Question 4

An arbitrary point i belong to graph $G(V,E)$. $opt(i,k)$ is the **optimal solution** for i with the length k. That is, the maximum weight path of length k which ends at i. Obviously, $opt(i,0) = 0$ because this path does not go through any edge in G.

In general, for every vertex $i_1 \dots i_n$ in G, we need to find $opt(i,K)$ where K is the required length in this question and then the maximum total weight would be:

$$\max\left(opt(i_1,K) \dots opt(i_n,K)\right)$$

First, go through graph G and record the adjacent vertex(the vertex that can reach i) for any vertex i. The adjacent vertex for any vertex i could be in set $(v_1, \dots v_n)$.
For obtaining the $opt(i,K)$, we can follow the method of dynamic programming.

- Known that the Base case $opt(i,0) = 0$
- $opt(i,1) = \max\left(E(v_1,i), E(v_2,i) \dots E(v_n,i)\right)$ where $E(v,i)$ is the weight of edge which has vertex v, i on its end.
- Therefore, $opt(i,K) = \max\left[(E(v_1,i) + opt(v_1,K-1), \dots, (E(v_n,i) + opt(v_n,K-1)\right]$ we can recursively solve this equation until $k = 1$ and $opt(i,k-1)$ = 0
- During the recursion, for every optimal solution for the subproblem, we record its path information to an array or a linked list structure.

Overrall, we obtain the optimal solution (path that maximum the weight) for every vertex in G by applying the dynamic programming. Choosing the vertex with the maximum $opt()$ and then check our linked list structure to obtain the target path.