NBD HW1

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1 Part 2

1.1 Exercise 1

Let n be the number of ports of each switch. We know (slide 53, book pag. 17) that:

Fat-Tree

- **Number of servers:**

$$N_{fat} = \underbrace{\Bigg(\underbrace{\frac{n}{2}}_{\text{ports per switch}} \cdot \underbrace{\frac{n}{2}}_{\text{aggregation layer}} \Bigg)}_{\text{pod}} \cdot \underbrace{n}_{\text{number of pods}} = \frac{n^3}{4} \tag{1}$$

- **Number of switches:**

$$S_{fat} = n \cdot \underbrace{\Bigg(\underbrace{\frac{n}{2}}_{\text{aggregation layer}} + \underbrace{\frac{n}{2}}_{\text{edge layer}} \Bigg)}_{\text{pod}} + \underbrace{\left(\frac{n}{2}\right)^2}_{\text{core layer}} = \frac{5}{4}n^2 \tag{2}$$

- **Number of links:**

$$l_{fat} = N_{fat} \log_{\frac{n}{2}} \frac{N_{fat}}{2} \tag{3}$$

- **Number of links <u>connecting switches</u>:**

$$\begin{aligned} L_{fat} = l_{fat} - \underbrace{N_{fat}}_{\text{links servers}} &= N_{fat} \log_{\frac{n}{2}} \frac{N_{fat}}{2} - N_{fat} \\ &= N_{fat} \Big(\log_{\frac{n}{2}} \frac{N_{fat}}{2} - 1 \Big) \end{aligned} \tag{4}$$

Jellyfish

- **Number of servers:**

$$N_{jelly} = S_{jelly} \cdot (n - r) \tag{5}$$

- **Number of switches:**

$$S_{jelly} \tag{6}$$

- **Number of links:** (*check slide 79*)

$$l_{jelly} = N_{jelly} \cdot r \tag{7}$$

- **Number of links <u>connecting switches</u>:**

$$L_{jelly} = \underbrace{\frac{S \cdot r}{2}}_{\text{each switch is connected to } r \text{ switches}} \tag{8}$$

We want that:

$$\begin{cases} N_{fat} = N_{jelly} \\ S_{fat} = S_{jelly} \\ L_{fat} = L_{jelly} \end{cases} \quad \begin{cases} \frac{n^3}{4} = S_{jelly} \cdot (n - r) \\ S_{jelly} = \frac{5}{4}n^2 \\ L_{fat} = L_{jelly} \end{cases} \implies \begin{aligned} &\frac{n^3}{4} = \frac{5}{4}n^2(n - r) \\ &\frac{n^3}{4} = \frac{5}{4}n^3 - \frac{5}{4}n^2 r \\ &\frac{5}{4}n^2 r = n^3 \\ &r = \frac{4}{5}n \qquad (\star) \end{aligned} \tag{9}$$

Let's check the $(\star)$ result by substituting it in the third equation:

Proof.

$$\begin{gathered} L_{fat} = L_{jelly} \Leftrightarrow N_{fat}\left(\log_{\frac{n}{2}} \frac{N_{fat}}{2} - 1\right) = \frac{S_{jelly} \cdot r}{2} \Leftrightarrow^{\text{by } S_{fat} = S_{jelly}} \\ \frac{n^3}{4}\left(\log_{\frac{n}{2}} \frac{n^3}{8} - 1\right) = \frac{\frac{5}{4}n^2 \cdot \frac{4}{5}n}{2} \Leftrightarrow \frac{n^3}{4}(3 - 1) = \frac{n^3}{2} \Leftrightarrow \frac{n^3}{2} = \frac{n^3}{2} \end{gathered} \tag{10}$$

□

To summarize,

$$\mathbf{r = \frac{4}{5}n}$$

1.2 Exercise 2

From slide 77 we know that:

$$TH \leq \frac{l}{\bar{h} \cdot \nu_f} \qquad \text{(Application-oblivious throughput bound)}$$

having $\bar{h}$ equal to

$$\begin{aligned} &\bar{h} \geq \frac{\sum_{j=1}^{k-1} j \cdot r \cdot (r-1)^{j-1} + k \cdot R}{N-1} \\ &\text{where} \quad R = N - 1 - \sum_{j-1}^{k-1} r \cdot (r-1)^{j-1} \end{aligned} \tag{11}$$

TO BE CONTINUED