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Question 2

Let $n = x^{100}$, therefore $n^2 = x^{200}$. This problem could be described in such way

$$Pc(n) = P(n) * P(n)$$
$$= (A0 + A1n + A2n^2)^2$$

The square of the polynomial $P(n)$ of degree 2 is a polynomial $Pc(n)$ of degree 4. Therefore, we need 5 inputs to uniquely determine $Pc(n)$.

$$Pc(n) = C4 + C3n + C2n^2 + C1n^3 + C0n^4$$
$$n = -2, -1 \ldots .1,2$$

Substitute n into $P(n)$ and compute all five results. This procedure would be done in linear time as it only involves addition and multiple with constant which is also basically addition. For example, $P(-2) = A0 - 2A1 + 4A2$ involves 1+2*1+4*1=7 times addition.

Then, we perform 5 times large number multiplication that generate 5 values of $\boldsymbol{Pc(n) = P(n) * P(n)}$ with substituting $\boldsymbol{n = -2 \ldots 2}$ into $\boldsymbol{P(n)}$. We now use the results of $Pc(n)$ to solve the linear equations and produce the expression of coefficient of $Pc(n)$. The expression of $C4 \ldots C0$ is linear combination of the 5 values of $Pc(n)$. Therefore, we can get the value of each coefficient in linear time. Finally, we substitute x^{100} back to $Pc(n)$ and compute the result.

In total, we have only performed 5 large integer multiplications.