NBD HW1

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1 Part 2

1.1 Exercise 1

Let n be the number of ports of each switch. We know (slide 53, book pag. 17) that:

Fat-Tree

- **Number of servers:**

$$N_{fat} = \underbrace{\Big(\underbrace{\frac{n}{2}}_{\text{ports per switch}} \cdot \underbrace{\frac{n}{2}}_{\text{edge layer}} \Big)}_{\text{pod}} \cdot \underbrace{n}_{\text{number of pods}} = \frac{n^3}{4} \tag{1}$$

- **Number of switches:**

$$S_{fat} = n \cdot \underbrace{\Big(\underbrace{\frac{n}{2}}_{\text{aggregation layer}} + \underbrace{\frac{n}{2}}_{\text{edge layer}} \Big)}_{\text{pod}} + \underbrace{\left(\frac{n}{2}\right)^2}_{\text{core layer}} = \frac{5}{4}n^2 \tag{2}$$

- **Number of links:**

$$l_{fat} = N_{fat} \log_{\frac{n}{2}} \frac{N_{fat}}{2} \tag{3}$$

- **Number of links <u>connecting switches</u>:**

$$\begin{aligned} L_{fat} = l_{fat} - \underbrace{N_{fat}}_{\text{links servers}} &= N_{fat} \log_{\frac{n}{2}} \frac{N_{fat}}{2} - N_{fat} \\ &= N_{fat}\Big(\log_{\frac{n}{2}} \frac{N_{fat}}{2} - 1 \Big) \end{aligned} \tag{4}$$

Jellyfish

- **Number of servers:**

$$N_{jelly} = S_{jelly} \cdot (n - r) \tag{5}$$

- **Number of switches:**

$$S_{jelly} \tag{6}$$

- **Number of links:** (*check slide 79*)

$$l_{jelly} = S_{jelly} \cdot r \tag{7}$$

- **Number of links <u>connecting switches</u>:**

$$L_{jelly} = \underbrace{\frac{S_{jelly} \cdot r}{2}}_{\text{each switch is connected to } r \text{ switches}} \tag{8}$$

We want that:

$$\begin{cases} N_{fat} = N_{jelly} \\ S_{fat} = S_{jelly} \\ L_{fat} = L_{jelly} \end{cases} \quad \begin{cases} \frac{n^3}{4} = S_{jelly} \cdot (n - r) \\ S_{jelly} = \frac{5}{4} n^2 \\ L_{fat} = L_{jelly} \end{cases} \implies \begin{aligned} &\frac{n^3}{4} = \frac{5}{4} n^2 (n - r) \\ &\frac{n^3}{4} = \frac{5}{4} n^3 - \frac{5}{4} n^2 r \\ &\frac{5}{4} n^2 r = n^3 \\ &r = \frac{4}{5} n \qquad (\star) \end{aligned} \tag{9}$$

Let's check the $(\star)$ result by substituting it in the third equation:

Proof.

$$\begin{aligned} &L_{fat} = L_{jelly} \Leftrightarrow N_{fat} \left(\log_{\frac{n}{2}} \frac{N_{fat}}{2} - 1 \right) = \frac{S_{jelly} \cdot r}{2} \Leftrightarrow^{\text{by } S_{fat} = S_{jelly}} \\ &\frac{n^3}{4} \left(\log_{\frac{n}{2}} \frac{n^3}{8} - 1 \right) = \frac{\frac{5}{4} n^2 \cdot \frac{4}{5} n}{2} \Leftrightarrow \frac{n^3}{4} (3 - 1) = \frac{n^3}{2} \Leftrightarrow \frac{n^3}{2} = \frac{n^3}{2} \end{aligned} \tag{10}$$

$\square$

To summarize,

$$\mathbf{r} = \mathbf{\frac{4}{5} n}$$

1.2 Exercise 2

From slide 77 we know that:

$$TH \leq \frac{l}{\bar{h} \cdot \nu_f} \qquad \text{(Application-oblivious throughput bound)}$$

We aim to derive the expression for the throughput as a function of just $\bar{h}$ (average path length) and ν_f (number of flows).
According to the slide 79,

$$l = S_{jelly} \cdot r \tag{11}$$

(remind that in the slide the professor indicates the number of switches with N and the number of servers with S, so that $l = N \cdot r$).
Assuming as true that

$$\nu_f = N_{jelly} \cdot (N_{jelly} - 1) \tag{12}$$

(may check in the book the last sentence of the paragraph at page 27 and this paper), we have

$$\begin{aligned} TH \leq \frac{l}{\bar{h} \cdot \nu_f} &= \frac{S_{jelly} \cdot r}{\bar{h} \cdot N_{jelly} \cdot (N_{jelly} - 1)} = \\ &= \frac{S_{jelly} \cdot r}{\bar{h} \cdot [S_{jelly} \cdot (n - r)] \cdot [S_{jelly} \cdot (n - r) - 1]} = \\ &= \frac{\cancel{S_{jelly}} \cdot r}{\bar{h} \cdot [\cancel{S_{jelly}} \cdot (n - r)] \cdot [S_{jelly} \cdot (n - r) - 1]} = \\ &= \frac{\frac{4}{5}n}{\bar{h} \cdot (n - \frac{4}{5}n) \cdot [\frac{5}{4}n^2 \cdot (n - \frac{4}{5}n) - 1]} = \\ &= \frac{\frac{4}{5}n}{\bar{h} \cdot \frac{1}{5}n \cdot [\frac{5}{4}n^2 \cdot \frac{1}{5}n - 1]} = \\ &= \frac{\frac{4}{\cancel{5}}\cancel{n}}{\bar{h} \cdot \frac{1}{\cancel{5}}\cancel{n} \cdot [\frac{5}{4}n^2 \cdot \frac{1}{5}n - 1]} = \\ &= \frac{4}{\bar{h} \cdot [\frac{1}{4}n^3 - 1]} \qquad \textit{Right almost for sure!} \end{aligned} \tag{13}$$

1.3 Exercise 3

For $\bar{h}_{fat}$ we have:

$$
\begin{aligned}
&\bar{h}_{fat} = \\
&= \frac{\overbrace{n\left(\frac{n^3}{4}-\frac{n^2}{4}\right)}^{1}\overbrace{\left(\left(\frac{n}{2}\right)^2\cdot 6\right)}^{2}+\frac{n^2}{2}\left(\overbrace{\left(\frac{n}{2}\right)^2}^{3}-\overbrace{\frac{n}{2}}^{4}\right)\left(\overbrace{\frac{n}{2}}^{5}\cdot\overbrace{4}^{6}\right)+n\left(\frac{n}{2}\right)^2\overbrace{\left(\frac{n}{2}-1\right)}^{7}\overbrace{(1\cdot 2)}^{8}}{n\left(\frac{n^3}{4}-\frac{n^2}{4}\right)\left(\frac{n}{2}\right)^2+n\frac{n}{2}\left(\left(\frac{n}{2}\right)^2-\frac{n}{2}\right)\left(\frac{n}{2}\right)+n\left(\frac{n}{2}\right)^2\left(\frac{n}{2}-1\right)} \\
&= \frac{n}{n}\,\frac{\frac{n^2}{4}(n-1)\left(\frac{n^2}{4}\cdot 6\right)+\left(\frac{n}{2}\right)^2\left(\frac{n}{2}-1\right)\left(\frac{n}{2}\cdot 4\right)+\left(\frac{n}{2}\right)^2\left(\frac{n}{2}-1\right)\cdot 2}{\frac{n^2}{4}(n-1)\left(\frac{n^2}{4}\right)+\left(\frac{n}{2}\right)^2\left(\frac{n}{2}-1\right)\left(\frac{n}{2}\right)+\left(\frac{n}{2}\right)^2\left(\frac{n}{2}-1\right)} \\
&= \frac{\left(\frac{n}{2}\right)^4(n-1)\cdot 6+\left(\frac{n}{2}\right)^3\left(\frac{n}{2}-1\right)\cdot 4+\left(\frac{n}{2}\right)^2\left(\frac{n}{2}-1\right)\cdot 2}{\left(\frac{n}{2}\right)^4(n-1)+\left(\frac{n}{2}\right)^3\left(\frac{n}{2}-1\right)+\left(\frac{n}{2}\right)^2\left(\frac{n}{2}-1\right)}
\end{aligned}
\tag{14}
$$

On the other hand, $\bar{h}_{jelly}$ is bounded by

$$
\begin{aligned}
\bar{h}_{jelly} &\geq \frac{\sum_{j=1}^{k-1} j\cdot r\cdot (r-1)^{j-1}+k\cdot R}{S_{jelly}-1} \\
&= \frac{\sum_{j=1}^{k-1} j\cdot \frac{4}{5}n\cdot (\frac{4}{5}n-1)^{j-1}+k\cdot R}{\frac{5}{4}n^2-1}
\end{aligned}
\tag{15}
$$

where

$$
\begin{aligned}
R &= S_{jelly}-1-\sum_{j-1}^{k-1} r\cdot (r-1)^{j-1} = \\
&= \frac{5}{4}n^2-1-\sum_{j=1}^{k-1}\frac{4}{5}n\cdot(\frac{4}{5}n-1)^{j-1}
\end{aligned}
\tag{16}
$$

[1] Number of servers in a different pod
[2] See slide 75
[3] Number of servers in the same pod
[4] Number of servers in the same edge switch
[5] Number of paths for servers in different edge switches
[6] Path length between servers in different edge switches
[7] Number of servers in the same edge switch
[8] There is a unique path between a server and one another connected to the same edge switch. The path length is 2

and

$$
\begin{aligned}
k &= 1 + \left\lfloor \frac{\log\left(\frac{S_{jelly}-2(S_{jelly}-1)}{r}\right)}{\log(r-1)} \right\rfloor = \\
&= 1 + \left\lfloor \frac{\log\left(\frac{\frac{5}{4}n^2-2(\frac{5}{4}n^2-1)}{\frac{4}{5}n}\right)}{\log(\frac{4}{5}n-1)} \right\rfloor
\end{aligned}
\tag{17}
$$