Notes on the Implicit Solutions Homework

Goals: This problem will give you practice working with implicit solutions. In particular, we will learn how to derive differential equation (DE) from a given implicit solution. We will also study what the Existence and Uniqueness Theorem tells us about uniqueness of solutions and when uniqueness fails. We will use Maple to plot solutions and see this "bad behavior."

Directions: For this problem you will need to print out and turn in your Maple sheet.

Problem 1. *The curve defined by the equation*

$$x^{\frac{2}{3}} + t^{\frac{2}{3}} = 1$$

is called the astroid.

a) *Use Maple to create a plot of the astroid. Use Maple to plot the equation*

$$|x|^{\frac{2}{3}} + |t|^{\frac{2}{3}} = 1$$

on the interval $-1 \leq x \leq 1$. To do so, first load the 'plots' package in Maple. Use the 'implicitplot' command, and add the option 'numpoints=10000' at the end, otherwise you'll miss some important features.)

To plot an implicit curve $g(t, x) = 1$ in Maple (we'll call $g(t, x)$ 'g'), we type:

`with(plots):`
- *calls the plots package*

`implicitplot(g=1,t=a..b,x=c..d)`
- *plots $g = 1$ on the range $a \leq t \leq b$ and $c \leq x \leq d$*

`implicitplot(g=1,t=a..b,x=c..d,numpoint=X)`
- *plots $g = 1$ on the range $a \leq t \leq b$ and $c \leq x \leq d$ using X points*

b) *Find a first order differential equation in normal form (i.e., $x' = f(t, x)$) for which the astroid is an implicit solution. Note that you cannot solve for x explicitly, because the graph in part a) is not a graph of a function.*

c) *Determine where the Existence and Uniqueness Theorem guarantees that the initial value problem to your DE in part a) with initial conditions $x(t_0) = x_0$.*

d) *Use your plot in part a) to help explain where the existence and uniqueness theorem doesn't apply.*