

# Master Thesis Experiment Report II

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## Abstract

Experiment report on the PDFs of a bounceback-node and his neighboring site before and after the collision.

## 1 Formalism

The particle density functions are divided into two categories : particles incoming on a node ( $f_{in}$ ) and particles going out of a node ( $f_{out}$ ) for a given node. Bvery node has a coordinate  $(x,y)$  and 9 directions in which the PDFs flow (represented Figure 1) to which we assign a direction vector as illustrated in Figure 2.

### 1.1 Streaming

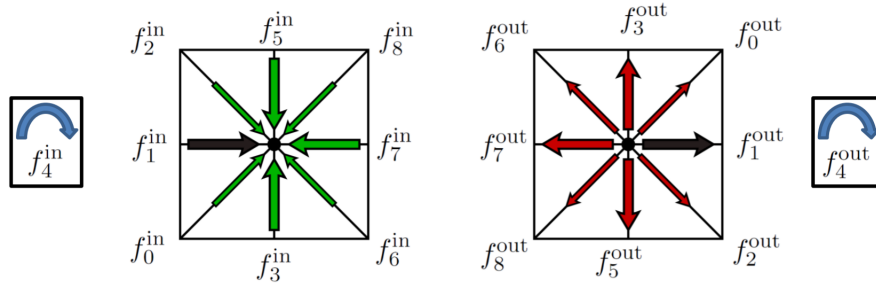


Figure 1: Directional velocity in a D2Q9 lattice for  $f_{in}$  and  $f_{out}$

The streaming step is performed using a directional vector for each direction of each node of the lattice. When doing the streaming step, we need to propagate a PDF value assigned to a specific direction of a node to a neighboring cell following said direction. The population flowing out of a cell ( $f_{out}$ ) will flow in a neighbor cell ( $f_{in}$ ) in the same direction,  $f_{in}$  and  $f_{out}$  having their direction indexes described as in Figure 1. Example : a PDF stored in  $f_7^{out}$  will flow into the cell on its left and enter  $f_7^{in}$ . To do so, we compute :

```
for a direction i :  
next_x_coordinate = x + v[i,0]  
next_y_coordinate = y + v[i,1]  
fin[i,next_x_coordinate,next_y_coordinate] = fout[i,x,y]
```

### 1.2 Bounceback

The bounceback step simply consists in inverting the direction of a PDF in a cell, meaning that for a given PDF in a specific  $f_{in}$  direction, the bounceback step will assign the given PDF to the opposite direction on the  $f_{out}$ . For a given cell at the coordinate  $(x, y)$ , for a direction  $i$ ,  $f_{out}(i,x,y) = f_{in}(8-i,x,y)$ .

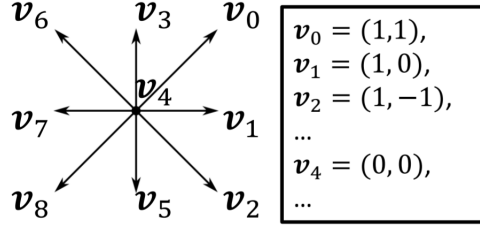


Figure 2: Directions vectors for a node of the lattice

## 2 Context

We are running a Lattice-Boltzmann experiment on a branching lattice. The lattice size is 201x301 and the branching is illustrated in Figure 3. The color scheme represents the defined flags : purple for open parts, blue for bounceback nodes, and yellow for inside obstacles. The purple parts flow normally, the bounceback nodes flow and have a point-based bounceback condition during collision and there is no update inside the obstacles.

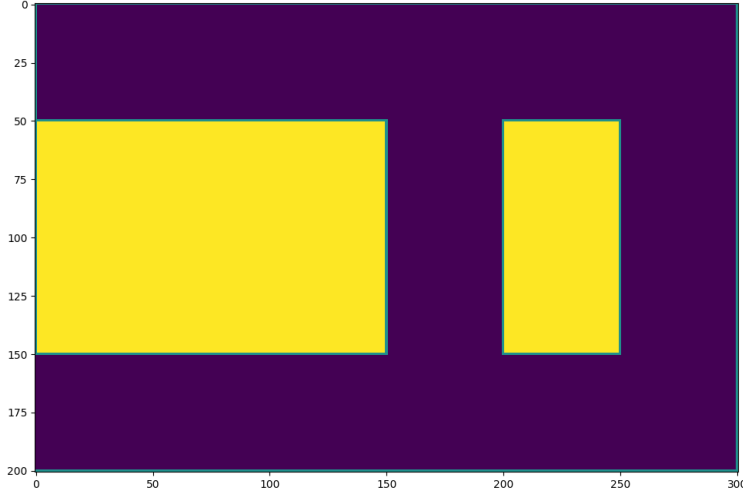


Figure 3: Flags for the branch system of the lattice

The streaming step is made by brute force, going through every node and updating those that have a flag open path or bounceback.

The upper left open path is defined as an inlet (right in front of the bounceback node to avoid leaks made by the cyclic behavior of the streaming step) following the Zu-He method, and the lower left is defined as an outlet.

## 3 Problem

When running the experiment long enough, the bounceback nodes see a constant increase in velocity compared to their neighboring nodes. This phenomena is illustrated in Figure

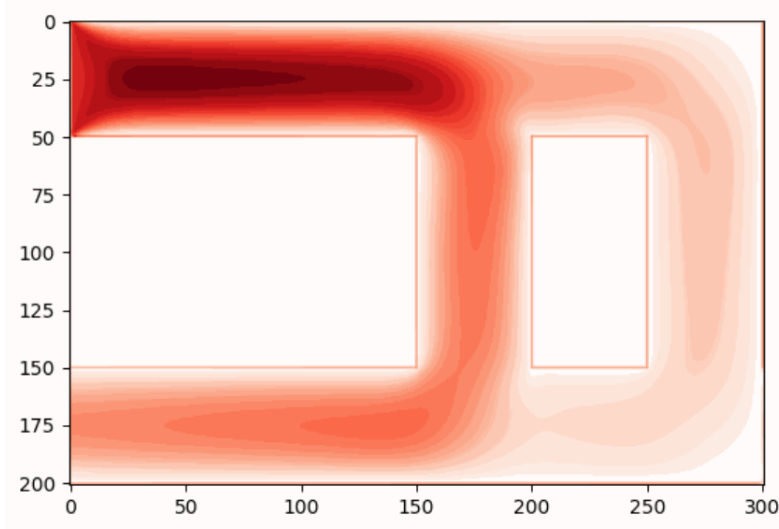


Figure 4: Norm of the velocity over each node of the lattice at 10000 iterations

## 4 Experiments

### 4.1 Experiment 1

#### 4.1.1 Description

We want to assess what is happening inside a bounceback node during the collision process. To do so we checked the directional PDFs for 4 points : one bounceback node and it's three nodes above. Formally :

1. The 3 points above the bounceback node before propagation :  $f_{out}[:,49,49]$ ,  $f_{out}[:,50,49]$ ,  $f_{out}[:,51,49]$
2. the bounceback node after streaming and before collision :  $f_{in}[:,50,50]$
3. the bounceback node after collision :  $f_{out}[:,50,50]$

The simulation has run for approximately 2000 iterations (enough to see the phenomena appear).

#### 4.1.2 Results

A snippet of the results are shown in Figure 5. For further clarity the results of iteration 1865 have been re-drawn in Figure 6.

We can observe that the PDFs have propagated in an incorrect direction in the bounceback node. Analysis is required on the streaming step to assess where the inconsistency comes from.

### 4.2 Experiment 2

#### 4.2.1 Description

We pushed further testing by studying specifically the streaming and bounceback steps. We created a new very small system of dimension  $5 \times 4$ , set bounceback nodes on the top and bottom borders, and went through a few iterations. Every PDF value was set to 0 with the exception of a random set to 1. The main idea here is to follow the value throughout the system as iterations go on to understand the exact behavior of these two steps.

We started a new code from scratch with only the two aforementioned steps and output the results in a generated file. The file shows every direction for every cell correctly drawn to add a better visual understanding of what is going on as illustrated in Figure 7.

```

1865 | [array([0.03194192, 0.12654402, 0.03130778, 0.12555123, 0.50218111,
0.12554181, 0.03085006, 0.12455371, 0.03147897]), array([0.03193379, 0.1265122, 0.03129982, 0.12551921, 0.50205308,
0.12550983, 0.03084209, 0.12452146, 0.03146291]), array([0.03192565, 0.12648029, 0.03129182, 0.12548704, 0.50192448,
0.1254777, 0.03083407, 0.12448904, 0.03145481])) | [0.03194192 0.11111111 0.02777778 0.12551921 0.44444444 0.11111111
0.03083407 0.11111111 0.02777778] | [0.02777778 0.11111111 0.03083407 0.11111111 0.44444444 0.12551921
0.02777778 0.11111111 0.03194192]
1866 | [array([0.03194297, 0.12654807, 0.0313087, 0.12555488, 0.50219611,
0.12554556, 0.03085082, 0.12455716, 0.03147193]), array([0.03193484, 0.12651623, 0.03130073, 0.12552283, 0.50206798,
0.12551356, 0.03084284, 0.12452488, 0.03146386]), array([0.0319267, 0.1264843, 0.03129273, 0.12549063, 0.50193929,
0.12548141, 0.03083481, 0.12449244, 0.03145576])) | [0.03194297 0.11111111 0.02777778 0.12552283 0.44444444 0.11111111
0.03083481 0.11111111 0.02777778] | [0.02777778 0.11111111 0.03083481 0.11111111 0.44444444 0.12552283
0.02777778 0.11111111 0.03194297]
1867 | [array([0.03194402, 0.12655209, 0.03130961, 0.12555849, 0.502211,
0.12554928, 0.03085158, 0.12456058, 0.03147288]), array([0.03193589, 0.12652024, 0.03130164, 0.12552642, 0.50208278,
0.12551725, 0.03084359, 0.12452827, 0.0314648 ]), array([0.03192774, 0.12648828, 0.03129363, 0.12549419, 0.50195399,
0.12548508, 0.03083555, 0.1244958, 0.03145669])) | [0.03194402 0.11111111 0.02777778 0.12552642 0.44444444 0.11111111
0.03083555 0.11111111 0.02777778] | [0.02777778 0.11111111 0.03083555 0.11111111 0.44444444 0.12552642
0.02777778 0.11111111 0.03194402]
1868 | [array([0.03194507, 0.12655609, 0.03131052, 0.12556208, 0.50222579,
0.12555298, 0.03085233, 0.12456397, 0.03147382]), array([0.03193693, 0.12652422, 0.03130254, 0.12552998, 0.50209746,
0.12552092, 0.03084433, 0.12453163, 0.03146573]), array([0.03192878, 0.12649224, 0.03129452, 0.12549773, 0.50196857,
0.12548872, 0.03083628, 0.12449913, 0.03145762])) | [0.03194507 0.11111111 0.02777778 0.12552998 0.44444444 0.11111111
0.03083628 0.11111111 0.02777778] | [0.02777778 0.11111111 0.03083628 0.11111111 0.44444444 0.12552998
0.02777778 0.11111111 0.03194507]
1869 | [array([0.03194611, 0.12656007, 0.03131142, 0.12556564, 0.50224046,
0.12555664, 0.03085307, 0.12456733, 0.03147475]), array([0.03193796, 0.12652818, 0.03130343, 0.12553352, 0.50211204,
0.12552456, 0.03084506, 0.12453496, 0.03146666]), array([0.03192981, 0.12649618, 0.03129541, 0.12550124, 0.50198305,
0.12549234, 0.03083701, 0.12450244, 0.03145854])) | [0.03194611 0.11111111 0.02777778 0.12553352 0.44444444 0.11111111
0.03083701 0.11111111 0.02777778]

```

Figure 5: PDFs for three nodes, from iteration 1865 through 1869

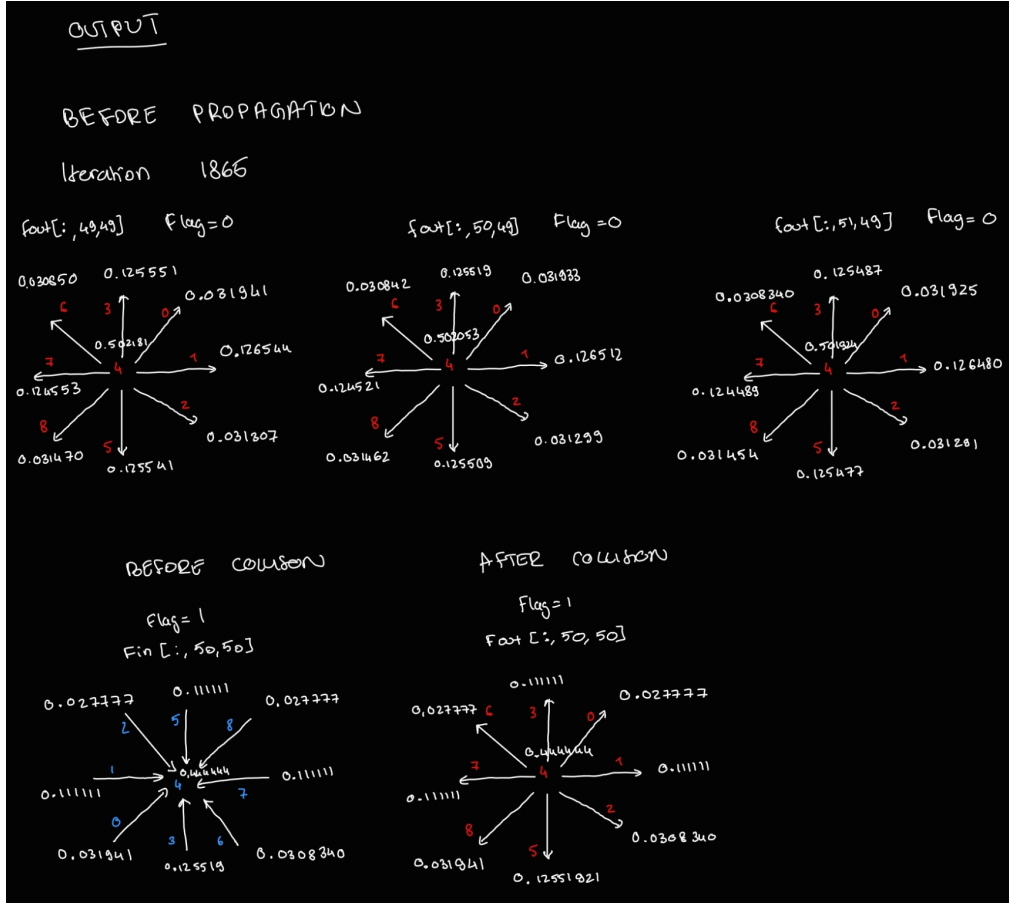


Figure 6: PDFs for three nodes at iteration 1865, simplified view

#### 4.2.2 Results

Note : the displayed values in Figures 8 and 9 (streaming occurred .../ bounceback occurred ...) do not reflect the actual displayed value because of a display convention. All the coordinates detailed below follow the lattice displayed on the figures and not the actual coordinate values.

Figure 8 shows the result of the streaming step. We can see that the value of interest situated in the  $f_{out}$  at the coordinate  $[2,1]$  in direction 2 correctly migrates in the  $f_{in}$  of the cell  $[3,2]$  in direction



```

After Streaming, Before Collision
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 | 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 | 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 | 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0

0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 | 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 | 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 | 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0

0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 | 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 | 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 | 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0

0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 1.0 0.0 0.0 | 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 | 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 | 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0

Streaming moved value on :
- fout[2,3,1] to fin[2,4,0]

-----

Iteration : 2
After Collision, Before Streaming
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 | 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 | 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 | 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0

0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 | 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 | 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 | 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0

0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 | 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 | 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 | 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0

0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 | 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 1.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 | 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 | 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0

Bounceback occured on :
- fin[2,4,0] to fout[6,4,0]

```

Figure 9: Output file bounceback step on the system

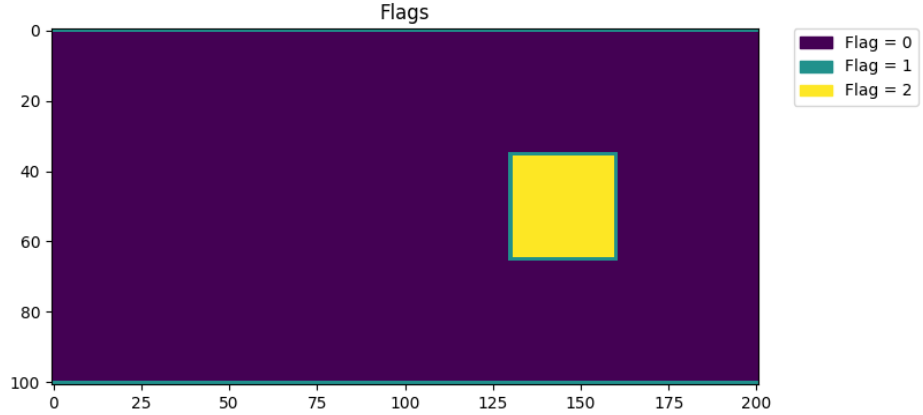


Figure 10: Flags of the system : 0 = open path, 1 = bounceback nodes, 2 = inside obstacle / no flow

We monitor all the PDFs of a single node known to be a bounceback node (here node with coordinate [130,50]) through every iteration as well as its horizontal and vertical velocity.

#### 4.3.2 Results

Figure 11 shows the results obtained.

We can observe a steady constant increase of the velocity inside of the node instead of a constant value. Further investigation of the velocity is needed.

```

1 Node : [130,50], maxiter = 5000
2
3 Fin | Fout   Horizontal Velocity  Vertical velocity
4
5 2 5 8 | 6 3 0      u[0]      u[1]
6 1 4 7 | 7 4 1
7 0 3 6 | 8 5 2
8
9 -----
10 iteration : 0
11
12 0.02778 0.11111 0.02778 | 0.02778 0.11111 0.02778      0.00000      0.00000
13 0.11111 0.44444 0.11111 | 0.11111 0.44444 0.11111
14 0.02778 0.11111 0.02778 | 0.02778 0.11111 0.02778
15

```

(a) Output format and Iteration 0

```

-----
iteration : 1881
-----
0.03050 0.11111 0.02778 | 0.03050 0.11111 0.02778      0.01624      0.00000
0.12218 0.44444 0.11111 | 0.12218 0.44444 0.11111
0.03050 0.11111 0.02778 | 0.03050 0.11111 0.02778
-----

```

(b) Iteration 1881

```

-----
iteration : 4623
-----
0.03253 0.11111 0.02778 | 0.03253 0.11111 0.02778      0.02793      -0.00000
0.13034 0.44444 0.11111 | 0.13034 0.44444 0.11111
0.03253 0.11111 0.02778 | 0.03253 0.11111 0.02778
-----

```

(c) Iteration 4623

Figure 11: Node monitoring

## 5 Conclusion

We eliminated potential sources for the problem and will now continue investigating to find the exact issue.