INSTITUTE OF TECHNOLOGY

TALLAGHT

Higher Certificate in Science
Bachelor of Science
Bachelor of Science (Honours)

IT Management
Computing

Full Time

Semester Three : January 2014

DISCRETE MATHEMATICS 2

Internal Examiners
Dr. Noel Gorman

External Examiners
Mr. Dermot Logue

Day	**Monday**
Date	**13th January 2014**
Time	**15.30 - 17.30**

Instructions to Candidates

1. ANSWER 4 OF THE 5 QUESTIONS.
2. ALL QUESTIONS ARE WORTH 25 MARKS.
3. START EACH QUESTION ON A NEW PAGE.

4. GRAPH PAPER AND LOG TABLES ARE PROVIDED ON REQUEST.

Question 1 **(25 Marks)**

(a) The GCD of two positive integers is their greatest common divisor. Use the Euclidean algorithm to find,

GCD(14812, 3335) **[8 Marks]**

(b) Divide through by the given amount c in each of the congruence identities below to obtain a second **true** congruence identity:

(i) $348 \equiv 282 \bmod 11 \quad c = 6$ (ii) $3405 \equiv 2685 \bmod 36 \quad c = 15$

[5 Marks]

[You may assume the division rule

$$ac \equiv bc \bmod m \;\Rightarrow\; a \equiv b \bmod \left(m/d\right), \qquad \text{where} \qquad d = \mathrm{GCD}(c,m)\,.]$$

(c) The modular encoding $y \equiv (19x + 8) \bmod 26$ was used to construct the encrypted message

N G X L W K D X

(i) Express x in terms of y (that is, find the decoding function). **[5 Marks]**

(ii) Decypher the message. **[7 Marks]**

The following tables may be helpful:

Table of Inverses mod 26

Least Residue	1	3	5	7	9	11	15	17	19	21	23	25
Inverse	1	9	21	15	3	19	7	23	11	5	17	25

Table of mod 26 correspondence for the letters of the alphabet

A	B	C	D	E	F	G	H	I	J	K	L	M
1	2	3	4	5	6	7	8	9	10	11	12	13

N	O	P	Q	R	S	T	U	V	W	X	Y	Z
14	15	16	17	18	19	20	21	22	23	24	25	0

Question 2 **(25 Marks)**

(a) Find the inverse, z, of the number 5 in mod 13 arithmetic, by solving,

$$5z \equiv 1 \bmod 13.$$

[5 Marks]

(b) Solve the following congruence identities for x, finding the **least residues** in each case:

(i) $4x + 8 \equiv 3 \bmod 11$ (ii) $-8 \equiv (9x - 2) \bmod 17$ **[8 Marks]**

[You may assume the division rule

$$ac \equiv bc \bmod m \Rightarrow a \equiv b \bmod \left(m/d \right), \quad \text{where} \quad d = \mathrm{GCD}(c, m).]$$

(c) Given a remainder r between 0 and $m-1$, we can generate a whole number between a and b, inclusive, by carrying out the steps indicated in the following table:

r	Division r/m	Scaling $\times(b-a+1)$	Truncation	Addition of a

Use the pseudorandom number generator,

$$r_{i+1} \equiv (23r_i + 9) \bmod 37, \qquad r_1 = 15,$$

to generate three pseudorandom remainders r_2, r_3 and r_4. Then use r_2, r_3 and r_4 to generate three random whole numbers between 3 and 18, inclusive, making use of the arithmetic operations shown in the table above. **[12 Marks]**

Question 3 **(25 Marks)**

(a) Calculate AB for the following two matrices:

$$A = \begin{pmatrix} 3 & -4 \\ -2 & 1 \end{pmatrix}, \qquad B = \begin{pmatrix} -2 & 5 \\ 4 & 3 \end{pmatrix}.$$

[6 Marks]

(b) Determine if the matrix,

$$M = \begin{pmatrix} 1 & 1 & 3 \\ 3 & -1 & -1 \\ -2 & 2 & 3 \end{pmatrix},$$

has an inverse, or not. (Do **not** attempt to find an inverse.) **[7 Marks]**

(c) Write the system of linear equations

$$\begin{aligned} x - 3y - z &= 11 \\ 2x - 2y - 5z &= 4 \\ 4x - 4y - 8z &= 12 \end{aligned}$$

in matrix form and use Gaussian elimination to solve them for x, y and z. **[12 Marks]**

Question 4 **(25 Marks)**

(a) The table below is useful for applying the Bisection Method for solving $f(x) = 0$ in the interval $x = a$ to $x = b$.

a	b	$m = \dfrac{a+b}{2}$	$f(a)$	$f(m)$

Taking $f(x) = x^4 - 35$, and using the interval $x = 2.42$ to $x = 2.48$ as the starting interval, copy the table into your exam script and fill in the first 4 rows, in order to find an approximate value for the fourth root of 35. **[12 Marks]**

(b) Newton's method for the solution of the equation $f(x) = 0$ says that if x_n is the n^{th} approximation to the solution then,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

should be a better approximation, where $f'(x_n)$ is the derivative of $f(x)$ at x_n. The following table is useful for carrying out the calculations needed to apply Newtons Method.

x_n	$f(x_n)$	$f'(x_n)$	$\dfrac{f(x_n)}{f'(x_n)}$

Using this method to finding an approximate value for the fifth root of 40, we take

$$f(x) = x^5 - 40 \qquad \text{for which} \qquad f'(x) = 5x^4.$$

Starting with $x_1 = 2.15$, copy the above table into your exam script and use Newton's formula to find x_2 and x_3, working to a minimum of 4 decimal places throughout. Calculate the value of $f(x)$ at x_3, to check how good an approximation it is to the true solution. **[13 marks]**

Question 5 **(25 Marks)**

(a) A multi-national company compares yearly sales figures in several countries and the corresponding decrease in GDP of those countries. The table below shows the percentage decrease in GDP and the sales figures in millions of dollars for last year.

Decrease, X, in GDP (%)	Sales, Y, in millions of dollars
0.65	305
0.72	280
0.83	232
0.92	224
1.18	170
1.22	157

Use linear interpolation to estimate the sales figure the company should expect if there is a decrease in GDP of 1.1% in a particular country. **[10 Marks]**

[You may assume that the equation of the line through the points (x_1, y_1) and (x_2, y_2) is,

$$y = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1) + y_1 .]$$

(b) The trapezoidal rule for approximating $\int_a^b f(x)dx$ is

$$\int_a^b f(x)dx \approx \frac{1}{2}h\left[y_1 + 2y_2 + 2y_3 + \ldots + 2y_n + y_{n+1}\right] ,$$

where the interval $a \le x \le b$ is divided into n equal subintervals of width h by the points $x_1, x_2, x_3, \ldots, x_{n+1}$, such that,

$$a = x_1,\ b = x_{n+1}, \qquad x_{i+1} = x_i + h,\ i = 2,\ldots,n, \qquad h = \frac{b-a}{n} = x_{i+1} - x_i$$

and $y_i = f(x_i)$, for $i = 1,\ldots, n+1$. Apply this rule to approximate

$$\int_0^2 \sqrt{1+x^2}\, dx ,$$

correct to 4 decimal places, using 5 equal subintervals. You may use the table headings shown to help your calculations.

x	y	Coeff.	Coeff. $\times y$

[15 Marks]