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Question 1

To compute M^n, we can recursively compute $M^{\lfloor n/2 \rfloor}$. If n is odd, $M^n = x(x^2)^{\lfloor n/2 \rfloor}$. If n is even, $M^n = (x^2)^{\lfloor n/2 \rfloor}$. The time complexity could be reduced by repeating square.
For example,

$$x * x = x^2$$
$$(x^2)^2 = x^4$$
$$(x^4)^2 = x^8$$

In such way, we compute x^8 and using only three multiplication. For M^n, it has at most $\log n$ times multiplications.
Pseudo code:

```
 if M == 0:
    return 0.0
num = 1

while n:
    if n & 1:
        res *= M    // when n is odd
    M *= M
    n >>= 1         // n/2
return res
```

This method is the implementation of divide and conquer. It is using $O(\log n)$ times multiplications.