Master Thesis Experiment Report I

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Abstract

Experiment report on the velocity profile and density gradient in a simple tube

1 Context

We are running a simulation of the Lattice-Boltzmann method for fluid inside a simple tube. The lattice is of size 301x201 and is unobstructed. The top and lower borders are made using point-based bounceback. The streaming step is made by hand in a brute-force fashion using flags (different for inside the tube and the borders) instead of the roll function available from the Numpy library. The left border is an inlet following Zu-He equation and the right border is an outlet.

2 Problem

The theoretical velocity profile is flat and not the expected parabola.

3 Experiment

3.1 Description

We want to check upon the theoretical velocity profile inside the tube and compare it against the real profile and the expected profile using the maximum velocity. The theoretical velocity is obtain using Poiseuille's equation :

$$u(r) = \frac{\Delta P}{4\eta L}(R^2 - r^2) \tag{1}$$

where $u(r)[m/s]$ the velocity with $r[m]$ the distance to the center of the tube radius-wise, $R[m/s]$ the radius of the tube, $\Delta P[Pa]$ the pressure gradient along the tube, $\eta[m^2/s]$ the kinematic viscosity and $L[m]$ the length of the tube section. For our simulation we can re-write the kinematic viscosity as $\eta = \nu * \rho$, with $\nu[m^2/s]$ the viscosity of the fluid (ρ taken here as the mean of a vertical slice at the center coordinate x = 150 of the tube), and the pressure gradient as $\Delta P = c_s^2 * \Delta\rho$ with c_s^2 the lattice speed of sound squared ($c_s[m/s] = \sqrt{\frac{1}{3}}$) and $\Delta\rho$ the density gradient across the tube. From there, using the fact that the profile is a parabola, the maximum velocity u_{max} is obtained at the center of the tube when $r = 0$. We measure u_{max} directly at the end of the simulation and we can derive equation 1 to get the expected velocity profile :

$$u(r) = u_{max}(1 - (\frac{r}{R})^2) \tag{2}$$

Finally, the real profile is measured directly at the end of the runtime.

In addition, we will measure the gradient of the density $\Delta\rho$ at the end of the simulation to see if it is coherent to our expectations (expected to be slightly higher upstream of the tube).

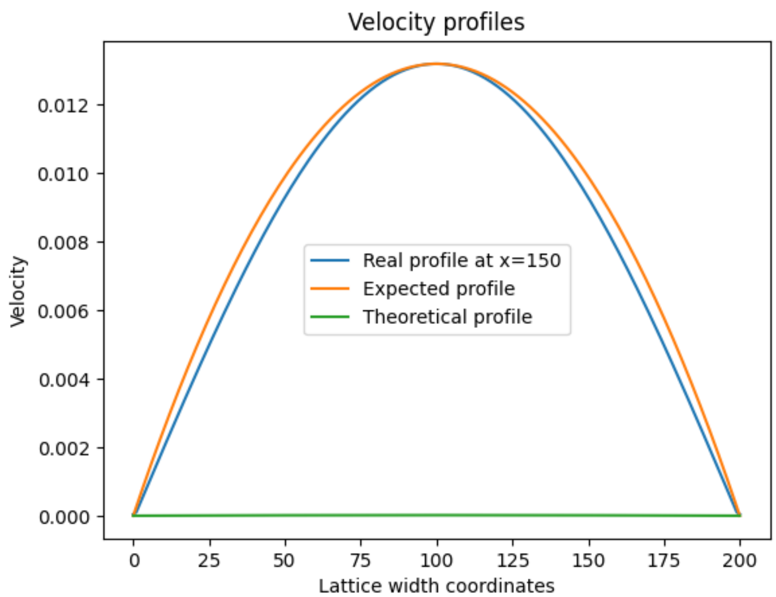

Figure 1: Velocity profiles

3.2 Parameters

The simulation will run for 30'000 iterations. At half the execution time the system removes the inlet and outlet and becomes fully cyclic letting the fluid flow from one side to the other freely. The velocity profiles have been measured after the final iteration at x = 150. The tube measured here goes from x = 50 to x = 301 - 50 = 251 : we started and ended further from the boundaries to avoid any influence of the inlet and outlet even after the system was made cyclic.

4 Results

4.1 Velocity profile

We have the follow-up results for the velocity profiles : Figure 1 shows the real, expected and calculated velocity profiles. The Real and expected profile yield good results but the theoretical profile however is definitely not as expected. We have the following values for each of its parameters from equation 1 taken at the very last iteration :

1. $\Delta\rho = 1.8526416047937033e - 06$
2. $\Delta P = 6.175472015979011e - 07$
3. $R = 100$
4. $\nu = 0.4$
5. ρ at the center $= 1.125075450967681$
6. $L = 201$
7. u_{max} theoretical $= 1.7067600525482712e - 05$

Further investigation is necessary.

4.2 Density Gradient

The density gradient $\Delta\rho$ is illustrated in Figure 2. As expected the upper stream density is slightly higher the the lower-stream one, however we have an unexpected curve. This could be a side effect of a ripple traveling through the tube.

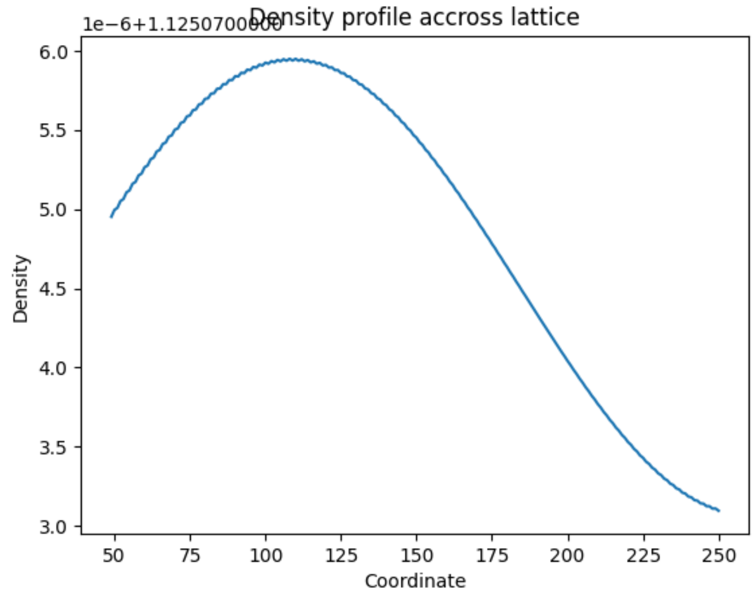

Figure 2: Density gradient

5 Conclusion

The obtained results are not yet as expected, and there is a need to investigate the individual values obtained to better grasp how the simulation is unfolding and why the theoretical velocity profile is not the expected parabola.