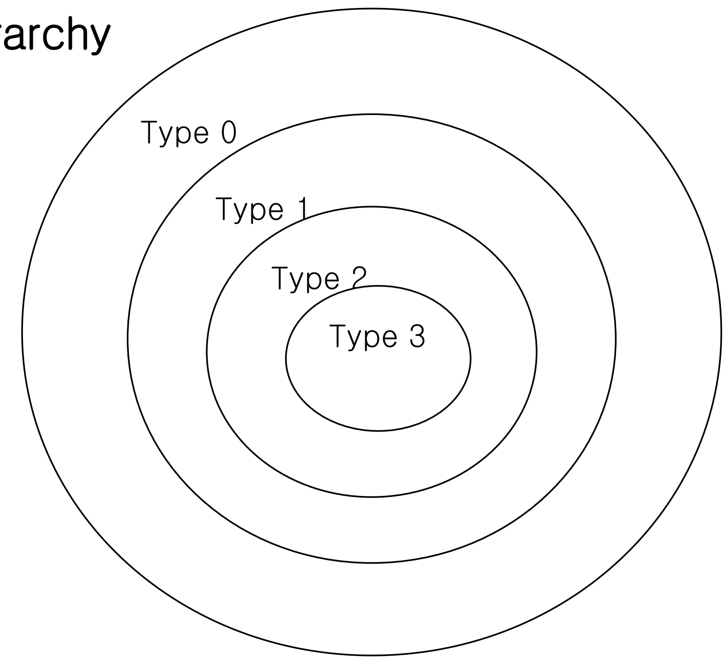
3.3 Chomsky Hierarchy of Grammars

Chomsky Hierarchy shows different types of grammars based on the forms of productions. => 4 types (0,1,2 and 3)

New Convention:

- Capital Letter: A, B …. Non-terminal
- Lowercase letters: a,b,c … terminals
- Greek Letters: α, β, δ, γ … strings of N and T

Chomsky Hierarchy



a) Type 0: Unrestricted Grammar

Def: Production form: α -> β

where both (α, β) are any string of N and T

Ex.

R1) S -> ACaB

R2) Ca -> aa C

R3) CB -> DB

R4) CB -> E

R5) aD -> D a

R6) AD -> AC

R7) aE -> Ea

R8) AE -> ε

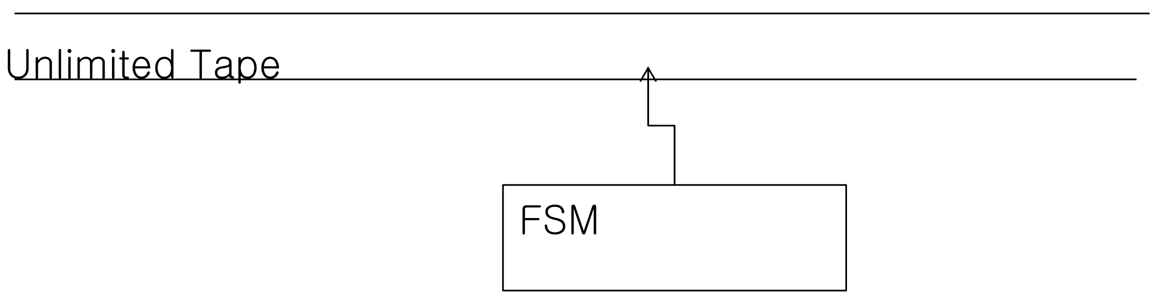
A Simple Strings: aa, aaaa, aaaaaaaa etc

$$S \overset{R1}{\Rightarrow} A\,C\,a\,B \overset{R2}{\Rightarrow} A\,aa\,CB \overset{R4}{\Rightarrow} AaaE \overset{R7}{\Rightarrow} AaEa \overset{R7}{\Rightarrow} AE\,aa \overset{R8}{\Rightarrow} aa$$

L(G) = {w| a^i where i is a positive power of 2}

Machine for Unrestricted languages:

Turing Machine (By Allen Turing 1912–1954)



The Transition function:

N (q, a) –> (qi, b $\in\Sigma$)   /* goto a state and write to the tape */

–> (qi, {L,R})   /* goto a state and move to the write/read */

b) Type 1: Context Sensitive Grammar

Def: production Form: $\gamma A \delta \rightarrow \gamma \alpha \delta$,

Similar to type 0 but

1) α cannot be ε i.e., -> $|\gamma\alpha\delta| >= |\gamma A \delta|$, RHS >= LHS

2) A -> α in the context of γ, δ.

Ex.

R1) S -> aSBC

R2) S -> abC

R3) CB -> BC /* Strictly speaking is not context sensitive */

R4) bB -> bb

R6) bC -> bc

R6) cC -> cc

Sample string: abc, aabbcc etc

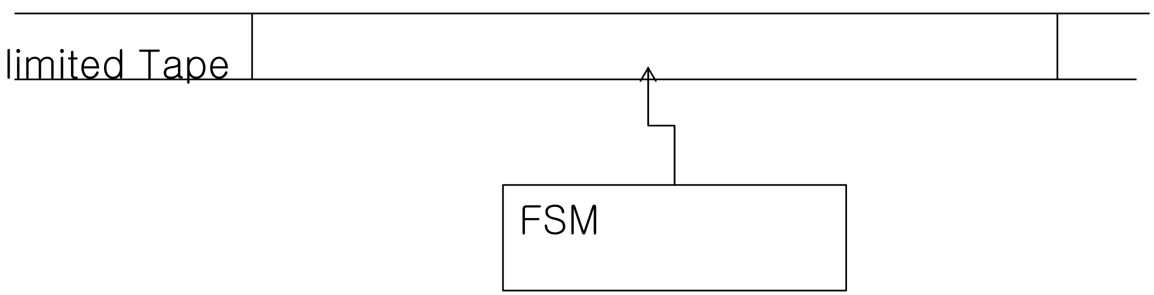
R1	R2	R3	R4	R5	R6

S => aSBC =>aabCBC =>aabBCC =>aabbCC => aabbcC => aabbcc

L(G) = {w | w = $a^n b^n c^n$} Equal number of a,b and c.

Machine for Context Sensitive languages:

LBA (Lineary Bounded Automaton): Similar to Turing but with



The Transition function:

N (q, a) -> (qi, b ∈∑)   /* goto a state and write to the tape */

-> (qi, {L,R})   /* goto a state and move to the write/read */

c) Type 2: Context Free Grammar

Def: Production Form: α -> β

where α is a <u>single</u> non terminal

Ex.

R1) S -> a B

R2) S -> bA

R3) A -> a

R4) A -> aS

R5) A -> bAA

R6) B -> b

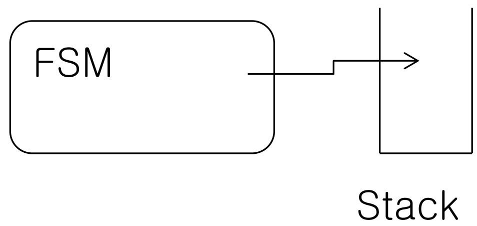
R7) B -> bS

R8) B -> a BB

Simple String: ab, ba, abab, aaabbb etc

L(G) ={ w | w has same number of a's and b's}

c) Machine: Push Down Automaton (PDA)



More in Detail:
PDA = (Σ, Q, q0, F, N, Γ)

Same as FSM, except Γ with Stack Symbols
and N: (Q x Σ x Γ) -> (Q x Γ*)
i.e.,
State x input x Stack Symbol – > State x StackOperation

Acceptance: 1) entire string read,
2) FSM in F
3) Stack empty

EX:

Given the following productions, build a PDA that accepts L(G)

R1) S -> a S a
R2) S -> b S b
R3) S -> c

Sample strings: aca, abcba, abacaba

L(G) =
$\{wcw^r\}$ (Palindrom)

PDA = (Σ = {a,b,c}, Q = {s,f}, q0 = s, F = {f}, Γ = {1,2}

N =

{ N1) (s,a,ε) -> (s,1)

N2) (s,b, ε) -> (s,2)

N3) (s,c, ε) -> (f, ε)

N4) (f,a,1) -> (f, ε)

N5) (f,b,2) -> (f, ε) }

w = abbcbba

State	Input	Stack	N-used
S	abbcbba	ε	1
S	bbcbba	ε 1	2
S	bcbba	ε21	2
S	cbba	ε221	3
f	bba	221	5
f	ba	21	5
f	a	1	4
f	ε	ε	
accepting state	empty	empty => accepted	

d) Type 3: Regular Grammar

Def: α -> β where

α is a single nonterminal and,

β is either all terminals or at most one Non-terminal

(the last symbol on the RHS, first or last symbol)

Ex. R1) S -> bA

R2) S -> aB

R3) S -> ε

R4) A -> abaS

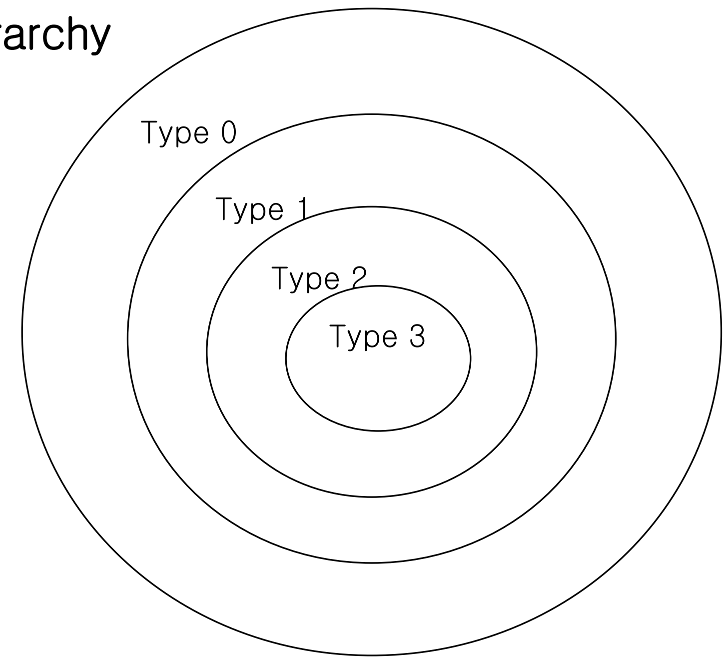
R5) B -> babS

String: abab, baba, abababab, babaabab etc

(abab | baba)* which is an RE

<u>Machine: FSM</u>

Chomsky Hierarchy

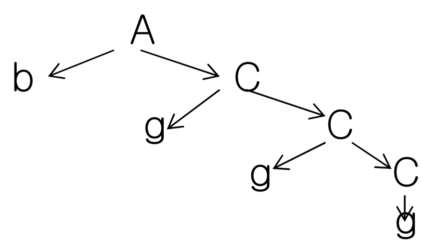


3.4 Left-Recursion and Back-Tracking

Q: How do we build a Parser for CFL?

Initially let's briefly look at again how a parser works.
Ex. A -> Ba | bC
B -> d | eBF
C -> gC | g

Let's Consider a string bggg



➢ Starts from the Top of the Tree and continues to parse until all token are matched. (Top-Down Parser)

➢Now before we build a such parser from the grammar, we have to consider 2 issues

a) Left Recursion

We have to eliminate all left-recursive productions in the grammar.

A left-recursive production is when the LHS of a production occurs as the first symbol on the RHS of the same production

ex. E -> E + T

Why?

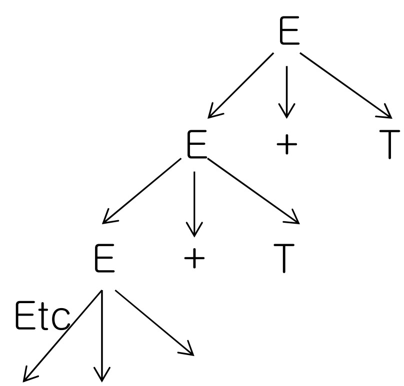
Let's consider the following rules:

R1) E -> E + T

R2) E -> T

R3) T -> id

Given string **a + b + c** , the parser tries to match.



=> the parser does not know when to stop expanding E -> E +T

Since it does not know how many + the string has

Q: How do we eliminate left recursions?

A: By changing the productions as follows:

Assume we have the following productions.

$A \rightarrow A\alpha$

$A \rightarrow \delta$

Steps:

1. Introduce a new Non-terminal A'
2. Change $A \rightarrow \delta A'$
3. $A' \rightarrow \alpha A' \mid \varepsilon$

=>

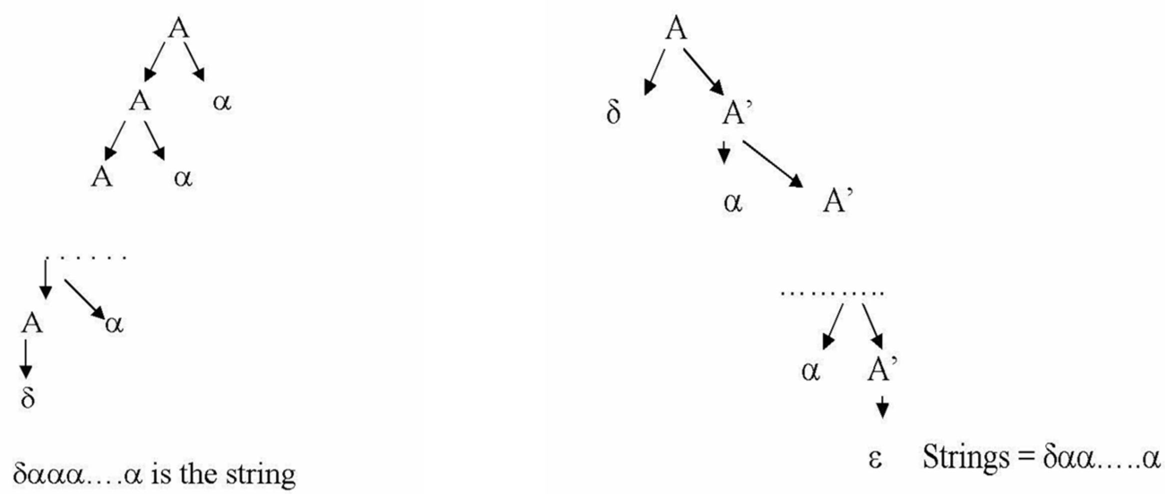
$A \rightarrow \delta A$

$A' \rightarrow \alpha A' \mid \varepsilon$

Let's See what we have done

$A \rightarrow A\alpha$
$A \rightarrow \delta$

$A \rightarrow \delta\ A$
$A' \rightarrow \alpha\ A' \mid \varepsilon$



Note: We have two parser trees with same strings but the shapes are different

Ex. of Removing Left recursion

E -> E + T

E -> T

T -> id

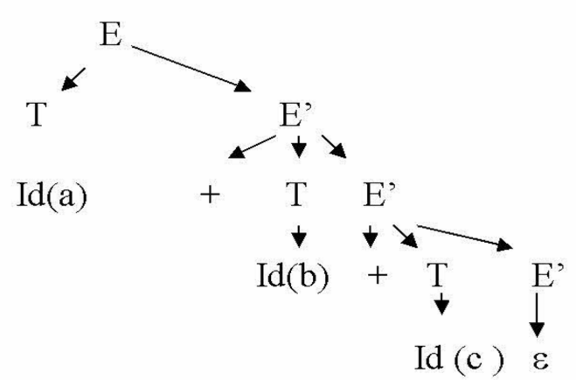
Remove left recursion =>

E -> T E'

E' -> +TE' | ε

T -> id

Let's further consider the string a + b + c



But there can be "indirect (non-immediate)" left-recursion.

Ex.

A -> B C

B -> A C

Thus, we need an algorithm to remove all left-recursions:

Method:

1. Make a list of all NT (ex. In the sequence they occur)
2. For each NT (N) do

   If RHS begins with a NT (A) earlier in the list then

   - Substitute A
   - Remove any direct recursion

Ex. A -> N | β

...

N -> Aɣ

Then replace A =>

N -> Nɣ | βɣ (remove direct left-recursion)

Ex.

R1) E -> E + T   R2) E -> T   R3) T -> E   R4) T -> id

1) List of NT => 1. E, 2.T
2) Direct Left-recursion in (R1) and (R2)
   E -> TE'
   E' -> + TE' | ε
3) Need to replace E in (R3)
   T -> TE'
4) Here is Left-Recursion
   T -> id T'
   T' -> E'T' | ε

Therefore:

R1) E -> TE'
R2) E' -> TE' | ε
R3) T -> id T'
R4) T' -> E'T' | ε
=> E, E', T, T'
Another Round…

b)Backtracking

Backtracking is reparsing of the same/previous tokens

Ex) Assuming the following productions:

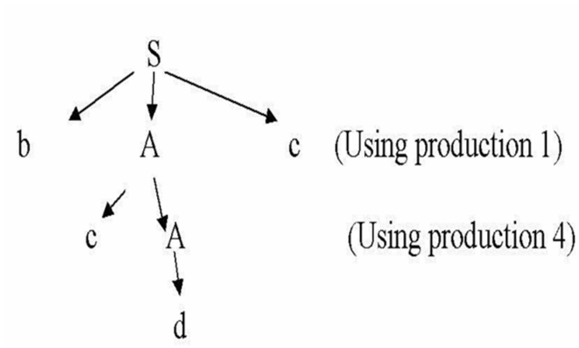
S -> b A c

S -> b A e

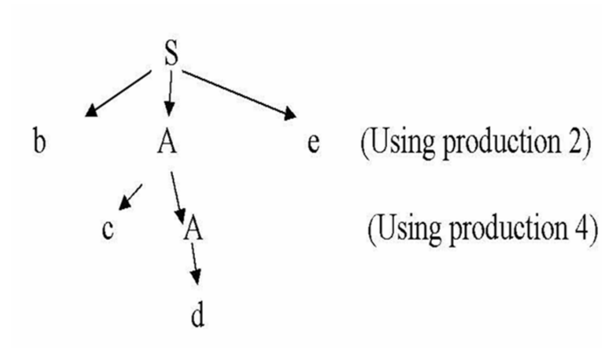
A -> d

A -> c A

and the following string   bcde



Does not work.
String wrong?
=>
Consider other
Possibilities



Works!!

How to solve backtracking? =>

use of Left-Factorization =>

Factor out same symbols of RHSs of the productions for the same Non-terminal

Ex)

R1) S -> b A c

R2) S -> b A e

R3) A --> d

R4) A -> c A

=>

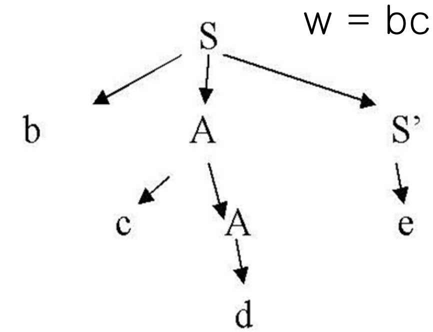
R1) S -> b A S' (Factor out bA from R1 and R2)

R2) S' -> c | e

R3) A -> d

R4) c A

w = bcde



CONCLUSION:

1) We need to eliminate Left-Recursion
2) Remove Back-Tracking

before
constructing the Top-Down parsers

3.5 Top-Down Parsers

a) Recursive Descent Parser (RDP)

The basic idea is that each <u>non-terminal has an associated parsing procedure</u> that can recognize any sequence of tokens generated by that non-terminal.

For example.

1. E -> E + T
2. E -> E - T
3. E -> T
4. T -> id

1.Remove Left-Recursion

E -> TE'
E' -> + TE' | - TE' | ε
T -> id

Productions:
E -> TE'
E' -> + TE' | - TE' | ε
T -> id

```
Procedure E ()
{
  T ();
   E'();
   If not eof marker then
      error-message
}
```

```
Procedure E'()
{
If token = + or - then
        {
        Lexer();
       T();
        E'();
        }
}
```

```
Procedure T();
{
   If token is id then
        lexer()
    else error-message
    (id expected)
}
```

<u>Ex 1) Assume we have a string a+b-c</u>

E -> T E' -> T -> id (a) and lexer()

+ T E' (match + and lexer())

Id (b)

- T E' (match – and lexer())

Id (c)

ε

Ex 2) String a +

E -> T E' -> T -> id (a) and lexer()

+ T E' (match + and lexer())

Error-message (id expected)

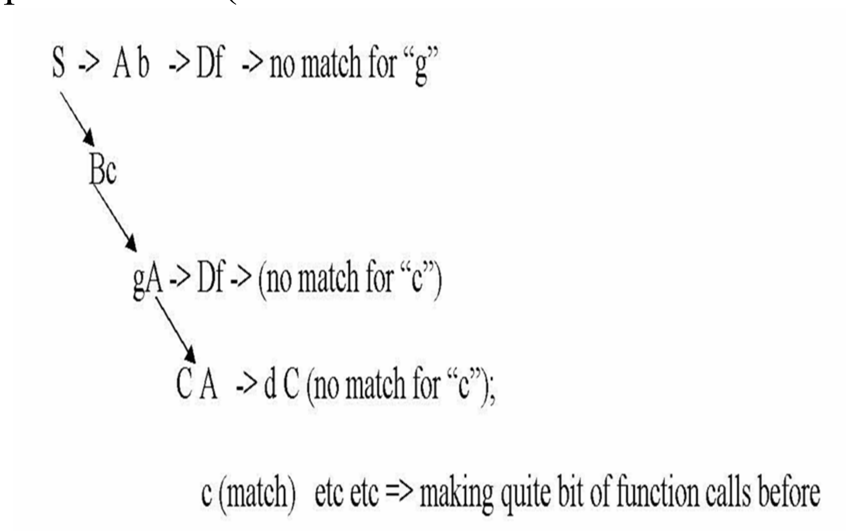
b)Predictive Recursive Descent Parser (PRDP)

A more efficient way of implementing RDP.

Assume we have the following productions (No Left-Recursion or Back-Tracking)

1. S -> Ab | B c
2. A -> Df | CA
3. B -> gA | e
4. C -> dC | c
5. D -> h | i

and a string “gchfc” and parse

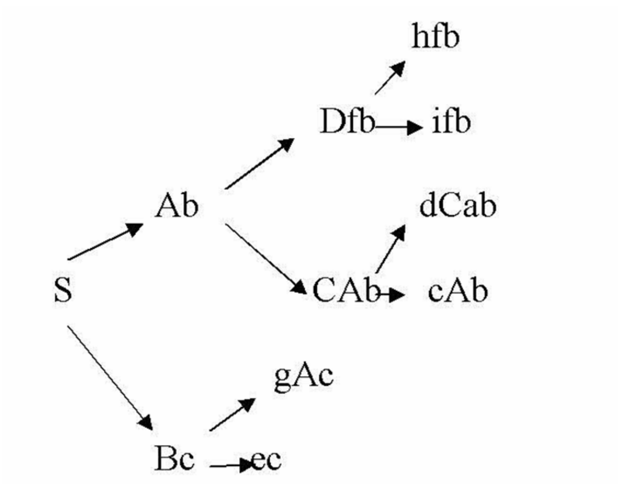


making quite bit of function calls before matching

=> a better way? A more efficient way?

Yes: anticipate what terminal symbols are derivable from each nonterminal symbol on the RHS of productions.

Ex). Let's compute before we construct the parser.



if the parser goes (Ab) =>
 it will encounter terminals {h, i, d,c}
else (BC) => {g, e}

So, before paring look ahead of the sets which way to go:
Ex. string "gchfc" => "g" is in (BC) route, so no need to go to the first route.
<u>These sets are called First sets.</u>
First (Ab) = { h, i, d, c}
First (Bc) = {g, e}

Def: First (α) Consider every string derivable from α by a left most derivation.
If α => β where β begins with some terminal, then that terminal is in First (α).

Computation of First (α)

1) if α begins with a terminal t , then First(α) = t
2) If α begins with a nonterminal A, then First (α) includes First(A) – ε
   – and if A => ε then include First (γ) where α = Aγ
   – and if α => ε, then First(α) includes ε
3) First (ε) = ε

Examples

E -> TE'

E ' -> + TE' | ε

T -> FT'

T' -> * FT' | ε

F -> id | (E)

First (F) = First (id) U First ((E)) = { id, (}

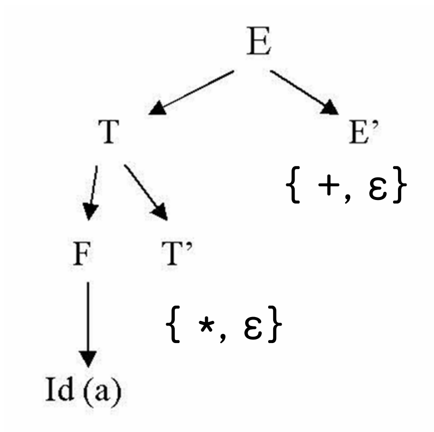
First (T') = First (*FT') | First (ε) = { *, ε }

First (T) = First (FT') = First (F) – ε = {id, (}

First (E') = First (+TE') U First (ε) = { +, ε}

First (E) = First (TE') = First (T) = First (id) U First ((E)) =
{ id, (}

However, there is a problem using just the First sets.
Consider the previous arithmetic grammar example and a string “ a +b”



Next token = + but is not in First (T’) = { *, ε}
Does that mean it is wrong?
NO, because T’ => ε

In that case, we have to consider what can follow after T’ = { + } => acceptable tokens

So, because of the ε, we need to consider also Follow (N = nonterminal) = { terminals that can follow right after N}

Follow (A):

Definition: Follow (A) is the set of all terminal symbols that can come right after A in any sentential form of L(G).

If A comes at the end of, the Follow(A) includes “\$” = end of file marker

Computation:

IF A is the starting symbol, then include \$ in Follow(A)

For all occurrences of A on the RHS of productions do as follows:

Let $Q \to \alpha A \beta$ (means α before A and β after A), then

If β begins with a terminal t, then t is in Follow(A)

If β begins with a nonterminal , then include First (β) – ε

If $\beta \Rightarrow \varepsilon$ or $\beta = \varepsilon$, then include Follow (Q) in Follow(A)

(** we ignore the case Q = A, e.g, $A \to \alpha A \beta$)

Examples

E -> TE'

E' -> + TE' | ε

T -> FT'

T' -> * FT' | ε

F -> id | (E)

Follow (E) = {$} U {) } = { $,)}

Follow (E') = Follow (E) U Follow (E') (ignore) = { $,)}

Follow (T) = First (E') – ε U Follow (E) U Follow (E') = {+, $,)}

Follow (T') = Follow (T) U Follow (T') (ignore) = {+, $,)}

Follow (F) = First (T') – ε U Follow (T) U First (T') – Follow (T')

= {*} U {+ $,)} U {*} U {+, $,)} = { *, +, $,)}

Predictive RDP with First and Follow Sets :

```
Procedure E ()
{
If token in First (E) then
     T();
     E'();
else error-message (token in First
       of (E) expected)
}
```

```
Procedure T()
{
      If token in First (T) then
          F();
          T'();
     else error-message (……)
}
```

```
Procedure E' ()
{
     If token = + then
        Lexer();
        T();
        E'();
    else if token not in Follow E' then
             error-message (……)
}
```

```
Procedure T'()
{
     If token = '*' then
         Lexer();
         F();
         T'();
    else if token NOT in folllow (T') then
           error-message (……..)
}
```

```
Procedure F() {
 …… same ….}
```

Productions:

E -> TE'
E' -> +TE' | ε
T -> FT'
T' -> * FT' | ε
F -> (E) | id

Example:

E -> TE'
E' -> +TE' | ε
T -> FT'
T' -> * FT' | ε
F -> (E) | id

C) Table Driven Predictive Parser

Consist of 3 components:
Parsing table, Stack, Program Driver

Table : to generate a table with all t (columns) and NT (rows)
Ex. For expression grammar

	id	+	*	(	)	$
E	TE'			TE'		
E'		+TE'			ε	ε
T	FT'			FT'		
T'		ε	*FT'		ε	ε
F	id			(E)		

•<u>Stack</u> => well known with pop(), push(), etc

•<u>Driver:</u>

```
Push $ onto the stack
Put end-of-file marker ($) at the end of the input string
Push (Starting Symbol) on to the stack
While stack not empty do
        let t =  TOS symbol and  i=incoming token
        if t = terminal symbol then
              if t=i then
                 pop(t); lexer()
              else error-message (…._
       else begin
              if Table [t, i]  has entry then
                  pop(t);
                  push  Table[t, i] in reverse order
              else error
              end
 endwhile
```

	id	+	*	(	)	$
E	TE'			TE'		
E'		+TE'			ε	ε
T	FT'			FT'		
T'		ε	*FT'		ε	ε
F	id			(E)		

Ex: String b + c

Stack	*Input*	*Action*
$ E	b+c$	pop(E), Push (E', T)
$E'T	b+c$	pop(T), push(T', F)
$E'T'F	b+c$	pop(F), push (id);
$E'T' id	b+c$	pop(id), lexer();
$E'T'	+c$	pop(T'); push (ε)
$E'	+c$	pop(E'); push (E', T, +)
$E'T+	+c$	pop(+), lexer()
$E'T	c$	pop(T), push (T',F)
$E'T'F	c$	pop(F); push (id);
$E'T'id	c$	pop(id), lexer()
$E'T'	$	pop(T'), push (ε)
$E'	$	pop(E'), push (ε)
$	$	Stack empty

Q: How do we construct such a table?

```
For each Non-terminal N do
{
        Let N -> β a typical production
        Computer First (β);
        Each terminal t in First (β) except ε  do
              Table [N, t] = β
       If First (β) has ε then
           For each terminal t in  Follow (N) do
                Table [N, t] = ε
}
```

Example:

E -> TE'
E' -> +TE' | ε
T -> FT'
T' -> * FT' | ε
F -> (E) | id

Computer First Sets:

First (F) = {(, id}
First (T') = (*, ε)
First (T) = First (F) – ε = {(, id}
First(E') = {+, ε}
First (E) = First(T) – ε = {(, id}

If a First set contains ε, then compute Follow sets

Follow (T') = { +, $,)}
Follow (E') = {), $}

	id	+	*	(	)	$
E	TE'			TE'		
E'		+TE'			ε	ε
T	FT'			FT'		
T'		ε	*FT'		ε	ε
F	id			(E)		

Conflict:

In a table driven predictive parser, a conflict occurs if a cell Table [N, t] has More than one entry.

Ex. “Dangling else” problem

1) S -> if C then S E | a | b

2) C -> x | y

3) E -> else S | ε

Let’s try to construct the table in particular for E

First (E) = {else, ε}

Since it contains ε

We need to computer Follow (E) = Follow (S) =

{ $} U First (E) – ε U Follow(E)

= {S} U { else} = {$, else}

							else	$
S								
C								
E							else S ε	ε

Two entries in Table [E, else] => conflict

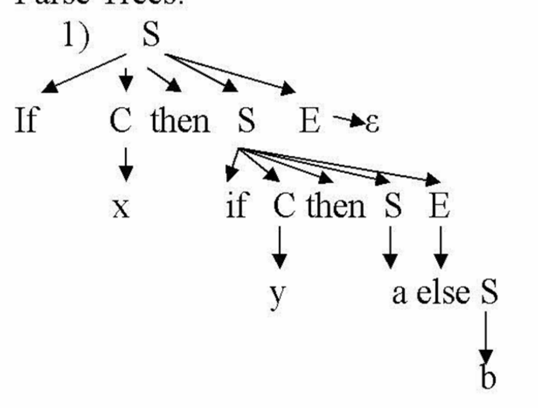
Why Conflict?

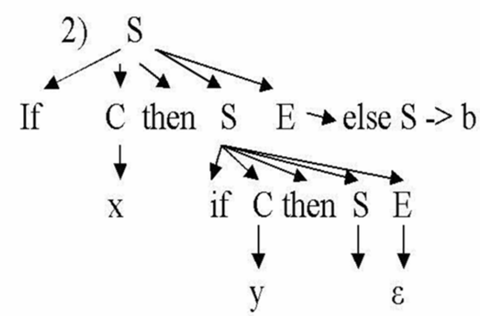
a sample sentence:

If x then
    If y then a
    else b

(where does this else belong????)

Parse Trees:





Grammar is Ambiguous

=> If a grammar is ambiguous, it will create a conflict

END