Homework # 1

Remember to submit a single pdf file. Please include all code in your submission.

1. Consider the queue model we discussed in class (i.e. a single server model). As we did in class, assume that the interarrival times, T_i are iid as are the service times S_i. Assume further that each T_i is exponentially distributed with rate λ and each S_i is exponentially distributed with rate μ. Let $Q(t)$ be the number of customers waiting in line at time t. Let W_i be the waiting time of the ith individual. Assume that initially the queue is empty, so $Q(0) = 0$ and $W_1 = 0$.

   (a) Determine $P(W_2 \geq c)$ for c a positive number. (write down an integral and evaluate it, you won't need a computer). Note this is a little different than what we did in class when we considered $P(W_2 = 0)$.

   (b) Write down an integral expression for $P(W_3 \geq c)$ (You don't need to evaluate the integral, unless you want to. Your answer may be the sum of two integrals.)

   (c) Write a function **WaitingTimes(n λ, μ)** that samples the waiting times of the first n customers. Your function should return a vector of length n with the sampled waiting time. Show the output of your function for $n = 10, \lambda = 1, \mu = 1$.

   (d) Write a function **plotQ(t, λ, μ)** that simulates (in other words samples) the queue and plots $Q(t)$ up to a time t. Show a single simulation for $t = 20$, $\lambda = 1$, $\mu = 1$.

   (e) Using a Monte Carlo approach, estimate $P(W_2 \geq 1)$. Assume $\lambda = 1$, $\mu = 1$. Compare your estimate to the exact answer you derived in part (a). Repeat for $P(W_{100} \geq 1)$, except in this case you won't have the exact answer.