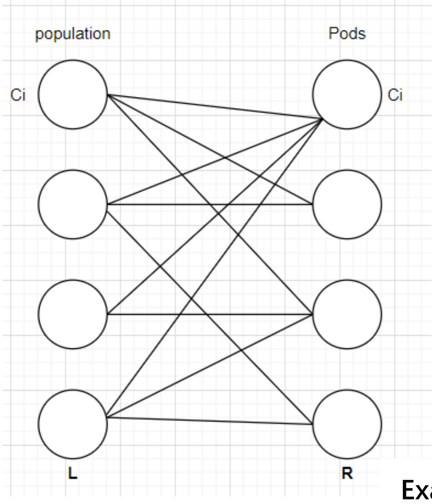
Question 1

A directed graph $G = (V, E)$ could be constructed where $v \in V$ is the city of Krypton and $e \in E$ is the time cost between two cities. For each city C_i, Using Breadth-first search to obtain the minimum time cost $t(C_i, C_j)$ from city C_i to city C_j.

Then, constructing a bipartite graph which has a set of $v \in L$ representing the population of each city and a set of $v \in R$ representing the number of pods that each city has. Connecting the two vertexes from L, R respectively only when minimum time cost $t(C_i, C_j)$ is less than X days. It could be represented by following diagram.

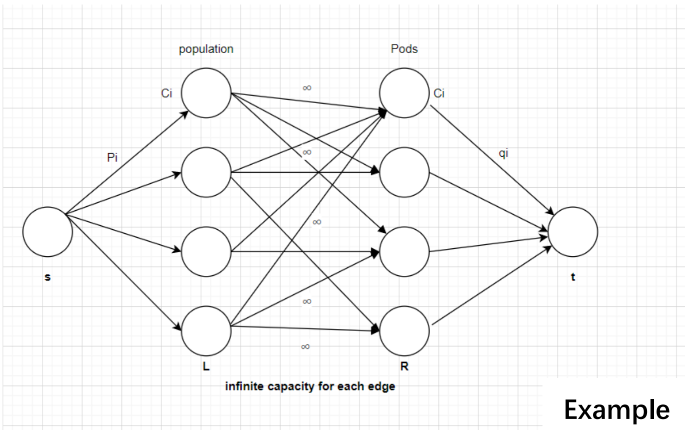


Example

Where set L is the set of population of each city and set R is the number of pods of each city.

To obtain the largest population that could take the pods to earth, we need to find the maximum matching of G. Thus, constructing a network flow G' that respect to G. The capacity of the path from s to the vertex in L and the path from the vertex in R to sink t should be the population of the city and the number of pods of each city. It makes sense because source s can only "transfer" P_i to C_i where P_i is the population of C_i and each vertex in set R could only "transfer" q_i to sink t where q_i is the number of pods of C_i.

In addition, the capacity of edge between L and R should be infinity because there is not such a limitation about the maximum population flow between each city. Therefore, G' could be represented by following diagram.



Example

Finally, applying ***Ford-Fulkerson algorithm*** to obtain the maximum flow of G' and the number of edge from L to R is the maximum number of invaders the Earth will have to deal with.