Question 4

(a)

To compute $< 1,0,0,0, \dots ,1 > * < 1,0,0, \dots ,0,1 >$, we need to compute $P(x) * P(x) = (1 + x^{k+1})^2 = 1 + 2x^{k+1} + x^{2(k+1)} \quad = 1 + 0 * x^1 + \dots + 2x^{k+1} + \dots + x^{2(k+1)}$

Therefore, the convolution of $< 1,0,0,0, \dots ,1 > * < 1,0,0, \dots ,0,1 >$ is

$$< 1, \underbrace{0, \dots ,0}_{k}, 2, \underbrace{0, \dots ,0}_{k}, 1 >$$

(b)

Sequence $A =< 1,0,0 \dots ,0,1 >$ could be written as $P(x) = 1 + x^{k+1}$. The length of the sequence is $k + 2$. We evaluate it at all complex roots of unity which is $P\left(w_{k+2}^{k}\right)$

$$\begin{aligned} DFT(A) &=< P(w_{k+2}^{0}), P(w_{k+2}^{1}), \dots , P\left(w_{k+2}^{k+1}\right) > \\ &=< 1 + (w_{k+2}^{0})^{k+1}, 1 + (w_{k+2}^{1})^{k+1}, \dots ,1 + (w_{k+2}^{k+1})^{k+1} > \\ &=< 2, 1 + (w_{k+2}^{k+1}) \ , \dots ,1 + (w_{k+2}^{(k+1)^2}) \quad > \end{aligned}$$