Institute of Technology Tallaght

Bachelor of Science

IT Management

ACCS

Semester Three : May 2015

Discrete Maths 2 (Out of Semester)

Internal Examiners
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Day **Wednesday**
Date **20th May 2015**
Time **19:00-21:00**

Instructions to Candidates

Answer any four questions, all questions carry equal marks.

Question 1

(a) The GCD of two positive integers is their greatest common divisor. **8%**

Use the Euclidean algorithm to find,

$$\text{GCD}(\ 189764703\ , 99507\ ).$$

Use a table, or other clear method, to show the steps in the algorithm.

(b) The letters of the alphabet are encrypted as follows, **5%**

$$y \equiv (7x - 10)\ mod\ 26$$

Two letters are encrypted as themselves using this encryption. Find these two letters. Use the table of mod 26 correspondences later on in this question to assist you if you wish. Show your method.

(c) The modular encoding **5%**

$$y \equiv (\,11\,x + 19)\ \text{mod}\ 26$$

was used to construct the following encrypted message.

VUOBL

(i) Express x in terms of y (that is, find the decoding function).

(ii) Decipher the message. **7%**

The following tables may be helpful:

Table of Inverses mod 26.

Least Residue	1	3	5	7	9	11	15	17	19	21	23	25
Inverse	1	9	21	15	3	19	7	23	11	5	17	25

Table of mod 26 correspondence for the letters of the alphabet.

A	B	C	D	E	F	G	H	I	J	K	L	M
1	2	3	4	5	6	7	8	9	10	11	12	13

N	O	P	Q	R	S	T	U	V	W	X	Y	Z
14	15	16	17	18	19	20	21	22	23	24	25	0

Question 2

(a) **5%**

Divide through by the given amount c in each of the congruences below to obtain a second true congruence:

(i) $150 \equiv 33\ mod\ 9$, $c = 3$

(ii) $2408 \equiv 616 \mod 32$, $c = 8$

You may assume the division rule
$ac \equiv bc \bmod m \;\; => a \equiv \; b \bmod \left(\frac{m}{d}\right)$, where $d = \mathrm{GCD}\ (c, m)$.

(b) Solve the following congruence equations for x, find the least residue in each case: **8%**

(i)

$$5x + 20 \equiv \; 19 \bmod 11$$

(ii)

$$-7 \equiv 10x + 9 \bmod 19$$

You may assume the division rule
$ac \equiv bc \bmod m \;\; => a \equiv \; b \bmod \left(\frac{m}{d}\right)$, where $d = \mathrm{GCD}\ (c, m)$.

(c) Given a remainder r between 0 and $m - 1$, we can generate a whole number between a and b, inclusive, by carrying out the steps indicated in the following table: **12%**

r	Division r/m	Scaling $\times(b-a+1)$	Truncation	Addition of a

Use the pseudorandom number generator,

$$r_{i+1} = (\, 15r_i + 23) \bmod 31, \qquad r_1 = 4$$

to generate three pseudorandom remainders r_2, r_3 and r_4. Then use r_2, r_3 and r_4 to generate three random whole numbers between 6 and 12 inclusive, making use of the arithmetic operations shown in the table above.

Question 3

(a) The table below is useful for applying the Bisection Method for solving $f(x) = 0$ in the interval $x = a$ to $x = b$. **12%**

a	b	$m = \frac{a+b}{2}$	$f(a)$	$f(m)$

Taking $f(x) = x^5 - 10$ and using the interval $x = 1$ to $x = 2$ as the starting interval, copy the table into your exam script and fill in the first four rows, in order to find an approximate value to the fifth root of 10.

(b) Newton's method for the solution of the equation. $f(x) = 0$ states that if x_n is the n^{th} approximation to the solution then, **13%**

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

should be a better approximation, where $f'(x_n)$ is the derivative of $f(x)$ and x_n. The following table is useful for carrying out the calculations needed to apply Newton's Method.

x	$f(x_n)$	$f'(x_n)$	$\frac{f(x_n)}{f'(x_n)}$

Use this method to find an approximate value for the cube root of 9, we take

$$f(x) = x^3 - 9 \quad \text{for which } f'(x) = 3x^2$$

(i) Starting with $x = 3$, copy the above table into your exam script and use Newton's formula to find x_2 and x_3 to a minimum of four decimal places throughout.

(ii) Also calculate the value of $f(x) = 0$, to check how good an approximation it is to the true solution.

Question 4

(a) The Sunspot Number between June 1st 2014 and June 15th 2014 is shown below. Data is missing for three dates June 7, June 12 and June 13 **10%**

Date	Sunspot Number
Sunday June 1, 2014	44
Monday June 2, 2014	43
Tuesday June 3, 2014	51
Wednesday June 4, 2014	44
Thursday June 5, 2014	60
Friday June 6, 2014	82
Saturday June 7, 2014	???
Sunday June 8, 2014	98
Monday June 9, 2014	108
Tuesday June 10, 2014	112
Wednesday June 11, 2014	120
Thursday June 12, 2014	
Friday June 13, 2014	???
Saturday June 14, 2014	95
Sunday June 15, 2014	51

(i) Use linear interpolation to estimate the Sunspot Number on June 7.

(ii) Use linear interpolation to estimate the Sunspot Number on June 13. No data is available for Thursday June 12, 2014.

In both calculations clearly show the formula used and the values you use in it.

You may assume that the equation of the line through the points (x_1, y_1) and (x_2, y_2) is $\quad y = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1) + y_1$

(b) The trapezoidal rule for approximating $\int_a^b f(x)dx$ is **15%**

$$\int_a^b f(x)dx \approx \frac{1}{2}h\,[y_1 + 2y_2 + 2y_3 + \ \dots + 2y_n + 2y + y_{n+1}\,],$$

where the interval $a \le x \le b$ is divided into n equal subintervals of width h by the points $x_1, x_2, x_3, \dots, x_{n+1}$., such that,

$$a = x_1, b = x_{n+1},$$
$$x_{i+n} = x_i + h\,, \qquad i = 2, \dots, n$$

and $y_i = f(x_i)$, for $i = 1, \cdots n + 1$. Apply this rule to approximate

$$\int_2^8 \frac{\sqrt{x}}{(x+1)}dx$$

Correct to 4 decimal places, using 5 equal subdivisions. You may use the table headings shown to help your calculations.

x	y	*Coeff.*	*Coeff.* × x

Question 5

(a) **11%**

(1) Calculate AB for the following two matrices (4%)

$$A = \begin{pmatrix} 6 & -2 \\ 3 & -5 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 4 \\ -2 & 3 \end{pmatrix}$$

(2) Now calculate or write down the matrix product $A\,I$. (2%)
where $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(3) Consider the following potential matrix operations, state which ones are **defined** , that is valid and allowed by definition, **there is no need to calculate them.** (Just state which of (a) to (e) , if any, is defined.) 5%

(a)
$$\begin{pmatrix} 6 & -2 \\ 3 & -5 \end{pmatrix} \times \begin{pmatrix} 6 & -2 \\ 3 & -5 \end{pmatrix}$$

(b)
$$3 \times \begin{pmatrix} 6 & -2 \\ 3 & -5 \end{pmatrix} - 5 \times \begin{pmatrix} 1 & 4 \\ -2 & 3 \end{pmatrix}$$

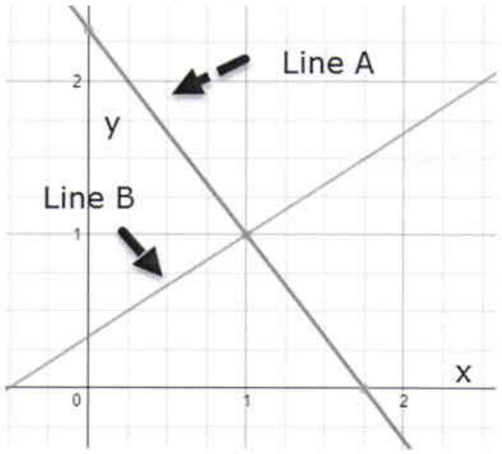
(c)
$$\begin{pmatrix} 6 & -2 \\ 3 & -5 \end{pmatrix}^T \times \begin{pmatrix} 6 & -2 \\ 3 & -5 \end{pmatrix}$$

(d)
$$(1\ 2\ 3) \times (4\,5\,6)$$

(e)
$$\begin{pmatrix} 8 & 1 \\ 6 & -2 \\ 3 & -5 \end{pmatrix} \times \begin{pmatrix} 6 & 1 \\ 12 & 3 \\ 6 & -2 \\ 3 & -5 \end{pmatrix}$$

The question continues on the next page.

(b) A plot of two linear equations is shown in the following graph. The lines are called Line A and Line B. **9%**



The equations may be written as

$$4x + 3y = 7$$
$$-2x + 3y = 1$$

(1) From the graph what is the solution to these two linear equations? (1%)

(2) Which one of the two equations is the equation for Line A and which one is the equation for line B. (2%)

(3) If these linear equations are written in matrix form $X = B$, what are the matrix values of or$A$, X and B? (4%)

(4) Find the inverse of the A matrix and confirm that the solution is the same as the graphical solution. Show your work. (4%)

Note : You may assume that the inverse of the matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is given by $A^{-1} = \frac{1}{\Delta}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ where $\Delta = \det(A)$ is the determinant of the matrix A.

(c) Without attempting to find solutions, determine if the following system of linear equations has a unique solution or not. Show your work. **5%**

$$\begin{array}{cccccc} 1x & -3y & +1z & = & 4 \\ 2x & -8y & +8z & = & -2 \\ -6x & +3y & -15z & = & 9 \end{array}$$