Master Thesis Experiment Report I

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Abstract

Experiment report on the velocity profile and density gradient in a simple tube

1 Context

We are running a simulation of the Lattice-Boltzmann method for fluid inside a simple tube. The lattice is of size 301x201 and is unobstructed. The top and lower borders are made using point-based bounceback. The streaming step is made by hand in a brute-force fashion using flags (different for inside the tube and the borders) instead of the roll function available from the Numpy library. The left border is an inlet following Zu-He equation and the right border is an outlet.

2 Issue

The theoretical velocity profile is flat and not the expected parabola.

3 Experiments

3.1 Experiment 1

3.1.1 Description

We want to check upon the theoretical velocity profile inside the tube and compare it against the real profile and the expected profile using the maximum velocity. The theoretical velocity is obtain using Poiseuille's equation :

$$u(r) = \frac{\Delta P}{4\eta L}(R^2 - r^2) \tag{1}$$

where $u(r)[m/s]$ the velocity with $r[m]$ the distance to the center of the tube radius-wise, $R[m/s]$ the radius of the tube, $\Delta P[Pa]$ the pressure gradient along the tube, $\eta[m^2/s]$ the kinematic viscosity and $L[m]$ the length of the tube section. For our simulation we can re-write the kinematic viscosity as $\eta = \nu * \rho$, with $\nu[m^2/s]$ the viscosity of the fluid (ρ taken here as the mean of a vertical slice at the center coordinate x = 150 of the tube), and the pressure gradient as $\Delta P = c_s^2 * \Delta\rho$ with c_s^2 the lattice speed of sound squared ($c_s[m/s] = \sqrt{\frac{1}{3}}$) and $\Delta\rho$ the density gradient across the tube. This gives us the following function :

$$u(r) = \frac{c_s^2 * \Delta\rho}{4\nu\rho L}(R^2 - r^2) \tag{2}$$

From there, using the fact that the profile is a parabola, the maximum velocity u_{max} is obtained at the center of the tube when $r = 0$. We measure u_{max} directly at the end of the simulation and we can derive equation 1 to get the expected velocity profile :

$$u(r) = u_{max}(1 - (\frac{r}{R})^2) \tag{3}$$

Finally, the real profile is measured directly at the end of the runtime.

In addition, we will measure the gradient of the density $\Delta\rho$ at the end of the simulation to see if it is coherent to our expectations (expected to be slightly higher upstream of the tube).

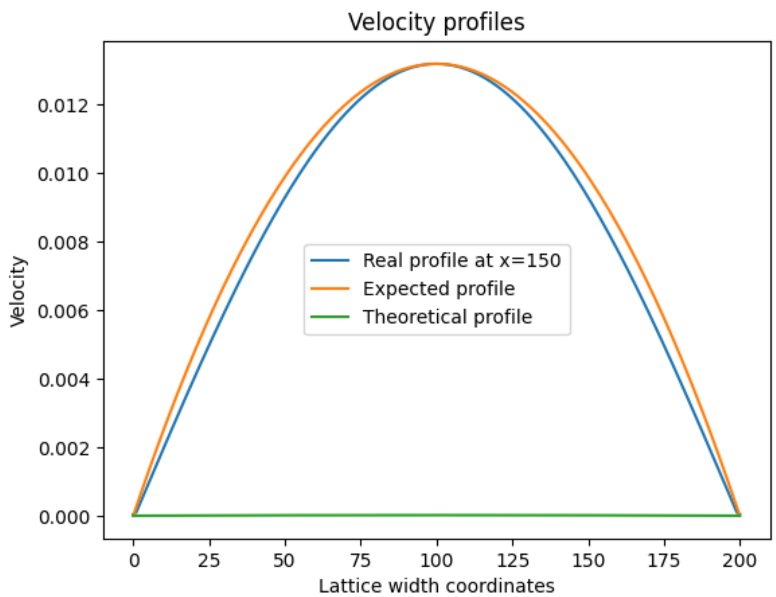


Figure 1: Velocity profiles

3.1.2 Parameters

The simulation will run for 30'000 iterations. At half the execution time the system removes the inlet and outlet and becomes fully cyclic letting the fluid flow from one side to the other freely. The velocity profiles have been measured after the final iteration at x = 150. The tube measured here goes from x = 50 to x = 301 - 50 = 251 : we started and ended further from the boundaries to avoid any influence of the inlet and outlet even after the system was made cyclic.

3.1.3 Velocity profile Results

We have the follow-up results for the velocity profiles : Figure 1 shows the real, expected and calculated velocity profiles. The Real and expected profile yield good results but the theoretical profile however is definitely not as expected. We have the following values for each of its parameters from equation 1 taken at the very last iteration :

1. $\Delta\rho = 1.8526416047937033e - 06$
2. $\Delta P = 6.175472015979011e - 07$
3. $R = 100$
4. $\nu = 0.4$
5. ρ at the center $= 1.125075450967681$
6. $L = 201$
7. u_{max} theoretical $= 1.7067600525482712e - 05$

Further investigation is necessary.

3.1.4 Density Gradient Results

The density gradient $\Delta\rho$ is illustrated in Figure 2. As expected the upper stream density is slightly higher the the lower-stream one, however we have an unexpected curve. This could be a side effect of a ripple traveling through the tube.

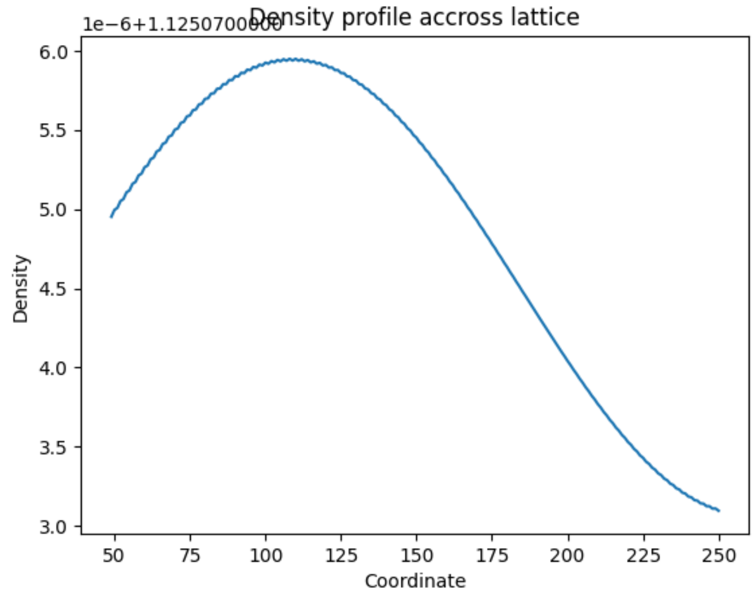


Figure 2: Density gradient

3.2 Experiment 2

3.2.1 Description

For the second experiment, we slightly modified the system. Firstly, we reduced the lattice size to 201x101 instead of 301x201. Additionally, we let the system flow until the last iteration (20'000 in this case) and removed the transition between inlet/outlet and cyclic. This should hopefully remove any ripple effect that came from the system-state transition. Finally, we changed the velocity input. Until now, the input defines a constant input of velocity ($u = 0.04$) in every iteration at the input site. We then had to wait for the system to stabilize to get the expected velocity parabola. What we did instead was define the velocity parabola directly as initial horizontal velocity (with a max value of 0.04) and velocity input on the inlet. Figure 3 shows the initial/input velocity gradient on the full lattice. This should help the system stabilize much faster.

Furthermore, we changed the calculation of the fluid kinematic velocity ν. In our system, the Reynolds number Re is fixed at 10 and used to calculate the viscosity using the following formula :

$$Re = \frac{\rho * u * D}{\nu} \equiv \nu = \frac{\rho * u * D}{Re} \tag{4}$$

where ρ is the fluid density (set initially as $\rho = 1$) and D the characteristic length. Until now the characteristic length was defined as the radius of the tube. We changed that to be the diameter instead.

3.2.2 Velocity profile results

Figure 4 shows the resulting velocity profiles. The theoretical velocity now shows a curve, which indicates that we are progressing towards the objective. The amplitude is still off, as there is a factor 2 difference as compared to the expected and real profiles. More research will be necessary to determine the cause of it.

The parameters we obtained in the last iteration are the following :

1. $\Delta\rho = 0.0021821604444840492$
2. $\Delta P = 0.0021821604444840492$

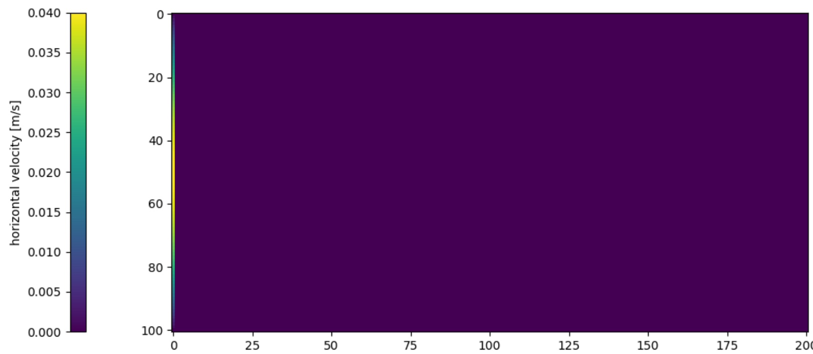


Figure 3: Initial horizontal velocity of the lattice

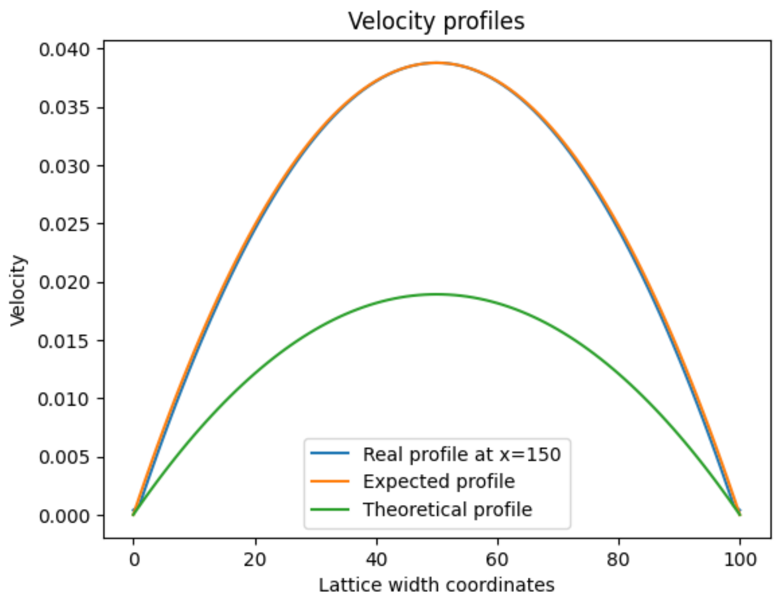


Figure 4: Horizontal velocity on the left side of the lattice

3. $R = 50$
4. $\nu = 0.2$
5. ρ at the center $= 1.1672435283160743$
6. $L = 101$
7. u_{max} theoretical $= 0.019281135098025287$

The density and pressure gradient now have the correct order of magnitude. The viscosity has updated correctly with its new calculation as explained in the previous section. And factor 2 shown in Figure 4 comes from the maximal velocity (0.019 instead of 0.04).

3.2.3 Density gradient results

The newly obtained density profile is shown in Figure 5. The results is now a much better straight line with a magnitude difference of 1e-3. The ripple from the transition of the system has disappeared and the gradient is what we expected.

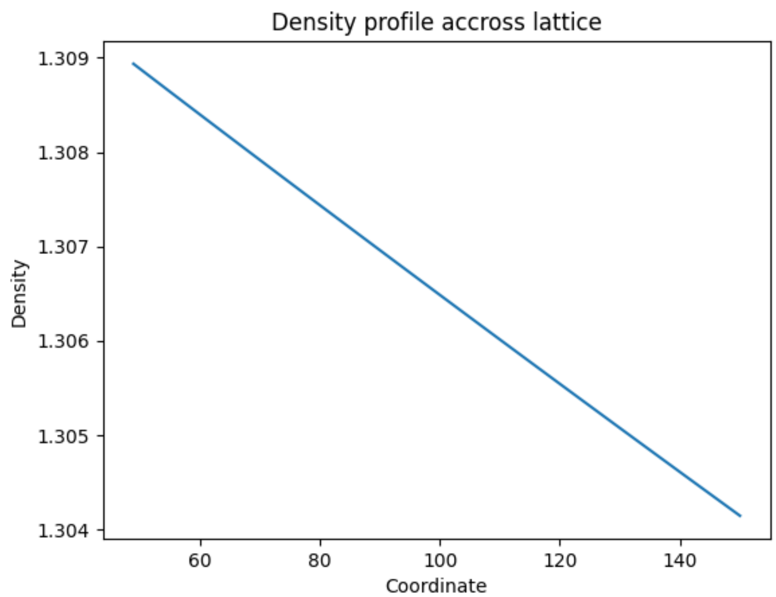


Figure 5: Horizontal velocity on the left side of the lattice

3.3 Experiment 3

3.3.1 Description

To further assess where the erroneous factor comes from, we tried a different approach. Up to now the formula used for the theoretical velocity profile detailed in equation 2 has been used for a tube. Instead we will try using another equation : the Poiseuille velocity profile for a plane. The equation goes :

$$u(r) = \frac{\Delta P}{2\eta L}[y(H-y)] \tag{5}$$

where H[m] is the height of the lattice (H = 101 in this experiment) and y[m] is the distance to one of the top/bottom edges of the lattice. Applying the same changes we made on equation 1 we get :

$$u(r) = \frac{c_s^2 * \Delta\rho}{2\nu\rho L}[y(H-y)] \tag{6}$$

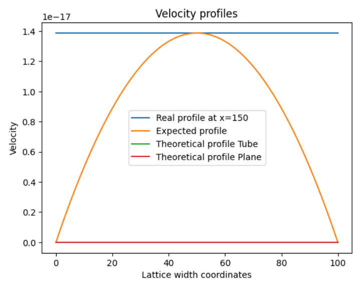
We will monitor the evolution of the velocity and density profiles to assess their convergence to the expected parabola/line.

In addition, we cleaned up properly the code as to remove any unnecessary test used for troubleshooting to greatly reduce the execution time.

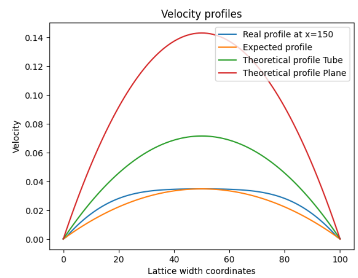
3.3.2 Results

Numerical results :
Iteration 2000 :

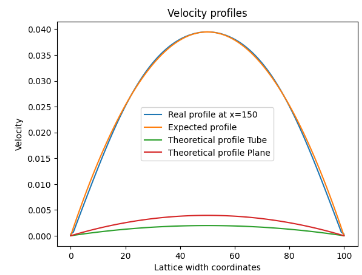
1. $\Delta\rho = 0.004185256402624349$
2. $\Delta P = 0.0013950854675414495$
3. $R = 50$
4. $H = 101$
5. $\nu = 0.404$



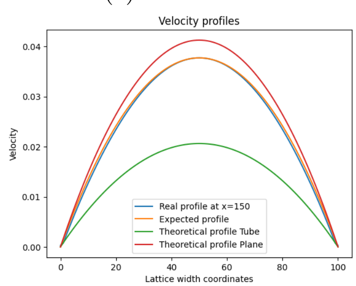
(a) Iteration 0



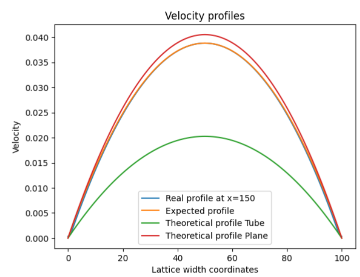
(b) Iteration 500



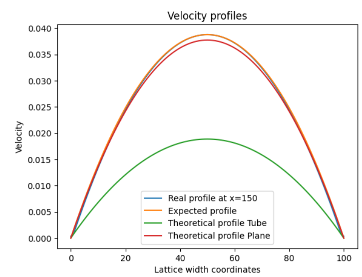
(c) Iteration 1'000



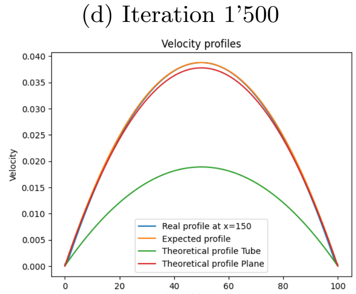
(d) Iteration 1'500



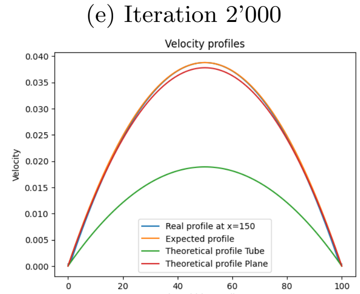
(e) Iteration 2'000



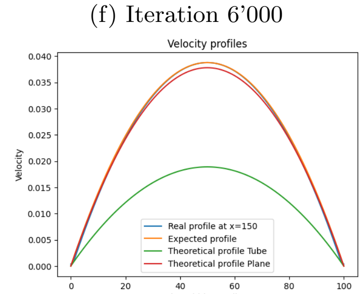
(f) Iteration 6'000



(g) Iteration 10'000

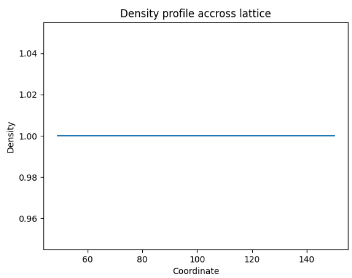


(h) Iteration 14'000

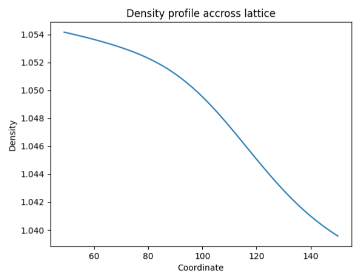


(i) Iteration 15'000

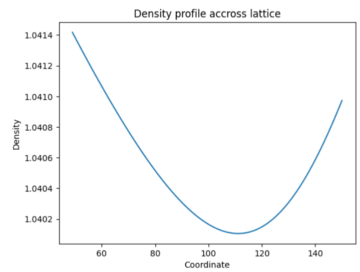
Figure 6: Velocity Profile evolution for 15'000 total iterations



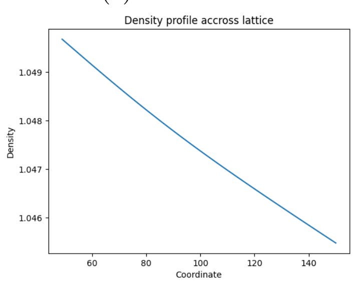
(a) Iteration 0



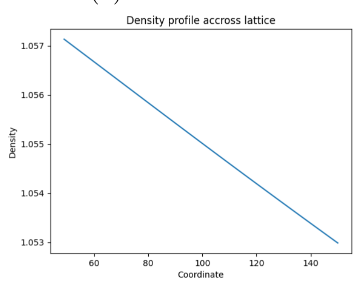
(b) Iteration 500



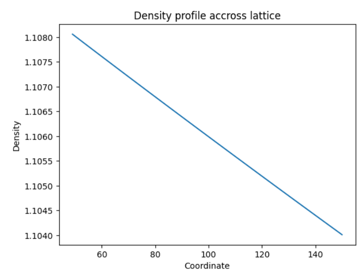
(c) Iteration 1'000



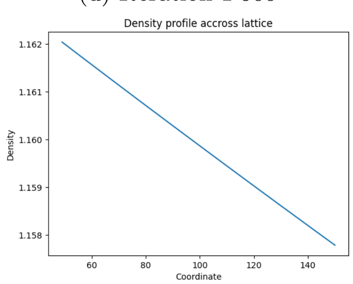
(d) Iteration 1'500



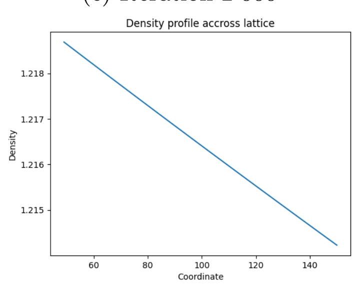
(e) Iteration 2'000



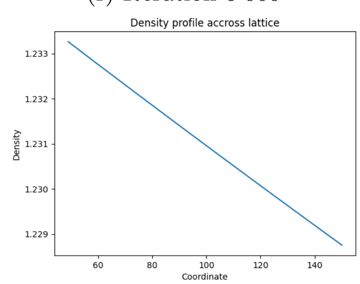
(f) Iteration 6'000



(g) Iteration 10'000



(h) Iteration 14'000



(i) Iteration 15'000

Figure 7: Density Profile evolution for 15'000 total iterations

6. ρ at the center = 1.0550118393931964

7. $L = 101$

8. u_{max} theoretical Tube = 0.02025446423548077

Iteration 4000 :

1. $\Delta\rho = 0.003985818683550857$

2. $\Delta P = 0.0013286062278502855$

3. ρ at the center = 1.0800096060720537

4. u_{max} theoretical tube = 0.01884282193325715

The results of the experiment are shown in Figure 6 for the velocity profiles and in Figure 7 for the density profile. We can see in Figure 6 the newly added formula for the velocity profile is quite coherent with the expected results. The convergence happens fast - almost complete at iteration 2'000 as we started with a good initial velocity profile, which helped speed up the process a lot.

4 Conclusion

After trial and error, we ended up having a good velocity profile, both real and theoretical. We can now look further into the experiment.