

February 17, 2025

Diff Eq 1: SMAT 365
Practice Exam 1

Name: _____

Directions:

- No technology (other than something to write with)
- Show all work on all problems

Academic integrity pledge: By signing below, I pledge that I have followed the above instructions and therefore have neither given nor received any aid on this exam. I recognize that giving or receiving aid on an exam lessens the value of my Spelman degree.

Signature: _____

1. (10 points) Consider the differential equation:

$$x' = t - tx$$

- (a) (8 points) Use separation of variables to find a general solution.

- (b) (2 points) What is the limit of $x(t)$ as $t \rightarrow \infty$?

2. (10 points) Show that $x_1(t) = e^{3t}$ and $x_2(t) = e^{-2t}$ are a solutions to the differential equation:

$$x'' - x' - 6x = 0.$$

3. (10 points) Consider the following differential equation:

$$x' = x^2 (x^2 - 4x + 3).$$

(a) (3 points) What are the equilibrium solutions of this differential equation?

(b) (5 points) Draw a phase line that includes labelled equilibria and arrows that indicate the direction of flow. Determine the stability of the equilibrium solutions.

(c) (2 points) Given the initial condition, $x(0) = 2$, find the limit of the solution as $t \rightarrow \infty$. Briefly explain your answer.

4. (10 points) Solve the following initial value problem using the integrating-factor method:

$$x' = 2x + te^{2t}, \quad x(0) = 1$$

5. (10 points) Newton's Law of Cooling. The temperature of a cup of coffee, initially at $T(0) = 210$ F, is placed in a 70 F room.

(a) (3 points) Set up an initial value problem for $T(t)$.

(b) (2 points) Find the solution to the IVP.

(c) (5 points) If the temperature of the coffee has dropped to 185 F after five minutes, at what time will the coffee reach a nice drinkable temperature of 160 F?