Homework # 2

1. (a) Let X be an exponential r.v. with rate 1. Using cdf inversion, write a function that generates n independent samples of X (we discussed this example in class, but you should do the cdf inversion yourself, not just quote our result). Compare the speed of your sampler for $n = 10^6$ with that of your language's exponential sampler (in R **rexp**).

   (b) Let X be a r.v. with the following pdf

$$f(x) = \frac{1}{2}\exp[-|x|], \tag{1}$$

   where $\exp[-|x|] = e^{-|x|}$. Write a function that samples from X using cdf inversion. Either theoretically or through numerical examples, show that your sampler correctly samples X.

2. Write a function **MarkovChain(P, s_0, s)** that simulates a Markov chain $X(t)$ until the first time the chain is in state s, assuming $X(0) = s_0$. The function should return the path of the chain from $t = 0$ to when it "hits" state s. You may use your language's discrete sampler (in R **sample**) or write your own.

3. Chutes and Ladders is a popular children's game. The board is shown in the image file attached to this assignment. One version of the rules are as follows:

   A board contains squares labeled with the numbers 1,2,..,100. A player starts with her/his piece off the board and rolls the die so as to start on one of squares $1, 2, 3, 4, 5$ *or* 6*. During each subsequent turn, each player rolls a six-sided die and moves forward the corresponding number of squares. If after moving the piece forward according to the die roll, the piece is on a square with a ladder bottom, then that player must slide her/his piece up the ladder to the square it ends on. If the token lands on a square with a chute top, then that player must slide down to the square the chute ends on. The goal is to reach* 100 *and the player is assumed to reach* 100 *if the player's piece goes past*

or lands on the 100 *square. So, for example, if the piece is on* 99 *and the player roles a* 5 *then the player wins. The game is played with two players who take turns until one of them wins.*

Use your function from problem 2 and a Monte Carlo approach to compute $E[L]$ where L is the average length, in terms of the number of die rolls, of a Chutes and Ladders game. Use a CLT argument to determine roughly how many games you have to simulate to estimate $E[L]$ to an accuracy of ± 5.

(The time consuming part of this problem is to specifiy the transition probability matrix P. Don't do it entry by entry, think of a clever way to automate the definition of P. Here's one way. First, define the transition matrix P as if there are no chutes and ladders. Then, modify the matrix to include the effect of the ladders and chutes. Feel free to ignore this hint, try whatever method makes most sense to you)

To save you time, the attached csv file contains the start and end squares connected by the chutes and ladders: