INSTITUTE OF TECHNOLOGY
TALLAGHT

Bachelor of Science

IT Management

ACCS

Semester Three : May 2016

Discrete Mathematics 2

Internal Examiners
Ms. Gerard Stockil

External Examiners
Mr. Dermot Boyle
Mr. Thomas Dowling

Day	**Wednesday**
Date	**18 May 2016**
Time	**19.00 -21.00**

Instructions to Candidates

This examination paper contains FIVE questions.

All questions carry equal marks.

Answer any FOUR questions.

Please start each question on a new page.

Question 1

(a) The modular encryption equation $y \equiv (11x + 12) mod\ 26$ is used to encipher a message. By solving the equation, $x \equiv (11x + 12) mod\ 26$ identify the two letters of the alphabet that are encrypted as themselves in this scheme. **(8%)**

You may assume the following rule

$$ac \equiv bc \bmod m \;\; => a \equiv \; b \bmod \left(\tfrac{m}{d}\right), \text{ where } d = \text{GCD } (c, m)\ .$$

You can use the table of Mod 26 correspondences for the letters in the alphabet shown below.

Table of mod 26 correspondence for the letters of the alphabet.

A	B	C	D	E	F	G	H	I	J	K	L	M
1	2	3	4	5	6	7	8	9	10	11	12	13

N	O	P	Q	R	S	T	U	V	W	X	Y	Z
14	15	16	17	18	19	20	21	22	23	24	25	0

(b) Given a remainder r between 0 and $m - 1$, we can generate a whole number between a and b, inclusive, by carrying out the steps indicated in the following table: **(9%)**

r	Division r/m	Scaling $\times(b - a + 1)$	Truncation	Addition of a

Use the pseudorandom number generator,

$$r_{i+1} \equiv (\,9r_i + 3)\bmod 17, \qquad\qquad r_1 = 1$$

to generate three pseudorandom remainders r_2, r_3 and r_4. Then use r_2, r_3 and r_4 to generate three random whole numbers between 3 and 12 inclusive, making use of the arithmetic operations shown in the table above.

(c) The GCD of two positive integers is their greatest common divisor. **(8%)**

Use the Euclidean algorithm to find,

$$\text{GCD}(\ 2442\ \ ,1406\ \ ).$$

Use a table, or other clear method, to show the steps in the algorithm.

Question 2

The two most frequency occurring letters in an encrypted message are R and S respectively. Because E and T are the most frequently occurring letters in English, and the method of encryption is a modular cypher of the form

$$y \equiv (ax + b) mod\ 26$$

the coefficients of a and b must satisfy the simultaneous congruence equations.

$$5a + b \equiv 18\ mod\ 26$$
$$20a + b \equiv 19\ mod\ 26$$

(a) By solving these congruence identities show that $a = 7$ and $b = 9$ .Clearly show your method. You may assume the following rule **(11%)**

$$ac \equiv bc \bmod m \;\; => a \equiv \;\; b \bmod \left(\frac{m}{d}\right), \;\; \text{where} \;\; d = \text{GCD} \;\; (c, m) \;.$$

(b) Construct the decryption map $x \equiv cy + d\ mod\ 26$, (solve for x). **(8%)**

You may use the following information

Table of Inverses mod 26.

Least Residue	**1**	**3**	**5**	**7**	**9**	**11**	**15**	**17**	**19**	**21**	**23**	**25**
Inverse	1	9	21	15	3	19	7	23	11	5	17	25

Table of mod 26 correspondence for the letters of the alphabet.

A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**	**L**	**M**
1	2	3	4	5	6	7	8	9	10	11	12	13

N	**O**	**P**	**Q**	**R**	**S**	**T**	**U**	**V**	**W**	**X**	**Y**	**Z**
14	15	16	17	18	19	20	21	22	23	24	25	0

(c) Decypher the message HYVXMS . **(6%)**

Question 3

(a) The table below is useful for applying the Bisection Method for solving $f(x) = 0$ in the interval $x = a$ to $x = b$. **(12%)**

a	b	$m = \dfrac{a+b}{2}$	$f(a)$	$f(m)$

Taking $f(x) = x^4 - 7$ and using the interval $x = 0$ to $x = 4$ as the starting interval, copy the table into your exam script and fill in the first four rows, in order to find an approximate value to the fourth root of 7.

(b) Newton's method for the solution of the equation. $f(x) = 0$ states that if x_n is the n^{th} approximation to the solution then, **(13%)**

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

should be a better approximation, where $f'(x_n)$ is the derivative of $f(x)$ and x_n. The following table is useful for carrying out the calculations needed to apply Newton's Method.

x	$f(x_n)$	$f'(x_n)$	$\dfrac{f(x_n)}{f'(x_n)}$

Use this method to find an approximate value for x, given

$$f(x) = 2x^3 - 9 \quad \text{for which } f'(x) = 6x^2$$

(i) Starting with $x = 3$, copy the above table into your exam script and use Newton's formula to find x_2 and x_3 to a minimum of four decimal places throughout.

(ii) Also calculate the value of $f(x) = 0$ using a calculator.

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Question 4

(a) A record of windspeed in Tallaght on 17th March 2016 is shown. Time is shown in hours, and windspeed is shown in km per hour (km/h). The speed at 06:00 is not given. **(10%)**

Windspeed	5 km/h	3 km/h	????	5 km/hr	11 km/hr	16 km/h
TIME	00.00	03.00	06.00	09.00	12.00	15.00

(i) Use linear interpolation to estimate the windspeed when the time is 06.00.

(ii) Use linear interpolation to estimate the windspeed when the time is 13.00.

In both calculations clearly show the formula used and the values you use in it.

You may assume that the equation of the line through the points (x_1, y_1) and (x_2, y_2) is

$$y = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1) + y_1$$

(b) The trapezoidal rule for approximating $\int_a^b f(x)dx$ is **(15%)**

$$\int_a^b f(x)dx \approx \frac{1}{2}h\,[y_1 + 2y_2 + 2y_3 + \ \dots + 2y_n + 2y + y_{n+1}\,],$$

where the interval $a \le x \le b$ is divided into n equal subintervals of width h by the points $x_1, x_2, x_3, \dots, x_{n+1}$., such that,

$$a = x_1, b = x_{n+1}, \ \ x_{i+n} = x_i + h\,, \qquad i = 2, \dots, n$$
$$h = \frac{b-a}{n} = x_{i+1} - x_i$$

and $y_i = f(x_i)$, for $i = 1, \cdots n+1$. Apply this rule to approximate

$$\int_2^{4.5} \frac{12}{(3+x)}dx$$

correct to 4 decimal places, using 5 equal subdivisions. You may use the table headings shown to help your calculations.

x	y	Coeff.	Coeff. × y

Question 5

(a) Consider the following matrices

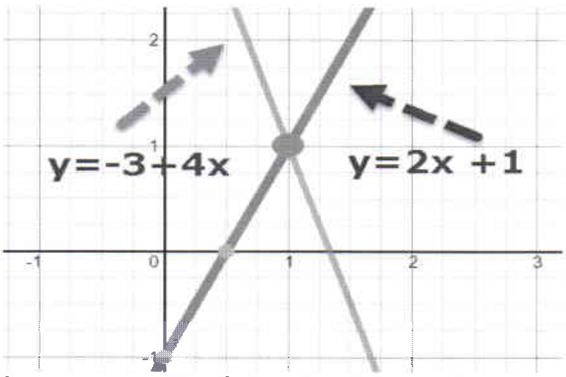
$$A = \begin{pmatrix} 4 & 7 \\ 2 & -1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 4 \\ 3 & -2 \end{pmatrix}$$ **(8%)**

(i) Find $A + B$.**(1%)**
(ii) Find $3A + B$.**(1%)**
(iii) Find AI where I is the identity matrix. **(2%)**
(iv) Find AB . **(2%)**
(v) Is matrix multiplication commutative in general, yes or no? Illustrate your answer by finding the multiplication of BA. **(2%)**

(b) (i) What is the condition for an $n \times n$ matrix to have an inverse?

(ii) Determine if the following matrix $\begin{pmatrix} 3 & 1 & 4 \\ -2 & 2 & 1 \\ 1 & 0 & 1 \end{pmatrix}$ has an inverse, or not. (Do **not** attempt to find the inverse). **(5%)**

(c) Two linear equations are plotted and their equations are given in the following diagram. The equations are $\boldsymbol{y = -3 + 4x}$ and $\boldsymbol{y = 2x + 1}$ **(5%)**



(i) Write the equations in matrix form $\boldsymbol{Ax = b}$. Where **A** is a 2×2 matrix, $\boldsymbol{x} = \binom{x}{y}$ and **b** is a 2×1 matrix. **(2%)**

(ii) Find the solution by matrix methods, that is find the values of $\boldsymbol{x} = \binom{x}{y}$. **(3%)**

You can use the following information.

In the case of a 2×2 matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then the inverse , if it exists, is given by

$$A^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

(d) Write the following system of linear equations **(7%)**

$$\begin{array}{rrrcl} 1x & +1y & -1z & = & 9 \\ 0x & +1y & +3z & = & 3 \\ -1x & +0y & -2z & = & 2 \end{array}$$

in matrix form as an augmented matrix

Perform suitable operations on the rows to form the following matrix

$$\left[\begin{array}{ccc|c} 1 & a & b & c \\ 0 & 1 & d & e \\ 0 & 0 & 1 & f \end{array}\right]$$

where $a, b, c, d, e\ f, g$ are integers. Show the operations you used to do this and clearly state the values for a, b, c, d, e and f.