

NBD HW1

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1 Part 2

1.1 Exercise 1

Let n be the number of ports of each switch. We know (slide 53, book pag. 17) that:

Fat-Tree

- **Number of servers:**

$$N_{fat} = \underbrace{\left(\underbrace{\frac{n}{2}}_{\text{ports per switch}} \cdot \underbrace{\frac{n}{2}}_{\text{edge layer}} \right)}_{\text{pod}} \cdot \underbrace{n}_{\text{number of pods}} = \frac{n^3}{4} \quad (1)$$

- **Number of switches:**

$$S_{fat} = n \cdot \underbrace{\left(\underbrace{\frac{n}{2}}_{\text{aggregation layer}} + \underbrace{\frac{n}{2}}_{\text{edge layer}} \right)}_{\text{pod}} + \underbrace{\left(\frac{n}{2} \right)^2}_{\text{core layer}} = \frac{5}{4}n^2 \quad (2)$$

- **Number of links:**

$$l_{fat} = N_{fat} \log_{\frac{n}{2}} \frac{N_{fat}}{2} \quad (3)$$

- **Number of links connecting switches:**

$$\begin{aligned} L_{fat} &= l_{fat} - \underbrace{N_{fat}}_{\text{links servers}} = N_{fat} \log_{\frac{n}{2}} \frac{N_{fat}}{2} - N_{fat} \\ &= N_{fat} \left(\log_{\frac{n}{2}} \frac{N_{fat}}{2} - 1 \right) \end{aligned} \quad (4)$$

Jellyfish

- Number of servers:

$$N_{jelly} = S_{jelly} \cdot (n - r) \quad (5)$$

- Number of switches:

$$S_{jelly} \quad (6)$$

- Number of links: (*check slide 79*)

$$l_{jelly} = S_{jelly} \cdot r \quad (7)$$

- Number of links connecting switches:

$$L_{jelly} = \underbrace{\frac{S_{jelly} \cdot r}{2}}_{\text{each switch is connected to } r \text{ switches}} \quad (8)$$

We want that:

$$\begin{aligned} \begin{cases} N_{fat} = N_{jelly} \\ S_{fat} = S_{jelly} \\ L_{fat} = L_{jelly} \end{cases} \quad \begin{cases} \frac{n^3}{4} = S_{jelly} \cdot (n - r) \\ S_{jelly} = \frac{5}{4}n^2 \\ L_{fat} = L_{jelly} \end{cases} &\implies \begin{aligned} \frac{n^3}{4} &= \frac{5}{4}n^2(n - r) \\ \frac{n^3}{4} &= \frac{5}{4}n^3 - \frac{5}{4}n^2r \\ \frac{5}{4}n^2r &= n^3 \\ r &= \frac{4}{5}n \quad (\star) \end{aligned} \end{aligned} \quad (9)$$

Let's check the (\star) result by substituting it in the third equation:

Proof.

$$\begin{aligned} L_{fat} = L_{jelly} &\Leftrightarrow N_{fat} \left(\log_{\frac{n}{2}} \frac{N_{fat}}{2} - 1 \right) = \frac{S_{jelly} \cdot r}{2} \Leftrightarrow^{\text{by } S_{fat}=S_{jelly}} \\ \frac{n^3}{4} \left(\log_{\frac{n}{2}} \frac{n^3}{8} - 1 \right) &= \frac{\frac{5}{4}n^2 \cdot \frac{4}{5}n}{2} \Leftrightarrow \frac{n^3}{4} (3 - 1) = \frac{n^3}{2} \Leftrightarrow \frac{n^3}{2} = \frac{n^3}{2} \end{aligned} \quad (10)$$

□

To summarize,

$$\mathbf{r} = \frac{4}{5}\mathbf{n}$$

1.2 Exercise 2

From slide 77 we know that:

$$TH \leq \frac{l}{\bar{h} \cdot \nu_f} \quad (\text{Application-oblivious throughput bound})$$

We aim to derive the expression for the throughput as a function of just \bar{h} (average path length) and ν_f (number of flows).

According to the slide 79,

$$l = S_{jelly} \cdot r \quad (11)$$

(remind that in the slide the professor indicates the number of switches with N and the number of servers with S , so that $l = N \cdot r$).

Assuming as true that

$$\nu_f = N_{jelly} \cdot (N_{jelly} - 1) \quad (12)$$

(may check in the book the last sentence of the paragraph at page 27 and [this paper](#)), we have

$$\begin{aligned} TH &\leq \frac{l}{\bar{h} \cdot \nu_f} = \frac{S_{jelly} \cdot r}{\bar{h} \cdot N_{jelly} \cdot (N_{jelly} - 1)} = \\ &= \frac{S_{jelly} \cdot r}{\bar{h} \cdot [S_{jelly} \cdot (n - r)] \cdot [S_{jelly} \cdot (n - r) - 1]} = \\ &= \frac{S_{jelly} \cdot r}{\bar{h} \cdot [S_{jelly} \cdot (n - r)] \cdot [S_{jelly} \cdot (n - r) - 1]} = \\ &= \frac{\frac{4}{5}n}{\bar{h} \cdot (n - \frac{4}{5}n) \cdot [\frac{5}{4}n^2 \cdot (n - \frac{4}{5}n) - 1]} = \\ &= \frac{\frac{4}{5}n}{\bar{h} \cdot \frac{1}{5}n \cdot [\frac{5}{4}n^2 \cdot \frac{1}{5}n - 1]} = \\ &= \frac{\frac{4}{5}n}{\bar{h} \cdot \cancel{\frac{1}{5}}n \cdot [\frac{5}{4}n^2 \cdot \frac{1}{5}n - 1]} = \\ &= \frac{4}{\bar{h} \cdot [\frac{1}{4}n^3 - 1]} \quad \text{Right almost for sure!} \end{aligned} \quad (13)$$

1.3 Exercise 3

For \bar{h}_{fat} we have:

$$\begin{aligned}
\bar{h}_{fat} &= \\
&= \frac{\overbrace{n\left(\frac{n^3}{4} - \frac{n^2}{4}\right)}^1 \overbrace{\left(\left(\frac{n}{2}\right)^2 \cdot 6\right)}^2 + \overbrace{\frac{n^2}{2}}^3 \left(\overbrace{\left(\frac{n}{2}\right)^2}^4 - \overbrace{\frac{n}{2}}^5\right) \left(\overbrace{\frac{n}{2}}^6 \cdot \overbrace{4}^6\right) + n\left(\frac{n}{2}\right)^2 \overbrace{\left(\frac{n}{2} - 1\right)}^7 \overbrace{(1 \cdot 2)}^8}{n\left(\frac{n^3}{4} - \frac{n^2}{4}\right)\left(\frac{n}{2}\right)^2 + n\frac{n}{2}\left(\left(\frac{n}{2}\right)^2 - \frac{n}{2}\right)\left(\frac{n}{2}\right) + n\left(\frac{n}{2}\right)^2\left(\frac{n}{2} - 1\right)} \\
&= \frac{n\frac{n^2}{4}(n-1)\left(\frac{n^2}{4} \cdot 6\right) + \left(\frac{n}{2}\right)^2\left(\frac{n}{2} - 1\right)\left(\frac{n}{2} \cdot 4\right) + \left(\frac{n}{2}\right)^2\left(\frac{n}{2} - 1\right) \cdot 2}{n\frac{n^2}{4}(n-1)\left(\frac{n^2}{4}\right) + \left(\frac{n}{2}\right)^2\left(\frac{n}{2} - 1\right)\left(\frac{n}{2}\right) + \left(\frac{n}{2}\right)^2\left(\frac{n}{2} - 1\right)} \\
&= \frac{\left(\frac{n}{2}\right)^4(n-1) \cdot 6 + \left(\frac{n}{2}\right)^3\left(\frac{n}{2} - 1\right) \cdot 4 + \left(\frac{n}{2}\right)^2\left(\frac{n}{2} - 1\right) \cdot 2}{\left(\frac{n}{2}\right)^4(n-1) + \left(\frac{n}{2}\right)^3\left(\frac{n}{2} - 1\right) + \left(\frac{n}{2}\right)^2\left(\frac{n}{2} - 1\right)}
\end{aligned} \tag{14}$$

On the other hand, \bar{h}_{jelly} is bounded by

$$\begin{aligned}
\bar{h}_{jelly} &\geq \frac{\sum_{j=1}^{k-1} j \cdot r \cdot (r-1)^{j-1} + k \cdot R}{S_{jelly} - 1} \\
&= \frac{\sum_{j=1}^{k-1} j \cdot \frac{4}{5}n \cdot \left(\frac{4}{5}n - 1\right)^{j-1} + k \cdot R}{\frac{5}{4}n^2 - 1}
\end{aligned} \tag{15}$$

where

$$\begin{aligned}
R &= S_{jelly} - 1 - \sum_{j=1}^{k-1} r \cdot (r-1)^{j-1} = \\
&= \frac{5}{4}n^2 - 1 - \sum_{j=1}^{k-1} \frac{4}{5}n \cdot \left(\frac{4}{5}n - 1\right)^{j-1}
\end{aligned} \tag{16}$$

¹Number of servers in a different pod

²See slide 75

³Number of servers in the same pod

⁴Number of servers in the same edge switch

⁵Number of paths for servers in different edge switches

⁶Path length between servers in different edge switches

⁷Number of servers in the same edge switch

⁸There is a unique path between a server and one another connected to the same edge switch. The path length is 2

and

$$\begin{aligned}
k &= 1 + \left\lfloor \frac{\log \left(\frac{S_{jelly} - 2(S_{jelly} - 1)}{r} \right)}{\log(r - 1)} \right\rfloor = \\
&= 1 + \left\lfloor \frac{\log \left(\frac{\frac{5}{4}n^2 - 2(\frac{5}{4}n^2 - 1)}{\frac{4}{5}n} \right)}{\log(\frac{4}{5}n - 1)} \right\rfloor
\end{aligned} \tag{17}$$

N	S	L	TH Fat-Tree	TH Jellyfish
250	125	500	2.0×10^{-2}	6.6×10^{-3}
2000	500	4000	5.0×10^{-3}	8.1×10^{-4}
6750	1125	13500	2.2×10^{-3}	2.4×10^{-4}
16000	2000	32000	1.2×10^{-3}	1.0×10^{-4}
31250	3125	62500	8.0×10^{-4}	5.1×10^{-5}