# Question 3

Consider the shore as a sequence of length 100n:

$$A = < A_0, A_1, \dots, A_{100n-1} >$$

and the net as a sequence of length n:

$$N = < N_0, N_1, \dots, N_{n-1} >$$

If we want to catch the maximum number of fishes, we need to examine every possible number when the net goes through the whole shore.

Now, We transfer the shore to polynomial $P_A = A_0 + A_1x + A_2x^2 + \cdots + A_{100n-1}x^{100n-1}$ and the net to polynomial $P_N = N_0 + N_1x + N_2x^2 + \cdots + N_{n-1}x^{n-1}$

We multiply them together and choose the large coefficient which represent the largest fish number. These could be done in such way:

1. For simplicity, inverse the net N and form polynomial $P_N$. This could be done in $O(n)$
   Note: if we do not reverse the net, we will obtain a "backward" of the multiplication between $N_n$ and $A_n$.
2. DFT both $P_N, P_A$ and produce two sequence with length $101n - 1$
   $$\{P_A(1), P_A(w_{101n-1}), \dots, P_A(w_{101n-1}^{101n-2})\}$$
   $$\{P_N(1), P_N(w_{101n-1}), \dots, P_N(w_{101n-1}^{101n-2})\}$$
   This could be done in $O(n \log n)$ using $FFT$
3. Multiply these two sequences form"
   $$\{P_A(1)P_N(1), P_A(w_{101n-1})P_N(w_{101n-1}), \dots, P_A(w_{101n-1}^{101n-2})P_N(w_{101n-1}^{101n-2})\}$$
4. Then using IDFT to change the polynomial from value representation to coefficient representation.
   $$Result(x) = \sum_{j=0}^{101n-2} (\sum_{i=0}^{j} A_i N_{j-i}) x^j$$
   The time complexity of IDFT is $O(n \log n)$.

   So far, we have already got **the convolution of A and N** which is the middle part of above equation.
5. Next, find the maximum value of the convolution of A and N. This could be done in such way: reserve a number that represent the maximum number and initialize it to be zero. Then traversing the whole sequence and replace the number whenever there exist a number bigger than it. The time complexity would be $O(n)$.

   In summary, we find the maximum number of fishes that we can catch by following above procedure and the time complexity would be $O(n \log n)$.